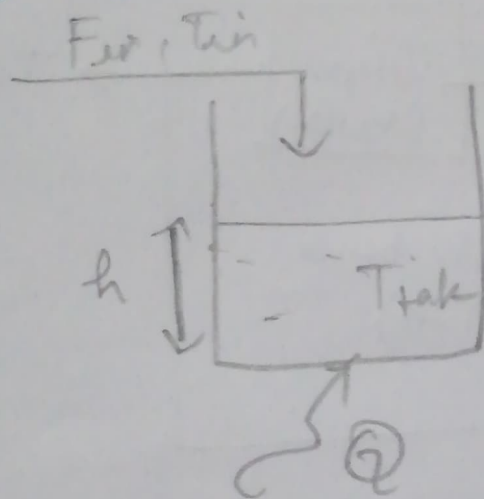


# CH3050 ASSIGNMENT-1

## ① a) Feed Batch

We can imagine the system to have water flowing in, with heat being supplied to the stored water



There is no flow out because it is a storage geyser

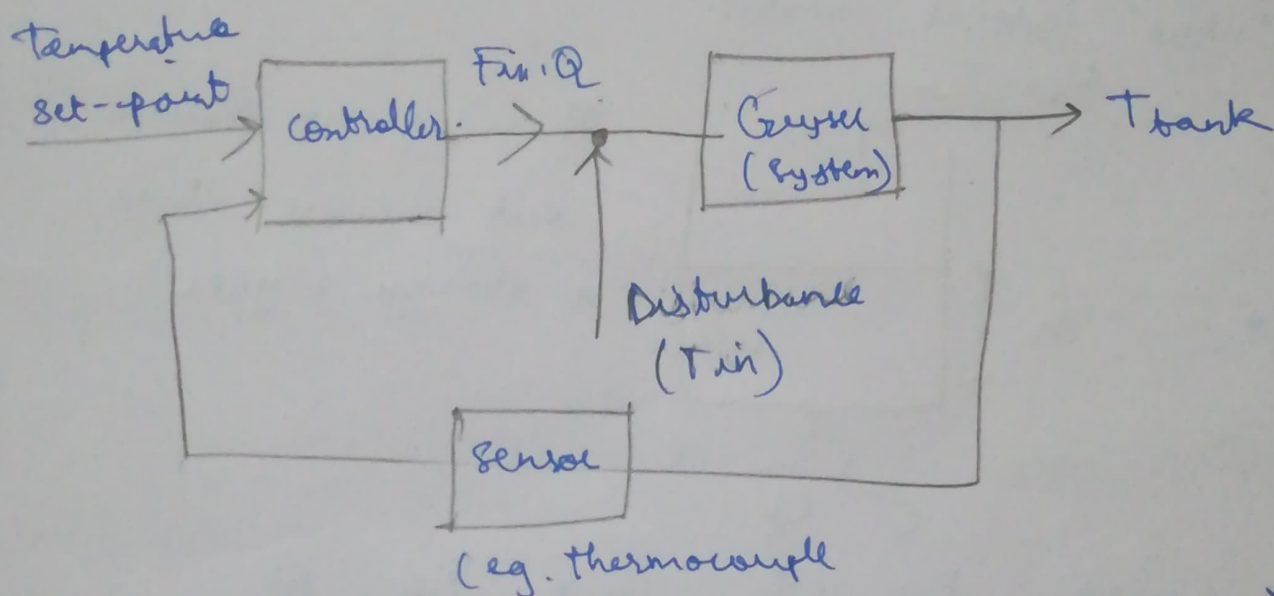
## b) Variables: $F_{in}$ , $h_{tank}$ , $T_{tank}$ , $T_{in}$ , $Q$

→ Manipulated variables:  $F_{in}$ ,  $Q$  (where  $F_{in}$  is flow in, and  $Q$  is the heat added - I assume that  $Q$  can be modified) by the

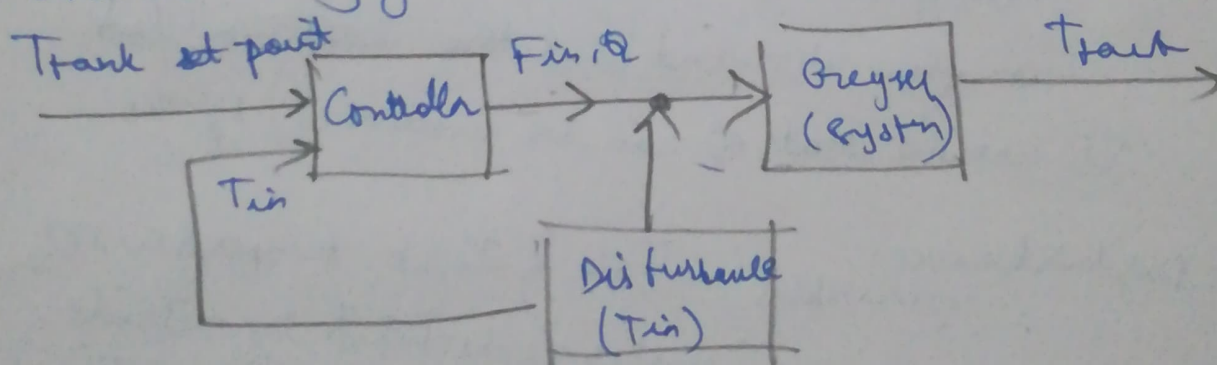
→ Disturbance variables:  $T_{in}$  (Inlet temperature of water is dictated by outside conditions)

→ Controlled variables:  $T_{tank}$  (temperature of water in the tank)

- c) Feedback system: Measure the temperature of tank ( $T_{\text{tank}}$ ) and manipulate  $F_{\text{in}}$  and  $Q$  accordingly.  $F_{\text{in}}$  manipulated by a valve and  $Q$  by using a rheostat.



- d) Feedforward system: We measure  $T_{\text{in}}$  (the disturbance) and accordingly change the manipulated variable.





2) a) At steady state  $\frac{dy}{dt} = \frac{d^2y}{dt^2} = 0$

$$\Rightarrow y(t) \quad y_{ss} = \frac{b_0}{a_0} u_{ss} \quad \text{--- (1)}$$

consider the ODE

$$\frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y(t) = b_0 u(t)$$

$$\frac{d^2y}{dt^2} (y - y_{ss}) + a_1 \frac{d}{dt} (y - y_{ss}) + a_0 (y(t) - y_{ss}) = b_0 (u(t) - u_{ss})$$

( $\because \frac{dy_{ss}}{dt} = 0 \rightarrow y_{ss}$  is a constant)

But (1)  $\Rightarrow a_0 y_{ss} = b_0 u_{ss}$

$$+ b_0 u_{ss} \quad \text{--- (2)}$$

$$\therefore \frac{d^2 \tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 \tilde{u}(t) \quad \text{--- (3)}$$

(this is expected because the ODE is linear)

b)  ~~$y_{ss} = \frac{b_0}{a_0} u_{ss} \Rightarrow y_{ss} = \frac{3}{15} \times u_{ss}$~~

(3)  $\Rightarrow a_0 \tilde{y}_{ss} = b_0 \tilde{u}_{ss}$  (@ steady state)

$$\Rightarrow \cancel{y_{ss}} \quad 2a_0 = b_0 \tilde{u}$$

$$\Rightarrow \tilde{u} = \frac{2 \times 15}{3}$$

$$\Rightarrow \boxed{\Delta u = 10 \text{ units}}$$

c) Substituting  $\tilde{u}(t) = K_c (2 - \tilde{y}(t))$  in eqn (3),

$$\frac{d^2 \tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 K_c (2 - \tilde{y}(t))$$

$$\Rightarrow \frac{d^2 \tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + \tilde{y}(t) (a_0 + b_0 K_c) = 2b_0 K_c \quad \text{--- (4)}$$

Now to answer whether the system will achieve the control objective is eqvt to asking whether ~~the~~ <sup>the</sup> system <sup>forced</sup> is <sup>stable</sup> in a <sup>stable</sup> saturated (whether it is stable)

To analyse the stability we can see the poles of this new system.

$$\text{Also } \left. \frac{d^2 \tilde{y}}{dt^2} \right|_{t=0} = 0 \quad \& \quad \left. \frac{d\tilde{y}}{dt} \right|_{t=0} = 0 \quad \& \quad \tilde{y}(t) \Big|_{t=0} = 0$$

because the system is initially assumed to be in steady state.

$$\text{So } \mathcal{L} \left\{ \frac{d^2 \tilde{y}}{dt^2} \right\} = s^2 \tilde{Y}(s)$$

$$\text{and } \mathcal{L} \left\{ \frac{d\tilde{y}}{dt} \right\} = s \tilde{Y}(s)$$



a) taking Laplace transform,

$$s \tilde{Y}(s) + a_1 s \tilde{Y}(s) + a_0 \tilde{Y}(s) = \frac{2b_0 K_c}{s} + b_0 K_c \tilde{Y}(s)$$

$$\Rightarrow \tilde{Y}(s) = \frac{2b_0 K_c}{s(s^2 + a_1 s + a_0 + b_0 K_c)}$$

to apply

By Final Value theorem, we need the conditions  $s\tilde{Y}(s)$  to be stable.

$$\Rightarrow \text{'roots' of } s^2 + a_1 s + a_0 + b_0 K_c < 0$$

$$\Rightarrow \frac{-a_1 \pm \sqrt{a_1^2 - 4(a_0 + b_0 K_c)}}{2} < 0$$

$$\Rightarrow 0 \leq a_1^2 - 4(a_0 + b_0 K_c) < a_1^2$$

$$\Rightarrow K_c \leq \frac{a_1^2 - 4a_0}{4b_0} \text{ and}$$

$$K_c > -\frac{a_0}{b_0}$$

$$\Rightarrow K_c \leq \frac{1}{3} \text{ and } K_c > -5$$

$$\Rightarrow -5 < K_c \leq 1/3 \Rightarrow 0 < K_c \leq 1/3$$

Under such conditions we apply FVT

$$\text{to get } \tilde{y}(t) \text{ as } \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \tilde{Y}(s)$$

$$\lim_{t \rightarrow \infty} \tilde{y}(t) = \lim_{s \rightarrow 0} s \times \frac{2b_0 K_c}{s^2 + a_1 s + a_0 + b_0 K_c} = 2$$

$\therefore$  Required objective is attained  
for  $0 < K_c \leq \frac{1}{3}$

d) ③  $\Rightarrow \frac{d^2 \tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y}(t) = b_0 \tilde{u}(t)$

Differentiate wrt time,

$$\frac{d^3 \tilde{y}}{dt^3} + a_1 \frac{d^2 \tilde{y}}{dt^2} + a_0 \frac{d\tilde{y}}{dt} = b_0 \frac{d\tilde{u}}{dt}$$

$$= b_0 \left( K_c \left( -\frac{d\tilde{y}}{dt} \right) + K_I (2 - \tilde{y}) \right)$$

For notational convenience,  $\tilde{y}$  is dropped  
( $a_0 + b_0 K_c$ )

$$\Rightarrow \frac{d^3 y}{dt^3} + (a_1 + b_0 K_c) \frac{d^2 y}{dt^2} + \frac{a_0}{b_0 K_I} \frac{dy}{dt} + b_0 K_I y = 2 K_I b_0$$

Similar to previous part, take Laplace transform to get,

$$(s^3 + a_1 s^2 + (a_0 + b_0 K_c)s + b_0 K_I) Y(s) = \frac{2 K_I b_0}{s}$$



$$\Rightarrow Y(s) = \frac{2K_I b_0}{s(s^3 + a_1 s^2 + (a_0 + b_0 K_C) s + b_0 K_I)}$$

Let We want the poly(sY(s)) to be stable

$$\Rightarrow \text{roots}(s^3 + a_1 s^2 + (a_0 + b_0 K_C) s + b_0 K_I) < 0 \quad \textcircled{5}$$

$$\Rightarrow \text{roots}(s^3 + 8s^2 + (15 + 3K_C)s + 3K_I) < 0$$

Assuming roots obey this property, apply RVT

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \times \frac{2K_I b_0}{s(s^3 + 8s^2 + (15 + 3K_C)s + 3K_I)}$$

$\therefore$  objective is achieved.

Conclusion: The objective can't be achieved for any  $\{K_C, K_I\}$ , it is achieved only when  $\textcircled{5}$  is satisfied.

eg.  $K_C = K_I = 1$ , roots are  $-3.9 + 1.14j$   
 $-3.9 - 1.14j$   
 $-0.181$

$\Rightarrow$  objective is not achieved.

But for  $K_C = 0.25$ ,  $K_I = 0.5$

roots are  $-4.25$ ,  $-3.14$ ,  $-0.1$

$\Rightarrow$  objective is achieved.

$$\textcircled{3} \textcircled{a) } \frac{dw}{dt} = -\left(\frac{L+Va}{M}\right)w + \frac{Va}{M}z \quad \textcircled{1} \quad \begin{array}{l} a = 0.5 \\ 2.5 = 0.1 \\ M = 10 \end{array}$$

$$\frac{dz}{dt} = -\frac{L}{M}w - \left(\frac{L+Va}{M}\right)z + \frac{Vz_f}{M} \quad \textcircled{2}$$

At steady state,  $L = L_{ss} = 4$ ,  $V = V_{ss} = 200$   
 $- 89$

$$\frac{dw}{dt} = 0, \quad \frac{dz}{dt} = 0$$

$$\textcircled{1} \Rightarrow -6.5w + 2.5z = 0 \quad \textcircled{3}$$

$$\textcircled{2} \Rightarrow 4w - 6.5z = -0.5 \quad \textcircled{4}$$

Solving  $\textcircled{3}$  &  $\textcircled{4}$ ,

$$w_{ss} = 0.0388 \text{ and } z_{ss} = 0.1008$$

$\Rightarrow$  Steady state values:  $w = 0.0388$   
 $z = 0.1008$

b) In this model,  $\text{state}_{(M)} = \begin{bmatrix} w \\ z \end{bmatrix}$

$$\text{Inputs}_{(M)} = \begin{bmatrix} L \\ V \end{bmatrix} \quad \text{Let } \frac{dw}{dt} = f(\cdot) \text{ \& } \frac{dz}{dt} = g(\cdot)$$

$$\frac{dw}{dt} = \frac{dw}{dt} \Big|_{\text{steady state}} + \frac{\partial f}{\partial L} (L - L_{ss}) + \frac{\partial f}{\partial V} (V - V_{ss}) + \frac{\partial f}{\partial w} (w - w_{ss}) + \frac{\partial f}{\partial z} (z - z_{ss})$$



$$a_{11} = \frac{\partial f}{\partial \omega} = - \frac{(V_a + L)}{M}$$

$$a_{12} = \frac{\partial f}{\partial z} = \frac{V}{M}$$

$$a_{21} = \frac{\partial g}{\partial \omega} = \frac{L}{M} ; a_{22} = - \left( \frac{L + V_a}{M} \right)$$

$$b_{11} = \frac{\partial f}{\partial L} = - \frac{\omega}{M} ; b_{12} = \frac{\partial f}{\partial g} = - \frac{\omega}{M} + \frac{q_2}{M}$$

$$b_{21} = \frac{\partial g}{\partial L} = \frac{\omega}{M} ; b_{22} = \frac{\partial g}{\partial g} = - \frac{q_2}{M} + \frac{2f}{M}$$

$$\therefore A = \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix}$$

(all values evaluated at steady state)

$$B = \begin{bmatrix} -0.0019 & 0.0016 \\ -0.0031 & 0.0025 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{dw}{dt} \\ \frac{dz}{dt} \end{bmatrix} = A \begin{bmatrix} w - w_{ss} \\ z - z_{ss} \end{bmatrix} + B \begin{bmatrix} L - L_{ss} \\ V - V_{ss} \end{bmatrix}$$

$$\text{Let } y = \begin{bmatrix} w - w_{ss} \\ z - z_{ss} \end{bmatrix}$$

$$\Rightarrow y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \cdot w_{83} \\ z - 2u \end{bmatrix}$$

c) eigenvalues: ~~De~~ -3.338, -9.162

eigenvectors:  $\begin{bmatrix} 0.6202 \\ 0.7845 \end{bmatrix}$  &  $\begin{bmatrix} -0.6202 \\ 0.7845 \end{bmatrix}$

Since  $|\lambda_2| > |\lambda_1|$

~~-9.162~~  
fastest dec: eigenvector 2:  $\begin{pmatrix} -0.6202 \\ 0.7845 \end{pmatrix}$

slowest dec: eigenvector 1:  $\begin{pmatrix} 0.6202 \\ 0.7845 \end{pmatrix}$

④ a)  $\mathcal{L}\{x(t)\} =$

For  $0 \leq t < 3$

$$\mathcal{L}\{t-2\} = \frac{e^{-2s}}{s} \int_0^3 t e^{-st} dt$$

$$= \frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2} \Big|_0^3 = \frac{1}{s} (1 - e^{-3s})$$

$$= \frac{1 - 2s - (1 + se^{-3s})}{s^2}$$

$$= \frac{te^{-st}}{-s} - \int_0^3 \frac{e^{-st}}{s} - \frac{2}{s} (1 - e^{-3s})$$

$$= \frac{1}{s} \left( \frac{e^{-3s}}{s} + \frac{e^{-3s}}{s^2} - \frac{1}{s^2} - \frac{2}{s} (1 - e^{-3s}) \right)$$



$$\Rightarrow X(s) = \boxed{\frac{-(1+s)e^{-3s} - 2s + 1}{s^2}} \quad 0 \leq t < 3$$

$$3 \leq t \leq 4$$

$$X(s) = \int_3^4 e^{-st} dt$$

$$= \boxed{\frac{e^{-3s} - e^{-4s}}{s}}$$

$$4 \leq t < 5$$

$$X(s) = \int_4^5 -\cos(3\pi(t-4)) e^{-st} dt$$

$$\Rightarrow X(s) = \cancel{\frac{-\cos(3\pi(t-4))}{-s}} e^{-st} \left( + \int_4^5 3\pi \sin(3\pi(t-4)) e^{-st} dt \right)$$

$$= \frac{-\cos(3\pi(t-4))}{-s} e^{-st} + \left( \frac{3\pi \sin(\pi(t-4))}{-s^2} e^{-st} \right) \Big|_4^5$$

$$- \int_4^5 \left( \frac{9\pi^2 \cos(3\pi(t-4))}{-s^2} e^{-3t} \right) dt$$

$$\Rightarrow X(s) \left( 1 + \frac{9\pi^2}{s^2} \right)$$

$$= \frac{-e^{-5s}}{s} + \frac{3\pi e^{-4s}}{s^2}$$

$$\Rightarrow X(s) = \frac{(e^{-5s} + 3\pi e^{-4s})}{\frac{9\pi^2}{s^2} + 1}$$

$$t \geq 5$$

$$\mathcal{L}^{-1}\{e^{-2s} X(s)\} = \int_5^{\infty} e^{-2(t-\tau)} \cos(5\pi(t-\tau)) e^{-st} dt$$

$$= \int_5^{\infty} e^{-t(s+2)-5} \cos(5\pi(t-\tau)) dt$$

By part 1

$$\Rightarrow X(s) = \frac{e^{-t(s+2)+5} \cos(5\pi(t-\tau))}{-(s+2)} \Big|_5^{\infty} - \int_5^{\infty} \frac{\sin(5\pi(t-\tau)) e^{-t(s+2)+5}}{s+2} dt$$

$$= \frac{e^{-5(s+2)+50}}{s+2} - 5\pi \left[ \frac{\sin(5\pi(t-\tau))}{(s+2)^2} + \int_5^{\infty} \frac{\cos(5\pi(t-\tau)) e^{-t(s+2)+5}}{(s+2)^2} dt \right]$$

$$\Rightarrow X(s) \left( 1 + \frac{25\pi^2}{(s+2)^2} \right) = -e^{-5s}$$

$$\Rightarrow X(s) = \frac{e^{-5s}}{1 + \frac{25\pi^2}{(s+2)^2}}$$



Find soln:

Ans:

$$\textcircled{b) \quad \frac{s-2}{s(s^2 + 2\frac{q}{T}s + 1)} = \frac{1(s-2)}{T^2 s \left( s^2 + 2\frac{q}{T}s + 1 \right)}$$

Roots of  $s^2 + 2\frac{q}{T}s + 1$

$$= \frac{-2\frac{q}{T}}{2T} \pm \frac{\sqrt{4\frac{q^2}{T^2} - 4}}{2T} \quad (\text{call these } \alpha, \beta)$$

They are real if  $q \geq 1$

i)  $q > 1$

$$\frac{A_1}{s} + \frac{A_2}{s - \frac{-q + \sqrt{q^2 - 1}}{T}} + \frac{A_3}{s - \frac{-q - \sqrt{q^2 - 1}}{T}}$$

$$= \frac{s-2}{T^2 \left( s^2 + 2\frac{q}{T}s + 1 \right)}$$

put  $s=0$

$$A_1 = \frac{-2}{T^2}$$

let  $s = \alpha$

$$A_2 = \frac{\alpha - 2}{T^2}$$

$$A_3 = \frac{\beta - 2}{T^2}$$

### Q3 Part d)

linsys =

A =

	Integ1	Integ2
Integ1	-6.5	2.5
Integ2	4	-6.5

B =

	FR	FR1
Integ1	-0.001938	0.00155
Integ2	-0.003101	0.002481

C =

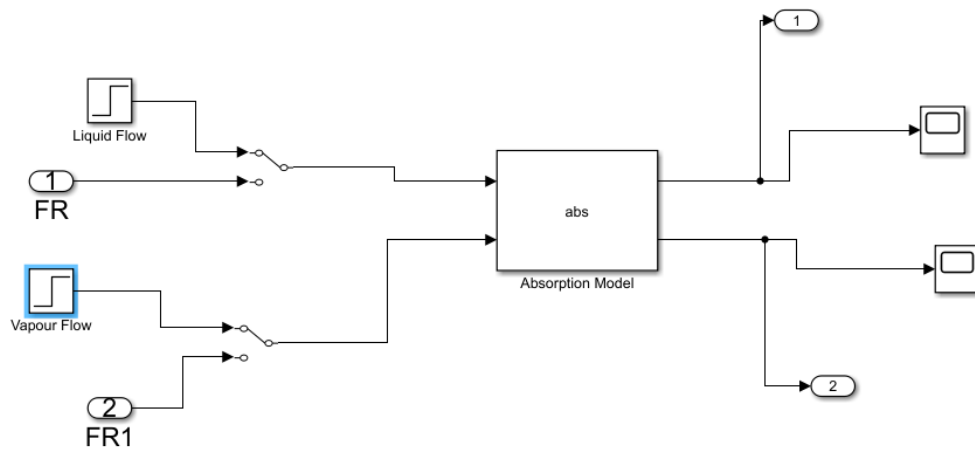
	Integ1	Integ2
Out1	1	0
Out2	0	1

D =

	FR	FR1
Out1	0	0
Out2	0	0

Continuous-time state-space model.



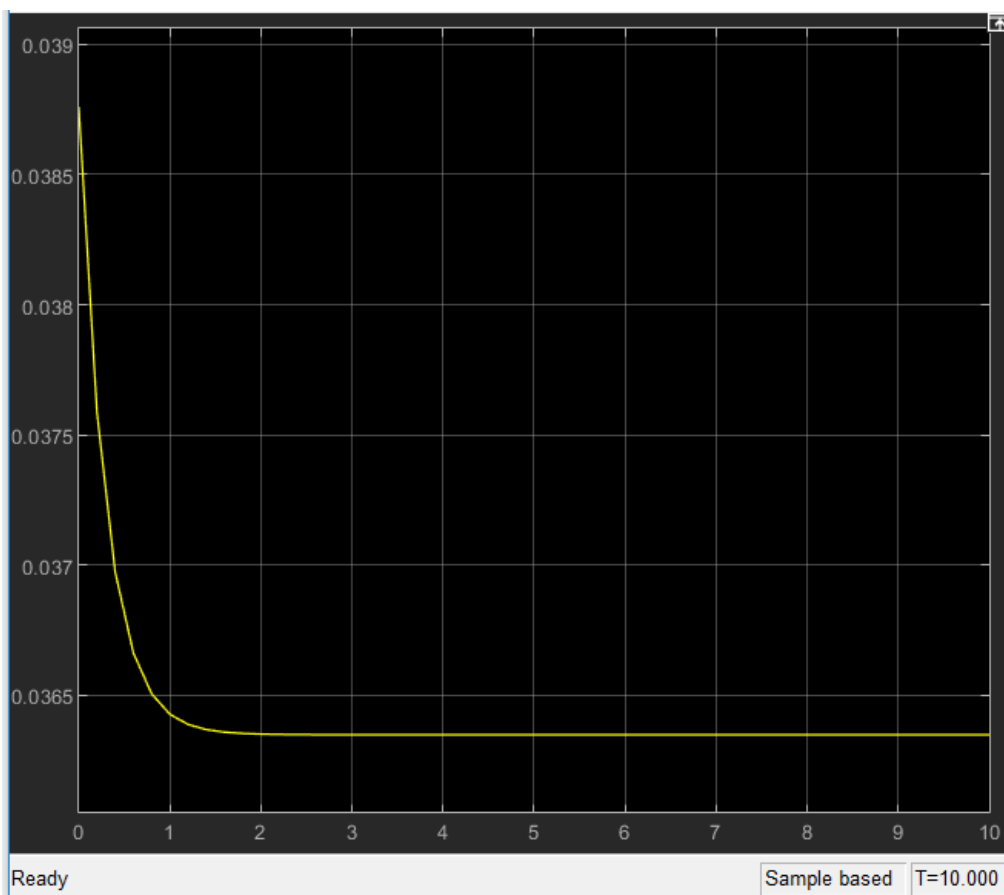


### Part e)

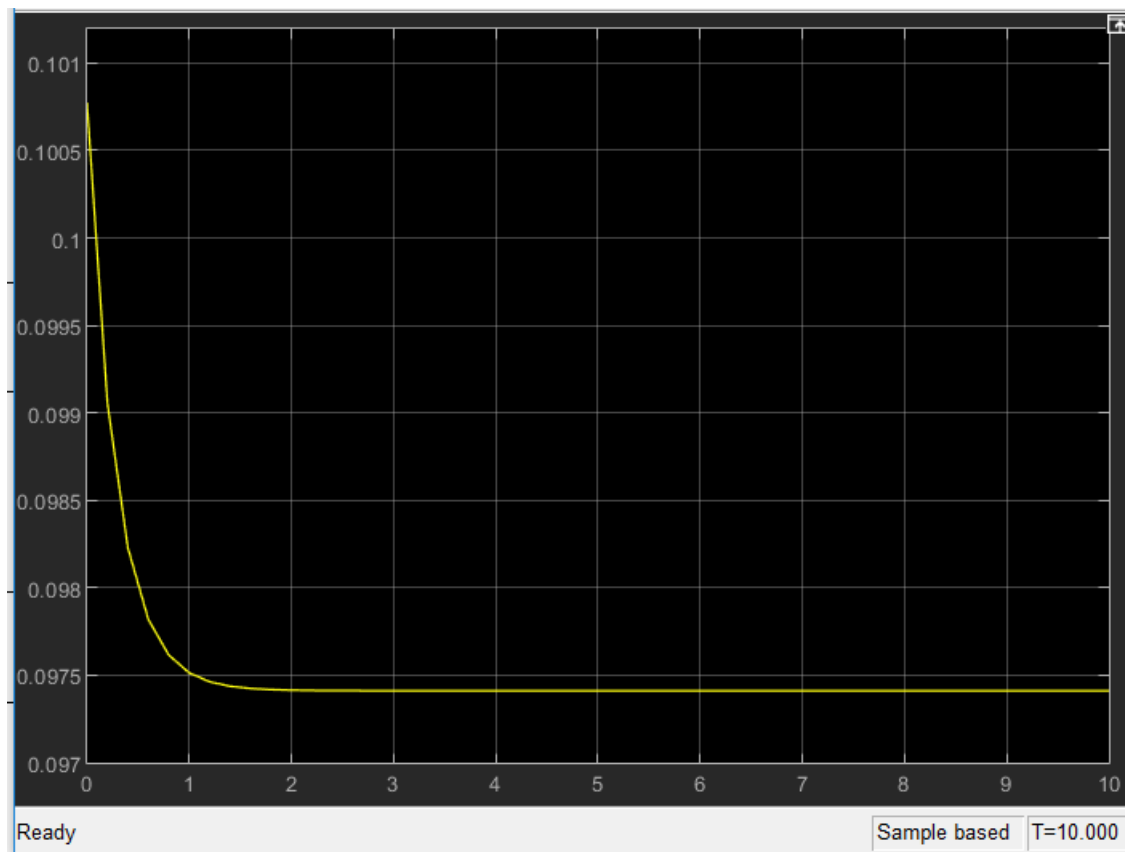
Since if we change both the flow rates, steady state values are same as original, I am changing only  $L$  in both cases.

ALL GRAPHS ARE FROM NON LINEAR MODEL. I DID NOT PLOT FOR LINEAR MODEL.

i)  $L = 1.05L_{ss}$



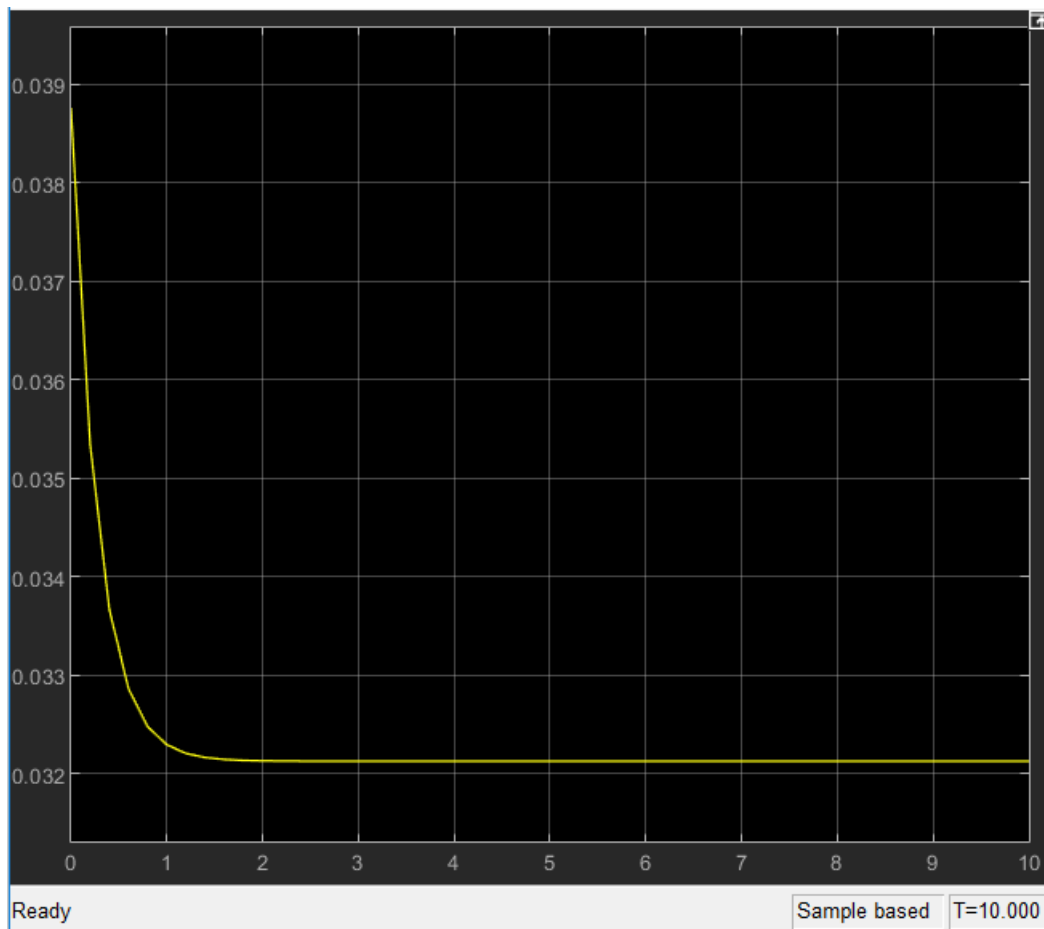
W graph



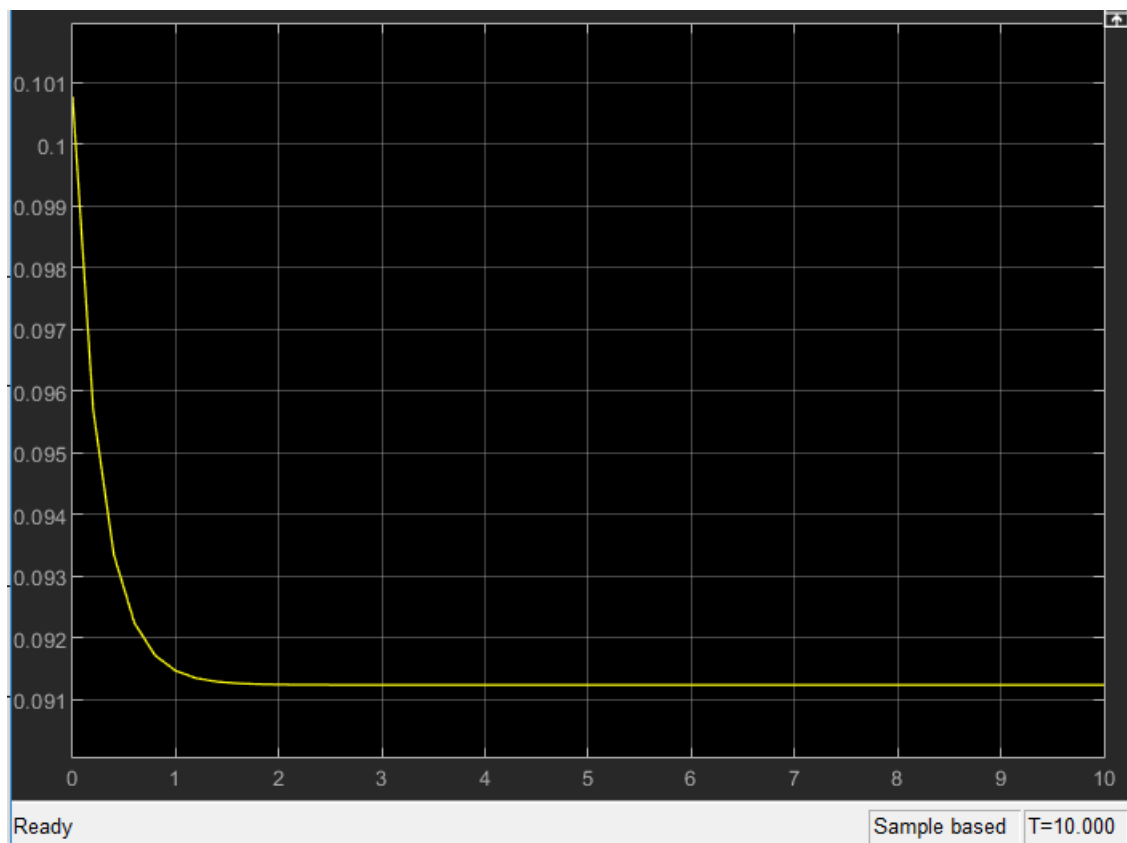
Z graph

ii)  $L = 1.15L_{ss}$





W graph



## Z graph

Code:

```
clear;close all;

%% System charecterstics

Lss = 80; Vss = 100;

M = 20; a=0.5; zf = 0.1;

%% Part a) Finding steady state (by hand)

% Equate derivatives to zero, solve the linear eqn

Ass = [-(a*Vss+Lss)/M Vss*a/M;Lss/M -(a*Vss+Lss)/M];

bss = [0;-Vss*zf/M];

x_ss = inv(Ass)*bss;

%% Part b) Linearisation (by Taylor Expansion)

w_ss = x_ss(1);z_ss=x_ss(2);

A = [-(Vss*a+Lss)/M Vss*a/M;Lss/M -(Lss+Vss*a)/M];

B = [-w_ss/M (-a*w_ss+a*z_ss)/M;(w_ss-z_ss)/M -a*z_ss/M+zf/M];

%% Part c) Finding the eigenvalues-eignvectors of the system

[V,D] = eig(A);

% Second eigen value is faster (more negative)

%% Part d) Find steady-state and linearise

open_system('Q3_model')

% Read the operating conditions into an object

opc = operspec('Q3_model');

% Operating conditions

opc.Inputs(1).u = 80;

opc.Inputs(2).u = 100;

opc.Inputs(1).Known = 1;

opc.Inputs(2).Known = 1;

% Constraints

opc.States(1).Min = 0;opc.States(2).Min = 0;
```



```
% Find the steady state point
ss_point = findop('Q3_model',opc);

% Linearize
linsys = linearize('Q3_model',ss_point)

%% Part e) Give step changes and plot
% Done in SIMULINK. Use the manual switch to step input(s)
[Y,T,X]=step(linsys);

plot()
```