Influence of Magnetic Fields on Direct and Alternating Currents - I

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Contents

1 Introduction

Here I discuss the impact of a magnetic field in direct current (DC) and alternating current (AC) when generated by a third electrical conductor - i.e., under the influence of external electromagnetic field generated by a body acting as an inductor. This involves the *Biot-Savart Law*, *Faraday's Law*, *Lenz and Faraday-Lenz Laws*, *Ohm's Laws* and *Gauss' Laws* and *Maxwell's Laws* mainly, among others. It also takes into account: to consider voltage level, type of current, signal type, operating frequency, electric field magnitude, and the type of electric field.

For the sake of simplicity but maintaining formalism, I am identifying the variables as follows:

- $\vec{B}$: Magnetic field;
- μ_0: Magnetic permeability of free space;
- I: Current;
- $d\vec{l}$: Differential length element of the wire;
- $\vec{r}$: Position vector from the wire element to the point where the magnetic field is being calculated;
- $\vec{r}_p$: Position vector of the point to be determined in the field;
- $\vec{r}_l$: Position vector that goes from the origin to a point on the wire;
- L: Length of the wire;
- ε: Electromotive force;
- Φ_B: Magnetic flux;
- Q_{enc}: Charge enclosed by the surface;
- ε_0: Vacuum permittivity;
- V: Voltage;
- R: Resistance;
- f: Frequency;
- t: Time;
- θ: Angle.
- $\hat{\mathbf{r}}$: Unit vector from the current element to the point where the field is calculated.
- r: Distance between the current element and the point.
- $\mathbf{E}$: Electric field.
- $d\mathbf{A}$: Differential area element.
- Q: Electric charge.
- sin: Sine function.
- cos: Cosine function.
- tan: Tangent function.
- π: Pi constant as $\approx$ 3.14159265358979323846 . . . .

2 Basic Concepts

2.1 Direct Current - DC

Direct current is the unidirectional flow of electric charge. It is represented by the equation: $I_{DC} = \frac{dQ}{dt}$ where I_{DC} is the direct current, Q is the electric charge, and t is the time.

2.2 Alternating Current - AC

Alternating current is the flow of electric charge that periodically reverses direction. It is represented by the equation: $I_{AC} = I_{max} \sin(2\pi f t)$ where I_{AC} is the alternating current, I_{max} is the maximum current, f is the frequency, and t is the time.

3 Influence of Electromagnetic Fields

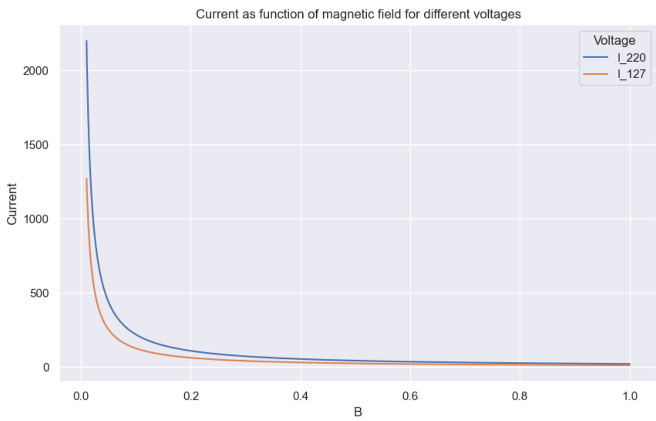
Let's start with an example of the Hall effect, in which the electrical current can be influenced by a magnetic field.

The full form of the Hall Effect equation is given by:

$$V_H = \frac{IB}{ne}, \tag{1}$$

where V_H is the Hall voltage, I is the current, B is the magnetic field, n is the charge carrier density, and e is the elementary charge.

Below, in Python code, I simulated the effect of a magnetic field $\vec{B}$ on a current I in a conductor with charge carrier density n and elementary charge e as constants. I also considered the voltages of either 220V of 127V, and the resistance of the conductor as $R = R\ \Omega$.




```
␣␣import␣pandas␣as␣pd
␣␣import␣seaborn␣as␣sns
␣␣import␣matplotlib.pyplot␣as␣plt

␣␣#␣load␣seaborn
␣␣sns.set_theme()

␣␣#␣constants␣definition

␣␣d␣=␣0.001␣#␣cable␣width␣in␣meters
␣␣R_H␣=␣1e-3␣#␣Hall␣coefficient␣in␣m^3/C

␣␣#␣magnetic␣field␣space
␣␣B␣=␣np.linspace(0.01,␣1,␣1000)␣#␣0.01␣T␣to␣1␣T

␣␣#␣voltages

␣␣V_220␣=␣220
␣␣V_127␣=␣127

␣␣#␣lets␣assume␣that␣V_H␣is␣proportional␣to␣V␣-␣add␣a␣factor
```

```
  k = 0.1
  V_H_220 = k * V_220
  V_H_127 = k * V_127

  # current as function of B for either 220 or 127 V
  I_220 = (V_H_220 * d) / (R_H * B)
  I_127 = (V_H_127 * d) / (R_H * B)

  # create a dataframe

  data = np.column_stack((B, I_220, I_127))
  data = pd.DataFrame(data, columns=['B', 'I_220', 'I_127'])
  data_melted = pd.melt(data, id_vars='B', value_vars=['I_220', 'I_127'], var_name='Voltage', value_nam

  # plot
  plt.figure(figsize=(10, 6))
  sns.lineplot(x='B',y='Current', hue='Voltage', data=data_melted)
  plt.title('Current as function of magnetic field for different voltages')
  plt.grid(True)
  plt.show()
```

The Biot-Savart Law describes the magnetic field generated by an electric current. It is given by:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

where $\mathbf{B}$ is the magnetic field, μ_0 is the vacuum permeability, I is the current, $d\mathbf{l}$ is the element of length through which the current flows, $\hat{\mathbf{r}}$ is the unit vector from the current element to the point where the field is calculated, and r is the distance between the current element and the point.

3.1 Faraday-Lenz's Law

Faraday-Lenz's Law states that the induced electromotive force (EMF) in any closed circuit is equal to the negative of the time rate of change of the magnetic flux through the circuit:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

where ε is the EMF and Φ_B is the magnetic flux.

3.2 Gauss's Law

Gauss's Law relates the distribution of electric charge to the resulting electric field:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

where $\mathbf{E}$ is the electric field, $d\mathbf{A}$ is the differential area element, Q_{enc} is the charge enclosed by the surface, and ε_0 is the vacuum permittivity.

3.3 Ohm's Law

Ohm's Law states the relationship between voltage, current, and resistance in an electrical circuit:

$$V = IR$$

where V is the voltage, I is the current, and R is the resistance.

3.4 Tesla's Postulates

Nikola Tesla's contributions to the understanding of electromagnetic fields and alternating current systems are numerous. One of his fundamental postulates emphasizes the efficiency and necessity of alternating current for long-distance electrical power transmission.

4 The Biot-Savart Law

The *Biot-Savart Law* mathematically describes magnetic field generated by a current-carrying wire, allowing us to calculate the magnetic field at any point in space. The Law is fundamental to the study of magnetism and electromagnetism, and it is used to calculate the magnetic field produced by various current distributions, such as straight wires, loops, and solenoids, *a cornerstorne in the power generation, transmission and distribution systems to all end-users. The Law is also used in the design of electric motors, transformers, and inductors and in the study of magnetic materials.*

Some of the applications of the *Biot-Savart Law* include:

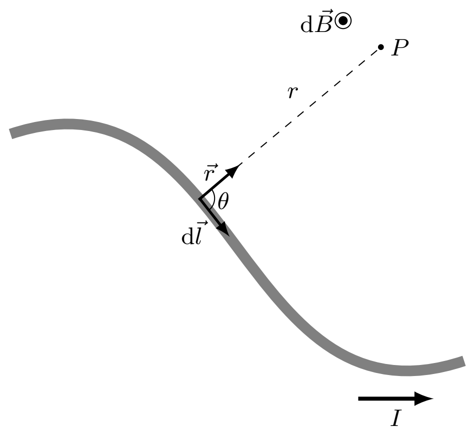
- Calculation of the magnetic field produced by a current-carrying wire.
- Calculation of the magnetic field produced by a current loop.
- Calculation of the magnetic field produced by a solenoid.
- Calculation of the magnetic field produced by a toroid.
- Calculation of the magnetic field produced by a straight wire.
- Calculation of the magnetic field produced by a circular arc.
- Calculation of the magnetic field produced by a current sheet.
- Calculation of the magnetic field produced by a finite wire.
- Calculation of the magnetic field produced by a semi-infinite wire.
- Calculation of the magnetic field produced by a coaxial cable.

The canonical form of the *Biot-Savart Law* is given by:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^2} \tag{2}$$

where $\vec{B}$ is the magnetic field, μ_0 is the magnetic permeability of free space, I is the current, $d\vec{l}$ is the differential length element of the wire, $\vec{r}$ is the position vector that goes from the wire element to the point where the magnetic field is being calculated, and $|\vec{r}|$ is the magnitude of the position vector.

Below there is a plot of a current-carrying wire, where the magnetic field is calculated at a point P. This, together with Faraday's Law, is the basis for the operation of electric generators; the Lenz's Law, which is the basis for the operation of electric motors; and the Ampère's Law, which is the basis for the operation of transformers and Tesla's discoveries on the generation of high-voltage currents gives us the basis for the operation of the power transmission and distribution systems, and also the alternating current (AC) systems. Below there is a plot of a direct-current (DC) system versus an alternating-current (AC) system given the presence and influence of a magnetic field induced by a current-carrying wire.



4.1 Biot-Savart Law in the Finite Wire

For infinitesimal components, a possible solution is to integrate the equation of a curve C. In this case, we assume that I is constant and we remove it from the integration.

Case I. Integration of *Biot-Savart* for the magnetic field in an energized conductor.

The magnetic field $\vec{B}$ is given by:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_c \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^2} \tag{3}$$

assuming that $d\vec{l}$ can be equivalent to $dl\hat{\eta}$.

Given that $\vec{r} = \vec{r}_p - \vec{r}_l$, where the latter is the position vector that goes from the origin to a point on the wire, and; $\vec{r}_p = x_p\hat{i} + y_p\hat{j} + z_p\hat{k}$ is the position vector of the point to be determined in the field, we can write $\vec{r}_l = x\hat{i}$, such that:

$$\vec{r} = (x_p - x)\hat{i} + y_p\hat{j} + z_p\hat{k} \tag{4}$$

,

which gives us:

$$|\vec{r}|^3 = \left[(x_p - x)^2 + y_p^2 + z_p^2\right]^{3/2} \tag{5}$$

This allows us to calculate the cross product between $d\vec{l}$ and $\vec{r}$:

$$d\vec{l} \times \vec{r} = -z_p \; dx\hat{j} + y_p \; dx\hat{k} \tag{6}$$

With equation 1.3, we integrate:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^L \frac{-z_p \; dx\hat{j} + y_p \; dx\hat{k}}{\left[(x_p - x)^2 + y_p^2 + z_p^2\right]^{3/2}} \tag{7}$$

with the components of $\vec{B}$ in R^3, such as:

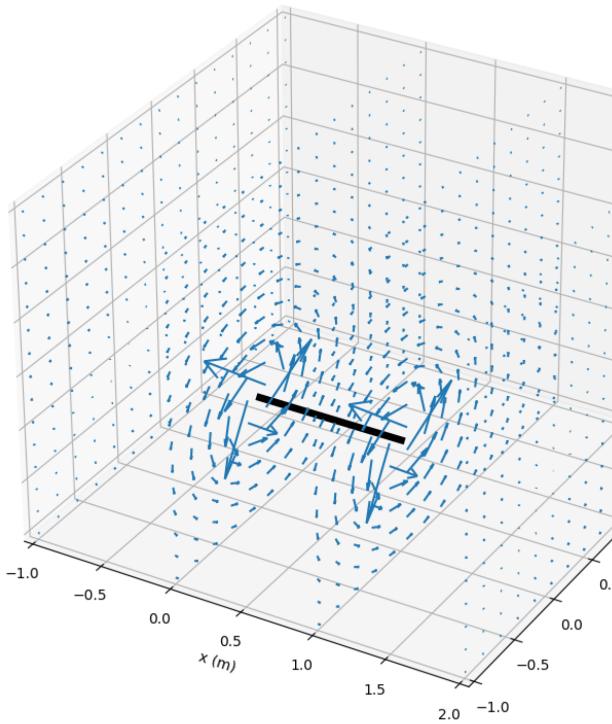
$$B_x = 0 \tag{8}$$

$$B_y = -\frac{\mu_0 I}{4\pi} z_p \int_0^L \frac{dx}{[(x_p - x)^2 + y_p^2 + z_p^2]^{3/2}} \tag{9}$$

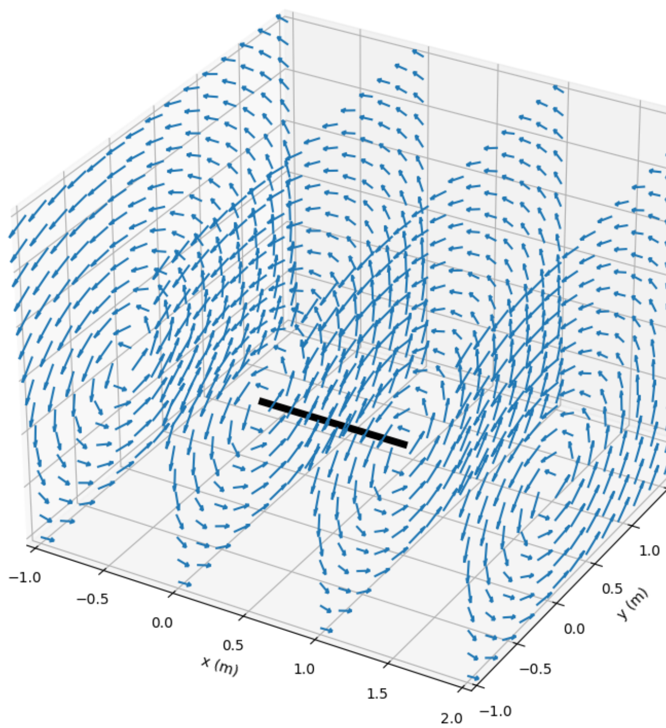
$$B_z = \frac{\mu_0 I}{4\pi} y_p \int_0^L \frac{dx}{[(x_p - x)^2 + y_p^2 + z_p^2]^{3/2}} \tag{10}$$

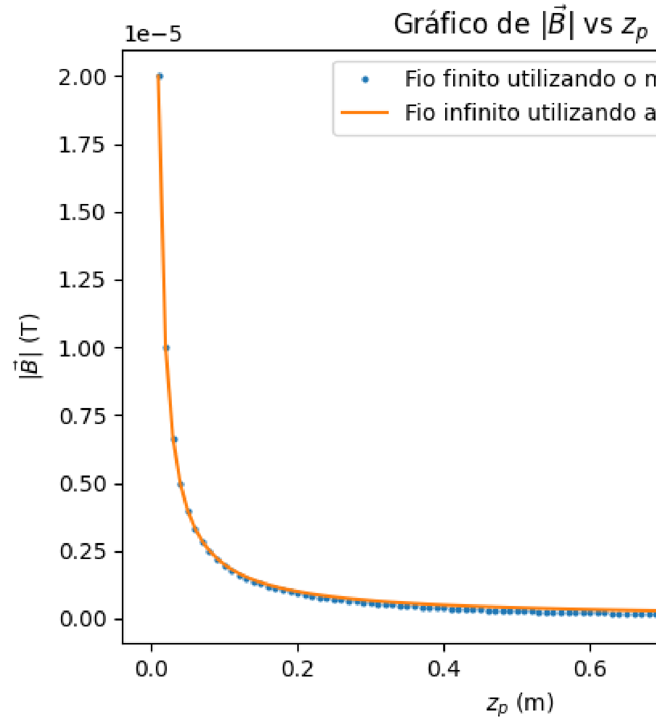
Note that the integrals B_y and B_z are the same, and can be solved numerically. For this, we will consider the following Python code, and the parameters $L = 1 \; m, \; I = 1A$.

4.2 Plots of the Magnetic Field in the Finite Wire



Plot 1 - Non-normalized magnetic field in the finite wire.





Plot 3 - Magnetic field in the finite wire as z_p component under R^3.