Fault Detection And Diagnosis - Midterm Exam Simulation

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A Homework Solution
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Abstract

This report contains the solution to the question 4 simulation of midterm exam for the corresponding *Fault Detection And Diagnosis* course with professor Mahdi Siahi. The question is also provided for convenience.

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Chapter 1

Problem Formulation And Analytic Solution

In this chapter, a problem is defined and then a corresponding observer is designed analytically for the system in that problem.

Consider the problem below :

The the state space equation of a nonlinear system is given below:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\sin(x_1) \\ \mathrm{y} &= \mathrm{x1} + \mathrm{x2} \end{aligned} \tag{1.1}$$

Design a nonlinear observer to have a valid estimation of the system states. And also discuss the advantages and disadvantages of your suggested observer.

1.1 Analytic solution

In order to solve this problem, a *Thau observer (Lipschitz observer)* is proposed which has the general form of equation 1.3 for a general nonlinear system of equation 1.2.

$$
\begin{aligned}
\dot{x} &= Ax + f(x) \\
y &= Cx
\end{aligned}
\tag{1.2}
$$

$$
\begin{aligned}
\dot{\hat{x}} &= A\hat{x} + f(\hat{x}) + K(y - \hat{y}) \\
\hat{y} &= C\hat{x}
\end{aligned}
\tag{1.3}
$$

The conditions for the observer are that A, C and $f(.)$ should be known and K is derived such that $A_0 = A - KC$ is stable. For this particular problem in equation 1.1, A, f and C are represented by the following matrices.

$$
A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{1.4}
$$

$$
f(x) = \begin{bmatrix} 0 \\ -\sin(x_1) \end{bmatrix} \tag{1.5}
$$

$$
C = \begin{bmatrix} 1 & 1 \end{bmatrix} \tag{1.6}
$$

In the given observer, the observer error is estimated by equation 1.7 since the state error is $e = e_x = \hat{x} - x$ and the output error is $e_y = \hat{y} - y = Ce_x$.

$$
e_o = K(y - \hat{y}) = -Ke_y = -KCe \tag{1.7}
$$

Intuitively, $f(\hat{x})$ can be found from equation 1.8. In order to find K, A_0 is formed in equation 1.9 with $K = [k_1 \quad k_2]^T$.Then, we need A_0 to be stable (the eigenvalues of A_0 should be negative), therefore, a simple substitution of $k_1 = 1, k_2 = 1$ gives the equation 1.10 which is stable with two eigenvalues at -1. As noted previously, the designed K matrix is in the form of equation 1.11.

$$f(\hat{x}) = \begin{bmatrix} 0 \\ -\sin(\hat{x_1}) \end{bmatrix} \tag{1.8}$$

$$A_0 = A - KC = \begin{bmatrix} -k_1 & 1 - k_1 \\ -k_2 & -k_2 \end{bmatrix} \tag{1.9}$$

$$A_0 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} \tag{1.10}$$

$$K = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{1.11}$$

Subsequently, with the previously designed matrices in equations 1.4, 1.8, 1.11 and 1.6, we can achieve our desired observer in equation 1.3 and next we just need to prove that the observer error converges to zero. For that, the state error derivative is calculated from equations 1.2 and 1.3 and simplified in terms of our parameters which yields equation 1.12. Then, a Lyapunov function like equation 1.13 is utilized to prove the stability of the observer system with a P.D. value for P. By taking the derivative of 1.13, equation 1.14 is derived. You can see that because A_0 is stable, the Lyapunov relation ${A_0}^T P + P A_0 = -2Q$ with a P.D. Q is valid and the term associated with it becomes negative definite. Note that the reason behind substituting with $-2Q$ and not $-Q$ is simply for later factorization.

$$\dot{e} = \dot{\hat{x}} - \dot{x} = (A - KC)e + f(\hat{x}) - f(x) = A_0 + f(\hat{x}) - f(x) \tag{1.12}$$

$$V = e^T P e \tag{1.13}$$

$$\dot{V} = \dot{e}^T Pe + e^T P\dot{e} = e^T({A_0}^T P + PA_0)e + 2e^T P[f(\hat{x}) - f(x)] = -2e^T Qe + 2e^T P[f(\hat{x}) - f(x)] \tag{1.14}$$

By applying the Lipschitz condition we simplify equation 1.14 even further to find a negative upper bound and the resultant inequalities 1.15 are acquired.

$$\begin{aligned} \dot{V} &\leq -2e^T Qe + 2L\|e\|\|P\|\|e\| \\ &\leq -2\lambda_{\min}(Q)\|e\|^2 + 2L\|e\|\|P\|\|e\| \\ &\leq -2\left[\lambda_{\min}(Q) - L\|P\|\right]\|e\|^2 \end{aligned} \tag{1.15}$$

Only if $\lambda_{\min}(Q) > L\|P\|$ then we can conclude that $\dot{V} < 0$ and $e \longrightarrow 0$ as $t \longrightarrow \infty$ and therefore, the stability of our suggested observer is proved.

In the next chapter this designed observer is simulated with its corresponding system to show that the observer error convergence to zero holds.

Chapter 2

Simulation

In this chapter, the system and the designed observer in the previous chapter are simulated to see if the design is valid.

2.1 MATLAB Code

The MATLAB code used to initiate the desired Simulink model is given in the following. As shown, the code is very simple and only includes the declaration of 4 design parameters obtained in previous chapter.

```
A = [0  1;0  0];
C = [1  1];
K = [1;1];
x1filter = [1  0];
```

As reflected in the code, K is the observer error controller, A and C are system parameters and x1filter is only a mask to filter out X_1 out of the X state vector which is used for both the system and the observer.

2.2 Simulink Simulation

In order to check the validity of our design, a Simulink model illustrated in figure 2.1 is created.

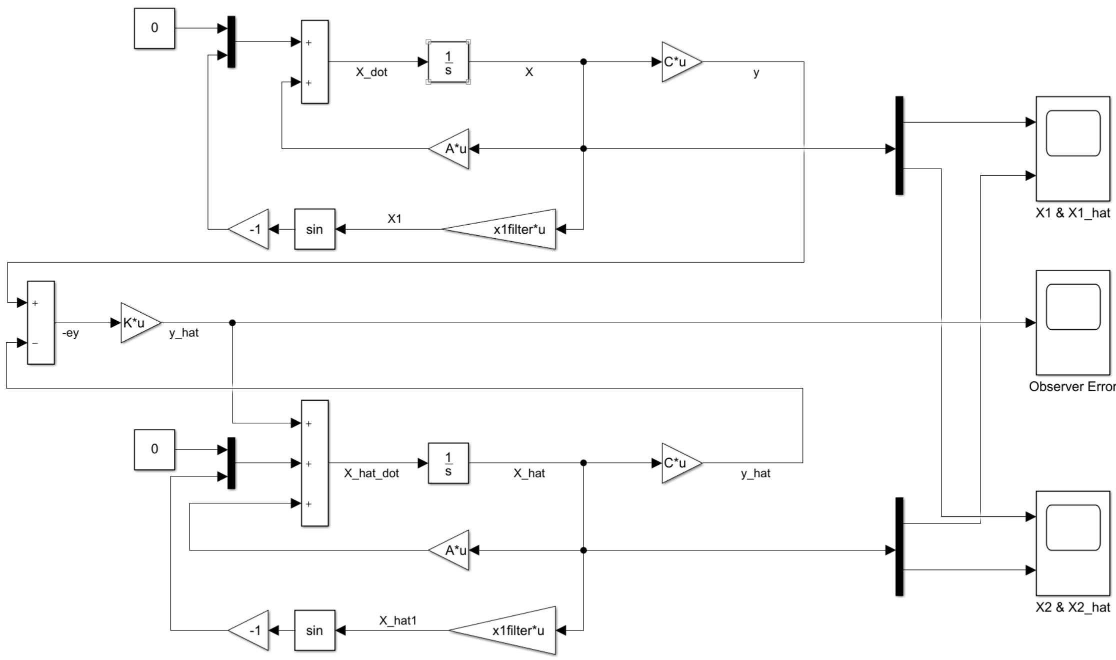

Figure 2.1: Simulink model for the observer system design

In this model, the initial conditions for the system integrator and observer integrator are respectively $X_0 = [1 \quad 2]$ and $\hat{X}_0 = [-1 \quad 3]$. The reason for this selection is that first, we need to see the dynamics of the system and second, with these initial conditions the original nonlinear system becomes a little unstable and therefore we can see the state tracking quality of the observer in a better view. After running the Simulink model, the state tracking diagrams for both the system and the observer are shown in figures 2.2 and 2.3 and the observer output error is shown in figure 2.4.

As expected, the system states are tracked reliably and the observer output error goes to zero as time passes.

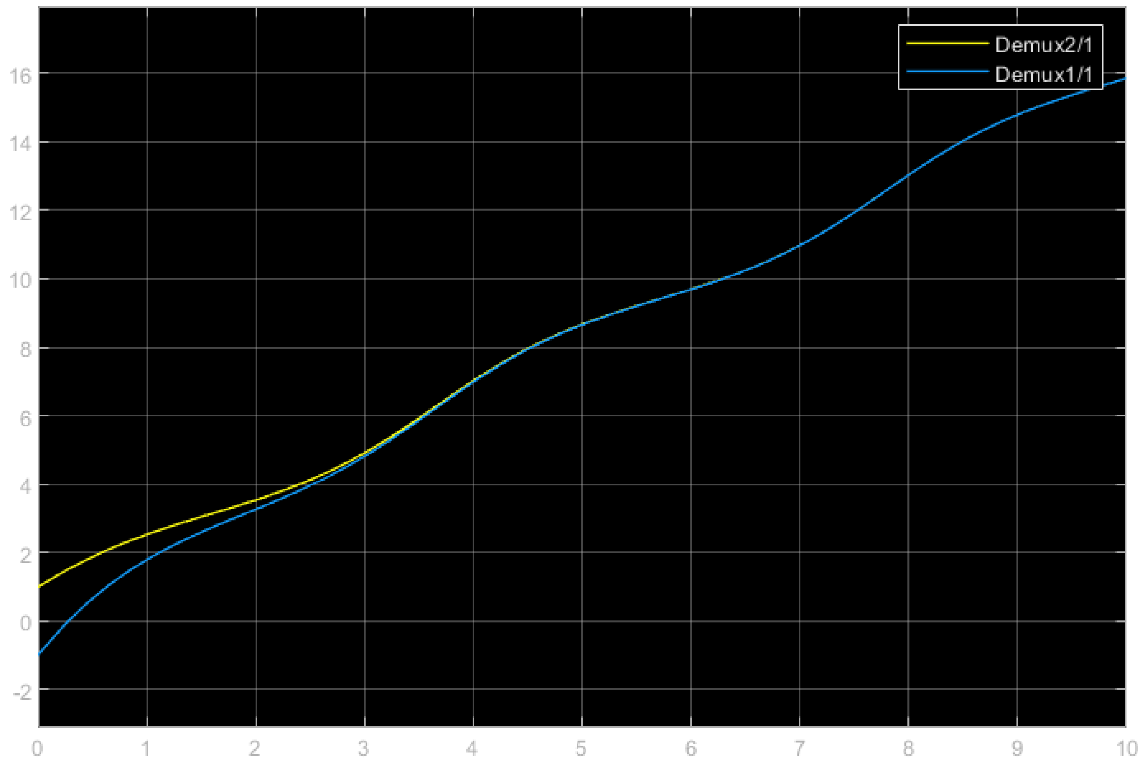

Figure 2.2: X_1 (yellow) and $\hat{X}_1$ (blue) vs time diagrams

2.3 Discussion

Because the calculations for K were very simple and based on the naked eye, the observer is not very fast but still reliable. However, if need be, K can be changed in a way that the observer error eigenvalues are designed further than that of the original system to increase observer speed. It is also worth noting that the observer is nonetheless, very powerful even with lower speed and the goal of tracking the system states is achieved sufficiently.

Also, note that because the problem didn't have a controller design, therefore, it is obvious that this nonlinear system will become unstable in some regions. But, if controller output is provided to the system, it can be stabilized using a feedback linearization method or by utilizing a sliding mode control method and assuming that the nonlinear part is an unknown parameter.

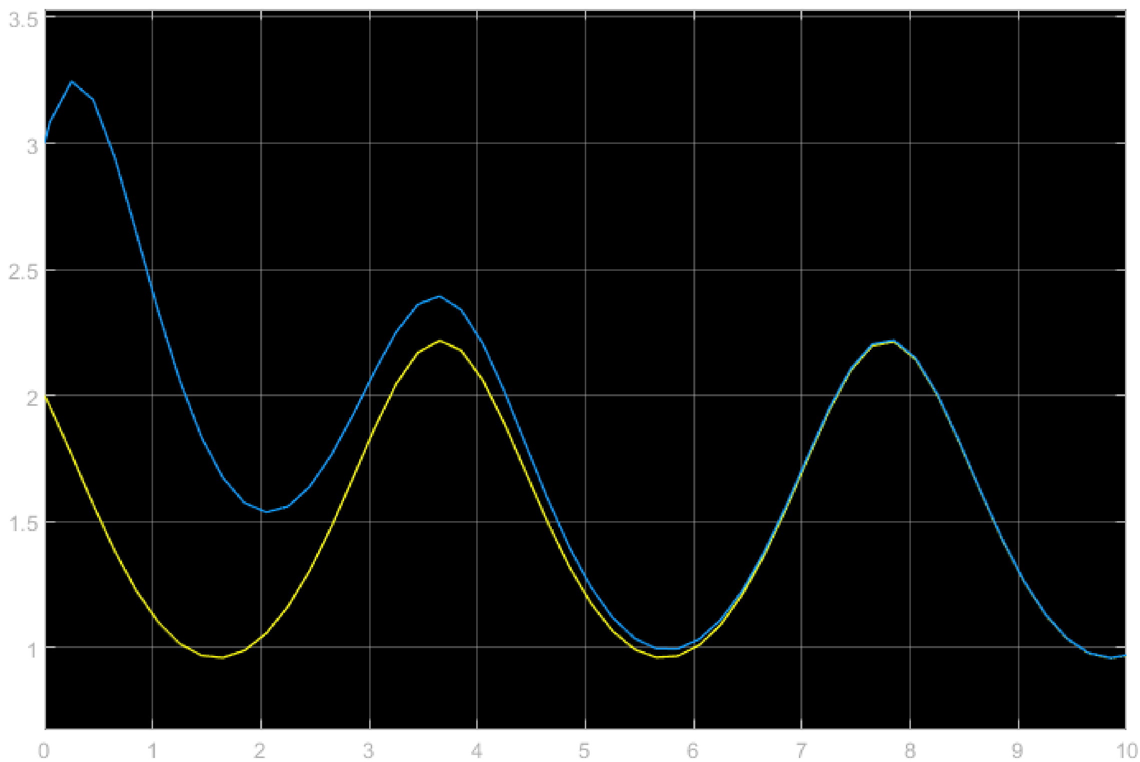

Figure 2.3: X_2 (yellow) and $\hat{X}_2$ (blue) vs time diagrams

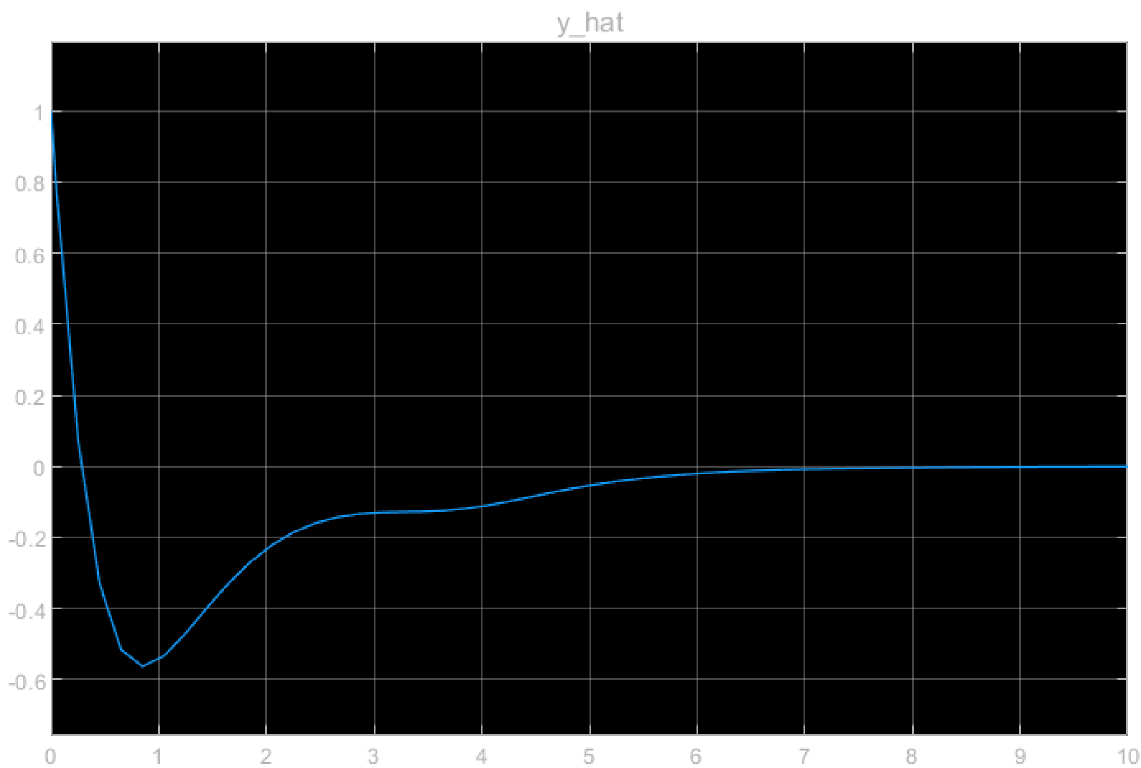

Figure 2.4: Observer output error diagram vs time

Chapter 3

Conclusion

A nonlinear system was given and the goal was to design an observer that follows the system reliably. So, a Thau observer was proposed and its stability was analytically proven. We also simulated our system together with the observer and saw the observer output error approaching zero which is all we wanted but it came at the cost of a slower observer since the calculations were naive.

3.1 Future Works

This system can be controlled using a sliding mode method or feedback linearization depending on the design preference.