- Residual

$$\begin{pmatrix} r_p \\ r_q \\ r_{bg} \end{pmatrix} = \begin{pmatrix} q^*_{wb_i}(p_{wb_j} - p_{wb_i}) - \alpha_{b_ib_j} \\ 2[q^*_{b_ib_j} \otimes q^*_{wb_i} q_{wb_j}]_{xyz} \\ bg_j - bg_i \end{pmatrix}$$

so we need to optimize:

p_{wb_i} q_{wb_i} bg_i

p_{wb_j} q_{wb_j} bg_j

- Jacobian

Position:

$$\frac{\partial r_p}{\partial \delta p_{wi}} = \frac{\partial q^*_{wb_i}(p_{wb_j} - p_{wb_i} - \delta p_{wb_i}) - \alpha_{b_ib_j}}{\partial \delta p_{wi}}$$

$$= -R_{b_iw}$$

$$\frac{\partial r_p}{\partial \delta p_{wj}} = R_{b_iw}$$

$$\frac{\partial r_p}{\partial \delta\theta_{b_ib_i'}} = \frac{\partial (R_{wb_i} R_{b_ib_i'})^{-1}(p_{wb_j} - p_{wb_i})}{\partial \delta\theta_{b_ib_i'}}$$

$$= \frac{\partial (I - [\delta\theta_{b_ib_i'}]_\times) R_{b_iw}(p_{wb_j} - p_{wb_i})}{\partial \delta\theta_{b_ib_i'}}$$

$$= \frac{\partial -[\delta\theta_{b_ib_i'}]_\times R_{b_iw}(p_{wb_j} - p_{wb_i})}{\partial \delta\theta_{b_ib_i'}}$$

$$= [R_{b_iw}(p_{wb_j} - p_{wb_i})]_\times$$

$$\frac{\partial r_p}{\partial \delta\theta_{b_jb_j'}} = 0$$

$$\frac{\partial r_p}{\partial \delta b_{g_i}} = -J^{\alpha}_{b_{g_i}}$$

$$\frac{\partial r_p}{\partial \delta b_{g_j}} = 0$$

Orientation:

$$\frac{\partial r_q}{\partial \delta p_{wi}} = \frac{\partial r_q}{\partial \delta p_{wj}} = 0$$

$$\frac{\partial r_q}{\partial \delta \theta_{b_ib_i'}} = \frac{\partial 2\left[q^*_{b_ib_j} \otimes \left(q_{wb_i} \otimes \begin{pmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_ib_i'} \end{pmatrix}\right)^* \otimes q_{wb_j}\right]_{xyz}}{\partial \delta\theta_{b_ib_i'}}$$

$$= -2\begin{pmatrix} 0 & I \end{pmatrix} \left[q^*_{wb_j} \otimes q_{wb_i}\right]_L \left[q_{b_ib_j}\right]_R \begin{pmatrix} 0 \\ \frac{1}{2} I \end{pmatrix}$$

$$\frac{\partial r_q}{\partial \delta \theta_{b_jb_j'}} = 2\begin{pmatrix} 0 & I \end{pmatrix} \left[q^*_{b_ib_j} \otimes q^*_{wb_i} \otimes q_{wb_j}\right]_L \begin{pmatrix} 0 \\ \frac{1}{2} I \end{pmatrix}$$

$$\frac{\partial r_q}{\partial \delta b_{g_i}} = \frac{\partial 2\left[\left(q_{b_ib_j} \otimes \begin{pmatrix} 1 \\ \frac{1}{2} J^q_{bg_i}\delta b_{g_i} \end{pmatrix}\right)^* \otimes q^*_{wb_i} \otimes q_{wb_j}\right]_{xyz}}{\partial \delta b_{g_i}}$$

$$= \frac{\partial -2\left[q^*_{wb_j} \otimes q_{wb_i} \otimes q_{b_ib_j} \otimes \begin{pmatrix} 1 \\ \frac{1}{2} J^q_{bg_i}\delta b_{g_i} \end{pmatrix}\right]_{xyz}}{\partial \delta b_{g_i}}$$

$$= -2\begin{pmatrix} 0 & I \end{pmatrix} \left[q^*_{wb_j} \otimes q_{wb_i} \otimes q_{b_ib_j}\right]_L \begin{pmatrix} 0 \\ \frac{1}{2} J^q_{bg_i} \end{pmatrix}$$

$$\frac{\partial r_q}{\partial \delta b_{g_j}} = 0$$

Gyro. Bias:

$$\frac{\partial r_{gb}}{\partial \delta p_{wb_i}} = \frac{\partial r_{gb}}{\partial \delta p_{wb_j}} = \frac{\partial r_{gb}}{\partial \delta \theta_{b_ib_i'}} = \frac{\partial r_{gb}}{\partial \delta \theta_{b_jb_j'}} = 0$$

$$\frac{\partial r_{gb}}{\partial \delta b_{g_i}} = \frac{\partial\, b_{g_j} - b_{g_i} - \delta b_{g_i}}{\partial \delta b_{g_i}} = -I$$

$$\frac{\partial r_{gb}}{\partial \delta b_{g_j}} = \frac{\partial\, b_{g_j} + \delta b_{g_j} - b_{g_i}}{\partial \delta b_{g_j}} = I$$

- Covariance

$$P_{k+1} = F_k P_k F_k^T + B_k Q B_k^T$$

Cont. time: $\dot{x} = F_t x + B_t w$

Disc. time $x_{k+1} = F_k x_k + B_k w_k$

where $x_k = \begin{pmatrix} \delta\alpha_k \\ \delta\theta_k \\ \delta b_{g_k} \end{pmatrix}$ $x_{k+1} = \begin{pmatrix} \delta\alpha_k \\ \delta\theta_k \\ \delta b_{g_k} \end{pmatrix}$ $w_k = \begin{pmatrix} n_{\phi_k} \\ n_{w_k} \\ n_{w_{k+1}} \\ n_{b_{g_k}} \end{pmatrix}$

① $\delta\dot{\theta} = -[w_t - b_{g_t}]_\times \delta\theta + n_w - \delta b_{g_t}$

$$\delta\dot{\theta}_k = -\left[\frac{w_k + w_{k+1}}{2} - b_{g_t}\right]_\times \delta\theta_k + \frac{n_{w_k} + n_{w_{k+1}}}{2} - \delta b_{g_k}$$

$$\delta\theta_{k+1} = \left(I - \left[\frac{w_k + w_{k+1}}{2} - b_{g_k}\right]_\times \Delta t\right)\delta\theta_k + \frac{n_{w_k} + n_{w_{k+1}}}{2}\Delta t - \delta b_{g_k}\Delta t$$

② $\alpha_{b_ib_{k+1}} = \alpha_{b_ib_k} + q_{b_ib_k}\phi_k$ $\quad\tilde{\alpha}_{b_ib_{k+1}} = \tilde{\alpha}_{b_ib_k} + \tilde{q}_{b_ib_k}\tilde{\phi}_k$

Since $\tilde{\alpha}_{b_ib_{k+1}} = \alpha_{b_ib_{k+1}} + \delta\alpha_{b_ib_{k+1}}$

$$\tilde{\alpha}_{b_ib_k} = \alpha_{b_ib_k} + \delta\alpha_{b_ib_k}$$

$$\tilde{R}_{b_ib_k} = R_{b_ib_k}\exp([\delta\theta_k]_\times) = R_{b_ib_k}(I + [\delta\theta]_\times)$$

$$\tilde{\phi}_k = \phi_k + n_{\phi_k}$$

$$\alpha_{b_ib_{k+1}} + \delta\alpha_{b_ib_{k+1}} = \alpha_{b_ib_k} + \delta\alpha_{b_ib_k} + R_{b_ib_k}(I + [\delta\theta_k]_\times)(\phi_k + n_{\phi_k})$$

$$\alpha_{b_ib_k} + R_{b_ib_k}\phi_k + \delta\alpha_{b_ib_{k+1}} = \alpha_{b_ib_k} + \delta\alpha_{b_ib_k} + R_{b_ib_k}(I + [\delta\theta_k]_\times)(\phi_k + n_{\phi_k})$$

$$\delta\alpha_{b_ib_{k+1}} = \delta\alpha_{b_ib_k} + R_{b_ib_k}(I + [\delta\theta_k]_\times)(\phi_k + n_{\phi_k}) - R_{b_ib_k}\phi_k$$

$$= \delta\alpha_{b_ib_k} + R_{b_ib_k}[\delta\theta_k]_\times(\phi_k + n_{\phi_k})$$

$$= \delta\alpha_{b_ib_k} - R_{b_ib_k}[\phi_k]_\times\delta\theta_k + R_{b_ib_k}n_{\phi_k}$$

③ $\delta \dot{b}_g = n_{b_g}$

$\delta \dot{b}_{g_k} = n_{b_g}$

$\delta b_{g_{k+1}} = \delta b_{g_k} + n_{b_g} \Delta t$

$$F_k = \begin{pmatrix} I & -R_k[\phi_k]_\times & 0 \\ 0 & I - [\frac{\omega_k + \omega_{k+1}}{2} - b_{g_k}]_\times \Delta t & -I\Delta t \\ 0 & 0 & I \end{pmatrix}$$

$$G_k = \begin{pmatrix} R_k & 0 & 0 & 0 \\ 0 & \frac{1}{2}I\Delta t & \frac{1}{2}I\Delta t & 0 \\ 0 & 0 & 0 & I\Delta t \end{pmatrix}$$

- Bias Update

$\alpha_{b_i b_j} = \bar{\alpha}_{b_i b_j} + J_{\alpha b_{g_i}} \delta b_{g_i}$

$q_{b_i b_j} = \bar{q}_{b_i b_j} \otimes \left(\frac{1}{2} J_{q b_{g_i}} \delta b_{g_i} \right)$

We compute J as $J_{k+1} = F_k J_k$