- Jacobian:

Orientation:

$$\frac{\partial r_q}{\partial \delta\theta_{b_ib_i'}} = \frac{\partial 2\left[q_{b_ib_j}^* \otimes \left(q_{wb_i} \otimes \begin{pmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_ib_i'} \end{pmatrix}\right)^* \otimes q_{wb_j}\right]_{xyz}}{\partial \delta\theta_{b_ib_i'}}$$

$$= \frac{\partial -2\left[\left(q_{b_ib_j}^* \otimes \left(q_{wb_i} \otimes \begin{pmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_ib_i'} \end{pmatrix}\right)^* \otimes q_{wb_j}\right)^*\right]_{xyz}}{\partial \delta\theta_{b_ib_i'}}$$

$$= \frac{\partial -2\left[q_{wb_j}^* \otimes q_{wb_i} \otimes \begin{pmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_ib_i'} \end{pmatrix} \otimes q_{b_ib_j}\right]_{xyz}}{\partial \delta\theta_{b_ib_i'}}$$

$$= -2\begin{pmatrix} 0 & I \end{pmatrix} \frac{\partial [q_{wb_j}^* \otimes q_{wb_i}]_L [q_{b_ib_j}]_R \begin{pmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_ib_i'} \end{pmatrix}}{\partial \delta\theta_{b_ib_i'}}$$

$$= -2\begin{pmatrix} 0 & I \end{pmatrix} [q_{wb_j}^* \otimes q_{wb_i}]_L [q_{b_ib_j}]_R \begin{pmatrix} 0 \\ \frac{1}{2}I \end{pmatrix}$$

$$\frac{\partial r_q}{\partial \delta\theta_{b_jb_j'}} = \frac{\partial 2\left[q_{b_ib_j}^* \otimes q_{wb_i}^* \otimes q_{wb_j} \otimes \begin{pmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_jb_j'} \end{pmatrix}\right]_{xyz}}{\partial \delta\theta_{b_jb_j'}}$$

$$= 2\begin{pmatrix} 0 & I \end{pmatrix} [q_{b_ib_j}^* \otimes q_{wb_i}^* \otimes q_{wb_j}]_L \begin{pmatrix} 0 \\ \frac{1}{2}I \end{pmatrix}$$

$$\frac{\partial r_q}{\partial \delta b_{g_i}} = \frac{\partial 2\left[\left(q_{b_ib_j} \otimes \begin{pmatrix} 1 \\ \frac{1}{2}J_{q_{bg_i}}\delta b_{g_i} \end{pmatrix}\right)^* \otimes q_{wb_i}^* \otimes q_{wb_j}\right]_{xyz}}{\partial \delta b_{g_i}}$$

$$= \frac{\partial -2\left[q_{wb_j}^* \otimes q_{wb_i} \otimes q_{b_ib_j} \otimes \begin{pmatrix} 1 \\ \frac{1}{2}J_{q_{bg_i}}\delta b_{g_i} \end{pmatrix}\right]_{xyz}}{\partial \delta b_{g_i}}$$

$$= -2\begin{pmatrix} 0 & I \end{pmatrix} [q_{wb_j}^* \otimes q_{wb_i} \otimes q_{b_ib_j}]_L \begin{pmatrix} 0 \\ \frac{1}{2}J_{q_{bg_i}} \end{pmatrix}$$

$$\frac{\partial r_q}{\partial \delta b_{g_j}} = 0$$

$$\frac{\partial r_q}{\partial \delta b_{a_i}} = -2\begin{pmatrix} 0 & I \end{pmatrix} [q_{wb_j}^* \otimes q_{wb_i} \otimes q_{b_ib_j}]_L \begin{pmatrix} 0 \\ \frac{1}{2}J_{q_{ba_i}} \end{pmatrix}$$

$$\frac{\partial r_q}{\partial \delta b_{a_j}} = 0$$

$$\frac{\partial r_q}{\partial \delta p_{wb_i}} = \frac{\partial r_q}{\delta p_{wb_j}} = \frac{\partial r_q}{\partial \delta V_{w_i}} = \frac{\partial r_q}{\partial V_{wj}} = 0$$

Velocity:

$$\frac{\partial r_v}{\partial \delta\theta_{b_ib_i'}} = \frac{\partial (q_{wb_i} \otimes (\frac{1}{2}\delta\theta_{b_ib_i'}))^* (v_{wj} - v_{w_i} + g_w \Delta t)}{\partial \delta\theta_{b_ib_i'}}$$

$$= \frac{\partial (R_{wb_i} \exp([\delta\theta_{b_ib_i'}]_\times))^{-1} (v_{wj} - v_{w_i} + g_w \Delta t)}{\partial \delta\theta_{b_ib_i'}}$$

$$= \frac{\partial (I - [\delta\theta_{b_ib_i'}]_\times) R_{b_iw} (v_{wj} - v_{w_i} + g_w \Delta t)}{\partial \delta\theta_{b_ib_i'}}$$

$$= [R_{b_iw} (v_{wj} - v_{w_i} + g_w \Delta t)]_\times$$

$$\frac{\partial r_v}{\partial \delta\theta_{b_jb_j'}} = 0$$

$$\frac{\partial r_v}{\partial \delta V_{w_i}} = \frac{\partial q^*_{wb_i} (v_{wj} - v_{w_i} - \delta V_{w_i} + g_w \Delta t) - \beta_{b_ib_j}}{\partial \delta V_{w_i}}$$

$$= -R_{b_iw}$$

$$\frac{\partial r_v}{\partial \delta V_{wj}} = R_{b_iw}$$

$$\frac{\partial r_v}{\partial \delta b_{a_i}} = -J_{\beta b_{a_i}}$$

$$\frac{\partial r_v}{\partial \delta b_{a_j}} = 0$$

$$\frac{\partial r_v}{\partial \delta b_{g_i}} = -J_{\beta b_{g_i}}$$

$$\frac{\partial r_v}{\partial \delta b_{g_j}} = 0$$

$$\frac{\partial r_v}{\partial \delta p_{wb_i}} = \frac{\partial r_v}{\partial \delta p_{wb_j}} = 0$$

Position:

$$\frac{\partial r_p}{\partial \delta p_{wb_i}} = \frac{\partial q^*_{wb_i}(p_{wb_j} - p_{wb_i} - \delta p_{wb_i} - v_{w_i}\Delta t + \frac{1}{2} g_w \Delta t^2) - \alpha_{b_i b_j}}{\partial \delta p_{wb_i}}$$

$$= -R_{b_i w}$$

$$\frac{\partial r_p}{\partial \delta p_{wb_j}} = R_{b_i w}$$

$$\frac{\partial r_p}{\partial \delta \theta_{b_i b_i'}} = \frac{\partial (R_{wb_i} \exp([\delta\theta_{b_i b_i'}]_\times))^{-1} (p_{wb_j} - p_{wb_i} - v_{w_i}\Delta t + \frac{1}{2} g_w \Delta t^2)}{\partial \delta\theta_{b_i b_i'}}$$

$$= \frac{\partial - [\delta\theta_{b_i b_i'}]_\times R_{b_i w} (p_{wb_j} - p_{wb_i} - v_w \cdot \Delta t + \frac{1}{2} g_w \Delta t^2)}{\partial \delta\theta_{b_i b_i'}}$$

$$= [R_{b_i w}(p_{wb_j} - p_{wb_i} - \delta p_{wb_i} - v_w \cdot \Delta t + \frac{1}{2} g_w \Delta t^2)]_\times$$

$$\frac{\partial r_p}{\partial \delta \theta_{b_j b_j'}} = 0$$

$$\frac{\partial r_p}{\partial \delta v_{w_i}} = \frac{\partial q^*_{wb_i}(p_{wb_j} - p_{wb_i} - \delta p_{wb_i} - (v_{w_i} + \delta v_{w_i})\Delta t + \frac{1}{2} g_w \Delta t^2)}{\partial \delta v_{w_i}}$$

$$= -R_{b_i w} \Delta t$$

$$\frac{\partial r_p}{\partial \delta v_{w_j}} = 0$$

$$\frac{\partial r_p}{\partial \delta b_{a_i}} = -J_{\alpha b_{a_i}}$$

$$\frac{\partial r_p}{\partial \delta b_{a_j}} = 0$$

$$\frac{\partial r_p}{\partial \delta b_{g_i}} = -J_{\alpha b_{g_i}}$$

$$\frac{\partial r_p}{\partial \delta b_{g_j}} = 0$$

Acc. Bias:

$$\frac{\partial r_{ba}}{\partial \delta p_{wb_i}} = \frac{\partial r_{ba}}{\partial \delta p_{wb_j}} = \frac{\partial r_{ba}}{\partial \delta \theta_{b_i b_i'}} = \frac{\partial r_{ba}}{\partial \theta_{b_j b_j'}} = \frac{\partial r_{ba}}{\partial \delta v_{w_i}} = \frac{\partial r_{ba}}{\partial \delta v_{w_j}} = \frac{\partial r_{ba}}{\partial \delta b_{g_i}} = \frac{\partial r_{b}}{\partial \delta b_{g_j}} = 0$$

$$\frac{\partial r_{ba}}{\partial \delta b_{a_i}} = \frac{\partial b_{a_j} - b_{a_i} - \delta b_{a_i}}{\partial \delta b_{a_i}} = -I$$

$$\frac{\partial r_{ba}}{\partial \delta b_{a_j}} = I$$

Gyro. Bias:

$$\frac{\partial r_{bg}}{\partial \delta p_{wb_i}} = \frac{\partial r_{bg}}{\partial \delta p_{wb_j}} = \frac{\partial r_{bg}}{\partial \delta \theta_{b_i b_i'}} = \frac{\partial r_{bg}}{\partial \theta_{b_j b_j'}} = \frac{\partial r_{bg}}{\partial \delta v_{w_i}} = \frac{\partial r_{bg}}{\partial \delta v_{w_j}} = \frac{\partial r_{bg}}{\partial \delta b_{a_i}} = \frac{\partial r_{bg}}{\partial \delta b_{a_j}} = 0$$

$$\frac{\partial r_{bg}}{\partial \delta b_{g_i}} = \frac{\partial b_{g_j} - b_{g_i} - \delta b_{g_i}}{\partial \delta b_{g_i}} = -I$$

$$\frac{\partial r_{bg}}{\partial \delta b_{g_j}} = I$$