

Block CG Solvers Documentation

github.com/lkeegan/blockCG

Liam Keegan
liam@keegan.ch

May 25, 2018

1 Introduction

Formulations of block CG solvers. Notation conventions used: A is the hermitian positive definite $L \times L$ matrix to be inverted, lowercase Roman letters represent L -component vectors, uppercase Roman letters represent $L \times n_{\text{RHS}}$ block-vectors, and greek letters represent scalars or $n_{\text{RHS}} \times n_{\text{RHS}}$ matrices.

2 Solvers

2.1 CG

Algorithm 1 CG: Solve $Ax = b$

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1:  $x, p, r, t \in \mathcal{C}^L$ ;  $\alpha, \beta \in \mathcal{R}$ 
2:  $x_0 = t_0 = p_0 = 0$ ,  $r_0 = b$ ,  $\alpha_0 = b^\dagger b$ 
3: for  $k = 1, 2, \dots$  until  $|Ax_k - b| / |b| < \epsilon$  do
4:    $p_k \leftarrow r_{k-1} + p_k \alpha_k$ 
5:    $t_k \leftarrow Ap_k$ 
6:    $\beta_k \leftarrow (p_k^\dagger t_k)^{-1} (r_{k-1}^\dagger r_{k-1})$ 
7:    $r_k \leftarrow r_{k-1} - t_k \beta_k$ 
8:    $x_k \leftarrow x_{k-1} + p_k \beta_k$ 
9:    $\alpha_k \leftarrow (r_{k-1}^\dagger r_{k-1})^{-1} (r_k^\dagger r_k)$ 
10: end for
```

2.2 SCG

3 Block Solvers

3.1 BCG

3.2 SBCGrQ