(3)

$$\frac{d^2h'}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh'}{dt} + \frac{3}{2}\frac{g}{L}h' = \frac{3}{4\rho L}p'(t)$$

$$\Rightarrow s^2H(s) + \frac{6\mu}{R^2\rho}sH(s) + \frac{3}{2}\frac{g}{L}H(s) = \frac{3}{4\rho L}P(s)$$

(Taking laplace transform)

$$\Rightarrow H(s)\left[s^2 + \frac{6\mu}{R^2\rho}s + \frac{3}{2}\frac{g}{L}\right] = \frac{3}{4\rho L}P(s)$$

$$\Rightarrow \frac{H(s)}{P(s)} = \frac{\frac{3}{4\rho L}}{s^2 + \frac{6\mu}{R^2\rho}s + \frac{3g}{2L}}$$

$$\therefore \omega_n^2 = \frac{3g}{2L} \Rightarrow \boxed{\omega = \sqrt{\frac{3g}{2L}}}$$

$$T = \frac{1}{\omega} \Rightarrow \boxed{T = \sqrt{\frac{2L}{3g}}}$$

$$2\zeta\omega = \frac{6\mu}{R^2\rho} \Rightarrow \zeta = \frac{3\mu}{R^2\rho}\sqrt{\frac{2L}{3g}}$$

$$\Rightarrow \boxed{\zeta = \frac{\mu}{R^2\rho}\sqrt{\frac{6L}{g}}}$$

$$K_p\omega^2 = \frac{3}{4\rho L} \Rightarrow \boxed{K_p = \frac{1}{2g\rho}}$$

b) We get osillatory responses for underdamped systems

$\Rightarrow \quad 0 < \zeta \cancel{\leq 1} < 1$

$$\Rightarrow \quad \frac{3\mu}{R^2\rho}\sqrt{\frac{2L}{3g}} < 1$$

($\because$ these physical constants are always +ve)

$$\Rightarrow \boxed{\frac{3\mu}{R^2\rho}\sqrt{\frac{2L}{3g}} < 1}$$

c) More osillatory responses have lower damping (lower ζ), less osillatory responses have higher damping (higher ζ)

i) As L ↑ses, ζ ↑ses ($\zeta \propto \sqrt{L}$)

So higher $L \Rightarrow$ less osillatory response

lower $L \Rightarrow$ more osillatory response

ii) $\zeta \propto \mu$ (As μ ↑ses, ζ ↑ses)

So higher $\mu \Rightarrow$ less osillatory response

lower $\mu \Rightarrow$ more osillatory response