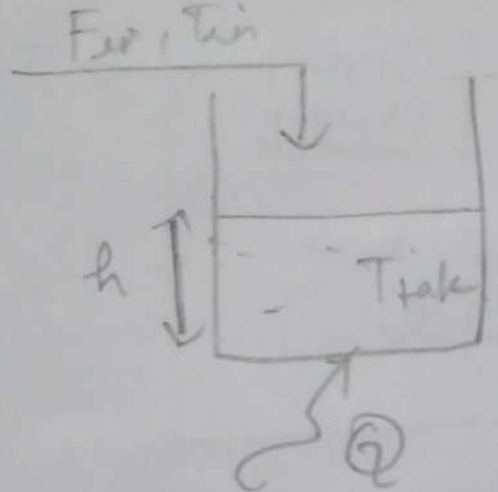
CH3050 ASSIGNMENT - 1

1. a) Fed Batch

We can imagine the system to have water flowing in, with heat being supplied to the stored water



There is no flow out because it is a storage geyser

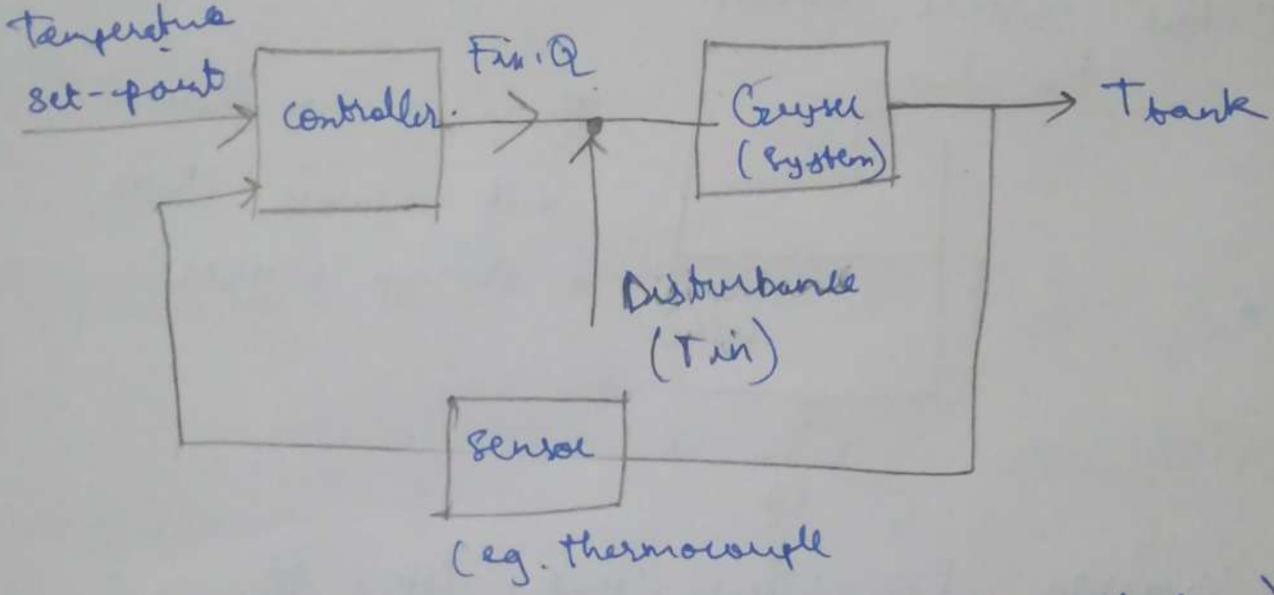
b) Variables: F_{in}, h_{tank}, T_{tank}, T_{in}, Q

→ Manipulated variables: F_{in}, Q (where F_{in} is flow in, and Q is the heat added by the
I assume that Q can be modified)

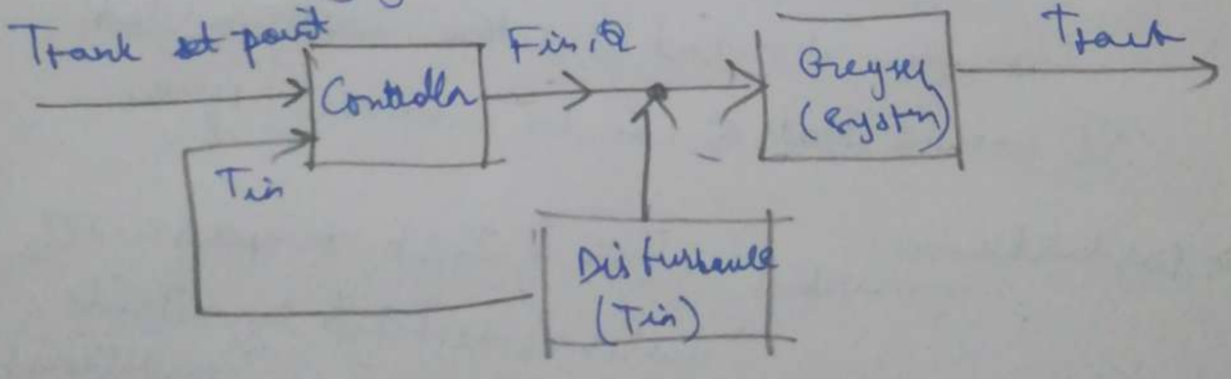
→ Disturbance variables: T_{in} (Inlet temperature of water is dictated by outside conditions)

→ controlled variables: T_{tank} (temperature of water in the tank.

c) Feedback system: Measure the temperature of tank (T_{tank}) and manipulate F_{in} and Q accordingly. F_{in} manipulated by a valve
Q : by using a rheostat



d) Feedforward system: We measure T_{in} (the disturbance) and accordingly change the manipulated variable.



(2) a) At steady state $\frac{dy}{dt} = \frac{d^2y}{dt^2} = 0$

$$\Rightarrow \cancel{y(t)}\ y_{ss} = \frac{b_0}{a_0} u_{ss}. \quad (1)$$

Consider the ODE

$$\frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y(t) = b_0 u(t)$$

$$\frac{d^2}{dt^2}(y - y_{ss}) + a_1 \frac{d}{dt}(y - y_{ss}) + a_0 (y(t) - y_{ss}) + a_0 y_{ss}$$
$$= b_0 (u(t) - u_{ss}) + b_0 u_{ss} \quad (2)$$

$\left(\because \frac{d y_{ss}}{dt} = 0 \rightarrow y_{ss} \text{ is a constant}\right)$

But (1) $\Rightarrow a_0 y_{ss} = b_0 u_{ss}$

$$\therefore \frac{d^2 \tilde{y}}{dt} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 \tilde{u}(t) \quad (3)$$

(This is expected because the ODE is linear)

b) $\cancel{y_{ss} = \frac{b_0}{a_0} u_{ss} \Rightarrow y_{ss} = \frac{3}{15} \times u_{ss}}$

(3) $\Rightarrow \quad a_0 \tilde{y}_{ss} = b_0 \tilde{u}_{ss}$ (@ steadystate)

$$\Rightarrow \tilde{y}_{ss}\ 2a_0 = b_0 \tilde{u}$$

$$\Rightarrow \tilde{u} = \frac{2 \times 15}{3}$$

$$\Rightarrow \boxed{\Delta u = 10 \text{ units}}$$

c) Substituting $\tilde{u}(t) = K_c(2 - \tilde{y}(t))$ in eqn ③,

$$\frac{d^2\tilde{y}}{dt^2} + a_1\frac{d\tilde{y}}{dt} + a_0\tilde{y}(t) = b_0 K_c(2 - \tilde{y}(t))$$

$$\Rightarrow \frac{d^2\tilde{y}}{dt^2} + a_1\frac{d\tilde{y}}{dt} + \tilde{y}(t)(a_0 + b_0K_c) = 2b_0K_c \quad \text{④}$$

Now to answer whether the system will achieve the control objective is equal to asking whether ~~the system~~ the system found in 4 saturates (whether it is stable)

To analyse the stability we can see the poles of this new system.

Also $\left.\frac{d^2\tilde{y}}{dt^2}\right|_{t=0} = 0$ & $\left.\frac{d\tilde{y}}{dt}\right|_{t=0} = 0$ & $\left.\tilde{y}(t)\right|_{t=0} = 0$

because the system is initially assumed to be in steady state.

So $\mathcal{L}\left\{\frac{d^2\tilde{y}}{dt^2}\right\} = s^2\tilde{Y}(s)$

and $\mathcal{L}\left\{\frac{dy}{dt}\right\} = s\tilde{Y}(s)$

~~a)~~ taking Laplace transform,

$$s^2 \tilde{Y}(s) + a_1 s \tilde{Y}(s) + a_0 \tilde{Y}(s) = \frac{2b_0K_c}{s}$$
$$+ b_0K_c \tilde{Y}(s)$$

$$\Rightarrow \tilde{Y}(s) ~~+~~ = \frac{2b_0K_c}{s(s^2+a_1s+a_0 ~~)~~ + b_0K_c)}$$

To apply ~~By~~ Final value theorem, we need the conditions

$s\tilde{Y}(s)$ to be stable.

$\Rightarrow$ roots of $s^2 + a_1 s + a_0 + b_0K_c < 0$

$$\Rightarrow \frac{-a_1 \pm \sqrt{a_1^2 - 4(a_0 + b_0 ~~2~~ K_c)}}{2} < 0$$

~~Note that~~ $\Rightarrow 0 \leq a_1^2 - 4(a_0 + b_0K_c) < a_1^2$

$$\Rightarrow K_c \leq \frac{a_1^2 - 4a_0}{4b_0} \text{ and}$$

$$K_c > -\frac{~~4~~ a_0}{b_0}$$

$\Rightarrow K_c \leq \frac{1}{3}$ and $K_c > -5$

$\Rightarrow -5 < K_c \leq 1/3 \Rightarrow 0 < K_c \leq 1/3$

Under such conditions we apply FVT

to get $\lim_{t \to \infty} \tilde{y}(t) = \lim_{s \to 0} s\tilde{Y}(s)$

$\rightarrow \lim_{t\to\infty} \tilde{y}(t) = \lim_{s\to 0} \cancel{s} \times \frac{2 b_0 K_c}{\cancel{s}(s^2 + a_1 s + a_0 + b_0 K_c)}$

$= 2$

$\therefore$ Required objective is attained

for $0 < K_c \leq 1/3$

d) (3) $\Rightarrow \frac{d^2\tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 \tilde{u}(t)$

Differentiate wrt time,

$$\frac{d^3\tilde{y}}{dt^3} + a_1 \frac{d^2\tilde{y}}{dt^2} + a_0 \frac{d\tilde{y}}{dt} = b_0 \frac{d\tilde{u}}{dt}$$

$$= b_0 \left(K_c \left(- \frac{d\tilde{y}}{dt} \right) + K_I (2 - \tilde{y}) \right)$$

For notational convenience, $\sim$ is dropped

$$\rightarrow \frac{d^3 y}{dt^3} + (a_1 \cancel{+ b_0 K_c}) \frac{d^2 y}{dt^2} + \overset{(a_0 + b_0 K_c)}{\cancel{a_0}} \frac{dy}{dt} + \overset{b_0 K_I y}{\cancel{b_0 y K_I}} = 2 K_I b_0$$

Similar to previous part, take Laplace transform to get,

$$(s^3 + a_1 s^2 + (a_0 + b_0 K_c) s + b_0 K_I) Y(s) = \frac{2 K_I b_0}{s}$$

$$\Rightarrow Y(s) = \frac{2K_I b_0}{s(s^3 + a_1 s^2 + (a_0 + b_0 K_c)s + b_0 K_I)}$$

~~Let~~ We want the poles $(sY(s))$ to be stable

$$\Rightarrow \text{roots}(s^3 + a_1 s^2 + (a_0 + b_0 K_c)s + b_0 K_I) < 0$$

$$\Rightarrow \text{roots}(s^3 + 8s^2 + (15 + 3K_c)s + 3K_I) < 0 \quad (5)$$

Assuming roots obey this property, apply FVT

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s \times \frac{2K_I b_0}{s(s^3 + a_1 s^2 + (a_0 + b_0 K_c)s + b_0 K_I)}$$

$$= 2$$

$\therefore$ objective is achieved.

Conclusion: The objective can't be achieved for any $\{K_c, K_I\}$, it is achieved only when (5) is satisfied.

e.g. $K_c = K_I = 1$, roots are $-3.9 + 1.14j$, $-3.9 - 1.14j$, -0.181

$\Rightarrow$ objective is not achieved.

But for $K_c = 0.25$, $K_I = 0.5$

roots are -4.25, -3.14, -0.1

$\Rightarrow$ objective is achieved.

(3)

a) $$\frac{dw}{dt} = -\left(\frac{L+Va}{M}\right)w + \frac{Va}{M}z \quad \text{— (1)}$$

$a = 0.5$
$z_f = 0.1$
$M = 40$

$$\frac{dz}{dt} = \frac{L}{M}w - \left(\frac{L+Va}{M}\right)z + \frac{Vz_f}{M} \quad \text{— (2)}$$

At steady state, $L = L_{ss} - 4$, $V = V_{ss} = 200$
$= 83$

$$\frac{dw}{dt} = 0, \quad \frac{dz}{dt} = 0$$

(1) $\Rightarrow$ $-6.5w + 2.5z = 0$ — (3)

(2) $\Rightarrow$ $4w - 6.5z = -0.5$ — (4)

Solving (3) & (4),

$w_{ss} = 0.0388$ and $z_{ss} = 0.1008$

$\Rightarrow$ Steady state values: $w = 0.0388$
$z = 0.1008$

b) In this model, states $(x) = \begin{bmatrix} w \\ z \end{bmatrix}$

Inputs $(u) = \begin{bmatrix} L \\ V \end{bmatrix}$ Let $\frac{dw}{dt} = f(\cdot)$ & $\frac{dz}{dt} = g(\cdot)$

$$\frac{dw}{dt} = \frac{dw}{dt}\Big|_{\text{@ steady state}} + \frac{\partial f}{\partial L}(L - L_{ss}) + \frac{\partial f}{\partial V}(V - V_{ss}) + \frac{\partial f}{\partial w}(w - w_{ss}) + \frac{\partial f}{\partial z}(z - z_{ss})$$

$$a_{11} = \frac{\partial f}{\partial w} = -\frac{(Va + L)}{M}$$

$$a_{12} = \frac{\partial f}{\partial z} = \frac{V}{M}$$

$$a_{21} = \frac{\partial g}{\partial w} = \frac{L}{M} \; ; \; a_{22} = -\left(\frac{L + Va}{M}\right)$$

$$b_{11} = \frac{\partial f}{\partial L} = \frac{-w}{M} \; ; \; b_{12} = \frac{\partial f}{\partial g} = \frac{-w}{M} + \frac{az}{M}$$

$$b_{21} = \frac{\partial g}{\partial L} = \frac{w}{M} \; ; \; b_{22} = \frac{-az}{M} + \frac{zf}{M}$$

$$\therefore A = \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix}$$

(all values evaluated at steady state)

$$B = \begin{bmatrix} -0.0019 & 0.0016 \\ -0.0031 & 0.0025 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{dw}{dt} \\ \frac{dz}{dt} \end{bmatrix} = A \begin{bmatrix} w - w_{ss} \\ z - z_{ss} \end{bmatrix} + B \begin{bmatrix} L - L_{ss} \\ V - V_{ss} \end{bmatrix}$$

Let $y = \begin{bmatrix} w - w_{ss} \\ z - z_{ss} \end{bmatrix}$

$\Rightarrow y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4.485 \\ 2-2y \end{bmatrix}$

c) eigenvalues : -3.338, -9.162

eigenvectors : $\begin{bmatrix} 0.6202 \\ 0.7845 \end{bmatrix}$ & $\begin{bmatrix} -0.6202 \\ 0.7845 \end{bmatrix}$

Since $|\lambda_2| > |\lambda_1|$

fastest dir : eigenvector 2 : $\begin{pmatrix} -0.6202 \\ 0.7845 \end{pmatrix}$

slowest dir : eigenvector 1 : $\begin{pmatrix} 0.6202 \\ 0.7845 \end{pmatrix}$

4 a) $\mathcal{L}\{x(t)\} =$

For $0 \le t < 3$

$$\mathcal{L}(t-2) = \int_0^3 t e^{-st}\,dt - 2\int_0^3 e^{-st}\,dt$$

$$= \cancel{\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2} - \frac{2}{s}\left(1 - e^{-3st}\right)\Big|_0^3}$$

$$\cancel{= \frac{1-2s-(1+se^{-3s})}{s^2}}$$

$$= \frac{te^{-st}}{-s}\Big|_0^3 - \int_0^3 \frac{e^{-st}}{s} - \frac{2}{s}\left(1 - e^{-3st}\right)$$

$$= \frac{-3e^{-3s}}{s} + \frac{e^{-3s}}{s^2} - \frac{1}{s^2} - \frac{2}{s}\left(1 - e^{-3s}\right)$$

$$\Rightarrow X(s) = \boxed{\frac{-(1+s)e^{-3s} - 2s + 1}{s^2}} \quad 0 \le t < 3$$

$3 \le t \le 4$

$$X(s) = \int_{3}^{4} e^{-st}\, dt$$

$$\boxed{= \frac{e^{-3s} - e^{-4s}}{s}}$$

$4 \le t < 5$

$$X(s) = \int_{4}^{5} -\cos(3\pi(t-4))\, e^{-st}\, dt$$

$$\Rightarrow X(s) = \cancel{e - \cos(3\pi} \; \frac{-\cos(3\pi(t-4))\, e^{-st}}{-s} \Bigg|_4^5 + \int 3\pi \sin(3\pi(t-4)) \frac{e^{-st}}{+s}\, dt$$

$$= \frac{-\cos(3\pi(t-4))\, e^{-st}}{-s} + \left(\frac{3\pi \sin(3\pi(t-4))\, e^{-st}}{-s^2} \right) \Bigg|_4^5$$

$$- \int \left(\frac{9\pi^2 \cos(3\pi(t-4))}{-s^2} e^{-3t} \right) dt$$

$$\downarrow$$

$$X(s) \times \frac{9\pi^2}{s^2}$$

$$\Rightarrow X(s)\left(1 + \frac{9\pi^2}{s^2}\right) = \frac{-e^{-5s}}{s} + \frac{-e^{-4s}}{s}\ \ldots - \frac{3\pi e^{-4s}}{s^2}$$

$$\Rightarrow \boxed{X(s) = \frac{\left(e^{-5s} + 3\pi e^{-4s}\right)}{\frac{9\pi^2}{s^2} + 1}}$$

$t \geq 5$

$$L\{e^{-2t} x(t)\} = \int_5^\infty e^{-2(t-5)} \cos(5\pi(t-5))\, e^{-st}\, dt$$

$$= \int_5^\infty e^{-t(s+2)-5} \cos(5\pi(t-5))\, dt$$

By part 1

$$\Rightarrow X(s) = \frac{e^{-t(s+2)+10}\cos(5\pi(t-5))}{-(s+2)}\Bigg|_5^\infty - 5\pi \int \frac{\sin(5\pi(t-5))\, e^{-t(s+2)-5}}{s+2}\, dt$$

$$= \frac{e^{-5(s+2)+10}}{s+2} - 5\pi\left[\frac{\sin(5\pi(t-5))}{(s+2)^2} + 5\pi \int \frac{\cos(5\pi(t-5))\, e^{-t(s+2)-5}}{(s+2)^2}\, dt\right]$$

$$\Rightarrow X(s)\left(1 + \frac{25\pi^2}{(s+2)^2}\right) = -e^{-5s}$$

$$\Rightarrow \boxed{X(s) = \frac{e^{-5s}}{1 + \frac{25\pi^2}{(s+2)^2}}}$$

~~Final soln~~

~~$x(s)$~~

(4) b) $\dfrac{s-2}{s(T^2s^2+2\xi Ts+1)} = \dfrac{(s-2)}{T^2 s\left(s^2+\dfrac{2\xi s}{T}+1\right)}$

Roots of $s^2+\dfrac{2\xi s}{T}+\dfrac{1}{T^2}$

$$= -\frac{\xi}{T} \pm \frac{\sqrt{\xi^2-1}}{T}$$ (call these as α, β)

They are real if $\xi \geq 1$

i) $\xi > 1$

$$\frac{A_1}{s} + \frac{A_2}{s+\frac{\xi}{T}-\sqrt{\xi^2-1}} + \frac{A_3}{s-\frac{\xi}{T}+\sqrt{\xi^2-1}}$$

$$= \frac{s-2}{T^2\left(s^2+\frac{2\xi s}{T}+1\right)}$$

Put $s=0$,

$$A_1 = \frac{-2}{T^2}$$

Put $s=$ root α

$$A_2 = \frac{\alpha-2}{T^2}, \quad A_3 = \frac{\beta-2}{T^2}$$

Q3 Part d)

linsys =

A =

	Integ1	Integ2
Integ1	-6.5	2.5
Integ2	4	-6.5

B =

	FR	FR1
Integ1	-0.001938	0.00155
Integ2	-0.003101	0.002481

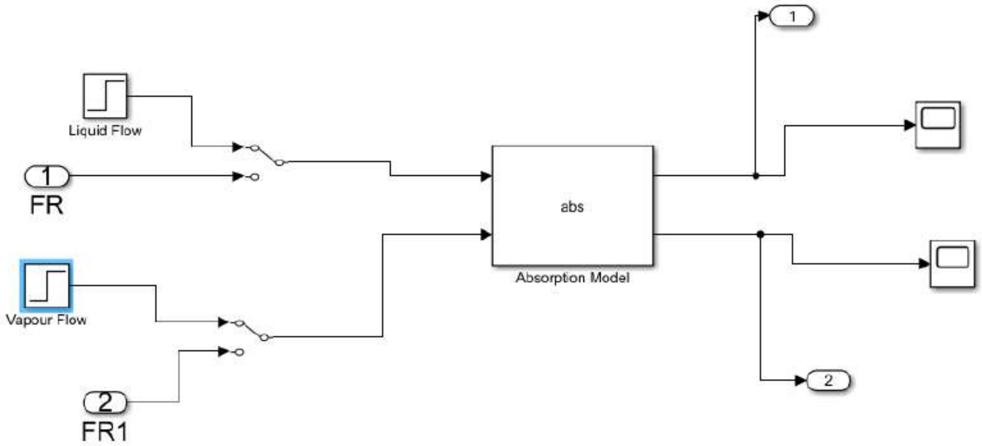
C =

	Integ1	Integ2
Out1	1	0
Out2	0	1

D =

	FR	FR1
Out1	0	0
Out2	0	0

Continuous-time state-space model.

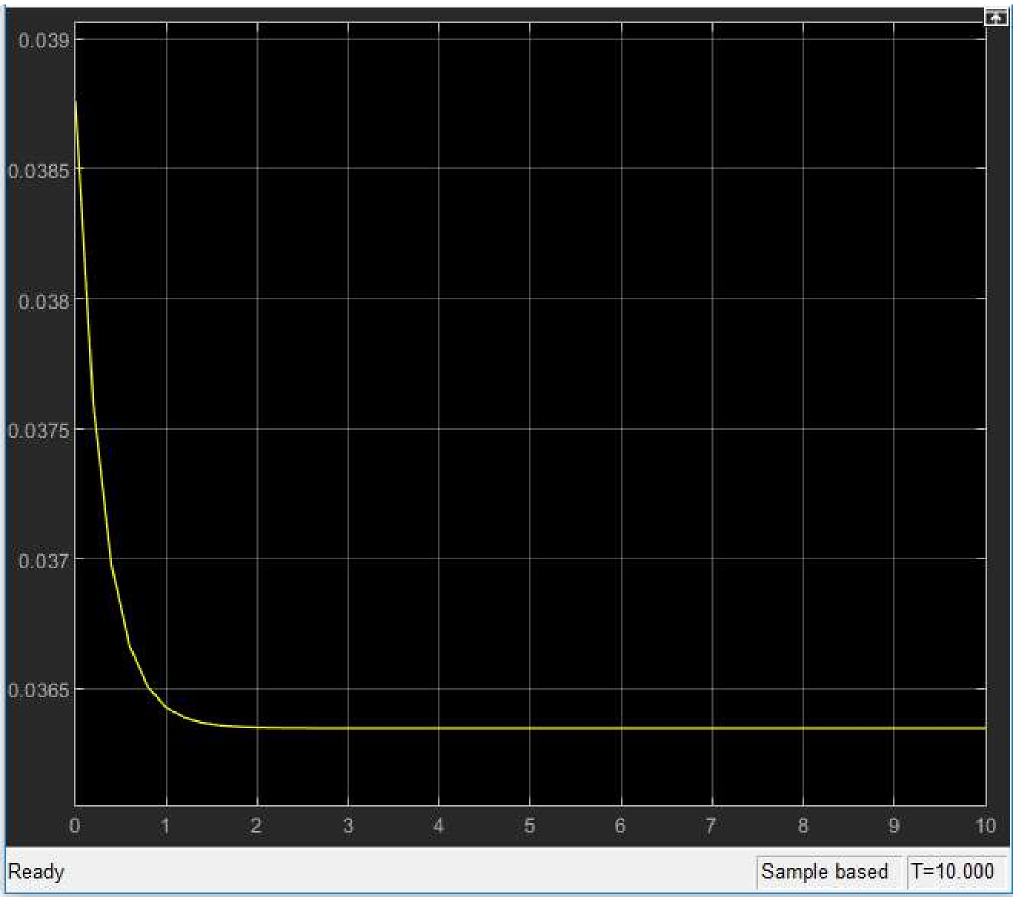


Part e)

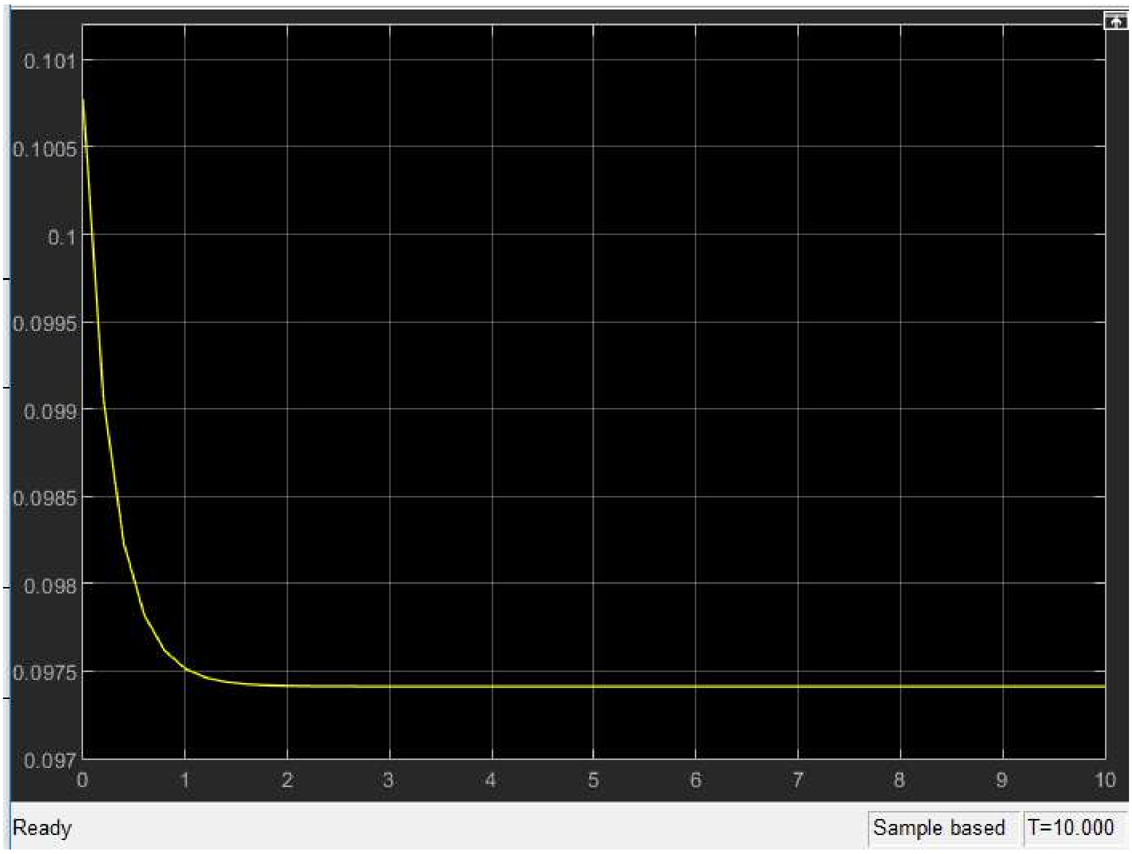
Since if we change both the flow rates, steady state values are same as original, I am changing only L in both cases.

ALL GRAPHS ARE FROM NON LINEAR MODEL. I DID NOT PLOT FOR LINEAR MODEL.

i) $L = 1.05L_{ss}$

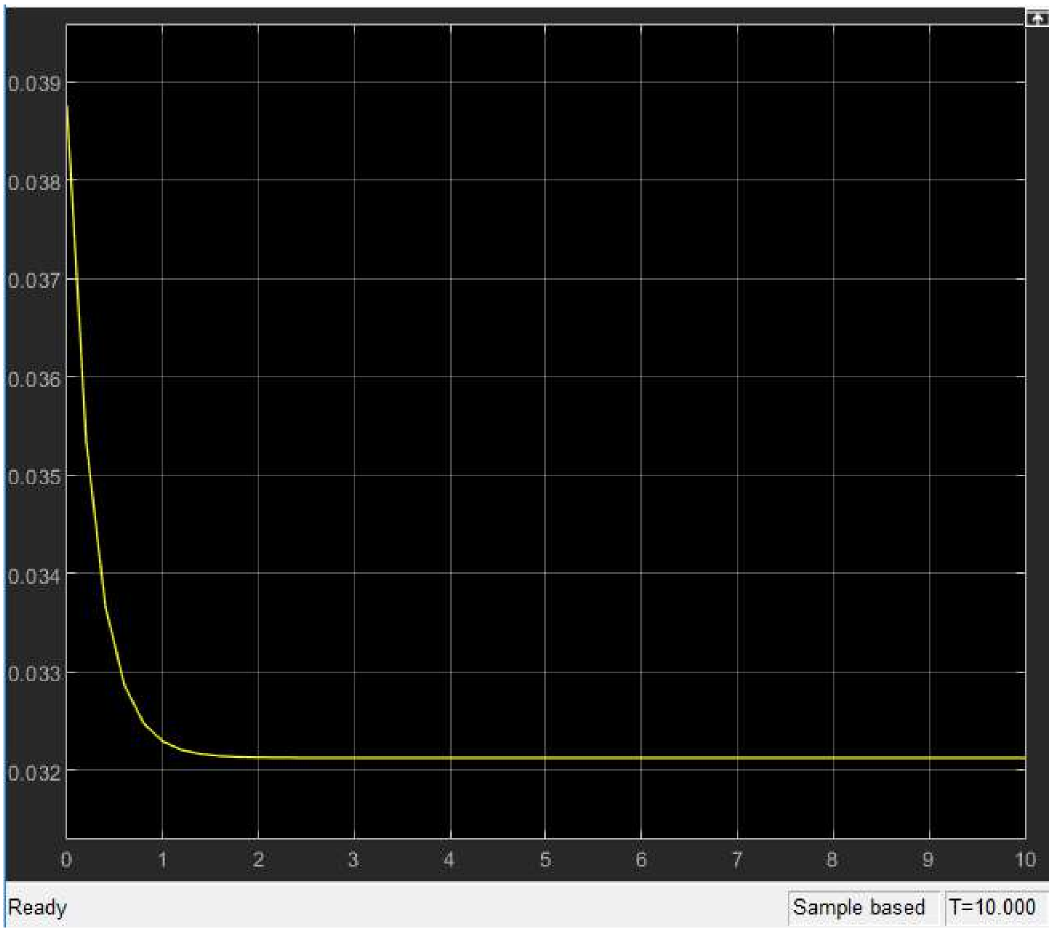


W graph



Z graph

ii) L= $1.15L_{ss}$


0.039
0.038
0.037
0.036
0.035
0.034
0.033
0.032
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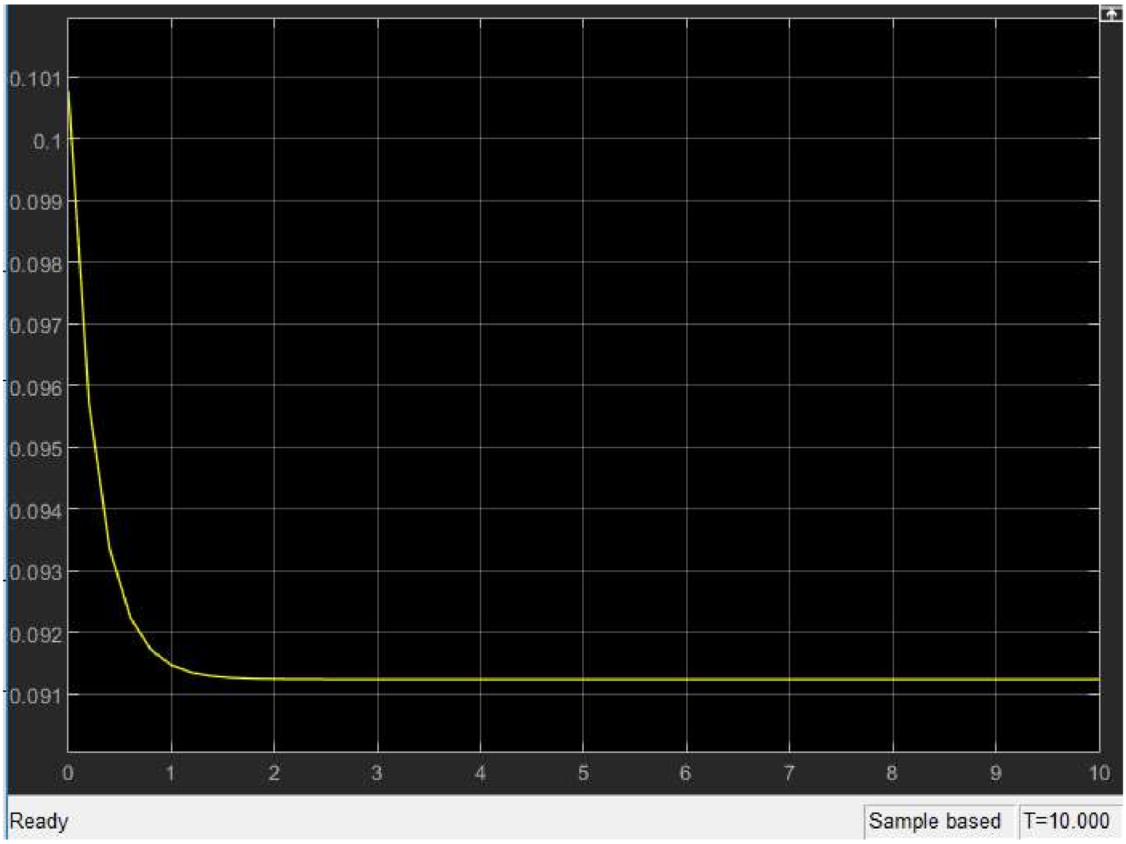
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000

W graph


0.101
0.1
0.099
0.098
0.097
0.096
0.095
0.094
0.093
0.092
0.091
0
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000

Z graph

Code:

```
clear;close all;

%% System charecterstics

Lss = 80; Vss = 100;

M = 20; a=0.5; zf = 0.1;

%% Part a) Finding steady state (by hand)

% Equate derivatives to zero, solve the linear eqn

Ass = [-(a*Vss+Lss)/M Vss*a/M;Lss/M -(a*Vss+Lss)/M];

bss = [0;-Vss*zf/M];

x_ss = inv(Ass)*bss;

%% Part b) Linearisation (by Taylor Expansion)

w_ss = x_ss(1);z_ss=x_ss(2);

A = [-(Vss*a+Lss)/M Vss*a/M;Lss/M -(Lss+Vss*a)/M];

B = [-w_ss/M (-a*w_ss+a*z_ss)/M;(w_ss-z_ss)/M -a*z_ss/M+zf/M];

%% Part c) Finding the eigenvalues-eignvectors of the system

[V,D] = eig(A);

% Second eigen value is faster (more negative)

%% Part d) Find steady-state and linearise

open_system('Q3_model')

% Read the operating conditions into an object

opc = operspec('Q3_model');

% Operating conditions

opc.Inputs(1).u = 80;

opc.Inputs(2).u = 100;

opc.Inputs(1).Known = 1;

opc.Inputs(2).Known = 1;

% Constraints

opc.States(1).Min = 0;opc.States(2).Min = 0;
```

```
% Find the steady state point

ss_point = findop('Q3_model',opc);

% Linearize

linsys = linearize('Q3_model',ss_point)

%% Part e) Give step changes and plot

% Done in SIMULINK. Use the manual switch to step input(s)

[Y,T,X]=step(linsys);

plot()
```