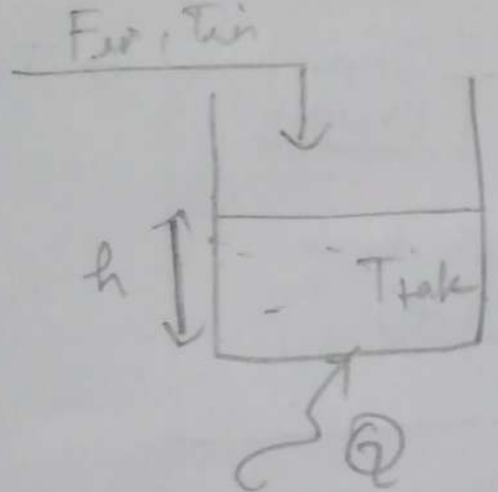
CH3050 ASSIGNMENT - 1

① a) Fed Batch

We can imagine the system to have water flowing in, with heat being supplied to the stored water



There is no flow out because it is a storage geyser

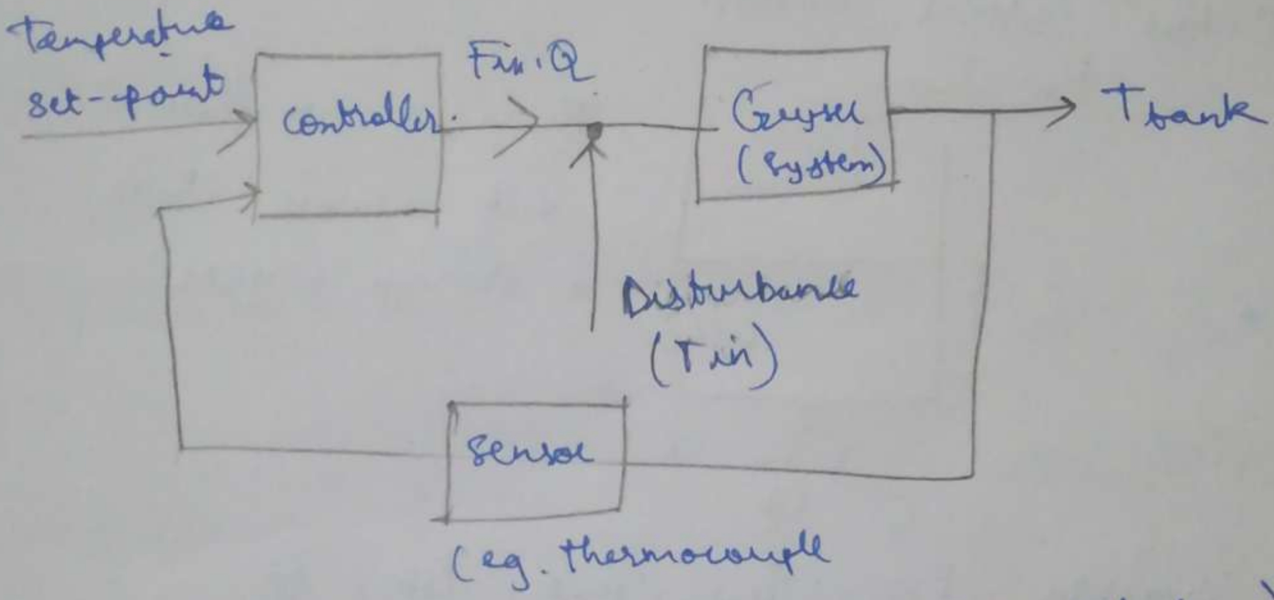
b) Variables: F_{in}, h_{tank}, T_{tank}, T_{in}, Q

→ Manipulated variables: F_{in}, Q (where F_{in} is flow in, and Q is the heat added
I assume that Q can be modified) by the

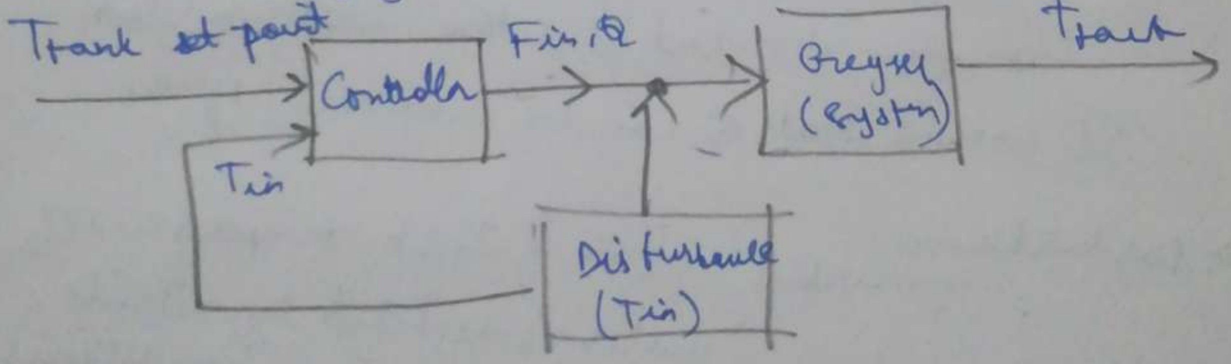
→ Disturbance variables: T_{in} (Inlet temperature of water is dictated by outside conditions)

→ controlled variables: T_{tank} (temperature of water in the tank.

c) Feedback system: Measure the temperature of tank (T_{tank}) and manipulate F_{in} and Q accordingly. F_{in} manipulated by a valve
Q · by using a rheostate



d) Feedforward system: We measure T_{in} (the disturbance) and accordingly change the manipulated variable



(2) a) At steady state $\frac{dy}{dt} = \frac{d^2y}{dt^2} = 0$

$$\Rightarrow \cancel{y(t)} \quad y_{ss} = \frac{b_0}{a_0} u_{ss}. \quad \text{—— (1)}$$

Consider the ODE

$$\frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y(t) = b_0 u(t)$$

$$\frac{d^2}{dt^2}(y - y_{ss}) + a_1 \frac{d}{dt}(y - y_{ss}) + a_0 (y(t) - y_{ss}) + a_0 y_{ss} = b_0 (u(t) - u_{ss}) + b_0 u_{ss} \quad \text{—— (2)}$$

$\left(\because \frac{d y_{ss}}{dt} = 0 \rightarrow y_{ss} \text{ is a constant}\right)$

But (1) $\Rightarrow a_0 y_{ss} = b_0 u_{ss}$

$$\therefore \frac{d^2 \tilde{y}}{dt} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 \tilde{u}(t) \quad \text{—— (3)}$$

(This is expected because the ODE is linear)

b) $\cancel{y_{ss} = \frac{b_0}{a_0} u_{ss} \Rightarrow y_{ss} = \frac{3}{15} \times u_{ss}}$

(3) $\Rightarrow \quad a_0 \tilde{y}_{ss} = b_0 \tilde{u}_{ss}$ (@ steadystate)

$$\Rightarrow \tilde{y}_{ss}\ 2a_0 = b_0 \tilde{u}$$

$$\Rightarrow \tilde{u} = \frac{2 \times 15}{3}$$

$$\Rightarrow \boxed{\Delta u = 10 \text{ units}}$$

c) Substituting $\tilde{u}(t) = K_c(2 - \tilde{y}(t))$ in eqn (3),

$$\frac{d^2\tilde{y}}{dt^2} + a_1\frac{d\tilde{y}}{dt} + a_0\tilde{y}(t) = b_0 K_c (2 - \tilde{y}(t))$$

$$\Rightarrow \frac{d^2\tilde{y}}{dt^2} + a_1\frac{d\tilde{y}}{dt} + \tilde{y}(t)(a_0 + b_0 K_c) = 2b_0 K_c \qquad (4)$$

Now to answer whether the system will achieve the control objective is equt to asking whether ~~the~~ ~~system~~ the found system saturates (whether it is stable) in 4

To analyse the stability we can see the poles of this new system.

Also $\left.\frac{d^2\tilde{y}}{dt^2}\right|_{t=0} = 0$ & $\left.\frac{d\tilde{y}}{dt}\right|_{t=0} = 0$ & $\left.\tilde{y}(t)\right|_{t=0} = 0$

because the system is initially assumed to be in steady state.

So $\mathcal{L}\left\{\frac{d^2\tilde{y}}{dt^2}\right\} = s^2\tilde{Y}(s)$

and $\mathcal{L}\left\{\frac{dy}{dt}\right\} = s\tilde{Y}(s)$

~~a)~~ taking Laplace transform,

$$s^2 \tilde{Y}(s) + a_1 s \tilde{Y}(s) + a_0 \tilde{Y}(s) = \frac{2 b_0 K_c}{s}$$
$$+ b_0 K_c \tilde{Y}(s)$$

$$\Rightarrow \tilde{Y}(s) \cancel{+} = \frac{2 b_0 K_c}{s(s^2 + a_1 s + a_0 + b_0 K_c)}$$

To apply ~~By~~ Final value theorem, we need the conditions $s\tilde{Y}(s)$ to be stable.

$\Rightarrow$ roots of $s^2 + a_1 s + a_0 + b_0 K_c < 0$

$$\Rightarrow \frac{-a_1 \pm \sqrt{a_1^2 - 4(a_0 + b_0 K_c)}}{2} < 0$$

~~Note that~~ $\Rightarrow 0 \leq a_1^2 - 4(a_0 + b_0 K_c) < a_1^2$

$$\Rightarrow K_c \leq \frac{a_1^2 - 4a_0}{4b_0} \text{ and}$$

$$K_c > -\frac{a_0}{b_0}$$

$$\Rightarrow K_c \leq \frac{1}{3} \text{ and } K_c > -5$$

$$\Rightarrow -5 < K_c \leq 1/3 \Rightarrow 0 < K_c \leq 1/3$$

Under such conditions we apply FVT

to get $\lim_{t \to \infty} \tilde{y}(t) = \lim_{s \to 0} s\tilde{Y}(s)$

$$\rightarrow \lim_{t\to\infty} \tilde{y}(t) = \lim_{s\to 0} \cancel{s} \times \frac{2 b_0 K_c}{\cancel{s}\,(s^2 + a_1 s + a_0 + b_0 K_c)}$$

$$= 2$$

$\therefore$ Required objective is attained for $0 < K_c \leq 1/3$

d) (3) $\Rightarrow \frac{d^2\tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 \tilde{u}(t)$

Differentiate wrt time,

$$\frac{d^3\tilde{y}}{dt^3} + a_1 \frac{d^2\tilde{y}}{dt^2} + a_0 \frac{d\tilde{y}}{dt} = b_0 \frac{d\tilde{u}}{dt}$$

$$= b_0 \left(K_c \left(- \frac{d\tilde{y}}{dt} \right) + K_I (2 - \tilde{y}) \right)$$

For notational convenience, $\sim$ is dropped

$$\rightarrow \frac{d^3 y}{dt^3} + (a_1 \cancel{+ b_0 K_c}) \frac{d^2 y}{dt^2} + \overset{(a_0 + b_0 K_c)}{\cancel{a_0}} \frac{dy}{dt} + \overset{b_0 K_I y}{\cancel{b_0 y K_I}} = 2 K_I b_0$$

Similar to previous part, take Laplace transform to get,

$$(s^3 + a_1 s^2 + (a_0 + b_0 K_c) s + b_0 K_I)\, Y(s) = \frac{2 K_I b_0}{s}$$

$\Rightarrow Y(s) = \dfrac{2 K_I b_0}{s(s^3 + a_1 s^2 + (a_0 + b_0 K_c) s + b_0 K_I)}$

~~Let~~ We want the poles $(sY(s))$ to be stable

$\Rightarrow$ roots $(s^3 + a_1 s^2 + (a_0 + b_0 K_c) s + b_0 K_I) < 0$

$\Rightarrow$ roots $(s^3 + 8 s^2 + (15 + 3K_c) s + 3 K_I) < 0$ — (5)

Assuming roots obey this property, apply FVT

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s \times \frac{2 K_I b_0}{s(s^3 + a_1 s^2 + (a_0 + b_0 K_c) s + b_0 K_I)}$$

$= 2$

$\therefore$ objective is achieved.

Conclusion: The objective can't be achieved for any $\{K_c, K_I\}$, it is achieved only when (5) is satisfied.

e.g. $K_c = K_I = 1$, roots are $-3.9 + 1.14j$, $-3.9 - 1.14j$, -0.181

$\Rightarrow$ objective is not achieved.

But for $K_c = 0.25$, $K_I = 0.5$

roots are -4.75, -3.14, -0.1

$\Rightarrow$ objective is achieved.

3) a) $\frac{dw}{dt} = -\left(\frac{L+Va}{M}\right)w + \frac{Va}{M}z$ — ① $\quad a = 0.5$, $z_f = 0.1$, $M = 40$

$\frac{dz}{dt} = \frac{L}{M}w - \left(\frac{L+Va}{M}\right)z + \frac{V z_f}{M}$ — ②

At steady state, $L = L_{ss} = 8$, $V = V_{ss} = 200$
$= 83$

$\frac{dw}{dt} = 0$, $\frac{dz}{dt} = 0$

① $\Rightarrow -6.5w + 2.5z = 0$ — ③

② $\Rightarrow 4w - 6.5z = -0.5$ — ④

Solving ③ & ④,

$w_{ss} = 0.0388$ and $z_{ss} = 0.1008$

$\Rightarrow$ steady state values: $w = 0.0388$
$z = 0.1008$

b) In this model, states $(x) = \begin{bmatrix} w \\ z \end{bmatrix}$

Inputs $(u) = \begin{bmatrix} L \\ V \end{bmatrix}$ Let $\frac{dw}{dt} = f(\cdot)$ & $\frac{dz}{dt} = g(\cdot)$

$$\frac{dw}{dt} = \frac{dw}{dt}\Big|_{@\text{ steady state}} + \frac{\partial f}{\partial L}(L - L_{ss}) + \frac{\partial f}{\partial V}(V - V_{ss}) + \frac{\partial f}{\partial w}(w - w_{ss}) + \frac{\partial f}{\partial z}(z - z_{ss})$$

$$a_{11} = \frac{\partial f}{\partial \omega} = -\frac{(Va + L)}{M}$$

$$a_{12} = \frac{\partial f}{\partial z} = \frac{V}{M}$$

$$a_{21} = \frac{\partial g}{\partial \omega} = \frac{L}{M} ; \quad a_{22} = -\left(\frac{L + Va}{M}\right)$$

$$b_{11} = \frac{\partial f}{\partial L} = \frac{-\omega}{M} ; \quad b_{12} = \frac{\partial f}{\partial \cancel{g}} = \frac{-\omega}{M} + \frac{a_2}{M}$$

$$b_{21} = \frac{\partial g}{\partial L} = \frac{\omega}{M} ; \quad b_{22} = \cancel{\quad} \frac{-a_2}{M} + \frac{z_f}{M}$$

$$\therefore A = \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix}$$

(all values evaluated at steady state)

$$B = \begin{bmatrix} -0.0019 & 0.0016 \\ -0.0031 & 0.0025 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{d\omega}{dt} \\ \frac{dz}{dt} \end{bmatrix} = A \begin{bmatrix} \omega - \omega_{ss} \\ z - z_{ss} \end{bmatrix} + B \begin{bmatrix} L - L_{ss} \\ V - V_{ss} \end{bmatrix}$$

Let $y = \begin{bmatrix} \omega - \omega_s \\ z - z_s \end{bmatrix}$

$\Rightarrow y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4.485 \\ 2-2y \end{bmatrix}$

c) eigenvalues : -3.338 , -9.162

eigenvectors : $\begin{bmatrix} 0.6202 \\ 0.7845 \end{bmatrix}$ & $\begin{bmatrix} -0.6202 \\ 0.7845 \end{bmatrix}$

Since $|\lambda_2| > |\lambda_1|$

fastest dir : eigenvector 2 : $\begin{pmatrix} -0.6202 \\ 0.7845 \end{pmatrix}$

slowest dir : eigenvector 1 : $\begin{pmatrix} 0.6202 \\ 0.7845 \end{pmatrix}$

(4) a) $\mathcal{L}\{x(t)\} =$

For $0 \le t < 3$

$L(t-2) = \int_0^3 t e^{-st} dt - 2\int_0^3 e^{-st} dt$

$= \cancel{\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2} - \frac{2}{s}\left(1 - e^{-3st}\right)\Big|_0^3}$

$= \cancel{\frac{1-2s-(1+se^{-3s})}{s^2}}$

$= \frac{te^{-st}}{-s}\Big|_0^3 - \int_0^3 \frac{e^{-st}}{s} - \frac{2}{s}\left(1-e^{-3st}\right)$

$= \frac{-3e^{-3s}}{s} + \frac{e^{-3s}}{s^2} - \frac{1}{s^2} - \frac{2}{s}\left(1-e^{-3s}\right)$

$$\Rightarrow X(s) = \boxed{\frac{-(1+s)e^{-3s} - 2s + 1}{s^2}} \quad 0 \le t < 3$$

$3 \le t \le 4$

$$X(s) = \int_3^4 e^{-st}\,dt$$

$$\boxed{= \frac{e^{-3s} - e^{-4s}}{s}}$$

$4 \le t < 5$

$$X(s) = \int_4^5 -\cos(3\pi(t-4))\, e^{-st}\,dt$$

$$\Rightarrow X(s) = \cancel{-\cos(3\pi} \; \frac{-\cos(3\pi(t-4))}{-s} e^{-st} \Big|_4^5 + \int 3\pi \sin(3\pi(t-4)) \frac{e^{-st}}{+s}\,dt$$

$$= \frac{-\cos(3\pi(t-4))}{-s} e^{-st} + \left(\frac{3\pi \sin(3\pi(t-4))}{-s^2} e^{-st} \right|_4^5$$

$$- \underbrace{\int \left(\frac{9\pi^2 \cos(3\pi(t-4))}{-s^2} e^{-st} \right) dt}$$

$$\downarrow$$

$$X(s) \times \frac{9\pi^2}{s^2}$$

$$\Rightarrow X(s)\left(1 + \frac{9\pi^2}{s^2}\right) = \frac{-e^{-5s}}{s} + -\frac{3\pi e^{-4s}}{s^2}$$

$$\Rightarrow \boxed{X(s) = \frac{\left(e^{-5s} + 3\pi e^{-4s}\right)}{\frac{9\pi^2}{s^2} + 1}}$$

$t \geq 5$

$$\mathcal{L}\{e^{-2t}\} X(s) = \int_5^\infty e^{-2(t-5)} \cos(5\pi(t-5)) \, e^{-st} \, dt$$

$$= \int_5^\infty e^{-t(s+2)-5} \cos(5\pi(t-5)) \, dt$$

By parts I

$$\Rightarrow X(s) = \left.\frac{e^{-t(s+2)+10} \cos(5\pi(t-5))}{-(s+2)}\right|_5^\infty - 5\pi \int \frac{\sin(5\pi(t-5)) e^{-t(s+2)-5}}{s+2} dt$$

$$= \frac{e^{-5(s+2)+10}}{s+2} - 5\pi \left[\frac{\sin(5\pi(t-5))}{(s+2)^2} + 5\pi \int \frac{\cos(5\pi(t-5)) e^{-t(s+2)-5}}{(s+2)^2} dt \right]$$

$$\Rightarrow X(s)\left(1 + \frac{25\pi^2}{(s+2)^2}\right) = e^{-5s}$$

$$\Rightarrow \boxed{X(s) = \frac{e^{-5s}}{1 + \frac{25\pi^2}{(s+2)^2}}}$$

④ b) $\dfrac{s-2}{T^2 s\left(s^2+\dfrac{2\zeta s}{T}+1\right)}$

Roots of $s^2+\dfrac{\zeta s}{T}+\dfrac{1}{T^2}$:

$$\alpha = -\frac{\zeta}{T} + \frac{\sqrt{\zeta^2-1}}{T}$$

$$\beta = -\frac{\zeta}{T} - \frac{\sqrt{\zeta^2-1}}{T}$$

i) ~~Real~~ $\zeta > 1$, real unique roots

Let $\dfrac{A_1}{s}+\dfrac{A_2}{s-\alpha}+\dfrac{A_3}{s-\beta} = X(s) = \dfrac{s-2}{T^2 s\left(s^2+\dfrac{2\zeta s}{T}+1\right)}$

$\Rightarrow A_1(s-\alpha)(s-\beta)+A_2\, s(s-\beta)+A_3\, s(s-\alpha)$

$= \dfrac{s-2}{T^2}$

Put $s=0$,

$$A_1 = \frac{-2}{T^2\alpha\beta}$$

Put $s=\alpha$,

$$A_2 = \frac{\alpha-2}{T^2\alpha(\alpha-\beta)}$$

Put $s=\beta$,

$$A_3 = \frac{\beta-2}{T^2\beta(\beta-\alpha)}$$

$$\therefore X(s) = \frac{A_1}{s} + \frac{A_2}{s-\alpha} + \frac{A_3}{s-\beta}$$

$$= \left(\frac{-2}{T^2\alpha\beta}\right) \times \frac{1}{s} + \left(\frac{\alpha-2}{T^2(\alpha)(\alpha-\beta)}\right)\frac{1}{s-\alpha} + \left(\frac{\beta-2}{T^2\beta(\beta-\alpha)}\right)\frac{1}{s-\beta}.$$

Take Laplace inverse.

& use the property

$$\mathcal{L}^{-1}\{F(s)+G(s)\} = f(t) + g(t)$$

$$\Rightarrow x(t) = \frac{-2}{T^2\alpha\beta}\exp(0t) + \frac{\alpha-2}{T^2(\alpha)(\alpha-\beta)}\exp(\alpha t) + \frac{\beta-2}{T^2\beta(\beta-\alpha)}\exp(\beta t)$$

$$\alpha\beta = \frac{1}{T^2} \ ; \ \alpha - \beta = \frac{2\sqrt{\zeta^2-1}}{T}$$

$$\Rightarrow x(t) = -2 + \frac{(\alpha-2)\beta}{T^2(\alpha\beta)(\alpha-\beta)}\exp(\alpha t) + \frac{(\beta-2)\alpha}{T^2\alpha\beta(\beta-\alpha)}\exp(\beta t)$$

$$\Rightarrow x(t) = -2 + \frac{(\alpha-2)\beta\exp(\alpha t)}{\alpha-\beta} + \frac{(\beta-2)\alpha\exp(\beta t)}{(\beta-\alpha)} \quad \text{---} \ (\star)$$

$$= -2 + \frac{1}{T}\frac{\exp(\alpha t)}{2\sqrt{\zeta^2-1}} - 2\left(\frac{-\zeta-\sqrt{\zeta^2-1}}{2\sqrt{\zeta^2-1}}\right)\exp(\alpha t)$$

$$- \frac{1}{T}\frac{\exp(\beta t)}{2\sqrt{\zeta^2-1}} + \frac{2\left(-\zeta+\sqrt{\zeta^2-1}\right)\exp(\beta t)}{2\sqrt{\zeta^2-1}}$$

$$\Rightarrow x(t) = -2 + \frac{1}{2T\sqrt{\zeta^2-1}}\exp\left(\left(\frac{-\zeta+\sqrt{\zeta^2-1}}{T}\right)t\right)$$

$$+ \left(\frac{\zeta+\sqrt{\zeta^2-1}}{\sqrt{\zeta^2-1}}\right)\exp\left(\left(\frac{-\zeta+\sqrt{\zeta^2-1}}{T}\right)t\right) - \frac{1}{T}$$

$$- \frac{1}{2T\sqrt{\zeta^2-1}}\exp\left(\left(\frac{-\zeta-\sqrt{\zeta^2-1}}{T}\right)t\right) + \left(\frac{-\zeta+\sqrt{\zeta^2-1}}{2\sqrt{\zeta^2-1}}\right)\exp\left(\left(\frac{-\zeta-\sqrt{\zeta^2-1}}{T}\right)t\right)$$

$$\Rightarrow \boxed{\begin{aligned} x(t) = -2 &+ \frac{1}{2T\sqrt{\zeta^2-1}}\exp\left(\left(\frac{-\zeta+\sqrt{\zeta^2-1}}{T}\right)t\right) - \frac{1}{2T\sqrt{\zeta^2-1}}\exp\left(\frac{-\zeta-\sqrt{\zeta^2-1}}{T}\right) \\ &+ \left(\frac{\zeta+\sqrt{\zeta^2-1}}{\sqrt{\zeta^2-1}}\right)\exp\left(\left(\frac{-\zeta+\sqrt{\zeta^2-1}}{T}\right)t\right) \\ &+ \left(\frac{-\zeta+\sqrt{\zeta^2-1}}{\sqrt{\zeta^2-1}}\right)\exp\left(\frac{-\zeta-\sqrt{\zeta^2-1}}{T}t\right) \end{aligned}}$$

ii) $q_1 = 1$

$$X(s) = \frac{A_1}{s} + \frac{A_2}{s-\alpha} + \frac{A_3}{(s-\alpha)^2}$$

$$\alpha = \beta = -\frac{q_1}{T} = -\frac{1}{T}$$

$$\Rightarrow A_1(s-\alpha)^2 + A_2(s-\alpha)s + A_3 s = \frac{s-2}{T^2}$$

$$2A_1(s-\alpha) + A_2(2s-\alpha) = \frac{1}{T^2} \Rightarrow A_1 = -A_2$$

$$\Rightarrow A_2 = \frac{2}{\alpha^2 T^2} \quad ; \quad A_3 = \frac{\alpha-2}{T^2 \alpha}$$

$$A_1 = \frac{-2}{T^2\alpha^2}$$

$$\therefore X(s) = \left(\frac{-2}{T^2\alpha^2}\right)\left(\frac{1}{s}\right) + \left(\frac{2}{\alpha^2 T^2}\right)\frac{1}{(s-\alpha)} + \left(\frac{\alpha-2}{\alpha T^2}\right)\frac{1}{(s-\alpha)^2}$$

take inverse Laplace transform.

$$\Rightarrow x(t) = \frac{-2}{T^2\alpha^2}\exp(0t) + \frac{2}{\alpha^2 T^2}\exp(\alpha t) + \left(\frac{\alpha-2}{T^2}\right)\frac{t}{\alpha}\exp(\alpha t)$$

Subst. α

$$x(t) = -2 + \frac{2}{\alpha^2 T^2}\left(\exp\left(-\frac{t}{T}\right)\right) + \frac{t}{T^2}\exp\left(-\frac{t}{T}\right) + \frac{2t}{T}\exp\left(-\frac{t}{T}\right)$$

$$\boxed{\Rightarrow x(t) = -2 \times \left(\exp\left(-\frac{t}{T}\right)\right)\left(\frac{t}{T^2} + \frac{2t}{T} + 2\right)}$$

iii) $\zeta < 1$, complex roots

Simplification

~~Soln~~ same as.. in (i) till the eqn marked as '$\circledast$'

Now we know that the solution will be real so we just take the real part by simplifying using euler's formula.

$$\alpha - \beta = \frac{2j\sqrt{1-\zeta^2}}{T}$$

~~$x(t) = -2 + (\alpha\beta)\exp(\alpha t)$~~

$$\alpha\beta = \frac{1}{T^2} \qquad = \frac{2\sqrt{1-\zeta^2}}{T}\exp\left(\frac{-\pi}{2}j\right)$$

$$\alpha = \frac{1}{T}\exp\left(\tan^{-1}\left(-\frac{\sqrt{-\zeta^2+1}}{T}\right) \times j\right)$$

$$\beta = \frac{1}{T}\exp\left(\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{T}\right)j\right)$$

$$\therefore x(t) = -2 + \frac{\alpha\beta}{\alpha-\beta}\exp(\alpha t) - \frac{2\beta}{\alpha-\beta}\exp(\alpha t)$$

$$+ \frac{\alpha\beta}{\beta-\alpha}\exp(\beta t) - \frac{2\alpha}{(\beta-\alpha)}\exp(\beta t)$$

term 1 : $\dfrac{\alpha\beta\frac{1}{T^2}}{\alpha-\beta}\exp(\alpha t)$

$$= \frac{1}{2T^2 \frac{\sqrt{1-\zeta^2}}{T}\exp\left(\frac{\pi}{2}j\right)} \exp\left(-\frac{\zeta t}{T}\right) \exp\left(j\left(\sqrt{1-\zeta^2}\right)t\right)$$

$$= \frac{1}{2T\sqrt{1-\zeta^2}}\exp\left(\frac{-\zeta t}{T}\right)\exp\left(j\left(\frac{\sqrt{1-\zeta^2}}{T}t - \frac{\pi}{2}\right)\right)$$

term2 : $\dfrac{-2\beta}{\alpha-\beta}\exp(\alpha t)$

$$= \frac{\frac{1}{T}\;-\beta}{\frac{2}{T}\sqrt{1-\zeta^2}}\exp\left(\frac{-\zeta t}{T}\right)\exp\left(j\left(\frac{\pi}{2}+\frac{\sqrt{1-\zeta^2}}{T}\right)t\right)$$

$$= \frac{1}{\zeta\sqrt{1-\zeta^2}}\exp\left(\frac{-\zeta t}{T}\right)\exp\left(j\left(\left(\frac{\pi}{2}+\frac{\sqrt{1-\zeta^2}}{T}\right)t + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)\right)$$

term 3 : $\dfrac{\alpha\beta}{\beta-\alpha}\exp\left(-\frac{\zeta t}{T}\right)\exp\left(-j\left(\sqrt{1-\zeta^2}\,t\right)\right)$

$$= \frac{-1}{2T\sqrt{1-\zeta^2}}\exp\left(\frac{-\zeta t}{T}\right)\exp\left(j\left(-\sqrt{1-\zeta^2}\,\frac{t}{T}+\frac{\pi}{2}\right)\right)$$

$$\Rightarrow x(t) = -2 + \frac{\exp\left(\frac{-\zeta}{T}\right)}{T\sqrt{1-\zeta^2}}\left(\sin\left(\frac{\sqrt{1-\zeta^2}\,t}{T}\right) + \sin\left(\frac{\sqrt{1-\zeta^2}\,t}{T} + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)\right)$$

Didnot show some steps to reduce pdf size.

Will send if required. (I have written but I am not including in scan.)

Q3 Part d)

```
>> ss_point

 Operating point for the Model Q3_model.
 (Time-Varying Components Evaluated at time t=0)

States:
----------
(1.) Q3_model/Absorption Model/Integ1
      x: 0.0388
(2.) Q3_model/Absorption Model/Integ2
      x: 0.101
```

linsys =

A =

	Integ1	Integ2
Integ1	-6.5	2.5
Integ2	4	-6.5

B =

	FR	FR1
Integ1	-0.001938	0.00155
Integ2	-0.003101	0.002481

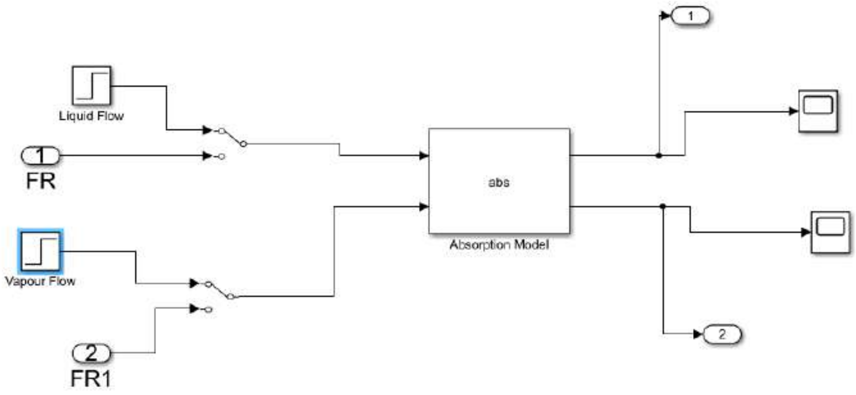
C =

	Integ1	Integ2
Out1	1	0
Out2	0	1

D =

	FR	FR1
Out1	0	0
Out2	0	0

Continuous-time state-space model.

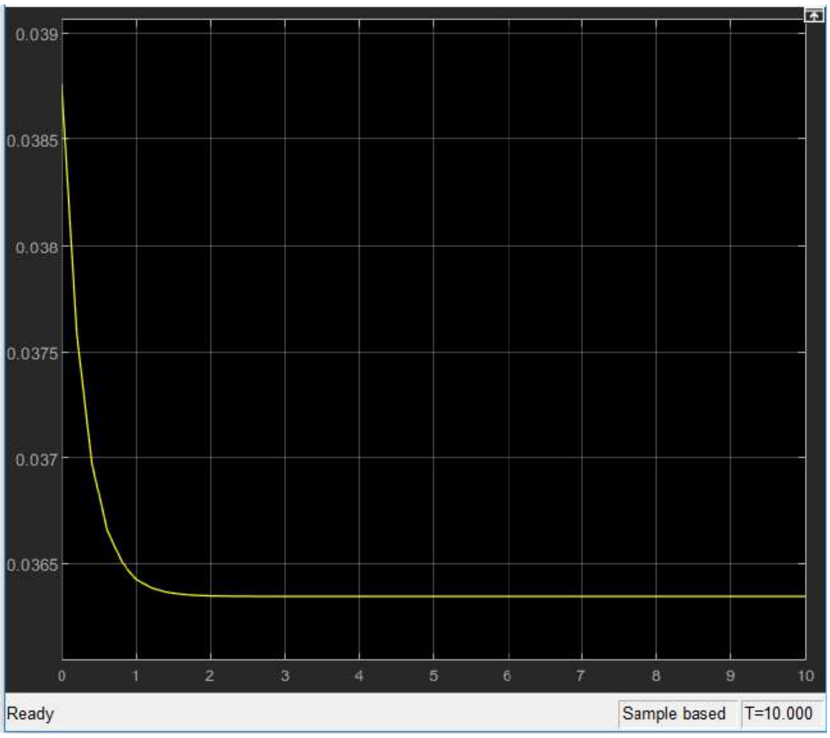


Part e)

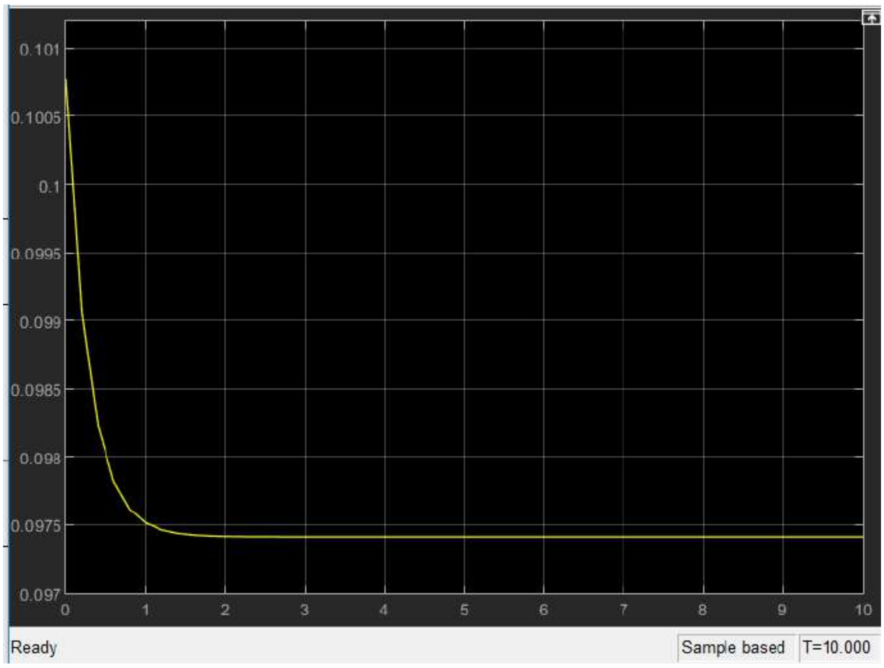
Since if we change both the flow rates, steady state values are same as original, I am changing only L in both cases.

ALL GRAPHS ARE FROM NON LINEAR MODEL. I DID NOT PLOT FOR LINEAR MODEL.

i) $L = 1.05L_{ss}$

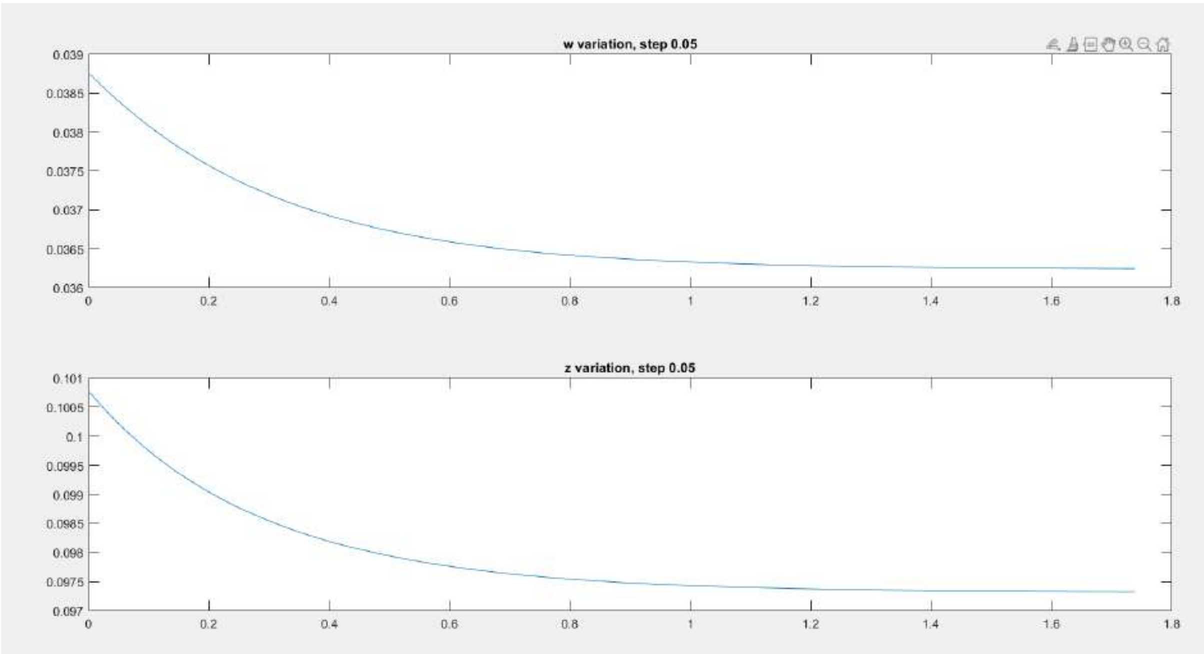


W graph

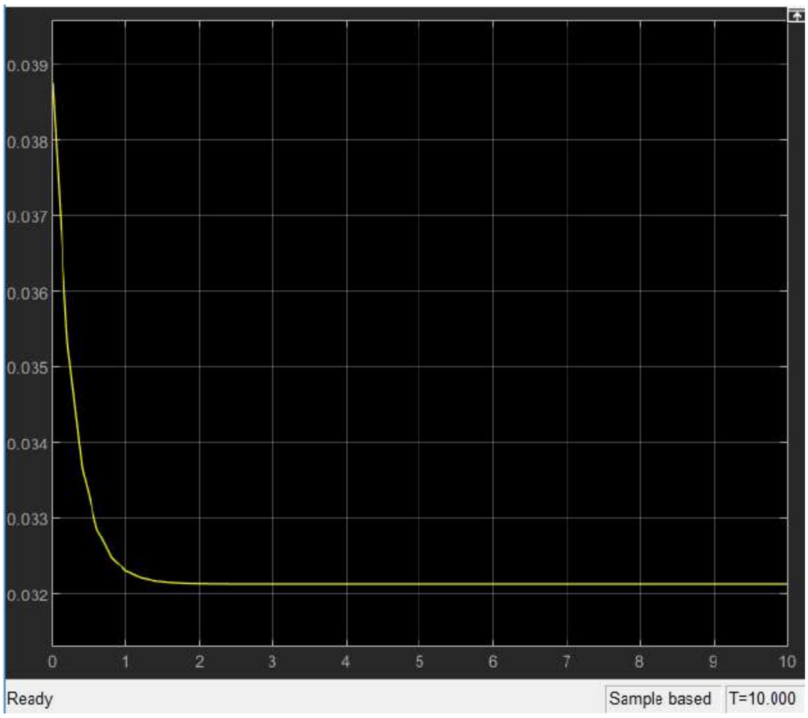


Z graph

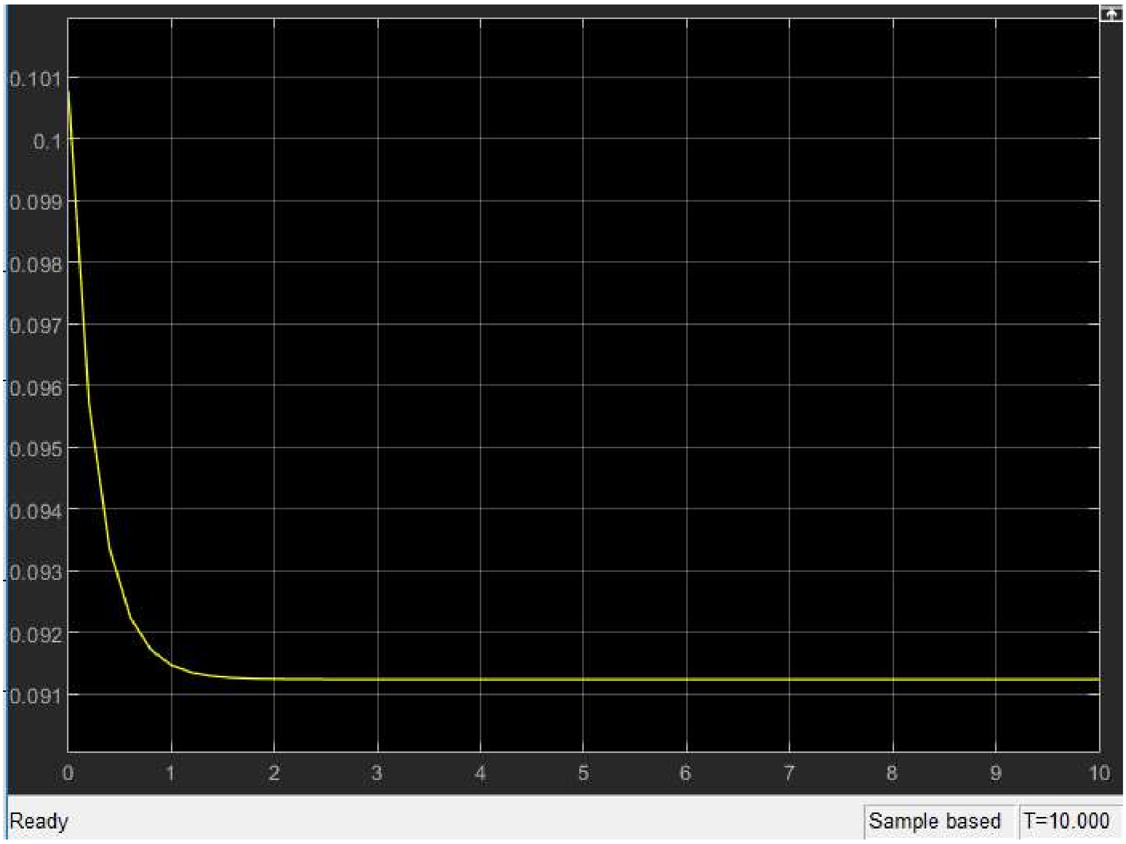
Linear model



ii) L= 1.15L_{ss}

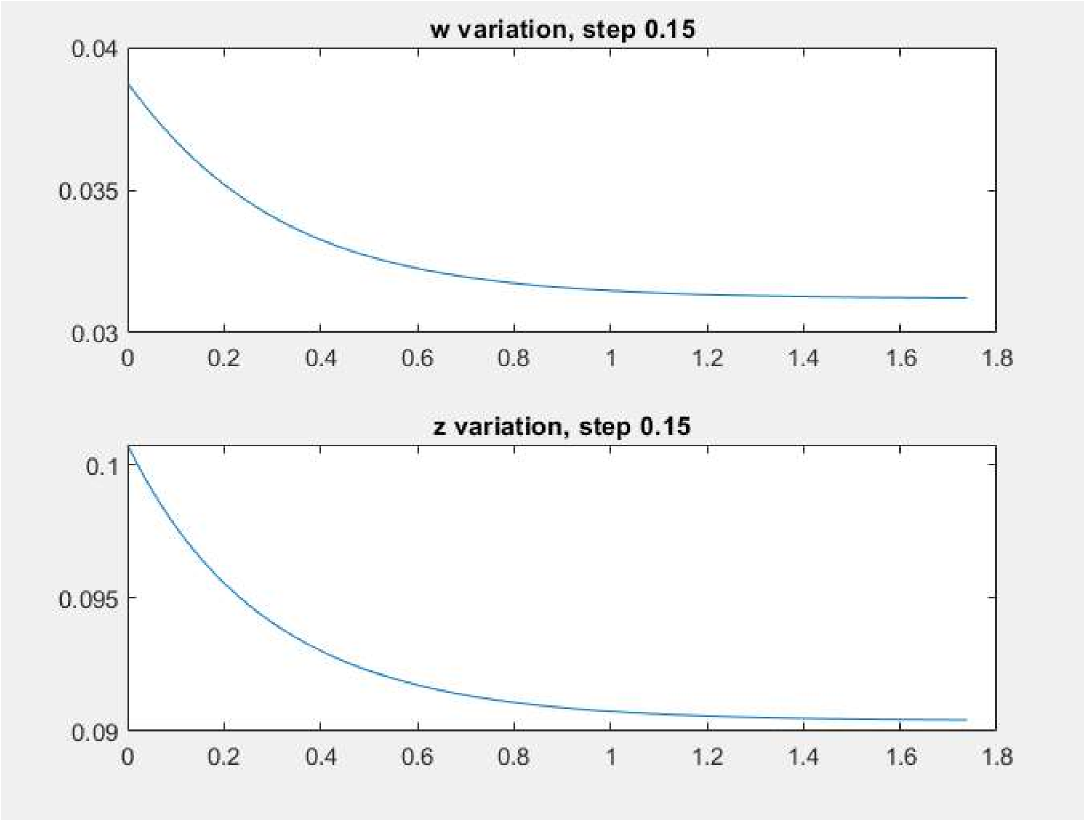


W graph



Z graph

Linear model:



Code:

```
clear;close all;

%% System charecterstics

Lss = 80; Vss = 100;

M = 20; a=0.5; zf = 0.1;

%% Part a) Finding steady state (by hand)

% Equate derivatives to zero, solve the linear eqn

Ass = [-(a*Vss+Lss)/M Vss*a/M;Lss/M -(a*Vss+Lss)/M];

bss = [0;-Vss*zf/M];

x_ss = inv(Ass)*bss;

%% Part b) Linearisation (by Taylor Expansion)

w_ss = x_ss(1);z_ss=x_ss(2);

A = [-(Vss*a+Lss)/M Vss*a/M;Lss/M -(Lss+Vss*a)/M];

B = [-w_ss/M (-a*w_ss+a*z_ss)/M;(w_ss-z_ss)/M -a*z_ss/M+zf/M];

%% Part c) Finding the eigenvalues-eignvectors of the system
```

```
[V,D] = eig(A);

% Second eigen value is faster (more negative)

%% Part d) Find steady-state and linearise

open_system('Q3_model')

% Read the operating conditions into an object

opc = operspec('Q3_model');

% Operating conditions

opc.Inputs(1).u = 80;

opc.Inputs(2).u = 100;

opc.Inputs(1).Known = 1;

opc.Inputs(2).Known = 1;

% Constraints

opc.States(1).Min = 0;opc.States(2).Min = 0;

% Find the steady state point

ss_point = findop('Q3_model',opc);

% Linearize

linsys = linearize('Q3_model',ss_point)

%% Part e) Give step changes and plot

% Done in SIMULINK. Use the manual switch to step input(s)

[Y,T,X]=step(linsys);

% Y(:,:,1) contains responses for change in L

% Since linear system, changes in input and output are proportional

figure();

subplot(2,1,1);plot(T,Y(:,1,1)*.05*Lss+w_ss); title('w variation, step 0.05');

subplot(2,1,2);plot(T,Y(:,2,1)*.05*Lss+z_ss); title('z variation, step 0.05');

figure();

subplot(2,1,1);plot(T,Y(:,1,1)*.15*Lss+w_ss); title('w variation, step 0.15');

subplot(2,1,2);plot(T,Y(:,2,1)*.15*Lss+z_ss); title('z variation, step 0.15');
```