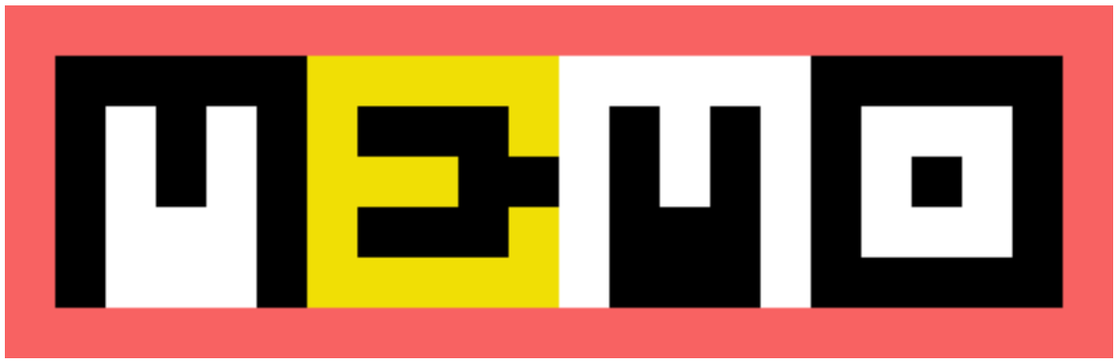
The **memo** handbook

v1.2.0



Anatomy of a memo

axes of array to compute

name

scalar* free parameters

default value

```
@memo                def model[x: X, y: Y](a, b, c=3.14):
```

optional comment for documentation

```
    ''' This is a model of ____! '''
    alice: …
    bob: …
    return …
```

sequence of **statements**

expression** whose value to compute for each cell in returned array

*Advanced: you can pass arrays, too. Array-valued parameters must be declared with an ellipsis annotation (`a: ...`).

**Advanced: you can have models with multiple return values by writing multiple return statements.

Statements

chooses

Domain of choice (name of Python list/enum or JAX array)

Agent making choice

```
bob: chooses(a in Actions, wpp=exp(β*utility(a)))
```

Name of choice

"With probability proportional to"
For softmax, use wpp=exp(…)
For uniform choice, use wpp=1

Can make multiple choices simultaneously

```
bob: chooses(x in X, y in Y, wpp=joint(x, y))
```

more ways to choose

```
bob: chooses(a in Actions, to_maximize=utility(a))
```

For argmax use to_maximize
For argmin use to_minimize

Aliases of "chooses" that don't imply agency/goal-orientation

(These all mean the same, but can make your model easier to read.)

```
bob: given(r in Roles, wpp=1)
bob: draws(r in Roles, wpp=1)
bob: assigned(r in Roles, wpp=1)
bob: guesses(r in Roles, wpp=1)
```

thinks

Agent doing the thinking

```
bob: thinks[
    alice: chooses(...),
    charlie: chooses(...),
    ...
]
```

What that agent thinks
(notice the commas!)

observes

Agent observing

Choice being observed (square brackets are a mnemonic for "someone else's choice")

```
bob: observes [alice.x] is y
```

What the choice is observed to actually be. Can create false beliefs this way!

```
bob: observes [alice.x] is charlie.y
```

This value can also be another agent's choice.

advanced use of observes

Agent observing

Condition that was observed (boolean expression)

```
bob: observes_that [coin.bias > 0.5]
```

like "condition(…)" in WebPPL

Agent observing

Probability of event observed

```
bob: observes_event(wpp=coin.bias)
```

like "factor(log(…))" in WebPPL

You generally shouldn't need these advanced constructs — most models should only use `observes` as documented on the previous page.

knows

Agent who knows

Choices that are known

```
bob: knows(x, alice.y)
```

This utility is useful for the common case of "pushing" a variable into an agent's frame of mind. Roughly shorthand for this:

```
bob: thinks[ alice: chooses(y in Y, wpp=...) ]
bob: observes [alice.y] is alice.y
```

snapshots_self_as

Agents can remember "snapshots" of their past selves. Useful for counterfactuals and hypotheticals, especially when used with "imagine" expressions (see below…).

Agent who snapshots

"aliases" of snapshots

```
alice: snapshots_self_as(past_alice, …)
```

```
alice: observes [bob.x] is x
return alice[ past_alice[ E[bob.x] ] ]
```

not affected by "observe" statement

wants / EU

wants sets up a predicate or utility function ("goal") that is defined over *future* states of the world. You can reference choices that have not yet been made yet. The expected future value of the goal expression can be evaluated using an EU expression.

name of goal (here, "win")

predicate/utility to evaluate in the future

Agent who wants

```
alice: wants(win=score > bob.score)
alice: chooses(a in A, to_maximize=EU[win])
alice: given(score in N, wpp=…)
```

Even though "score" is only defined here, the wants and EU expressions above are okay. Conceptually, the EU expression is evaluated here.

name of goal to evaluate when making goal-directed choice

INSPECT

At compile-time, print out the current state of an agent.
This can be helpful for debugging tricky models.

Agent to inspect

```
alice: INSPECT()
```

Expressions

literals

floating-point numbers only

↓

```
3.14
```

also references to declared free parameters

↓

```
a, b, c, …
```

operators

memo supports most Python unary/binary ops

```
1 + 1
```

also some free bonus functions

```
exp(…), log(…), abs(…)
```

can also call any function tagged with @jax.jit

useful for calling deep learning, etc.
JAX is a big ecosystem

```
@jax.jit
def f(x):
  return np.cos(x)
```

note: can only take scalar inputs and can only return one scalar output

choices

```
alice: chooses(x in X, wpp=1)
alice: chooses(y in Y, wpp=f(x, y))
```

you can refer to an agent's own choice as if it were simply a variable

or refer to other agents' choices with "dot" notation

```
alice.x + alice.y
```

equivalent to either of these

```
alice[x] + alice[y]
alice[x + y]
```

probabilistic operators

expectation

```
E[alice.x + bob.z]
```

variance

```
Var[alice.y * 2]
```

probability

```
Pr[alice.y >= 0]
Pr[a.x > 0, b.y < 2]
```

can use both commas and "and" for joint

information-theoretic operators

(mutual) entropy between choices

H[alice.x, bob.y, …]

KL divergence

KL[alice.x | bob.y]

queries

```
Var[alice[abs(x) * 2]]
alice[bob.y == 7]
```

can "query" another agent for the value
of an expression using square brackets

hypotheticals

set up hypothetical world by running statements

```
imagine[
  bob: chooses(y in Y, wpp=1),
  alice: observes [bob.y] is bob.y,
  alice[Pr[bob.x == 7]]
]
```

last line = expression to evaluate in that world

memo calls

```
@memo
def f[x: X](a, b, c): …

@memo
def g():
  alice: chooses(x in X, wpp=f[x](1.0, 0.0, 3.1))
```

can reference one memo from another. (don't forget to pass parameters!)

shorthand: can use "..." if f should be called with all of g's parameters; in this case, (a, b, c).

```
@memo
def g(a, b, c):
  alice: chooses(x in X, wpp=f[x](...))
```

When calling a model with multiple return values, first indicate which one you want.

```
g[0][x, y](3.14)
```

cost reflection

```
@memo def f[…](a, b, c): …
cost @ f(3, 4, 5)
```

get number of FLOPs needed
to evaluate f
(note: no axes, params only!)

reference to Python variable

```
class Action(IntEnum): WAIT = 0; …

@memo def f[…](…):
    return {Action.WAIT}
```

use braces for inline reference
to a global Python variable

```
@memo def f[…](x):
    return {x[0] << 4}
```

You can also refer to model parameters, and write raw Python expressions. You cannot refer to memo constructs, e.g. agents or their choices, in curly braces.

Things to do with a memo

Running a memo

call it like a function with params
(returns an array with prescribed axes)

```
f(a, b)
```

pretty-print table of results

```
f(a, b, print_table=True)
```

```
f(a, b, return_pandas=True)
f(a, b, return_xarray=True)
```

get outputs in other formats

save "comic book" visualization of model via graphviz

```
f(a, b, save_comic="file")
```

@memo def options

cache results (keyed by scalar parameters)

```
@memo(cache=True)
```

trace execution, showing time taken

```
@memo(debug_trace=True)
```

Automatic differentiation (useful for fitting by gradient descent)

```
@memo
def f[…](a, b): …
```

returns tuple of value + gradient wrt params a & b

↓

```
jax.value_and_grad(f)(a, b)
```