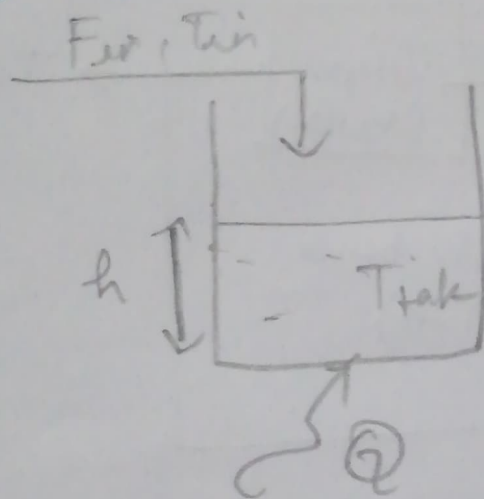


CH3050 ASSIGNMENT-1

① a) Feed Batch

We can imagine the system to have water flowing in, with heat being supplied to the stored water



There is no flow out because it is a storage geyser

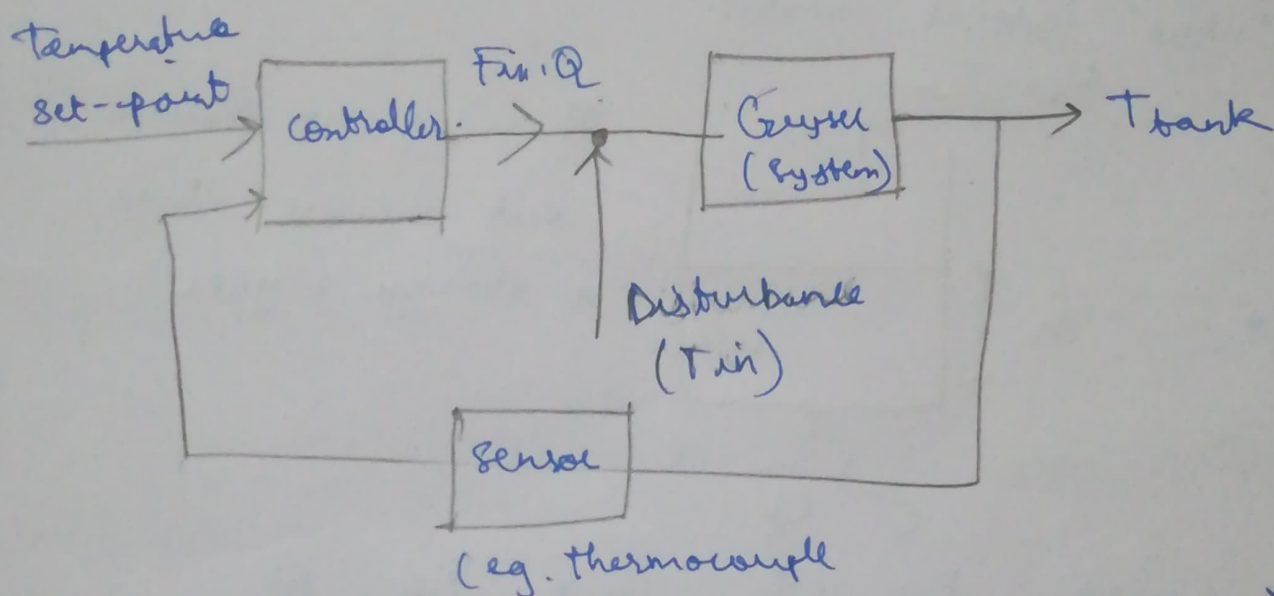
b) Variables: F_{in} , h_{tank} , T_{tank} , T_{in} , Q

→ Manipulated variables: F_{in} , Q (where F_{in} is flow in, and Q is the heat added - I assume that Q can be modified) by the

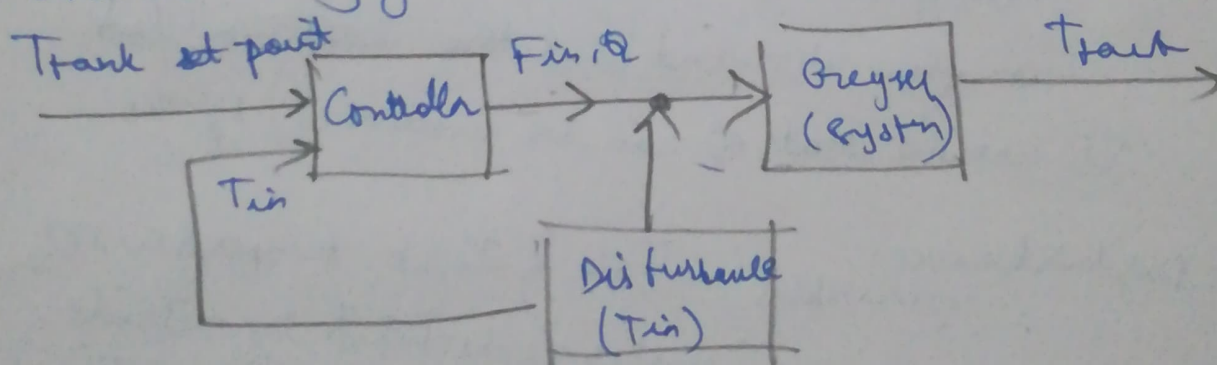
→ Disturbance variables: T_{in} (Inlet temperature of water is dictated by outside conditions)

→ Controlled variables: T_{tank} (temperature of water in the tank)

- c) Feedback system: Measure the temperature of tank (T_{tank}) and manipulate F_{in} and Q accordingly. F_{in} manipulated by a valve and Q by using a rheostat.



- d) Feedforward system: We measure T_{in} (the disturbance) and accordingly change the manipulated variable.



2) a) At steady state $\frac{dy}{dt} = \frac{d^2y}{dt^2} = 0$

$$\Rightarrow y(t) \quad y_{ss} = \frac{b_0}{a_0} u_{ss} \quad \text{--- (1)}$$

consider the ODE

$$\frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y(t) = b_0 u(t)$$

$$\frac{d^2y}{dt^2} (y - y_{ss}) + a_1 \frac{d}{dt} (y - y_{ss}) + a_0 (y(t) - y_{ss}) = b_0 (u(t) - u_{ss})$$

($\because \frac{dy_{ss}}{dt} = 0 \rightarrow y_{ss}$ is a constant)

But (1) $\Rightarrow a_0 y_{ss} = b_0 u_{ss}$

$$\therefore \frac{d^2 \tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 \tilde{u}(t) \quad \text{--- (3)}$$

(this is expected because the ODE is linear)

b) ~~$y_{ss} = \frac{b_0}{a_0} u_{ss} \Rightarrow y_{ss} = \frac{3}{15} \times u_{ss}$~~

(3) $\Rightarrow a_0 \tilde{y}_{ss} = b_0 \tilde{u}_{ss}$ (@ steady state)

$$\Rightarrow \cancel{2a_0} \tilde{y}_{ss} = b_0 \tilde{u}$$

$$\Rightarrow \tilde{u} = \frac{2 \times 15}{3}$$

$$\Rightarrow \boxed{\Delta u = 10 \text{ units}}$$

c) Substituting $\tilde{u}(t) = K_c (2 - \tilde{y}(t))$ in eqn (3),

$$\frac{d^2 \tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 K_c (2 - \tilde{y}(t))$$

$$\Rightarrow \frac{d^2 \tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + \tilde{y}(t) (a_0 + b_0 K_c) = 2b_0 K_c \quad \text{--- (4)}$$

Now to answer whether the system will achieve the control objective is eqvt to asking whether ~~the~~ ^{found} system is ^{stable} in a ^{stable} saturated (whether it is stable)

To analyse the stability we can see the poles of this new system.

$$\text{Also } \left. \frac{d^2 \tilde{y}}{dt^2} \right|_{t=0} = 0 \quad \& \quad \left. \frac{d\tilde{y}}{dt} \right|_{t=0} = 0 \quad \& \quad \left. \tilde{y}(t) \right|_{t=0} = 0$$

because the system is initially assumed to be in steady state.

$$\text{So } \mathcal{L} \left\{ \frac{d^2 \tilde{y}}{dt^2} \right\} = s^2 \tilde{Y}(s)$$

$$\text{and } \mathcal{L} \left\{ \frac{d\tilde{y}}{dt} \right\} = s \tilde{Y}(s)$$

a) taking Laplace transform,

$$s \tilde{Y}(s) + a_1 s \tilde{Y}(s) + a_0 \tilde{Y}(s) = \frac{2b_0 K_c}{s} + b_0 K_c \tilde{Y}(s)$$

$$\Rightarrow \tilde{Y}(s) = \frac{2b_0 K_c}{s(s^2 + a_1 s + a_0 + b_0 K_c)}$$

to apply

By Final Value theorem, we need the condition $s\tilde{Y}(s)$ to be stable.

$$\Rightarrow \text{'roots' of } s^2 + a_1 s + a_0 + b_0 K_c < 0$$

$$\Rightarrow \frac{-a_1 \pm \sqrt{a_1^2 - 4(a_0 + b_0 K_c)}}{2} < 0$$

$$\Rightarrow 0 \leq a_1^2 - 4(a_0 + b_0 K_c) < a_1^2$$

$$\Rightarrow K_c \leq \frac{a_1^2 - 4a_0}{4b_0} \text{ and}$$

$$K_c > -\frac{a_0}{b_0}$$

$$\Rightarrow K_c \leq \frac{1}{3} \text{ and } K_c > -5$$

$$\Rightarrow -5 < K_c \leq 1/3 \Rightarrow 0 < K_c \leq 1/3$$

Under such conditions we apply FVT

$$\text{to get } \tilde{y}(t) \text{ as } \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \tilde{Y}(s)$$

$$\lim_{t \rightarrow \infty} \tilde{y}(t) = \lim_{s \rightarrow 0} s \times \frac{2b_0 K_c}{s^2 + a_1 s + a_0 + b_0 K_c} = 2$$

\therefore Required objective is attained
for $0 < K_c \leq \frac{1}{3}$

d) ③ $\Rightarrow \frac{d^2 \tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 \tilde{u}(t)$

Differentiate wrt time,

$$\frac{d^3 \tilde{y}}{dt^3} + a_1 \frac{d^2 \tilde{y}}{dt^2} + a_0 \frac{d\tilde{y}}{dt} = b_0 \frac{d\tilde{u}}{dt}$$

$$= b_0 \left(K_c \left(-\frac{d\tilde{y}}{dt} \right) + K_I (2 - \tilde{y}) \right)$$

For notational convenience, \tilde{y} is dropped
($a_0 + b_0 K_c$)

$$\Rightarrow \frac{d^3 y}{dt^3} + (a_1 + b_0 K_c) \frac{d^2 y}{dt^2} + \frac{a_0}{b_0 K_I} \frac{dy}{dt} + b_0 K_I y = 2 K_I b_0$$

Similar to previous part, take Laplace transform to get,

$$(s^3 + a_1 s^2 + (a_0 + b_0 K_c)s + b_0 K_I) Y(s) = \frac{2 K_I b_0}{s}$$

$$\Rightarrow Y(s) = \frac{2K_I b_0}{s(s^3 + a_1 s^2 + (a_0 + b_0 K_C) s + b_0 K_I)}$$

Let We want the poly(sY(s)) to be stable

$$\Rightarrow \text{roots}(s^3 + a_1 s^2 + (a_0 + b_0 K_C) s + b_0 K_I) < 0 \quad \text{--- (5)}$$

$$\Rightarrow \text{roots}(s^3 + 8s^2 + (15 + 3K_C)s + 3K_I) < 0$$

Assuming roots obey this property, apply RVT

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \times \frac{2K_I b_0}{s(s^3 + 8s^2 + (15 + 3K_C)s + 3K_I)}$$

\therefore objective is achieved.

Conclusion: The objective can't be achieved for any $\{K_C, K_I\}$, it is achieved only when (5) is satisfied.

eg. $K_C = K_I = 1$, roots are $-3.9 + 1.14j$
 $-3.9 - 1.14j$
 -0.181

\Rightarrow objective is not achieved.

But for $K_C = 0.25$, $K_I = 0.5$

roots are -4.25 , -3.14 , -0.1

\Rightarrow objective is achieved.

$$\textcircled{3} \textcircled{a) } \frac{dw}{dt} = -\left(\frac{L+Va}{M}\right)w + \frac{Va}{M}z \quad \textcircled{1} \quad \begin{array}{l} a = 0.5 \\ 2.5 = 0.1 \\ M = 10 \end{array}$$

$$\frac{dz}{dt} = -\frac{L}{M}w - \left(\frac{L+Va}{M}\right)z + \frac{Vz_f}{M} \quad \textcircled{2}$$

At steady state, $L = L_{ss} = 4$, $V = V_{ss} = 200$
 $- 89$

$$\frac{dw}{dt} = 0, \quad \frac{dz}{dt} = 0$$

$$\textcircled{1} \Rightarrow -6.5w + 2.5z = 0 \quad \textcircled{3}$$

$$\textcircled{2} \Rightarrow 4w - 6.5z = -0.5 \quad \textcircled{4}$$

Solving $\textcircled{3}$ & $\textcircled{4}$,

$$w_{ss} = 0.0388 \text{ and } z_{ss} = 0.1008$$

\Rightarrow Steady state values: $w = 0.0388$
 $z = 0.1008$

b) In this model, $\text{state}_{(n)} = \begin{bmatrix} w \\ z \end{bmatrix}$

$$\text{Inputs}_{(n)} = \begin{bmatrix} L \\ V \end{bmatrix} \quad \text{Let } \frac{dw}{dt} = f(\cdot) \text{ \& } \frac{dz}{dt} = g(\cdot)$$

$$\frac{dw}{dt} = \frac{dw}{dt} \Big|_{\text{steady state}} + \frac{\partial f}{\partial L} (L - L_{ss}) + \frac{\partial f}{\partial V} (V - V_{ss}) + \frac{\partial f}{\partial w} (w - w_{ss}) + \frac{\partial f}{\partial z} (z - z_{ss})$$

$$a_{11} = \frac{\partial f}{\partial \omega} = - \frac{(V_a + L)}{M}$$

$$a_{12} = \frac{\partial f}{\partial z} = \frac{V}{M}$$

$$a_{21} = \frac{\partial g}{\partial \omega} = \frac{L}{M} ; a_{22} = - \left(\frac{L + V_a}{M} \right)$$

$$b_{11} = \frac{\partial f}{\partial L} = - \frac{\omega}{M} ; b_{12} = \frac{\partial f}{\partial g} = - \frac{\omega}{M} + \frac{q_2}{M}$$

$$b_{21} = \frac{\partial g}{\partial L} = \frac{\omega}{M} ; b_{22} = \frac{\partial g}{\partial g} = - \frac{q_2}{M} + \frac{2f}{M}$$

$$\therefore A = \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix}$$

(all values evaluated at steady state)

$$B = \begin{bmatrix} -0.0019 & 0.0016 \\ -0.0031 & 0.0025 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{dw}{dt} \\ \frac{dz}{dt} \end{bmatrix} = A \begin{bmatrix} w - w_{ss} \\ z - z_{ss} \end{bmatrix} + B \begin{bmatrix} L - L_{ss} \\ V - V_{ss} \end{bmatrix}$$

$$\text{Let } y = \begin{bmatrix} w - w_{ss} \\ z - z_{ss} \end{bmatrix}$$

$$\Rightarrow y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \cdot w_{83} \\ z - 2u \end{bmatrix}$$

c) eigenvalues: ~~De~~ -3.338, -9.162

eigenvectors: $\begin{bmatrix} 0.6202 \\ 0.7845 \end{bmatrix}$ & $\begin{bmatrix} -0.6202 \\ 0.7845 \end{bmatrix}$

Since $|\lambda_2| > |\lambda_1|$

~~fastest dec~~ fastest dec: eigenvector 2: $\begin{pmatrix} -0.6202 \\ 0.7845 \end{pmatrix}$

slowest dec: eigenvector 1: $\begin{pmatrix} 0.6202 \\ 0.7845 \end{pmatrix}$

④ a) $\mathcal{L}\{x(t)\} =$

For $0 \leq t < 3$

$$\mathcal{L}\{t-2\} = \frac{e^{-2s}}{s} \int_0^3 t e^{-st} dt = \frac{e^{-2s}}{s} \left[-\frac{1}{s} e^{-st} (t+1) \right]_0^3$$

$$= \frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2} \Big|_0^3 = \frac{1}{s} (1 - e^{-3s})$$

$$= \frac{1 - 2s - (1 + se^{-3s})}{s^2}$$

$$= \frac{te^{-st}}{-s} - \int_0^3 \frac{e^{-st}}{s} - \frac{2}{s} (1 - e^{-3s})$$

$$= \frac{1}{s} \left(\frac{e^{-3s}}{s} + \frac{e^{-3s}}{s^2} - \frac{1}{s^2} - \frac{2}{s} (1 - e^{-3s}) \right)$$

$$\Rightarrow X(s) = \boxed{\frac{-(1+s)e^{-3s} - 2s + 1}{s^2}} \quad 0 \leq t < 3$$

$$3 \leq t \leq 4$$

$$X(s) = \int_3^4 e^{-st} dt$$

$$= \boxed{\frac{e^{-3s} - e^{-4s}}{s}}$$

$$4 \leq t < 5$$

$$X(s) = \int_4^5 -\cos(3\pi(t-4)) e^{-st} dt$$

$$\Rightarrow X(s) = \cancel{\frac{-\cos(3\pi(t-4))}{-s}} e^{-st} \left(+ \int_4^5 3\pi \sin(3\pi(t-4)) e^{-st} dt \right)$$

$$= \frac{-\cos(3\pi(t-4))}{-s} e^{-st} + \left(\frac{3\pi \sin(\pi(t-4))}{-s^2} e^{-st} \right) \Big|_4^5$$

$$- \int_4^5 \left(\frac{9\pi^2 \cos(3\pi(t-4))}{-s^2} e^{-3t} \right) dt$$

$$\Rightarrow X(s) \left(1 + \frac{9\pi^2}{s^2} \right)$$

$$= \frac{-e^{-5s}}{s} + \frac{3\pi e^{-4s}}{s^2}$$

$$\Rightarrow X(s) = \frac{(e^{-5s} + 3\pi e^{-4s})}{\frac{9\pi^2}{s^2} + 1}$$

$$t \geq 5$$

$$\mathcal{L}\{e^{-2t} X(s)\} = \int_5^{\infty} e^{-2(t-\tau)} \cos(5\pi(t-\tau)) e^{-st} dt$$

$$= \int_5^{\infty} e^{-t(s+2)-5} \cos(5\pi(t-\tau)) dt$$

By part 1

$$\Rightarrow X(s) = \frac{e^{-t(s+2)+5} \cos(5\pi(t-\tau))}{-(s+2)} \Big|_5^{\infty} - \int_5^{\infty} \frac{\sin(5\pi(t-\tau)) e^{-t(s+2)+5}}{s+2} dt$$

$$= \frac{e^{-5(s+2)+50}}{s+2} - 5\pi \left[\frac{\sin(5\pi(t-\tau))}{(s+2)^2} + \int_5^{\infty} \frac{\cos(5\pi(t-\tau)) e^{-t(s+2)+5}}{(s+2)^2} dt \right]$$

$$\Rightarrow X(s) \left(1 + \frac{25\pi^2}{(s+2)^2} \right) = -e^{-5s}$$

$$\Rightarrow X(s) = \frac{e^{-5s}}{1 + \frac{25\pi^2}{(s+2)^2}}$$

Find soln:

Ans:

$$\textcircled{b) \quad \frac{s-2}{s \left(s^2 + 2\frac{q}{T}s + 1 \right)} = \frac{1(s-2)}{T^2 s \left(\frac{s^2}{T^2} + 2\frac{q}{T}s + 1 \right)}$$

Roots of $s^2 + 2\frac{q}{T}s + 1$

$$= \frac{-2\frac{q}{T}}{2T} \pm \frac{\sqrt{4\frac{q^2}{T^2} - 4}}{2T} \quad \left(\begin{array}{l} \text{call these} \\ \alpha \text{ and } \beta \end{array} \right)$$

They are real if $q \geq 1$

i) $q > 1$

$$\frac{A_1}{s} + \frac{A_2}{s - \frac{-q + \sqrt{q^2 - 1}}{T}} + \frac{A_3}{s - \frac{-q - \sqrt{q^2 - 1}}{T}}$$

$$= \frac{s-2}{T^2 \left(s^2 + 2\frac{q}{T}s + 1 \right)}$$

put $s=0$

$$A_1 = \frac{-2}{T^2}$$

let $s = \text{root } \alpha$

$$A_2 = \frac{\alpha - 2}{T^2}$$

$$A_3 = \frac{\beta - 2}{T^2}$$