2023-01-0541 Published 11 Apr 2023

Moments of Power: Statistical Analysis of the Primary Energy Consumption of a Vehicle

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Citation: Steffen, T., Jegede, T., and Knowles, J., "Moments of Power: Statistical Analysis of the Primary Energy Consumption of a Vehicle," SAE Technical Paper 2023-01-0541, 2023, doi:10.4271/2023-01-0541.

Received: 23 Oct 2022 Revised: 21 Dec 2022 Accepted: 22 Jan 2023

Abstract

The energy consumption of a vehicle is typically determined either by testing or in simulation. While both approaches are valid, they only work for a specific drive cycle, they are time intensive, and they do not directly result in a closed-form relationship between key parameters and consumption. This paper presents an alternative approach that determines the consumption based on a simple analytical model of the vehicle and statistical parameters of the drive cycle, specifically the moments of the velocity. This results in a closed-form solution that can be used for analysis or synthesis.

The drive cycle is quantified via its moments, specifically the average speed, the standard deviation of the speed as well as the higher order moments skewness, and the kurtosis. A mixed quadratic term is added to account for acceleration or aggressiveness, but it is noticeably distinct from the conventional metric of positive kinetic energy (PKE). The vehicle is quantified using a polynomial model of the traction force and of the primary energy consumption of the powertrain. This model form fits both conventional and electrified powertrains, including all the component efficiencies.

Through a statistical analysis of the model, the primary energy consumption can be related to both the model parameters and the statistical properties of the drive cycle. This result can be useful for the analysis of a drive cycle, for the analysis of a powertrain, for economy optimization, and for control purposes. An example of a Nissan LEAF powertrain is presented over different cycles.

Introduction

Drive cycles and simulations have been used for decades to establish the primary energy consumption of vehicles, covering conventional, hybrid, and electric powertrains. Many cycles have regulatory significance, specifying tests to determine headline consumption figures and for emission compliance [1]. However, while the impact of the cycles on the results has been long known in the industry, it is difficult to analyze from a simulation or experimental point of view. One of the few papers looking at this problem is [2].

The aim of this paper is to propose a different methodology for calculating energy consumption. It is based on statistical metrics of the drive cycle, and it use a high-level train model to calculate the consumption from these statistical parameters. This approach is time free in that it does not require a simulation of the vehicle model over time. The advantage of this approach is an analytical relationship that is easier to analyze, easier to use constructively, and much faster to evaluate than conventional approaches.

The paper is organized as follows: the Background section introduces three important background area, specifically the use of drive cycles for measuring primary energy consumption, the method of moments, and the statistical modelling of powertrains. Based on this, the main result will be derived in the Analysis section, followed by the Numerical Results section. The paper finishes with a Conclusion section.

Background

Drive Cycles

Drive cycles are driving cycles define how a vehicle is driven longitudinally, using a speed profile over time. Drive cycles are used extensively when testing vehicle, typically using a wheel or chassis dynamometer. Table 1 lists all the cycles used in this paper.

Drive cycles are often specified in regulatory tests for primary energy consumption and for vehicle emissions. This approach is well established with early cycles dating to the 1970s. It has several advantages: it simplifies testing, and it allows for a comparison of different vehicles under identical conditions. [3]

However, drive cycles are not always representative of real-world driving, and this has led to the introduction of Real Driving Emission under the EURO 6d regulation package by the UNECE WP.29 forum [4]. Real-world driving is not (yet)

TABLE 1 Common Drive Cycles and Their Properties

Cycle	Duration in s	Stopped in s	Mean Speed in m/s	Absolute Accel. in *m/s²*	PKE in *m/s²*
us ftp75	1877	340.3	9.47	0.40	1.64
us us06	600	39.6	21.48	0.61	4.48
eu nedc	1100	213.4	10.01	0.27	1.12
wltp 3b	1800	226.9	12.92	0.36	1.99
cadc urm150	3145	307.9	16.43	0.41	2.68
cadc urban	993	262.3	4.90	0.50	1.53
cadc rural	1083	30.5	15.95	0.41	2.79
cadc mw150	1069	15.3	27.64	0.34	3.62

used for establishing consumption, because it is not considered reproducible enough for this purpose.

Powertrain models and cycles are used for control design in many papers, such as [5]. One of the challenges is that cycles may not be representative, and control structures that work well on one cycle may not perform on another. This is a regulatory challenge, and different regulatory regimes take different approaches to prevent it.

Typical measures calculated for cycles are the average speed, average absolute (or positive) acceleration, and the positive kinetic energy (PKE), which is the product *va* of speed and acceleration for positive accelerations [6, 7, 8, 9]. These are listed in Table 1 for a selection of common cycles.

However, while correlations between cycle properties and outcomes have been observed and used for inductive economy models, the validity of these remains limited. So far, no analytical (deductive) relationship has been demonstrated between the cycle and the test result. Because of this, there is no accepted way of measuring how "challenging" a cycle is, although there is general agreement that it depends on both speed and acceleration.

For the purpose of this document, a drive cycle will be defined by its duration *T*, the speed profile *v*(*t*) and the acceleration profile

$$\mathrm{a}(\mathrm{t}) = \frac{\mathrm{d}}{\mathrm{dt}}\mathrm{v}(\mathrm{t}) \tag{1}$$

All the data is handled in SI units (meters *m* and seconds *s*). Since the cycle is usually defined only at discrete sample times, linear interpolation is used for the speed, and thus constant acceleration is assumed between sampling points.

Using Moments

The analysis of drive cycles is performed using the method of moment (sometimes called higher-order statistics), which has been shown to help qualify the impact of uncertainties on a system. [10,11]

The nth (raw) moment μ_n' of the velocity profile $v(t)$ is defined as

$$\mu_n = \mathbb{E}\left[v^n\right] = \frac{1}{T}\int_{t=0}^{T} v(t)^n \, dt \tag{2}$$

$\mathbb{E}[x]$ denotes the expected value or mean of *x*. This equation uses the deterministic interpretation of the mean, calculated as the integral over time from a known profile.

It is also possible to calculate the moment from a probability distribution:

$$\mu_n = \mathbb{E}\left[v^n\right] = \int_{v=-\infty}^{+\infty} v^n p(v)\, dv$$

where $p(v)$ is the probability density function (PDF) of speed *v*. This probabilistic interpretation will not be used here, but it may be useful for future applications. Note that using the probabilistic definition can lead to wide uncertainties for higher moments, but this is not a problem for the deterministic definition.

It is easy to show that $\mu_0' = 1$. The first raw moment is known as the expected value or mean of the signal, and the shorthand $\mu = \mu_1'$ is used. The second raw moment is the mean square or signal energy.

It is common to use central moments μ_n instead of raw moments, because they are numerically more stable. Central moments are defined with respect to the average μ_1':

$$\mu_n = E\left[\left(v - \mu_1'\right)^n\right] = \frac{1}{T}\int_{t=0}^{T} \left(v(t) - \mu_1'\right)^n dt \tag{3}$$

and they can be calculated from the raw moments using binomial expansion:

$$\mu_n = \sum_{i=0..n} \binom{n}{i} \mu_i' \left(-\mu\right)^{n-i} \tag{4}$$

Note that $\mu_1 = 0$. The second central moment is known as variance or square of the standard deviation: $\mathbb{V}(v) = \mu_2 = \sigma^2$.

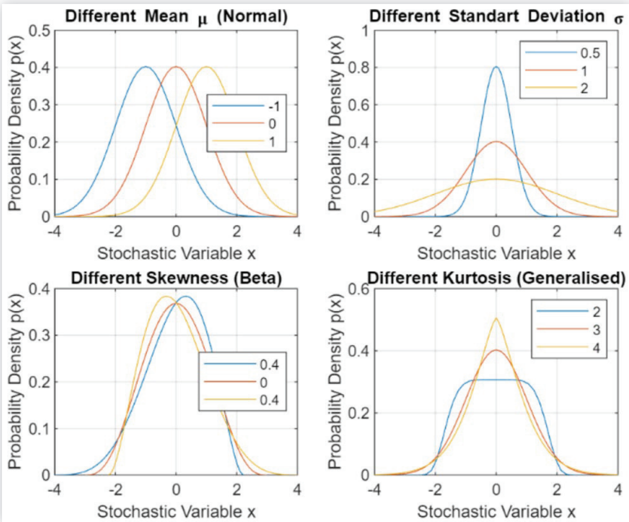
Sometimes, normalized central moments are used $\frac{\mu_n}{\sigma^n}$. They have the advantage of being dimensionless numbers and thus easier to compare. The third and fourth normalized moments are called skewness and kurtosis, and they are 0 and 3 for a normal distribution. Higher moments do not have a name.

Figure 1 shows how the first four moments affect a distribution, starting from a normal distribution $\mathcal{N}(0,1)$.

Moments are one way of dealing with unspecified distributions. An alternative would be to use binning and discretize the distribution, but that often causes higher errors. If a sufficient number of moments is known, this gives a pretty good idea of the distribution and its effect on the vehicle.

Moments can be defined across several variables, using a product of monomial terms. These are called mixed moments, and they will be used here to deal with the impact

FIGURE 1 Distributions Showing Different Moments



of acceleration. The most notable mixed moment is the covariance, denoted as $\mathbb{E}[va]$ for the raw covariance. Note that $\mathbb{E}\left[v^n a\right] = 0$ as long as $v(0) = v(T)$. Usually, both are zero, but this may not hold for cycle segments.

Polynomial Vehicle Models

This paper uses polynomial models, which have been found to have distinct advantages over conventional look-up tables. [12] For a vehicle model on a flat road without wind, the tractive effort is calculated as

$$F = ma + c_0 + c_1 v + c_2 v^2 \tag{5}$$

Here, a is the acceleration, v is the speed, m is the weight of the vehicle, c_0 represents tire friction, c_1 is viscous friction coefficient, and c_2 is a coefficient for aerodynamic drag. Note how the tractive effort is a polynomial in speed v and acceleration a, and the analysis will stay within this polynomial class as a matter of convenience.

From the tractive effort, the tractive power can be calculated:

$$P = Fv = mav + c_0 v + c_1 v^2 + c_2 v^3 \tag{6}$$

A polynomial model is used for the powertrain, to calculate the primary power demand from the tractive power. To keep the equations simple, a quadratic model is used here:

$$Y = \alpha_0 + \alpha_1 P + \alpha_2 P^2 \tag{7}$$

where α_i are the coefficients of this polynomial. This is known a backward-facing model, because it takes the power demand as the model input. This is distinct from a forward-facing model, where the accelerator pedal position would be the model input, and therefore some kind of driver model is required.

Quadratic models have a long history in powertrains, because they capture the diminishing efficiency at very low or very higher operating points appropriately. Obviously, this is a very simple model, but the method can be extended to higher order polynomials, models in two variables, dynamic models, and even piecewise defined models. However, the equations do get a lot more complicated, and so these extensions are best resolved using a computer (either algebraically or numerically).

Inserting the vehicle model into the powertrain model leads to another polynomial model:

$$\begin{aligned} Y = \alpha_0 + \alpha_1\left(mav + c_0 v + c_1 v^2 + c_2 v^3\right) + \\ \alpha_2\left(mav + c_0 v + c_1 v^2 + c_2 v^3\right)^2 \end{aligned} \tag{8}$$

$$\begin{aligned} Y &= \alpha_0 + \alpha_1 c_0 v + \left(\alpha_1 c_1 + \alpha_2 c_0^2\right) v^2 + \left(\alpha_1 c_2 + 2\alpha_2 c_0 c_1\right) v^3 \\ &+ \alpha_2\left(c_1^2 + 2c_0 c_2\right) v^4 + 2\alpha_2 c_1 c_2 v^5 + \alpha_2 c_2^2 v^6 \\ &+ \left(\alpha_1 v + 2\alpha_2 c_0 v^2 + 2\alpha_2 c_1 v^3 + 2\alpha_2 c_2 v^4\right) ma + \alpha_2 m^2 v^2 a^2 \end{aligned} \tag{9}$$

Aims and Objectives

The aim of the paper is summarized in Figure 2. Instead of running a simulation to calculate a running total of primary energy consumption, the aim is to find the consumption directly from statistical properties of the cycle.

This leads to the following two objectives:

O1: Calculate the primary energy consumption using the moments of the cycle.

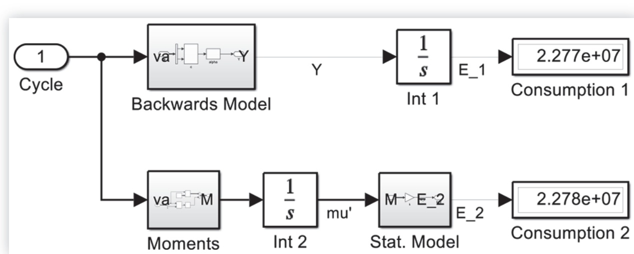
O2: Determine the sensitivity of the consumption to the different moments.

Consumption Estimation

Usually, the consumption is estimated by simulating the model, resulting in a primary power profile $Y(t)$. The primary energy consumption can then be calculated based on the definition of energy as

$$E = \int_{t=0}^{T} Y(t)\, dt \tag{10}$$

FIGURE 2 Aim of the Paper: Reformulate Consumption in Terms of Statistical Properties of the Drive Cycle



The answer from this analysis is that four factors are key – the first three moments and a measure for the aggressiveness:

1. the average speed μ
2. the standard deviation of the speed σ
3. the skewness of the speed s
4. the mixed moment $\mu_{av} = \mathbb{E}\left[\left(av\right)^2\right]$

This result is notably different from conventional wisdom, where the skewness is not considered, and the aggressiveness is measured using the positive kinetic energy (PKE) or $\frac{1}{2}\mathbb{E}\left[\left|av\right|\right]$. The later point requires further analysis, because PKE and μ_{av} are closely correlated. But the inclusion of the skewness is without doubt a significant contribution to knowledge.

This result has many potential implications from assessing different drivers for how economically they drive over statistics for the variation of typical usage cycles to testing of vehicles in the laboratory and on the road in real driving emissions (RDE). This result is also helpful for the development of representative synthetic drive cycles, and in the synthesis of complex or predictive controllers.

Extensions and Limitations

The quadratic powertrain model used here is very simple, with only one variable, three parameters, and no dynamics. This approach can easily be extended to more sophisticated models with several variables (such as torque and speed), higher order polynomials, and even derivatives to include fast dynamic effects such as turbo-lag. The equations would get longer, but still manageable with a computer algebra system, especially if central moments are used to eliminate insignificant terms as demonstrated. This would also enable to use of this approach for other outputs such as losses or toxic emissions, which requires more sophisticated models.

Slow dynamics such as the engine temperature, battery state of charge, or the condition of the aftertreatment system, are much more challenging to include without a simulation model. There is no easy approach from a statistical perspective, and they may require the segmentation of drive cycles into different segments (such as a cold and a warm segment). There is present in the regulation for such a segmentation.

Implementing limits and piecewise defined models is clearly indicated. Modelling conventional vehicle requires different models for acceleration and braking, which are hard to combine into one polynomial model. Further research in this area is ongoing using different approaches, but they either require a relaxation of the separation between model and cycle, or additional assumptions about the intermediate distributions.

Finally, most powertrains include discrete states, such as gear number, engine modes etc., which may show significant hysteresis. It is not obvious how the impact of that hysteresis can be included in the framework. There are a few potentially promising approaches such as discrete/continuous Markov Chains, or the describing function analysis for nonlinear systems. This remains an interesting area for further study.

Conclusion

This paper has introduced a new methodology for estimating the primary energy consumption over a drive cycle, which separate the analysis of the cycle from the analysis of the vehicle and powertrain. This leads to a closed loop form of consumption based on the moments of the cycles. For numerical analysis, this represents an extreme improvement in computational efficiency, but it also opens further opportunities through algebraic analysis, with many applications in analysis, design, calibration, and optimization.

One interesting result is that the difficulty of cycle for consumption can be measured using the first three moments plus the aggressiveness term $\mathbb{E}\left[\left(av\right)^2\right]$. This term is directly relevant, and more suitable to measure aggressiveness than conventional measures such as the absolute acceleration $\mathbb{E}\left[\left|a\right|\right]$ or the positive kinetic energy $0.5\,\mathbb{E}\left[\left|av\right|\right)\right]$.

The method is generally applicable to a wide range of powertrain technologies, but further work is necessary to ensure that it works with switching control functions such as gear shifts or mode transitions.

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All data and all the code used in the creation of this paper will be available as a data publication at https://github.com/LU-Centre-for-Autonomous-Systems/moments_of_power. The integration of the core functions into the ELVIO library [16] is planned for the future.

Acknowledgments

This paper is not part of a funded project, but it is based on previous results sponsored by the Advanced Propulsion Centre (APC)/UKRI Innovate UK as Project ViVID under grant number 113210.

Symbols

$\boldsymbol{a(t)}$ - Acceleration
$\boldsymbol{\alpha_n}$ - Powertrain model coefficients, n=0..2
$\boldsymbol{c_i}$ - Vehicle model coefficients
$\boldsymbol{E}$ - Primary (electric) energy consumption
$\mathbb{E}[\boldsymbol{x}]$ - Expected value of x
$\boldsymbol{F}$ - Tractive effort (force)
$\boldsymbol{k_n'}$ - Coefficient for raw moments, $n = 0..6$
$\boldsymbol{k_n}$ - Coefficients for central moments, $n = 0..6$
$\boldsymbol{m}$ - Vehicle mass
$\boldsymbol{\mu}$ - Mean, short for μ_1'
$\boldsymbol{\mu_n'}$ - nth raw moment, $n = 0..6$
$\boldsymbol{\mu_n}$ - nth central moment, $n = 0..6$
$\boldsymbol{\mu_{av}}$ - Mixed moment $\mathbb{E}\left[(av)^2\right]$
$\boldsymbol{P(t)}$ - Tractive (mechanical) power
$\boldsymbol{\sigma}$ - Standard deviation
$\boldsymbol{s}$ - Skewness
$\boldsymbol{t}$ - Time
$\boldsymbol{T}$ - Cycle duration
$\boldsymbol{v(t)}$ - Velocity
$\boldsymbol{Y(t)}$ - Primary (electric) power

Definitions/Abbreviations

CADC - Common Artemis Driving Cycles
EU - European Union
EUDC - Extra-Urban Driving Cycle
FTP-75 - Federal Test Procedure 1975
MW - Moterway
NEDC - New European Drive Cycle
RDE - Real Driving Emissions
PKE - Positive Kinetic Energy ($\int av\, dt$ for $a > 0$)
UNECE - United Nations Economic Commission for Europe
URM - Urban, Rural, Motorway
WLTP - World Light-Duty Test Protocol

Appendix

The coefficients for k_n' can be calculated according to Table 2, and the coefficients for k_n can be calculated according to Table 5.

TABLE 5 Coefficients k_n for the Central Moments

Coefficient	Coefficient Value
$\boldsymbol{k_6}$	$\alpha_2 c_2^2$
$\boldsymbol{k_5}$	$2\alpha_2 c_1 c_2 v_0 + 6\alpha_2 c_2^2 v_0^2$
$\boldsymbol{k_4}$	$\alpha_2\left(c_1^2 + 2c_0c_2\right) + 10\alpha_2 c_1 c_2 v_0 + 15\alpha_2 c_2^2 v_0^2$
$\boldsymbol{k_3}$	$c_2\left(2\alpha_2 c_0 c_1 + \alpha_1\right) + 4\alpha_2\left(c_1^2 + 2c_0c_2\right)v_0 + 20\alpha_2 c_1 c_2 v_0^2 + 20\alpha_2 c_2^2 v_0^3$
$\boldsymbol{k_2}$	$\alpha_1 c_1 + \alpha_2 c_0^2 + 3c_2\left(2\alpha_2 c_0 c_1 + \alpha_1\right)v_0 + 6\alpha_2\left(c_1^2 + 2c_0c_2\right)v_0^2 + 20\alpha_2 c_1 c_2 v_0^3 + 15\alpha_2 c_2^2 v_0^4$
$\boldsymbol{k_1}$	$\alpha_1 c_0 + 2\left(\alpha_2 c_0^2 + \alpha_1 c_1\right)v_0 + 3c_2\left(2\alpha_2 c_0 c_1 + \alpha_1\right)v_0^2 + 4\alpha_2\left(c_1^2 + 2c_0c_2\right)v_0^3 + 10\alpha_2 c_1 c_2 v_0^4 + 6\alpha_2 c_2^2 v_0^5$
$\boldsymbol{k_0}$	$\alpha_0 + \alpha_1 c_0 v_0 + \left(\alpha_2 c_2 + \alpha_1 c_1\right)v_2^2 + c_2\left(2\alpha_2 c_0 c_1 + \alpha_1\right)v_0^3 + \alpha_2\left(c_1^2 + 2c_0c_2\right)v_0^4 + 2\alpha_2 c_1 c_2 v_0^5 + \alpha_2 c_2^2 v_0^6$
$\boldsymbol{k_{av}}$	$\alpha_2 m^2$

ISSN 0148-7191