CH3050 Process Dynamics and Control Assignment 2 Solutions

March 2021

Marks Distribution

	Question 1	Question 2	Question 3	Question 4
(a)	10	5	15	10
(b)	20	5	15	20
(c)	10	10	–	–
(d)	15	10	–	–
(e)	5	–	–	–

1

Given that an exothermic reaction $A \longrightarrow 2B$, takes place adiabatically in a stirred-tank reactor. This liquid reaction occurs at constant volume in a 12 00 -gallon reactor. The reaction can be considered to be first order and irreversible with the rate constant given by $k = 2.4 \times 10^{15} e^{-20000/T} \left(\min^{-1}\right)$ where T is in $^\circ R$. The steady-state conditions are $c_{Ai,\mathrm{ss}} = 0.8 \mathrm{~mol/ft}^3$ and $F_{ss} = 20$ gallons /min. The physical property data for the mixture: $T_i = 90^\circ\mathrm{F}, C = 0.8 \mathrm{Btu} / (\mathrm{lb}^\circ\mathrm{F}), \rho = 52 \mathrm{lb/ft}^3$ and $\Delta H_R = -500$ kJ/mol

(a)

The first-principles model for the given stirred-tank reactor assuming

1. perfectly mixed reactor
2. constant fluid properties and heat of reaction

is given below. Component balance is

$$V \frac{dc_A}{dt} = F c_{A_i} - F c_A - V k(T) c_A$$
$$\frac{dc_A}{dt} = -\left(\frac{F}{V} + k(T)\right) c_A + \frac{F c_{A_i}}{V}$$

The energy balance is

$$V \rho c_p \frac{dT}{dt} = F \rho c_p T_i - F \rho c_p T + (-\Delta H_R)(k(T) C_A) V$$

$$\frac{dT}{dt} = (T_i - T)\frac{F}{V} - \frac{\Delta H_R}{\rho c_p}(k(T)C_A)$$

where,

$$k(T) = -2.4 \times 10^{15} e^{\frac{-20000}{T}} \left(\frac{-1}{\min}\right), c_p = 0.8\frac{Btu}{lb^\circ F}$$

$$\rho = 52\frac{lb}{ft^3}, \Delta H_R = -500\frac{KJ}{mol}, F = F_{ss} = 20\frac{gallons}{\min}$$

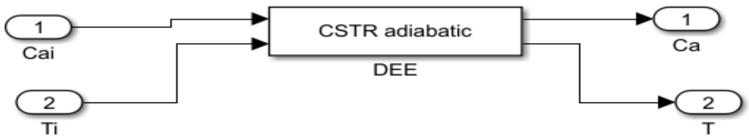
Therefore the model is

$$\frac{dc_A}{dt} = -\left(\frac{F}{V} + k(T)\right)c_A + \frac{Fc_{A_i}}{V}$$

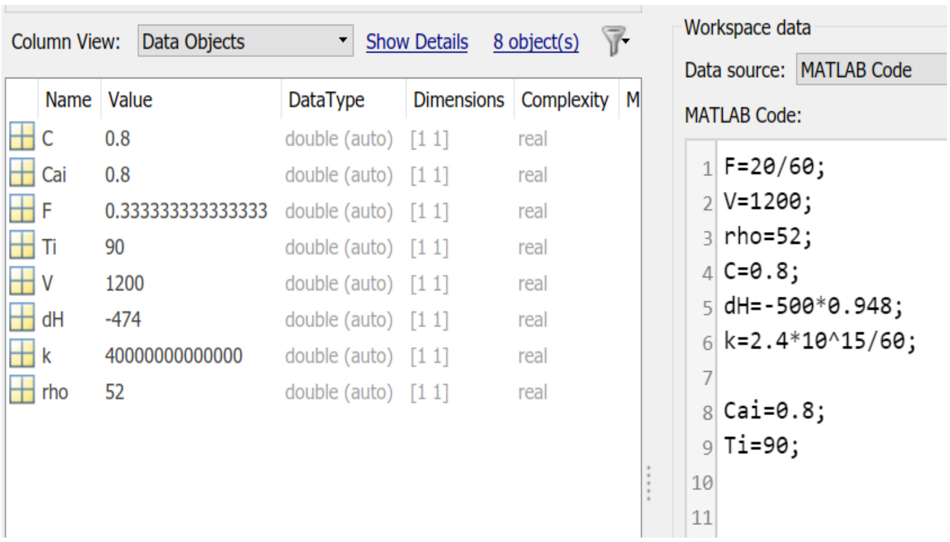
$$\frac{dT}{dt} = (T_i - T)\frac{F}{V} - \frac{\Delta H_R}{\rho c_p}(k(T)C_A)$$

(b)

The Simulink block diagram to determine the steady-state exit temperature using the findop routine of MATLAB is given in Fig 1. F=q here in the block diagram. Remember 500 KJ is 473.909 BTU.

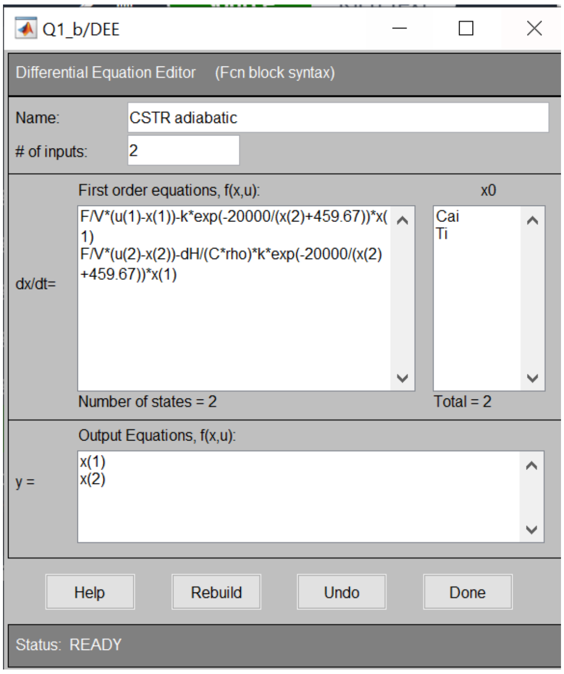


(a) Simulink block diagram



(b) DEE Block

Figure 1: Simulink block diagram



The code for to determine steady state temperature using findop is given below. MATLAB code:

```
% Assignment 2, ch3050, 2021
% Raghav Moar
% Question 1 (b)
% Steady−atate exit temperature using findop, with CAiss= 0.8
open_system('Q1_b')
opspec = operspec('Q1_b');
opspec.Inputs(1).u = 0.8;
opspec.Inputs(1).Known = 1;
opspec.Inputs(2).u = 90;
opspec.Inputs(2).Known = 1;
Steady_state = findop('Q1_b',opspec);
```

The steady-state exist temperature is $Tss = 98.9°F$ Range valid= $98.7 - 99.0°F$.

(c)

Transfer function using MATLAB.
The code in MATLAB to obtain the transfer function relating the exit temperature T and outltet concentration C_A to the inlet concentration C_{Ai} is given below.

```
% Assignment 2, ch3050, 2021
% Raghav Moar
% Question 1 (c)
%Steady−atate exit temperature using findop, with CAiss= 0.8
open_system('Q1_b')
```

```
opspec = operspec('Q1_b');
opspec.Inputs(1).u = 0.8;
opspec.Inputs(1).Known = 1;
opspec.Inputs(2).u = 90;
opspec.Inputs(2).Known = 1;
Steady_state = findop('Q1_b',opspec);
Lin_system = linearize('Q1_b', Steady_state);
% Transfer function
%add 1 as the fifth input because transfer function on cai
[num,den] =ss2tf(Lin_system.A,Lin_system.B,Lin_system.C,Lin_system.D,1);
%for CA
Gs1 = tf(num(1,:),den)
%for T
Gs2 = tf(num(2,:),den)
```

```
Outputs:
----------
(1.) Q1_b/Ca
      y:          0.0193     [-Inf Inf]
(2.) Q1_b/T
      y:            98.9     [-Inf Inf]


Gs1 =

    0.0002778 s + 3.316e-08
  ----------------------------
  s^2 + 0.01166 s + 3.162e-06

Continuous-time transfer function.


Gs2 =

           3.565e-05
  ----------------------------
  s^2 + 0.01166 s + 3.162e-06

Continuous-time transfer function.
```

Transfer function by hand The transfer function relating the exit temperature T to the inlet concentration

cA_i is obtained as follows assuming the other inputs, namely q and T_i, to be constant. Linearizing the above first-principles model, we get

$$\frac{dc_A}{dt} = \frac{dc'_A}{dt}, \frac{dT}{t} = \frac{dT'}{dt}$$

$$V\frac{dc'_A}{dt} = qc'_{A_1} - \left(q + Vk(\bar{T})c'_A - Vc_A k(\bar{T})\frac{20000}{T^2}T'\right).$$

Note that $\bar{c_A}$ and $\bar{T}$ represent the steady state values.

$$V\rho c_p \frac{dT'}{dt} = -\left(q\rho c_p + \Delta H_R V\overline{c_A}k(T)\frac{20000}{T^2}\right)T' + (-\Delta H_R)\, Vk(T)c_A$$

Taking the Laplace transforms and rearranging, we get

$$[Vs + q + Vk(\bar{T})]C'_A(s) = qC'_{A_t}(s) - V\overline{CA}k(\bar{T})\frac{20000}{T^2}T'(s)$$

$$\left[V\rho c_p s + q\rho c_p - \left(-\Delta H_R V c_A k(\bar{T})\frac{20000}{T^2}\right)\right]T'(s) = (-\Delta H_R)\, Vk(\bar{T})C'_A(s)$$

Substituting C'_A(s) and rearranging, we get

$$\frac{T'(s)}{C_{A_i}(s)} = \frac{\Delta H_R Vk(\bar{T})q}{[Vs + q + Vk(\bar{T})]\left[V\rho c_p s + q\rho c_p - \left(-\Delta H_R V c_A k(\bar{T})\frac{20000}{T^2}\right)\right] + -\Delta H_R \bar{A}_A V^2 k^2(\bar{T})\frac{20000}{T^2}}$$

Similarly substituting for T'(s) and rearranging, we get equation for $\frac{C'_A(s)}{C_{A_i}(s)}$.
Now putting $C_{Ass} = 0.0193$ $T_{ss} = 98.9$) we can verify that we get similar results.

(d)

At 10% $C'_A(i)$ increase. the result for non-linear change is $C_{Ass} = 0.02$ and $T_{ss} = 99.8$.

For linearised model at same 10% increase in $C'_A(i)$ the result is $C_{Ass} = 0.0204$ and $T_{ss} = 99.802$.

Therefore, error percent in Ca = —0.02-0.0204—/0.0204 * 100 = 1.96%

error percent in T = —99.8-99.802—/99.8 * 100 = 0.002%

(e)

$$Ca_{Cai=0.8} = 0.0193 \;\;\&\;\; Ca_{Cai=0.88} = 0.02$$

$$T_{Cai=0.8} = 98.9 \;\;\&\;\; T_{Cai=0.88} = 99.8$$

On the basis percent change in the value of the variable, Ca is affected more by a unit change in step Cai.

2

MATLAB code:

```
% Run the script
a2q2_datagen

% Construct transfer functions
G1s = tf(Kp1,[tau1 1]);
G2s = tf(Kp2,[tau2 1]);

% TF between Flow 1 and Level 2
G12s = Cv1*G2s*G1s;

% Gain and poles
gain12 = dcgain(G12s);
poles12 = pole(G12s);

% Amplitude of oscillation in Level 2
h2w = freqresp(G12s,w0);

if (abs(h2w) < 0.1)
    oscillh2 = 0;
else
    oscillh2 = 1;
end

% Approximating G12 with a first-order

% Approximate gain
Kphat = dcgain(G12s);

% Step response
[ystep,tvec] = step(G12s);

% Approximate time constant
errhat = ystep - 0.632*ystep(end);
[~,ind_min] = min(abs(errhat));
tauphat = tvec(ind_min);

% Approximate TF
G12approx = tf(Kphat,[tauphat 1]);
```

3

(a)

(i)Partial fraction expansion method (SS1)

$$
\begin{aligned}
G(s) &= \frac{s+1}{s^3+10s^2+31s+30} \\
&= \frac{s+1}{(s+2)(s+3)(s+5)} \\
\frac{Y(s)}{U(s)} &= \frac{-1/3}{s+2} + \frac{1}{s+3} + \frac{-2/3}{s+5}
\end{aligned}
$$

Let $X_1(s) = \frac{U(s)}{s+2}$, $X_2(s) = \frac{U(s)}{s+3}$, $X_3(s) = \frac{U(s)}{s+5}$.

$$
Y(s) = \frac{-X_1(s)}{3} + X_2(s) + \frac{-2X_3(s)}{3}
$$

$$
\begin{aligned}
\dot{x}_1 &= u(t) - 2x_1(t) \\
\dot{x}_2 &= u(t) - 3x_2(t) \\
\dot{x}_3 &= u(t) - 5x_3(t)
\end{aligned}
$$

The SS representation (SS1) is:

$$
\begin{aligned}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t) \\
y(t) &= \begin{bmatrix} -1/3 & 1 & -2/3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}
\end{aligned}
$$

(ii) Nested integral method (SS2)

$$
\begin{aligned}
G(s) &= \frac{s+1}{s^3+10s^2+31s+30} \\
\frac{Y(s)}{U(s)} &= \frac{s+1}{s^3+10s^2+31s+30} \\
s^3Y(s) + 10s^2Y(s) + 31sY(s) + 30Y(s) &= sU(s) + U(s)
\end{aligned}
$$

Applying inverse Laplace transform:

$$
y''' + 10y'' + 31' + 30y = u' + u
$$

Integrating on both sides:

$$
y(t) = \int -10y'' + \int (u' - 31y') \int (u - 30y) dtdtdt
$$

$$y(t) = x_3(t)$$

$$\begin{aligned}
\dot{x}_1(t) &= u(t) - 30x_3(t) \\
\dot{x}_2(t) &= u(t) - 31x_3(t) + x_1(t) \\
\dot{x}_3(t) &= -10x_3(t) + x_2(t)
\end{aligned}$$

The SS representation (SS2) is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -30 \\ 1 & 0 & -31 \\ 0 & 1 & -10 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

SS1 ($\mathbf{x}$) $\longrightarrow$ Diagonal canonical form
SS2 ($\tilde{\mathbf{x}}$) $\longrightarrow$ Observer canonical form

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} 0 & 0 & -30 \\ 1 & 0 & -31 \\ 0 & 1 & -10 \end{bmatrix}$$

$$\exists \mathbf{T}, \text{ s.t. } \mathbf{x} = \mathbf{T}\tilde{\mathbf{x}}$$
$$T^{-1}\tilde{A}T = A$$

A is a diagonal matrix, hence T is the eigenvector matrix for $\tilde{A}$

$$\mathbf{T} = \begin{bmatrix} -0.8808 & -0.8165 & 0.7620 \\ -0.4698 & -0.5715 & 0.6350 \\ -0.0587 & -0.0816 & 0.1270 \end{bmatrix}$$

(b)

$$\frac{Y_1(s)}{U(s)} = \frac{4s+1}{(s+1)(s+3)}$$
$$\implies \frac{Y_1(s)}{U(s)} = -\frac{1.5}{s+1} + \frac{5.5}{s+3}$$
$$\implies Y_1(s) = -\underbrace{\frac{1.5U(s)}{s+1}}_{X_1} + \underbrace{\frac{5.5U(s)}{s+3}}_{X_2}$$
$$\implies Y_1(s) = X_2 - X_1$$

$$\frac{Y_2(s)}{U(s)} = \frac{10s}{(s+2)(s+3)}$$
$$\implies \frac{Y_2(s)}{U(s)} = -\frac{20}{s+2} + \frac{30}{s+3}$$
$$\implies Y_2(s) = -\underbrace{\frac{20U(s)}{s+2}}_{X_3} + \frac{30U(s)}{s+3}$$
$$\implies Y_2(s) = \frac{60}{11}X_2 - X_3$$

$$\dot{x}_1 = 1.5u(t) - x_1(t)$$
$$\dot{x}_2 = 5.5u(t) - 3x_2(t)$$
$$\dot{x}_3 = 20u(t) - 2x_3(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1.5 \\ 5.5 \\ 20 \end{bmatrix} u(t)$$
$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & \frac{60}{11} & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

4

(i)

The block diagram relating R(s) to Y(s) for the given signal graph of the system is given in Fig 3.

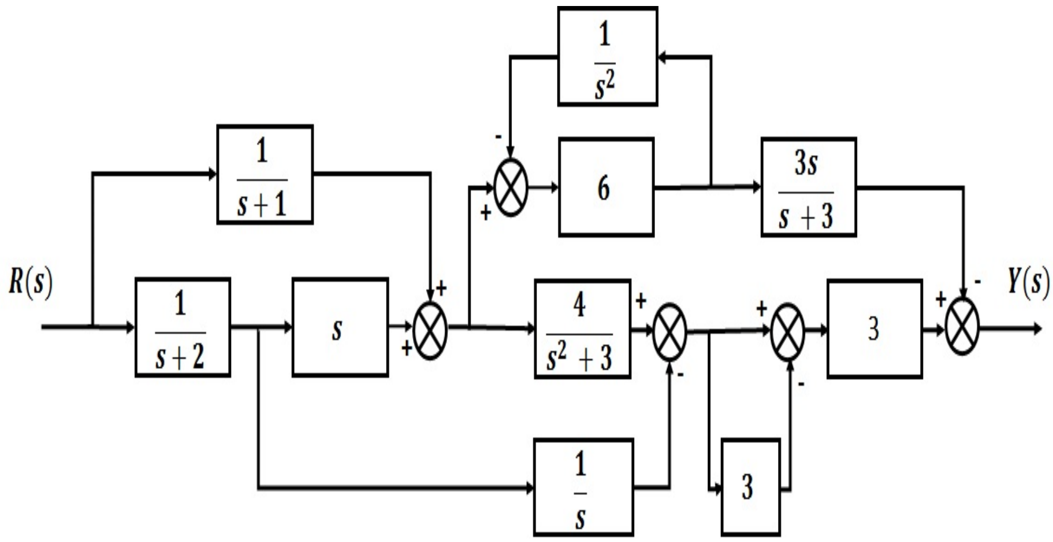


Figure 2: The block diagram relating R(s) to Y(s)

ii

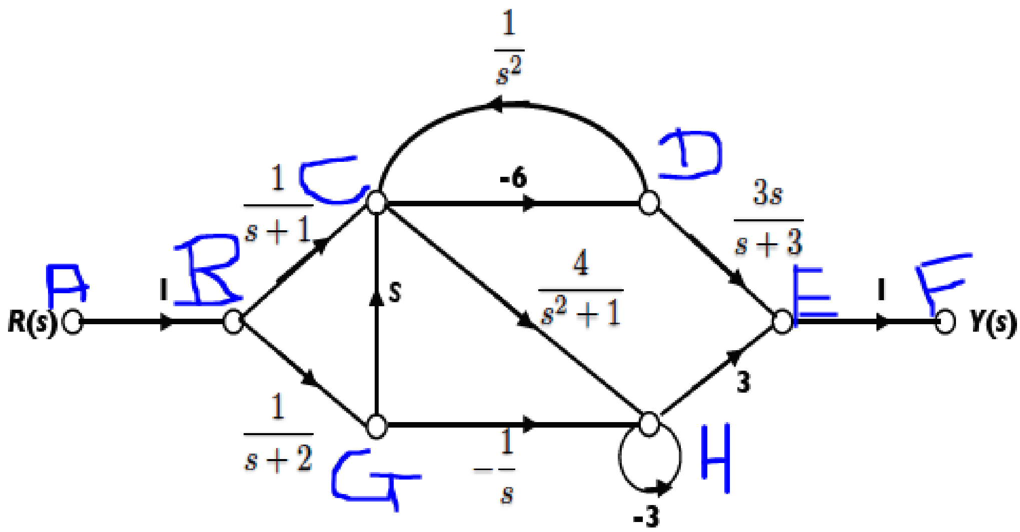


Figure 3: Signal flow graph for Q.4

1st forward path ABCDEF transfer function G_1 is

$$1.\ \frac{1}{s+1} \cdot (-6) \cdot \frac{3s}{s+3} \cdot 1 = \frac{-18s}{(s+1)(s+3)}$$

2nd forward path ABCHEF transfer function G_2 is

$$1 \cdot \frac{1}{s+1} \cdot \frac{4}{s^2+1} 3.1 = \frac{12}{(s^2+1)(s+1)}$$

3rd forward path ABGHEF transfer function G_3 is

$$1.\ \frac{1}{s+2} \cdot \frac{-1}{s} \cdot 3 \cdot 1 = \frac{-3}{s(s+2)}$$

4th forward path ABGCDEF transfer function G_4 is

$$1.\ \frac{1}{s+2} \cdot s \cdot (-6) \cdot \frac{3s}{s+3} \cdot 1 = \frac{-18s^2}{(s+2)(s+3)}$$

5th forward path ABGCHEF transfer function G_5 is

$$1.\ \frac{1}{s+2} \cdot \frac{4}{s^2+1} \cdot 3 \cdot 1 = \frac{12s}{\left(s^2+1\right)(s+2)}$$

1st independent loop CDC transfer function L_1 is

$$-6 \cdot \frac{1}{s^2}$$

$2nd$ independent loop HH transfer function L_2 is

$$-3$$

According to the Mason's gain formula

$$\frac{Y(s)}{R(s)} = \frac{\sum G_k \Delta_k}{\Delta}, \quad k = 1, 2 \ldots$$

where,

$$\Delta = 1 - \sum L_1 + \sum L_1 L_2 + \ldots.$$
$$\sum G_k \Delta_k = G_1 \Delta_1 + G_2 \Delta_2 + \ldots$$
$$\sum G_k \Delta_k = G_1\left(1 - L_2\right) + G_2(1) + G_3\left(1 - L_1\right) + G_4\left(1 - L_2\right) + G_5(1)$$
$$= \tfrac{-72}{(s+1)(s+3)} + \tfrac{12}{(s^2+1)(s+1)} - \tfrac{3\left(1+\frac{6}{s^2}\right)}{s(s+2)} + \tfrac{-72s^2}{(s+2)(s+3)} + \tfrac{12s}{(s^2+1)(s+2)}$$

$$\begin{aligned}\Delta &= 1 - \left(L_1 + L_2\right) + \left(L_1 L_2\right) + \ldots \\ &= 1 - \left(\frac{-6}{s^2} - 3\right) + \frac{18}{s^2} \\ &= \frac{4\left(s^2+6\right)}{s^2}\end{aligned}$$

Substituting the values of Δ and $\sum G_k \Delta_k$, we get the transfer function of the system as

$$\frac{Y(s)}{R(s)} = \frac{-72s^3\left(s^2+1\right)(s+2) + 12s^3(s+2)(s+3) - 3\left(s^2+6\right)\left(s^2+1\right)(s+3)(s+1)}{4s\left(s^2+6\right)\left(s^2+1\right)(s+1)(s+2)(s+3)}$$

$$\frac{-72s^5\left(s^2+1\right)(s+1) + 12s^4(s+1)(s+3)}{4s\left(s^2+6\right)\left(s^2+1\right)(s+1)(s+2)(s+3)}$$