Multidisciplinary Optimization and Machine Learning for Engineering Design

19 July 2021 – 5 August 2021

https://mdoml2021.ftmd.itb.ac.id/

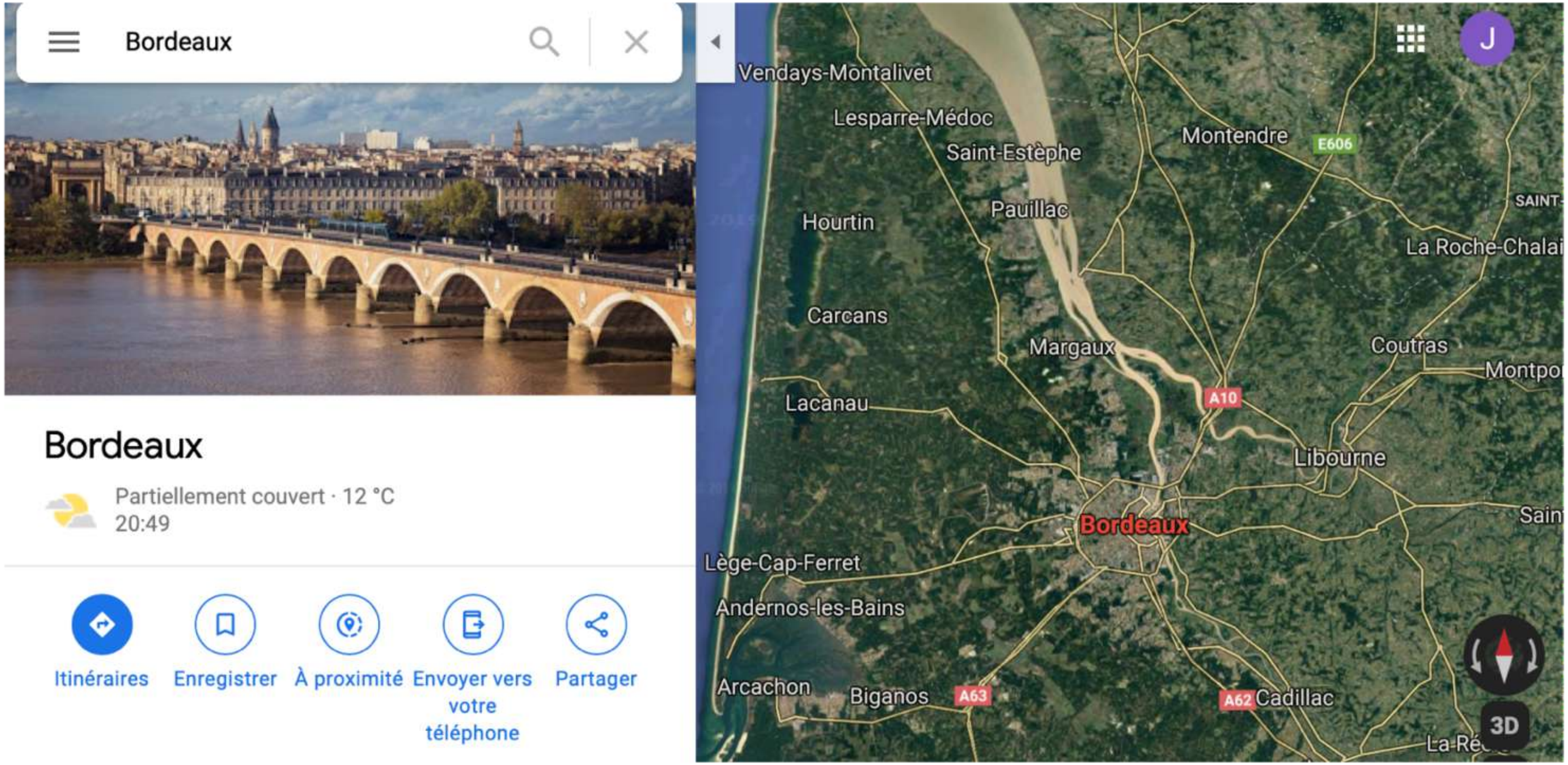
Jointly organized by

香港科技大學 THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

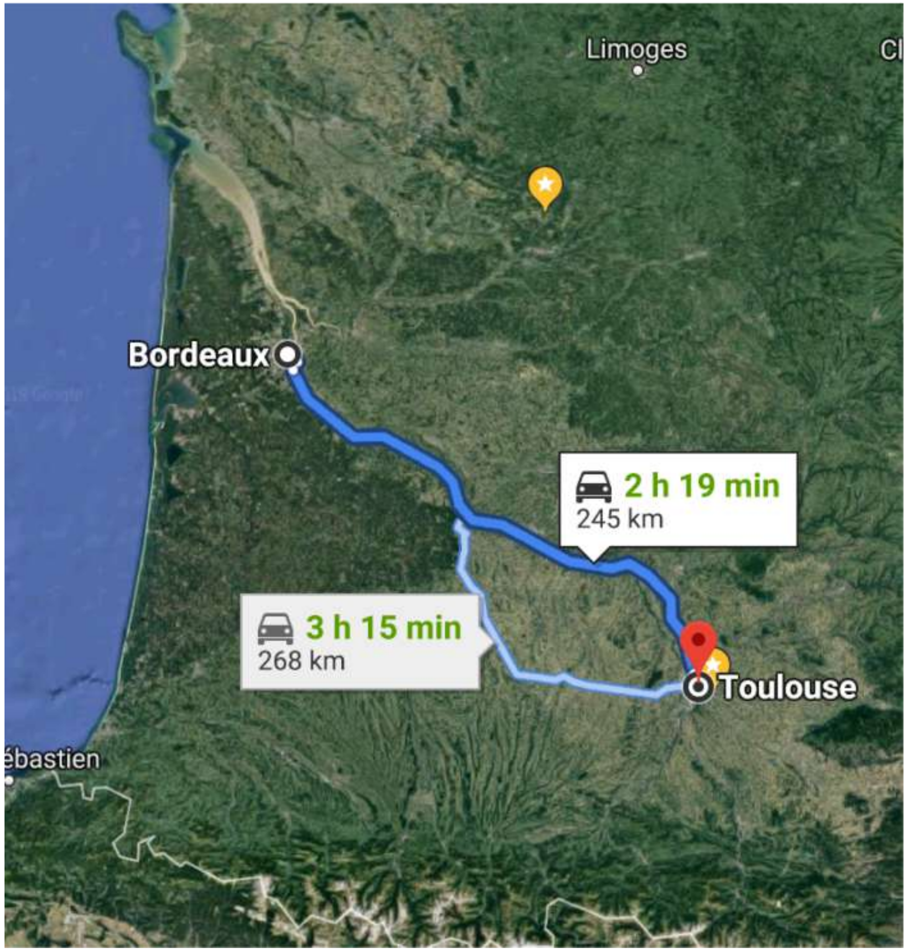
Design for Additive Manufacturing: Topology Optimization

Prof. Joseph Morlier

Who am I ?


Bordeaux
Bordeaux
Partiellement couvert · 12 °C
20:49
Itinéraires
Enregistrer
À proximité
Envoyer vers votre téléphone
Partager
Vendays-Montalivet
Lesparre-Médoc
Saint-Estèphe
Montendre
E606
Hourtin
Pauillac
La Roche-Chalai
Carcans
Margaux
Coutras
A10
Lacanau
Libourne
Bordeaux
Lège-Cap-Ferret
Andernos-les-Bains
Arcachon
Biganos
A63
A62
Cadillac
3D
J

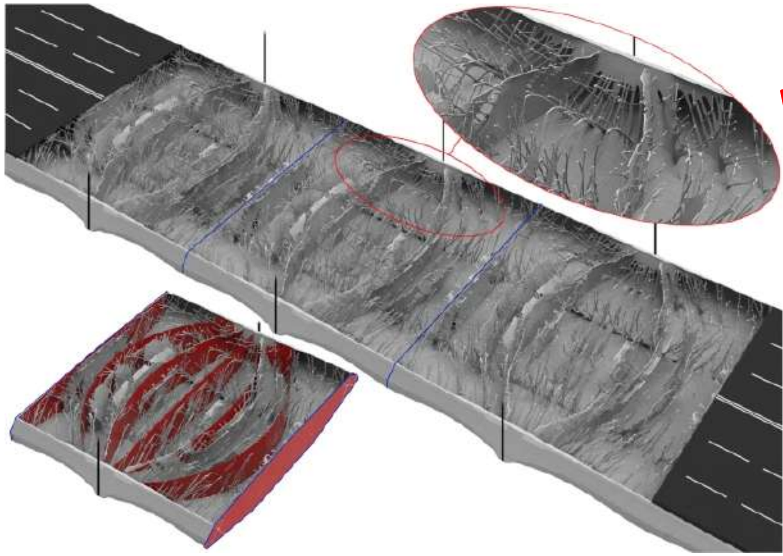
PhD in Bordeaux then…
Toulouse



« Vision » of a Famous Bridge

https://www.engineering.com/story/generative-design-takes-on-the-golden-gate

https://www.nature.com/articles/s41467-020-16599-6



Can you give me the type of structural elements here?

Element recognition?

Wait for Part4

ISAE SUPAERO
Institut Supérieur de l'Aéronautique et de l'Espace

Key Figures at a Glance

1909

200
PhD students

60 M€
Budget

30%
Foreign Students

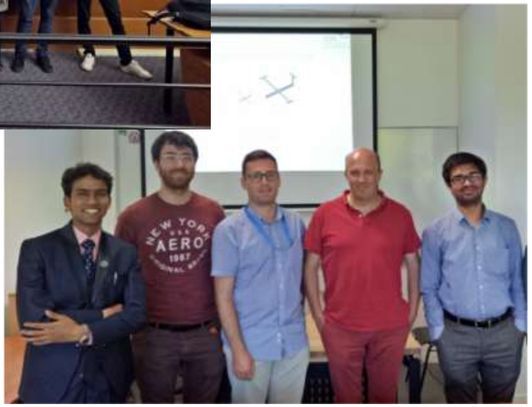
130
Professors & Researchers

1 700
Students on campus

650
students graduated a year

> 130
Academic Agreements

MDO courses & seminars



- CSM
- Sensitivity Of Finite Element Code
- Structural/Topology Optimization
- Continuous Constrained Optimization
- Surrogate/DOE using **SMT**
- UQ
- Artificial Intelligence For Engineers
- Reduced Order Modeling
- MDO using **Openmdao**
- ALM

https://github.com/jomorlier/mdocourse

https://github.com/jomorlier/feacourse

https://github.com/jomorlier/almcourse

https://github.com/jomorlier/OptimizationCourse

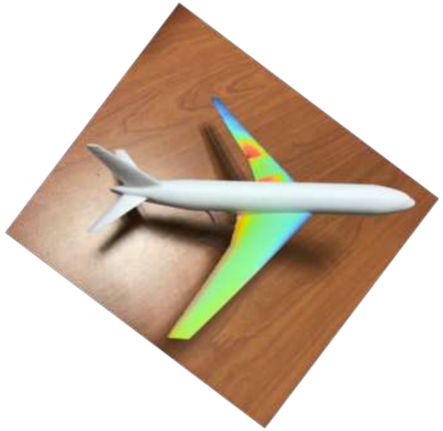
Toulouse: European Capital for Aerospace Activities

INSTITUT CLÉMENT ADER

https://ica.cnrs.fr/author/jmorlier/

Agenda for today

Slides in gray→ Complementary slides



- Part1 The Big Picture for today 20'
- Part2 Theoretical Background On Sensitivities And SIMP 30'
- Q&A10'
- Part3 3D Printing 15'
- Part4 Ecoptimization & Computational Fabrication 35'
- Q&A10'

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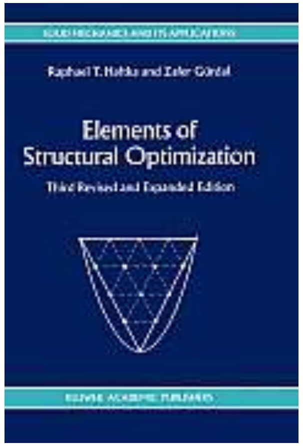
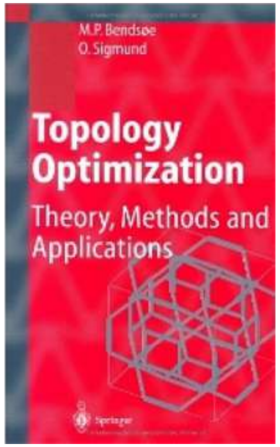
Part 1

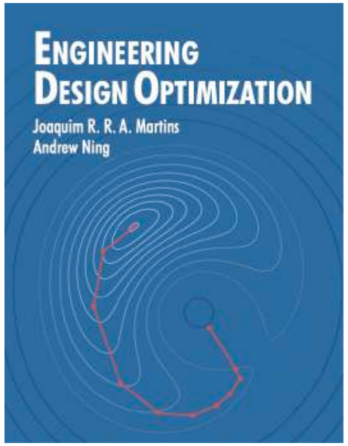
The Big Picture

Slides inspired by Prof. de Weck and Prof. Willcox (MIT) and prof Fred van Keulen and Matthijs Langelaar (DELFT)

Multistart: Here are Some Good X_0







Recipes for today:
Optimization+ CSM + material selection

Kanye West - Stronger - YouTube

Engineering view

- **Abaqus** http://www.simulia.com/products/unified_fea.html
- **BESO** http://www.isg.rmit.edu.au/downloads.html
- **BOSS quattro** http://caesam.com/en/pss.php?ID=3&W=products
- **CATOPO** http://www.catopo.de/index.htm
- **Dynaflow** http://www.princeton.edu/~dynaflow/index.htm
- **Eyeshot** http://www.devdept.com
- **FELyX** http://felyx.sourceforge.net/idea.html
- **Femap** http://www.plm.automation.siemens.com/en_in/products/velocity/femap/topography-optimization/index.shtml
- **FEMTools** http://www.femtools.com/products/ftopt.htm
- **Fomd** http://www.mkpsd.com/fomd3d.html
- **Forcepad** http://forcepad.sourceforge.net/
- **FreeFEM++** http://www.cmap.polytechnique.fr/~allaire/freefem_en.html
- **GENESIS** http://www.vrand.com/Genesis.html
- **GPUTop** (Link no longer available)
- **Impact** http://impact.sourceforge.net/
- **LS-OPT** http://www.lstc.com/lsopt.htm
- **Nastran** http://www.mscsoftware.com/Solutions/Applications/Design-Optimization.aspx
- **ODESSY** http://www.ime.auc.dk/research/design/odessy/Ods_index.htm
- **OPTISHAPE-TS** http://www.quint.co.jp/eng/pro/ots/ots_fnc_e.htm
- **Optistruct**http://www.altairhyperworks.com/%28X%281%29S%28qi3xmn30qwTniy452OTu4u55%29%29/Product,19,OptiStruct.aspx
- **PareTO** http://sciartsoft.com/Products.html
- **PERMAS** http://www.intes.de/
- **ProTOp** http://www.edibo.si/ess/index.html
- **SFE** http://www.sfe-berlin.de/
- **SmartDO** http://www.fea-optimization.com/SmartDO/index_e.htm
- **solidThinking** http://www.solidthinking.com/InspiredNav.aspx?pid=4
- **TOPLSM** http://www2.mae.cuhk.edu.hk/~cmdl/download.htm
- **TopOpt** http://www.topopt.dtu.dk/?q=node/11
- **Topostruct** http://sawapan.eu/sections/section79_topostruct/download.html
- **TOPO4ABQ** http://www.fema.se/
- **ToPy** http://code.google.com/p/topy/
- **TOSCA** http://www.processopt.com/toscatopo.htm
- **Trinitas** http://www.solid.iei.liu.se/Offered_services/Trinitas/index.html

Researcher view (Reproducible Research)

- https://www.topopt.mek.dtu.dk
- https://www.top3d.app

- https://github.com/topggp/blog
- https://github.com/mid2SUPAERO/EMTO
- https://smt.readthedocs.io/en/latest/

Crash intro

https://ntopology.com/blog/2019/10/18/5-techniques-for-lightweighting

5 Techniques for Lightweighting: Doing More With Less

Materials Selection (*Ecodesign*)

To get started, you need to select the right material. Why use steel if you can get away with aluminum?

There's a weight reduction of approximately two thirds on the table, albeit with compromises: reduced stiffness, higher cost, more difficulty in welding.

A great way of visualizing these tradeoffs and rankings of mechanical properties is with Ashby Charts, which essentially represent the menu of materials that an engineer can select.

Light stiff beam:

Index $\mathbf{M} = \dfrac{\mathbf{E^{1/2}}}{\boldsymbol{\rho}}$

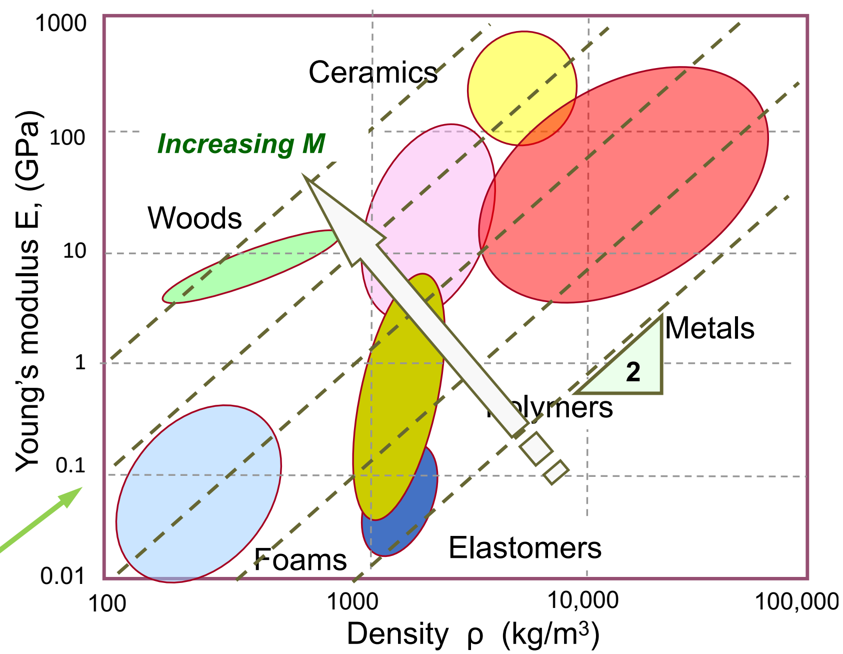
Rearrange:

$$E = \rho^2 M^2$$

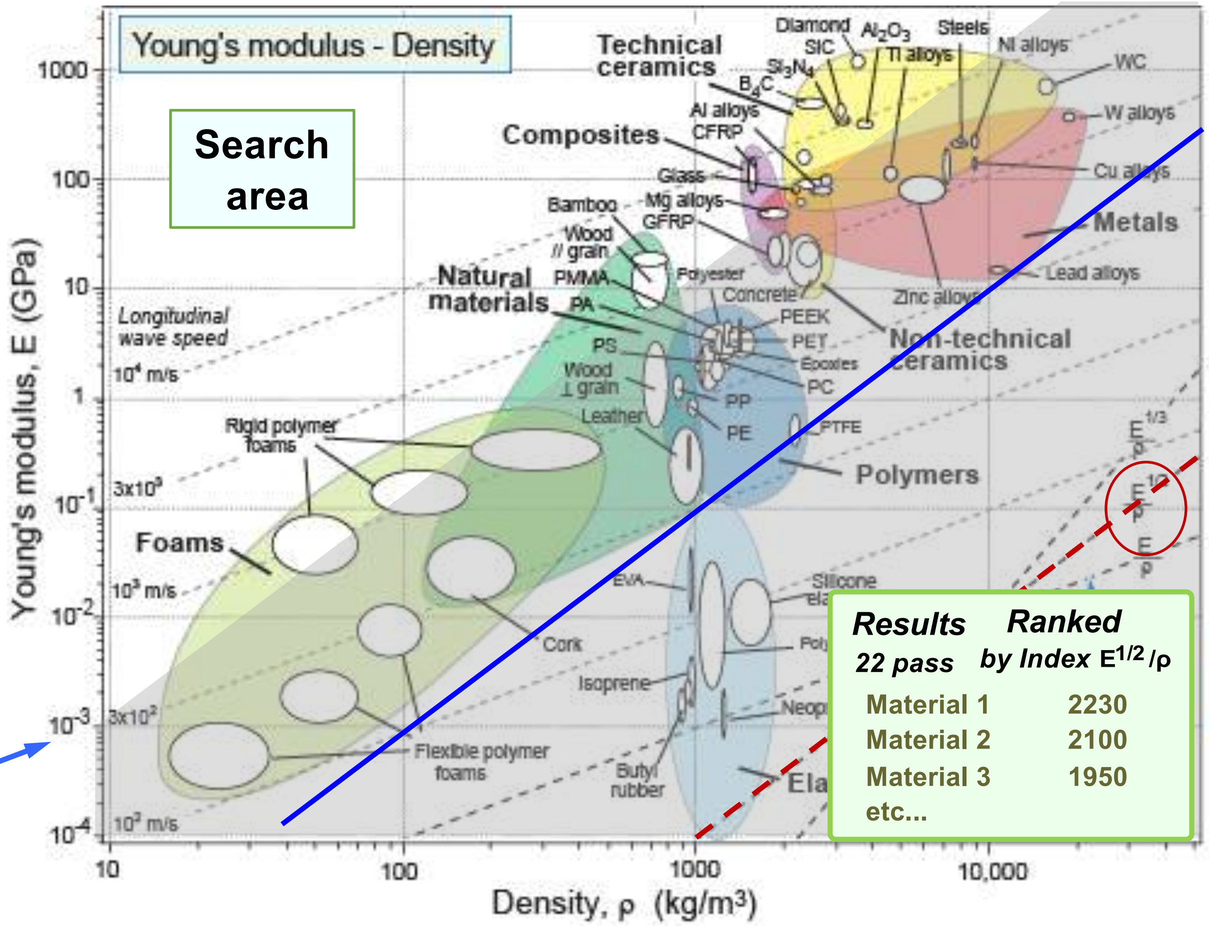
Take logs:

$$\text{Log } E = 2 \log \rho + 2 \log M$$

Function	***Index***	***Slope***
Tie	**E/ρ**	***1***
Beam	$\mathbf{E^{1/2}/\rho}$	***2***
Panel	$\mathbf{E^{1/3}/\rho}$	***3***



Selection using index in a bubble chart

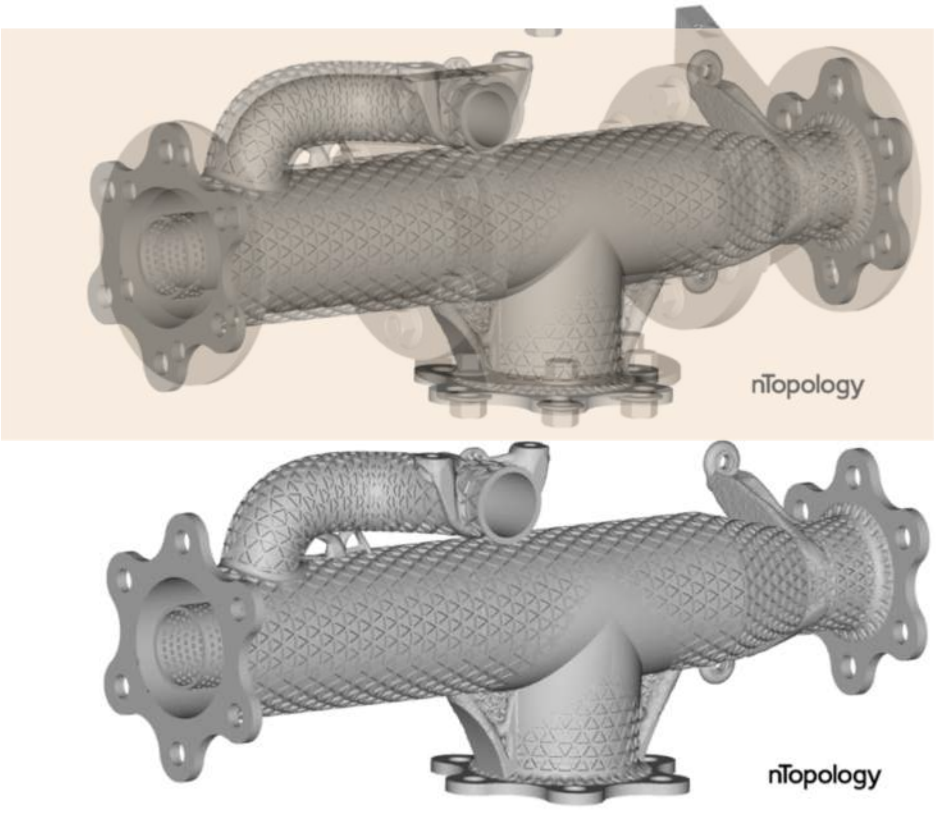


$$\frac{E^{1/2}}{\rho} = M$$

Part or Function Consolidation

Many assemblies are designed as such due to manufacturing rules, fastened together with nuts and bolts. These alone can add up significantly in weight, not to mention assembly time.

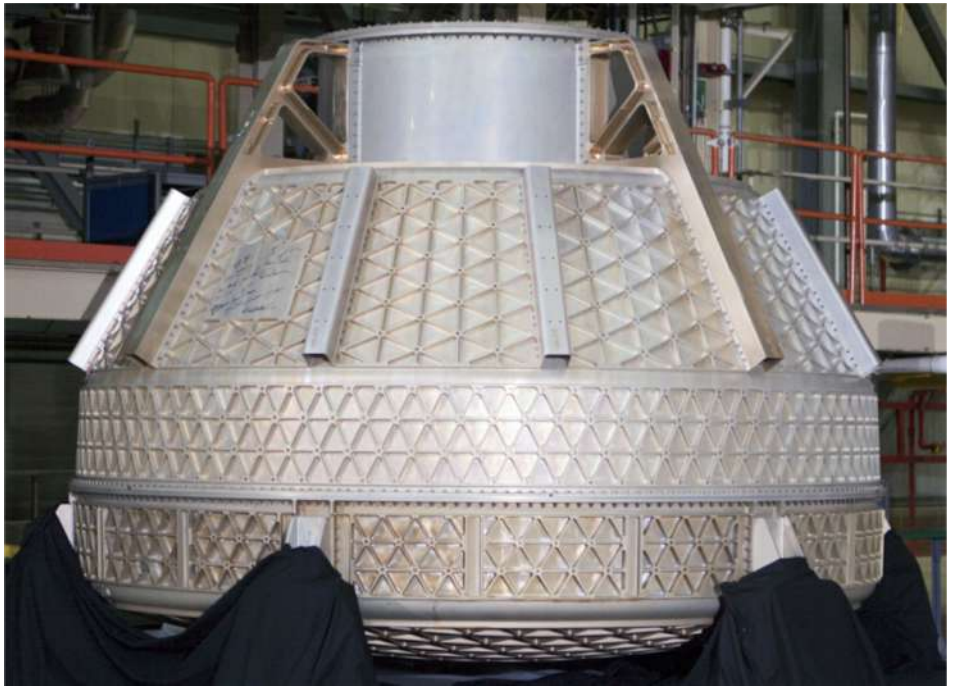
Approaching these problems from a *function-first* mindset, without the shackles of traditional manufacturing rules, can be used to trim a lot of the excess which is not directly contributing to the overall function. The bulky flanges and bolts in this pipe system, for example, don't help us at all with controlling fluid flow (ie. why we're even building a part).



A pipe assembly lightweighted through part-consolidation and conformal ribbing.

Conformal Ribbing

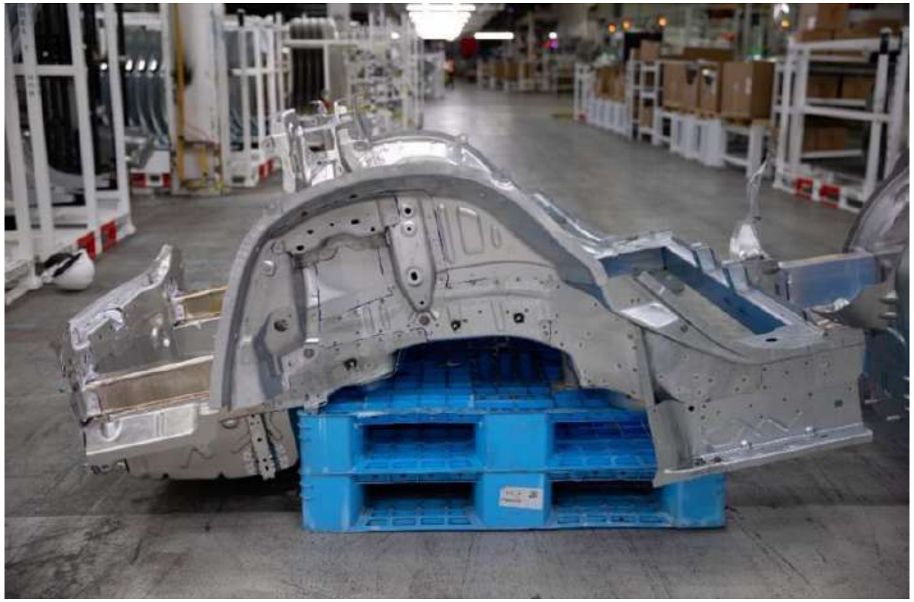
But what can we do with stiffer and stronger materials? Isogrid, shown below, has long been popular in the space industry, with design handbooks dating back to the 1970s. They are very expensive to produce via conventional machining: the cost-per-pound for machined metal isogrid is likely only justifiable for high-end aerospace applications, but additive manufacturing is making this technique much more accessible.



Machined metal isogrid on a spacecraft capsule. The high cost to get vehicles to orbit motivates advanced lightweighting techniques.

Tesla

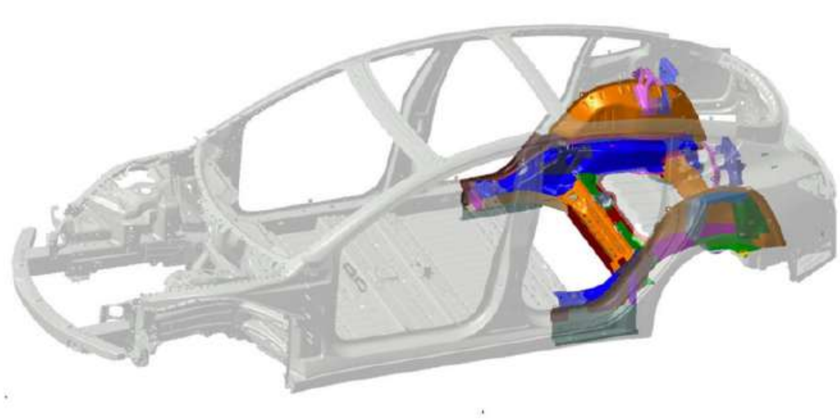
https://www.3dprintingmedia.network/tesla-shows-massive-generatively-designed-3d-printed-part-in-model-y-underbody



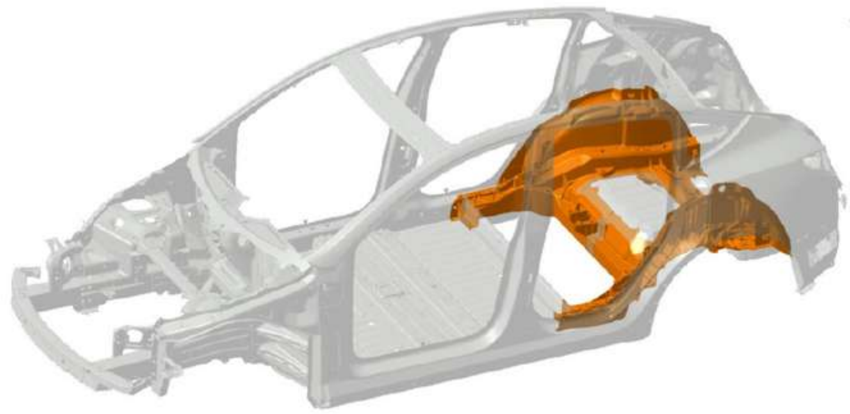
The current underbody part made of 70 different components

The generatively designed underbody, made of 2 and eventually 1 single piece.

Think different!



Model 3 rear underbody
70 pieces of metal

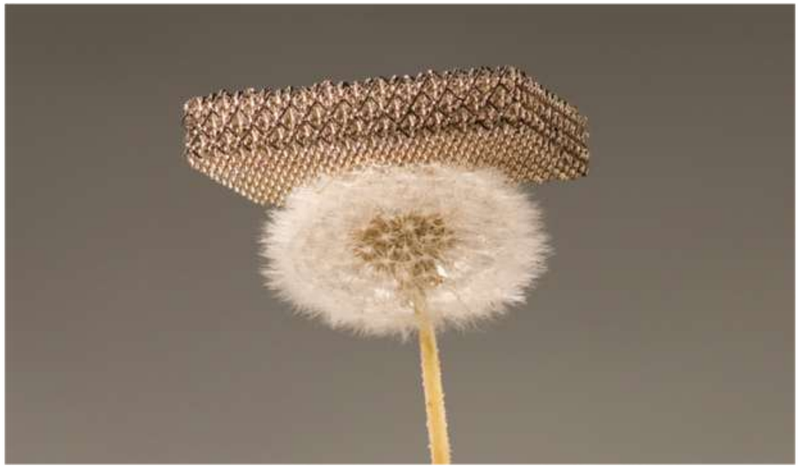


Model Y rear underbody
2 pieces of metal (eventually a single piece)

The use of 3D printing for sand casts such as that offered by voxeljet and ExOne for to enable the reduction of subassemblies (form 70 to 1) in a custom cast can bring about a significant transition even before metal AM can be used to produce such large metal parts directly. Producing a complex cast that can reduce the number of parts to this degree needs digital casting technology

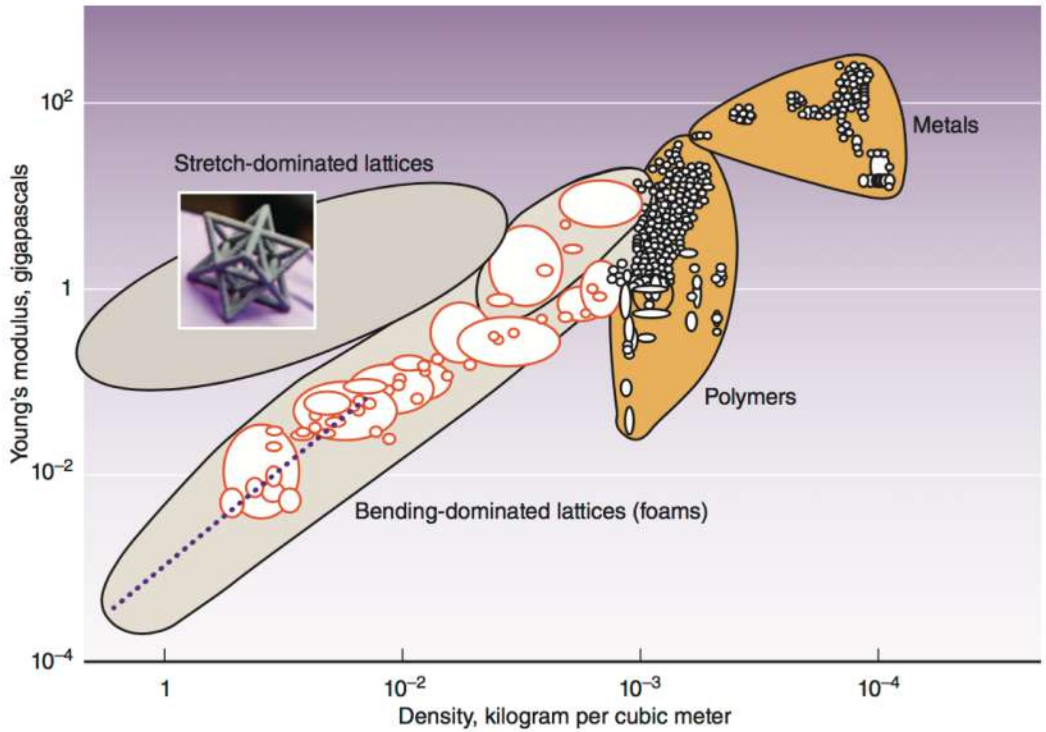
Lattices and Architected Materials

Lattices, cellular materials, periodic structures, architected materials… are as diverse as the assortment of names used to describe them. Fundamentally, they are hybrids of solid material and empty space (or gas) — with the precise arrangement dictating the effective mechanical properties. The amount of material is of course important as well and is referred to as ***relative density*** (the mass of the lattice divided by the mass of a solid block of its parent material).



The most extreme example of lightweighting on this list: a nickel microlattice structure fabricated by HRL, with a density 100 times less than styrofoam. Each strut is a hollow nickel tube of approximately 100 nm wall thickness, manufactured through a combination of additive manufacturing and electroless plating.

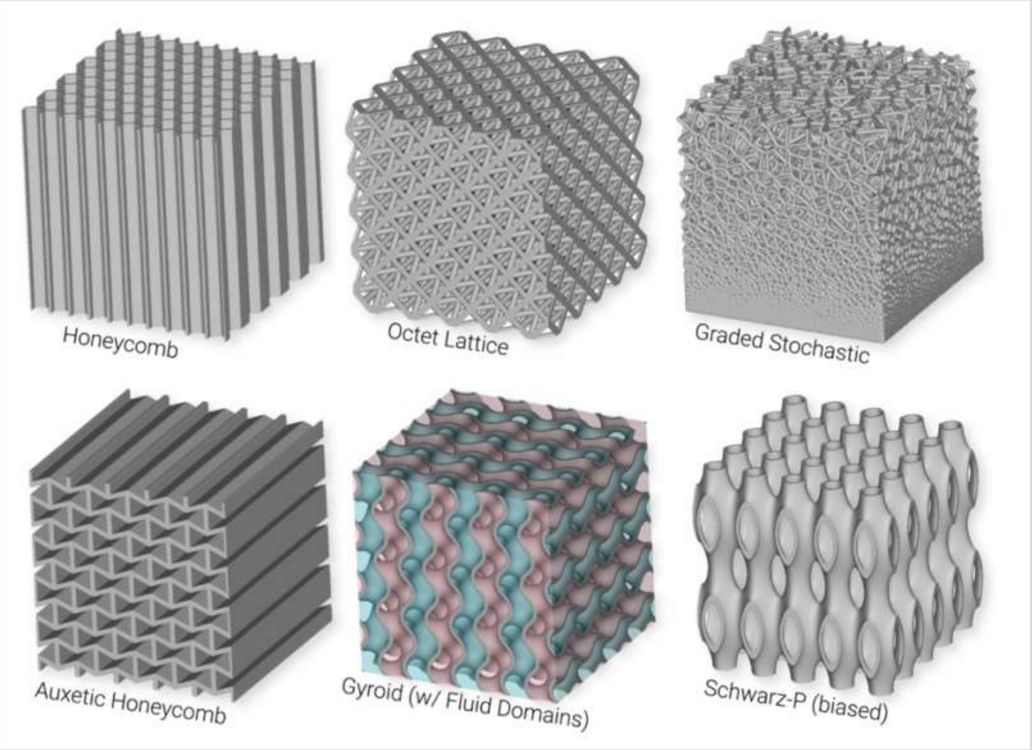
Lattices and Architected Materials 2



Materials designed with new additive manufacturing techniques exhibit high stiffness and low density, occupying a previously unsettled area of the Ashby material selection chart for Young's modulus (stiffness) versus density. The octet truss structure recently fabricated by Livermore researchers is a stretch-dominated lattice.

Lattices and Architected Materials 3

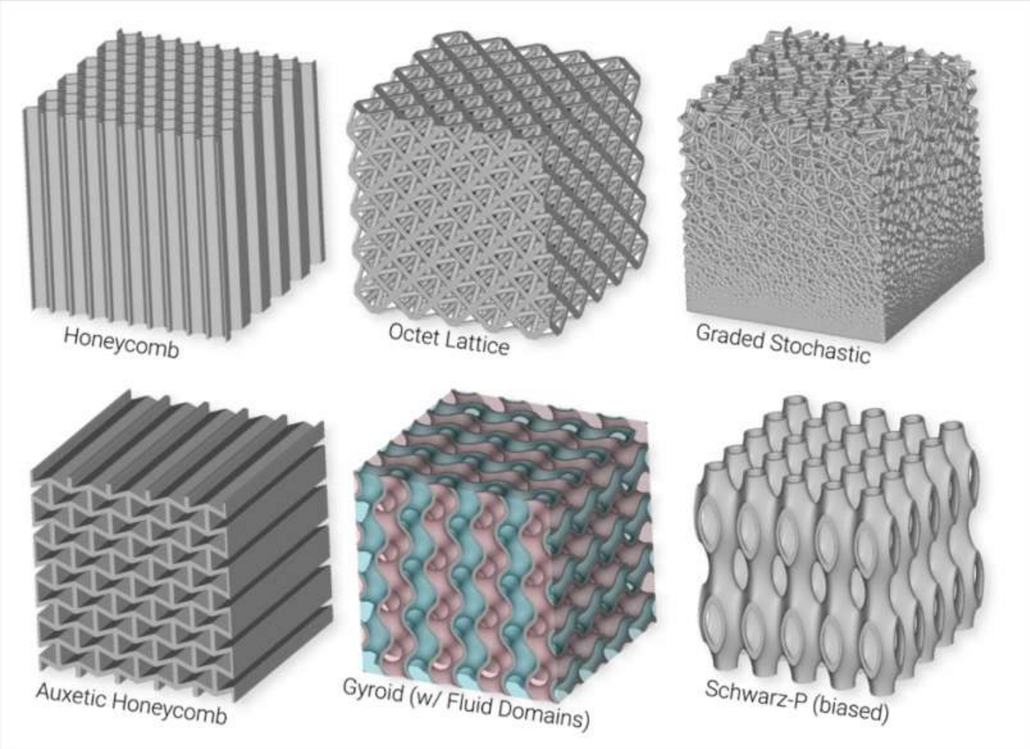
Some typical suspects in the cellular world include honeycombs, foams, lattices, and shell-like continuous structures such as gyroids, each having its own pros and cons.



An assortment of the many topologies available for lightweighting, with brief descriptions below.

Lattices and Architected Materials 3

- **Honeycombs** are hard to beat in the stiffness-to-weight ratio in their extruded direction but are much softer in the other two orthogonal directions (by around an order of magnitude).
- **Octet lattices** are stiff in all three principal directions but more difficult to manufacture.
- **Stochastic foams** are much more compliant but the easiest to finely tailor to a design's functional requirements through cell size and relative density.
- **Auxetic honeycombs** can be designed to exhibit special properties like a negative Poisson's ratio (contracts inwards when compressed, instead of barreling outwards).
- The **gyroid** and broader family of **triply periodic** structures inherently have two sides or fluid domains to them, making them natural candidates for heat exchanger applications. They can be easily tailored or biased in different directions to alter stiffness or flow properties as needed.



An assortment of the many topologies available for lightweighting, with brief descriptions below.

Sandwich structures

Cellular materials are perhaps most applicable to engineering in the form of ***sandwich structures***. These are prevalent throughout all levels of the lightweighting value spectrum, from cardboard packaging to commercial aircraft to interplanetary spacecraft. The effect is all the same: separate two thin sheets of material (the bread) by a lightweight core (the filling, available in the flavors shown above) and you can attain tremendous strength and stiffness increases for a minor increase in mass. Turn this around by keeping strength and stiffness constant, and you have significant lightweighting potential.

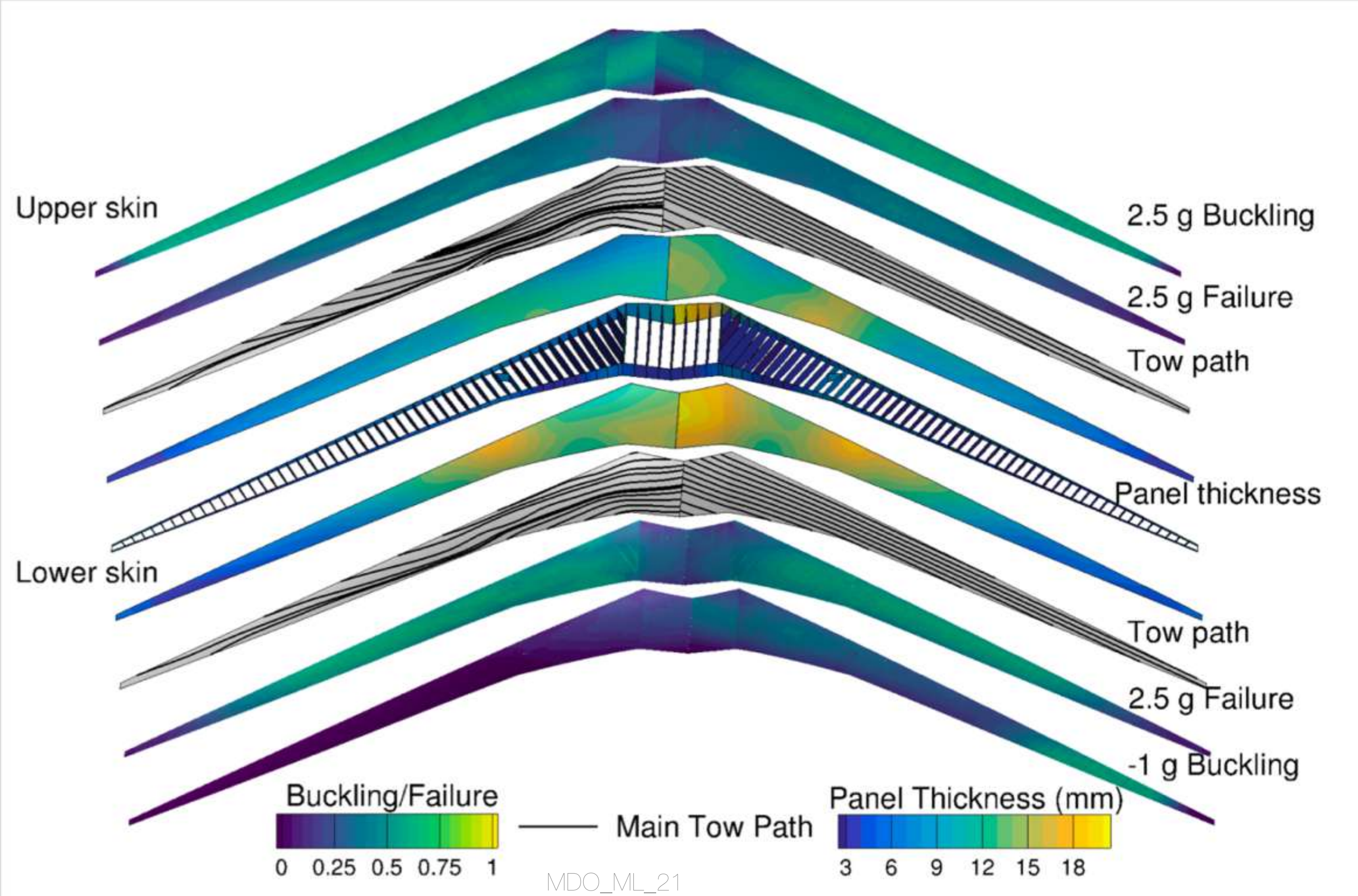
	Solid Material	Core Thickness t	Core Thickness 3t
	t	2t	4t
Stiffness	1.0	7.0	37.0
Flexural Strength	1.0	3.5	9.2
Weight	1.0	1.03	1.06

Bending stiffness and strength benefits of lightweighting with sandwich structures, with a few examples shown.

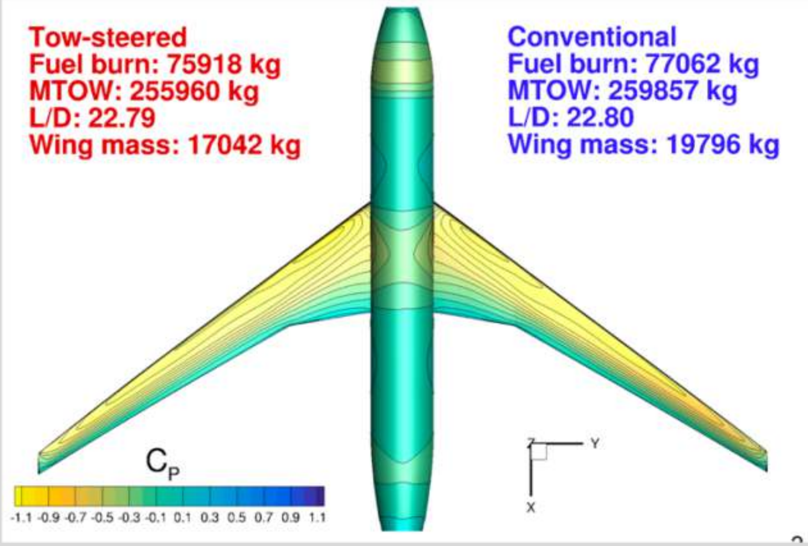
AFP: Automatic Fiber Placement

Figure 17: AFP machine manufacturing the tow-steered optimized uCRM-13.5 wingbox (left; courtesy of Aurora Flight Sciences). Static test of the same wing (right; courtesy of NASA)

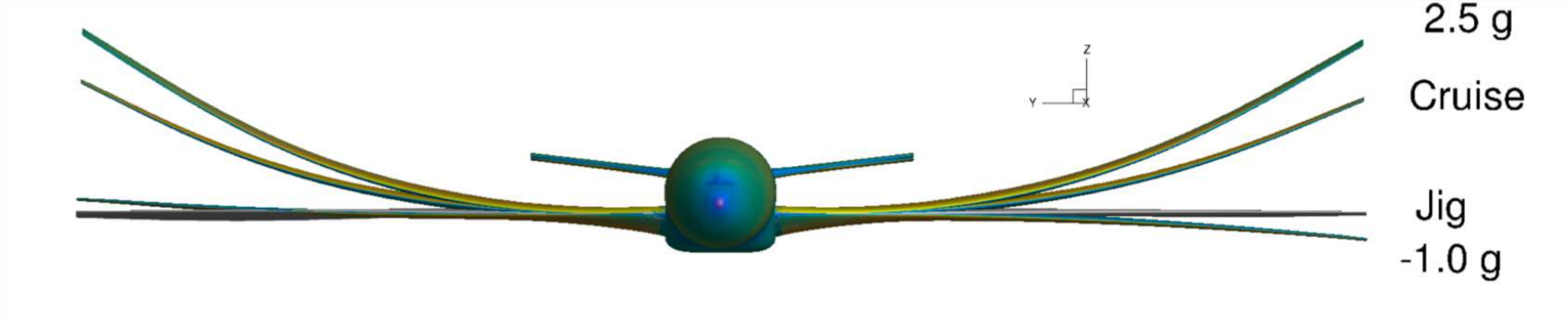
Prof Martins 'sTow Steered Wing Structure Design



MDOlabAerostructural Optimization of Tow-steered Composite Wings



Less fuel burn, lower wing mass



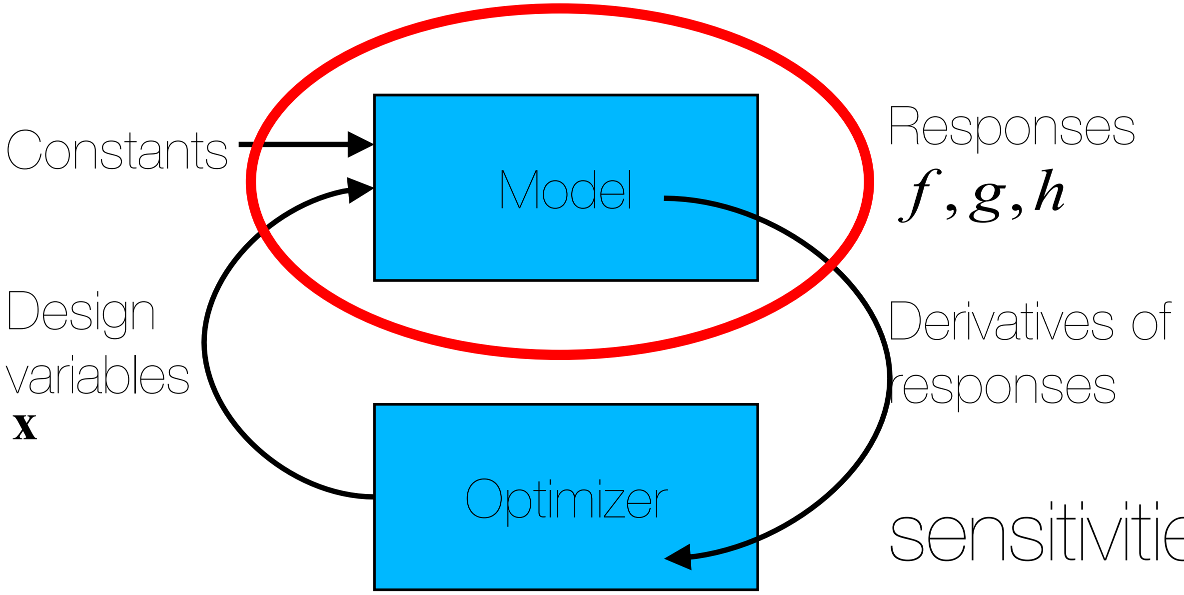
Topology Optimization

Last stop for today: topology optimization. Some refer to this as '[generative design]', but we'll stick to the engineering term here. Several methods and algorithms exist, but the general idea is to apply loads and restraints to a design region, and then whittle away material from this region until some constraints and objectives are met.

This is usually minimizing compliance (or maximizing stiffness) subject to a volume target. Many cases require additional constraints like maximum stress or displacement, or some restriction on manufacturing technique. The result is usually something exotic and organic looking

A titanium spacecraft bracket, lightweighted with topology optimization and built with electron beam melting (EBM).

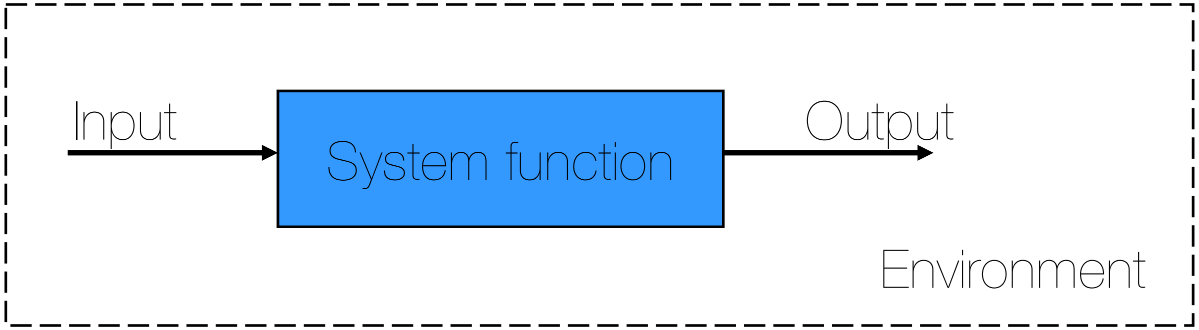
How? Back to the basics



sensitivities*:

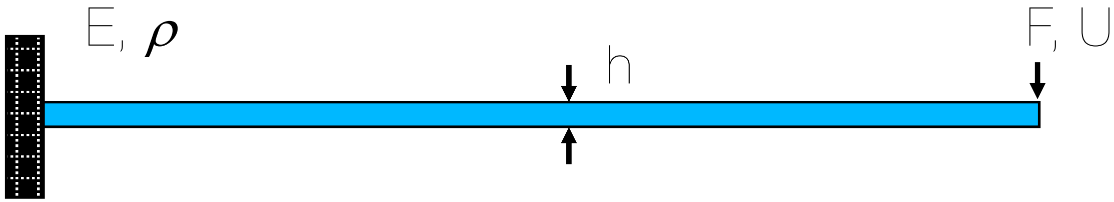
$$\frac{\partial f}{\partial x_i}, \frac{\partial g}{\partial x_i}, \frac{\partial h}{\partial x_i}$$

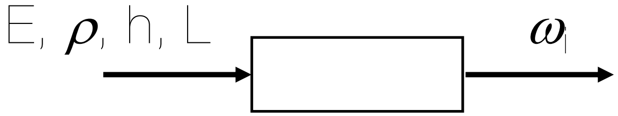
System approach

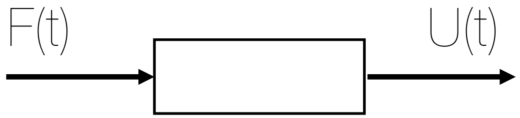


- Wonder:
    - What are the inputs / outputs?
    - What is the system / environment?
    - Hierarchize the system

Example #1 cantilever beam



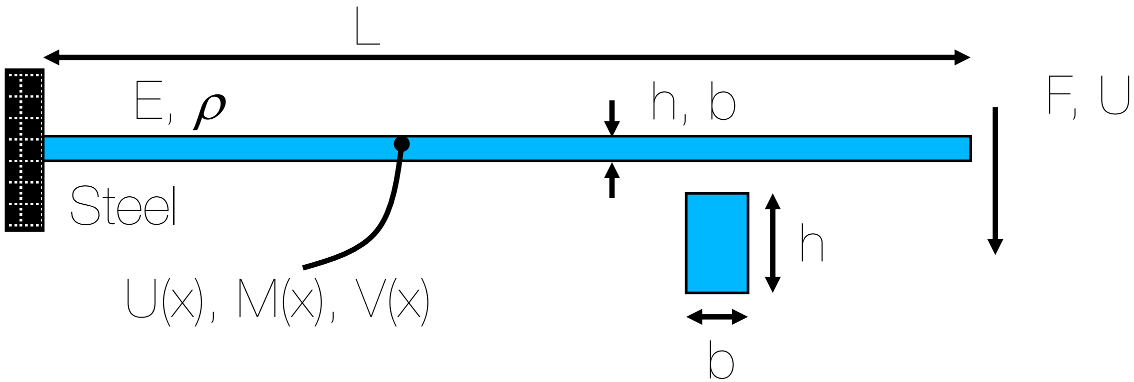




Etc.

Y=f(X1,...,Xk)

Exemple # 1.1: section

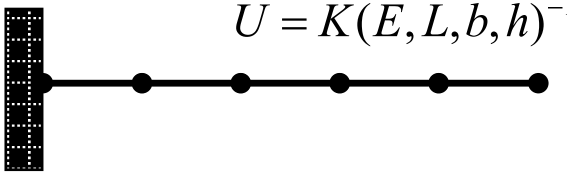


SoM:

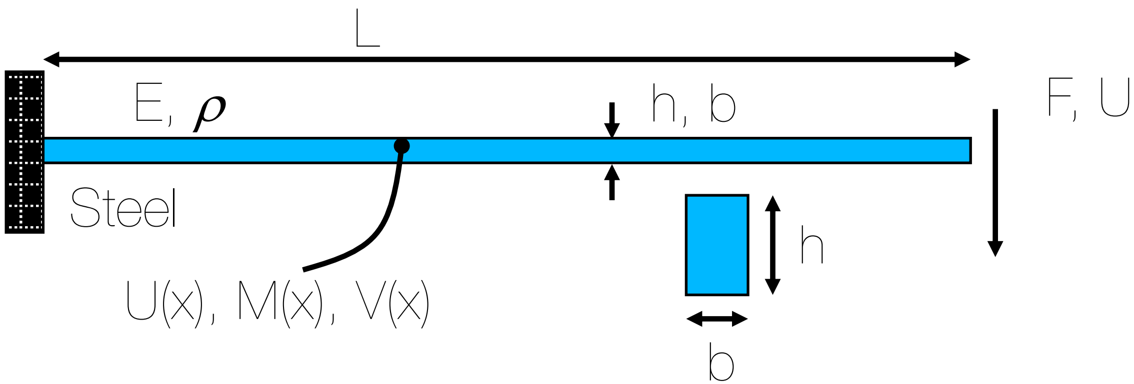
$$U = \frac{FL^3}{3EI} = \frac{FL^3}{3E\left(\frac{bh^3}{12}\right)}$$

FEM:

$$U = K(E, L, b, h)^{-1} F$$



States



- System state variables : U(x), M(x), V(x)
- System parameters: h, b, L
- System constants: E, ρ

Negative null form [MATLAB]

$$\begin{array}{rl} \underset{\mathbf{x}}{\text{Minimize}} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{h}(\mathbf{x}) = 0 \\ & \mathbf{g}(\mathbf{x}) \leq 0 \\ & \mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^n \end{array}$$

$\mathbf{x}$ = (column) vector of design variables

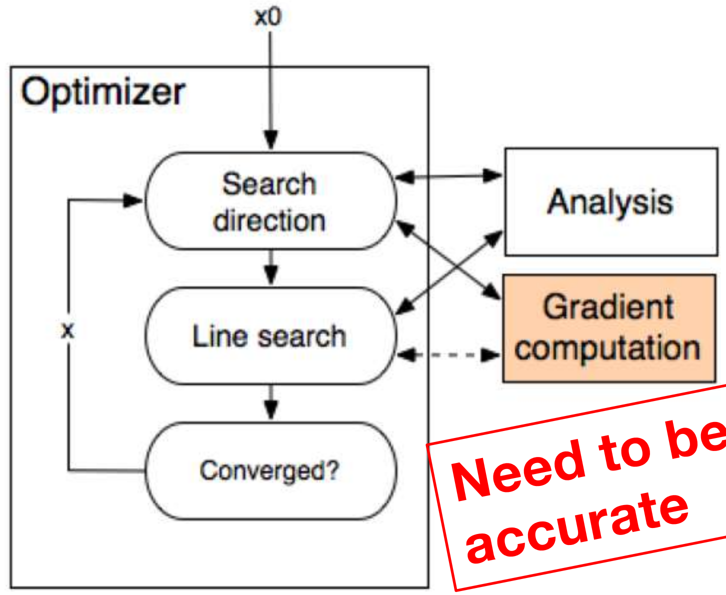
$\mathbf{h} = [h_1, h_2, \ldots, h_{m_h}]^T$

$\mathbf{g} = [g_1, g_2, \ldots, g_{m_g}]^T$

Other formulations:

- positive null form ($\mathbf{g}(\mathbf{x}) \geq 0$)
- negative unity form ($\mathbf{g}(\mathbf{x}) \leq 1$)
- positive unity form ($\mathbf{g}(\mathbf{x}) \geq 1$)

Optimizer solver



Need to be accurate

- Requirements
- Problem to solve

$$\begin{cases} \min f(x) \\ wrt\ x \in R^d \\ st\ g_i(x) \leq 0 \text{ for } i = 1, \dots m \end{cases}$$

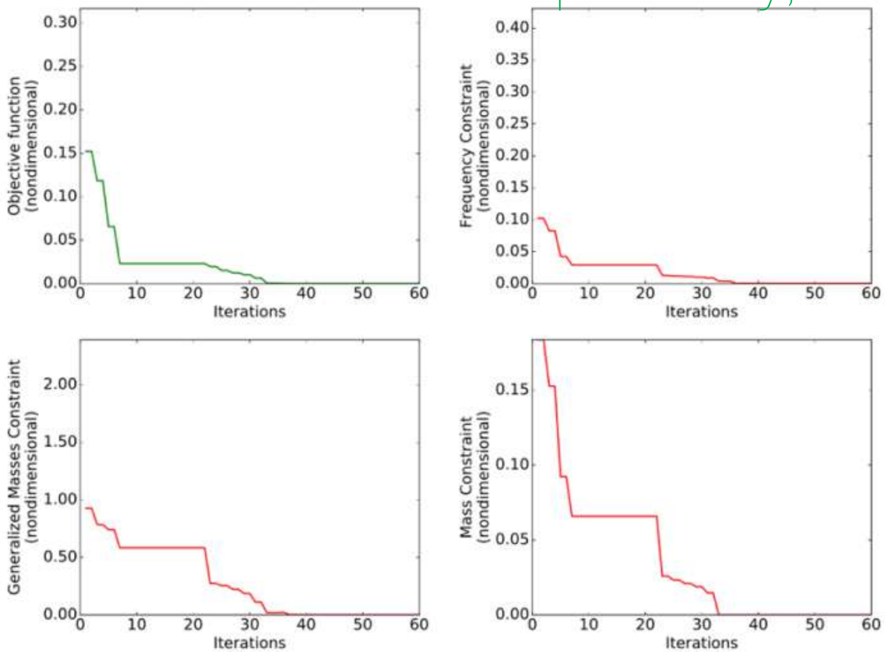
- Derivative Fee Optimizer (DFO)
- Evolutionnary Strategies (ES)
- **Bayesian Optimizer (BO)** https://github.com/SMTorg/smt

- Gradient based Optimizer
- Computation of the derivatives of $f(x)$ and $g_i(x)$ to iterate and satisfy the KKT optimality conditions

➔ Focus : computation of sensitivities (adjoint vs direct) $\frac{\partial f}{\partial x_i}, \frac{\partial g}{\partial x_i}, \frac{\partial h}{\partial x_i}$

Optimizer's outputs

Gradient based Optimality, Feasibility



Stopping criteria: tolfun, tolx, maxiter

Gradient's definition

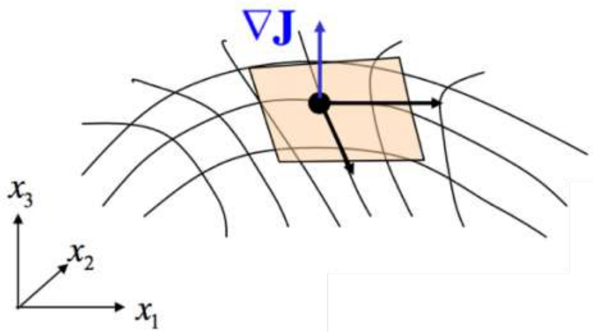
"How does the function J value change locally as we change elements of the design vector x?"

⇩

Compute partial derivatives of J with respect to x_i

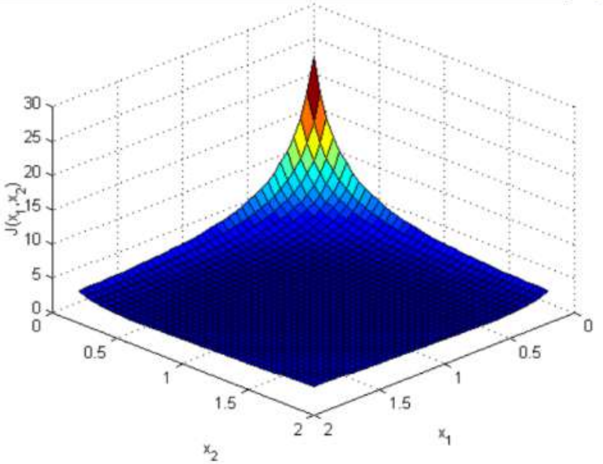
$$\frac{\partial J}{\partial x_i}$$

$$\nabla \mathbf{J} = \begin{bmatrix} \dfrac{\partial J}{\partial x_1} \\ \dfrac{\partial J}{\partial x_2} \\ \vdots \\ \dfrac{\partial J}{\partial x_n} \end{bmatrix}$$

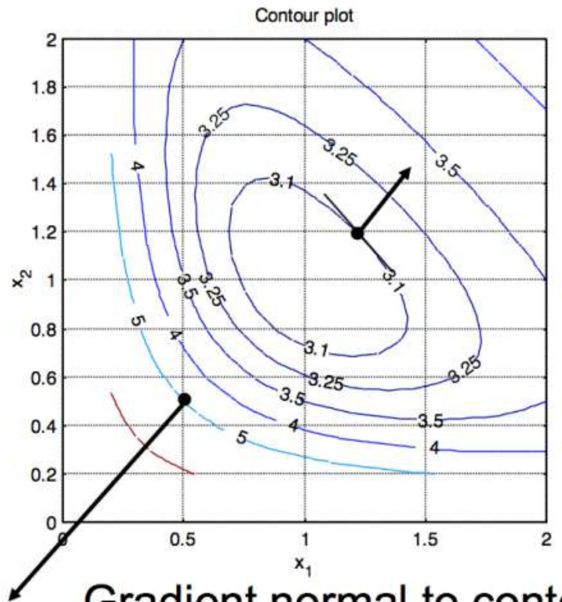


Geometry of Gradient (2D)

Example function: $J \;\; x_1, x_2 \;\; = x_1 + x_2 + \dfrac{1}{x_1 \cdot x_2}$



$$\nabla J = \begin{bmatrix} \dfrac{\partial J}{\partial x_1} \\[2ex] \dfrac{\partial J}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{1}{x_1^2 x_2} \\[2ex] 1 - \dfrac{1}{x_1 x_2^2} \end{bmatrix}$$



Gradient normal to contours

Other Gradient related ...

- Jacobian: Matrix of derivatives of multiple functions w.r.t. vector of variables

$$\mathbf{J} = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_z \end{bmatrix} \quad \Rightarrow \quad \nabla \mathbf{J} = \begin{bmatrix} \frac{\partial J_1}{\partial x_1} & \frac{\partial J_2}{\partial x_1} & \cdots & \frac{\partial J_z}{\partial x_1} \\ \frac{\partial J_1}{\partial x_2} & \frac{\partial J_2}{\partial x_2} & \cdots & \frac{\partial J_z}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial J_1}{\partial x_n} & \frac{\partial J_2}{\partial x_n} & \cdots & \frac{\partial J_z}{\partial x_n} \end{bmatrix}$$

z x 1   *n* x *z*

- Hessian: Matrix of second-order derivatives

$$\mathbf{H} = \nabla^2 J = \begin{bmatrix} \frac{\partial^2 J}{\partial x_1^2} & \frac{\partial^2 J}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 J}{\partial x_1 \partial x_n} \\ \frac{\partial^2 J}{\partial x_2 \partial x_1} & \frac{\partial^2 J}{\partial x_2^2} & \cdots & \frac{\partial^2 J}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 J}{\partial x_n \partial x_1} & \frac{\partial^2 J}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 J}{\partial x_n^2} \end{bmatrix} \quad n \times n$$

Analytical sensitivities

If the objective function is known in closed form, we can often compute the gradient vector(s) in closed form (analytically):

Example:
$$J(x_1, x_2) = x_1 + x_2 + \frac{1}{x_1 \cdot x_2}$$

Analytical Gradient:
$$\nabla J = \begin{bmatrix} \frac{\partial J}{\partial x_1} \\ \frac{\partial J}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1 - \frac{1}{x_1^2 x_2} \\ 1 - \frac{1}{x_1 x_2^2} \end{bmatrix}$$

Example

$x_1 = x_2 = 1$

$J(1,1)=3$

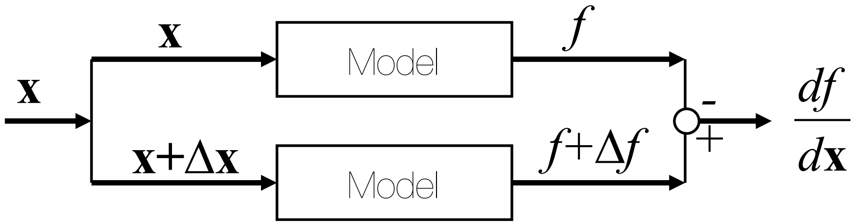
$$\nabla J(1,1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Minimum

For complex systems analytical gradients are rarely available

Sensitivity analysis approaches

Simpler approach (*default with fmincon*):



GFD

Matlab

How to proceed with PDE such as Kq=f?

Discrete

Nastran SOL200

Symbolic differentiation

- Use symbolic mathematics programs
- e.g. MATLAB®, Maple®, Mathematica®

construct a symbolic object

```
» syms x1 x2
» J=x1+x2+1/(x1*x2);
» dJdx1=diff(J,x1)
dJdx1 =1-1/x1^2/x2
» dJdx2=diff(J,x2)
 dJdx2 = 1-1/x1/x2^2
```

difference operator

Automatic Differentiation

- Mathematical formulae are built from a finite set of basic functions, e.g. additions, sin x, exp x, etc.
- Using chain rule, differentiate analysis code: add statements that generate derivatives of the basic functions
- Tracks numerical values of derivatives, does not track symbolically as discussed before
- Outputs modified program = original + derivative capability
- e.g., ADIFOR (FORTRAN), TAPENADE (C, FORTRAN), TOMLAB (MATLAB), many more…
- Resources at http://www.autodiff.org/

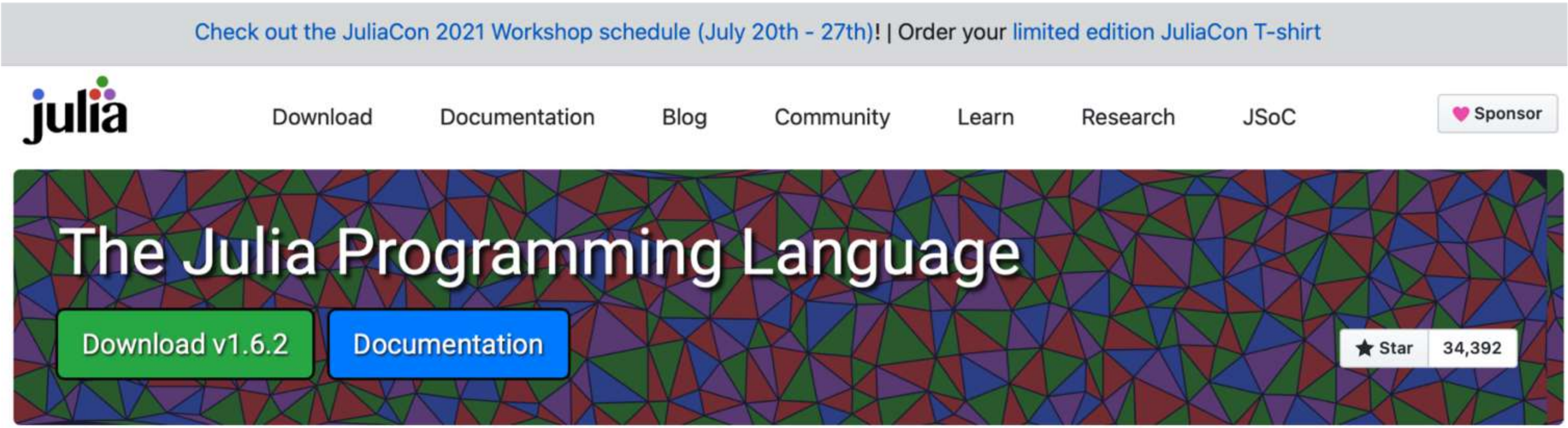
•OR…

Use differentiable programming (see tutorial on Github)

https://github.com/mid2SUPAERO/TopoOpti-Julia

Testcase	Resolution	solved using top88 julia		
MBB	100*100			
	time (s)	Allocation	GiB	percent gc time
Analytical	12.845634	156.44	6.358	10.76
AD	58.238602	182.39	70.522	13.31
FD	66.370186	106.86	192.019	9.99

https://sinews.siam.org/Details-Page/scientific-machine-learning-how-julia-employs-differentiable-programming-to-do-it-best



Use Complex Step Derivative
(see tutorial on Github)

- Similar to finite differences, but uses an imaginary step

$$f'(x_0) \approx \frac{\mathrm{Im}\left[f(x_0 + i\Delta x)\right]}{\Delta x}$$

Second order accurate

Canx use very small step sizes e.g. $\Delta x \approx 10^{-20}$

Doesn't have rounding error, since it doesn't perform subtraction

Limited application areas

Code must be able to handle complex step values

J.R.R.A. Martins, I.M. Kroo and J.J. Alonso, an automated method for sensitivity analysis using complex variables, AIAA paper 2000-0689, jan 2000

How Nastran is differentiating a FE code ?

?

Semi-analytic Diff.

- General case f is not depending on xi
- except for volumic force (i.e. gravity)

$$K(x)\, u(x) = f(x)$$

$$\frac{\partial K(x)}{\partial x_i} u(x) + K(x) \frac{\partial u(x)}{\partial x_i} = \frac{\partial f(x)}{\partial x_i}$$

$$K(x) \frac{\partial u(x)}{\partial x_i} = \frac{\partial f(x)}{\partial x_i} - \frac{\partial K(x)}{\partial x_i} u(x)$$

$$\widetilde{K}\ \widetilde{u} = \widetilde{f}$$

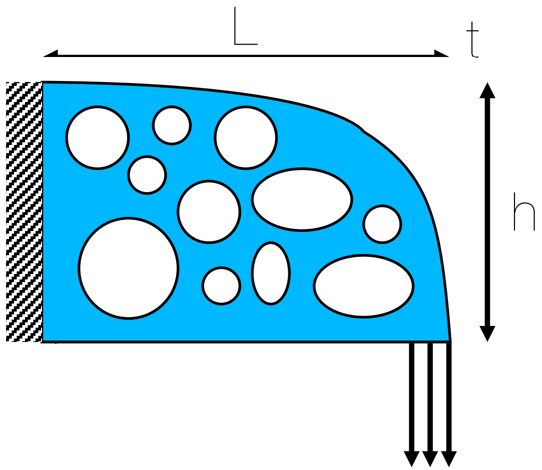
$$\widetilde{u} = \widetilde{K}^{-1} \widetilde{f} = K^{-1} \widetilde{f}$$

$$\frac{\partial u(x)}{\partial x_i} = K^{-1} \left\{ \frac{\partial f(x)}{\partial x_i} - \frac{\partial K(x)}{\partial x_i} u(x) \right\}$$

$$= K^{-1} \left\{ \frac{f(x+\Delta x) - f(x)}{\Delta x} - \frac{K(x+\Delta x) - K(x)}{\Delta x} u(x) \right\}$$

Structural Optimization

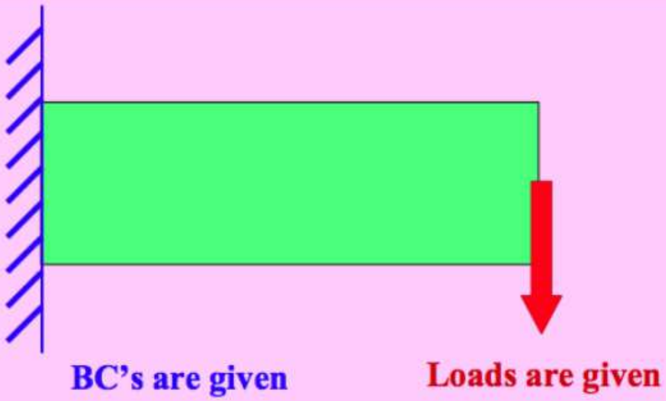
- Optimization techniques are applied to dimensioning (“sizing”) problems
- Different categories:
  - Parametric optimization
  - Material optimization
  - Shape optimization
  - Topology optimization



Structural Optimization MIT Course Number

ESD.77 / 16.888

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \le 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$



1. To make the structure strong
   e.g. Minimize displacement at the tip → *Min.* $f(\mathbf{x})$

2. Total mass $\le M_C$ → $g(\mathbf{x}) \le 0$

Sizing Optimization MIT Course Number

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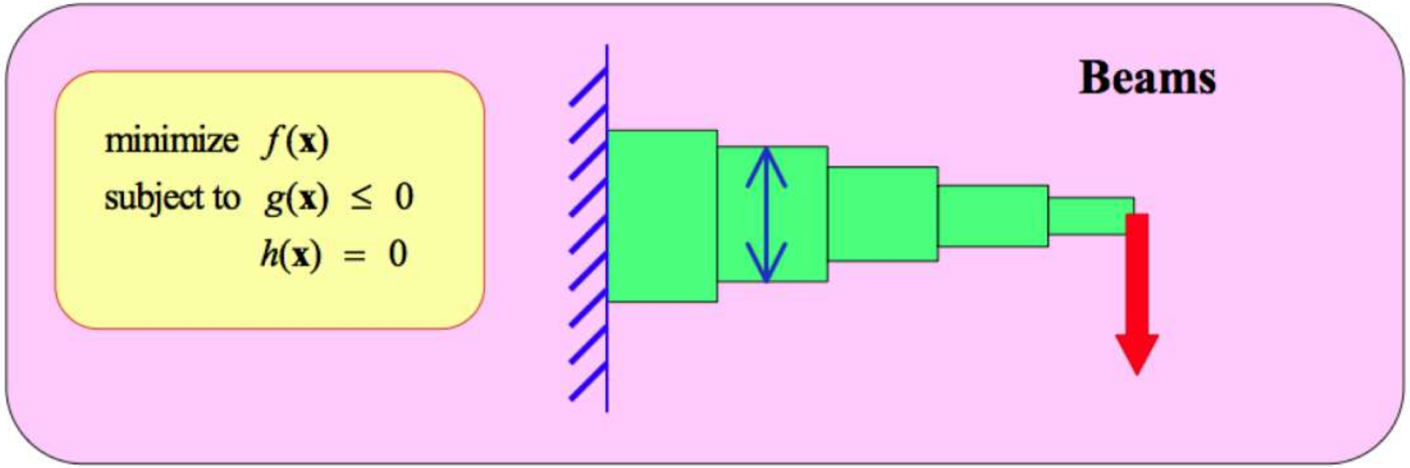


Design variables (x)

x : thickness of each beam

Number of design variables (ndv)

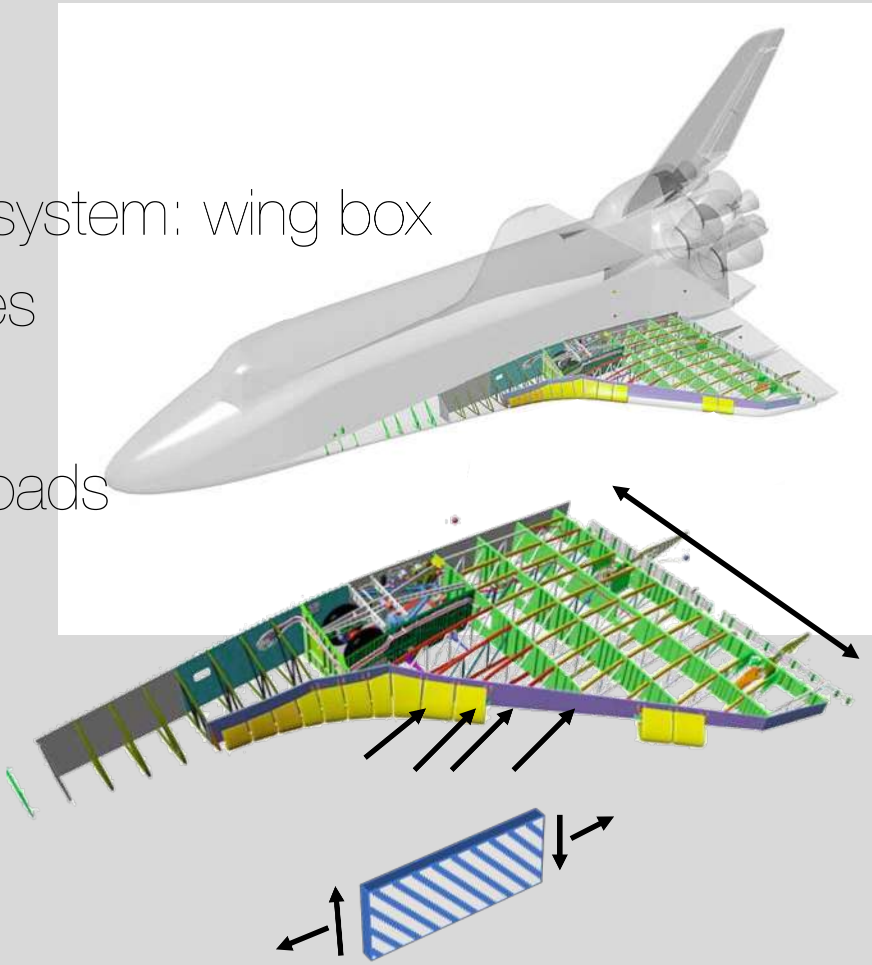
ndv = 5

***f*(x) : compliance**

***g*(x) : mass**

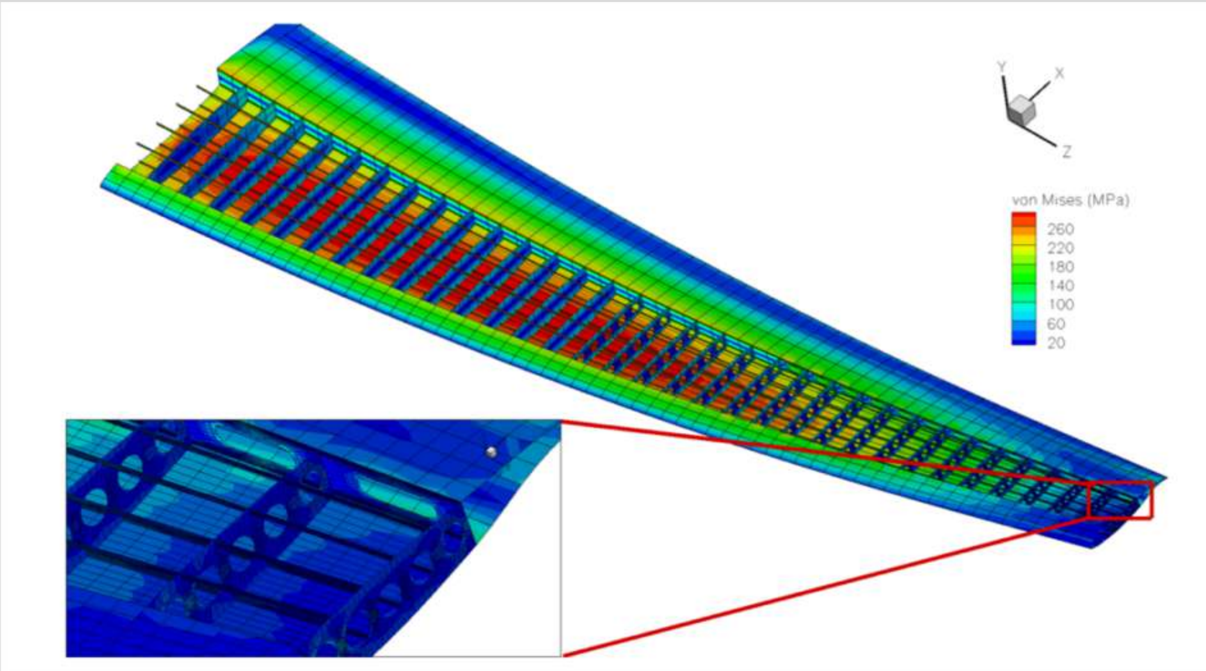
Hierarchical system

- Decomposition into sub system: wing box
- Too many design variables
- Global Level (GFEM):
- Overall dimensions and loads



- Local level (rib / stiffener)
- Thickness of skins
- fiber orientation

Using FE …and 10K DOFs…



How do you treat so many admissible stresses ?

Equivalent problem using KS function

Consequently, an optimization problem

$$\begin{array}{ll} \textbf{minimize} & f(\mathbf{x}) \\ \textbf{such that} & h_k(\mathbf{x}) = 0, \qquad k = 1, \ldots, n_e \,, \end{array}$$

may be reformulated as

$$\begin{array}{ll} \textbf{minimize} & f(\mathbf{x}) \\ \textbf{such that} & KS(h_1, -h_1, h_2, -h_2, \ldots, h_{n_e}, -h_{n_e}) \geq -\epsilon \;. \end{array}$$

where ϵ is a small tolerance.

Stress constraints aggregation
(see tutorial on Github)

Some of the numerical techniques offered in this chapter for the solution of constrained nonlinear optimization problems are not able to handle equality constraints, but are limited to inequality constraints. In such instances it is possible to replace the equality constraint of the form $h_i(\mathbf{x}) = 0$ with two inequality constraints $h_i(\mathbf{x}) \leq 0$ and $h_i(\mathbf{x}) \geq 0$. However, it is usually undesirable to increase the number of constraints. For problems with large numbers of inequality constraints, it is possible to construct an equivalent constraint to replace them. One of the ways to replace a family of inequality constraints ($g_i(\mathbf{x}) \geq 0, i = 1 \ldots m$) by an equivalent constraint is to use the Kreisselmeier-Steinhauser function [1] (KS-function) defined as

$$KS[g_i(\mathbf{x})] = -\frac{1}{\rho} \ln[\sum_i e^{-\rho g_i(\mathbf{x})}] \ ,$$

KS

$$KS[g_i(\mathbf{x})] = -\frac{1}{\rho}\ln[\sum_i e^{-\rho g_i(\mathbf{x})}] \ ,$$

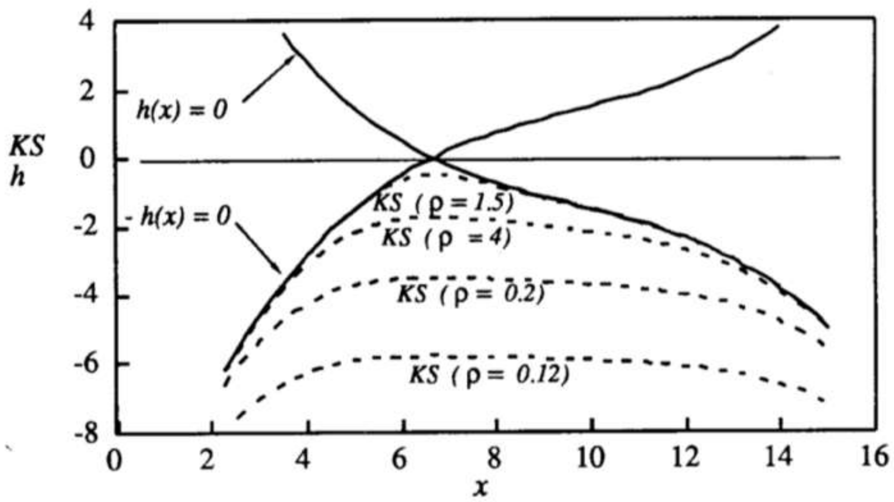
where ρ is a parameter which determines the closeness of the KS-function to the smallest inequality $min[g_i(\mathbf{x})]$. For any positive value of the ρ, the KS-function is always more negative than the most negative constraint, forming a lower bound envelope to the inequalities. As the value of ρ is increased the KS-functions conforms with the minimum value of the functions more closely. The value of the KS-function is always bounded by

$$g_{\min} \leq KS[g_i(\mathbf{x})] \leq g_{\min} - \frac{\ln(m)}{\rho} \ .$$

Stress constraints aggregation

For an equality constraint represented by a pair of inequalities, $h_i(\mathbf{x}) \leq 0$ and $-h_i(\mathbf{x}) \leq 0$, the solution is at a point where both inequalities are active, $h_i(\mathbf{x}) = -h_i(\mathbf{x}) = 0$, Figure 5.1 . Sobieski [2] shows that for a KS-function defined by such a positive and negative pair of h_i, the gradient of the KS-function at the solution point $h_i(\mathbf{x}) = 0$ vanishes regardless of the ρ value, and its value approaches to zero as the value of ρ tends to infinity, Figure 5.1 . Indeed, from Eq. (5.4) at $\mathbf{x}$ where $h_i = 0$, the KS-function has the property

$$0 \geq KS(h, -h) \geq -\frac{\ln(2)}{\rho} .$$



Kreisselmeier-Steinhauser function for replacing $h(\mathbf{x}) = 0$.

Max is not differentiable …

KS function is !

Shape Optimization MIT Course Number
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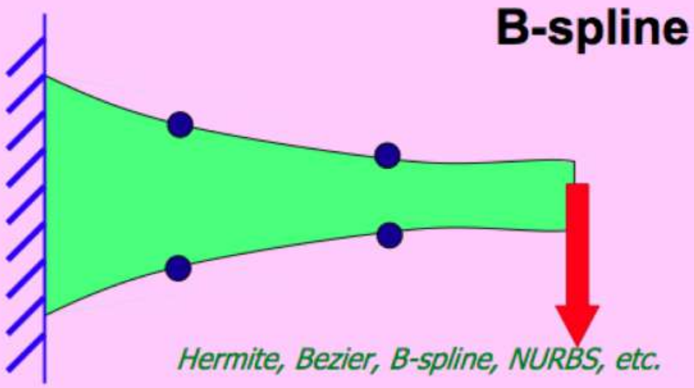
$$
\begin{aligned}
\text{minimize} \quad & f(\mathbf{x}) \\
\text{subject to} \quad & g(\mathbf{x}) \leq 0 \\
& h(\mathbf{x}) = 0
\end{aligned}
$$



Design variables (x)

x : control points of the B-spline

(position of each control point)

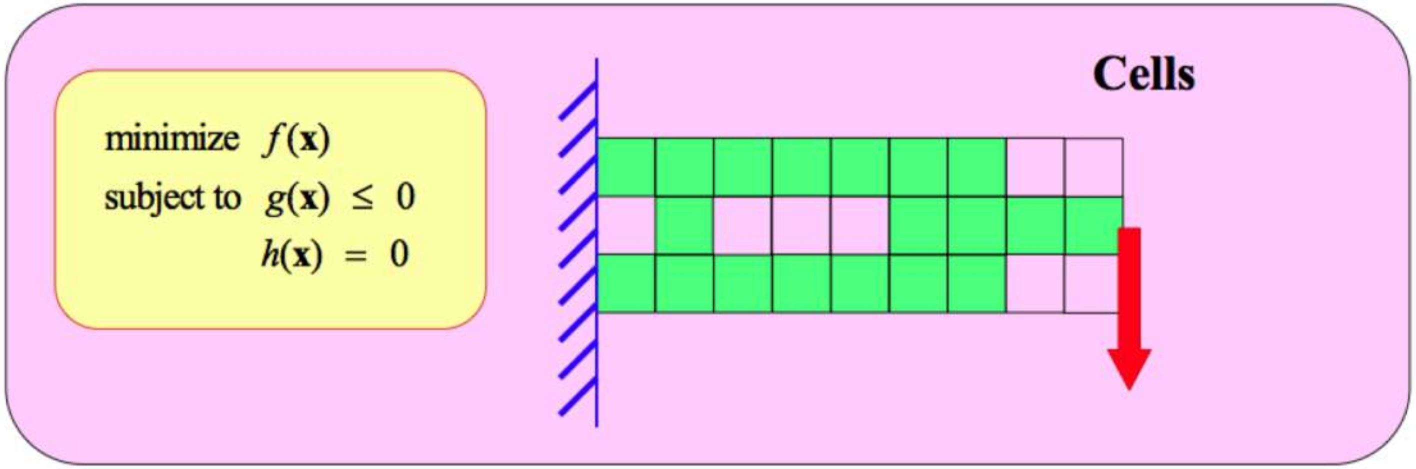
Number of design variables (ndv)

ndv = 8

***f*(x) : compliance**

***g*(x) : mass**

Topology Optimization MIT Course Number ESD.77 / 16.888



Design variables (x)

x : density of each cell

Number of design variables (ndv)

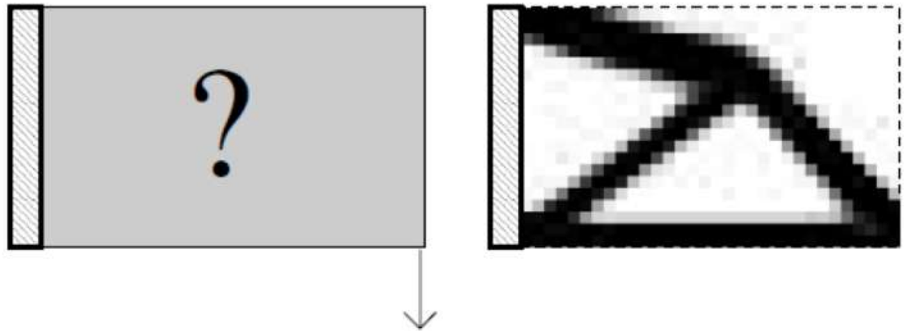
ndv = 27

***f*(x) : compliance**

***g*(x) : mass**

Topology optimization

According to Bendsøe (1989), Topology optimization:
"… should consist of a determination for every point in space whether there is material in that point or not."



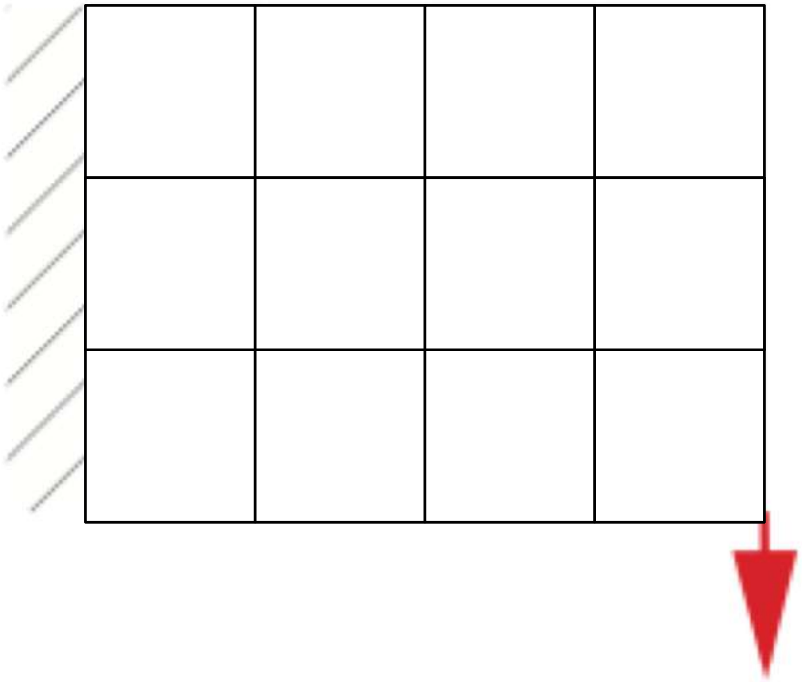
Bendsøe, M. P. and Sigmund, O. (2004)

Classic objective: Compliance minimization →

- Maximize the stiffness of the structure
- With respect to a set of loads
- With a fixed volume fraction

$$
\begin{array}{lll}
\min\limits_{\rho,U} : & \Phi(\rho,U) & \text{Objective function} \\
s.t. : & V(\rho) \leq V^{*} & \text{Constraints} \\
: & g_i(\rho,U) \leq g_i^{*}, \quad i = 2, \ldots, M & \text{Constraints} \\
: & \rho_{min} \leq \rho \leq \rho_{max} & \text{Box constraints} \\
: & \mathbf{K}(\rho)\mathbf{U} = \mathbf{F} & \text{State equation (FE)}
\end{array}
$$

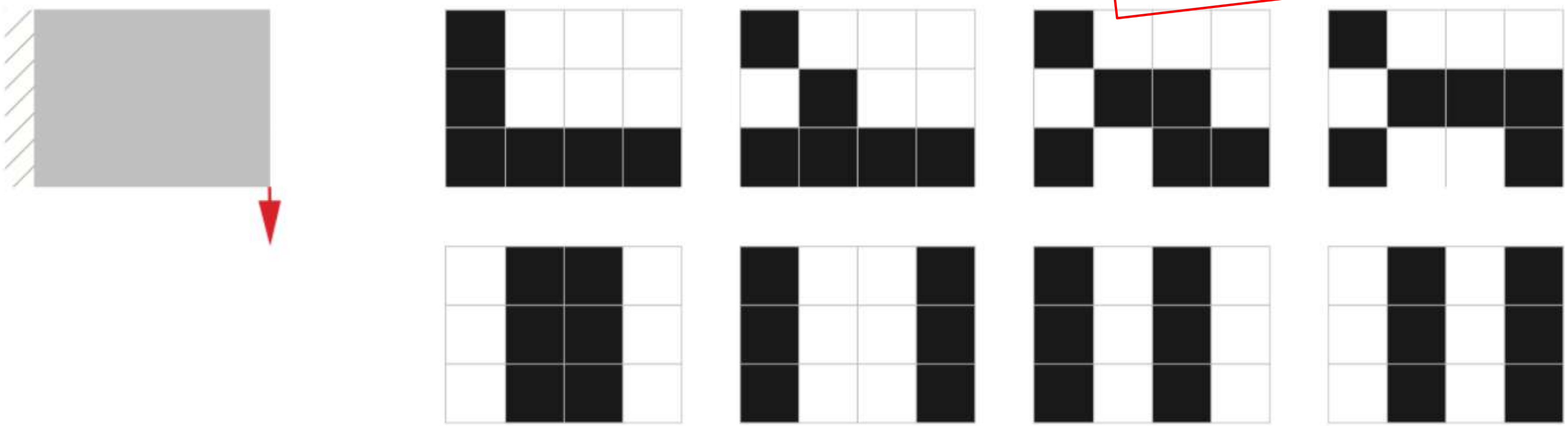
Quiz ! Put (black) or remove materials (white)



Goal: 50% gain in mass

Results

Is it discrete or continuous ?



The legal (top) and some illegal (bottom) topologies with 4 by 3 elements

Popularization

https://www.tripadvisor.fr/LocationPhotoDirectLink-g187529-d574612-i349532022-Museum_of_Natural_Science_Museo_de_Ciencias_Naturales-Valencia_Province_o.html

Is it possible to build an aircraft wing in LEGO® ?

Publié le 17 février 2020 Modifier l'article | Voir les stats

joseph morlier
Professor in Structural and Multidisciplinary Design Optimization, ... any idea?
5 articles

https://www.linkedin.com/pulse/possible-build-aircraft-wing-lego-joseph-morlier/?articleId=6627240732975480832

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Break before Part 2

Prof. J. Morlier