② a) $\frac{10(s-4)e^{-3s}}{(s+5)(s+2)} = G(s)$

(i) Impulse Response.

$u(t) = \delta(t) \Rightarrow U(s) = 1$

$\therefore Y(s) = G(s)U(s) = \frac{10(s-4)}{(s+5)(s+2)} e^{-3s}$

$= 10e^{-3s}\left(\frac{A_1}{s+5} + \frac{A_2}{s+2}\right)$

~~Subst.~~ Solving for A_1 & A_2, we have,

$\left[(s-4) = A_1(s+2) + A_2(s+5)\right]$

$A_1 = 3 \quad A_2 = -2$

$\Rightarrow Y(s) = \frac{30e^{-3s}}{s+5} - \frac{20e^{-3s}}{s+2}$

Take inverse Laplace transform. & by the linearity of the operator we just take the individual inverses & add them up,

$\Rightarrow y(t) = \begin{cases} 0 & t < 3 \\ 30e^{-5(t-3)} - 20e^{-2(t-3)} & t \geq 3 \end{cases}$

$= \{$

→ Initially $30e^{-5t} > 20e^{-2t}$ by the virtue of having a larger coeff.

→ Then ~~the~~ $30e^{-5t}$ keeps ~~increasing~~ decreasing fn @ a faster rate, so ∴ the value starts to fall

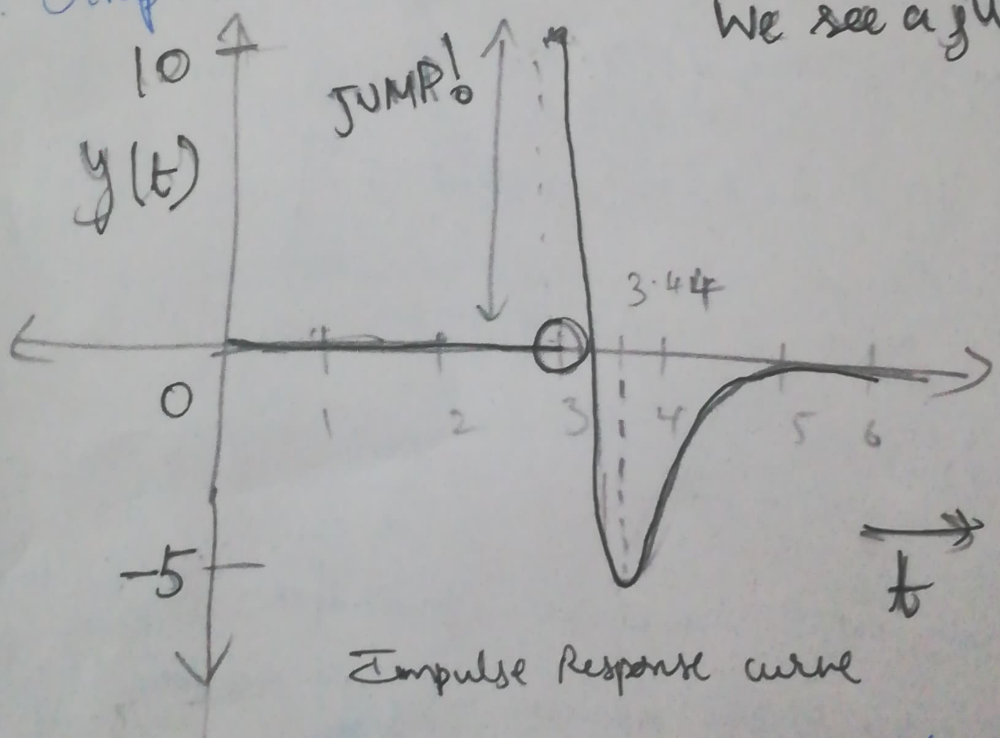
→ When derivative $= 0$ ($\Rightarrow e^{3(t-3)} = \frac{150}{40}$ ~~of~~ ~~...~~

We hit a minima $\Rightarrow 3(t-3) = \frac{1}{3} \ln \frac{150}{40}$ ~~4~~

→ as $t \to \infty$, $y \to 0$ $\Rightarrow t \simeq 3.445$)

∴ Graph looks like

We see a jump



Impulse Response curve

there is a jump because of presence of a zero

(ii) <u>Step response</u>

We can use $u(t) = 1 \text{ } \forall t \to U(s) = \frac{1}{s}$

& then solve for $y(t)$. ~~However,~~

However, since we have already found out the impulse response, we can get the step response by integrating the impulse respond

$$\Rightarrow y_{step}(t) = 0 \quad t < 3$$

For $t \geq 3$,

$$y_{step}(t) = \int_3^t \left(30e^{-5(t-3)} - 20e^{-2(t-3)}\right) dt$$

$$= \frac{30}{5}\left(1 - e^{-5(t-3)}\right) + -\frac{20}{2}\left(1 - e^{-2(t-3)}\right)$$

$$= -4 + 10e^{-2(t-3)} - 6e^{-5(t-3)} \quad \text{for } t \geq 3$$

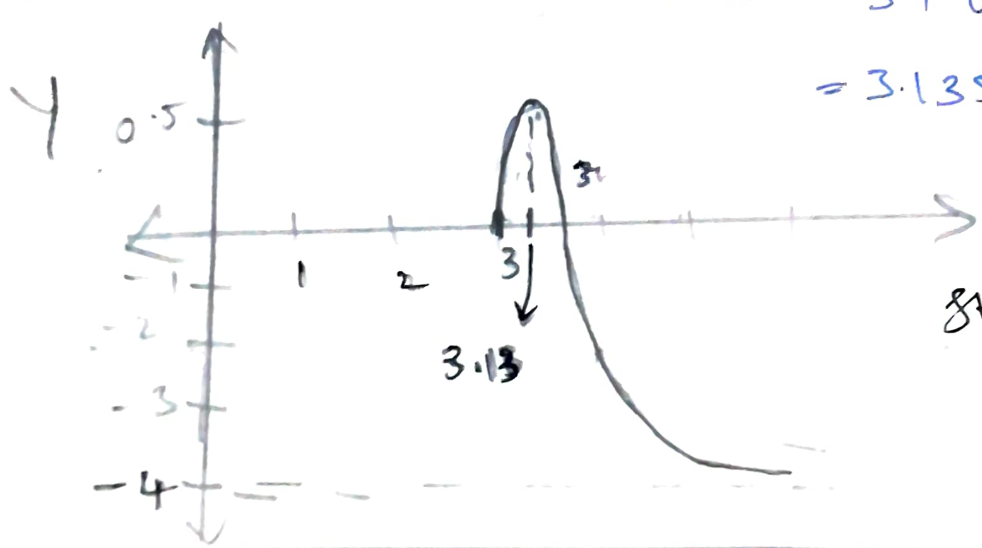
$$\Rightarrow y_{step}(t) = \begin{cases} 0 \\ -4 + 10e^{-2(t-3)} - 6e^{-5(t-3)} & \forall t \geq 3 \end{cases}$$

$t = 3$, $y_{step} = 0$ $\quad y' = 0$

$$\Rightarrow 10(-2)e^{-2(t-3)} + (6)(5)e^{-5(t-3)} = 0$$

$t \to \infty$, $y_{step} = -4$ $\quad \Rightarrow t = 3 + 0.1351$

$= 3.1351\, s$ (maxima)



Step response curve

b) $$G(s) = \frac{10(s-4)}{s^2 - 17s + 10} e^{-3s}.$$

Consider that we give a sinusoidal input

$$u(t) = A \sin w_0 t \Rightarrow U(s) = \frac{A w_0}{s^2 + w_0^2}$$

$$\therefore Y(s) = \frac{10(s-4)e^{-3s}}{s^2 + 7s + 10} \frac{A w_0}{s^2 + w_0^2}$$

By splitting into partial fractions, we can get

$$= \left(\frac{C_1}{s-a} + \frac{C_2}{s-b} + \frac{C_3}{s - jw_0} + \frac{C_4}{s + jw_0} \right) e^{-3s}$$

where a, b are the poles

For now, let's ignore the delay & taking $\mathcal{L}^{-1}\{\ \}$

$$\Rightarrow y(t) = C_1 e^{+at} + C_2 e^{bt} + C_3 e^{+jw_0} + C_4 e^{-jw_0}$$

Since we have stable poles after a long time $(t \to \infty)$, $C_1 e^{at}$ & $C_2 e^{bt} \to 0$

Also to have a real output ("physically meaningful") $C_3 = C_4^*$

$$C_3 = \frac{A}{2j} G(jw_0)$$ (By method of partial fraction)

So here too, we have $y_{ss}(t) = 2\,\mathrm{Re}\left(C_3 e^{jw_0 t} \right)$

This expression is similar to the one we got in the derivation for first order system done in class.

So we conclude,

$\cancel{|G(j\omega_0}\ \frac{B}{A} = |G(j\omega_0)|$

where $B \to$ amplitude of output

$A \to$ Amplitude of input.

& ϕ (phase) $= \angle G(j\omega_0) = \angle G(j\omega_0)$

Input $u(t) = 2\sin(5t) + 3\cos(0.1t)$

By the linearity property we can simply add the outputs of the individual inputs to get the output of the total input.

$$G(j\omega_0) = \frac{10\ \cancel{(s-4)}\ (j\omega_0 - 4)\ e^{-3j\omega_0}}{(j\omega_0)^2 + 7(j\omega_0) + 10}$$

$$= \frac{10\ (j\omega_0 - 4)\ e^{-3j\omega_0}}{(j\omega_0 + 5)(j\omega_0 + 2)}$$

$$\Rightarrow |G(j\omega)| = \frac{10\sqrt{\cancel{(j\omega)^2}\ (\cancel{16} + \omega^2)} \times (1)}{\sqrt{(\omega^2+25)(\omega^2+4)}}$$

$$= \frac{10\sqrt{16+\omega^2}}{\sqrt{(\omega^2+25)(\omega^2+4)}}$$

$\angle G(j\omega) = \cancel{\tan^{-1}(\omega)}\ \tan^{-1}\left(\frac{-\omega}{-4}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right) - 3\omega$

(for a CN $a+jb$, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$)

$\Rightarrow \angle G(j\omega) = \tan^{-1}\left(-\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right) - 3\omega.$

Input ①: $\omega = 5 \quad A = 2$

$\Rightarrow \frac{B}{A} = |G(j\omega)| = 1.6815$

$\Rightarrow B = 1.6815 \times A = 2 \times 1.6815$

$= 3.3631$

$\phi = -17.87$

$\therefore y_1 = 3.363 \sin(5t - 17.87)$

Input ②: $A = 3,\ \omega = 0.1$

$\frac{B}{A} = |G(j\omega)| = 3.9955$

$\Rightarrow B = 11.9863.$

$\phi = -0.3949.$

$\Rightarrow y_2 = 11.9863 \cos(0.1t - 0.3949)$

$$\boxed{\begin{aligned} \therefore y_{ss}(t) &= y_1 + y_2 \\ &= 3.363 \sin(5t - 17.87) + 11.9863 \cos(0.1t - 0.3949) \end{aligned}}$$

Also note that I used the $\frac{B}{A}$ & ϕ expns derived for sine input for cosine also. this holds true because cosine($\cos\omega t = \sin(\omega t + \frac{\pi}{2})$; so we will get back the same expressions again (for AR & ϕ)

c) $dB = 20 \log_{10} (AR(\omega))$

$$= 20 \log_{10} \left(\frac{10 \sqrt{16+\omega^2}}{\sqrt{(\omega^2+25)(\omega^2+4)}} \right)$$

$$= 20 \left[\log_{10} 10 + \log_{10} \sqrt{16+\omega^2} - \log_{10}(\sqrt{\omega^2+25}) - \log(\sqrt{\omega^2+4}) \right]$$

We can take each of these subsystems indep. and & add their dB together

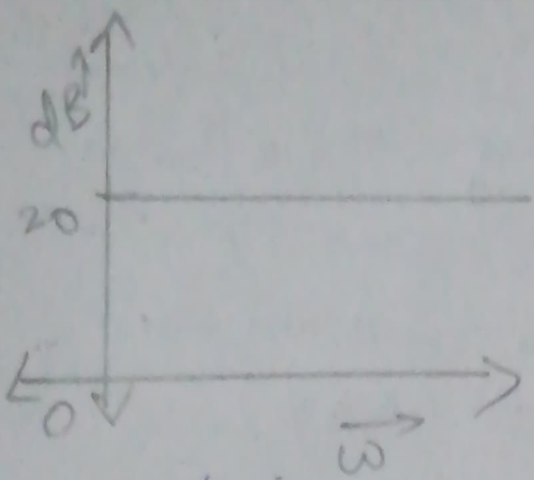
$dB = \sum_{\forall k} dB_k$ & $\phi = \sum_{\forall k} \phi_k$

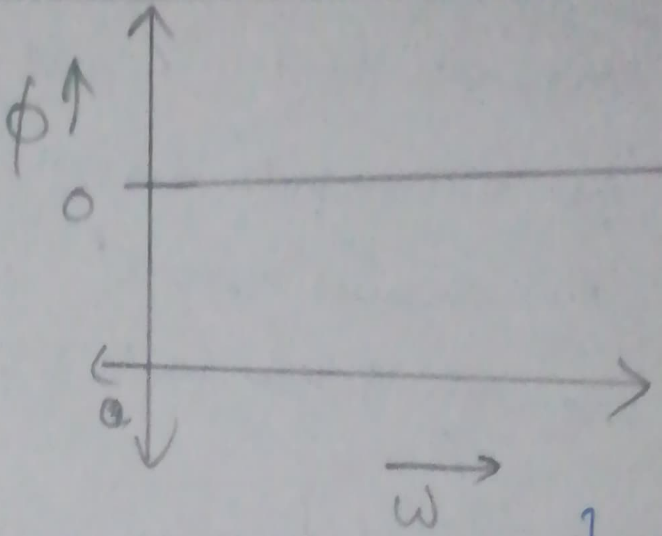
components :

(i) 10 (ii) $(j\omega - 4)$ (iii) $e^{-3j\omega}$ (iv) $\frac{1}{(j\omega+5)}$ (v) $\frac{1}{(j\omega+2)}$

i) 10 $dB = 20\log 10 = 20$

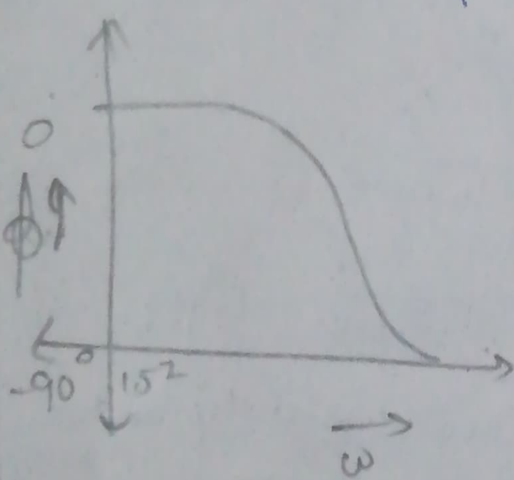
$\phi = 0$ (purely real)

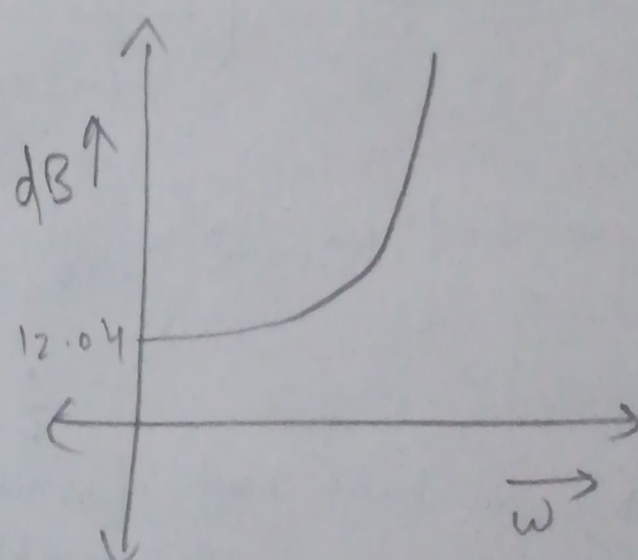




ii) $dB = 10 \log_{10}(16 + \omega^2)$ $\left[G_2(j\omega) = (j\omega - 4) \right]$

$\phi = \tan^{-1}\left(\frac{-\omega}{4}\right)$





(∵ x axis is in log scale,

$x = \log \omega$; $\omega \to \infty$, $x \to \infty$

$\omega \to 0^+$, $x \to -\infty$)

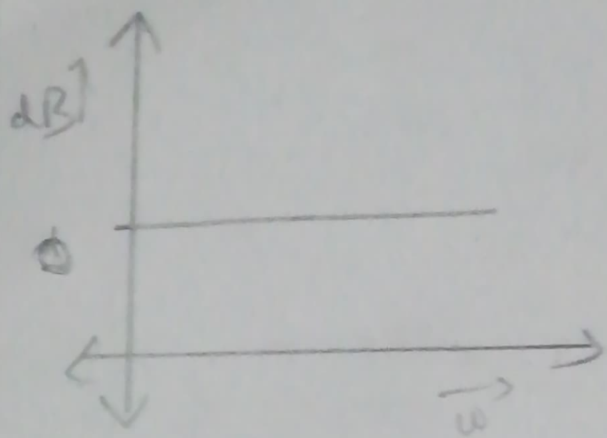
~~(looks simil~~

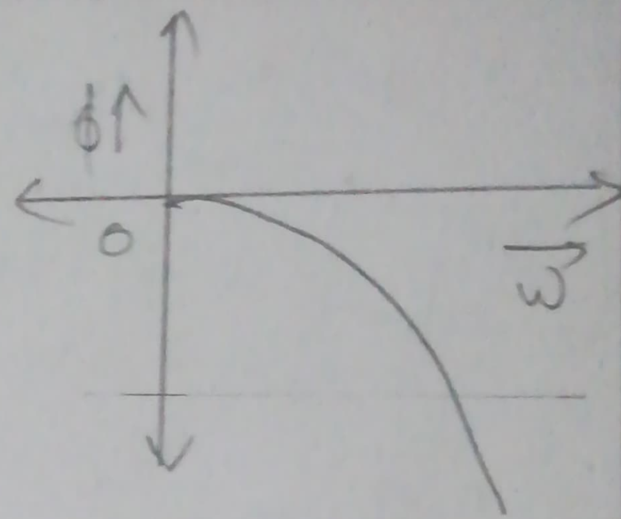
(looks similar to $\log x$)

iii) $G_3(j\omega) = e^{-3j\omega}$

$dB = 20 \log 1$ $= 0$ $\phi = -3j\omega$ -3ω → A linear variation with ω,

Although ϕ shows linear variation with ω, since Bode's plot is in semilog scale, the graph will look exponential.

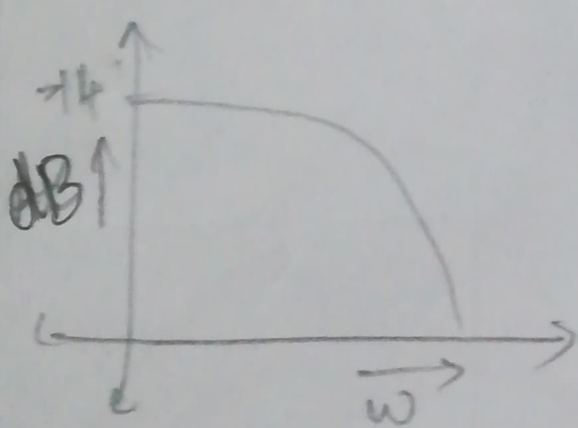


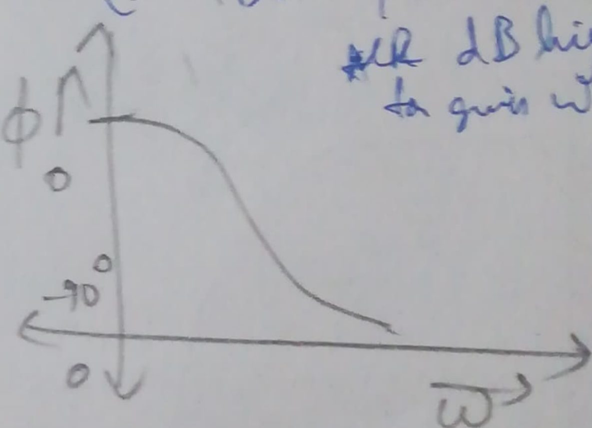


iv) $G(j\omega) = \dfrac{1}{(j\omega + 5)}$

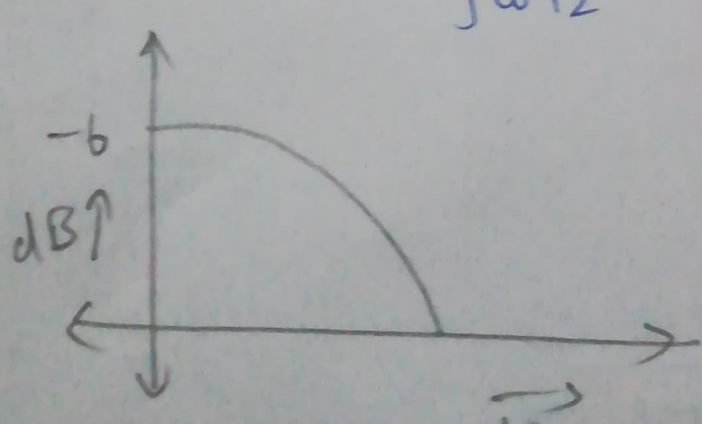
$\Rightarrow dB = -10 \log(\omega^2 + 25)$ & $\phi = \tan^{-1}\left(\dfrac{-\omega}{5}\right)$

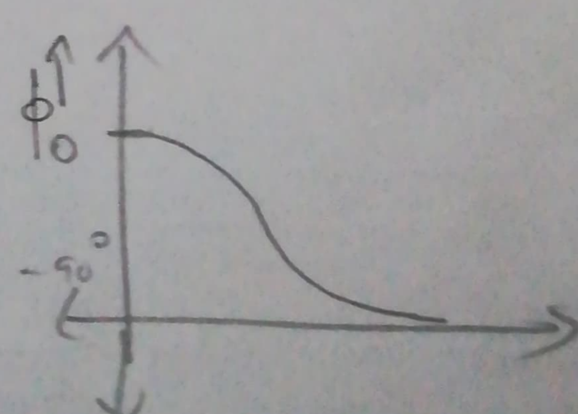
Plots will look similar to (iii) (But ϕ lower & dB higher for given ω)





v) $G(j\omega) = \dfrac{1}{j\omega + 2}$





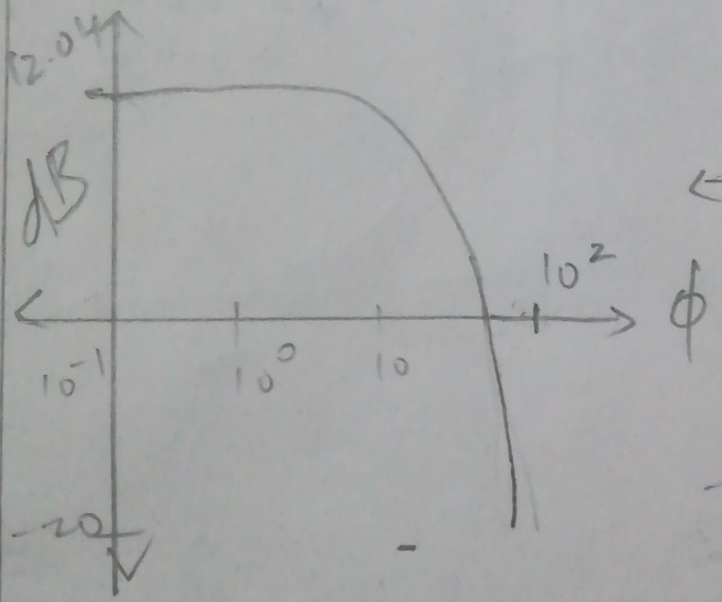
$\therefore \; dB = \sum_{\forall k} dB_k \quad ; \quad \phi = \sum_{\forall k} \phi_k$

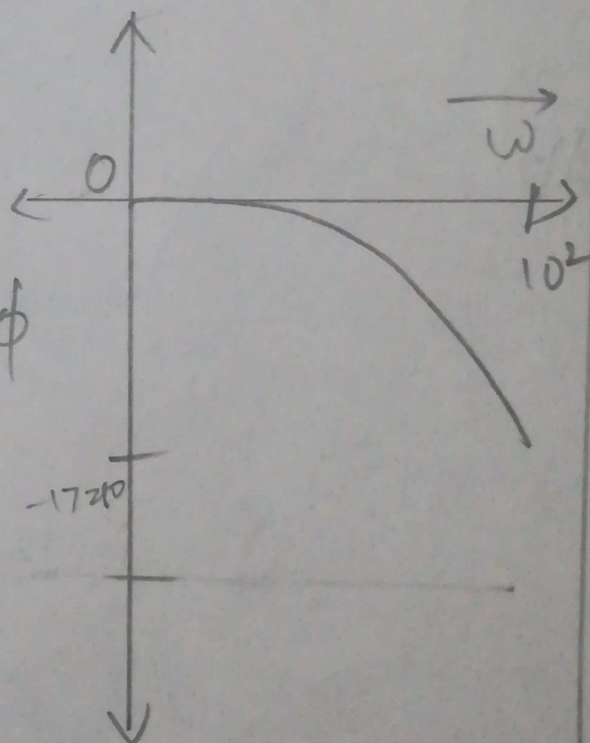
$$dB\Big|_{j\omega=0} = 12.04 + 20 - 14 - 6$$
$$= 12.04$$

$$\phi\, z\Big|_{j\omega=0} = 0$$

ϕ will be dominated by the linear fun. at high frequencies.

At lower ω ϕ will attempt to saturate at 90°





$$dB = 20 + 10\log\frac{(16+\omega^2)}{(\omega^2+25)(\omega^2-14)}$$

d) $G(j\omega) = \dfrac{10\,(j\omega - 4)\, e^{-3j\omega}}{(j\omega+5)(j\omega+2)}$

Same magnitude @ all ω & min phase.

$\Rightarrow |G_{new}| = \dfrac{10\sqrt{\omega^2+16}}{\sqrt{\omega^2+25}\,\sqrt{\omega^2+4}}$ $\forall\,\omega$.

Only thing we can now change is sign of the real & complex terms.

Note that @ minimum phase,

G_{new} is causal & G_{new} is stable.

for G_{new} to be causal $\dfrac{1}{G_{new}}$ should have stable poles

$\Rightarrow G_{new}$ should ~~hald~~ have all zeros in ~~RHP~~.

Notice $G(s)$ has 0 at $s = 4$ (RHP)

$\therefore$ the required LTI system is

$$G_{new}(s) = \frac{10(s+4)}{s^2+7s+10}\, e^{-3s}$$

(now the zero is at LHP, $s=-4$)