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CH18B020

# CH3050 PDC ASSIGNMENT-4

(1) FOPTD model : $G(s) = \dfrac{K_p e^{-Ds}}{(\tau_p s+1)}$ —— (1)

A step change of $\mu$ has been introduced

$\Rightarrow Y(s) = G(s)U(s) = \dfrac{K_p e^{-Ds} \mu}{(\tau_p s+1)(s)}$

$= \mu K_p e^{-Ds}\left(\dfrac{1}{s} - \dfrac{\tau_p}{\tau_p s+1}\right)$ —— (2)

Taking Inverse Laplace transform on both sides

$$y(t) = \begin{cases} \mu K_p \left(1 - e^{-(t-D)/\tau_p}\right) & t \geq D \\ 0 & t < D \end{cases}$$ —— (3)

Note that the values of $\mu$ & $y$ are deviation variables wrt the initial steady state values.

Step change given at 9:05 AM

First non-zero instant : 9:09 AM.

$\therefore$ Delay = 4 minutes.

Given input = 540 L/min

$\Rightarrow \mu = 540 - 500 = 40$ L/min

~~Out~~ Steadystate temperature $= 55.7^\circ C$

$\Rightarrow y = 55.7 - 50 = 5.7^\circ C$

Let us assume that ~~at~~ at settling time, response is 95% of the true steadystate response

$t_{settle} = 34 - 5 = 29$ minutes

Substituting the above values in the expression obtained (eqn 3)

$$0.95\, M K_p = M K_p \left(1 - e^{-\left(\frac{t-D}{\tau_p}\right)}\right)$$

$$\Rightarrow \frac{t-D}{\tau_p} = \ln \frac{1}{0.05} \Rightarrow \frac{29-4}{\tau_p} = \ln\left(\frac{1}{0.05}\right)$$

$$\Rightarrow \tau_p = \frac{25}{-\ln 0.05} = 8.3452 \text{ minutes}$$

Also $y_{steadystate} = M K_p$

$$\Rightarrow K_p = \frac{5.7}{40} = 0.1425$$

$$\therefore \boxed{G_{approx} = \frac{0.1425}{8.34528+1} e^{-4s}}$$

Since there were no maxima/minima, it can't be underdamped systems; However it can be overdamped 2nd order or higher order system. If operator had noted the rise time we can do a SOPTD approximation.

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And now, since the operator knows the gain, the person can note down time at which 35% & 85% of steady state value is reached, so that we can use Krishnaswamy and Sundaresan's method to get an approximate transfer function

(2) a) Krishnaswamy and Sundaresan's Method.

$t_1$ : time at which 35.3% of steady state output is reached ~~read at~~

$= 27$ units

$t_2$ : time at which 85.3% of steady state output is reached

$= 65.5$ units

From these values,

$$D = 1.3 t_1 - 0.29 t_2$$

$$= 16.105 \text{ units}$$

$$\tau = 0.67 (t_2 - t_1)$$

$$= 25.795 \text{ units}$$

$K_p = 1$ (identified from the steady state value)

$$\therefore G_{approx} = \frac{K_p e^{-Ds}}{\tau s+1}$$

$$\Rightarrow \boxed{G_{approx} = \frac{e^{-16.105s}}{(25.795s+1)}}$$

b) Skogestad's half rule Method.

i) FOPTD

$$G_{actual} = \frac{(Zs+1)e^{-Ds}}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)(\tau_4 s+1)}$$

$$\hat{\tau} = \tau_1 + \frac{\tau_2}{2} \; ; \; \hat{D} = D + \frac{\tau_2}{2} + \tau_3 + \tau_4 - Z$$

$K_p = 1$

$$\therefore G_{approx} = \cancel{\frac{}{27.5s+1}} \quad \boxed{\frac{e^{-13s}}{(27.5s+1)}}$$

ii) SOPTD

$$\hat{\tau}_1 = \tau_1 \, , \; \hat{\tau}_2 = \tau_2 + \frac{\tau_3}{2} \; ; \; K_p = 1$$

$$D = \frac{\tau_3}{2} + \tau_4 + D - Z$$

$$G_{approx} = \frac{e^{-\left(\frac{4}{2}+0.5+3-2\right)s}}{(20s+1)\left((15+2)s+1\right)}$$

$$= \boxed{\frac{e^{-3.5s}}{(20s+1)(17s+1)}}$$

c) The magnitude, phase data (bode plots) was generated using MATLAB.

Let Gapprox = $\dfrac{K_p e^{-Ds}}{(\tau_1 s+1)(\tau_2 s+1)}$

First using the magnitude data least squares was performed such that $\left(\|G_{approx}\| - \text{actual Magnitude}\right)$ is minimised in a least square sense.

$$|G_{approx}(j\omega)| = \frac{K_p}{\sqrt{((\tau_1\omega)^2+1)((\tau_2\omega)^2+1)}} \quad \left(\because |e^{-Dj\omega}| = 1\right)$$

$K_p = 1.0003$, $\tau_1 = 17.911$, $\tau_2 = 17.9335$ was found.

Next a least squares fit was done for $\cos(\phi)$

$\phi_{actual}$ is from data; $\phi_{approx} = \phi(G(j\omega))$

$$= \text{phase}\left(\frac{K_p}{(\tau_1 j\omega+1)(\tau_2 j\omega+1)} - D\right)$$

where D should be estimated.

It was estimated as $D = 1.006$

$$\therefore \boxed{G_{approx} = \frac{e^{-1.01s}}{(17.911s+1)(17.9335+1)}}$$