$$e = \|g\|^2 - \|a\|^2; \quad \text{where } a = (I - S_a) K_a' (A - b_a)$$

$$S_a = \begin{pmatrix} 0 & 0 & 0 \\ S_{ayx} & 0 & 0 \\ S_{azx} & S_{azy} & 0 \end{pmatrix} \quad K_a' = \begin{pmatrix} K'_{ax} & 0 & 0 \\ 0 & K'_{ay} & 0 \\ 0 & 0 & K'_{az} \end{pmatrix} \quad b_a = \begin{pmatrix} b_{ax} \\ b_{ay} \\ b_{az} \end{pmatrix}$$

$$\text{so } a = \begin{pmatrix} 1 & 0 & 0 \\ -S_{ayx} & 1 & 0 \\ -S_{azx} & -S_{azy} & 1 \end{pmatrix} \begin{pmatrix} K'_{ax} & 0 & 0 \\ 0 & K'_{ay} & 0 \\ 0 & 0 & K'_{az} \end{pmatrix} \begin{pmatrix} A_x - b_{ax} \\ A_y - b_{ay} \\ A_z - b_{az} \end{pmatrix}$$

$$= \begin{pmatrix} K'_{ax} & 0 & 0 \\ -S_{ayx} K'_{ax} & K'_{ay} & 0 \\ -S_{azx} K'_{ax} & -S_{azy} K'_{ay} & K'_{az} \end{pmatrix} \begin{pmatrix} A_x - b_{ax} \\ A_y - b_{ay} \\ A_z - b_{az} \end{pmatrix}$$

$$= \begin{pmatrix} K'_{ax}(A_x - b_{ax}) \\ -S_{ayx} K'_{ax}(A_x - b_{ax}) + K'_{ay}(A_y - b_{ay}) \\ -S_{azx} K'_{ax}(A_x - b_{ax}) - S_{azy} K'_{ay}(A_y - b_{ay}) + K'_{az}(A_z - b_{az}) \end{pmatrix}$$

Let $\theta = (S_{ayx}, S_{azx}, S_{azy}, K'_{ax}, K'_{ay}, K'_{az}, b_{gx}, b_{gy}, b_{gz})$

so $\dfrac{\partial e}{\partial \theta} = \dfrac{\partial e}{\partial a} \dfrac{\partial a}{\partial \theta} \qquad \dfrac{\partial e}{\partial a} = -2a^T$

$$\frac{\partial a}{\partial \theta_{1:3}} = \begin{pmatrix} 0 & 0 & 0 \\ -K'_{ax}(A_x - b_{ax}) & 0 & 0 \\ 0 & -K'_{ax}(A_x - b_{ax}) & -K'_{ay}(A_y - b_{ay}) \end{pmatrix}$$

$$\frac{\partial a}{\partial \theta_{4:6}} = \begin{pmatrix} A_x - b_{ax} & 0 & 0 \\ -S_{ayx}(A_x - b_{ax}) & A_y - b_{ay} & 0 \\ -S_{azx}(A_x - b_{ax}) & -S_{azy}(A_y - b_{ay}) & A_z - b_{az} \end{pmatrix}$$

$$\frac{\partial a}{\partial \theta_{7:9}} = \begin{pmatrix} -K'_{ax} & 0 & 0 \\ S_{ayx} K'_{ax} & -K'_{ay} & 0 \\ S_{azx} K'_{ax} & S_{azy} K'_{ay} & -K'_{az} \end{pmatrix}$$