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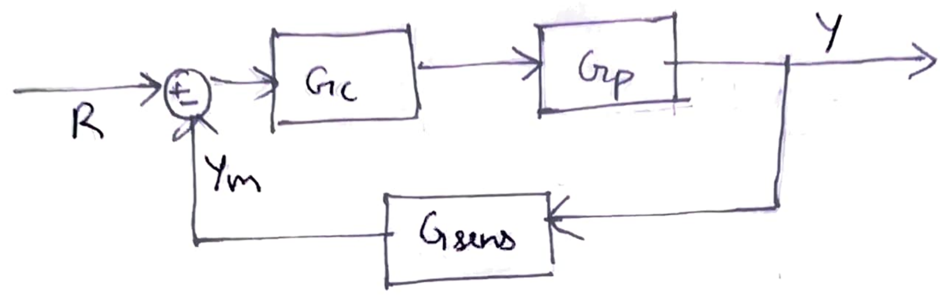
CH18B020

CH3050 ASSIGNMENT- 5

① a)



$$(R - G_{sens} Y)(G_c G_p) = Y$$

$$\Rightarrow \frac{Y}{R} = \frac{G_c G_p}{1 + G_c (G_{sens} G_p)}$$

where $G_c = K_c$.

$\therefore$ C.E. : $1 + K_c G_{sens} G_p = 0$

$$\Rightarrow 1 + K_c \left[\frac{s^2 - 4s + 8}{(s)(s+1)(s+3)(s+10)} \right] = 0$$

poles : $\boxed{0, -1, -3, -10}$

zeroes : $\frac{4 \pm \sqrt{16-32}}{2} = \boxed{2 \pm 2j}$

i) Asymptotes angles.

totally $p - z = 4 - 2 = 2$ asymptotes

$$\theta_1 = \frac{2-1}{2} \pi = \boxed{\frac{\pi}{2}}$$

$$\theta_2 = \frac{2(2)-1}{2} \pi = \boxed{\frac{3\pi}{2}}$$

ii) Centroid

$$\sigma = \frac{\sum P_i - \sum z_i}{P - Z}$$

$$= \frac{-14 - 4}{2} = -9$$

$\therefore$ centroid, $\boxed{\sigma = -9 + 0j}$

iii) Angles of arrival. $\left[180 + \sum_{\forall i} \angle z_j - P_i - \sum_{j \neq k} \angle (z_j - z_k)\right]$

The 'root locus 'arrives' at zeroes.

$$\theta_{z_1} : 180 + \tan^{-1}\left(\frac{2}{2}\right) + \tan^{-1}\left(\frac{2}{2-(-1)}\right)$$

$$+ \tan^{-1}\left(\frac{2}{2-(-3)}\right) + \tan^{-1}\left(\frac{2}{2-(-10)}\right)$$

$$- \tan^{-1}\left(\frac{2-(-2)}{2-2}\right)$$

$$\Rightarrow \boxed{\theta_{z_1} = 199.954^\circ}$$

$$\theta_{z_2} = 180 + \tan^{-1}\left(\frac{2}{-2}\right) + \tan^{-1}\left(\frac{2}{-3}\right)$$

$$+ \tan\left(\frac{2}{-8}\right) + \tan^{-1}\left(\frac{-2}{12}\right)$$

$$- \tan^{-1}\left(\frac{-2 - 2}{2 - 2}\right)$$

$$\Rightarrow \boxed{\theta_{z_2} = 160.046^\circ}$$

iv) Break-in points / Break-away points

$$\beta = \frac{-(s)(s+1)(s+3)(s+10)}{s^2 - 4s + 8}$$

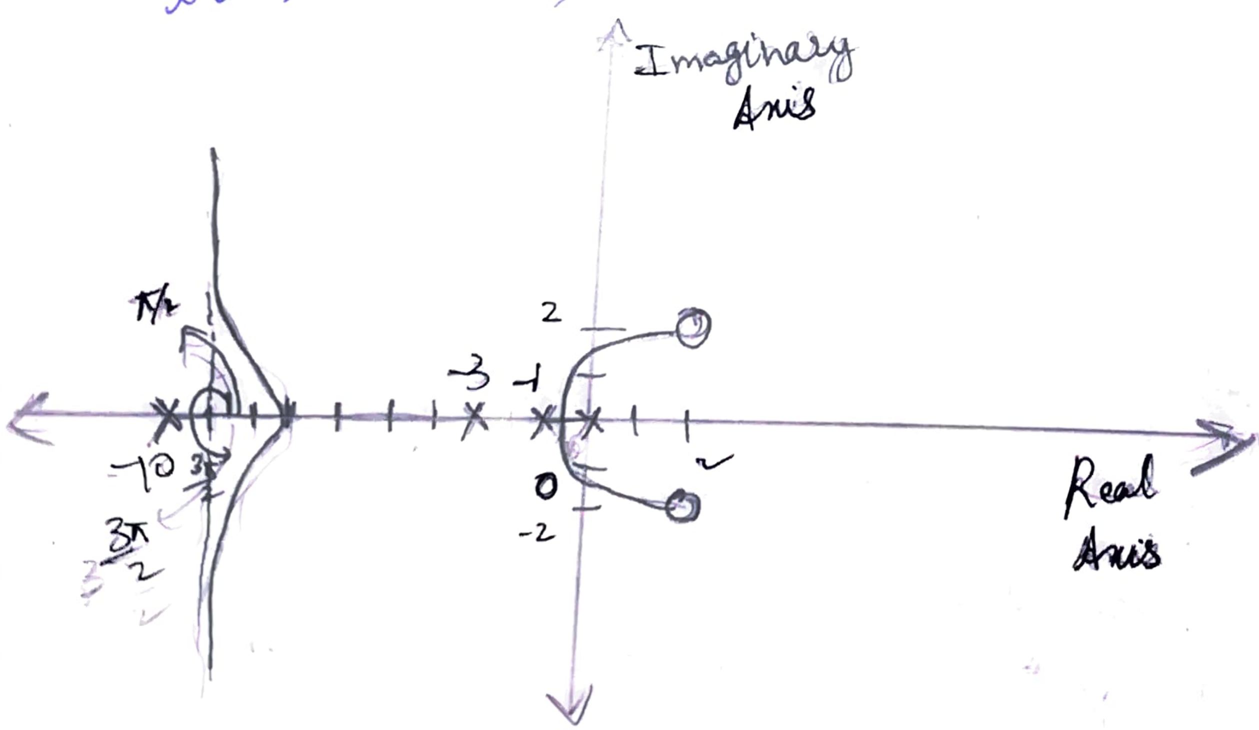
$$= \frac{-(s^4 + 14s^3 + 43s^2 + 30s)}{s^2 - 4s + 8}$$

$$\frac{d\beta}{ds} = 0 \Rightarrow (4s^3 + 42s^2 + 86s + 30)(s^2 - 4s + 8) - (2s-4)(s^4 + 4s^3 + 43s^2 + 30s) = 0$$

Solving for s,

$s = -0.384, -7.07$ are the relevant maximas.

(others are imaginary roots - don't lie on the real axis)



c) Sol. Recall, the C.E. cast in RL form:

$$1 + K_c \left[\frac{s^2 - 4s + 8}{(s)(s+1)(s+3)(s+10)} \right]$$

Call the parameter K_c as β

Now, we have the ultimate gain when the root locus intersects the imaginary axis (and goes to RHP)

Let the intersection be at some $s = j\omega$ (where ω is a real value)

$$\Rightarrow 1 + \beta \cdot \frac{\left((j\omega)^2 - 4(j\omega) + 8\right)}{\left((j\omega)^4 + 14(j\omega)^3 + 43(j\omega)^2 + 30j\omega\right)} = 0$$

$$\Rightarrow (\omega^4 - 14j\omega^3 - 43\omega^2 + 30j\omega) + \beta(-\omega^2 - 4j\omega + 8) = 0$$

$$\Rightarrow (\omega^4 - \beta\omega^2 - 43\omega^2 + 8\beta) + j(-14\omega^3 + 30\omega - 4\beta\omega) = 0$$

Equate the real and imaginary parts to 0

$$\omega^4 - \beta\omega^2 - 43\omega^2 + 8\beta = 0 \quad \text{——— (1)}$$

$$-14\omega^3 + 30\omega - 4\beta\omega = 0 \quad \text{——— (2)}$$

(2) $\Rightarrow$ $\omega^2 = \dfrac{30-4\beta}{14}$ ——— (3)

Substitute the above equation in (1),

$$\left(\frac{30-4\beta}{14}\right)^2 - \beta\left(\frac{30-4\beta}{14}\right) - 43\left(\frac{30-4\beta}{14}\right) + 8\beta = 0$$

$$\Rightarrow 900 + 16\beta^2 - 240\beta - 420\beta + 56\beta^2 - 18060 + 2408\beta + 8\times196\beta = 0.$$

$$\Rightarrow 72\beta^2 + 3316\beta - 17160 = 0$$

$$\Rightarrow \beta = \frac{-3316 \pm \sqrt{(3316)^2 + 4\times17160\times72}}{144}$$

$$= 4.696 \quad \text{or} \quad -50.252$$

Since $K_c > 0$ for a P-controller,

$\beta = 4.696 \Rightarrow \omega = \sqrt{\dfrac{30-4\beta}{14}}$

$= 0.895$

$\therefore K_{c,\,\text{ultimate}} = 4.696$

$\therefore$ Ultimate Gain $K_{cu} = \boxed{4.696}$

1e) Characterstic Equation : $1 + G_c G_p = 0$

$$1 + \left(K_c + \frac{K_I}{s}\right)\left(\frac{s^2 - 4s + 8}{s(s+1)(s+3)(s+10)}\right) = 0$$

$$\Rightarrow \left[1 + K_c \left(\frac{s^2 - 4s + 8}{s(s+1)(s+3)(s+10)}\right)\right] + \frac{K_I}{s}\,\frac{s^2 - 4s + 8}{s(s+1)(s+3)(s+10)} = 0$$

$$\Rightarrow 1 + \frac{K_I \frac{1}{s}\left(\frac{s^2 - 4s + 8}{s(s+1)(s+3)(s+10)}\right)}{1 + K_c\left(\frac{s^2 - 4s + 8}{s(s+1)(s+3)(s+10)}\right)} = 0$$

Substituting $K_c = 0.85$ & simplifying

$$\Rightarrow 1 + \frac{K_I (s^2 - 4s + 8)}{s^5 + 14s^4 + 43.85 s^3 + 26.6 s^2 + 6.8 s} = 0$$

To get the K_I, ultimate we need to find the gain at cross-over point.

For that purpose we can use the root locus plot.