Ejercicios tipo parcial

viernes, 7 de junio de 2024 18:45

1. Resolver lo siguientes ejercicios de vectores

a. $Si\ \vec{u} = (-3, 6, 7)$ y $\vec{v} = (1, -2, 4)$

Calcular:

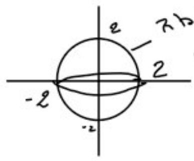
1) $\vec{u} - \vec{v}$ 2) $\vec{u} \bullet \vec{v}$ 3) $\vec{u}\ x\ \vec{v}$

b. Graficar en el espacio los siguientes conjuntos

1) en $\mathbb{R}^3$ $S_1 = \{\vec{u} \in \mathbb{R}^3\ /\ \|\vec{u}\| = 2\}$ ¿Qué Representa?

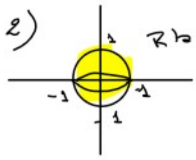
2) en $\mathbb{R}^3$ $S_1 = \{\vec{u} \in \mathbb{R}^3\ /\ \|\vec{u}\| \leq 1\}$ ¿Qué Representa?

b. 1)



Rb: cascara de la esfera

2)



Rb: esfera sólida

a. 1) $\vec{u} - \vec{v} = (\overset{x}{-3-1} \ ; \ \overset{y}{6-(-2)} \ ; \ \overset{z}{7-4})$

$\boxed{\vec{w} = (-4, 8 ; 3)}$

2) $\vec{u} \bullet \vec{v} = (-3, 6, 7) \bullet (1, -2, 4) =$ Nro!!!

$\alpha = (-3.1) + (6.-2) + (7.4) = -3 - 12 + 28$

$\boxed{\alpha = 13}$

3) $\vec{u} \times \vec{v} = \vec{w} = \begin{array}{ccc} x & y & z \\ \hline -3 & 6 & 7 \\ 1 & -2 & 4 \end{array}$

1+2 = (-)

$\vec{w} = \overset{x}{x} \cdot \left[(6.4) - (7.-2)\right] ; -\overset{y}{y} \cdot \left[(-3.4) - (7.1)\right] ; \overset{z}{z} \cdot \left[(-3.-2) - (6.1)\right]$

24 + 14 ; -12 - 7 ; 6 - 6

$\vec{w} = (38 ; +19 ; 0) = 38\check{i} + 19\check{j} + 0\check{k}$

$\boxed{\vec{w} = (\overset{x}{38}, \overset{y}{19}, \overset{z}{0})}$

2. Demuestre que:

$\alpha \in R \quad \wedge \quad \vec{u}\,y\,\vec{v} \in \mathbb{R}^n$ $\mathbb{R}^2$

1) escalar por vector

2) escalar por vector

3) producto escalar

$(\alpha\ .\ \vec{u}) \bullet \vec{v} = \alpha . (\vec{u} \bullet \vec{v})$

3) producto escalar

$\vec{u} = (a, b)$

$\vec{v} = (c, d)$

$[\alpha . (a, b)] . (c, d) = \alpha\ [(a, b) \bullet (c, d)]$

$(\alpha a, \alpha b) . (c, d) = \alpha . (ac + bd)$

Se cumple $\boxed{\alpha a c + \alpha b d = \alpha a c + \alpha b d}$

Compruebo

$a = 1 \quad c = 2$

$b = 2 \quad d = 3$

$\alpha = 2$

$2.1.2 + 2.2.3 = 2.1.2 + 2.2.3$

$4 + 12 = 4 + 12$

$\boxed{16 = 16}$ ✓

3. Resuelva las siguiente inecuación:

a. $|x+3| > 2$

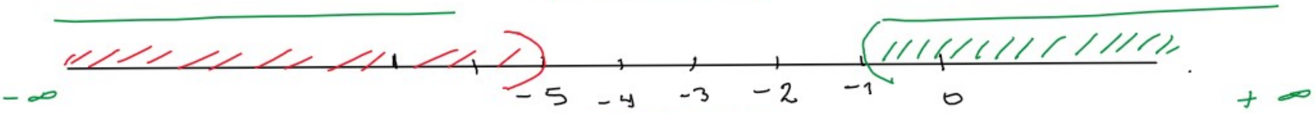
$$-2 > x + 3 > 2$$

* x es menor que -5

$$-2-3 > x > 2-3$$

* x es mayor que -1

$$-5 > x > -1$$



$$S = \{ x \in \mathbb{R} \,/\, -5 > x > -1 \}$$

$$S = \underbrace{(-\infty, -5)}_{\text{conjunto}} \cup \underbrace{(-1, +\infty)}_{\text{conjunto}}$$

Rta

otro caso

~~$S_1 = \emptyset$ conjunto vacío~~

otro caso

~~$S_2 = (-5, -1)$~~ ó ~~$S_3 = [-5, -1)$~~ ó ~~$S_4 [-5, -1]$~~

~~$S_5 = (-5, -1]$~~

$$|2(x+3)| > 5$$

$|2|$ → -2, 2

$$-5 > 2(x+3) > 5$$

$$-5 > 2x + 6 > 5$$

$$-5-6 > 2x > 5-6$$

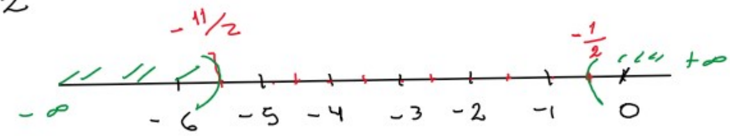
* x es menor que $-\frac{11}{2}$

$$\frac{-5-6}{2} > x > \frac{5-6}{2}$$

* x es mayor que $-\frac{1}{2}$

$$-\frac{11}{2} > x > -\frac{1}{2}$$

$-5{,}5$ $\qquad$ $-0{,}5$



4. Determine el valor de k ∈ R de modo que $\overline{AD} \bullet (\overline{AB} \times \overline{AC}) = 21$

A= (0, 1, 1) ; B= (2, k, 0) ; C= (1, -1, 0) ; D= (3, 0, -1)

$$\overline{AD} = D - A = (3, 0, -1) - (0, 1, 1) = (3\,;\, -1\,;\, -2)$$

$$\overline{AB} = B - A = (2, K, 0) - (0, 1, 1) = (2\,;\, K-1\,;\, -1)$$

$$\overline{AC} = C - A = (1, -1, 0) - (0, 1, 1) = (1\,;\, -2\,,\, -1)$$

$$\begin{vmatrix} 2 & K-1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$$\begin{vmatrix} 2 & K-1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$[(K-1).-1].-2$; $-[-2-(-1)]$; $[-4-(K-1).1]$

$(-K+1)-2$; 1 ; $-4-K+1$

$-K+1-2$; ; $-3-K$

$\boxed{(-K-1\ ;\ 1\ ;\ -3-K)}$ Producto vectorial

$(3\ ;\ -1\ ;\ -2)\cdot[(-K-1)\ ,\ 1\ ;\ (-3-K)] = 21$

$3.(-K-1) + (-1.1) + [(-2).(-3-K)] = 21$

$-3K-3-1+6+2K = 21$

$-3K+2K = 21+3+1-6$

$-K = 19$

① $-1.K = 19$

$K = \dfrac{19}{-1}$

$\boxed{K=-19}$

② $(-1)-K = 19.(-1)$

$\boxed{K=-19}$

5. Factorizar y determinar "TODAS" las raíces reales del polinomio, indicando la multiplicidad de ellas, conociendo que sus divisores son $\{+1\ y -1\}$ respectivamente. raíces del polinomio
"AYUDITA" Utilice la regla de Ruffini y recuerde que para la ecuación cuadrática la ecuación es: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ y se representa $a.(x-x_1).(x-x_2)$

grado del polinomio siempre debe ser 1

$\underbrace{(x-1)}_{Q(x)}\ \underbrace{(x+1)}_{Q(x)}$

P(x)= $8x^4 + 2x^3 - 9x^2 - 2x + 1$

	8	2	-9	-2	1
1		8	10	1	-1
	8	10	1	-1	$\boxed{0}$
-1		-8	-2	1	
	8	2	-1	$\boxed{0}$	

$8x^3 + 10x^2 + 1x - 1$

$$\boxed{P(x) = \underbrace{(8x^2+2x-1)}.(x-1).(x+1)}$$ Rta.

$$\frac{-b \pm \sqrt{b^2 - 4.a.c}}{2.a} = \frac{-2 \pm \sqrt{(2^2) - (4.8.1)}}{2.8}$$

$$= \frac{-2 \pm \sqrt{4 - 32}}{16}$$

raiz de un negativo ↳ valores imaginarios

siempre el grado debe ser menor que P(x)

$P(x) = (2x^4 + x^3 + 1)$ $Q(x) = (\underbrace{x^2 + 1}_{})$ → grado polinomio ≥ 1

$P(x) = 2x^4 + x^3 + 0x^2 + 0x + 1$

$$
\begin{array}{rrrrrr|l}
2x^4 & + x^3 & + 0x^2 & + 0x & + 1 & & x^2 + 1 \\
-2x^4 & & -2x^2 & & & & 2x^2 + x - 2 \\
0 & + x^3 & -2x^2 & + 0x & + 1 & & C(x) \\
& -x^3 & & -x & & & \\
& 0 & -2x^2 & -x & +1 & & \\
& & +2x^2 & & +2 & & \\
& & 0 & \boxed{-x} & \boxed{+3} & R(x) &
\end{array}
$$

$P(x) = C(x) . Q(x) + R(x)$

$P(x) = (2x^2 + x - 2) . (x^2 + 1) + (-x + 3)$ Rta

comprueba

$(2x^2 + x - 2) . (x^2 + 1) - x + 3$

$2x^4 + \cancel{2x^2} + x^3 + \cancel{x} - \cancel{2x^2} - 2 - \cancel{x} + 3$

$2x^4 + x^3 + 1$ ✓

1. Hallar $X \in R^{2x4} / A + X = B$ $X = \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix}$

1. Hallar $X \in R^{2x4}$ / $A + X = B$

$$X = \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 1 & 1 & 0 \\ 2 & 0 & 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & 1 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 1 & 0 \\ 2 & 0 & 1 & 3 \end{bmatrix} + \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix} = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & 1 & -1 & -3 \end{bmatrix}$$

$$\begin{cases} -2 + a = -1 \\ 1 + b = 2 \\ 1 + c = 3 \\ d = -2 \end{cases} \qquad \begin{cases} 2 + e = 0 \\ f = 1 \\ 1 + g = -1 \\ 3 + h = -3 \end{cases}$$

$$\boxed{X = \begin{bmatrix} 1 & 1 & 2 & -2 \\ -2 & 1 & -2 & -6 \end{bmatrix}}$$

Rta

$-2 + a = -1$
$a = -1 + 2$
$a = 1$

$1 + b = 2$
$b = 2 - 1$
$b = 1$

$1 + c = 3$
$c = 3 - 1$
$c = 2$

$2 + e = 0$
$e = -2$

$1 + g = -1$
$g = -1 - 1$
$g = -2$

$3 + h = -3$
$h = -3 - 3$
$h = -6$

2. Demuestre que $A \in R^{nxn}$ $/A \cdot A^{-1} = I_n$

$$A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a - 3c & 2b - 3d \\ -a + 2c & -b + 2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 2a - 3c = 1 \\ -a + 2c = 0 \end{cases}$$

$2c = a$

$c = \frac{a}{2}$

$$\begin{cases} 2b - 3d = 0 \longrightarrow 2b = 3d \\ -b + 2d = 1 \end{cases}$$

$b = \frac{3}{2} d$

$b = \frac{3}{\cancel{2}} \cdot \cancel{2}$

$-b + 2d = 1$

$-\frac{3}{2} d + 2d = 1$

$b = 3$

$c = \frac{a}{2}$

$2a - 3c = 1$

$2a - 3 \cdot \frac{a}{2} = 1$

$\frac{a}{2} = 1$

$a = 1 \cdot (2)$

$a = 2$

$c = \frac{a}{2} = \frac{\cancel{2}}{\cancel{2}}$

$c = 1$

$-\frac{3}{2}d + 2d = 1$

$\frac{d}{2} = 1$

$d = 1 \cdot 2$

$d = 2$

$b = 3$

$$A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Rta.

3. Hallar la Matriz $A \in R^{2x2}$ / $A.P = B.P$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 2 \\ 7 & 2 \end{bmatrix} \qquad P = \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix}$$

Nota: Los 2 valores arbitrarios que deberá considerar es 2.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 2 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix}}_{B.P}$$

$$\begin{bmatrix} 2a+2b & -a-b \\ 2c+2d & -c-d \end{bmatrix} = \begin{bmatrix} 4+4 & -2-2 \\ 14+4 & -7-2 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ 18 & -9 \end{bmatrix}$$

$\begin{cases} 2a + 2b = 8 \,* \\ -a - b = -4 \end{cases}$ $a = 2$

$-a - b = -4$

$-a + 4 = b$

$-2 + 4 = b$

$b = 2$

$2a + 2b = 8$

$2 \cdot 2 + 2 \cdot 2 = 8$

$8 = 8$ ✓

$\begin{cases} 2c + 2d = 18 \,* \\ -c - d = -9 \end{cases}$ $c = 2$

$-c - d = -9$

$-c + 9 = d$

$-2 + 9 = d$

$d = 7$

$2c + 2d = 18$

$2 \cdot 2 + 2 \cdot 7 = 18$

$4 + 14 = 18$

$18 = 18$ ✓

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 7 \end{bmatrix}$$ Rta

$$A = \begin{vmatrix} 2 & 2 \\ 2 & 7 \end{vmatrix}$$ Rta

4. Resuelva el siguiente Sistema de Ecuaciones por la regla de Cramer

$$\begin{cases} x - y + 3z = 11 \\ 4x + y - z = 4 \\ 2x - y + 3z = 10 \end{cases}$$

$$\overset{A}{\begin{pmatrix} 1 & -1 & 3 \\ 4 & 1 & -1 \\ 2 & -1 & 3 \end{pmatrix}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \overset{B}{\begin{pmatrix} 11 \\ 4 \\ 10 \end{pmatrix}}$$

5. Resuelva el ejercicio anterior por el Método de Gauss y compare resultados. Explique.

$$\det(A) = \begin{pmatrix} 1 & -1 & 3 \\ 4 & 1 & -1 \\ 2 & -1 & 3 \end{pmatrix} = \overset{1+1=(+)}{1.} \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} \overset{1+2=(-)}{-} (-1). \begin{vmatrix} 4 & -1 \\ 2 & 3 \end{vmatrix} \overset{1+3=(+)}{+} 3. \begin{vmatrix} 4 & 1 \\ 2 & -1 \end{vmatrix}$$

$$\det(A) = (3-1) + (12-(-2)) + 3.(-4-2)$$

$$= 2 + 14 - 18 =$$

$$\det(A) = -2$$

$$\begin{pmatrix} 1 & -1 & 3 \\ 4 & 1 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

Valor x

$$x = \frac{\det(x)}{\det(A)}$$

$$\det(x) = \begin{pmatrix} 11 & -1 & 3 \\ 4 & 1 & -1 \\ 10 & -1 & 3 \end{pmatrix}$$

$$\det(x) = -4^{2+1=(-)} \begin{vmatrix} -1 & 3 \\ -1 & 3 \end{vmatrix} + 1 \begin{vmatrix} 11 & 3 \\ 10 & 3 \end{vmatrix} + 1 \begin{vmatrix} 11 & -1 \\ 10 & -1 \end{vmatrix}$$

0

$$\det(x) = \cancel{-4.(-3-(-3))} + (33-30) + (-11-(-10))$$

$$-3+3=0$$

$$\det(x) = 3 - 1 = \boxed{2}$$

$$x = \frac{2}{-2}$$

$$x = -1$$

Valor y

$$\det(y) = \begin{pmatrix} 1 & 11 & 3 \\ 4 & 4 & -1 \\ 2 & 10 & 3 \end{pmatrix} = -4 \begin{vmatrix} 11 & 3 \\ 10 & 3 \end{vmatrix} + 4 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 11 \\ 2 & 10 \end{vmatrix}$$

$$-\det(y) = -4 \cdot (\overbrace{33-30}^{3}) + 4 \cdot (\overbrace{3-6}^{-3}) + (\overbrace{10-22}^{-12})$$

$$\det(y) = -12 - 12 - 12 = -36$$

$$y = \frac{-36}{-2} \qquad y = 18$$

Valor Z

$$\det Z = \begin{pmatrix} 1 & -1 & 11 \\ 4 & 1 & 4 \\ 2 & -1 & 10 \end{pmatrix} = -4 \cdot \begin{vmatrix} -1 & 11 \\ -1 & 10 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 11 \\ 2 & 10 \end{vmatrix} - 4 \cdot \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix}$$

$$\det(Z) = -4 \cdot (\overbrace{-10+11}^{1}) + (10-22) - 4 \cdot (-1+2)$$

$$\det(Z) = -4 - 12 - 4 = -20$$

$$Z = \frac{-20}{-2} = 10$$

$$P_1 = (-1, 18, 10) \quad \text{Rta.}$$

Gauss

$$A \quad \left(\begin{array}{ccc|c} 1 & -1 & 3 & 11 \\ 4 & 1 & -1 & 4 \\ 2 & -1 & 3 & 10 \end{array}\right) \xrightarrow[-2F_1+F_3]{-4F_1+F_2} \left(\begin{array}{ccc|c} 1 & -1 & 3 & 11 \\ 0 & 5 & -13 & -40 \\ 0 & 1 & -3 & -12 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 11 \\ 0 & 5 & -13 & -40 \\ 0 & 1 & -3 & -12 \end{array}\right) \xrightarrow[-5f_3+f_2]{} \left(\begin{array}{ccc|c} 1 & -1 & 3 & 11 \\ 0 & 5 & -13 & -40 \\ 0 & 0 & 2 & 20 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 0 & 5 & -13 & -40 \\ 0 & 1 & -3 & -12 \end{array}\right) \xrightarrow{} \left(\begin{array}{ccc|c} 0 & 5 & -13 & -40 \\ 0 & 0 & 2 & 20 \end{array}\right)$$

$-5f_3 + f_2$

$$\begin{cases} x - y + 3z = 11 \quad * \\ 5y - 13z = -40 \quad * \\ 2z = 20 \end{cases}$$

$z = \frac{20}{2} = 10$

* $5y - 13.10 = -40$

$5y = -40 + 130$

$y = \frac{90}{5}$

$y = 18$

* $x = 11 + y - 3.z$

$x = 11 + 18 - (3.10)$

$x = -1$

$P_2 = (-1, 18, 10)$

$P_1 = P_2$

¿que pasa?

$P_1 \neq P_2$

No solo se conectan en 1 punto sino que tienen más puntos de conexión.