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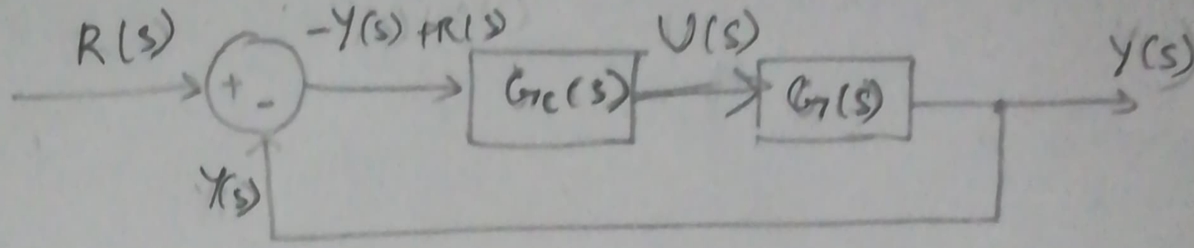
CH18B020

CH3050 ASSIGNMENT-3

① a)



From the block diagram, we can write,

$$U(s) = G_c(s) \left(R(s) - Y(s) \right) \quad \text{—— ①}$$

$$Y(s) = G(s)\, U(s) \quad \text{—— ②}$$

Subst. ① in ②,

$$Y(s) = G(s)\, G_c(s) \left(R(s) - Y(s) \right)$$

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{G(s)\, G_c(s)}{1 + G(s)\, G_c(s)}$$

$$\Rightarrow G_{CL}(s) = \frac{\frac{10}{s^2+7s+10} \times \left(K_C + \frac{K_I}{s} \right)}{1 + \frac{10}{s^2+7s+10} \left(K_C + \frac{K_I}{s} \right)}$$

$$= \frac{10 K_C s + 10 K_I}{s^3 + 7s^2 + 10s + 10 s K_C + 10 K_I}$$

$$\Rightarrow \boxed{G_{CL}(s) = \frac{10 \left(K_C s + K_I \right)}{s^3 + 7s^2 + s(10 + 10K_C) + 10K_I}}$$

b) Employing the R-H criterion for stability,

s^3	1	$10+10K_c$	0
s^2	7	$10K_I$	0
s	$\frac{7(10+10K_c) - 10K_I}{7}$	0	
1	$10K_I$		

To have no poles on RHP there should be no sign change.

$$\Rightarrow \frac{7(10+10K_c) - 10K_I}{7} > 0 \quad \& \quad 10K_I > 0$$

$$\Rightarrow \underline{K_c > -1 + \frac{K_I}{7}} \quad \text{— ①}$$

$$\underline{K_I > 0} \quad \text{— ②}$$

Sufficient conditions for stability

Note that the zero, $s = -\frac{K_I}{K_c}$ is predominantly negative in the admissible region given by the above eqns. So we can safely assume that there won't be any RHP pole cancelled out by the zero. (So ① & ② is sufficient and necessary for most parts) ~~this exists~~

① c) If the ~~d~~ K_c, K_I values are in the admissible region, then the stability is guaranteed.

$\Rightarrow$ We can use the Final Value theorem.

FVT

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s\,Y(s)$$

$$Y(s) = G_{CL}(s)\,R(s)$$

Let $r(t) = r$ (a constant value)

$$\Rightarrow R(s) = \frac{r}{s}$$

$$\therefore Y(s) = \frac{10(K_c s + K_I)}{(s^3 + 7s^2 + s(10 + 10K_c) + 10K_I)} \times \frac{r}{s}$$

Subst. in FVT, we get,

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} \cancel{s} \times \frac{r}{\cancel{s}} \times \frac{10(K_c s + K_I)}{(s^3 + 7s^2 + s(10 + 10K_c) + 10K_I)}$$

$$= \frac{r \times 10 K_I}{10 K_I}$$

$$\Rightarrow \boxed{\lim_{t \to \infty} y(t) = r} \quad (\because K_I \neq 0)$$

$\therefore$ Yes! Set point tracking is possible as long as G_{CL} is stable.