Potential Fixedpoint Semantics for PEGREG

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Progress on pegreg stalled because of concerns about the correctness of the algorithm that created an FST for possessive star.

Nonetheless, I believe it is still possible to define possessive star in finite state machines:

Parsing expression grammars are usually defined with respect to their input string, in effect embedding a "continuation" string into the semantics of a parsing expression grammar.

In the paper "Towards Typed Semantics for Parsing Expression Grammars" (2019), Rebeiro et. al. present an operational semantics for PEG that includes a left-recursive version of the star operator. I believe this semantics can be encoded as a fixpoint, viewing the strings "to be matched" as the set of all strings, then shown equivalent to the following rules:

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\begin{split} R_p[\![\langle \operatorname{ch}, \epsilon \rangle]\!] &= \{\operatorname{ch}\} & \operatorname{character\ literal} \\ R_p[\![\langle \operatorname{ch}, e_2 \rangle]\!] &= \{\operatorname{ch.s} \mid s \in R_p[\![\langle e_2, \epsilon \rangle]\!]\} & \operatorname{character\ literal} \\ R_p[\![\langle e_1, e_2, e_3 \rangle]\!] &= R_p[\![\langle e_1, e_2, e_3 \rangle]\!] & \operatorname{concatenation} \\ R_p[\![\langle e_1, e_2, e_3 \rangle]\!] &= \{s_1s_3, s_2s_3 \mid s_1 \in R_p[\![\langle e_1, \epsilon \rangle]\!], s_3 \in R_p[\![\langle e_3, \epsilon \rangle]\!], s_2 \in R_p[\![\langle e_2, \epsilon \rangle]\!] \setminus R_p[\![\langle e_1, \epsilon \rangle]\!]\} & \operatorname{ordered\ choice} \\ F_p[\![\langle e_1^*, \epsilon \rangle]\!] &= \{s_1s_1 \mid s_1 \in R_p[\![\langle e_1, \epsilon \rangle]\!]\} \cup \{s_1s_2 \mid s_1 \in X \land s_2 \in X\} & \operatorname{greedy\ repitition} \\ R_p[\![\langle e_1^*, \epsilon \rangle]\!] &= \|\operatorname{fp} F_p & *\epsilon\text{-}\operatorname{case} \\ R_p[\![\langle e_1^*, \epsilon \rangle]\!] &= \{s_1s_2 \mid s_1 \in R_p[\![\langle e_1^*, \epsilon \rangle]\!], s_2 \in R_p[\![e_2]\!] \setminus R_p[\![e_1]\!]\} & *\operatorname{general\ case} \end{split}
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The set minus operation can then be encoded in a finite state machine: $L_1 \setminus L_2 = L_1 \cap \neg(L_2) = \neg(L_1 \cup \neg(L_2))$.