CH 3050 PDC ASSGN-6

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1) a) $$G_c = \frac{1}{G_p}\,\frac{G_{cl}}{1-G_{cl}}$$

$$= \frac{1}{\frac{K}{(10s+1)(5s+1)}}\;\frac{\frac{1}{8s+1}}{\frac{8s}{5s+1}}$$

$$\Rightarrow G_c = \frac{50s^2+15s+1}{(\tau_c s)\,K}$$

$K_{nominal} = 1 \Rightarrow G_c = \dfrac{50s^2+15s+1}{\tau_c s}$

Now, consider '$1+G_pG_c$' (CCL eqn)

$$\Rightarrow \frac{50s^2+15s+1}{\tau_c s}K + 1 = 0$$

$$\Rightarrow K + \tau_c s = 0$$

$$\Rightarrow s = \frac{-K}{\tau_c}$$

$$= \frac{-(1+\delta)}{\tau_c}$$

~~For~~ For stability $s < 0$ (All LHP poles)

$\Rightarrow \eta > -1$

b) pole, $s = \dfrac{-1}{\tau_c}(1+\eta)$

~~For~~ ~~$|\eta|$~~ $|\eta| \geq 0.2$

$\Rightarrow \eta \geq -0.2 \Rightarrow \eta > -1$

$\therefore$ The pole remains in LHP irrespective of τ_c

$\Rightarrow$ Free to choose any $\tau_c > 0$ even when K is uncertain

c) $G_p = \dfrac{1.15}{80s^2+15s+1}$ $G_d = \dfrac{1}{5s+1}$

$G_c = \dfrac{50s^2+15s+1}{5s}$ (PID controller)

Now when ff & PID both are in combination

$Y = G_p U + G_d D_o$ — ①

$U = G_c(R-Y) + G_{ff} D_o$ — ②

~~where G_{ff}~~ =

Substituting ② in ①,

$$Y(1+G_cG_p) = G_pG_cR + (G_pG_{ff} + G_d)D_o$$

$R = 0$

$$\Rightarrow \frac{Y}{D_o} = \left(\frac{G_{ff}G_p + G_d}{1+G_pG_c}\right)$$

$$\text{Here } G_{ff} = \frac{-G_d}{G_{pm}} \times \frac{1}{(\lambda s+1)}$$ (filter added to make it realisable)

$$= \frac{-(10s+1)}{(\lambda s+1)(1.15)}$$

C.E. : Den $\left(\frac{G_{ff}G_p + G_d}{1+G_pG_c}\right)$

$$\Rightarrow (5s+1.15)^2(\lambda s+1)(5s+1) = 0$$

For settling time of 15, $\tau \approx 3$.

But here $\tau_{dominant} > 3$.

So the best t settling we can get is around 25 minuts.

The same was confirmed by search trying to solve the problem

$$\min_{\lambda} \left| t_{settle} - 15 \right|.$$

λ - came to be close to 0 (10^{-3} - lowest bound of search space)

$\lambda \approx 10^{-3}$,

$t_{settle} \approx 25$ minuts

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(2) a)

Since ~~chart~~ there is no delay or inverse response, we can fit a second order model

The model is estimated using lsqcurvefit on step response model as,

$$\hat{G}_p = \frac{\overset{\text{dcgain}(\hat{G}_p) \;=\; K_p \times K_{IV} \times \hat{K}}{1.781}}{4.785 s^2 + 4.361 + 1}$$

$\hat{K}$ is the estimate from ~~the lsqcurvefit~~ the step response curve

IMC

$$\tau_c = \frac{\max(\tau_1, \tau_2)}{2} = \frac{2.188}{2} = 1.094$$

$$K_c = \frac{\tau_1 + \tau_2}{K \tau_c} = 2.2385$$

$$\tau_I = \tau_1 + \tau_2 = 4.3611$$

$$\tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} = 1.0903$$

→ All relations from table 12.1

$$G_c = 2.238 \left(1 + \frac{1}{4.361 s} + 1.09 s \right)$$

b) Using Skogestad's half rule we obtain an FOPTD approximation (∵ tables are available only for them FOPTD)

$$\hat{G}_{p_{12}} = \frac{1.781\, e^{-\frac{2.1731}{2}s}}{\left(\left(2.188 + \frac{2.1731}{2}\right)s + 1\right)}$$

$$\Rightarrow \hat{G}_{p,FOPTD} = \frac{1.781\, e^{-1.09s}}{3.267s+1}$$

From table 12.4, [ITAE set-point]

P: $A = 0.965$, $B = -0.85$, $K_c = \frac{A\left(\frac{\theta}{\tau}\right)^{B}}{K}$
$= 1.3734$

I: $A = 0.796$, $B = -0.1465$, $\tau_I = \frac{\tau}{\left(A + \frac{B\theta}{\tau}\right)} = 4.374$

D: $A = 0.308$, 0.929, $\tau_D = A\left(\frac{\theta}{\tau}\right)^{B} \times \tau$
$= 0.3642$

$$\therefore G_{c1b} = 1.3734\left(1 + \frac{1}{4.374s} + 0.3642s\right)$$

c) From table 12.4, [I TAE disturbance]

P : $A = 1.357$ $B = -0.947$, $K_C = 2.1475$

I : $A = 0.842$ $B = -0.738$, $\tau_I = 2.3101$

D : $A = 0.381$, $B = 0.995$, $\tau_D = 0.4191$

$$\therefore G_{c}(s) = (2.1475)\left(1 + \frac{1}{2.31s} + 0.419s\right)$$

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③ a) Units of gain : K/radians

b) $$G_{pm} = \frac{-(0.5)(-10s+1)\,e^{-10s}}{(5s+1)(3s+1)}$$

All pass factorization : $$G_{pm} = \underbrace{\frac{-(0.5)(+10s+1)}{(5s+1)(3s+1)}}_{G_{m,i}} \underbrace{\frac{(-10s+1)e^{-10s}}{10s+1}}_{G_{m\,ni}}$$

$$\therefore Q = G_{ff} = \frac{+1}{G_{m,i}} \left[\frac{1}{\lambda s+1}\right]$$ filter added to ensure that it is biproper

$$= \frac{(5s+1)(3s+1)}{(-0.5)(10s+1)(\lambda s+1)}$$

c) $\frac{Y}{R} = \frac{Q G_p}{1 + Q \Delta G}$

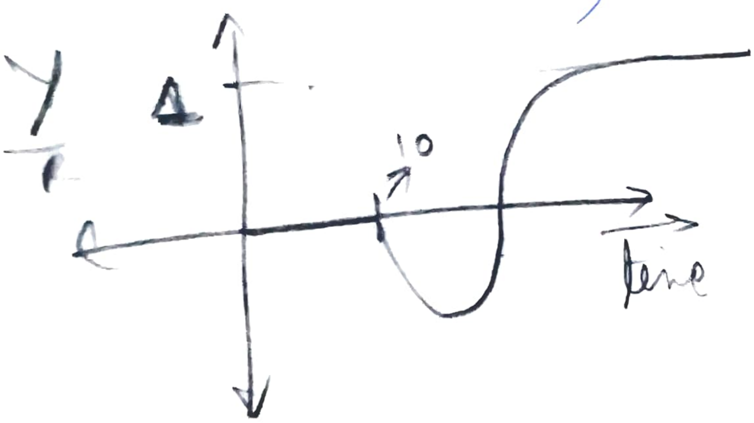
Here, $\Delta G = 0$ (given $G_p = G_m$ perfect model)

$\frac{Y}{R} = \frac{(-10s+1)e^{-10s}}{(\lambda s+1)(10s+1)}$

Since it is proper we won't have a jump.

But we have i) Delay

ii) Inverse Response ($\because$ RHP 0)



d) $\frac{U}{R} = \frac{Q}{1 + Q \Delta G} = Q = \frac{(5s+1)(3s+1)}{(-0.5)(10s+1)(\lambda s+1)}$

IVT : $U(0) = \lim_{s \to \infty} \frac{10(5s+1)(3s+1)}{(-0.5)(10s+1)(\lambda s+1)}$

$= \frac{10(5)(3)}{(-0.5)(10)(\lambda)} = \frac{-30}{\lambda}$

We want this to be less than 25% = $\frac{\pi}{4}$ rad.

45°

$\pi/4$ rad

$$\Rightarrow \left| \frac{30}{\lambda} \right| \leq \frac{\pi}{4}$$

$$\Rightarrow \lambda \geq \frac{120}{\pi}$$ $(\because \lambda$ +ve for stable $\theta)$

($\because$ Immediate control effort should be less than 25%.)

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(4) a) From the given data,

$$G_p = \frac{0.4}{25s+1} e^{-7.5s}$$

$$G_d = \exp(-10s) \left(\frac{0.5}{30s+1} \right)$$

So $Q = -\frac{G_d}{G_p}$

But we have non-invertible component $\longrightarrow$ delay.

Use Pade's first order approximation.

$$\Rightarrow Q = -\left(\frac{0.5 e^{-10s}}{30s+1} \right) \frac{(25s+1)(3.75s+1)}{(0.4)(-3.75s+1)}$$

$$= \frac{-250s^2 + 40s + 2}{240s^2 - 40s - 1.6} e^{-10s}$$

Question-1

Code

```
clear; close all;
tc = 5;
s = tf('s');
Gp = tf(1.15,[50 15 1]);
Gm = tf(1,[50 15 1]);
% PID controller
Gc  = tf([50 15 1],[tc 0]);
% Disturbance tf
Gd = tf(1,[5 1]);
lambdavec = 0.001:0.001:0.5;
r1 = ones(length(lambdavec),1);
r2 = r1;
for k = 1:length(lambdavec)
    % Feedforward controller
    Gff = -Gd*1/(lambdavec(k)*s+1)/Gp;
    % sys is Y/Do
    sys = (Gff*Gp+Gd)/(1+Gp*Gc);
    S = stepinfo(sys);
    % get settling time as close as possible to 15
    r2(k) = S.SettlingTime;
    r1(k) = abs(S.SettlingTime-15);
end
[val,loc] = min(r1);
lambda = lambdavec(loc);
```

Question-2

Tables

Table 12.1 IMC Controller Settings for Parallel-Form PID Controller (Chien and Fruehauf, 1990)

Case	Model	K_cK	τ_I	τ_D
A	$\dfrac{K}{\tau s+1}$	$\dfrac{\tau}{\tau_c}$	τ	–
B	$\dfrac{K}{(\tau_1 s+1)(\tau_2 s+1)}$	$\dfrac{\tau_1+\tau_2}{\tau_c}$	$\tau_1+\tau_2$	$\dfrac{\tau_1\tau_2}{\tau_1+\tau_2}$

Table 12.4 Controller Design Relations Based on the ITAE Performance Index and a First-Order-plus-Time-Delay Model (Lipták, 2006)*,†

Type of Input	Type of Controller	Mode	A	B
Disturbance	PI	P	0.859	−0.977
		I	0.674	−0.680
Disturbance	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03†	−0.165†
Set point	PID	P	0.965	−0.85
		I	0.796†	−0.1465†
		D	0.308	0.929

*Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.
†For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$.

Code

```
clear;close all;
s = tf('s');
%% Given Data
Kv = 0.9; Kip = 0.75;
t = (0:1:11)';
T =
([12,12.5,13.4,14,14.8,15.4,16.1,16.4,16.8,16.9,17,16.9]'
-12)/2;
plot(t,T);
% Can't see any inverse response, so mostly no zero
assume first order plus
% time delay.
%% Model Estimation
[X,RESNORM,RESIDUAL,EXITFLAG] = lsqcurvefit(@resp,[5 2
1],t,T);
K = X(1)*Kv*Kip;
tau1  = X(2);
tau2 = X(3);
Gp = tf(K,conv([tau1 1],[tau2 1]));
%% Part a) IMC
tauc = max(tau1 ,tau2)/2;
Kc = (tau1 +tau2)/(K*tauc);
tauI = tau1 + tau2;
tauD = (tau1*tau2)/(tau1 + tau2);
Gc_imc = Kc*(1+1/(tauI*s)+tauD*s);
%% Part b) ITAE (setpoint)
% FOPTD approximation
D = tau2/2;
```

```matlab
tau = tau1 + tau2/2;
% Use tables
AP = 0.965;
BP = -0.85;
Kc_b = AP*(D/tau)^BP/K;
AI = 0.796;
BI = -0.1465;
tauI_b = tau/(AI + BI*(D/tau));
AD = 0.308;
BD = 0.929;
tauD_b = AD*(D/tau)^BD*tau;
Gc_b = Kc_b*(1+1/(tauI_b*s)+tauD_b*s);
%% Part c) ITAE (disturbance)
AP = 1.357;
BP = -0.947;
Kc_c = AP*(D/tau)^BP/K;
AI = 0.842;
BI = -0.738;
tauI_c = tauI/(AI*(D/tau)^BI);
AD = 0.381;
BD = 0.995;
tauD_c = AD*(D/tau)^BD*tau;
Gc_c = Kc_c*(1+1/(tauI_c*s)+tauD_c*s);
%% Function to give step response for lsqcurvefit
function Y = resp(params,tvec)
    K = params(1);
    tau = params(2);
    tau2 = params(3);
    Gp = tf(K,conv([tau 1],[tau2 1]));
    Y = step(Gp,tvec);
end
```

Question-4

Part a) Feedforward controller

Model Workspace

Workspace data

Data source: MATLAB Code

MATLAB Code:

```
s = tf('s');
Kd = 0.25/0.5;
Dd = 10;
taud = 30;
Gd = tf(Kd,[taud 1],'iodelay',Dd);
Kp = 0.4;
Dp = 7.5;
taup = 25;
Gp = tf(Kp,[taup 1],'iodelay',Dp);
Gff = -1*Gd*(1-Dd*s/2)*(taup*s+1)/(1-Dd*s/2)/Kp;
```

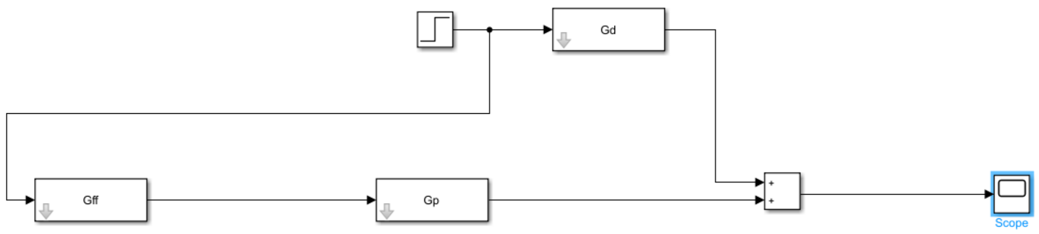

Figure 1: SIMULINK DIAGRAM of the system with just a feed-forward controller

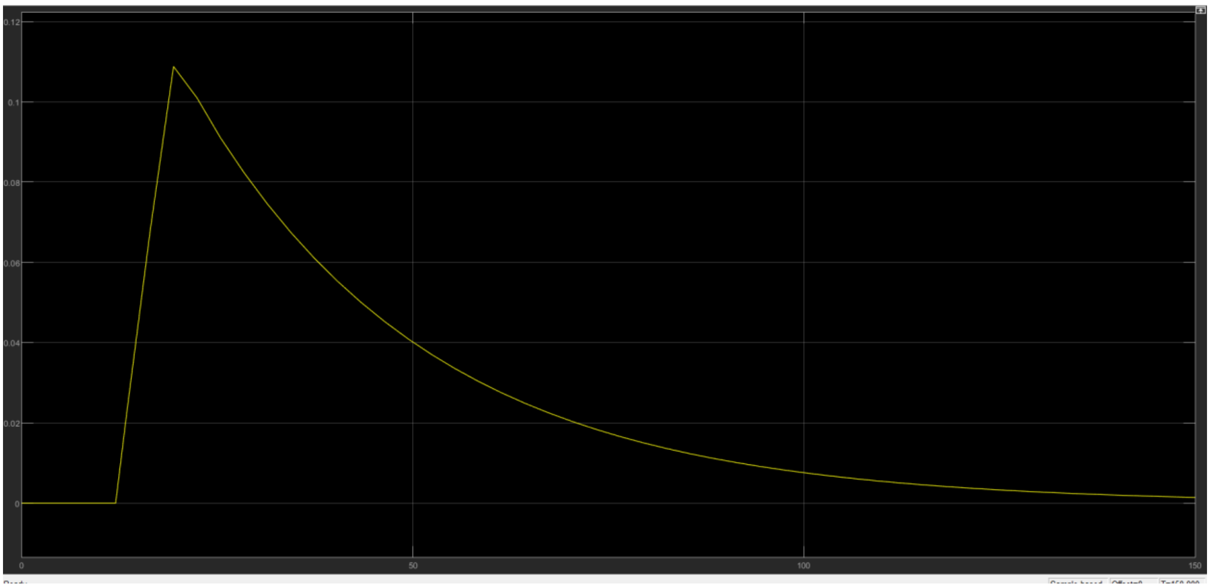

Figure 2: Disturbance rejection performance

Part b) Tuned PID Controller

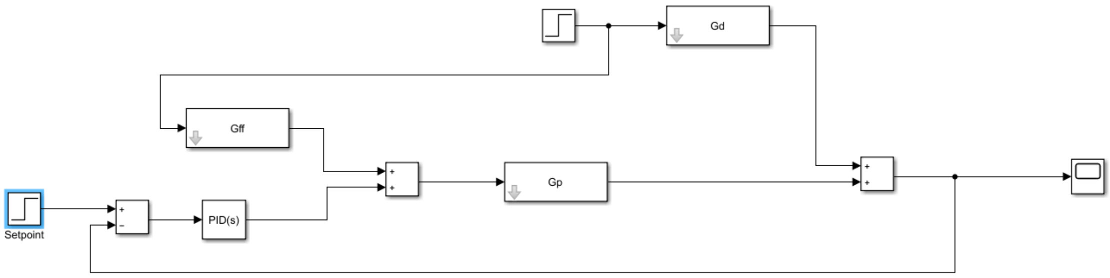

Figure 3: Feedforward in combination with a PID controller

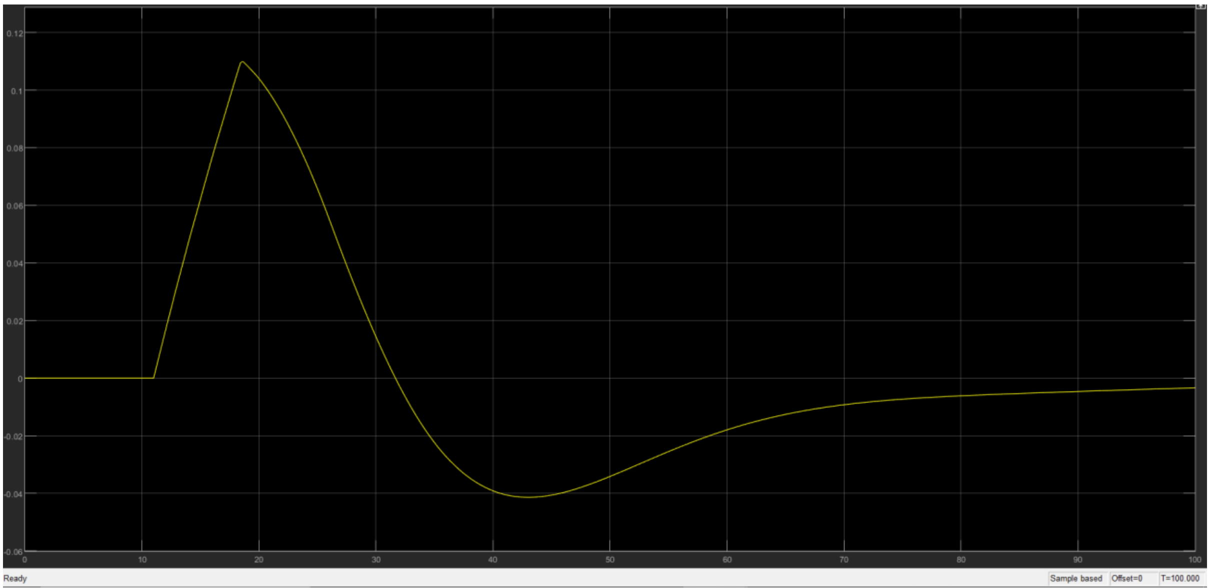

Figure 4: Response for combined efforts of feedforward and PID controller

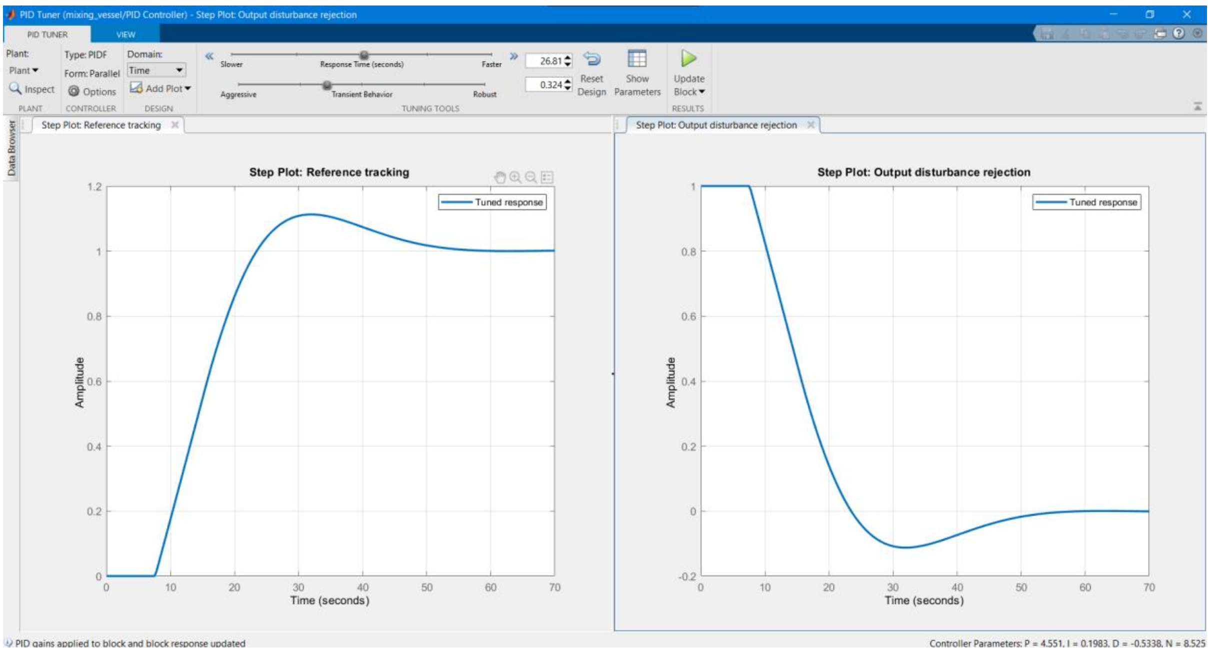

Figure 5: Tuning of the PID Controller (It linearizes the closed loop, and then we manually tune it on the basis of the responses/settling time requirements)

```
model = 'mixing_vessel';
load_system(model);
out = sim(model);
y = out.simout.data;
t = out.tout;
iae = trapz(t,abs(y));
```

IAE with FF controller alone was obtained to be 3.5326

IAE with FF + PID controller was obtained to be 2.3819

As expected, we see an improvement when we use a PID controller in addition.

Part c) MPC

- Firstly, I will obtain the step response models of the process and disturbance using the corresponding transfer functions we have.
- Given the time delay and time constants, I will choose a sampling interval of about 2.5 minutes.
- In this way I can obtain step response model length as about 5*25/2.5 = 50.
- Given this n, and multiple delays involved I would want to have larger prediction horizon, **p = 25**. Roughly half of what we have for n. (Note that obviously if we go for p > n, the system might exhibit instability)
- It is always safe to have the control horizon to be smaller than the predictive horizon. We can probably have **m = 10-15.** And tune as per the response we get.
- Higher m is more aggressive but we need more computational power (because we need to optimize more variables).
- Input constraints can be decided based on the expected disturbance inputs that might occur. Let's say if 0.5 is the maximum expected disturbance (this causes a change of 0.25 in output) we can constrain the valve to have absolute value of input moves within 0.25/0.4 = 0.625 psig.