INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering

CH3050 Process Dynamics & Control
Assignment 3

Due: Sunday, March 21, 2020

Exercise

1. A second-order process $G(s) = \dfrac{10}{s^2 + 7s + 10}$ is in negative feedback with what is known as a PI controller $G_c(s) = K_c + \dfrac{K_I}{s}$.
   (a) Determine the characteristic equation of the closed-loop (CL) system $G_{cl}(s) = Y(s)/R(s)$.
   (b) Identify the admissible regions of K_c and K_I that guarantee CL stability.
   (c) Is tracking of set-point guaranteed for any admissible values of K_c and K_I?
   (d) Demonstrate your findings in SIMULINK by simulating the CL system for a unit step change in set-point, for two different settings of K_c and K_I, one from the admissible and another from the non-admissible region. Report the chosen values and step responses in each case.

2. A process is given by the transfer function $G(s) = \dfrac{10(s-4)}{s^2 + 7s + 10} e^{-3s}$. For this process,
   (a) Compute the impulse and step response of the system. Sketch these responses by hand.
   (b) Determine the large-time response of the process to the input $u(t) = 2\sin(5t) + 3\cos(0.1t)$
   (c) Construct the Bode plot by hand. Show the working details neatly.
   (d) Determine the LTI system that has the same magnitude at all ω but has the lowest phase.
   (e) Verify your answers to all parts using MATLAB.

3. The dynamic behavior of the liquid level in a leg of a manometer tube, responding to a change in pressure, is given by

$$\frac{d^2h'}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh'}{dt} + \frac{3}{2}\frac{g}{L}h' = \frac{3}{4\rho L}p'(t)$$

   where $h(t)$ is the level of fluid measured with respect to the initial steady-state value, $p(t)$ is the pressure change, and R, L, g, ρ, and μ are constants.
   (a) Rearrange this equation into standard gain-time constant form and find expressions for K, τ, ζ in terms of the physical constants.
   (b) For what values of the physical constants does the manometer response oscillate?
   (c) How would you change the length L of the manometer leg so as to make the response more oscillatory, or less? Repeat the analysis for an increase in μ (viscosity).

4. The transfer function that relates the change in blood pressure y to change in u the infusion rate of a drug (sodium nitroprusside) is given by

$$G_p(s) = \frac{Ke^{-D_1 s}(1 + \alpha e^{-D_2 s})}{\tau s + 1}$$

The two time delays result from the blood recirculation that occurs in the body, and α is the recirculation coefficient. The following parameter values are available:

$$K = -1.2 \frac{\text{mm Hg}}{\text{ml/h}},$$
$$\alpha = 0.4, D_1 = 30 \text{ s}, D_2 = 45 \text{ s}, \text{and } \tau = 40 \text{ s}$$

Use Simulink to construct the block diagram and simulate the blood pressure response to a unit step change ($u = 1$) in sodium nitroprusside infusion rate. Is it similar to other responses discusses in the class?