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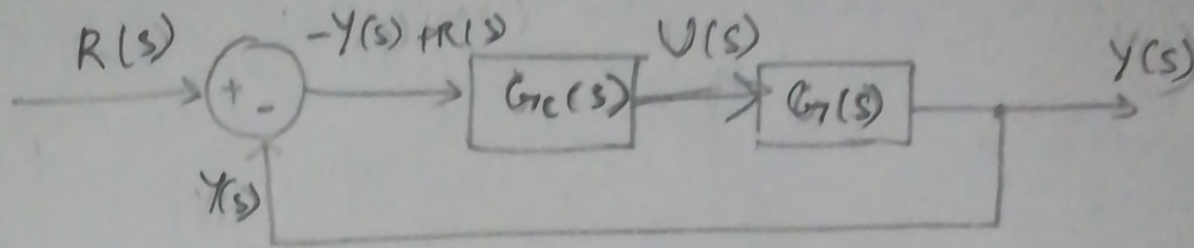
CH18B020

# CH3050 ASSIGNMENT-3

① a)




From the block diagram, we can write,

$$U(s) = G_c(s)\,(R(s) - Y(s)) \quad \text{—} ①$$

$$Y(s) = G(s)\,U(s) \quad \text{—} ②$$

Subst. ① in ②,

$$Y(s) = G(s)\,G_c(s)\,(R(s) - Y(s))$$

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{G(s)\,G_c(s)}{1 + G(s)\,G_c(s)}$$

$$\Rightarrow G_{CL}(s) = \frac{\frac{10}{s^2+7s+10} \times \left(K_c + \frac{K_I}{s}\right)}{1 + \frac{10}{s^2+7s+10}\left(K_c + \frac{K_I}{s}\right)}$$

$$= \frac{10K_c s + 10K_I}{s^3 + 7s^2 + 10s + 10sK_c + 10K_I}$$

$$\Rightarrow \boxed{G_{CL}(s) = \frac{10(K_c s + K_I)}{s^3 + 7s^2 + s(10 + 10K_c) + 10K_I}}$$

b) Employing the R-H criterion for stability,

| | | | |
|---|---|---|---|
| $s^3$ | $1$ | $10+10K_c$ | $0$ |
| $s^2$ | $7$ | $10K_I$ | $0$ |
| $s$ | $\frac{7(10+10K_c)-10K_I}{7}$ | $0$ | |
| $1$ | $10K_I$ | | |

To have no poles on RHP ~~there~~ there should be no sign change.

$$\Rightarrow \frac{7(10+10K_c)-10K_I}{7} > 0 \quad \& \quad 10K_I > 0$$

$$\Rightarrow \underline{K_c > -1 + \frac{K_I}{7}} \quad \text{——— ①}$$

$$\underline{K_I > 0} \quad \text{——— ②}$$

Sufficient conditions for stability

Note that the zero, $s = -\frac{K_I}{K_c}$ is predominantly negative in the admissible region given by the above eqns. So we can safely assume that there won't be any RHP pole cancelled out by the zero. (so ① & ② is sufficient and necessary for most parts) ~~this exist~~

①c) If the ~~a~~ $K_c$, $K_I$ values are in the admissible region, then the stability is guaranteed.

$\Rightarrow$ We can use the Final Value theorem.

FVT

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s\,Y(s)$$

$$Y(s) = G_{CL}(s)\,R(s)$$

Let $r(t) = r$ (a constant value)

$$\Rightarrow R(s) = \frac{r}{s}$$

$$\therefore Y(s) = \frac{10(K_c s + K_I)}{(s^3 + 7s^2 + s(10 + 10K_c) + 10K_I)} \times \frac{r}{s}$$

Subst. in FVT, we get,

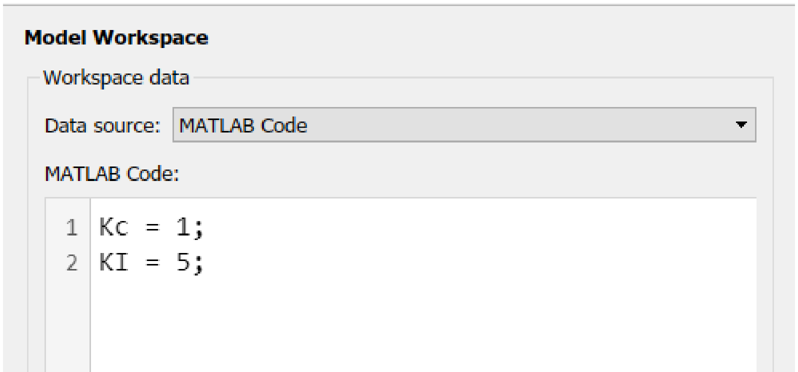
$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} \cancel{s} \times \frac{r}{\cancel{s}} \times \frac{10(K_c s + K_I)}{(s^3 + 7s^2 + s(10 + 10K_c) + 10K_I)}$$

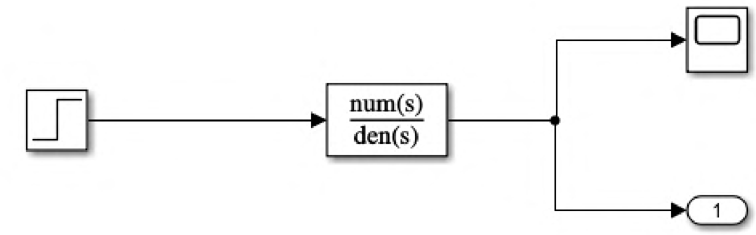
$$= \frac{r \times 10 K_I}{10 K_I}$$

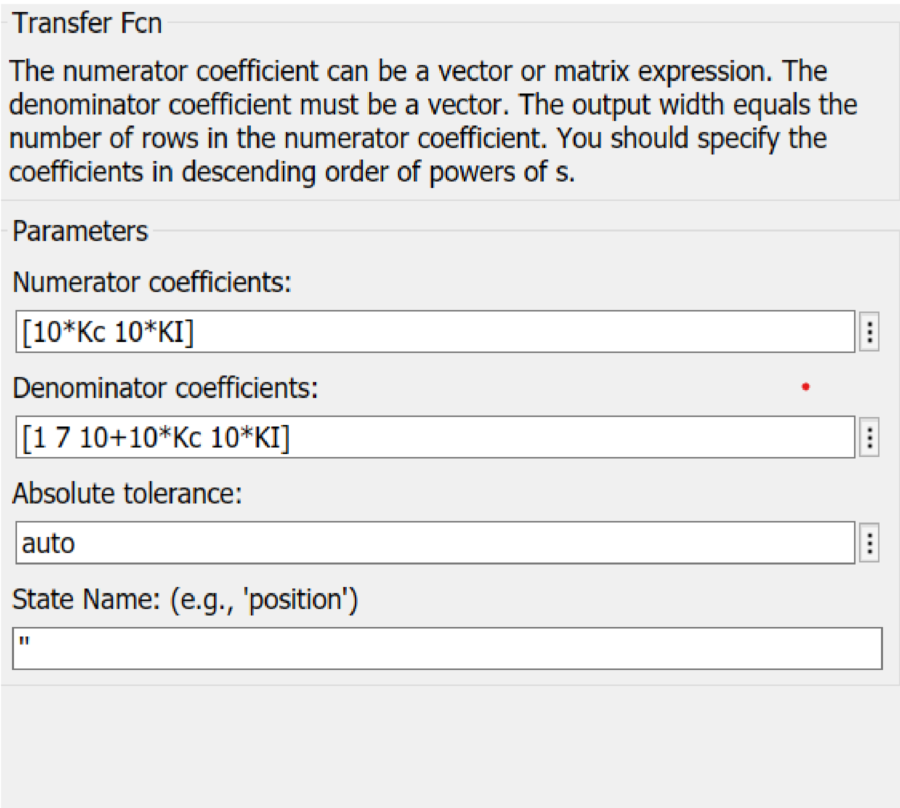
$$\Rightarrow \boxed{\lim_{t \to \infty} y(t) = r} \quad (\because K_I \neq 0)$$

$\therefore$ Yes! Set point tracking is ~~order~~ possible as long as $G_{CL}$ is stable.

## Q1 d)


Model Workspace
Workspace data
Data source: MATLAB Code
MATLAB Code:
Kc = 1;
KI = 5;


num(s)
den(s)
1


Transfer Fcn
The numerator coefficient can be a vector or matrix expression. The denominator coefficient must be a vector. The output width equals the number of rows in the numerator coefficient. You should specify the coefficients in descending order of powers of s.
Parameters
Numerator coefficients:
[10*Kc 10*KI]
Denominator coefficients:
[1 7 10+10*Kc 10*KI]
Absolute tolerance:
auto
State Name: (e.g., 'position')
''

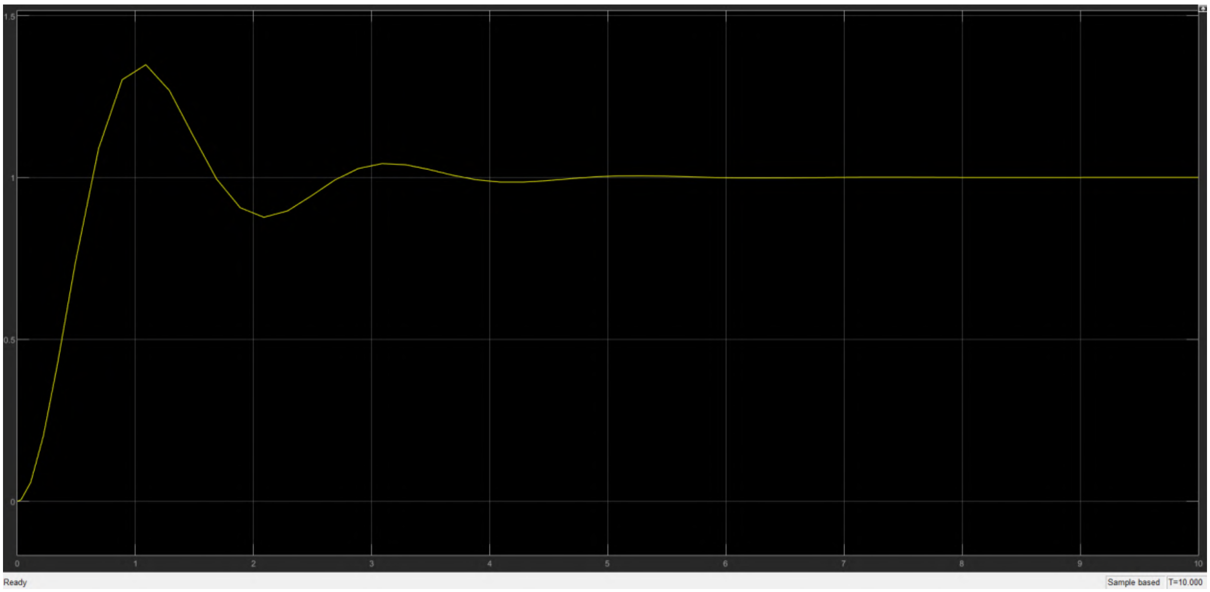


*Figure 1.1: Kc = 1, KI = 5; Acceptable region*

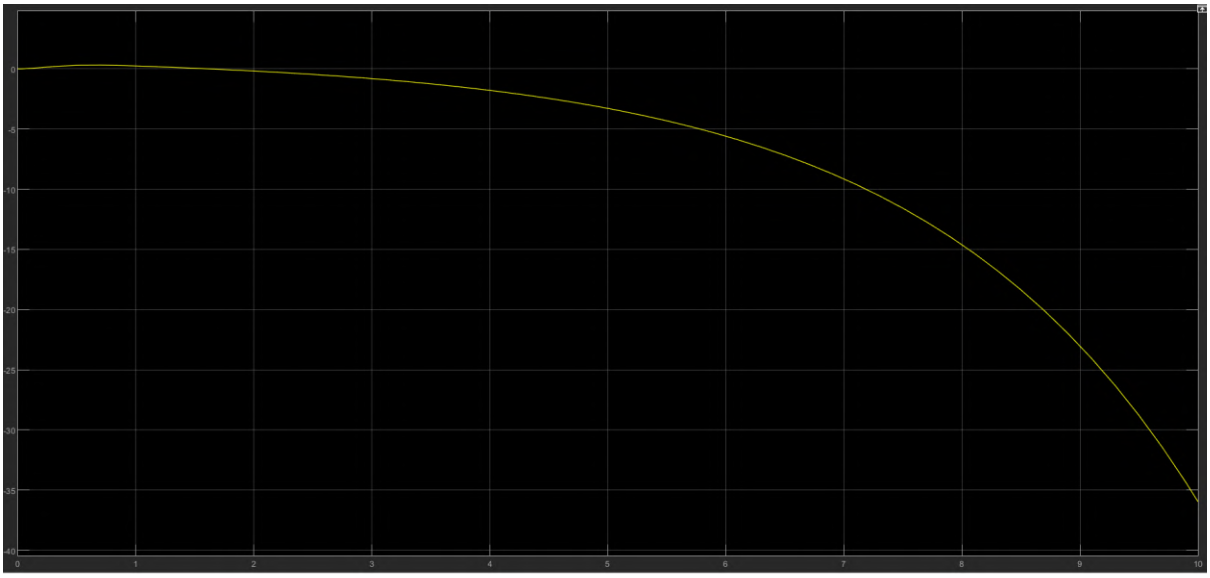


*Figure 1.2: Kc  = 1, Ki = -1; Kc acceptable, Ki not in acceptable region. Result: Setpoint not reached; unstable*

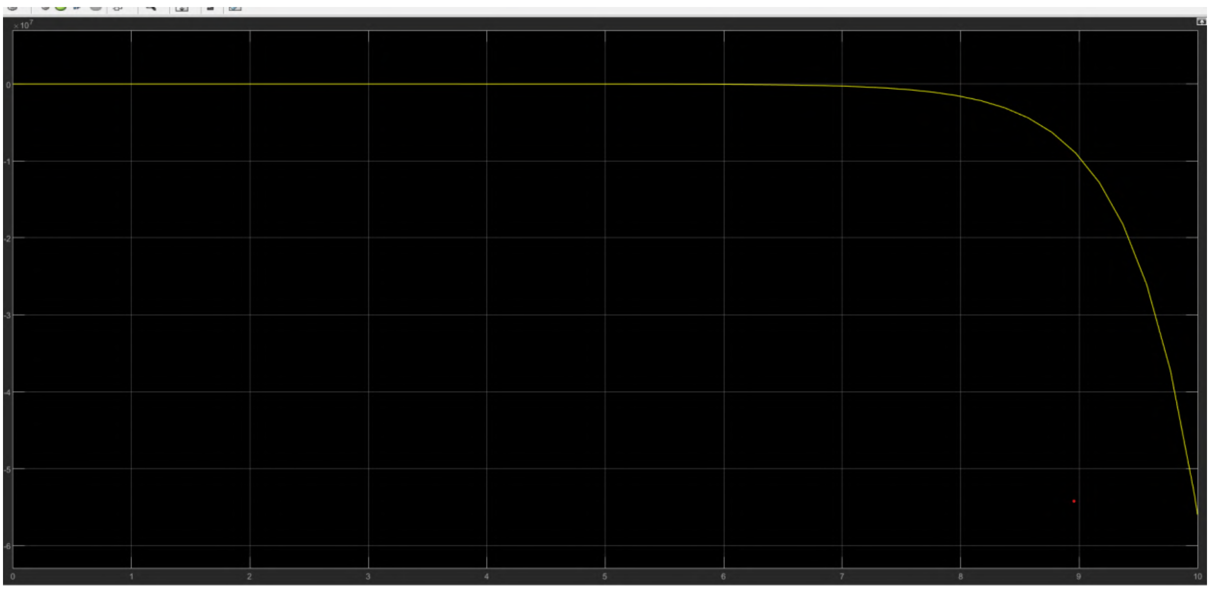


*Figure 1.3: Kc = -2; KI = -1; Both in unacceptable region. Setpoint not achieved, Gcl unstable*

(2) a) $\frac{10(s-4)e^{-3s}}{(s+5)(s+2)} = G(s)$

(i) Impulse Response.

$u(t) = \delta(t) \Rightarrow U(s) = 1$

$\therefore Y(s) = G(s)\,U(s) = \frac{10(s-4)}{(s+5)(s+2)} e^{-3s}$

$= 10e^{-3s}\left(\frac{A_1}{s+5} + \frac{A_2}{s+2}\right)$

~~Subst~~. Solving for $A_1$ & $A_2$, we have,

$[(s-4) = A_1(s+2) + A_2(s+5)]$

$A_1 = 3 \quad A_2 = -2$

$\Rightarrow Y(s) = \frac{30e^{-3s}}{s+5} - \frac{20e^{-3s}}{s+2}$

Take inverse Laplace transform. & by the linearity of the operation we just take the individual inverses & add them up,

$\Rightarrow y(t) = \begin{cases} 0 & t < 3 \\ 30e^{-5(t-3)} - 20e^{-2(t-3)} & t \geq 3 \end{cases}$

$= \{$

→ Initially $30e^{-5t} > 20e^{-2t}$ by the virtue of having a larger coeff.

→ Then ~~the~~ $30e^{-5t}$ keeps ~~increasing~~ decreasing ↓ @ a faster rate, so ∴ the value starts to fall

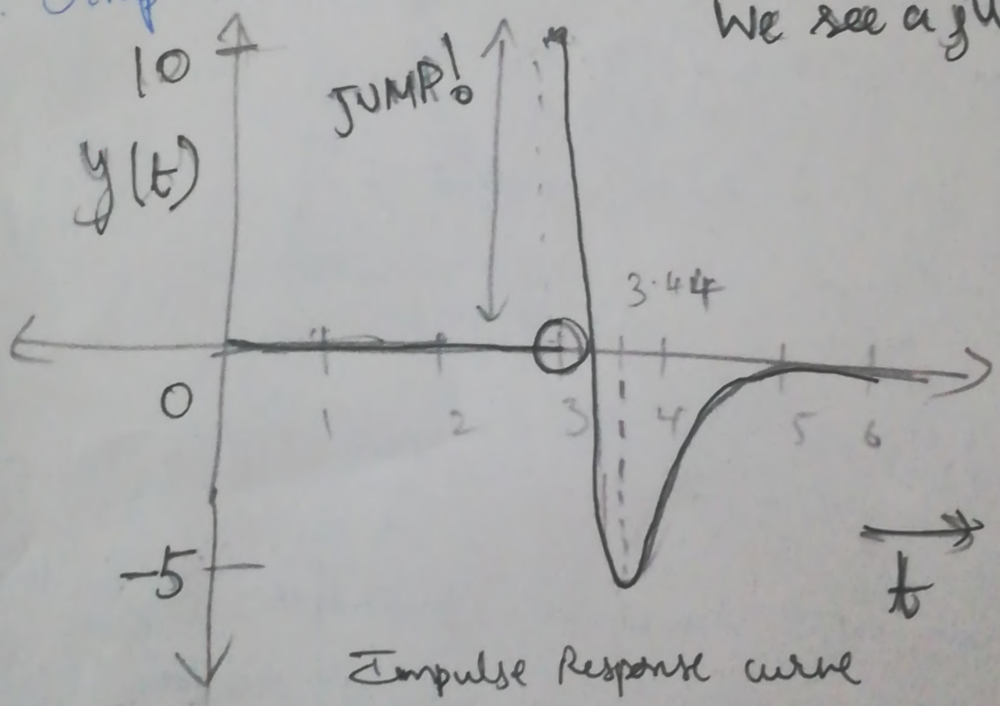
→ When derivative $= 0 \Leftrightarrow e^{3(t-3)} = \frac{150}{40}$ ~~of 3 ln~~

We hit a minima $\Rightarrow 3(t-3) = \frac{1}{3} \ln \frac{150}{40}$

→ as $t \to \infty$, $y \to 0$ $\Rightarrow t \simeq 3.445)$

∴ Graph looks like

We see a jump




Impulse Response curve

there is a jump because of presence of a zero

(ii) <u>Step response</u>

We can use $u(t) = 1 \; \forall t \to U(s) = \frac{1}{s}$

& then solve for $y(t)$. ~~However,~~

However, since we have already found out the impulse response, we can get the step response by integrating the impulse respond

$$\Rightarrow y_{step}(t) = 0 \quad t < 3$$

For $t \geq 3$,

$$y_{step}(t) = \int_3^t \left(30e^{-5(t-3)} - 20e^{-2(t-3)}\right) dt$$

$$= \frac{30}{5}\left(1 - e^{-5(t-3)}\right) + -\frac{20}{2}\left(1 - e^{-2(t-3)}\right)$$

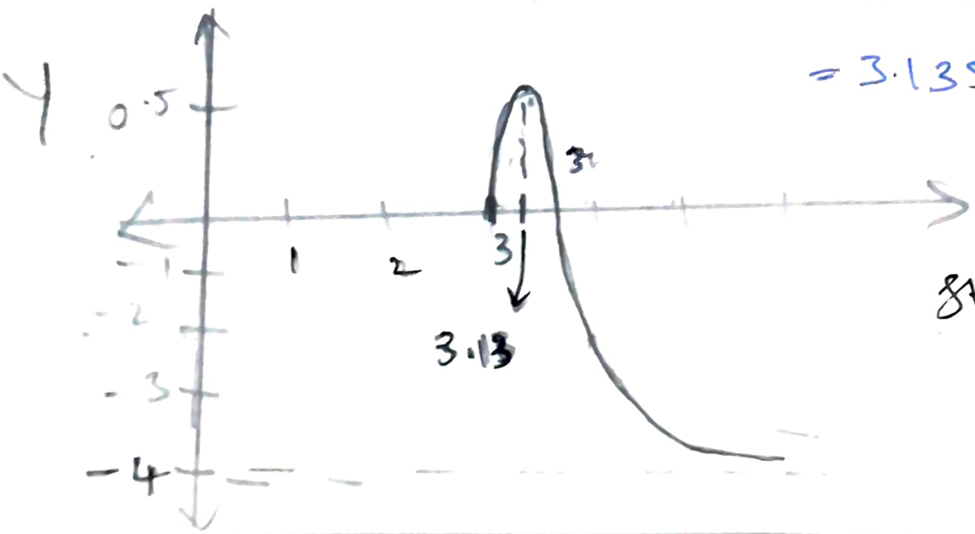
$$= -4 + 10e^{-2(t-3)} - 6e^{-5(t-3)} \quad \text{for } t \geq 3$$

$$\Rightarrow y_{step}(t) = \begin{cases} 0 \\ -4 + 10e^{-2(t-3)} - 6e^{-5(t-3)} & \forall t \geq 3 \end{cases}$$

$t = 3$, $y_{step} = 0$ $\quad$ $y' = 0$ $\Rightarrow 10(-2)e^{-2(t-3)} + (6)(5)e^{-5(t-3)} = 0$

$t \to \infty$, $y_{step} = -4$ $\quad \Rightarrow t = 3 + 0.1351$

$= 3.1351\,s$ (maximum)




Step response curve

b)

$$G(s) = \frac{10(s-4)}{s^2-17s+10} e^{-3s}.$$

Consider that we give a sinusoidal input

$$u(t) = A\sin w_0 t \Rightarrow U(s) = \frac{Aw_0}{s^2+w_0^2}$$

$$\therefore Y(s) = \frac{10(s-4)e^{-3s}}{s^2+7s+10} \frac{Aw_0}{s^2+w_0^2}$$

By ^{splitting into} partial fractions, we can get

$$= \left( \frac{C_1}{s-a} + \frac{C_2}{s-b} + \frac{C_3}{s-jw_0} + \frac{C_4}{s+jw_0} \right) e^{-3s}$$

where $a, b$ are the poles

For now, let's ignore the delay & taking $\mathcal{L}^{-1}\{\ \}$

$$\Rightarrow y(t) = C_1 e^{+at} + C_2 e^{bt} + C_3 e^{+jw_0} + C_4 e^{-jw_0}$$

Since we have stable poles after a long time $(t \to \infty)$, $C_1 e^{at}$ & $C_2 e^{bt} \longrightarrow 0$

Also to have a real output $C_3 = C_4^*$

("physically meaningful")

$$C_3 = \frac{A}{2j} G(jw_0) \quad \text{(By method of partial fraction)}$$

So here too, we have $y_{ss}(t) = 2\,\mathrm{Re}\left( C_3 e^{jw_0 t} \right)$

This expression is similar to the one we got in the derivation for first order system done in class.

So we conclude,

$$\frac{B}{A} = |G(j\omega_0)|$$

where $B \to$ amplitude of output

$A \to$ Amplitude of input.

ie $\phi$ (phase) $= \angle G(j\omega_0) = \angle G(j\omega_0)$

Input $u(t) = 2\sin(5t) + 3\cos(0.1t)$

By the linearity property we can simply add the outputs of the individual inputs to get the output of the total input.

$$G(j\omega_0) = \frac{10\,\cancel{(s-4)}\,(j\omega_0 - 4)\, e^{-3j\omega_0}}{(j\omega_0)^2 + 7(j\omega_0) + 10}$$

$$= \frac{10\,(j\omega_0 - 4)\, e^{-3j\omega_0}}{(j\omega_0 + 5)(j\omega_0 + 2)}$$

$$\Rightarrow |G(j\omega)| = \frac{10\sqrt{(16 + \omega^2)} \times (1)}{\sqrt{(\omega^2 + 25)(\omega^2 + 4)}}$$

$$= \frac{10\sqrt{16 + \omega^2}}{\sqrt{(\omega^2 + 25)(\omega^2 + 4)}}$$

$\angle G(j\omega) = \cancel{\tan^{-1}(\omega)}\ \tan^{-1}\left(\frac{-\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right) - 3\omega$

( for a CN $a+jb$, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ )

$\Rightarrow \angle G(j\omega) = \tan^{-1}\left(-\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right) - 3\omega.$

Input ①: $\omega = 5 \quad A = 2$

$\Rightarrow \frac{B}{A} = |G(j\omega)| = 1.6815$

$\Rightarrow B = 1.6815 \times A = 2 \times 1.6815$

$= 3.3631$

$\phi = -17.87$

$\therefore y_1 = 3.363 \sin(5t - 17.87)$

Input ②: $A = 3, \omega = 0.1$

$\frac{B}{A} = |G(j\omega)| = 3.9955$

$\Rightarrow B = 11.9863.$

$\phi = -0.3949.$

$\Rightarrow y_2 = 11.9863 \cos(0.1t - 0.3949)$

$\therefore \boxed{y_{ss}(t) = y_1 + y_2 = 3.363 \sin(5t - 17.87) + 11.9863 \cos(0.1t - 0.3949)}$

Also note that I used the $\frac{B}{A}$ & $\phi$ expressions derived for sine input for cosine also. this holds true because cosine( $\cos\omega t = \sin(\omega t + \frac{\pi}{2})$; so we will get back the same expressions again (for AR & $\phi$)

c) $dB = 20\log_{10}(AR(\omega))$

$$= 20\log_{10}\left(\frac{10\sqrt{16+\omega^2}}{\sqrt{(\omega^2+25)(\omega^2+4)}}\right)$$

$$= 20\left[\log_{10}10 + \log_{10}\sqrt{16+\omega^2} - \log_{10}(\sqrt{\omega^2+25}) - \log(\sqrt{\omega^2+4})\right]$$

We can take each of these subsystems indep. and & add their dB together

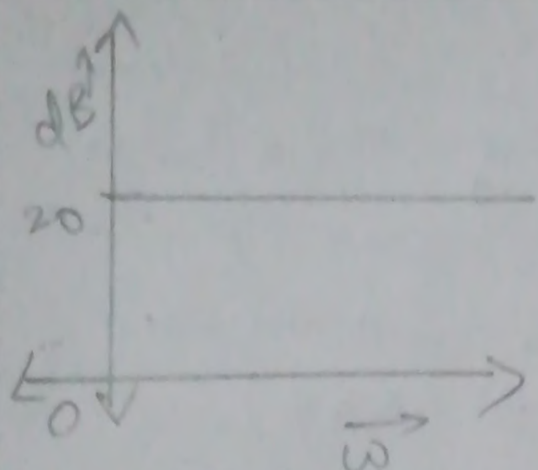
$$dB = \sum_{\forall k} dB_k \quad \& \quad \phi = \sum_{\forall k} \phi_k$$

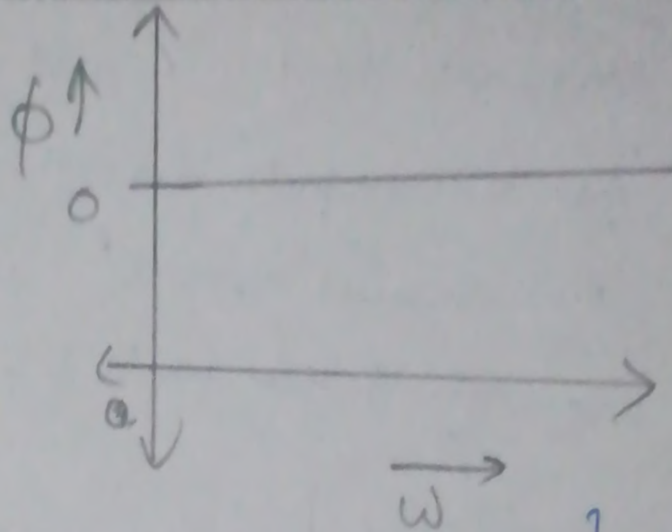
components:

(i) 10 (ii) $(j\omega - 4)$ (iii) $e^{-3j\omega}$ (iv) $\frac{1}{(j\omega+5)}$ (v) $\frac{1}{(j\omega+2)}$

i) 10 $dB = 20\log 10 = 20$

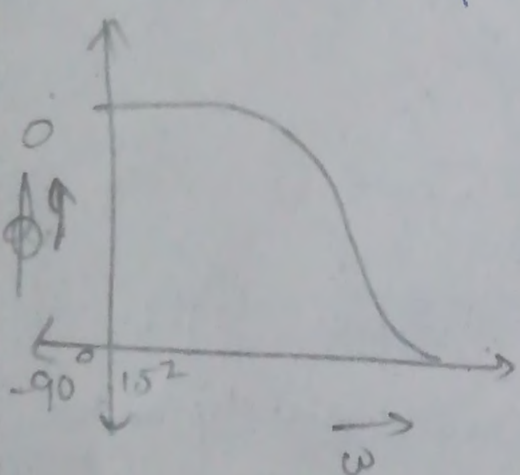
$\phi = 0$ (purely real)

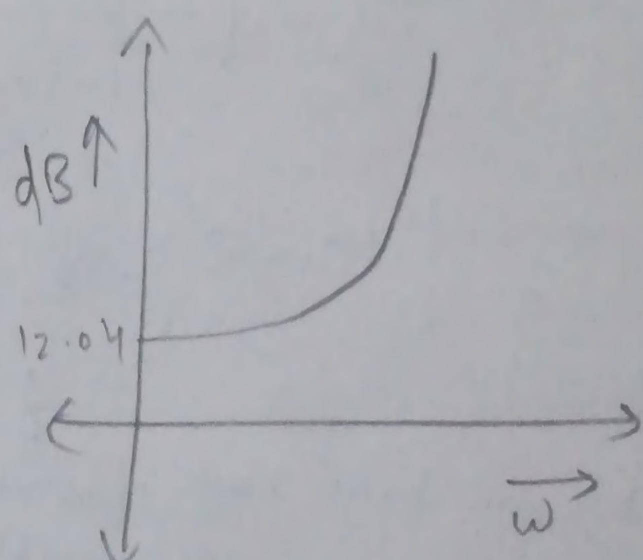



ii) $dB = 10 \log_{10}(16 + \omega^2)$ $\left[G_2(j\omega) = (j\omega - 4)\right]$

$\phi = \tan^{-1}\left(\frac{-\omega}{4}\right)$




($\because$ x axis is in log scale,

$x = \log \omega$; $\omega \to \infty$, $x \to \infty$

$\omega \to 0^+$, $x \to -\infty$)

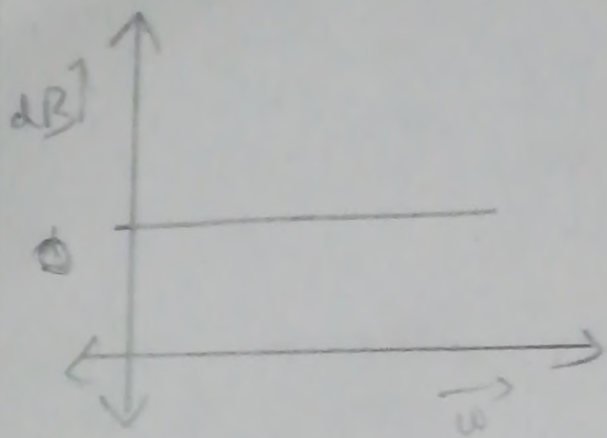
~~(looks similar~~

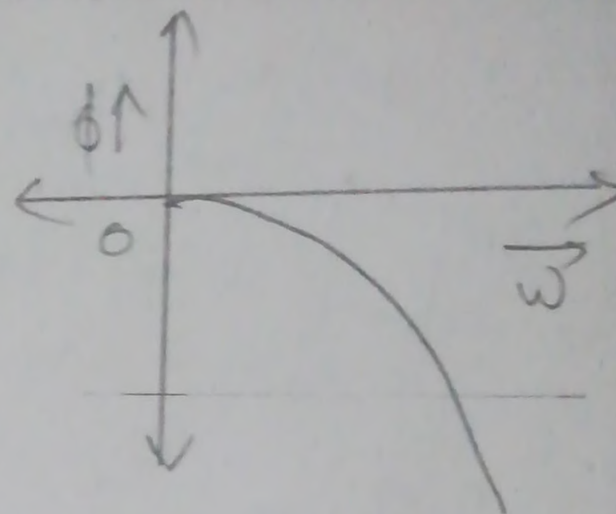
(looks similar to $\log x$)

iii) $G_3(j\omega) = e^{-3j\omega}$

$dB = 20 \log 1 = 0$ $\quad \phi = \cancel{-3j\omega}\ -3\omega$ → A linear variation

Although $\phi$ shows linear variation with $\omega$, since Bode's plot is in semilog scale, the graph will look exponential.

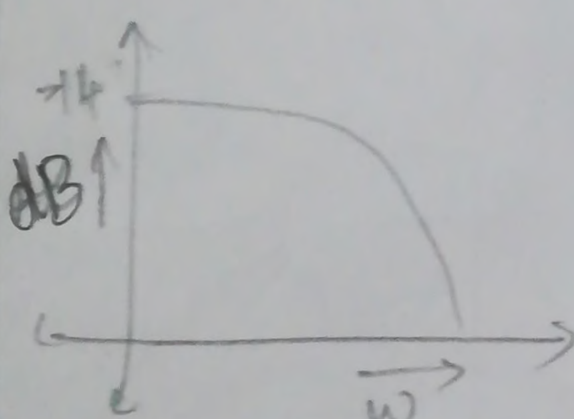


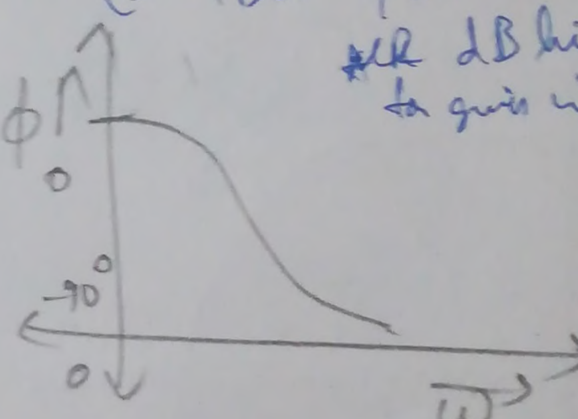




iv) $G(j\omega) = \dfrac{1}{(j\omega+5)}$

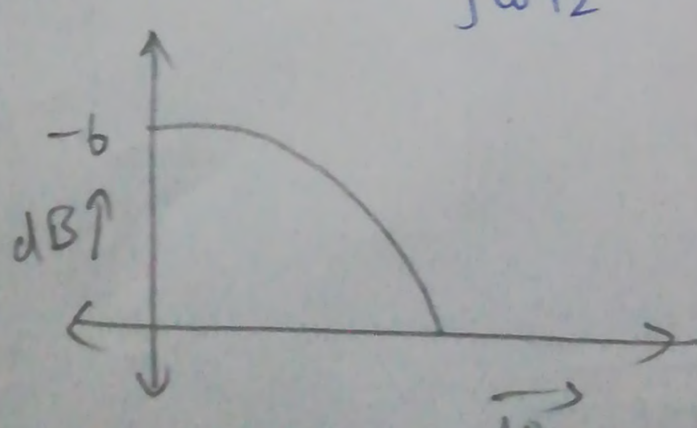
$\Rightarrow dB = -10\log(\omega^2+25)$ & $\phi = \tan^{-1}\left(\dfrac{-\omega}{5}\right)$

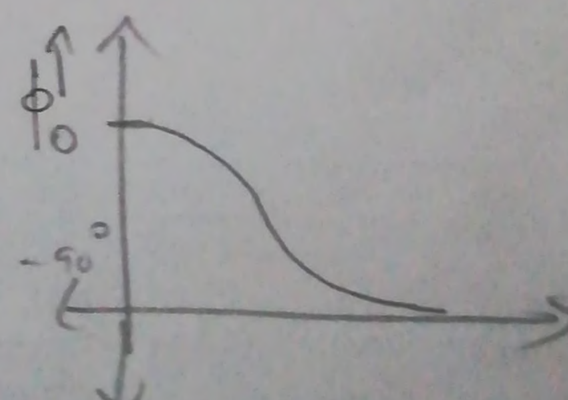
Plots will look similar to (iii) (But $\phi$ lower & dB higher for given $\omega$)




v) $G(j\omega) = \dfrac{1}{j\omega+2}$



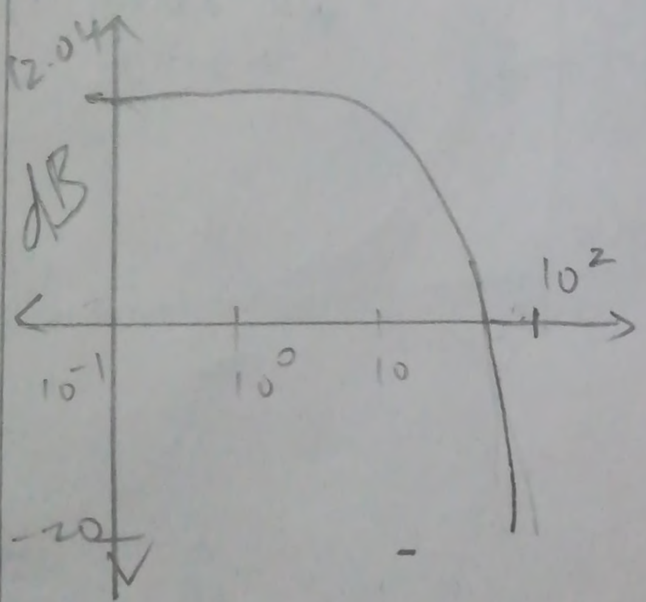
$\therefore\ dB = \sum_{\forall k} dB_k \quad ; \phi = \sum_{\forall k} \phi_k$

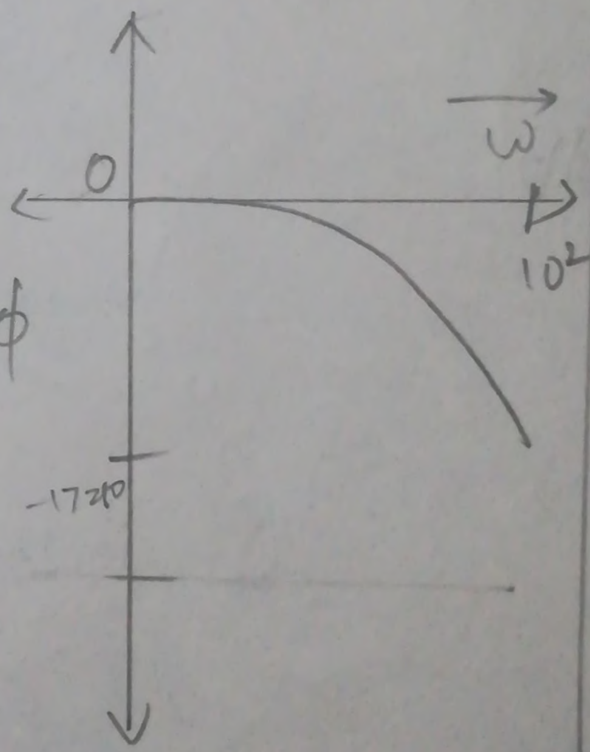
$$dB\Big|_{j\omega=0} = 12.04 + 20 - 14 - 6$$
$$= 12.04$$

$$\phi z\Big|_{j\omega=0} = 0$$

$\phi$ will be dominated by the linear ter. at high frequencies.

At lower $\omega$ $\phi$ will attempt to saturate at 90°




$$dB = 20 + 10\log\frac{(16+\omega^2)}{(\omega^2+25)(\omega^2-14)}$$

d) $G(j\omega) = \dfrac{10\,(j\omega - 4)\,e^{-3j\omega}}{(j\omega+5)(j\omega+2)}$

Same magnitude @ all $\omega$ & min phase.

$\Rightarrow |G_{new}| = \dfrac{10\sqrt{\omega^2+16}}{\sqrt{\omega^2+25}\,\sqrt{\omega^2+4}}$ $\forall \omega$.

Only thing we can now change is sign of the real & complex ~~exo~~ terms.

Note that @ minimum phase /

$G_{new}$ is causal & $G_{new}$ is stable.

for $G_{new}$ to be causal $\dfrac{1}{G_{new}}$ should have stable poles

$\Rightarrow$ $G_{new}$ should ~~have~~ have all zeros in RHP.

Notice $G(s)$ has 0 at $s = 4$ (RHP)

$\therefore$ the required LTI system is

$$G_{new}(s) = \frac{10(s+4)}{s^2+7s+10}\,e^{-3s}$$

(now the zero is at LHP, $s=-4$)

# Q2 e)

## Impulse and Step Responses

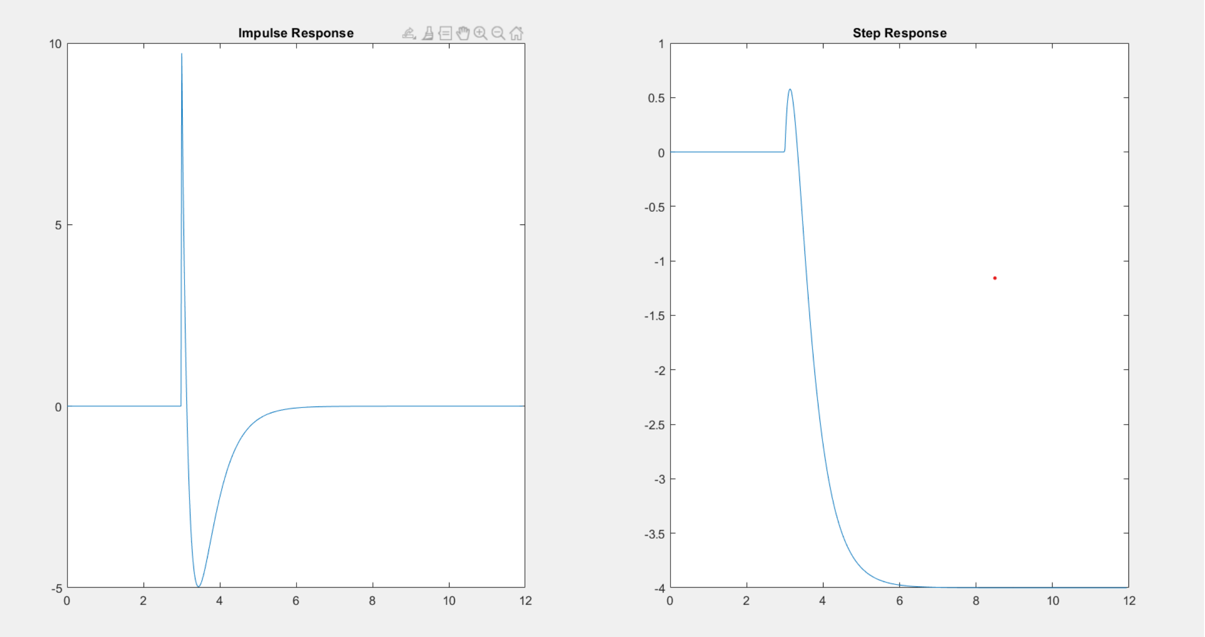



*Figure 1: Impulse and Step responses*

## Large time response

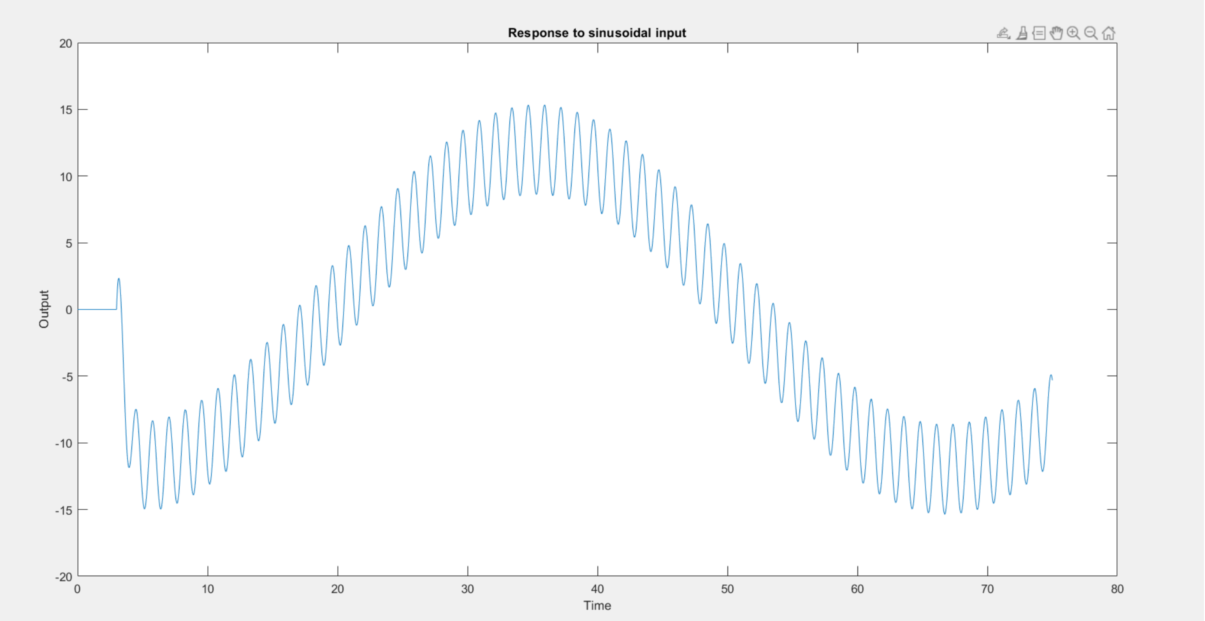



*Figure 2: Large time response from lsim*

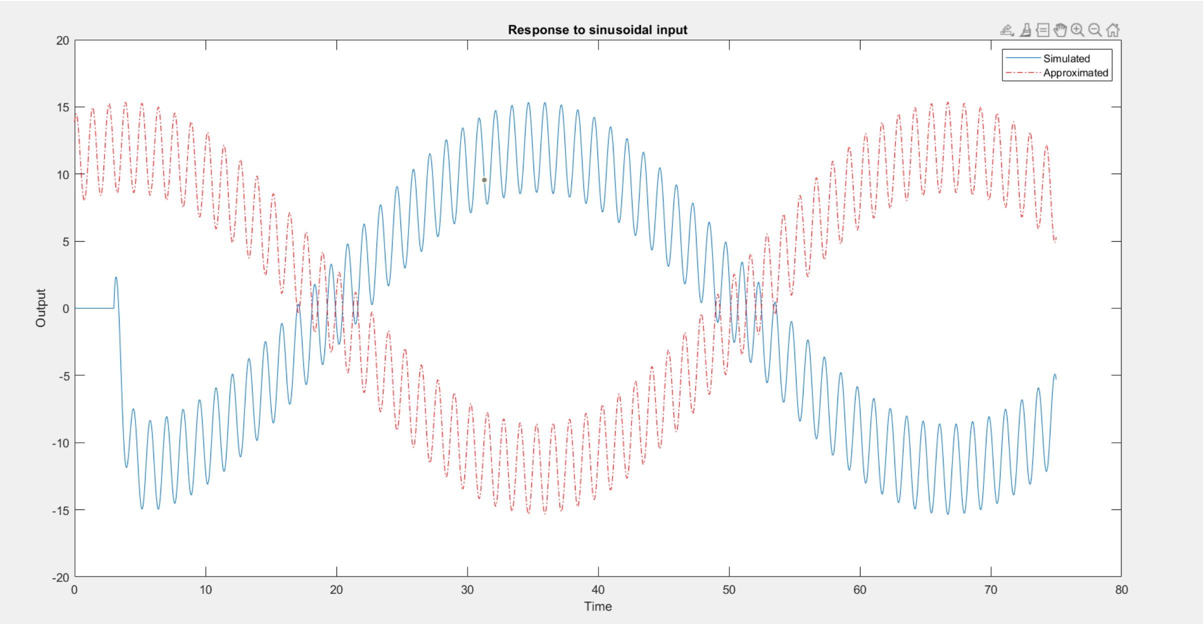



*Figure 3: lsim compared to handwritten approximation*

We see there is almost a 180 degree phase shift of the larger sinusoid. I am not sure whether that is a problem with my code or a problem with my derived expression. (or should they not match at all for small times?)

## Bode Diagram

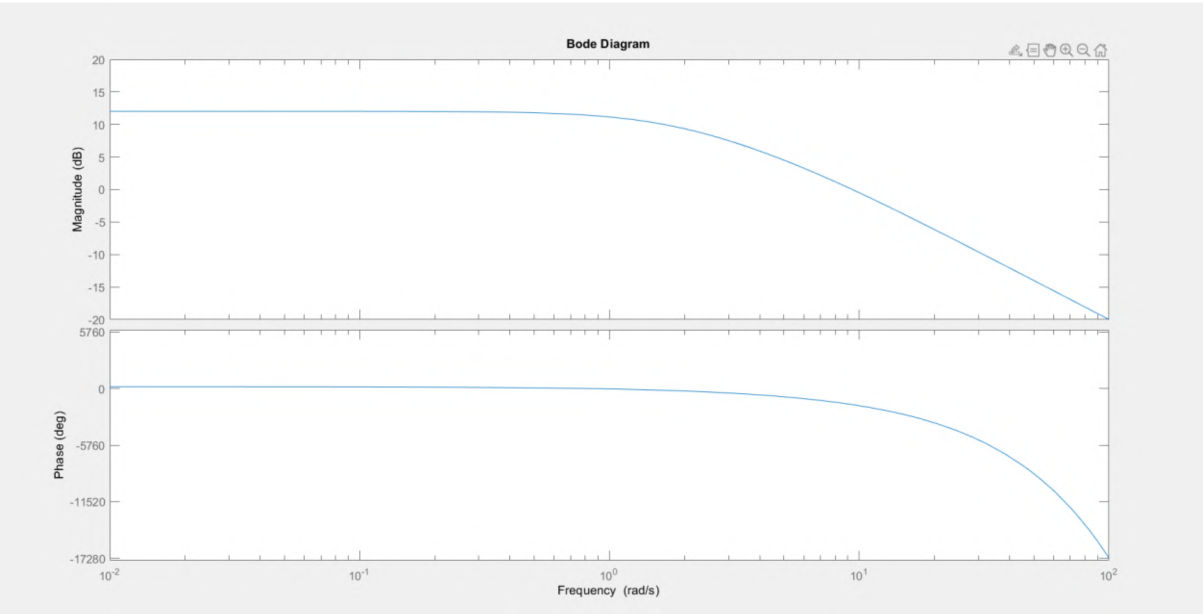



*Figure 4: Bode Diagram of the system*

## Minimal system

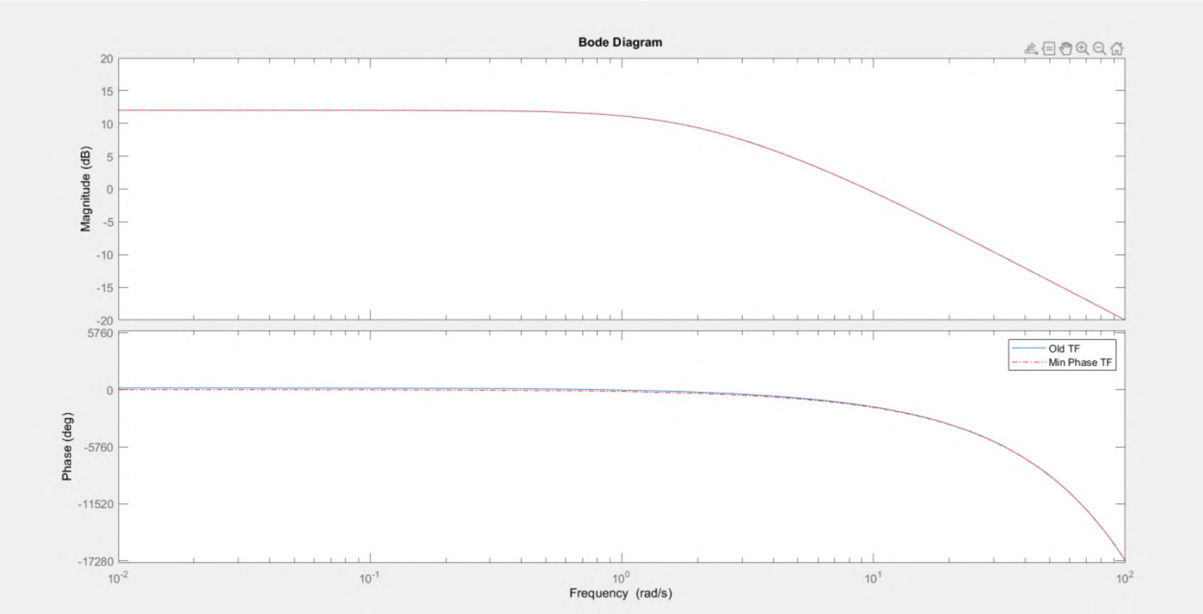



*Figure 5: Comparing the bode diagrams of original system and minimal system*

We can see that their magnitudes are same but the minimal system has lower phase than the original system. In fact, it will have the lowest phase among all systems having the same magnitudes.

## MATLAB code

```
close all; clear;
G = tf([10 -40],[1 7 10],'InputDelay',3);
%% Part a)- Impulse and Step responses
% Impulse response
[Yimpulse,Timpulse] = impulse(G);
subplot(1,2,1);
plot(Timpulse,Yimpulse);
title('Impulse Response');
% Step response
[Ystep,Tstep] = step(G);
subplot(1,2,2);
plot(Tstep,Ystep);
title('Step Response');
%% Part b)-Response to the given sinusoidal input
Tmax = 75;
t = 0:0.01:Tmax;
U = 2*sin(5*t) + 3*cos(0.1*t);
Y = lsim(G,U,t);
yhand = 3.363*sin(5*t-17.87) + 11.9863*cos(0.1*t-0.3949);
figure();
plot(t,Y); title('Response to sinusoidal input');
xlabel('Time');ylabel('Output');
figure();
plot(t,Y,t,yhand,'r-.');
legend('Simulated','Approximated'); title('Response to sinusoidal input');
xlabel('Time');ylabel('Output');
%% Part c) Bode Plot
```

```
figure();
bode(G);
[MAG,PHASE,W] = bode(G);
%% Part d) MinPhase
G2 = tf([10 40],[1 7 10],'InputDelay',3);
[MAG2,PHASE2,W2] = bode(G2);
figure();
bode(G);
hold on;
bode(G2,'r-.');
legend('Old TF','Min Phase TF');
```

(3)

$$\frac{d^2h}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh'}{dt} + \frac{3}{2}\frac{g}{L}h' = \frac{3}{4\rho L}p'(t)$$

$$\Rightarrow s^2H(s) + \frac{6\mu}{R^2\rho}sH(s) + \frac{3}{2}\frac{g}{L}H(s) = \frac{3}{4\rho L}P(s)$$

(Taking laplace transform)

$$\Rightarrow H(s)\left[s^2 + \frac{6\mu}{R^2\rho}s + \frac{3}{2}\frac{g}{L}\right] = \frac{3}{4\rho L}P(s)$$

$$\Rightarrow \frac{H(s)}{P(s)} = \frac{\frac{3}{4\rho L}}{s^2 + \frac{6\mu}{R^2\rho}s + \frac{3g}{2L}}$$

$$\therefore \omega^2 = \frac{3g}{2L} \Rightarrow \boxed{\omega = \sqrt{\frac{3g}{2L}}}$$

$$T = \frac{1}{\omega} \Rightarrow \boxed{T = \sqrt{\frac{2L}{3g}}}$$

$$2\xi\omega = \frac{6\mu}{R^2\rho} \Rightarrow \xi = \frac{3\mu}{R^2\rho}\sqrt{\frac{2L}{3g}}$$

$$\Rightarrow \boxed{\xi = \frac{\mu}{R^2\rho}\sqrt{\frac{6L}{g}}}$$

$$K_p\omega^2 = \frac{3}{4\rho L} \Rightarrow \boxed{K_p = \frac{1}{2g\rho}}$$

b) We get osillatory responses for underdamped systems

$\Rightarrow \quad 0 < \zeta < 1$

$$\Rightarrow \quad \frac{3\mu}{R^2 g}\sqrt{\frac{2L}{3g}} < 1$$

( $\because$ these physical constants are always +ve)

$$\Rightarrow \boxed{\frac{3\mu}{R^2 g}\sqrt{\frac{2L}{3g}} < 1}$$

c) More osillatory responses have lower damping (lower $\zeta$), less osillatory responses have higher damping (higher $\zeta$)

i) As L ↑ses, $\zeta$ ↑ses ( $\zeta \propto \sqrt{L}$ )

So higher L $\Rightarrow$ less osillatory response

lower L $\Rightarrow$ more osillatory response

ii) $\zeta \propto \mu$ ( As $\mu$ ↑ses, $\zeta$ ↑ses)

So higher $\mu$ $\Rightarrow$ less osillatory response

lower $\mu$ $\Rightarrow$ more osillatory response

## Q4

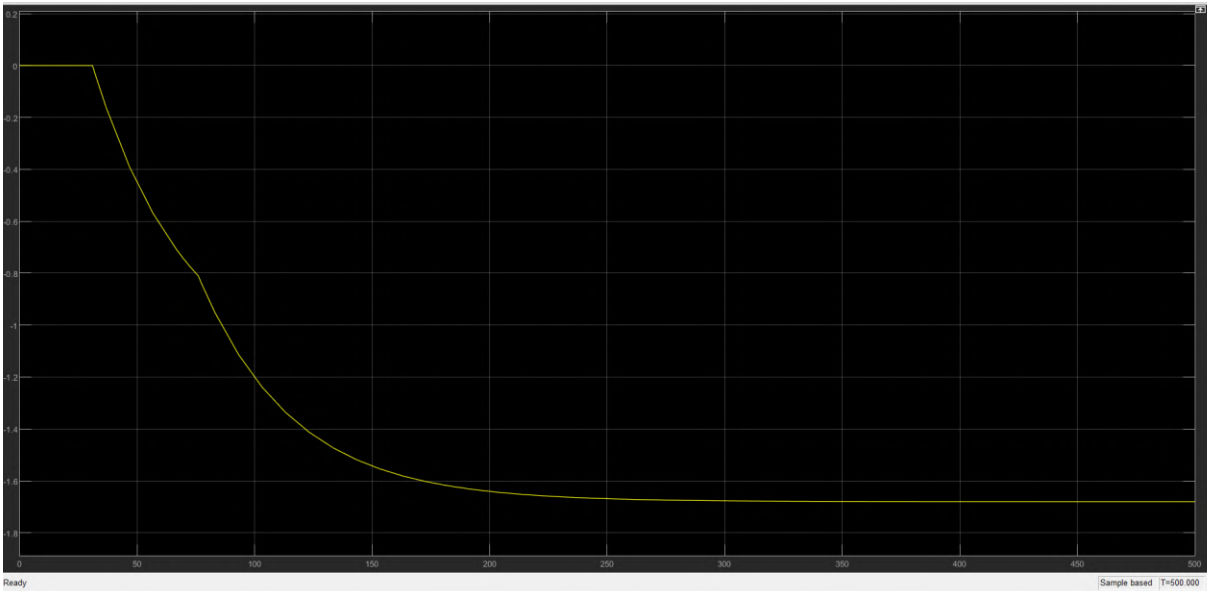



*Figure 4.1: Plot of output vs time for a unit step input*

A combination of two delayed subsystems. Initially we can't see any response, then we see a response. But after sometime a second subsystem kicks in, it shows a different kind of variation.

It is different than usual for two reasons:

1. Late response
2. The response curve is non-differentiable at one point (that is, the slope becomes discontinuous). As explained above this is because, the overall 2nd order system is composed of two 1st order systems with different delays.

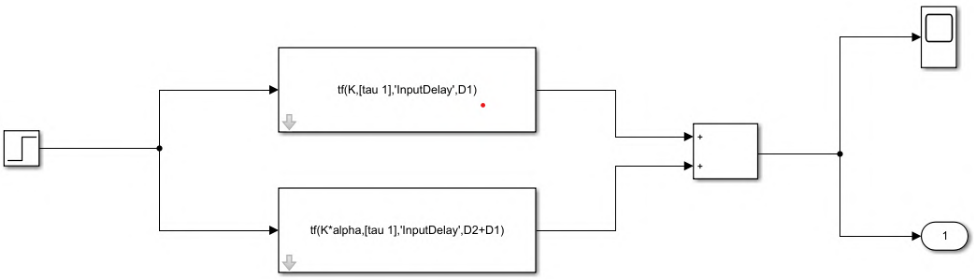



*Figure 4.2: SIMULINK Model. (LTI blocks have been used to simulate the system)*

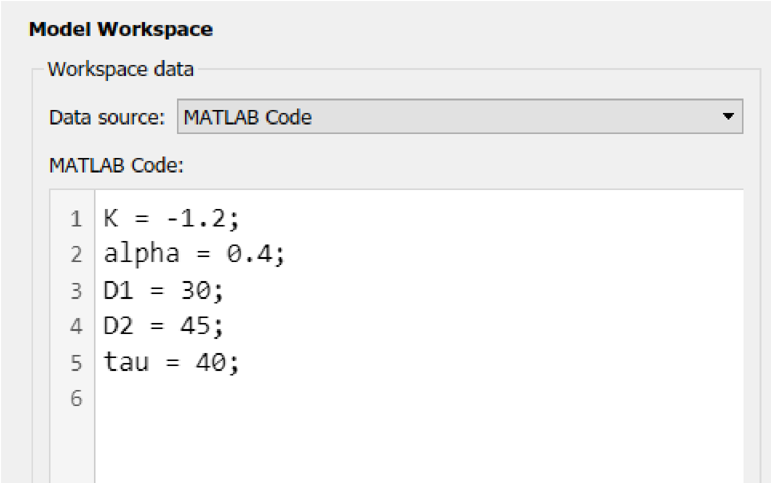



*Figure 4.3: Variables initialisation*