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A new approach of forecasting intermittent demand for spare parts inventories in the process industries

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Accurate demand forecasting is of vital importance in inventory management of spare parts in process industries, while the intermittent nature makes demand forecasting for spare parts especially difficult. With the wide application of information technology in enterprise management, more information and data are now available to improve forecasting accuracy. In this paper, we develop a new approach for forecasting the intermittent demand of spare parts. The described approach provides a mechanism to integrate the demand autocorrelated process and the relationship between explanatory variables and the nonzero demand of spare parts during forecasting occurrences of nonzero demands over lead times. Two types of performance measures for assessing forecast methods are also described. Using data sets of 40 kinds of spare parts from a petrochemical enterprise in China, we show that our method produces more accurate forecasts of lead time demands than do exponential smoothing, Croston's method and Markov bootstrapping method.

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Introduction

In process industries, the characteristics and requirements for inventory management of spare parts differ from those of other materials in several ways: service requirements are higher as the effect of stockouts may be financially remarkable, demand of parts may be extremely sporadic and difficult to forecast, the number of part types is usually very large, and prices of individual parts may be very high. As a result, enterprises in process industries may keep a large spare parts inventory, while the annual inventory turnover may be very low. To illustrate this point, we investigated a medium size petrochemical enterprise in China (its annual processing capacity is 400 million tons of petroleum). The enterprise has more than 35 000 types of spare parts, its annual average inventory in 2002 was 176.75 million Yuan RMB (1 US$ $\approx$ 8.3 Yuan RMB), while spare parts inventory accounted for 103.21 million Yuan RMB. The annual spare parts inventory turnover in 2002 was 0.58 times.

As Huiskonen (2001) pointed out, spare parts management is an important area of inventory research. Yet, besides a few publications on demand forecasting methods (eg, Croston, 1972; Rao, 1973; Willemain *et al*, 1994a, b, 2004; Johnston and Boylan, 1996; Ghobbar and Friend, 2003), spare parts research has mostly focused on inventory modelling. Accurate forecasting of demand is important in inventory control, but there are three main difficulties in forecasting the demand of spare parts.

Firstly, demand of spare part is often intermittent (Ghobbar and Friend, 2003). Intermittent demand is a random demand with a large proportion of zero values (Silver, 1981); this nature of demand makes forecasting especially difficult for spare parts (Willemain *et al*, 2004).

Secondly, historical data of spare part demand are usually very limited. This difficulty is further reinforced by the intermittent nature of spare part demand. Thus, it is hard to estimate the distribution of lead time demand (LTD) for spare part.

Thirdly, in some industries, spare parts inventory level is largely a function of how equipment is used and how it is maintained (Kennedy *et al*, 2002). This implies on one hand that demand of spare part at any time is a function of equipment maintenance operations and dependent on some explanatory variables and on the other hand that demand of spare part in consecutive periods may be a time series with autocorrelation. With the wide application of management information systems in equipment maintenance and spare parts inventory management, more information and data about explanatory variables (such as maintenance policy, preventive maintenance record, and emergent part replacement record of each equipment) are now available to improve the forecasting accuracy. There are some demand

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forecasting methods for spare parts dealing with autocorrelation (eg, Wang and Rao, 1992; Willemain *et al*, 2004), but we have not found any research result on regression analysis for evaluating the relationship of independent variables (or explanatory variables) with spare part demand.

During a project investigation on spare parts inventory management for the above mentioned petrochemical enterprise, we tried to deal with all these three difficulties and develop a new forecasting approach, which can synthetically evaluate autocorrelation and the relationship of explanatory variables with demand of spare part. Experimental results show that our method can considerably improve forecasting accuracy in comparison with other methods.

The remainder of the paper is structured as follows: We review and summarize forecasting methods for demand of spare part. Then we describe our forecasting method. Next, new performance measures for intermittent demand forecasting are introduced. We then describe the industrial data sets and illustrate our forecasting method by an example. We further assess relative accuracy of the various forecasting methods using the gathered industrial data sets. The paper ends with some concluding remarks.

Related research

Spare parts inventory management is often considered as a special case of general inventory management with some special characteristics, such as very low demand volumes and intermittent demand nature. The principal objective of any inventory management system is to achieve sufficient service level with minimum inventory investment and administrative costs. A presupposition of realizing this objective is that we know the accurate distribution of LTD for spare part. As elaborated in the previous section, demand forecasting for spare part is especially difficult.

The relevant literature to demand forecasting for spare part can be divided into three groups corresponding to the three difficulties described in the previous section.

Forecasting intermittent demand

As summarized by Willemain *et al* (2004), there are five types of forecasting methods for intermittent demand, that is, 'assessing or relaxing the standard assumptions for non-intermittent demand (Lau and Wang, 1987; Tyworth and O'Neill, 1997), non-extrapolative approaches, such as reliability theory and expert systems (Petrovic and Petrovic, 1992), variants of the Poisson model (Ward, 1978; Williams, 1982, 1984; Mitchell *et al*, 1983; Van Ness and Stevenson, 1983; Bagchi, 1987; Schultz, 1987; Watson, 1987; Dunsmuir and Snyder, 1989), simple statistical smoothing methods (Bier, 1984; Sani and Kingsman, 1997), and Croston's variant of exponential smoothing (ES) (Croston, 1972; Rao, 1973; Willemain *et al*, 1994a, b; Segerstedt, 1994; Johnston and Boylan, 1996)'.

Ghobbar and Friend (2003) evaluated some of the above methods by applying them to forecast the intermittent demand of aircraft's spare parts. They compared and evaluated 13 methods, that is, additive winter, multiplicative winter, seasonal regression model, component service life, weighted calculation of demand rates, weighted regression demand forecasters, Croston, single-ES, exponentially weighted moving average, trend-adjusted ES, weighted moving averages, double-ES, and adaptive-response-rate single-ES. They found that exponentially weighted moving average and Croston's methods outperformed other forecasting methods for intermittent demand, whereas the forecasting results of these methods measured by mean absolute percentage error (MAPE) are all greater than 80%, far above the acceptable level.

Bootstrap

The bootstrap, introduced by Efron (1979), is usually used as a tool for nonparametric estimation of sampling distribution and standard errors (Gamero *et al*, 1998). When historical data of spare part demand are limited, the bootstrap method is a powerful tool to estimate the distribution of LTD for spare part. Bookbinder and Lordahl (1989) found the bootstrap superior to the normal approximation for estimating high percentiles of LTD distributions for independent data. Wang and Rao (1992) also found the bootstrap effective to deal with smooth demand. All these papers do not consider the special problems of managing intermittent demand. Snyder (2002) proposed three forecasting methods (MCROST, LOG, and AVAR) specifically for intermittent demand, and developed a parametric bootstrap approach that can be used to forecast quantities rather than incidences of demand over lead time. Willemain *et al* (2004) provided an approach of forecasting the intermittent demand for service parts inventories. They developed a bootstrap-based approach to forecast the distribution of the sum of intermittent demands over a fixed lead time. Attributing to autocorrelation considered in the model and the patented jittering technique adopted in their method, they showed that their method was the most accurate forecasting method as compared with Croston's and ES method.

Regression analysis

An important point of Croston's variants of ES (eg, Croston, 1972; Johnston and Boylan, 1996; Willemain *et al*, 2004) is a compound of the forecasting of nonzero demand interval and the forecasting of nonzero demand volumes. Nonzero demand volumes in Croston (1972) were forecasted through ES, while Willemain *et al* (2004) forecasted them by the bootstrapping and jittering technique.

Nonzero demand of an equipment's spare part can be attributed to two factors: the first is a kind of statistic factor related to status of the equipment and the spare parts themselves (as described in Croston's method as a Bernoulli process, while in Willemain *et al* (2004) it was a process with autocorrelation and described as a Markov process); the second factor is related to the equipment maintenance policy. To estimate the distribution of LTD for spare part, it is important to recognize the impact of the second factor on the nonzero demand and establish the relationship between explanatory variables and the nonzero demand of spare part.

Some researchers used linear regression to study categorical variables (eg, Jovanis and Chang, 1986), which is doomed to larger error margins because the analysis violates the homoscedasticity assumption of linear regression. In a well-summarized review of models predicting the probability, Milton and Mannering (1997) pointed out that the use of linear regression models is inappropriate for making probabilistic statements about occurrence and they showed that the negative binomial regression is a powerful predictive tool. Some nonlinear regressions, such as loglinear analysis, logit modeling, negative binomial regression, and logistic modelling, have been used to explain the relationship between some categorical variables or to forecast the values of some categorical variables (James and Kim, 1996; Kim and Li 1996; Mercier *et al*, 1997; Nassar *et al*, 1997; Al-Ghamdi, 2002). But we have not found any research result that considers the relationship between explanatory variables (independent variables) and the nonzero demand of spare part (dependent variable).

Even if we have established a relationship between explanatory variables and the nonzero demand of spare part, we still need a mechanism to integrate this relationship with the Bernoulli process or the process with autocorrelation. Cointegration analysis can give us some hints on the integration mechanism although we cannot carry out cointegration analysis for a binary time series.

Cointegration analysis is often used to verify the relationship between economic variables (eg, Duy and Thoma, 1998; de Gooijer and Vidiella-i-Anguera, 2004), which is firstly conducted by a cointegration analysis to verify the stationary long-term relationship, and is then amended by adding the long-term trend of time series of economic variable. We adopt this two-phase integration in our forecasting approach for occurrences of nonzero demands of spare part, that is, we firstly carry out regression analysis to identify nonzero demand that can be attributed to explanatory variables; this regression result is then integrated into the forecast result of LTD.

The integrated forecasting method (IFM)

Let d_t be the observed demand in period t, $t = 1, 2, \ldots, n$, and L be the fixed lead time over which demand forecasts are desired. Our goal is to estimate the sum of the demands over the lead time, called the LTD:

$$LTD = \sum_{t=n+1}^{n+L} d_t \tag{1}$$

The main idea of the IFM is to firstly forecast the occurrence of nonzero demand, then estimate LTD.

Forecasting occurrence of nonzero demand

To forecast the occurrence of nonzero demand, we need at first to transform the demand time series $\{d_1, d_2, \ldots, d_n\}$ into a binary time series $\{y_1, y_2, \ldots, y_n\}$ in which 0 represents zero demand and 1 represents nonzero demand. We need to further discriminate which factor each nonzero demand should be attributed to: autocorrelation or explanatory variables.

We adopt the definition of autocorrelation function defined by Herzel and Große (1995) for symbol variable series to evaluate the autocorrelation of a binary time series. Denote by r_k the autocorrelation coefficient of order k for the binary time series $\{y_1, y_2, \ldots, y_n\}$, r_k is defined as

$$r_k = \frac{1}{n-k}\sum_{i=1}^{n-k} y_i y_{i+k} - \left(\frac{1}{n-k}\sum_{i=1}^{n-k} y_i\right) \times \left(\frac{1}{n-k}\sum_{i=1}^{n-k} y_{i+k}\right) \tag{2}$$

where $r_k \in [-1/4, 1/4]$. According to the above definition, $r_k = -1/4$ if the time series is completely negative autocorrelated of order k; $\lim_{n\to\infty} r_k = 1/4$ when the time series is completely positive autocorrelated of order k; and $r_k = 0$ indicates no autocorrelation. Similar to Willemain *et al* (2004), we assume that an autocorrelated binary time series is a first-order Markov process, that is, $k = 1$.

If the binary time series is strongly autocorrelated, we model autocorrelation using a two-state, first-order Markov process; otherwise, we need a mechanism to judge if each occurrence of nonzero demand can be attributed to explanatory variables.

Assume that there are m explanatory variables x_i, $i = 1, 2, \ldots, m$. Denote by $\{(x_1^t, x_2^t, \ldots, x_m^t) | t = 1, 2, \ldots, n\}$ the time series of explanatory variables corresponding to $\{y_1, y_2, \ldots, y_n\}$. As elaborated in the previous section, explanatory variables of spare parts demand are related to equipment maintenance policy, and are usually categorical variables. By performing the logistic regression, we can evaluate the probability of each occurrence of nonzero demand $\{\hat{p}_t | t = 1, 2, \ldots, n\}$ as follows (Al-Ghamdi, 2002):

$$\hat{p}_t = \frac{1}{1 + \exp(-(\alpha + \sum_{i=1}^{m} \beta_i x_i^t))} \tag{3}$$

Step 1: Solve $p_0 \in \arg\min_{p\in[0,1]} \sum_{t=1}^{n} (sign(\hat{p}_t - p) - y_t)^2$.

Step 2: For each $\hat{p}_t$ $(t=1,2,\cdots,n)$,

if $\hat{p}_t \geq p_0$, let $\hat{y}_t = 1$; otherwise, let $\hat{y}_t = 0$.

Step 3: Stop.

Figure 1 Main steps of Algorithm 1.

Step 0: Initialize $\bar{r}$, let $t = 1$.

Step 1: If $t \leq n$, goto Step 2; otherwise, goto Step 4.

Step 2: If $y_t = \hat{y}_t = 1$, then

let $S_t = \{y_1, y_2, \cdots, y_{t-1}, 1\}$,

and $S'_t = \{y_1, y_2, \cdots, y_{t-1}, 0\}$;

Calculate r_1 of S_t and r'_1 of S'_t;

If $r_1 < \bar{r}$ or $r_1 < r'_1$, let $y_t = 0$.

Step 3: Let $\bar{y}_t = y_t$, $t = t+1$, goto Step 1.

Step 4: Stop.

Figure 2 Main steps of Algorithm 2.

Time series $\{\hat{p}_t | t = 1, 2, \ldots, n\}$ can further be converted to a binary time series $\{\hat{y}_t | t = 1, 2, \ldots, n\}$ by Algorithm 1 (Figure 1).

In Step 1 of Algorithm 1,

$$sign(\hat{p}_t - p) = \begin{cases} 1 & \text{if } \hat{p}_t - p \geqslant 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Parameter p_0 is the threshold of probability, according to which $\{\hat{p}_t | t = 1, 2, \ldots, n\}$ is converted into $\{\hat{y}_t | t = 1, 2, \ldots, n\}$, and this conversion makes mismatches between $\{y_t | t = 1, 2, \ldots, n\}$ and $\{\hat{y}_t | t = 1, 2, \ldots, n\}$ minimal.

We then apply Algorithm 2 to judge whether an occurrence of nonzero demand can be attributed to explanatory variables or not, and obtain a 'purely' autocorrelated binary time series $\{\bar{y}_t | t = 1, 2, \ldots, n\}$ through replacing all occurrences of nonzero demands that can be attributed to explanatory variables by zeros (Figure 2).

In Step 0 of Algorithm 2, parameter $\bar{r}$ is a user-specified parameter and is introduced to balance the effect of explanatory variables and autocorrelation on the occurrence of nonzero demand. If the condition $y_t = \hat{y}_t = 1$ meets for most cases of $t \in \{1, 2, \ldots, n\}$, the user can set a larger $\bar{r}$ (eg, $\bar{r} = 0.1$); a half meets of the condition $y_t = \hat{y}_t = 1$ require a medium $\bar{r}$ (eg, $\bar{r} = 0.05$); rare meets of the condition $y_t = \hat{y}_t = 1$ require a smaller $\bar{r}$ (eg, $\bar{r} = 0.01$).

To obtain the purely autocorrelated time series $\{\bar{y}_t | t = 1, 2, \ldots, n\}$, Step 2 of Algorithm 2 lets $\bar{y}_t = 0$ for any $t \in \{1, 2, \ldots, n\}$ if nonzero demand ($y_t = 1$) can be attributed to explanatory variables ($\hat{y}_t = 1$) but cannot be attributed to autocorrelation ($r_1 < \bar{r}$ or $r_1 < r'_1$).

Step 0: Initialize r^*, let $\tilde{y}_{n+l} = 0$ for all $l \in \{1, 2, \cdots, \mathrm{L}\}$.

Step 1: Calculate r_1 of time series $\{y_1, y_2, \cdots, y_n\}$.

Step 2: If $|r_1| \geq r^*$, let $\tilde{y}_{t+l} = 1$ with probability p^A_{n+l}

for all $l \in \{1, 2, \cdots, \mathrm{L}\}$, goto Step 5;

otherwise, let $l = 1$, goto Step 3.

Step 3: If $l \leq L$, calculate $\hat{p}_{n+l}$, goto Step 4;

otherwise, go to Step 5.

Step 4: If $\hat{p}_{n+l} > p_0$, let $\tilde{y}_{n+l} = 1$ with probability $\hat{p}_{n+l}$;

otherwise, let $\tilde{y}_{n+l} = 1$ with probability p^A_{n+l}.

Let $l = l + 1$, goto Step 3.

Step 5: Stop.

Figure 3 Main steps of Algorithm 3.

We model the autocorrelated time series $\{\bar{y}_t | t = 1, 2, \ldots, n\}$ using a two-state, first-order Markov process. We are now ready to forecast the sequence of zero and nonzero values over the lead time periods. These forecasts are conditional on whether the last demand y_n is zero or not. We estimate the state transition probabilities from the autocorrelated time series $\{\bar{y}_t | t = 1, 2, \ldots, n\}$ using started counts (Mosteller and Tukey, 1977). Denote by p^A_{n+l} the state transition probability from state y_n to 1 in lead time l ($l = 1, 2, \ldots, L$) to be forecasted, then the process of forecasting occurrences of nonzero demands $\tilde{y}_{n+l}$($l = 1, 2, \ldots, L$) can be summarized in Figure 3.

In Step 0 of Algorithm 3, parameter r^* is a user-specified parameter that can be set in a similar way as $\bar{r}$. If we set $r^* = 0$, Algorithm 3 will be the same forecasting method as described in Willemain *et al* (2004), except for the patented jittering technique.

By applying Algorithm 3, we obtain a forecasting result of occurrence of nonzero demands, $(\tilde{y}_{n+1}, \tilde{y}_{n+2}, \ldots, \tilde{y}_{n+L})$, for the spare part over the fixed lead time. In a subsection of the case study section, detailed process of Algorithm 3 is illustrated through its application to a real data set.

Estimation of LTD

Once occurrences of nonzero demands over lead time have been forecasted, we need to give specific numerical values to the nonzero forecasts. We use the most direct way to do this by simply sampling from the nonzero values that have appeared in the past. Summing the forecasts over the lead time gives an estimation of *LTD* (denoted as $\widetilde{LTD}$). We repeat the process until we have enough bootstrap forecasts for estimating the entire distribution of *LTD*. The process can be summarized in Figure 4.

In Step 0 of Algorithm 4, parameter *Num* is the number of bootstrap forecasts, which is usually set 1000 for sufficient estimation of the distribution of *LTD*.

Step 0: Initialize *Num*, let *count* = 1.
Step 1: If *count* ≤ *Num*, let $\widetilde{\text{LTD}}=0$, goto Step 2; otherwise goto Step 4.
Step 2: For $l = 1$ to L do
  apply Algorithm 3 to obtain $\tilde{y}_{n+l}$;
  if $\tilde{y}_{n+l} = 1$, let $\tilde{d}_{n+l}$ equal to nonzero demand $d_t\ (t = 1, 2, \cdots, n)$ with equal probability;
  otherwise, let $\tilde{d}_{n+l}$ equal zero;
  $\widetilde{\text{LTD}} = \widetilde{\text{LTD}} + \tilde{d}_{n+l}$.
Step 3: Output $\widetilde{\text{LTD}}$, let *count* = *count* + 1, goto Step 1.
Step 4: Stop.

Figure 4 Main steps of Algorithm 4.

Forecasting accuracy measures

There are many forecasting accuracy measures for assessing the performance of forecasting methods, for example, mean error, sum of squared error, mean squared error, standard deviation of error, percentage error, mean percentage error, mean absolute error (MAE), and MAPE. These measures are inappropriate or insufficient to assess the performance of forecasting methods for intermittent demand.

As pointed out by Willemain *et al* (2004), during forecasting the intermittent demand of spare part, a more fundamental problem is that we need to assess the quality not of a point forecast of the mean but of a forecast of an entire distribution. They used the probability-integral transformation method to obtain the χ^2 statistic of fitted LTD distribution and used ANOVA to analyse the method's validity.

The forecasting accuracy-assessing method proposed by Willemain *et al* (2004) is applicable only when the number of spare part types is sufficiently large. We believe that the forecasting accuracy of occurrence of nonzero demands is more important than that of quantity of nonzero demands for intermittent process. Thus, measures such as mean square forecast error are not enough or inappropriate for intermittent demand. When one or more of the observed demands is zero, as is often true with intermittent demand, the MAPE is even undefined. To assess the performance of forecasting methods for intermittent demand of spare part, we propose two measures as follows:

The first measure is the error ratio of occurrences of nonzero demand judgements (*ERNJ*) over lead time, which is defined as

$$ERNJ = \frac{1}{L}\sum_{l=1}^{L}|\tilde{y}_{n+l} - y_{n+l}| \tag{5}$$

According to Algorithm 3, $\tilde{y}_{n+l} (l = 1, 2, \ldots, L)$ is a random binary variable, which is randomly set 1 with the probability of $\hat{p}_{n+l}$ or p^A_{n+l}. Therefore, *ERNJ* is also a random variable. For any given data set over a specific lead time, to compare the *ERNJ* of IFM with that of Markov bootstrapping (MB) method, we bootstrap $\tilde{y}_{n+l} (l = 1, 2, \ldots, L)$ *Num* times, and then calculate the average *ERNJ* (denote by *AERNJ*) of each forecasting method.

It is noteworthy that the measure *ERNJ* defined in Equation (5) is only applicable to binary forecast result. To assess the performance of forecasting methods on *LTD*, we introduce the next measure.

The second measure is the absolute percentage error of *LTD* (*APELTD*). When *LTD* is not zero, *APELTD* is defined as

$$\begin{aligned} APELTD &= \frac{\left|\widetilde{LTD} - LTD\right|}{LTD} \times 100\% \\ &= \frac{\left|\sum_{l=1}^{L} \tilde{d}_{n+l} - LTD\right|}{LTD} \times 100\% \end{aligned} \tag{6}$$

Considering the bootstrap in estimating the *LTD*, we denote by *MAPELTD* the average *APELTD* over bootstrap forecasts.

When *LTD* is zero, *APELTD* is not defined. In this situation, we can directly compare the statistics of $\widetilde{LTD}$ of different forecasting methods, which is equivalent to comparing MAE of forecasted *LTD*.

A case study

To illustrate the performance of our IFM method, we investigated a medium size petrochemical enterprise in China and gathered historical demand data sets of spare parts and their corresponding facility maintenance information from different points in its process flow. In this section, we first describe the main characteristics of the data sets; an illustrative example is then used to show what the data set looks like and how algorithms in our IFM method are applied to forecast nonzero demands of the spare part over its lead time; we further assess the relative accuracy of the various forecasting methods using the gathered industrial data sets.

Industrial data sets

The above-mentioned petrochemical enterprise has more than 16 000 types of machinery spare parts, of which only about 5000 types have available historical demand data. To describe facility maintenance information, we need the bill of material (BOM) of equipment, whereas only about 2000 of 5000 types of spare parts in the enterprise have BOM information of their corresponding equipments. We exclude many spare parts whose occurrences of nonzero demand in the past five years are less than five times, since these items lacked any basis for IFM. As demands of different spare parts in the same equipment may be similar, we thus choose

Table 1 Summary statistics for monthly intermittent demand data

	% Zero values	*Average of nonzero demands*	*CV of nonzero demands*
Mean	81.1460	2.0237	0.1095
Std. dev.	6.7937	2.2728	0.2346
Maximum	95.6044	12.0000	1.2122
75%ile	86.0233	2.0000	0.1675
50%ile	82.2070	1.0417	0
25%ile	79.0963	1.0000	0
Minimum	54.5455	1.0000	0

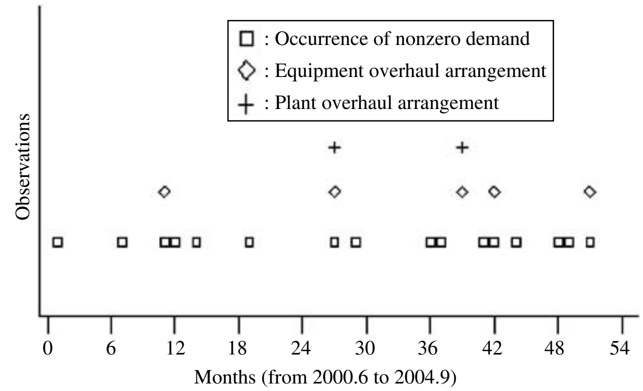

Figure 5 Raw data of the example spare part from 2000.6–2004.9.

and analyse historical data sets of 40 types of spare parts as representatives in this case study.

In our case study, we choose plant and equipment overhaul arrangements as two explanatory (predictor) variables. Each of the two explanatory variables is binary, whose value of 1 represents a plant or equipment overhaul arrangement in the corresponding month, while 0 represents no such overhaul arrangement.

All observations, including the demands of spare parts, plant, and equipment overhaul arrangements are monthly data. Table 1 summarizes the statistical characteristics of the intermittent demands of 40 types of spare parts. The total number of observations available per item, including observations we held out to assess accuracy, ranged from 36 to 72. Typically, well over half of the data values for an item were zeros. Nonzero demands were generally steady, as evidenced by the small values of the coefficient of variation (CV).

An illustrative example

To further illustrate the intermittent nature of spare part demands, we take one spare part as an example, whose demand series over 52 months (from 2000.6 to 2004.9) is {1,0,0,0,0,0,1,0,0,0,1,1,0,1,0,0,0,0,1,0,0,0,0,0,0,0,1,0,1,0,0,0, 0,0,0,1,1,0,0,0,1,1,0,1,0,0,0,2,2,0,1,0}. It can be found that only 16 of 52 months have nonzero demand as shown in Figure 5.

Plant and equipment overhaul arrangements corresponding to this spare part are also depicted in Figure 5, that is, there are two times of plant overhaul arrangement and five times of equipment overhaul arrangement during the period.

We split the 52 months into two periods: the first period is used to establish the relationship about the explanatory variables and estimate transition probabilities; the second period is held out for the purpose of assessing accuracy of the forecasting methods.

Considering the requirement of statistical analyses and the intermittent nature of spare part demand, we usually set a lower limitation of the length of the first period as 3 years (ie, 36 months). In this case study, the hold-out period varies from 1 month to 8 months. We set 8 months as the maximal length of the hold-out period because arrangement of plant or equipment overhaul beyond this period might be unavailable.

If we hold out the last 4 months (2004.6–2004.9) of this example data to assess the accuracy of the forecasting methods, the demand time series d_t $(t=1,2,\ldots,n)$ in the first period, is {1,0,0,0,0,0,1,0,0,0,1,1,0,1,0,0,0,0,1,0,0,0,0,0,0, 0,1,0,1,0,0,0,0,0,0,1,1,0,0,0,1,1,0,1,0,0,0,2}. To study the demand incidences, we first transform d_t $(t=1,2,\ldots,n)$ into a binary time series y_t $(t=1,2,\ldots,n)$ as {1,0,0,0,0,0,1,0, 0,0,1,1,0,1,0,0,0,0,1,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,1,1,0,0,0, 1,1,0,1,0,0,0,1}.

We then calculate $\hat{y}_t$ $(t=1,2,\ldots,n)$ by applying Algorithm 1. Results of $\hat{y}_t$ $(t=1,2,\ldots,n)$ show that occurrences of nonzero demand at $t=11$, 27, and 42 can be attributed to explanatory variables.

We further verify if y_{11}, y_{27}, and y_{42} can also be attributed to autocorrelation by applying Algorithm 2. Through calculating the first-order autocorrelation coefficient and by setting $\bar{r}=0.08$, it is found that none of them can be attributed to autocorrelation. As a result of Algorithm 2, we obtain a pure autocorrelated time series $\bar{y}_t$ $(t=1,2,\ldots,n)$ as {1,0,0,0,0,0,1,0,0,0,0,1,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0, 0,0,0,0,0,1,1,0,0,0,1,0,0,1,0,0,0,1}, which is identical with $\{y_1, y_2, \ldots, y_{48}\}$, except for $\bar{y}_{11}=\bar{y}_{27}=\bar{y}_{42}=0$. For the given lead time $(L=4)$, we can estimate the state transition probabilities p^A_{n+l} $(l=1,2,\ldots,L)$, by using the pure autocorrelated time series $\bar{y}_t$ $(t=1,2,\ldots,n)$.

As shown in Step 2 of Algorithm 3, if $\{y_1, y_2, \ldots, y_n\}$ is a very strongly autocorrelated time series, we forecast $\tilde{y}_{n+l}$ $(l=1,2,\ldots,L)$ just by using Markov chain, which does not occur in this illustrative example and occurred rarely in all our computation studies. The main purpose of Step 2 in Algorithm 3 is to avoid any possibly negative impact of regression on forecast result when a demand time series is very strongly autocorrelated.

In most cases, that is, when $\{y_1, y_2, \ldots, y_n\}$ is not a very strongly autocorrelated time series, Markov chain or the explanatory variables relationship is used to predict $\tilde{y}_{n+l}$ $(l=1,2,3,4)$. If an occurrence of nonzero demand at $t=n+l$ $(l=1,2,3,4)$ can be attributed to explanatory variables (ie, $\hat{p}_{n+l}>p_0$), we let $\tilde{y}_{n+l}=1$ with the probability $\hat{p}_{n+l}$; otherwise we let $\tilde{y}_{n+l}=1$ with the probability p^A_{n+l}. In this illustrative example, only $\tilde{y}_{n+3}$ should be attributed to the explanatory variables.

Results

We compared ES, Croston's method and MB method (it is the same as described in Willemain *et al* (2004) except for the patented jittering technique) with our IFM method. In this case study, we set $r^*=0.1$ and $\bar{r}=0.08$ in IFM, and set the smoothing constant of Croston's method $\alpha=0.2$ (following the suggestion of Croston, 1972).

Recall that the measure *ERNJ* is only applicable to binary forecast result. Since Croston's method was developed to provide an estimate of the mean demand per period rather than occurrence of nonzero demand in lead time, among the four methods, only MB method and our IFM method can predict the occurrence of nonzero demand in lead time. Thus, we only compare IFM with MB on *AERNJ*, and denote *AERNJ* improvement percentages of IFM over MB by $AERNJ_{IP}=(AERNJ_{MB}-AERNJ_{IFM})/AERNJ_{MB}\times 100\%$. Table 2 shows statistical results on *AERNJ*. As shown in Table 2, IFM is significantly better than MB in terms of *AERNJ*. This observation is confirmed by paired two-tailed t-test on *AERNJ* of IFM *versus* MB with 320 samples, where t-statistics is -30.15 and P-value is 0.000 (at the 0.01 significance level). Average *AERNJ*s over 40 datasets of IFM and MB when lead time varies from 1 to 8 months are illustrated in Figure 6.

For the 40 industrial data sets, we compared the four methods (ie, ES, Croston, MB, and IFM) on *MAPELTD* using lead times of 1–8 months ($L=1$–8 months). Table 3 shows statistical results on *MAPELTD* when *LTD* is nonzero; Table 4 shows statistical results of forecasted *LTD* (ie, $\widetilde{LTD}$) when *LTD* is zero. Tables 5 and 6 summarize all pairwise comparisons on forecasted *LTD* with different lead times (from 1 to 8 months) between the four methods by applying a paired two-tailed t-test for all 40 kinds of spare parts when *LTD* is nonzero and zero, respectively. In Tables 5 and 6, '$\approx$' indicates that there is no significant difference between the corresponding row and column methods, while '$<$' indicates that the column method is significantly better than the row method at the 0.01 significance level, and the values in brackets are the corresponding P-values of t-test. Comparison of the four methods on average *MAPELTD* over data sets with nonzero *LTD* under different lead times is depicted in Figure 7.

For the lead time of eight months ($L=8$ months), a detailed comparison on *MAPELTD* of four methods over all 40 data sets are reported in Table 7.

The above results, supplemented by paired two-tailed t-tests, support the following statements.

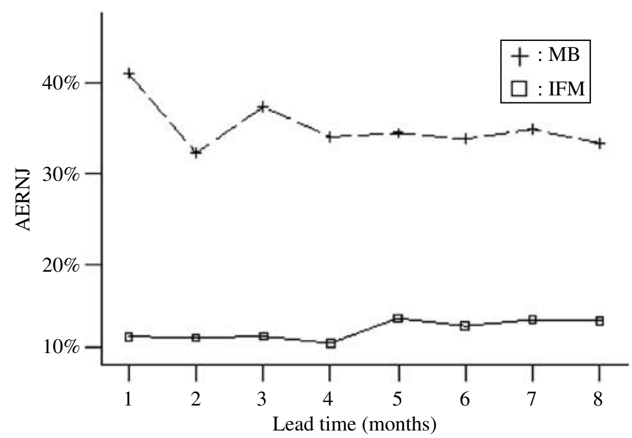

Figure 6 Comparison between IFM and MB on *AERNJ* over different lead times.

Table 2 Statistical results on *AERNJ* ($L=1$–8 months)

# of Samples = 320	$AERNJ_{MB}$	$AERNJ_{IFM}$	$AERNJ_{IP}$
Mean	0.3508	0.1187	0.6784
Std. dev.	0.1515	0.1161	0.2869
95% confidence interval			
Lower	0.3342	0.1060	0.6468
Upper	0.3675	0.1315	0.7099

Table 3 Statistical results on *MAPELTD* ($L=1$–8 months, $LTD>0$)

# of Samples = 266	*ES*	*Croston*	*MB*	*IFM*
Mean	0.4586	0.4318	0.4521	0.2229
Std. dev.	0.2816	0.3148	0.2259	0.2005
95% confidence interval				
Lower	0.4246	0.3938	0.4248	0.1987
Upper	0.4926	0.4698	0.4794	0.2471

Table 4 Statistical results on $\widetilde{LTD}$ ($L=1$–8 months, $LTD=0$)

# of Samples = 54	*ES*	*Croston*	*MB*	*IFM*
Mean	0.7576	0.4929	0.4672	0.1427
Std. dev.	1.0837	0.4594	0.7833	0.2466
95% confidence interval				
Lower	0.4618	0.3676	0.2534	0.0753
Upper	1.0534	0.6183	0.6810	0.2100

Table 5 Methods comparison on $MAPELTD$ ($LTD>0$)

# of Samples = 266	*MB*	*Croston*	*IFM*
ES	≈(0.679)	≈(0.115)	<(0.000)
MB		≈(0.214)	<(0.000)
Croston			<(0.000)

Table 6 Methods comparison on $\widetilde{LTD}$ ($LTD=0$)

# of Samples = 54	*MB*	*Croston*	*IFM*
ES	<(0.000)	<(0.007)	<(0.000)
MB		≈(0.729)	<(0.004)
Croston			<(0.000)

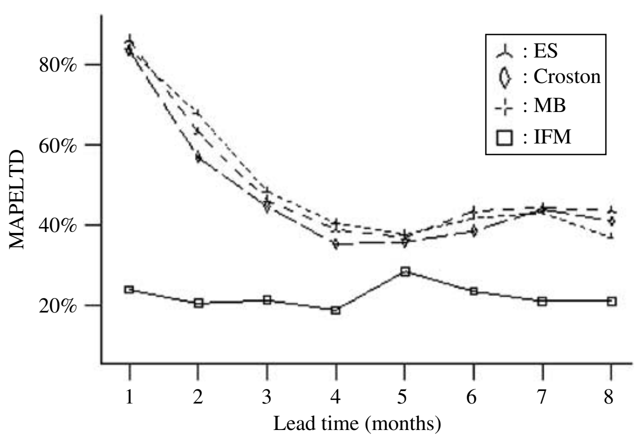

Figure 7 Comparison of four methods on $MAPELTD$ over different lead times.

(1) IFM is significantly better than MB on forecasting occurrences of nonzero demand.
(2) In comparison with ES, Croston's method, and MB method, IFM is the leading approach in terms of accuracy of forecasted LTD, no matter LTD is zero or not.
(3) Comparison of ES, Croston's method, and MB method shows that they have no significant difference on forecasting LTD in our case study, though Croston's method can provide a more accurate estimate of the mean demand over lead time.
(4) Lead time has no significant effect on the accuracy of forecasting results in this case study. This result is similar to that observed by Willemain *et al* (2004).

Conclusions

In process industries, inventory management of spare parts usually requires a high service level. To gain the end, we need exact estimation of LTD for spare parts. Owing to the intermittent nature, demand forecasting for spare parts is especially difficult.

In this paper, we develop a new approach of forecasting intermittent demand for spare parts inventories in process industries. The described approach provides a mechanism to integrate the autocorrelated demand process and the relationship between explanatory variables and nonzero demand of spare part in forecasting occurrences of nonzero demands over lead times. Two types of measures for assessing the performance of forecasting methods are described, which are simple and applicable for general intermittent demand forecasting.

Table 7 $MAPELTD$ comparisons of four methods over 40 data sets ($L=8$ months)

Data set	*ES*	*Croston*	*MB*	*IFM*	*Data set*	*ES*	*Croston*	*MB*	*IFM*
1	0.2930	0.1614	0.0785	0.0565	21	0.1782	0.0279	0.0645	0.0795
2	0.0276	0.1224	0.0450	0.0165	22	0.0081	0.2683	0.3555	0.2870
3	0.8099	0.3663	0.7280	0.0000	23	0.6878	0.9618	0.2575	0.1860
4	0.5065	0.0233	0.1685	0.2880	24	0.9998	2.8139	0.2200	0.0000
5	0.5103	0.5098	0.4930	0.3940	25	0.5899	0.2786	0.4150	0.0860
6	0.5094	0.5562	0.5820	0.3765	26	0.5702	0.3599	0.3995	0.1350
7	0.4256	0.4781	0.5554	0.5290	27	0.6108	0.1229	0.3080	0.0940
8	0.3670	0.3443	0.4060	0.2705	28	0.5051	0.5543	0.5757	0.1523
9	0.6763	0.6182	0.6953	0.6173	29	0.0803	0.1962	0.3155	0.1285
10	0.4370	0.2000	0.1170	0.0735	30	0.0098	0.1734	0.3485	0.1550
11	0.3825	0.2625	0.1735	0.0720	31	0.6149	0.6578	0.6137	0.3730
12	0.6603	0.7493	0.7536	0.6348	32	0.3981	0.3903	0.4405	0.1945
13	0.0059	0.5726	0.6310	0.5183	33	0.1827	0.4687	0.5085	0.3975
14	0.4500	0.2602	0.1560	0.0260	34	0.5976	0.4625	0.5295	0.2510
15	0.6769	0.2094	0.0020	0.0400	35	0.4820	0.5263	0.5344	0.1091
16	0.0881	0.2711	0.3460	0.0930	36	0.2780	0.2367	0.2820	0.4155
17	0.0906	0.1302	0.2643	0.0543	37	0.5354	0.4835	0.5003	0.1980
18	0.4737	0.5350	0.5470	0.3417	38	0.3189	0.2411	0.2595	0.2245
19	0.3298	0.0700	0.1555	0.0000	39	1.1009	0.0229	0.1210	0.2000
20	0.4882	0.1913	0.2345	0.0910	40	0.4848	0.5098	0.5470	0.2705

Based on the historical demand data sets of 40 kinds of spare parts from a petrochemical enterprise in China, we compared the performances of ES, Croston's method, and MB method with our IFM. Statistical results show that our method performs best across almost all the lead times.

Since our method can be easily implemented in practice, it has applications to a wide variety of enterprises both in process industries and in facility-based service industries. As shown in this paper, our IFM method performs better than other methods on forecasting the intermittent demand of spare part, because we bring together the special properties of intermittent demand with a logistic regression, which is reflective of corresponding equipment's BOM information and facility overhaul arrangements. To do the logistic regression, our IFM method requires, as usual regression analyses do, that the number of nonzero demands in the historical data set should not be too small.

With the wide application of management information systems in equipment maintenance, more data and information relative to spare part demand are now available to improve forecasting accuracy. In a further application study of IFM, potential explanatory variables other than plant and equipment overhaul arrangements might be defined as interaction between plant and equipment overhauls, workload of equipment, and equipment maintenance mode (outsourcing or in-house).

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