③ a) Let ~~us~~ the controller gain be β.

$$G_{approx}(s) = \frac{2(s+2)}{s^2+2s-3}\frac{\left(1-\frac{s}{2}\right)}{\left(1+\frac{s}{2}\right)} = \frac{2(s+2)(2-s)}{(s^2+2s-3)(2+s)}$$

C.E.:

$$1 + \beta G_p = 0$$

Case ①: cancel off $(s+2)$ in Nr & Dr

$$\Rightarrow \frac{2\beta(2-s)}{(s^2+2s-3)} + 1 = 0$$

$$\Rightarrow s^2 + 2(1-\beta)s + 4\beta - 3 = 0.$$

Conditions $P_1 = -2$ & $Re(P_2) < 0$ (& preferably $Re(P_2) < -2$)

condition ① $\Rightarrow$ $4 - 4(1-\beta) + 4\beta - 3 = 0$

$$\Rightarrow \beta = 3/8$$

condition ② $\Rightarrow$ product of roots > 0

sum of roots < 0

$$\Rightarrow 4\beta - 3 > 0 \ \& \ -2(1-\beta) < 0$$

$$\Rightarrow \beta > 3/4 \ \& \ \beta > 1$$

But $\beta = 3/8$ for pole $= -2$.

$\therefore$ We can't satisfy the condition —— ①

Case ② : dont cancel $(s+2)$ term.

$C.E \Rightarrow (s+2)(s^2 + 2(1-\beta)s + 4\beta - 3) = 0$

(same as previous except with $as+2$ factor)

This time we already have $p = -2$.

So we just need to impose

$Re(p) < 0$ & preferably $Re(p) < -2$

Roots : $-(-1+\beta) \pm \sqrt{\beta^2 - 6\beta + 4}$ —— ②

Now the term under squareroot has roots

$\beta = 3 \pm \sqrt{5}$

Subcase ① :

$\sqrt{\beta^2 - 6\beta + 4}$ is imaginary ($\beta > 3 - \sqrt{5}$, $\beta < 3 + \sqrt{5}$)

$\Rightarrow + (\beta - 1) < -2$

$\Rightarrow \beta < -1$

But $\beta > 3 - \sqrt{5}$

$\therefore$ No soln exist in subcase ① —— ③

Subcase 2 :

$\sqrt{\beta^2-6\beta+4}$ is real $\left(\beta > 3+\sqrt{5} \text{ or } \beta < 3-\sqrt{5} \right.$

$\Rightarrow \beta-1 \pm \sqrt{\beta^2-6\beta+4} < -2$

$\Rightarrow \cancel{(\beta-3)} \pm \sqrt{\beta^2-6\beta+4} > \cancel{3-\beta}\ \beta+1$

$\Rightarrow \beta^2-6\beta+4 > \cancel{9+\beta^2-6\beta}\ \beta^2+2\beta+1$

$\Rightarrow \beta < 3/8$

But $\beta > 3+\sqrt{5}$

not satisfied

$\therefore$ No solution exists in subcase 2 4

From 1, 3, 4

We can't solve for K_c such that $p=-2$ is the dominant pole.

$\rightarrow$ This was also verified using RLTOOL in MATLAB

b) $G(s) = \dfrac{2(s+2)}{s^2+2s-3} e^{-s}$; $\Rightarrow G_p G_c = L$

$\Rightarrow L(j\omega) = K_c \dfrac{2(j\omega)+2}{(j\omega)^2+2(j\omega)-3} e^{-j\omega}$

Solve for intersection with real axis (to get gain margin)

$\phi = k\pi$

$\Rightarrow$

$\Rightarrow -\omega + \tan^{-1}\left(\dfrac{\omega}{2}\right) - \tan^{-1}\left(\dfrac{2\omega}{\omega^2+3}\right) = k\pi$

$\Rightarrow \omega = \tan^{-1}\left(\dfrac{\frac{\omega}{2} + \frac{2\omega}{\omega^2+3}}{1 - \frac{2\omega^2}{\omega^2+3}}\right)$ (if $k=0$)

$\Rightarrow \tan\omega = \dfrac{4\omega + \omega^3 + 3\omega}{6}$

$\Rightarrow \tan\omega = 7\omega + \dfrac{\omega^3}{6}$

Sols $\omega = 0$, $\omega = 0.78$

$|L(j\omega)|_{\omega=0} = \dfrac{2K_c\sqrt{4}}{\sqrt{9}}$

$\Rightarrow GM = 1.092\,K_c$

In dB, $GM = -20\log\dfrac{1}{1.092K_c}$

$\Rightarrow 4.5 = -20\log\dfrac{1}{1.092K_c} \Rightarrow K_c = 0.224$

Similarly one can get.

$K_c = 0.2733$ by subst $\omega = 0.78$

We can use FVT to get offset provided the system is closedloop stable.

Stability is checked using the Nyquist plot

poles of G: $\frac{-2 \pm \sqrt{16}}{2} = 1, -3$

1 RHP pole.

$\therefore Z = N + P$ But $N = 0$ (no encirclements around -1)

$\Rightarrow Z = 1$

We have one RHP ~~zero~~ zero for $L + 1 = 0$

$\Rightarrow$ we have an RHP pole for the CL system

$\therefore$ the CL system is unstable ie mathematically

offset $\to \infty$.