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Abstract

1 Derivative of tan(x)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$for f(x) = tan(x)$ then $f(x+h) = tan(x+h)$

$$f'(x) = \lim_{h\to 0} \frac{tan(x+h) - tan(x)}{h}$$

$$\because tan(x+h) = \frac{tan(x) + tan(h)}{1 - tan(x)tan(h)}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{tan(x)+tan(h)}{1-tan(x)tan(h)} - tan(x)}{h}$$

$$= \lim_{h\to 0} \frac{tan(x) + tan(h)}{h(1 - tan(x)tan(h))} - \frac{tan(x)}{h}$$

$$= \lim_{h\to 0} \frac{h[tan(x) + tan(h)] - tan(x)h(1 - tan(x)tan(h))}{h^2(1 - tan(x)tan(h))}$$

$$= \lim_{h\to 0} \frac{(tan(x) + tan(h)) - tan(x)[1 - tan(x)tan(h)]}{h[1 - tan(x)tan(h)]}$$

$$= \lim_{h\to 0} \frac{(tan(x) + tan(h)) - [tan(x) - tan^2(x)tan(h)]}{h[-tan(x)tan(h)]}$$

$$= \lim_{h\to 0} \frac{tan(h) + tan^2(x)tan(h)}{h[1 - tan(x)tan(h)]}$$

$$= \lim_{h\to 0} \frac{tan(h)[1 + tan^2(x)]}{h[1 - tan(x)tan(h)]}$$

$$= \lim_{h\to 0} \frac{1 + tan^2(x)}{1 - tan(x)tan(h)} \lim_{h\to 0} \frac{tan(h)}{h}$$

$$\because \lim_{h\to 0} \frac{tan(h)}{h} = 1$$

By evaluate the $\lim_{h\to 0}$, we have

$$
\begin{aligned}
f'(x) &= \frac{1+tan^2(x)}{1-tan(x)tan(0)}(1) \\
\because tan(0) &= 0 \\
f'(x) &= \frac{1+tan^2(x)}{1-0} \\
&= 1+tan^2(x) \\
\because sec^2(x) &= 1+tan^2(x) \\
\therefore f'(x) &= sec^2(x)
\end{aligned}
$$

$\blacksquare$