An Event-B Specification of Poker
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CONTEXT Poker
EXTENDS Card
CONSTANTS

isA Check a set cards has or has not an ACE.
$result$ The set of value of fighting.
isK Check a set cards has or has not a KING.
$isPair$ Check a set cards has or has not a pair of card are the same value.
$isThree$ Check a set cards has or has not a three cards are the same value.
$getThree$ Get three cards are the same value in a set cards.
$getPair$ Get two cards are the same value in a set cards.
$isUnique$ Check a set cards has or has not two cards are the same id.
$isEmpty$ Check a set cards is or is not empty.
$getMax$ Get the max value of a set cards.
$compareSetCard$ Compare two set cards by value.
$getMax\_Card$ Get the card has the max value of a set cards.

AXIOMS

result_D : $result = \{-1, 0, 1, 2\}$

The set of value of fighting:
-1 : error
0 : fair
1 : hand 1 win 2
2 : hand 2 win 1

isEmpty_D : $isEmpty \in \mathbb{P}(SetID) \rightarrow BOOL$

Check a set cards is or is not empty?

isEmpty_F : $\forall a \cdot a \in \mathbb{P}(SetID) \land card(a) = 0 \Rightarrow isEmpty(a) = TRUE$

If the cardinate of a set cards a is 0, then it is empty.

isUnique_D : $isUnique \in \mathbb{P}(SetID) \rightarrow BOOL$

Check the set of cards has two cards same id or not?

isUnique_F1 : $\forall a \cdot a \in \mathbb{P}(SetID) \land isEmpty(a) = TRUE \Rightarrow isUnique(a) = TRUE$

If a is empty, then it is unique.

isUnique_F2 : $\forall a \cdot \exists x, y \cdot a \in \mathbb{P}(SetID) \land x \in a \land y \in a \land x = y \Rightarrow isUnique(a) = FALSE$

If the set of cards has two cards the same id, then it is not unique.
return FALSE

isA_D : $isA \in \mathbb{P}(SetID) \rightarrow BOOL$

Check the set of cards has a ACE card or not?

isA_F : $\forall a \cdot \exists x \cdot a \in \mathbb{P}(SetID) \land isUnique(a) = TRUE \land isEmpty(a) = FALSE \land x \in a \land getValue(x) = 1 \Rightarrow isA(a) = TRUE$

If the set a has a card with value 1,then it has a ACE.
return TRUE

isK_D : $isK \in \mathbb{P}(SetID) \rightarrow BOOL$

Check the set of cards has a KING card or not?

isK_F : $\forall a \cdot \exists x \cdot a \in \mathbb{P}(SetID) \land isUnique(a) = TRUE \land isEmpty(a) = FALSE$
$\land\, x \in a \land getValue(x) = 13$
$\Rightarrow isK(a) = TRUE$

If the set a has a card with value 13, then it has a King.
return TRUE

isPair_D : $isPair \in \mathbb{P}(SetID) \rightarrow BOOL$

Check the set of cards has a pair of card are the same value or not?

isPair_F : $\forall a \cdot \exists x, y \cdot a \in \mathbb{P}(SetID) \land isUnique(a) = TRUE \land card(a) > 1$
$\land\, x \in a \land y \in a \land x \neq y \land getValue(x) = getValue(y)$
$\Rightarrow isPair(a) = TRUE$

If the set of cards has two cards are the same value,
then it has a pair of a kind.
reuturn TRUE

isThree_D : $isThree \in \mathbb{P}(SetID) \rightarrow BOOL$

Check the set of cards has three cards are the same value or not?

isThree_F : $\forall a \cdot \exists x, y, z \cdot a \in \mathbb{P}(SetID) \land isUnique(a) = TRUE \land card(a) > 2 \land$
$x \in a \land y \in a \land z \in a \land getValue(x) = getValue(y) \land getValue(y) = getValue(z)$
$\Rightarrow isThree(a) = TRUE$

If the set of cards has 3 cards are the same value,
then it has three of a kind.
reuturn TRUE

getPair_D : $getPair \in \mathbb{P}(SetID) \rightarrow \mathbb{P}(SetID)$

Get two cards are the same value in the set of cards.

getPair_F : $\forall a \cdot \exists x, y \cdot a \in \mathbb{P}(SetID) \land isUnique(a) = TRUE \land card(a) > 1$
$\land\, x \in a \land y \in a \land x \neq y \land getValue(x) = getValue(y)$
$\Rightarrow getPair(a) = \{x, y\}$

If the set of cards has two cards are the same value,
return the pair.

getThree_D : $getThree \in \mathbb{P}(SetID) \rightarrow \mathbb{P}(SetID)$

Get three cards are the same value in the set of cards.

getThree_F : $\forall a \cdot \exists x, y, z \cdot a \in \mathbb{P}(SetID) \land isUnique(a) = TRUE \land card(a) > 2$
$\land\, x \in a \land y \in a \land z \in a \land x \neq y \land y \neq z \land z \neq x$
$\land\, getValue(x) = getValue(y) \land getValue(y) = getValue(z)$
$\Rightarrow getThree(a) = \{x, y, z\}$

If the set of cards has three cards are the same value,
return the three.

getMax_D : $getMax \in \mathbb{P}(SetID) \rightarrow Value$

Get the max value of a set of cards.

getMax_Func : $\forall a \cdot \exists x \cdot \forall y \cdot a \in \mathbb{P}(SetID) \land isUnique(a) = TRUE \land isEmpty(a) = FALSE$
$\land\, x \in a \land y \in a \land getValue(x) \geq getValue(y)$
$\Rightarrow getMax(a) = getValue(x)$

If the card x is the biggest value in the set of cards,

the value of x card is the max value of the set of cards,
return the value of x.

getMax_Card_D : $getMax\_Card \in \mathbb{P}(SetID) \rightarrow \mathbb{P}(SetID)$

Get the card has the value is the max value in a set of cards.

getMax_Card_F : $\forall a \cdot \exists x \cdot \forall y \cdot a \in \mathbb{P}(SetID) \land isUnique(a) = TRUE \land isEmpty(a) = FALSE$
$\land\ x \in a \land y \in a \land getValue(x) \geq getValue(y)$
$\Rightarrow getMax\_Card(a) = \{x\}$

If card x has the biggest value in the set of cards,
return x.

compareSetCard_D : $compareSetCard \in \mathbb{P}(SetID) \times \mathbb{P}(SetID) \rightarrowtail result$

Compare two set cards by value.

compareSetCard_F00 : $\forall a, b \cdot a \in \mathbb{P}(SetID) \land b \in \mathbb{P}(SetID) \land card(a) \neq card(b)$
$\Rightarrow compareSetCard(a \mapsto b) = -1$

If the number of cards of set a and b are different, then error.
return -1

compareSetCard_F01 : $\forall a, b \cdot a \in \mathbb{P}(SetID) \land b \in \mathbb{P}(SetID) \land isUnique(a) = FALSE$
$\Rightarrow compareSetCard(a \mapsto b) = -1$

If set a is not unique, then error.
return -1

compareSetCard_F02 : $\forall a, b \cdot a \in \mathbb{P}(SetID) \land b \in \mathbb{P}(SetID) \land isUnique(b) = FALSE$
$\Rightarrow compareSetCard(a \mapsto b) = -1$

If b is not unique, then error.
return -1

compareSetCard_F0 : $\forall a, b \cdot a \in \mathbb{P}(SetID) \land b \in \mathbb{P}(SetID)$
$\land\ isEmpty(a) = TRUE \land isEmpty(b) = TRUE$
$\Rightarrow compareSetCard(a \mapsto b) = 0$

If the two set of cards are empty, then the fighting result is fair.
return 0.

compareSetCard_F10 : $\forall a, b \cdot a \in \mathbb{P}(SetID) \land b \in \mathbb{P}(SetID)$
$\land\ isUnique(a) = TRUE \land isEmpty(a) = FALSE$
$\land\ isUnique(b) = TRUE \land isEmpty(b) = FALSE \land card(a) = card(b)$
$\land\ isA(a) = TRUE \land isA(b) = FALSE$
$\Rightarrow compareSetCard(a \mapsto b) = 1$

If set a and b are not empty,
and a has a ACE card, b has not a ACE card,
then a wins b.
return 1

compareSetCard_F11 : $\forall a, b \cdot a \in \mathbb{P}(SetID) \land b \in \mathbb{P}(SetID)$
$\land\ isUnique(a) = TRUE \land isEmpty(a) = FALSE$
$\land\ isUnique(b) = TRUE \land isEmpty(b) = FALSE \land card(a) = card(b)$
$\land\ getMax(a) > getMax(b) \land isA(a) = isA(b)$
$\Rightarrow compareSetCard(a \mapsto b) = 1$

If set a and b are not empty,
and both a and b have or have not ACE at the same time,
and the max value of a is bigger than the max value of b,
then a wins b.
return 1

compareSetCard_F20 : $\forall a, b \cdot a \in \mathbb{P}(SetID) \land b \in \mathbb{P}(SetID)$
$\land\ isUnique(a) = TRUE \land isEmpty(a) = FALSE$
$\land\ isUnique(b) = TRUE \land isEmpty(b) = FALSE$
$\land\ card(a) = card(b) \land isA(a) = FALSE \land isA(b) = TRUE$
$\Rightarrow compareSetCard(a \mapsto b) = 2$

If set a and b are not empty,
and a has not a ACE card, b has a ACE card,
then b wins a.
return 2

compareSetCard_F21 : $\forall a, b \cdot a \in \mathbb{P}(SetID) \land b \in \mathbb{P}(SetID)$
$\land\ isUnique(a) = TRUE \land isEmpty(a) = FALSE$
$\land\ isUnique(b) = TRUE \land isEmpty(b) = FALSE$
$\land\ card(a) = card(b) \land getMax(a) < getMax(b) \land isA(a) = isA(b)$
$\Rightarrow compareSetCard(a \mapsto b) = 2$

If set a and b are not empty,
there is or is not an ACE in a and b at the same time,
and the max value of b is bigger than the max value of a
then b wins a.
return 2

compareSetCard_FR : $\forall a, b \cdot \exists c, d \cdot a \in \mathbb{P}(SetID) \land b \in \mathbb{P}(SetID) \land c \in \mathbb{P}(SetID) \land d \in \mathbb{P}(SetID)$
$\land\ isUnique(a) = TRUE \land isEmpty(a) = FALSE$
$\land\ isUnique(b) = TRUE \land isEmpty(b) = FALSE$
$\land\ card(a) = card(b) \land getMax(a) = getMax(b) \land isA(a) = isA(b)$
$\land\ c = a \setminus getMax\_Card(a) \land d = b \setminus getMax\_Card(b)$
$\Rightarrow$
$compareSetCard(a \mapsto b) = compareSetCard(c \mapsto d)$

If set a and b are not empty,
and a and b have or have not a ACE card at the same time
and the max value of a equal the max value of b,
then remove the max card from a, b and compare a and b again.

END