Danmarks Tekniske Universitet

02417

Time Series Analysis

A2 - ARMA Processes and Seasonal Processes

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Contents

1 Question 2.1 Stability

Let the process Xt be given by

$$X_t + \phi_1 X_{t-1} + \phi_2 X_{t-1} = \epsilon_t$$

where ϵ_t is a white noise process. Investigate analytically for which values of f2 the process is stationary when $\phi_1 = -0.25$. In addition it should be investigated for which values of f2 the autocorrelation function shows damping harmonic oscillations. Still for $\phi_1 = -0.25$.

Solution

So, what we have here is a dynamic system, where the output at a given time instance t depends on a linear combination of previous outputs plus some noise ϵ_t. We therefore have an AR(2) model, which can be represented by an Infine Impulse Response Filter (IIR) that has some noise as input. In the book this is referenced as a filter whose transfer function in the Z domain only constains the $A(z)$ component described by equation (4.61).
This means that the system has all its zeros in the origin, and the poles are the points in the complex Z domain where the denominator, $A(z)$ becomes 0. The Z transform of the process is:

$$X(z)(1 + z^{-1}\phi_1 + z^{-2}\phi_2) = \epsilon(z)$$

We can consider the noise as our input so the transfer function of the system is:

$$H(z) = \frac{X(z)}{\epsilon(z)} = \frac{1}{1 + z^{-1}\phi_1 + z^{-2}\phi_2} = \frac{z^2}{z^2 + z^1\phi_1 + \phi_2}$$

So we have got 2 poles, in the points:

$$z_{p1} = \frac{-\phi_1 + \sqrt{\phi_1^2 - 4\phi_2}}{2}$$

$$z_{p2} = \frac{-\phi_1 - \sqrt{\phi_1^2 - 4\phi_2}}{2}$$

The system will be stable, and therefore the process **stationary** if the poles lie in the unit circle of the Z domain, that means that the modulus of the poles should be less than 1. The equation for the modulus of the poles depends on whether $\sqrt{\phi_1^2 - 4\phi_2}$ is real or imaginary.

Real roots
The roots are real if: $\phi_1^2 - 4\phi_2 \geq 0$, so we have $\phi_2 \leq \frac{\phi_1^2}{4} = \frac{1}{64}$
For the system to be stable $\|z_{p1}\|, \|z_{p2}\| \leq 1$ so:

$$-1 < \frac{-\phi_1 \pm \sqrt{\phi_1^2 - 4\phi_2}}{2} < 1$$

The bigger root is z_{p1}, so if it fullfills the upper bound, so it does z_{p2}. Taking that into account, the lower bound inequality simplifies to:

$$-\phi_1 + \sqrt{\phi_1^2 - 4\phi_2} < 2$$

Moving terms, squaring both sides and concelling terms we end up with the inequality:

$$\phi_2 > -1 - \phi_1$$

The smaller root is z_{p2}, so if it fullfills the lower bound, so it does z_{p1}. Taking that into account, the lower bound inequality simplifies to:

$$-2 < -\phi_1 - \sqrt{\phi_1^2 - 4\phi_2}$$

Moving terms, squaring both sides and concelling terms we end up with the inequality:

$$\phi_2 > -1 + \phi_1$$

Complex Roots
The roots are complex if: $\phi_1^2 - 4\phi_2 < 0$, so we have $\phi_2 > \frac{\phi_1^2}{4} = \frac{1}{64}$

For the system to be stable $\|z_{p1}\|, \|z_{p2}\| \leq 1$. The modulus will be the same for both poles. The equation for the poles becomes:

$$\frac{-\phi_1 \pm j\sqrt{-\phi_1^2 + 4\phi_2}}{2}$$

Note the change in sign inside the square root because we have put the negativeness of the root outside the root. So we have in modulus:

$$\|z_p\| = \sqrt{\frac{\phi_1^2 - \phi_1^2 + 4\phi_2}{4}} < 1$$

Squaring and moving the terms around we end up with the inequality:

$$\phi_2 < 1$$

Finally we have the stability condition for the real roots is $\phi_2 \leq \frac{\phi_1^2}{4} = \frac{1}{64}$ and $\phi_2 > -1 - \phi_1 = -0.75$ and $\phi_2 > -1 + \phi_1 = -1.25$. So end up with $-0.75 < \phi_2 \leq \frac{1}{64}$.
And the stability condition for the complex roots is: $\phi_2 \geq \frac{\phi_1^2}{2} = \frac{1}{32}$.
The system is stable for the union of both domains. Ans since we also have the constraint that the modulus of the coefficients must be lower than 1, $\phi_1, \phi_1 < 1$, then the final domain is:

$$-0.75 < \phi_2 < 1$$

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From equation (5.83) it can be seen that the autocorrelation will show **damping harmonic oscillations** if the characteristic polinomial contains complex conjugated roots. As we have just seen, this will happen for $\phi_2 \geq \frac{\phi_1^2}{4} = \frac{1}{64}$. Since the modulus of the coefficient must be lower than 1, we end up with:

$$\frac{1}{64} < \phi_2 < 1$$

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2 Question 2.2 University activity

A university counts the number of passed course modules every term. There are four terms per year. Based on historical data the following model has been identified:

$$(1 - 0.8B + 0.7B^2)(1 - 0.9B^4)(Y_t - \mu) = \epsilon_t$$

where ϵ_t is a white-noise process with variance σ_ϵ^2. Based on 60 observations, it is found that $\sigma_\epsilon^2 = 50^2$. Furthermore, μ was estimated to 1000. The table below shows the last ten observations:

t	51	52	53	54	55	56	57	58	59	60
count	998	714	1009	1296	1083	751	950	1267	1134	838

Table 1: Values

Predict the values of Y_t corresponding to t = 61 and 62, together with 95% confidence intervals for the predictions.

Solution

What we have here is an $AR(2)$ model with a periodic component with lag $d = 4$, in the general multiplicative seasonal model $(p, d, q) \times (P, D, Q)_s$ from equation 5.127 we have the particular case: $(2, 0, 0) \times (1, 0, 0)_4$.

We expand the polinomial and we express the system in the inverse form (AR form) given by equation (5.146).

$$(1.00 - 0.80B^1 + 0.70B^2 - 0.90B^4 + 0.72B^5 - 0.63B^6)(Y_t - \mu) = \epsilon_t$$

We apply the B operator and express everything in time domain, having:

$$(Y_t - \mu) = 0.80(Y_{t-1} - \mu) - 0.70(Y_{t-2} - \mu) + 0.90(Y_{t-4} - \mu) - 0.72(Y_{t-5} - \mu) + 0.63(Y_{t-6} - \mu) + \epsilon_t$$

Mean of the predictions

Since the optimal predictor is the conditional mean, given by equation (5.148), we apply the formula, and since $E[Y_{t-k}|Y_{t-k}] = Y_{t-k}$ and $E[\epsilon_t] = 0$ we end up with:

$$\hat{Y}_{t+1|t} = E[Y_{t+1}|\mathbf{Y_t}] = 0.80(Y_t - \mu) - 0.70(Y_{t-1} - \mu) + 0.90(Y_{t-3} - \mu) - 0.72(Y_{t-4} - \mu) + 0.63(Y_{t-5} - \mu) + \mu$$

For the time $t + 1 = 61$ we have:

$$\hat{Y}_{61} = 0.80(Y_{60} - \mu) - 0.70(Y_{59} - \mu) + 0.90(Y_{57} - \mu) - 0.72(Y_{56} - \mu) + 0.63(Y_{55} - \mu) + \mu$$

For the time instance $t = 62$, now we do not have Y_{61}, so we have the equation

$$\hat{Y}_{t+2|t} = E[Y_{t+2}|\mathbf{Y_t}]$$

So, we express Y_{t+2} in terms of Y_{t+1}.

$$Y_{t+2} = 0.80(Y_{t+1} + \mu) - 0.70(Y_t - \mu) + 0.90(Y_{t-2} - \mu) - 0.72(Y_{t-3} - \mu) + 0.63(Y_{t-4} - \mu) + \mu + \epsilon_{t+2}$$

Since we already calculated $E[Y_{t+1}|\mathbf{Y_t}]$ we have that:

$$\hat{Y}_{62} = 0.80(\hat{Y}_{61} - \mu) - 0.70(Y_{60} - \mu) + 0.90(Y_{58} - \mu) - 0.72(Y_{57} - \mu) + 0.63(Y_{56} - \mu) + \mu = 963.17$$

Variance of the predictions

The variance of the predictions is given by equation (5.149), in order to obtain it, for the first prediction we have:

$$\epsilon_{t+1|t} = Y_{t+1} - \hat{Y}_{t+1|t}$$

We have that:

$$Y_{t+1} = 0.80(Y_t - \mu) - 0.70(Y_{t-1} - \mu) + 0.90(Y_{t-3} - \mu) - 0.72(Y_{t-4} - \mu) + 0.63(Y_{t-5} - \mu) + \epsilon_{t+1} + \mu$$

$$\hat{Y}_{t+1|t} = 0.80(Y_t - \mu) - 0.70(Y_{t-1} - \mu) + 0.90(Y_{t-3} - \mu) - 0.72(Y_{t-4} - \mu) + 0.63(Y_{t-5} - \mu) + \mu$$

So end up with:

$$\epsilon_{t+1|t} = Y_{t+1} - \hat{Y}_{t+1|t} = \epsilon_{t+1}$$

Finally, the variance of the first estimator is:

$$VAR[\epsilon_{t+1|t}] = VAR[\epsilon_{t+1}] = \sigma_\epsilon^2 = 50^2 = 2500$$

For the estimation of $\epsilon_{t+2|t}$ we have that:

$$Y_{t+2|t} = 0.80(Y_{t+1} - \mu) - 0.70(Y_t - \mu) + 0.90(Y_{t-2} - \mu) - 0.72(Y_{t-3} - \mu) + 0.63(Y_{t-4} - \mu) + \epsilon_{t+2} + \mu$$

$$\hat{Y}_{t+2|t} = 0.80(\hat{Y}_{t-1} - \mu) - 0.70(Y_{t-2} - \mu) + 0.90(Y_{t-4} - \mu) - 0.72(Y_{t-5} - \mu) + 0.63(Y_{t-6} - \mu) + \mu$$

So we end up with:

$$\epsilon_{t+2|t} = 0.80(Y_{t+1} - \hat{Y}_{t+1} - \mu) + \epsilon_{t+2}$$

We have already calculated that $(Y_{t+1} - \hat{Y}_{t+1}) = \epsilon_{t+1}$ as we have already calculated. So we have that:

$$\epsilon_{t+2|t} = 0.80(\epsilon_{t+1} - \mu) + \epsilon_{t+2}$$

Finally, the variance of the second estimator is:

$$VAR[\epsilon_{t+2|t}] = VAR[\epsilon_{t+2}] = \sigma_\epsilon^2(1 + 0.8^2) = 1.64 \cdot 50^2 = 4100$$

Confidence interval of the predictions

The predictions obtained have a gaussian distribution with the mean and variance already calculated. The confidence interval at 95% is the range of values of the prediction around the mean, that contain the 95% of the probability distribution. For a normalized guassian with variance 1, this ranges goes from $\mu \pm 1.96$, so performing a normalization, the range of a gaussian with variance σ_ϵ^2 is $\mu \pm 1.96\sigma_\epsilon$
In our case, the 95% confidence interval of the predictions are:

$$IC(\hat{Y}_{61}) = 963.17 \pm 1.96 \cdot 50 = [865.17, 1061.17]$$

$$IC(\hat{Y}_{62}) = 1061.17 \pm 1.96 \cdot 64.03 = [935.6688, 1186.671]$$

Since we do not have many samples, we will use a t-student distribution as well:

3 Question 2.3 Random walk.

Let the process Y be given by

$$Y_t = \frac{1}{4} + \sum_{i=1}^{t} \epsilon_i$$

where ϵ_i is a white noise process with mean zero and variance σ_ϵ^2.
1. Find the mean value, variance and covariance functions of the process.
2. Is the process Y stationary? If so, in what sense? If not, in what sense?
3. Simulate a white noise process of 1000 values with mean zero and s2 e = 1. Then figure out a fast and compact way to calculate the process Yt based on this.
4. Next, simulate 10 realizations of the process Yt. Save the realizations and plot them in the same graph using plot and lines or matplot. Also plot their estimated autocorrelation functions (Preferably in one plot). Comment on the graphs, and confirm you answers in step 2.

1. Solution

So we have an additive white noise process where the output at time t is equal to a bias $b = \frac{1}{4}$ plus the sum of the noises in all the previous times, from time $i = 1$. We could see this an IIR filter (or AR model) where the transfer function is a train of unitary deltas.
Since the **mean** is a linear operator, the mean of a constant is the constant itself and the white noise random variables have all the same mean $E[\epsilon_i] = 0$, the mean of the process is:

$$E[Y_t] = E[\frac{1}{4}] + \sum_{i=1}^{t} E[\epsilon_i] = \frac{1}{4}$$

The **variance** of the process can also be easily calculated from the properties of the variance operator:

$$VAR[Y_t] = VAR[\frac{1}{4}] + \sum_{i=1}^{t} VAR[\epsilon_i] = 0 + t\sigma_\epsilon^2 = t\sigma_\epsilon^2$$

The **autocovariance** of the process is:

$$\gamma_{t,t-k} = \gamma(Y_t, Y_{t-k}) = Cov[Y_t, Y_{t-k}] = E[(Y_t - \mu_t)(Y_{t-k} - \mu_{t-k})] = E[(\sum_{i=1}^{t} \epsilon_i)(\sum_{j=1}^{t-k} \epsilon_j)]$$

Since the noise instances are independent: $E[\epsilon_i \epsilon_j] = 0$ when $j \neq i$ and it is equal to σ_ϵ^2 when $j = i$. So end up with:

$$\gamma(Y_t, Y_{t-k}) = (t-k)\sigma_\epsilon^2$$

2. Solution

Even though the mean is the same for every time instant, the varince of the process grows unbounded with time t. The covariance between two time instances depends on the number of instances they have in common, not in the difference between them. As time t grows, the variance of the signal increases to infinity so the process is not stationary. This can be easily seen as well because the impulse response of the characterizing linear filter is a train of deltas that go to infinity. So, the sum of the squared coefficients of the characteristic filter is unbounded.

3. Solution

To generate a white noise process we just use the R function *nrandom()* that generates independent random samples of a white noise distribution with mean 0 and variance 1. To generate the process Y_t we just add up the values using the *cumsum()* function and we add the bias $\mu = \frac{1}{4}$.

4. Solution

The next plot shows the 10 realizations of the process:

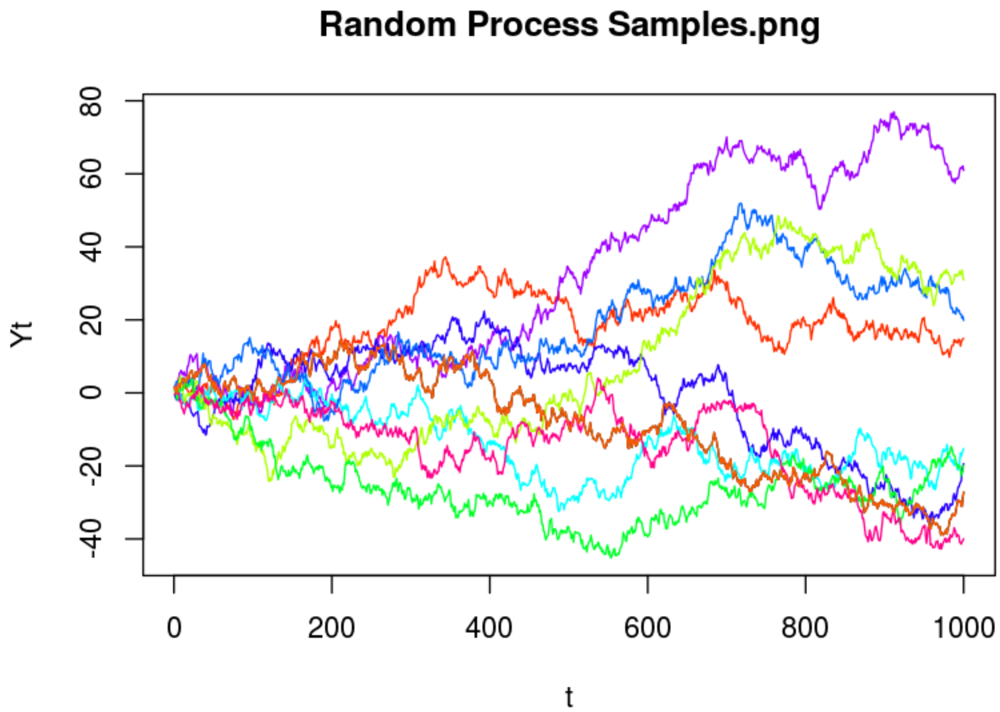

Figure 1: Random Process Realizations

As it can be observed in the graph, the mean value of the process remains constant but the standard deviation grows proportional to the square root of time (since the variance grows linearly as time increases), just as expected from the equations from step 1. Therefore the process is not stationary, as concluded in step 2.

The next plot shows the autocorrelation coefficients of the 10 realizations of the process in one graph.

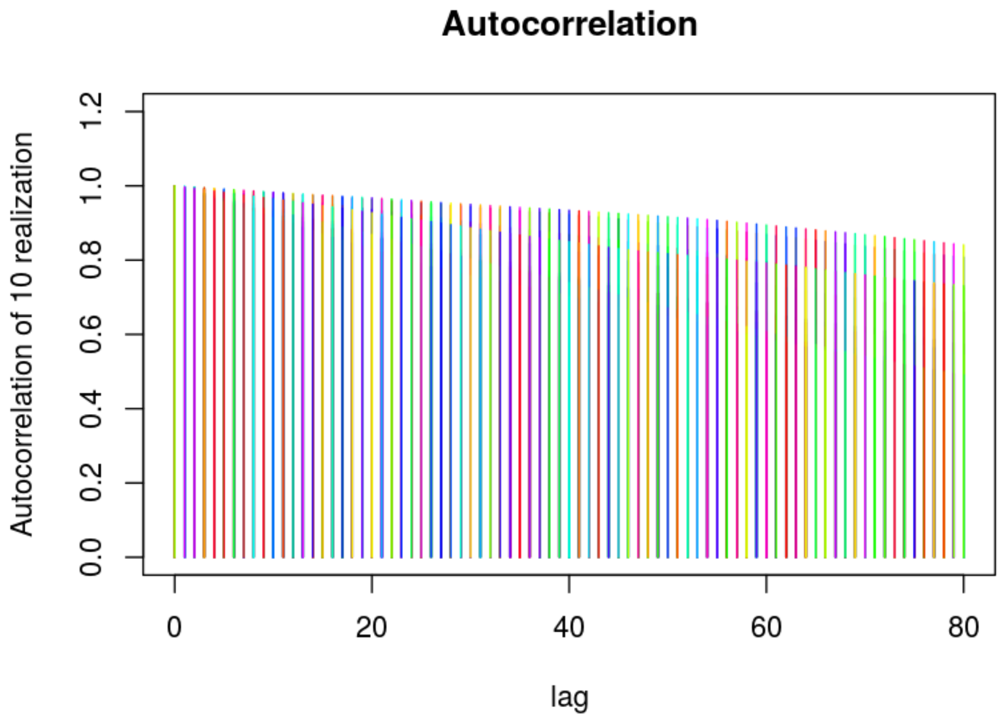

Figure 2: Random Process Realizations

The autocorrelacion function equals the impulse response of the characteristic filter of the system, and as we can see, it resembles a train of deltas. Due to lack of samples, as the lag grows, the correlation decreases but if we had enough samples then it would match the theoretical ones.
The next image shows the same process, but instead of having only 10 realizations with 1000 samples, it has 100 realizations with 10000 samples. In this picture the behaviour of the model can be more easily appreciated.

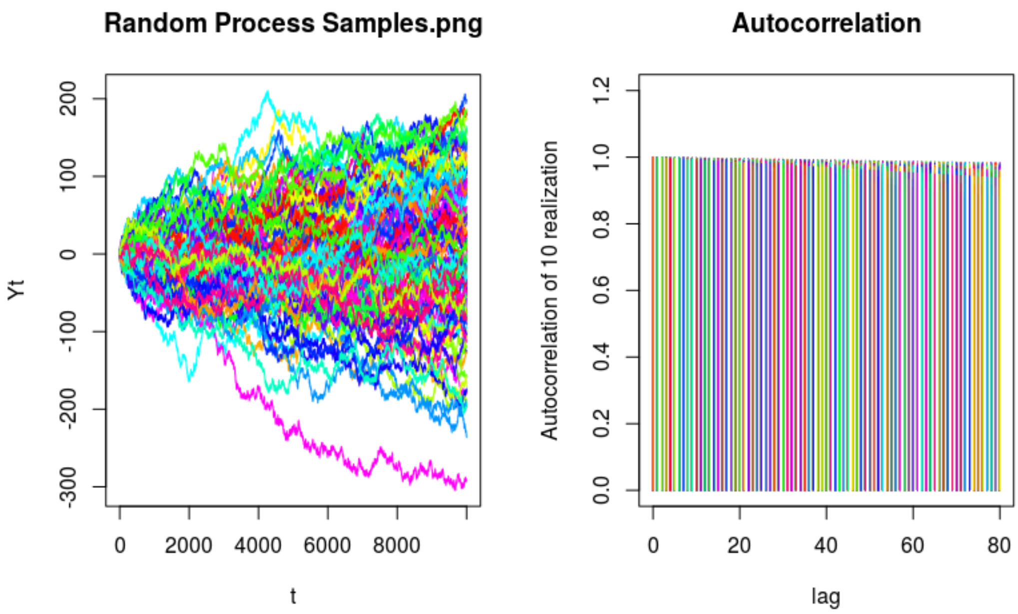

Figure 3: Random Process Realizations

It can be more clearly appreciated in this graph that the standard deviation grows as the square root of time and that the autocorrelaton of the process is a train of deltas.

4 Question 2.4 Simulating seasonal processes.

In the general multiplicative seasonal model $(p, d, q) \times (P, D, Q)_s$. Simulate the following models (where monthly data are assumed). Plot the simulations and the associated autocorrelation functions (ACF and PACF).
1. $A(1, 0, 0) \times (0, 0, 0)_{12}$ model with the parameter $\phi = 0.9$.
2. $A(0, 0, 0) \times (1, 0, 0)_{12}$ model with the parameter $\Phi = 0.7$.
3. $A(1, 0, 0) \times (0, 0, 1)_{12}$ model with the parameters $\phi = 0.9$. and $\Theta = 0.4$.
4. $A(1, 0, 0) \times (1, 0, 0)_{12}$ model with the parameters $\phi = 0.9$ and $\Phi = 0.7$.
5. $A(0, 0, 1) \times (0, 0, 1)_{12}$ model with the parameters $\theta = 0.4$. and $\Theta = 0.4$.
6. $A(0, 0, 1) \times (1, 0, 0)_{12}$model with the parameters $\theta = 0.4$ and $\Phi = 0.7$

Are all models seasonal? Summarize your observations on the processes and the autocorrelation functions. Which conclusions can you draw on the general behavior of the
autocorrelation function for seasonal processes? Note: arima.sim does not have a seasonal module, so model formulations as standard ARIMA processes have to be made.

Solution

In the following, each model will be expressed in the general multiplicative seasonal model $(p, d, q) \times (P, D, Q)_s$, and then expresses as normal ARIMA formulation, expanding the polinomials of the system. The realizations have been made using 1000 samples and the ACF and PACF coefficients have been plotted up to lag $k = 40$. Some of the plots shown in the assignment contain only a subset of the 1000 samples in order to have a better observation of the signal.

1. $A(1, 0, 0) \times (0, 0, 0)_{12}$ **model with the parameter** $\phi = 0.9$**.**
This model is a simple AR(1) model without seasonal component.

$$(1 - \phi B^1)Y_t = \epsilon_t$$

The following image shows a realization of the model where a certain correlation between nearby samples can be appreciated. No seasonal component is observed.

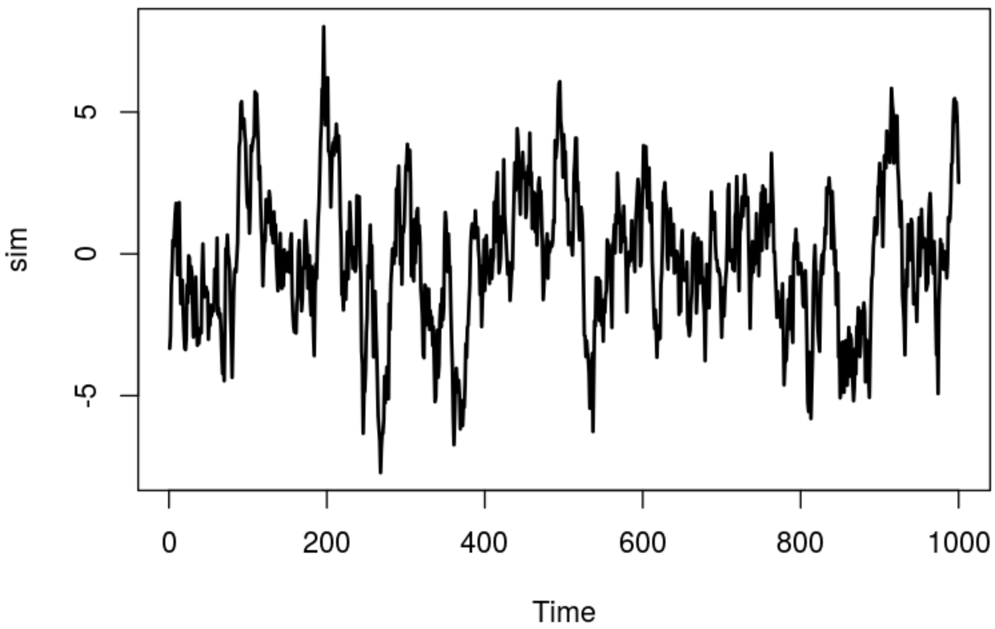

Figure 4: Realization of Model 1

The following image shows the ACF and PACF coefficients of the previous realization

- From the ACF graph we can see an exponential decay of the coefficients, which could indicate that the system contains an AR component with real positive roots.

- From the PACF graph we can see that there is only one significant coefficient at lag $k = 1$ which indicated we have an AR(1) system.

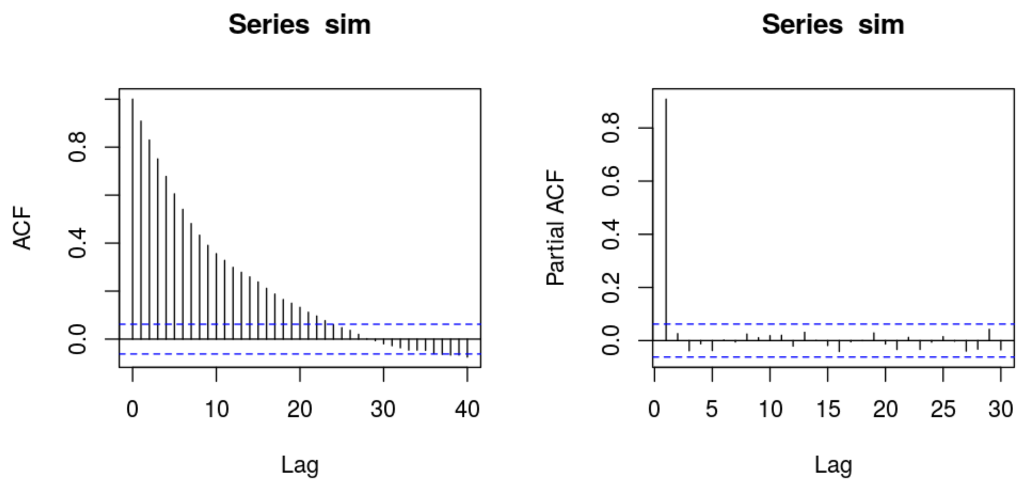

Figure 5: ACF and PACF of the realization of Model 1

So we can conclude that we are able to estimate the model for this case. This model is not periodic.

2. $A(0,0,0) \times (1,0,0)_{12}$ **model with the parameter** $\Phi = 0.7$**.**
This model contains periodic AR coefficients, every $s = 12$ samples.

$$(1 - \Phi B^s)Y_t = \epsilon_t$$

The following image shows a realization of the model where a certain correlation between nearby samples can be appreciated. It can be observed a seasonal component with lag $s = 12$, for example at the beggining there is a set of 6 high picks that are separated by 11 samples each.

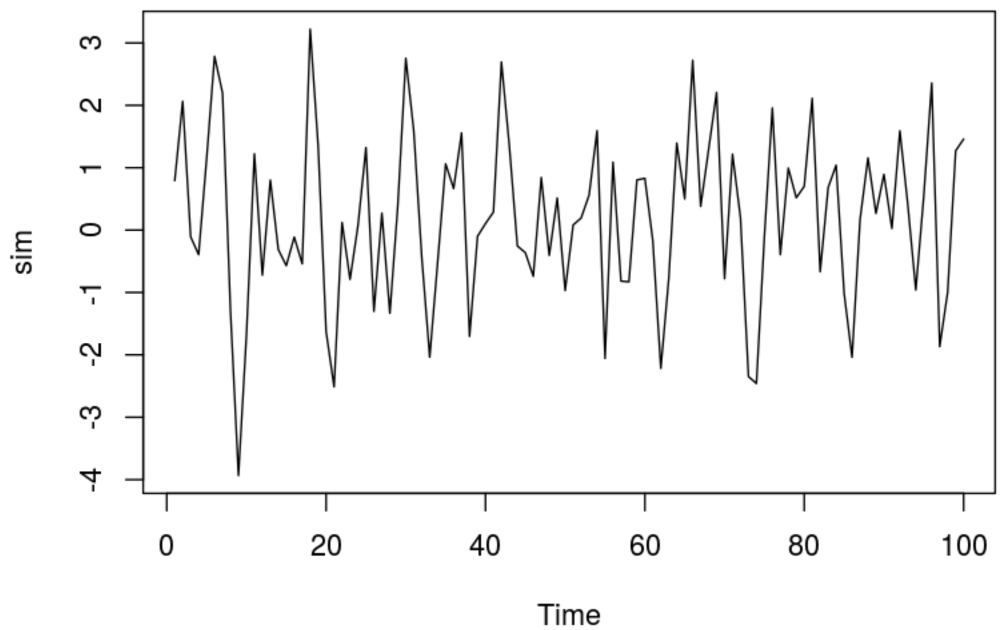

Figure 6: Realization of Model 1

The following image shows the ACF and PACF coefficients of the previous realization

- From the ACF graph we can see an exponential decay of the coefficients, but these coefficients are not contiguous, but rather they appear every 12 samples. This indicates that we can either have a MA model with several coefficients that are separated 12 samples from each other or it could be that the system contains a periodic AR component with positive real roots.
- From the PACF graph we can see that there is only one significant coefficient at lag $k = 12$ which indicated we have an AR system with a lag of $k = 12$.

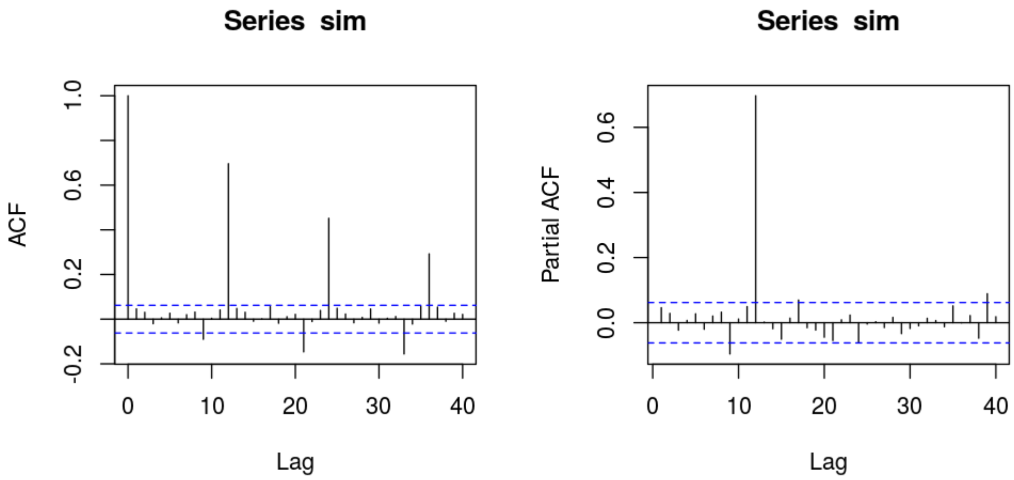

Figure 7: ACF and PACF of the realization of Model 1

So we can conclude that we are able to estimate the model for this case. This model is periodic.

3. $A(1,0,0) \times (0,0,1)_{12}$ **model with the parameters** $\phi = 0.9$**. and** $\Theta = 0.4$**.**
This model is an ARMA model with a periodic MA component in $s = 12$.

$$(1 - \phi B^1)Y_t = \epsilon_t(1 - \Theta B^s)$$

The following image shows a realization of the model where a certain correlation between nearby samples can be appreciated but it is hard to draw a pattern.

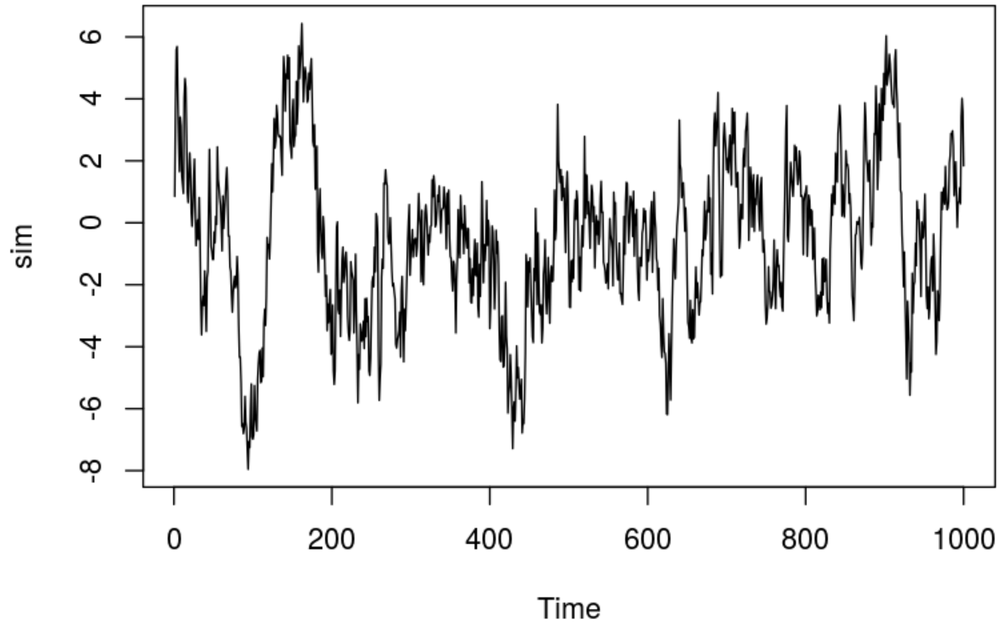

Figure 8: Realization of Model 1

The following image shows the ACF and PACF coefficients of the previous realization

- From the ACF graph we can see an exponential decay of the coefficients, but at $k = 12$, the coefficients seem to increase and then continue the exponential decrease. This indicates that we have an AR model but also, every 12 samples, the coefficients increase.
- From the PACF graph we can see that there is one big significant coefficient at lag $k = 1$ so we have an AR(1) and also we have a coefficient at lag $k = 12$ which indicates a periodic component.

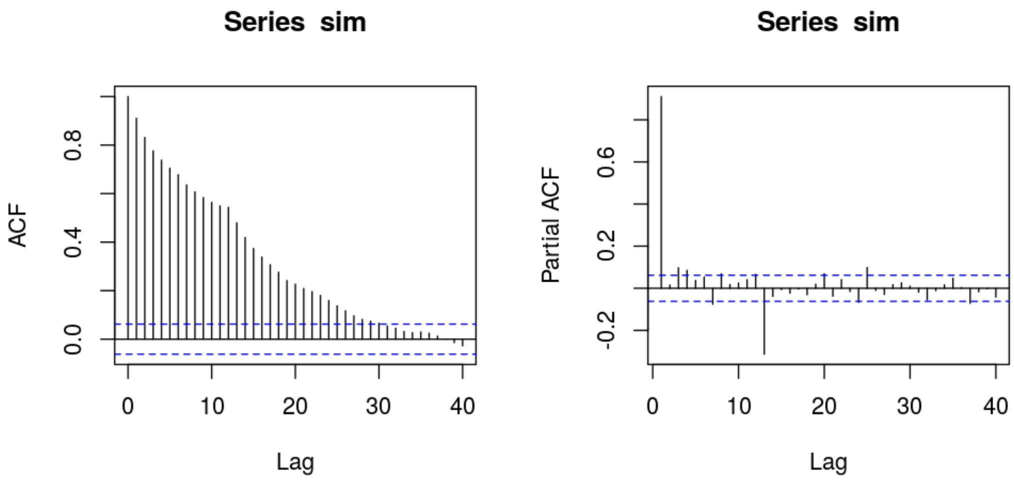

Figure 9: ACF and PACF of the realization of Model 1

So we can conclude that we are able to estimate the model for this case, since we can see that AR(1) component and a periodic increase of the ACF every 12 samples. This model is periodic.

4. $A(1, 0, 0) \times (1, 0, 0)_{12}$ **model with the parameters** $\phi = 0.9$ **and** $\Phi = 0.7$**.**
This model is a periodic AR model.

$$(1-\phi B^1)(1-\Phi B^s)Y_t = \epsilon_t$$

Expanding the polinomial we get:

$$(1-\phi B^1 - \Phi B^s + \phi\Phi B^{s+1})Y_t = \epsilon_t$$

The following image shows a realization of the model where a certain correlation between nearby samples can be appreciated but it is hard to draw a pattern.

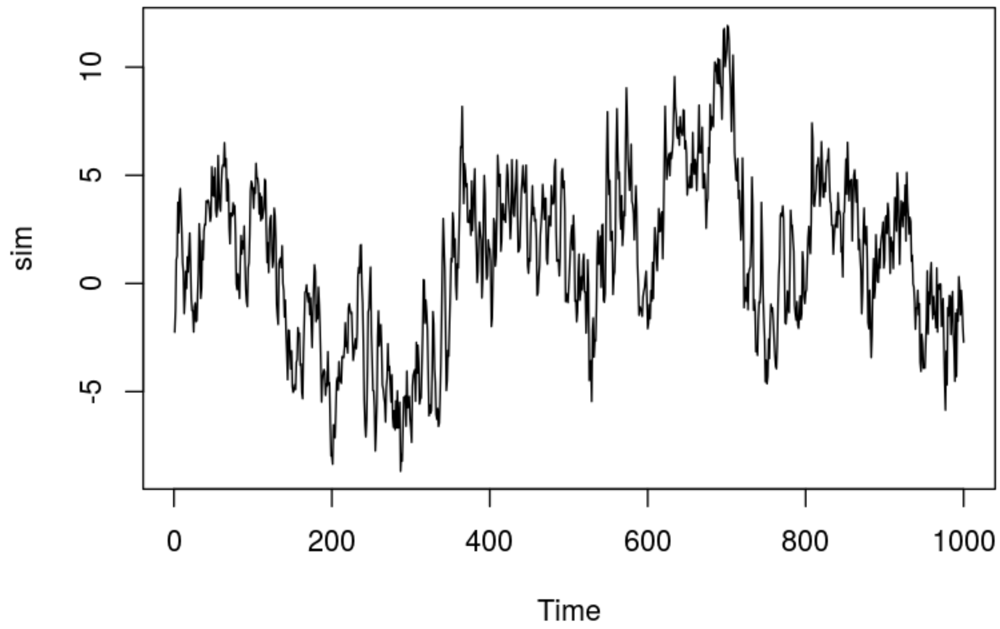

Figure 10: Realization of Model 1

The following image shows the ACF and PACF coefficients of the previous realization

- The ACF graph seems as a sum of decreassing AR(1) PAC graphs every 12 samples. As if, at every $k = 12n, n = 0, 1...$ we placed the coefficients of an AR(1) model. This indicates a periodic AR(1) model.
- From the PACF graph we can see that there is one big significant coefficient at lag $k = 1$ so we have an AR(1) and also we have a coefficient at lag $k = 12$ which indicates a periodic component.

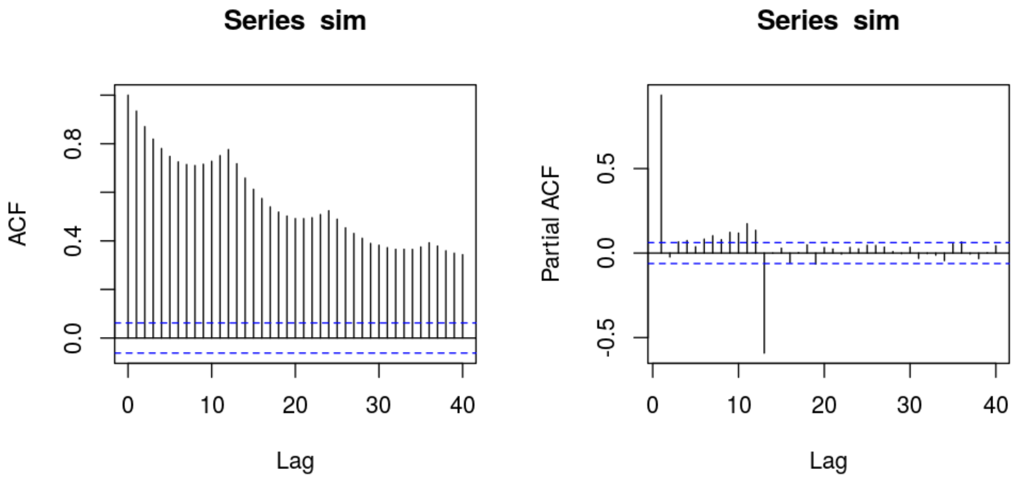

Figure 11: ACF and PACF of the realization of Model 1

So we can conclude that we are able to estimate the model for this case, since we can see that AR(1) component and a periodic increase of the ACF every 12 samples. We can diferentiatie this model from the previous one in the ACF function. This model is periodic.

5. $A(0, 0, 1) \times (0, 0, 1)_{12}$ **model with the parameters** $\theta = 0.4$**. and** $\Theta = 0.3$**.**
This model is a periodic MA(1) model.

$$Y_t = (1 - \theta B^1)(1 - \Theta B^s)\epsilon_t$$

Expanding the polinomial we get:

$$Y_t = (1 - \theta B^1 - \Theta B^s + \theta\Theta B^{s+1})\epsilon_t$$

The following image shows a realization of the model where a certain correlation between nearby samples can be appreciated but it is hard to draw a pattern.

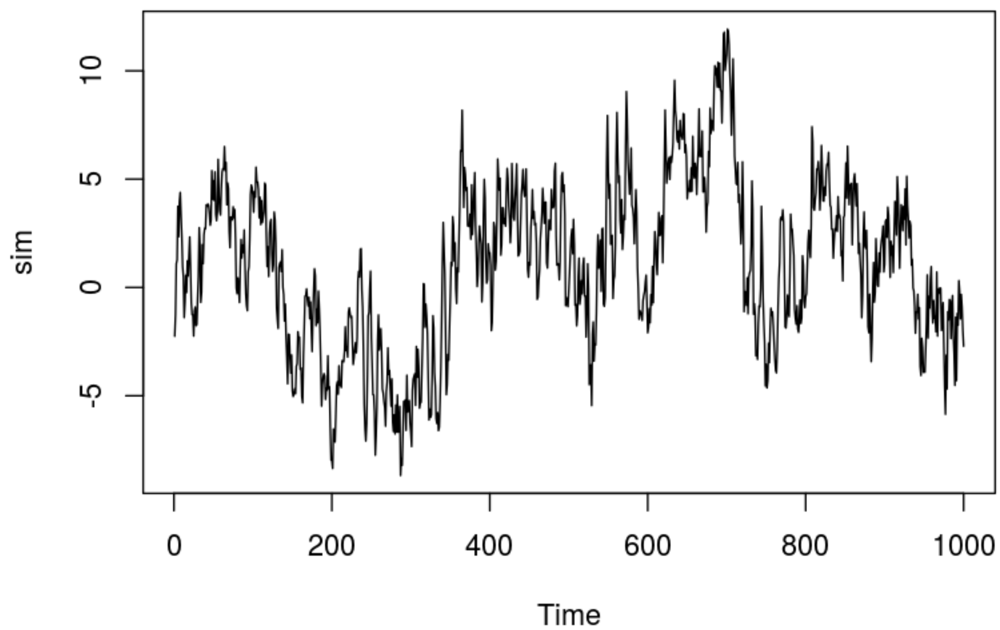

Figure 12: Realization of Model 1

The following image shows the ACF and PACF coefficients of the previous realization

- The PAC graph seems the sum of the PAC graph of 2 MA(1), the first one centered at 0 and the second one centered at $k = 12$ but where the second coefficient has been negated. This could indicate a periodic MA model.
- From the PACF graph we can see an oscilation of significant coefficients that decrease to 0, this vanishing of coefficients do not offer a clear result.

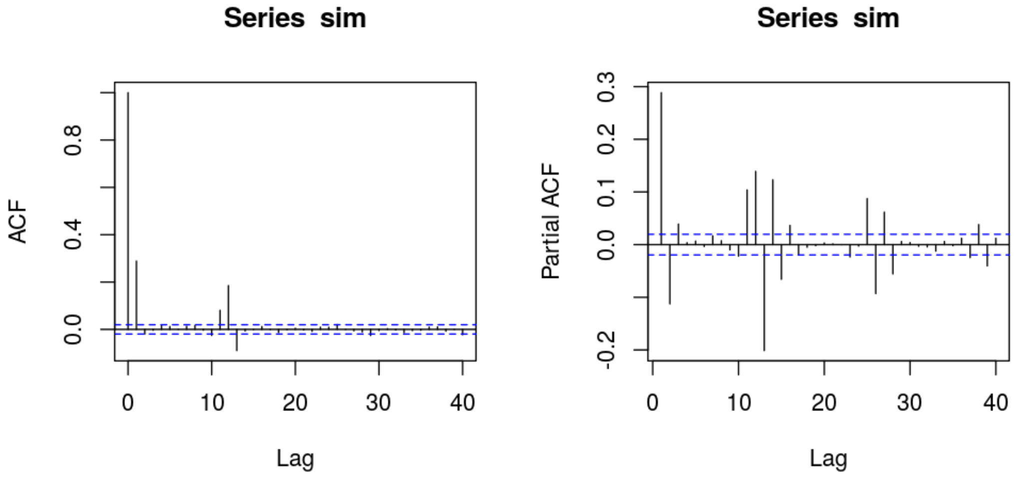

Figure 13: ACF and PACF of the realization of Model 1

So we can conclude that we are not completely able to estimate the model for this case, from the ACF we can see that it is most likely a MA model with a nonzero coefficient at $k = 1, 11, 12, 13$ so it is likely a periodic MA(1) model.

6. $A(0, 0, 1) \times (1, 0, 0)_{12}$**model with the parameters** $\theta = 0.4$ **and** $\Phi = 0.7$.
This model is an ARMA model with a periodic AR component.

$$(1 - \Phi B^s)Y_t = \epsilon_t(1 - \theta B^1)$$

The following image shows a realization of the model where a certain correlation between nearby samples can be appreciated but it is hard to draw a pattern.

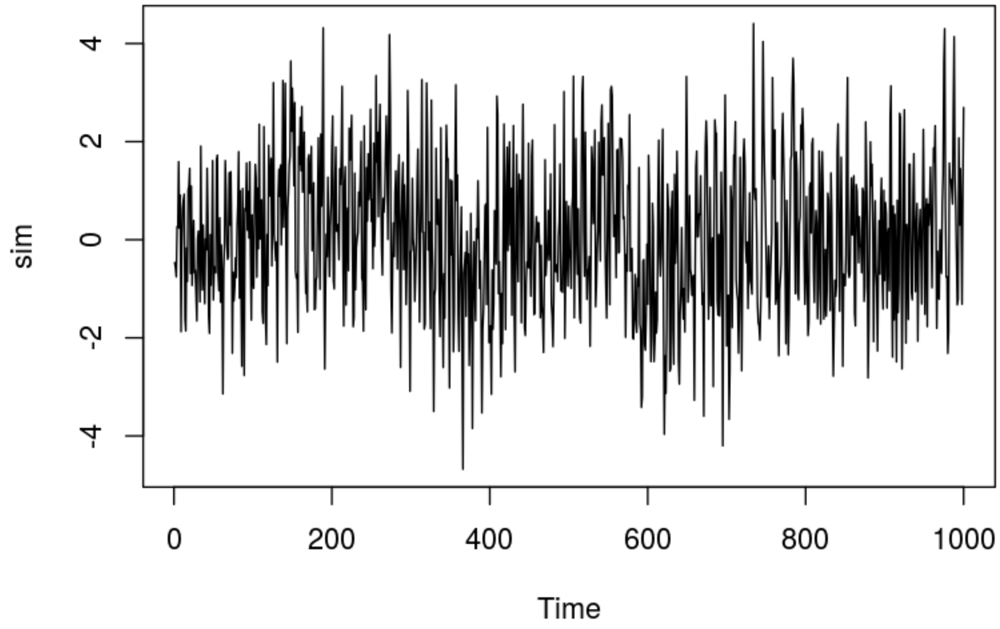

Figure 14: Realization of Model 1

The following image shows the ACF and PACF coefficients of the previous realization

- The ACF graph seems like the sum of the ACF graph of a MA(1) model replicated periodically each $k = 12$ coefficients and with exponentially decreasing coefficients.
- The PACF graph contains 2 significant coefficients at $k = 1, 2$ and another 3 at $k = 11, 12, 13$

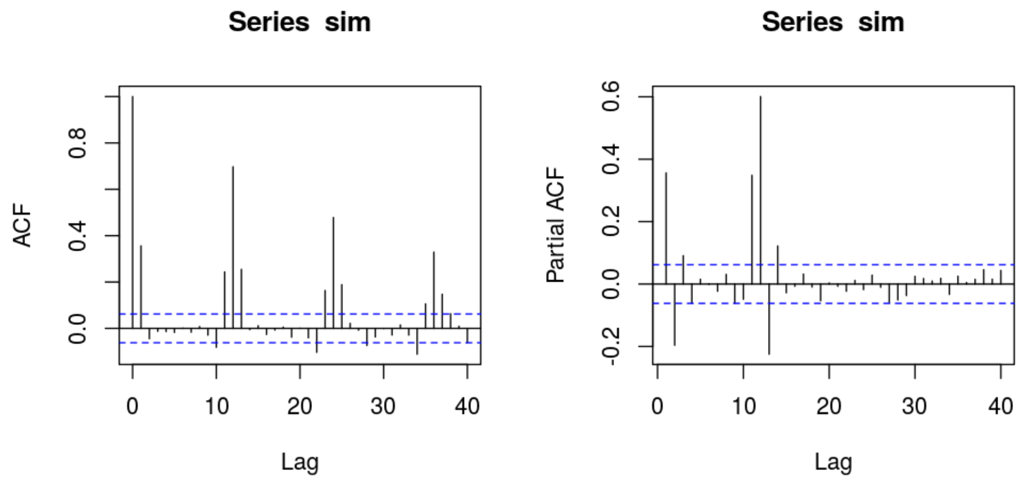

Figure 15: ACF and PACF of the realization of Model 1

The ACF coefficients seem like a periodic MA(1) model with period $s = 12$ so this could indicate that we can guess the true model from this graph. The PACF coefficient has its highest peak at $k = 12$ which supports the model guessed looking at the ACF graph.

General conclusions

If we consider a model as seasonal if the output at a given time t_1 depends in a special way on the input or output of a non-contiguous previous time instance t_2 with $t_2 - t_1 > 1$, then the models that contain a

value nonzero value at the P or Q components are seasonal. So the models 2,3,4,5,6 are seasonal in this sense.
For models with several components it is very hard to estimate the model looking at the process, some correlation between the samples is observed, but the exact type of model seems imposible to be estimated looking at the time series. The ACF and PACF graphs gives us a lot of information about the type of model but when the model has several components, these interfere with each other and it is still hard to determine.
The conclusion we can draw on the general behavior of the autocorrelation function for seasonal processes is that the autocorrelaiton funcion has nonzeros coefficients that are not contiguous. If it is periodic respect to the input (MA), then it only happens once, if it periodic respect to the output (AR) then ir happens periodically.

5 R code

This assignment has been entirely coded in R, since some variables are reused from one Question to the next, here I presen the code for all questions jointly. The code is fully commented and easy to follow.

```
################ Question 1 ######################
Nts = 10     # Number of timeSeries
Nsa = 1000   # Number of samples for the timeSeries
par(mfrow=c(1,2))
Processes = matrix(0,Nts,Nsa)
time_i = 1:Nsa

mu = 1.0/4

for (i in 1:Nts){
    noise <- rnorm(Nsa)
    process = cumsum(noise)
    Processes[i,] = process + mu
}

## PLOTTING
png(file="Random Process Samples",width=800, height = 600, res=130)
# Size and aspect ratio could be width=400,height=350,res=45
plot(time_i, Processes[i,],
     type = "l",                        # Draw as a line
     lwd= 1,                            # Line width
     main="Random Process Samples.png", # Title of the graph
     xlab="t",              # x label
     ylab="Yt",
     ylim = c(min(Processes),max(Processes)))                  # y label

for (i in 2:Nts){
  lines(time_i,
        Processes[i,], col=sample(rainbow(100)), type = "l", lwd=1)
}
dev.off()

### Autocorrelations !
Nlag = 80     # Maximum number of lag for autocorrelation
Autocorrelations = matrix(0,Nts,Nlag+1)
lag_i = 0:Nlag
for (i in 1:Nts){
  afc_obj = acf(Processes[i,],  lag.max = Nlag, plot = FALSE)
  Autocorrelations[i,] = afc_obj$acf
}
#sd = colMeans(Autocorrelations, na.rm = FALSE, dims = 1)
png(file="Autocorrelation.png",width=800, height = 600, res=130)
plot(lag_i, Autocorrelations[1,],
```

```
     type = "h",                        # Draw as a line
     lwd= 1,                            # Line width
     main="Autocorrelation", # Title of the graph
     xlab="lag",             # x label
     ylab="Autocorrelation of 10 realization",
     ylim = c(0,1.2))                 # y label

for (i in 2:Nts){
  lines(lag_i,
        Autocorrelations[i,], col=sample(rainbow(100)), type = "h", lwd=1)
}
dev.off()

################ Question 2.4 ######################

########## Model 1
# Simulating an AR(1)
phi = c(0.9)
s = 12
Nlag = 40
Nsa = 1000

sim <- arima.sim(model = list(ar = phi, order=c(1,0,0)), n = Nsa)

png(file="SM1_Realizations.png",width=800, height = 600, res=130)
par(mfrow=c(1,1))
plot(sim, lwd= 2)
dev.off()

png(file="SM1_ACFPACF.png",width=1000, height = 500, res=130)
par(mfrow=c(1,2))
acf(sim, lag.max = Nlag )
pacf(sim)
dev.off()

########## Model 2
# Simulating an AR(1)
Phi = c(0.7)
ar_coeff = c(matrix(0,1,s-1),Phi)
Nsa = 1000
sim <- arima.sim(model = list(ar = ar_coeff, order=c(12,0,0)), n = Nsa)

png(file="SM2_Realizations.png",width=800, height = 600, res=130)
par(mfrow=c(1,1))
plot(sim)
dev.off()

png(file="SM2_ACFPACF.png",width=1000, height = 500, res=130)
par(mfrow=c(1,2))
acf(sim, lag.max = Nlag , main = NULL, ylab = NULL)
pacf(sim, lag.max = Nlag, main = NULL , ylab = NULL)
dev.off()

########## Model 3
# Simulating an AR(1)
Theta = c(0.4)
phi = c(0.9)
s = 12
Nsa = 100000
ar_coeff = c(phi)
ma_coeff = c(matrix(0,1,s-1),Theta)
```

```
sim <- arima.sim(model = list(ar = ar_coeff, order=c(1,0,12), ma = ma_coeff), n = Nsa)

png(file="SM3_Realizations.png",width=800, height = 600, res=130)
par(mfrow=c(1,1))
plot(sim)
dev.off()

png(file="SM3_ACFPACF.png",width=1000, height = 500, res=130)
par(mfrow=c(1,2))
acf(sim, lag.max = Nlag )
pacf(sim, lag.max = Nlag )
dev.off()

########## Model 4
# Simulating an AR(1)
Phi = c(0.7)
s = 12
ar_coeff = c(matrix(0,1,s-1),Phi,-Phi*phi)
ar_coeff[1] = phi

sim <- arima.sim(model = list(ar = ar_coeff, order=c(13,0,0)), n = Nsa)

png(file="SM4_Realizations.png",width=800, height = 600, res=130)
par(mfrow=c(1,1))
plot(sim)
dev.off()

png(file="SM4_ACFPACF.png",width=1000, height = 500, res=130)
par(mfrow=c(1,2))
acf(sim, lag.max = Nlag )
pacf(sim, lag.max = Nlag )
dev.off()

########## Model 5
# Simulating an AR(1)
Theta = c(0.3)
theta = c(0.4)
s = 12
Nsa = 10000
ma_coeff = c(matrix(0,1,s-1),Theta,-Theta*theta)
ma_coeff[1] = theta

sim <- arima.sim(model = list(order=c(0,0,13), ma = ma_coeff), n = Nsa)

png(file="SM5_Realizations.png",width=800, height = 600, res=130)
par(mfrow=c(1,1))
plot(sim)
dev.off()

png(file="SM5_ACFPACF.png",width=1000, height = 500, res=130)
par(mfrow=c(1,2))
acf(sim, lag.max = Nlag )
pacf(sim, lag.max = Nlag )
dev.off()

########## Model 6
# Simulating an AR(1)
theta = c(0.4)
Phi = c(0.7)
```

```
ar_coeff = c(matrix(0,1,s-1),Phi)
ma_coeff = theta
s = 12
Nsa = 1000
sim <- arima.sim(model = list(order=c(12,0,1), ma = ma_coeff, ar = ar_coeff), n = Nsa)

png(file="SM6_Realizations.png",width=800, height = 600, res=130)
par(mfrow=c(1,1))
plot(sim)
dev.off()

png(file="SM6_ACFPACF.png",width=1000, height = 500, res=130)
par(mfrow=c(1,2))
acf(sim, lag.max = Nlag )
pacf(sim, lag.max = Nlag )
dev.off()
```