INDIAN INSTITUTE OF TECHNOLOGY MADRAS

Department of Chemical Engineering

CH3050: Process Dynamics and Control (Jan - May 2021)
Assignment-3 Solutions

Marks distribution

	Question 1	Question 2	Question 3	Question 4
(a)	5	10	12	20
(b)	5	5	8	-
(c)	5	10	5	-
(d)	5	5	-	-
(e)	-	5	-	-

Question 1

A second-order process $G(s) = \frac{10}{s^2+7s+10}$ is in negative feedback with what is known as a PI controller $G_c(s) = K_c + \frac{K_I}{s}$

(a) Determine the characteristic equation of the closed-loop (CL) system $G_{cl}(s) = Y(s)/R(s)$.

(b) Identify the admissible regions of K_c and K_I that guarantee CL stability.

(c) Is tracking of set-point guaranteed for any admissible values of K_c and K_I?

(d) Demonstrate your finding in SIMULINK by simulating the CL system for a unit step change in set-point, for two different setting of K_c and K_I, one from the admissible and another from the non-admissible region. Report the chosen values and step responses in each case.

Answer

$$G(s) = \frac{10}{s^2 + 7s + 10} \tag{1}$$

$$G_c(s) = K_c + \frac{K_I}{s} \tag{2}$$

(a)

$$G_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{G_c G(s)}{1 + G_c G(s)} \tag{3}$$

$$G_{cl}(s) = \frac{10(sK_c + K_I)}{s^3 + 7s^2 + 10s(1 + K_c) + 10K_I} \tag{4}$$

The characteristic equation is

$$s^3 + 7s^2 + 10s(1 + K_c) + 10K_I = 0 \tag{5}$$

(b)

s^3	1	$(10 + K_c)$
s^2	7	$10K_I$
s	$\frac{70+70K_c-10K_I}{7}$	0
s^0	$10K_I$	

All roots of the characteristic equation should be strictly be negative for an LTI system to be stable. In addition, no sign changes and no missing coefficients imply that roots are in LHP.

Therefore, admissible regions of K_c and K_I that guarantee closed loop stability can be obtained from RH test.

$$\frac{70 + 70K_c - 10K_I}{7} > 0 \tag{6}$$

$$K_I > 0 \tag{7}$$

$$K_c > \frac{K_I}{7} - 1 \tag{8}$$

(c)

The purpose of set-point tracking is to equalize set-point and process variable, and is achieved when the gain of the closed-loop system is 1.

$$Gain(G_{cl}(s)) = \lim_{s \to 0} G_{cl}(s) = 1 \tag{9}$$

Hence, set-point tracking is guaranteed for all the values of K_c and K_I for which the closed loop system is stable.

(d)

The CL system for a unit step change in set-point modelled is given below in Figure 1:

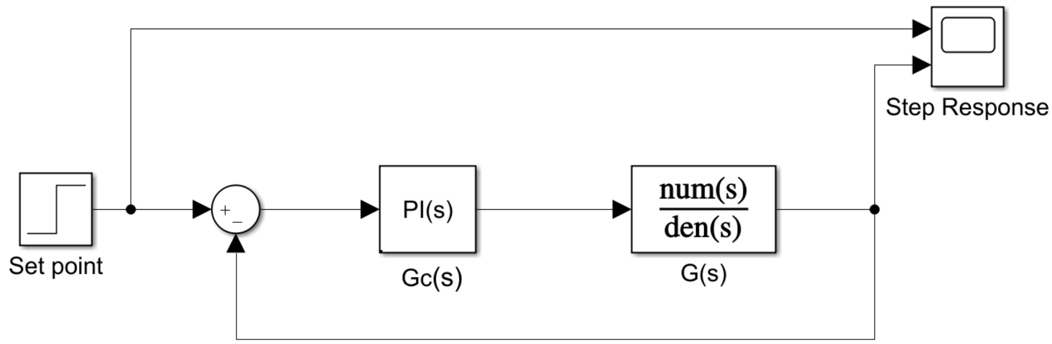


Figure 1: Simulink block diagram for the second order process

(1) Admissible region. $K_c = 7$ and $K_I = 14$. Set-point is achieved.

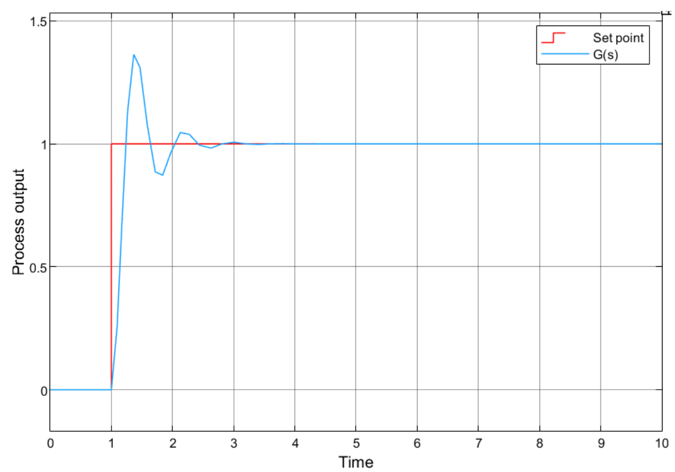


Figure 2: Response to step change for values of K_c and K_I from the admissible region

(2) Non-admissible region. $K_c = 1$ and $K_I = 21$. The system becomes more oscillatory.

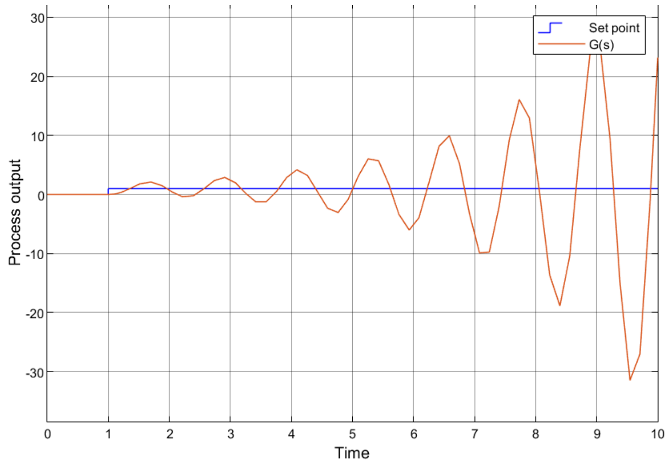


Figure 3: Response to step change for values of K_c and K_I from the non-admissible region

Question 2

A process is given by the transfer function $G(s) = \frac{10(s-4)e^{-3s}}{s^2+7s+10}$. For this process,

(a) Compute the impulse and step response of the system.Sketch these responses by hand.

(b) Determine the large-time response of the process to the input $u(t) = 2\sin(5t) + 3\cos(0.1t)$.

(c) Construct the Bode plot by hand.Show the working details neatly.

(d) Determine the LTI system that has the same magnitude at all ω but has the lowest phase.

(e) Verify your answers to all parts using MATLAB.

Answer

$$G(s) = \frac{10(s-4)e^{-3s}}{s^2+7s+10} \tag{10}$$

(a)

(1) Impulse Response:

$$U(s) = 1 \tag{11}$$

$$Y(s) = G(s)U(s) \tag{12}$$

$$Y(s) = \frac{10(s-4)e^{-3s}}{s^2+7s+10} \tag{13}$$

$$Y(s) = 10e^{-3s}(\frac{-2}{s+2} + \frac{3}{s+5} \tag{14}$$

$$y(t) = L^{-1}\{\frac{-20e^{-3s}}{s+2} + \frac{30e^{-3s}}{s+5}\} \tag{15}$$

$$y(t) = -20e^{-2(t-3)} + 30e^{-5(t-3)} \; t \geq 3 \tag{16}$$

(2) Step Response

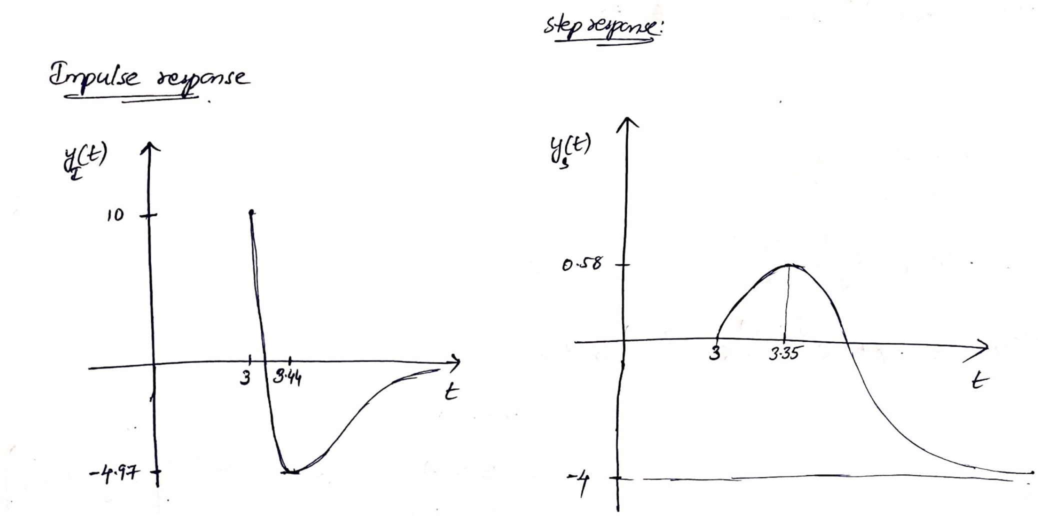
$$U(s) = \frac{1}{s} \tag{17}$$

$$Y(s) = G(s)U(s) \tag{18}$$

$$Y(s) = \frac{10(s-4)e^{-3s}}{(s^2+7s+10)s} \tag{19}$$

$$Y(s) = e^{-3s}(\frac{-4}{s} + \frac{10}{s+2} - \frac{6}{s+5} \tag{20}$$

$$y(t) = -4 + 10e^{-2(t-3)} - 6e^{-5(t-3)} \; t \geq 3 \tag{21}$$



(a) Impulse response

(b) Step response

Figure 4: Responses of the system

(b)

$$u(t) = 2\sin(5t) + 3\cos(0.1t) \tag{22}$$

$$G(j\omega) = \frac{10(j\omega - 4)}{-\omega^2 + 7j\omega + 10} e^{-3j\omega} \tag{23}$$

$$G(j\omega) = (10(j\omega - 4))(e^{-3j\omega})(\frac{1}{j\omega + 2})(\frac{1}{j\omega + 5}) \tag{24}$$

$$|G(j\omega)| = |(10(j\omega - 4))| * |(e^{-3j\omega})| * |(\frac{1}{j\omega + 2})| * |(\frac{1}{j\omega + 5})| \tag{25}$$

$$\frac{B}{A}(\omega) = |G(j\omega)| = 10\sqrt{(16 + \omega^2)} * 1 * \sqrt{\frac{1}{4 + \omega^2}} * \sqrt{\frac{1}{25 + \omega^2}} \tag{26}$$

$$\phi(\omega) = \tan^{-1}(\frac{\omega}{-4}) - 3\omega - \tan^{-1}(\frac{\omega}{2}) - \tan^{-1}(\frac{\omega}{5}) \tag{27}$$

$$\frac{B}{A}(\omega = 5) = 1.6815 \tag{28}$$

$$\frac{B}{A}(\omega = 0.1) = 3.995 \tag{29}$$

$$\phi(\omega = 5) = -17.8718 \tag{30}$$

$$\phi(\omega = 0.1) = -0.395 \tag{31}$$

The response of an LTI system to sinusoidal input is also sinusoidal with same frequency.

$$y(t) = 2 * 1.6815 \sin(4t - 17.8178) + 3 * 3.995 \sin(0.1t - 0.395) \tag{32}$$

(c)

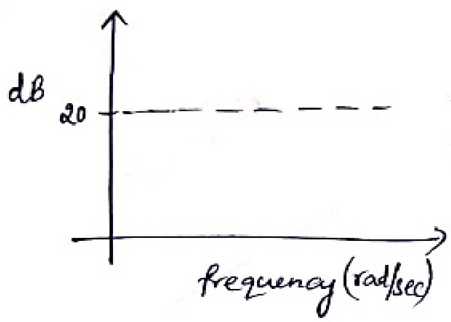
c) i) $G_0 = 10$

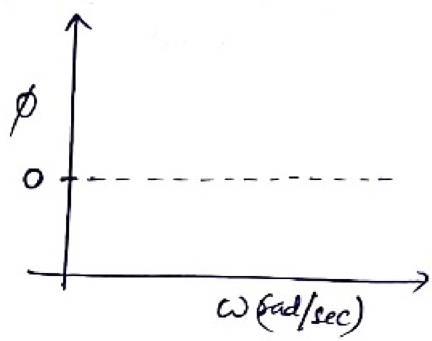
$G_0(j\omega) = 10$

$|G_0(j\omega)| = 10$

$dB = 20 \log_{10} 10 = 20 \ \forall \omega$

$\phi = 0 \ \forall \omega$





ii) $G_1 = s - 4$

$G_1(j\omega) = j\omega - 4$

$|G_1(j\omega)| = \sqrt{\omega^2 + 16}$

$\therefore \ dB = 20 \log_{10} \sqrt{\omega^2+16} = 10 \log_{10} (\omega^2 + 16)$

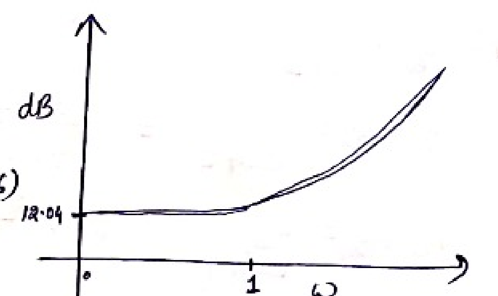
$\phi = -\tan^{-1}\left(\frac{\omega}{4}\right)$

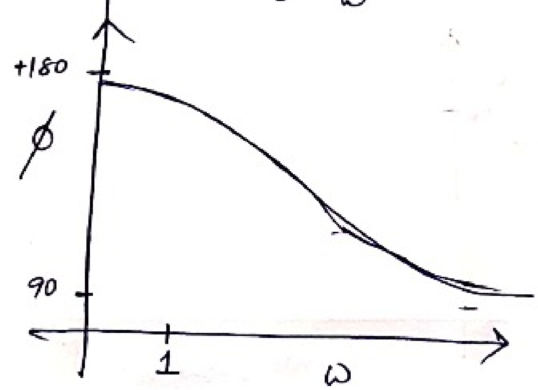
@$\omega = 0$, $dB = 12.04$,

as ω increases initially $(0<\omega<1)$, slow increase in dB as well,

But $(\omega > 1)$, there is square effect

Therefore, dB increases at faster rate





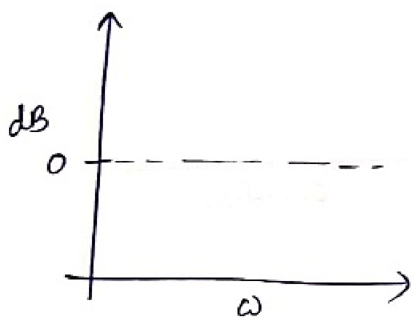
iii) $G_2 = e^{-3s}$

$$G_2(j\omega) = e^{-3j\omega} = \cos(-3\omega) + j\sin(-3\omega)$$
$$= \cos 3\omega - j\sin(3\omega)$$

$$|G_2(j\omega)| = 1$$

$\therefore$ $dB = 0 \;\forall\, \omega,$

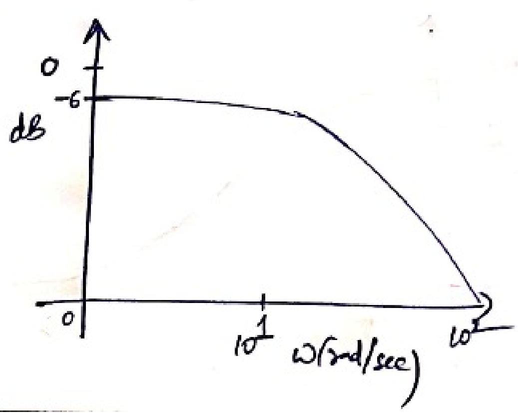
$$\phi = \tan^{-1}\left(\frac{-\sin 3\omega}{\cos 3\omega}\right) = -\tan^{-1}\tan(3\omega) = -3\omega$$

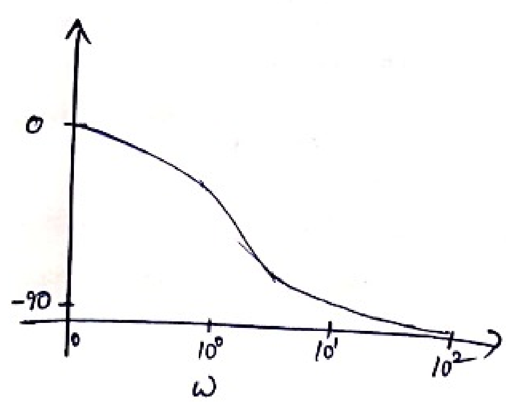


iv) $G_3 = \frac{1}{s+2}$

$$G_3(j\omega) = \frac{1}{j\omega+2} = \frac{2-j\omega}{4+\omega^2}$$

$$|G_3(j\omega)| = \frac{1}{\sqrt{\omega^2+4}}, \quad dB = -10\log(\omega^2+4), \quad \phi = \tan^{-1}\left(\frac{-\omega}{2}\right)$$

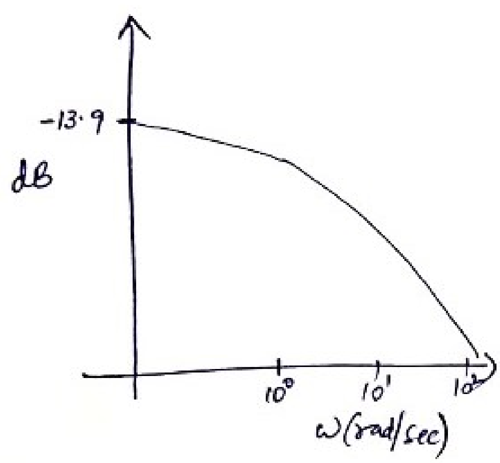


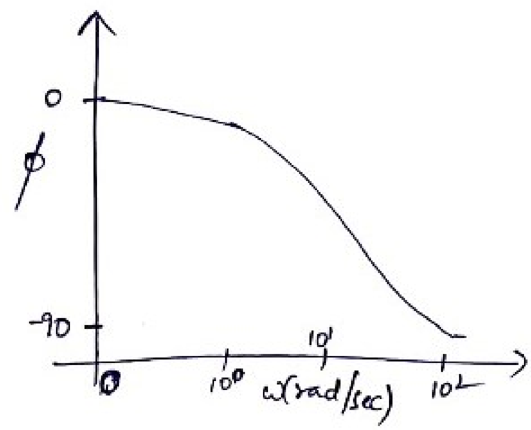


v) $G_4 = \frac{1}{s+5}$

$$G_4(j\omega) = \frac{1}{j\omega+5} = \frac{5-j\omega}{25+\omega^2}$$

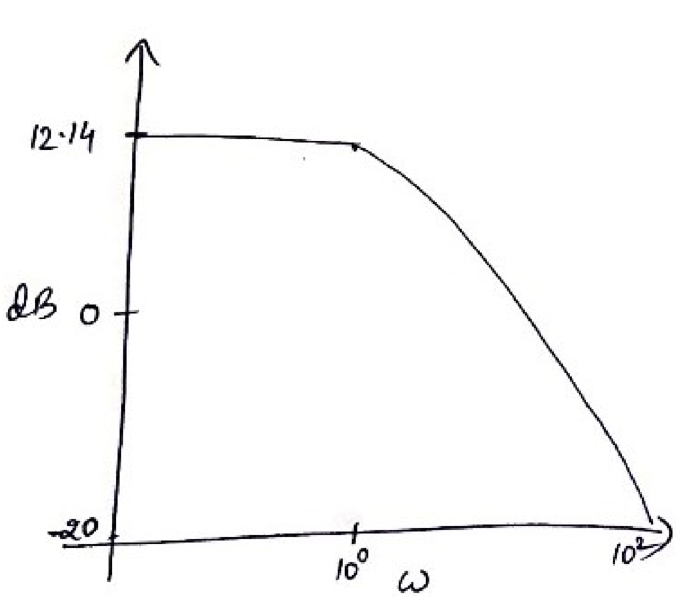
$$\therefore |G_4(j\omega)| = \frac{1}{\sqrt{\omega^2+25}}, \quad dB = -10\log(\omega^2+25), \quad \phi = \tan^{-1}\left(-\frac{\omega}{5}\right)$$

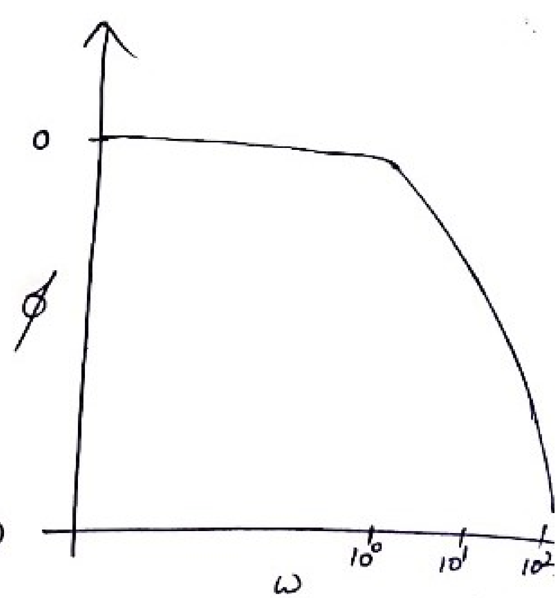




vi) Finally,

$$dB\Big|_{overall} = \sum_{i=0}^{4} dB_i$$



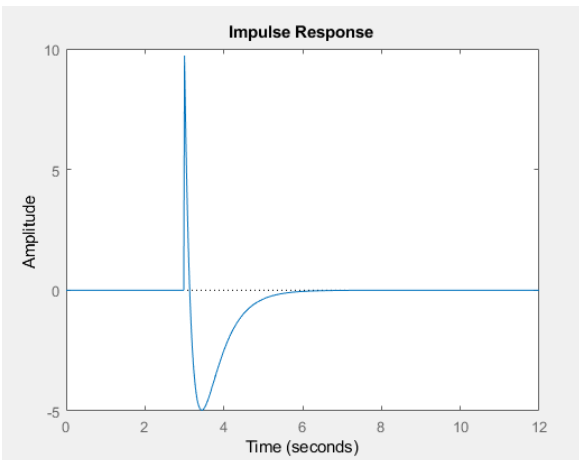


(d)

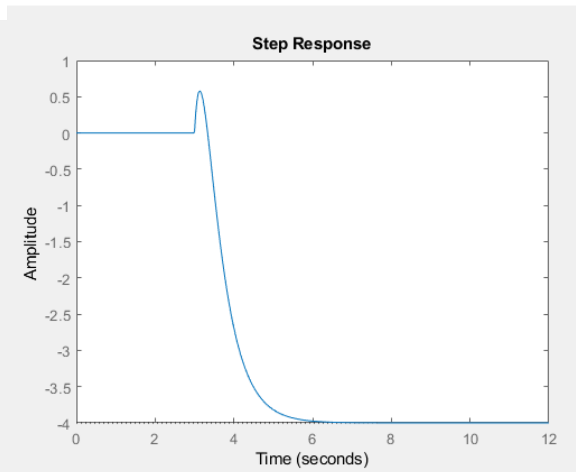
The LTI system that has same magnitude at all ω as that of of given system but with lowest phase is the same system with negative zeroes and without delay, i.e,

$$G(s) = \frac{10(s-4)}{s^2+7s+10} \tag{33}$$

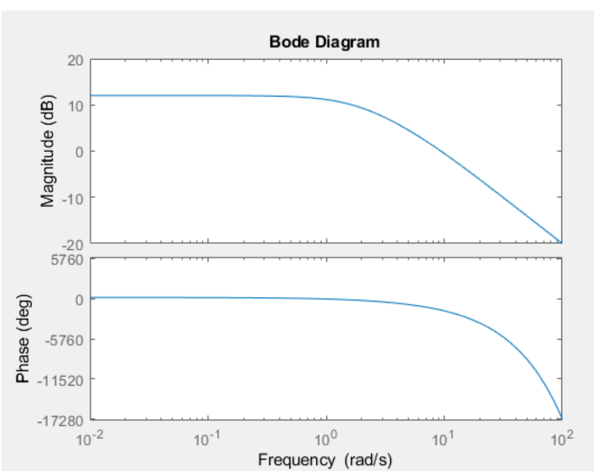
(e) Matlab Verification



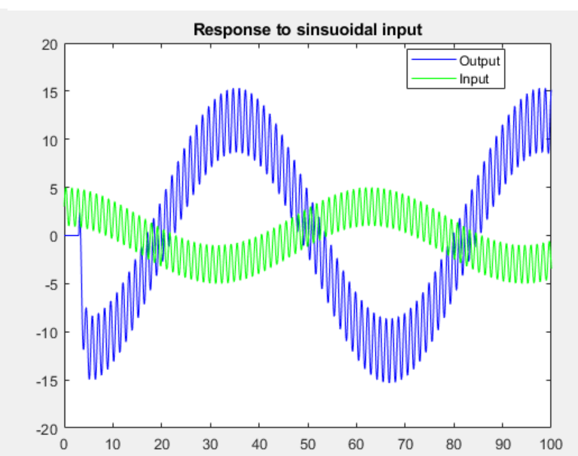
(a)



(b)



(c)



(d)

Question 3

The dynamic behaviour of the liquid level in a leg of a manometer tube,responding to a change in pressure, is given by

$$\frac{d^2h}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh}{dt} + \frac{3g}{2L}h = \frac{3}{4\rho L}p(t)$$

where h(t) is the liquid level of fluid measured with respect to the initial steady-state value, p(t) is the pressure change, and R,L,g,ρ, and μ are constant.

(a) Rearrange this equation into standard gain-time constant form and find the expression for K,τ,ζ in terms of the physical constants.

(b) For what values of the physical constants does the manometer response oscillate?

(c) How would you change the length L of the manometer leg so as to make the response more oscillatory, or less? Repeat the analysis for an increase in μ(viscosity).

Answer

(a)

$$\frac{d^2h}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh}{dt} + \frac{3g}{2L}h = \frac{3}{4\rho L}p(t) \tag{34}$$

Taking Laplace Transform,we get,

$$s^2H(s) + \frac{6\mu}{R^2\rho}sH(s) + \frac{3g}{2L}H(s) = \frac{3}{4\rho L}P(s) \tag{35}$$

$$\frac{H(s)}{P(s)} = \frac{\frac{3}{4\rho L}}{s^2 + \frac{6\mu}{R^2\rho}s + \frac{3g}{2L}} \tag{36}$$

Rearranging the expression, we get,

$$\frac{H(s)}{P(s)} = \frac{\frac{1}{2\rho g}}{\frac{2L}{3}s^2 + \frac{4\mu L}{R^2\rho g}s + 1} \tag{37}$$

Comparing the above expression with second order transfer function,

$$G(s) = \frac{K}{\tau^2 s^2 + 2\tau\zeta s + 1} \tag{38}$$

$$K = \frac{1}{2\rho g} \tag{39}$$

$$\tau = \sqrt{\frac{2L}{3g}} \tag{40}$$

$$\zeta = \frac{\mu}{\rho R^2}\sqrt{\frac{6L}{g}} \tag{41}$$

(b)

For the oscillation in the output, the system must be underdamped, that is,

$$0 < \zeta < 1 \tag{42}$$

$$0 < \frac{\mu}{\rho R^2}\sqrt{\frac{6L}{g}} < 1 \tag{43}$$

(c)

ζ gives the influence on oscillatory system that has the effect of reducing the oscillations. ζ close to 1 implies system dampens the oscillations and ζ close to 0 implies high oscillations. Thus increase in manometer leg (L) or viscosity (μ) restricts the oscillations and decrease in the same, results in more oscillatory response.

Question 4

The transfer function that relates the change in blood pressure y to change in u the infusion rate of drug(sodium nitroprusside) is given by

$$G_p(s) = \frac{Ke^{D_1 s}(1 + \alpha e^{-D_2 s})}{\tau s + 1}$$

The two time delays result from the blood recirculation that occurs in the body, and α is the recirculation coefficients. The following parameter values are available:

$$K = -1.2\frac{\text{mm HG}}{\text{ml/h}},$$

$$\alpha = 0.4, D_1 = 30 \text{ s}, D_2 = 45 \text{ s and } \tau = 40 \text{ s}$$

Use simulink to construct the block diagram and simulate the blood pressure response to a unit step change (u = 1)in sodium nitroprusside infusion rate. Is it similar to other responses discusses in the class?

Answer

$$G_p(s) = \frac{Ke^{D_1 s}(1 + \alpha e^{-D_2 s})}{\tau s + 1} \tag{44}$$

$$G_p(s) = \frac{Ke^{D_1 s}}{\tau s + 1} + \frac{K\alpha e^{-(D_1+D_2)s}}{\tau s + 1} \tag{45}$$

$$G_p(s) = G_1(s) + G_2(s) \tag{46}$$

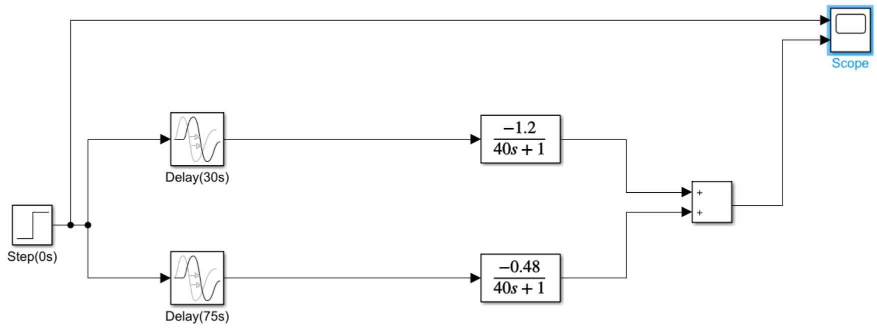


Figure 5: Simulink Block Diagram

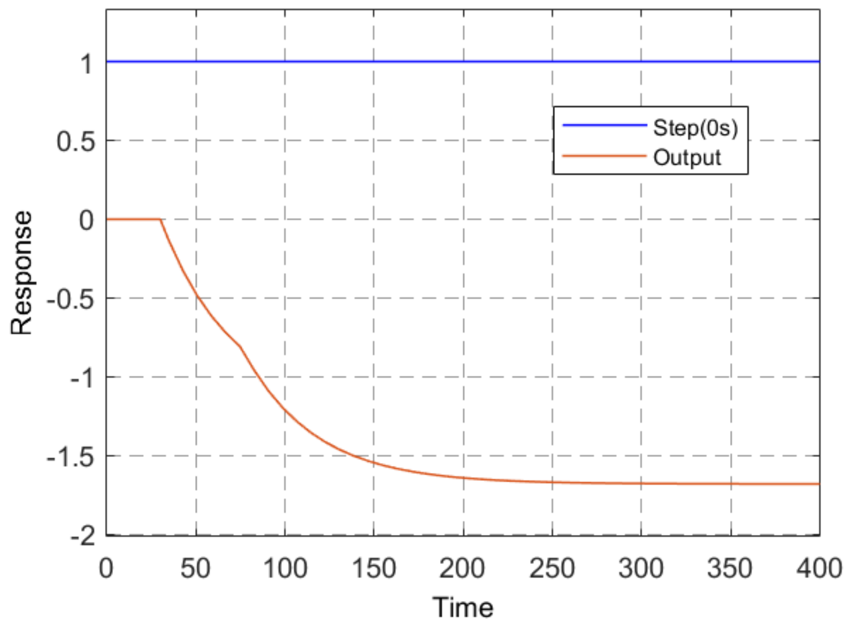


Figure 6: Blood Pressure Response to Unit Step change

In Figure 6, it is observed that there is discontinuity at two points, one at $t = 30s$ and other at $t = 75s$. Usually, first order system have a delay of order one, but here due to summing up of two parallel transfer function, the final transfer function is first order with two time delays.