Block CG Solvers Documentation
github.com/lkeegan/blockCG

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1 Introduction

Formulations of block CG solvers. Notation conventions used: A is the hermitian positive definite $L \times L$ matrix to be inverted, lowercase Roman letters respresent L–component vectors, uppercase Roman letters represent $L \times n_{\mathrm{RHS}}$ block–vectors, and greek letters represent scalars or $n_{\mathrm{RHS}} \times n_{\mathrm{RHS}}$ matrices.

2 Solvers

2.1 CG

Algorithm 1 CG: Solve $Ax = b$

$x, p, r, t \in \mathcal{C}^L$; $\alpha, \beta \in \mathcal{R}$
$x_0 = t_0 = p_0 = 0$, $r_0 = b$, $\alpha_0 = b^\dagger b$
for $k = 1, 2, \ldots$ until $|Ax_k - b| \,/\, |b| < \epsilon$ **do**
    $p_k \leftarrow r_{k-1} + p_k \alpha_k$
    $t_k \leftarrow A p_k$
    $\beta_k \leftarrow (p_k^\dagger t_k)^{-1} (r_{k-1}^\dagger r_{k-1})$
    $r_k \leftarrow r_{k-1} - t_k \beta_k$
    $x_k \leftarrow x_{k-1} + p_k \beta_k$
    $\alpha_k \leftarrow (r_{k-1}^\dagger r_{k-1})^{-1} (r_k^\dagger r_k)$
end for

2.2 SCG

3 Block Solvers

3.1 BCG

Algorithm 2 BCG: Solve $AX = B$

$X, P, R, T \in \mathcal{C}^{L \times n_{\mathrm{RHS}}}$; $\alpha, \beta \in \mathcal{C}^{n_{\mathrm{RHS}} \times n_{\mathrm{RHS}}}$
$X_0 = T_0 = P_0 = 0$, $R_0 = B$, $\alpha_0 = B^\dagger B$
for $k = 1, 2, \ldots$ until $|AX_k - B| \,/\, |B| < \epsilon$ **do**
    $P_k \leftarrow R_{k-1} + P_k \alpha_k$
    $T_k \leftarrow A P_k$
    $\beta_k \leftarrow (P_k^\dagger T_k)^{-1} (R_{k-1}^\dagger R_{k-1})$
    $R_k \leftarrow R_{k-1} - T_k \beta_k$
    $X_k \leftarrow X_{k-1} + P_k \beta_k$
    $\alpha_k \leftarrow (R_{k-1}^\dagger R_{k-1})^{-1} (R_k^\dagger R_k)$
end for

3.2 SBCGrQ