INDIAN INSTITUTE OF TECHNOLOGY MADRAS

Department of Chemical Engineering

CH3050: Process Dynamics and Control (Jan - May 2021)
Assignment-1 Solutions

Marks distribution

	Question 1	Question 2	Question 3	Question 4
(a)	5	2	5	14
(b)	5	8	7	14
(c)	5	5	5	-
(d)	5	5	5	-
(e)	-	-	10	-

Question 1

(a)

A household storage geyser that provides hot fluid stream to the user by heating the incoming cold water is a semi-batch process.

(b)

The controller variable is the temperature T of water, manipulated variable is in flow rate of waterF_i and heating coil rate Q and the disturbance variable is loss in the heat Q_L due to continuous flow of cold water and the temperate of inlet flow T_i.

(c)

The schematic diagram of feedback control mechanism for the storage geyser is shown in Fig. 1.

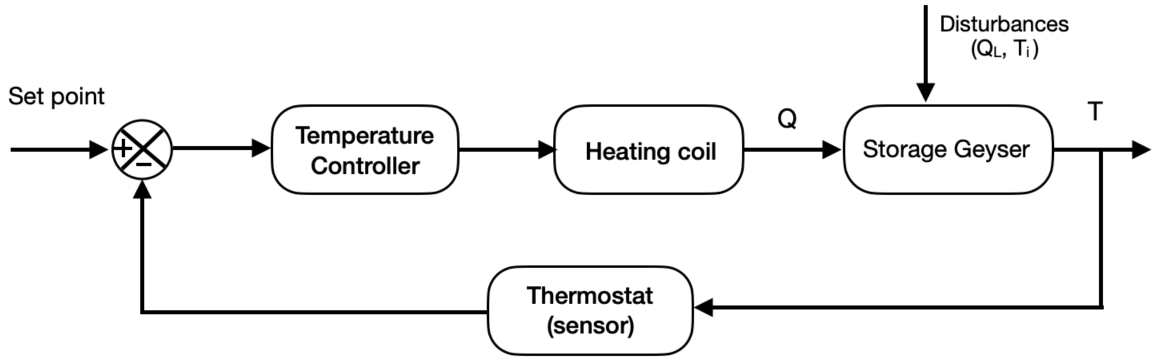


Figure 1: Schematic diagram of feedback control for storage geyser

(d)

The schematic diagram of feed-forward control mechanism for the home heating system is shown in Fig. 2.

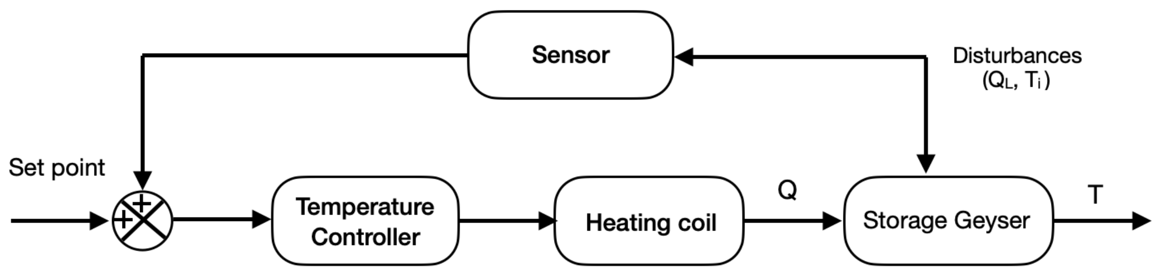


Figure 2: Schematic diagram of feed-forward control for storage geyser

Question 2

(a)

ODE in terms of deviations from steady-state:
$\tilde{y}(t) = y(t) - y(0)$; $\tilde{u}(t) = u(t) - u(0)$;
a_1=8; a_0= 15; b_0=3;

$$\frac{d^2\tilde{y}(t)}{dt^2} + 8\frac{d\tilde{y}(t)}{dt} + 15\tilde{y}(t) = 3\tilde{u}(t) \tag{1}$$

(b)

To increase the steady-state value of output $\tilde{y}(0) = 2$, AahaOohu decides to change the input by k units.

At steady state,

$$\frac{d^2\tilde{y}(t)}{dt^2} = 0, \frac{d\tilde{y}(t)}{dt} = 0$$

Substituting these values in Eq.(1),

$$15\tilde{y}(0) = 3k$$
$$\therefore k = 10$$

The amount of change in input required is k= 10 units.

(c)

Proportional Controller

The proportional controller produces an output, which is proportional to error signal. Our control objective is to achieve $\tilde{y}(t) = 2$ at steady-state.

$$\tilde{u}(t) = K_c e(t)$$
$$e(t) = 2 - \tilde{y}(t)$$
$$\frac{d^2\tilde{y}(t)}{dt^2} + 8\frac{d\tilde{y}(t)}{dt} + (15 + 3K_c)\tilde{y}(t) = 6K_c$$

At steady state,

$$K_c = \frac{5\tilde{y}(t)}{2 - \tilde{y}(t)}$$

Therefore, the control objective cannot be achieved for any finite value of $K_c > 0$

(d)

The rate of input is changed for implementing Proportional Derivative (PD) Controller

$$\frac{d\tilde{u}(t)}{dt} = K_c\frac{de(t)}{dt} + K_I e(t)$$
$$e(t) = 2 - \tilde{y}(t)$$
$$\frac{d^3\tilde{y}(t)}{dt^3} + 8\frac{d^2\tilde{y}(t)}{dt^2} + (15 + 3K_c)\frac{d\tilde{y}(t)}{dt} + 3K_I\tilde{y}(t) = 6K_I$$

Therefore, the control objective can be achieved for any value of K_c and $K_I \neq 0$.

Question 3

Given that

$$\frac{dw}{dt} = \frac{-(L+Va)}{M}w + \frac{Va}{M}z$$
$$\frac{dz}{dt} = \frac{L}{M}w - \frac{(L+Va)}{M}z + \frac{V}{M}z_f$$

where w and z are liquid concentrations on stage 1 and 2, respectively. L and V are the liquid and vapour molar flow rates, z_f si the concentration of the vapour stream entering the column.
The steady-state input values are $L = 80$ gmol inert liquid/min and $V = 100$ gmol inert vapour/min. The parameter values are $M = 20$ gmol inert liquid, $a = 0.5$ and $z_f = 0.1$ gmol solute / gmol inert vapour.

(a)

At steady state, the liquid concentrations are w_0 and z_0

$$0 = \frac{-(L+Va)}{M}w_0 + \frac{Va}{M}z_0 \tag{2}$$
$$z_0 = \frac{L+Va}{Va}w_0 \tag{3}$$

$$0 = \frac{L}{M}w_0 - \frac{(L+Va)}{M}z_0 + \frac{V}{M}z_f \tag{4}$$
$$w_0 = \frac{Va^2Z_f}{(L+Va^2) - LVa} \tag{5}$$

Substituting the given values in the above equations, we get the steady state values as $w_0 = 0.038$ and $z_0 = 0.100$

(b)

The given system is non-linear (non-linearity is caused by the product of state and inputs). The system is linearized around steady-state operations.

$$f_1 = \frac{dw}{dt} = \frac{-L}{M}w - \frac{Va}{M}w + \frac{Va}{M}z$$
$$f_2 = \frac{dz}{dt} = \frac{L(w-z) - Vaz + Vz_f}{M}$$

The state-space representation for the system is

$$\dot{x} = A\bar{x} + B\bar{u}$$
$$\bar{y} = C\bar{x} + D\bar{u} = C\bar{x}$$

State variables: w,z

Input variables : L,V

Output variables : w,z

D = 0

$$x = \begin{bmatrix} w \\ z \end{bmatrix}, \dot{x} = \begin{bmatrix} \frac{dw}{dt} \\ \frac{dz}{dt} \end{bmatrix} \tag{6}$$

$$A = \begin{bmatrix} \frac{-(L_0 + V_0 a)}{M} & \frac{V_0 a}{M} \\ \frac{L_0}{M} & \frac{-(L_0 + V_0 a)}{M} \end{bmatrix} \tag{7}$$
$$= \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} \frac{-w_0}{M} & \frac{-aw_0 + az_0}{M} \\ \frac{w_0 - z_0}{M} & \frac{-az_0 + zf_0}{M} \end{bmatrix} \tag{9}$$
$$= \begin{bmatrix} 1.9 \times 10^-3 & 1.55 \times 10^-3 \\ -3.1 \times 10^-3 & 2.5 \times 10^-3 \end{bmatrix} \tag{10}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{11}$$

Therefore, the linearized state space model around the steady-state operation is $A = \begin{bmatrix} -6.5 & 2.5 \\ 4 & -6.5 \end{bmatrix}$, $B = 10^{-3} \begin{bmatrix} 1.9 & 1.55 \\ -3.1 & 2.5 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $D = 0$.

(c)

Eigenvalues and eigenvectors of the system are

$$|A - \lambda I| = 0$$

The eigenvalues are $\lambda_1 = -3.34$ (slowest) and $\lambda_2 = -9.66$(fastest) and the eigenvectors are

$$V_1 = \begin{bmatrix} 0.79 \\ 1 \end{bmatrix} \tag{12}$$

$$V_2 = \begin{bmatrix} -0.79 \\ 1 \end{bmatrix} \tag{13}$$

Therefore the solution of the system is

$$x(t) = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t} \tag{14}$$

Expected fastest initial condition direction of the system is

$$x(0) = C_2 V_2 e^{\lambda_2 (0)} = C_2 V_2 \tag{15}$$

and the slowest one is

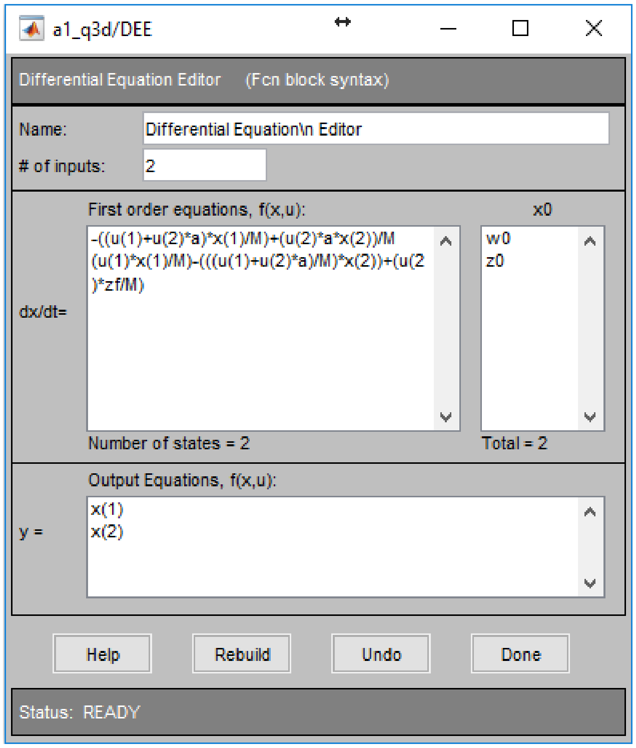
$$x(0) = C_1 V_1 \tag{16}$$

(d)

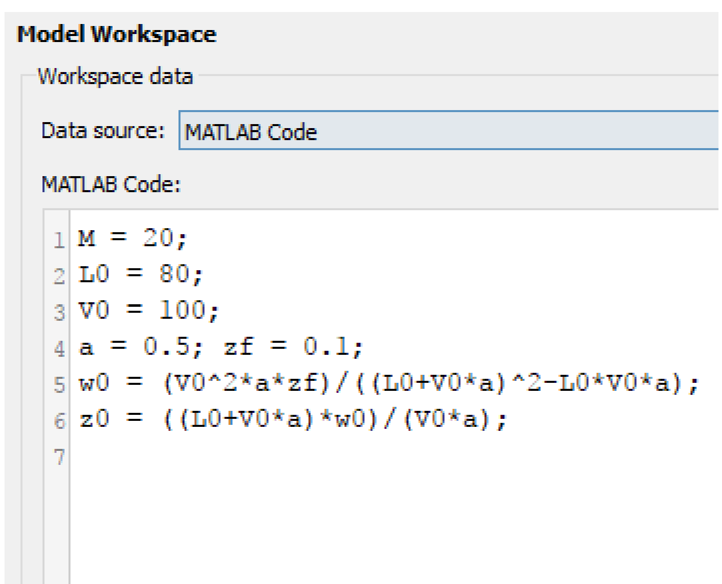
The Simulink model for the given non-linear system is given Fig. The MATLAB code to to obtain the linearized model for the above non-linear system is given below

```
%% Question 3(d)

%% Find the steady-state outputs at a fixed operating point

op = operspec('a1_q3d');
op.Inputs(1).u = 80; op.Inputs(1).Known = 1;
op.Inputs(2).u = 100; op.Inputs(2).Known = 1;
op_ss = findop('a1_q3d',op);

% Obtained steady-state outputs at a given operating condition
% y1_ss = 0.0388
% y2_ss = 0.101
```

(a) Simulink diagram



(b) DEE block



(c) Parameters

Figure 3: Simulink model for the given non-linear system

```
mod = linmod('a1_q3d',[0.038;0.1],[80;100]);
A = mod.a;
B = mod.b;
C = mod.c;
D = mod.d;
```

The linearized model is

```
A =
x1    x2
x1  -6.5   2.5
x2     4  -6.5

B =
u1         u2
x1  -0.0019    0.0015
x2  -0.0031   0.0025

C =
x1  x2
y1   1   0
y2   0   1

D =
u1  u2
y1   0   0
y2   0   0
```

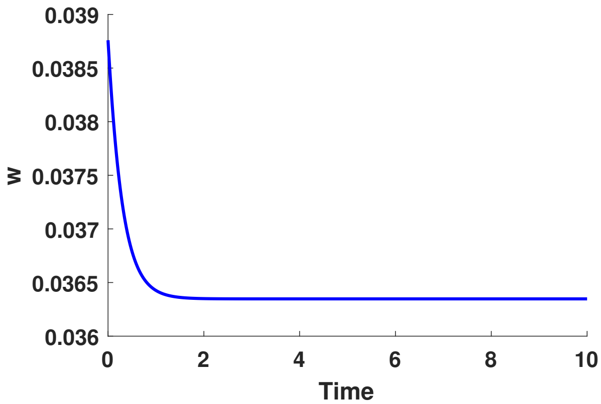
(e)

The step responses of the given non-linear system for two different magnitudes of steps (i) 5% and (ii) 15% change in the flow rate are illustrated in Fig. 4. Similarly, the step responses of the linearized system are shown in Fig. 5. As observed from Figure 4 and Figure 5,

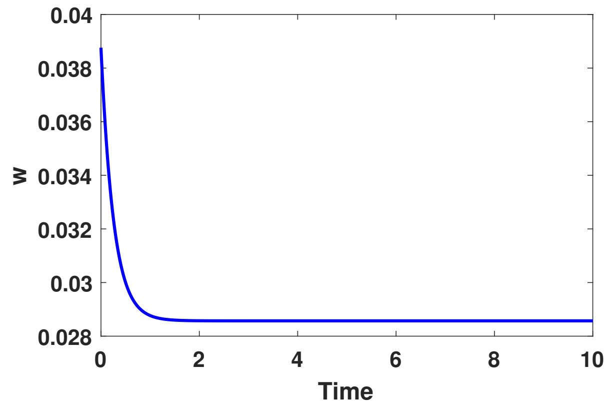
- For 5% step change in flow rate, the difference in responses between the nonlinear system and linearized model is 0.001.

- For 15% step change in flow rate, the difference in responses between the nonlinear system and linearized model is 0.002.

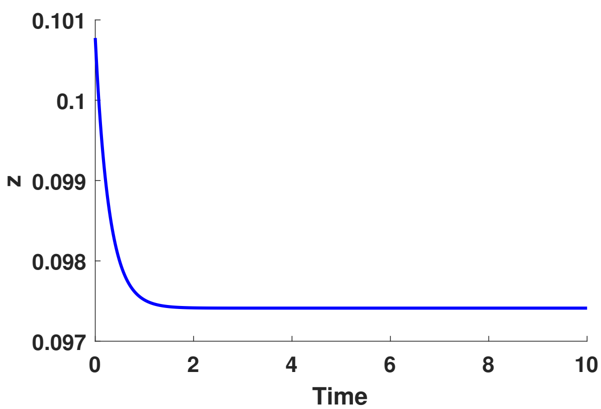
In conclusion, it is observed that the linearized model approximates the non-linearity model with high degree of accuracy. Furthermore, it is observed that with 5% change in flow rate, the significance of non-linearity in the system is less as compared to the 15% change in flow rate.



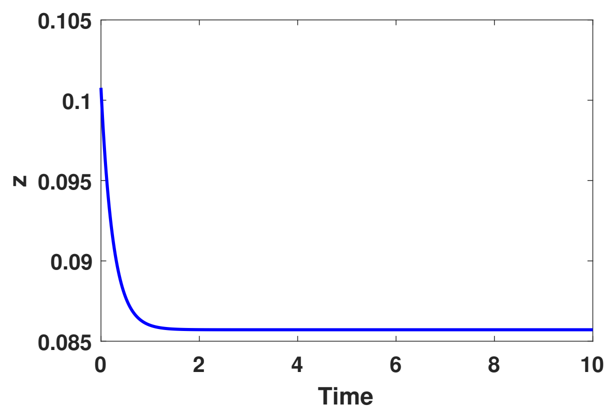
(a) w for 5% change in flow rate



(b) w for 15% change in flow rate

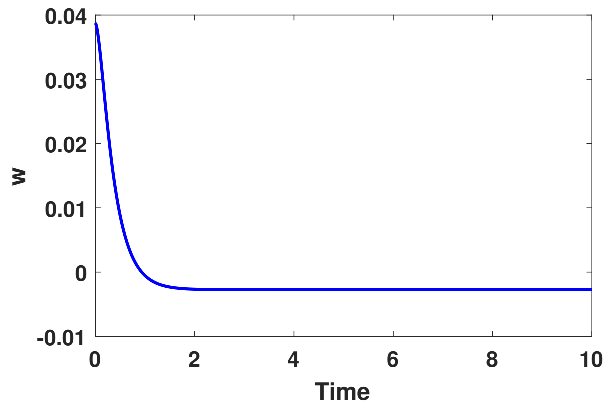


(c) z for 5% change in flow rate

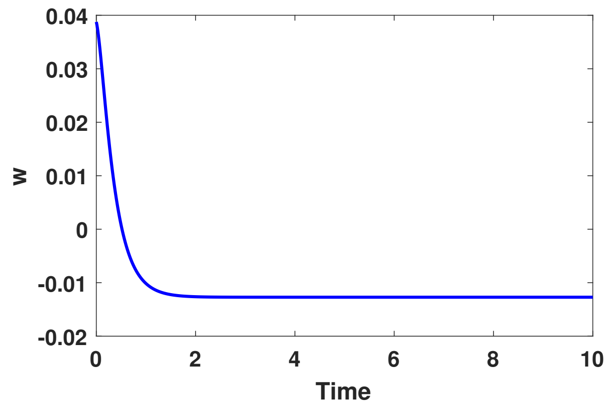


(d) z for 15% change in flow rate

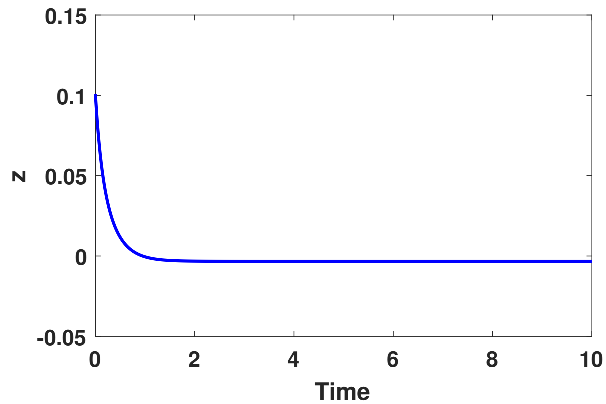
Figure 4: Step responses of non-linear system



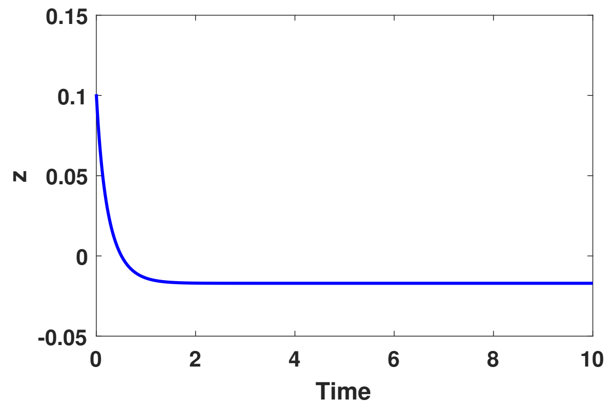
(a) w for 5% change in flow rate



(b) w for 15% change in flow rate



(c) z for 5% change in flow rate



(d) z for 15% change in flow rate

Figure 5: Step responses of linearized system

Question 4

(a)

Given signal is

$$x(t) = \begin{cases} t-2 & 0 \leq t < 3 \\ 1 & 3 \leq t < 4 \\ -\cos(3\pi(t-4)) & 4 \leq t < 5 \\ \exp^{-2(t-5)}\cos(5\pi(t-5)) & t \geq 5 \end{cases} \tag{17}$$

The Laplace transform of any signal $x(t)$ is

$$\mathcal{L}\{x(t)\} = X(s) = \int_0^\infty x(t)e^{-st}dt \tag{18}$$

Laplace transform of the given signal is

$$X(s) = \int_0^3 (t-2)e^{-st}dt + \int_3^4 1e^{-st}dt + \int_4^5 -\cos(3\pi(t-4))e^{-st}dt + \int_5^\infty \exp^{-2(t-5)}\cos(5\pi(t-5))e^{-st}dt \tag{19}$$

Using integration by parts,

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int (f'(x)\int g(x)dx)dx$$

we get

$$X(s) = \begin{cases} \dfrac{-e^{-3s}(s+1)-2s+1}{s^2} & 0 \leq t < 3 \\ \\ \dfrac{e^{-4s}(e^s-1)}{s} & 3 \leq 4 \\ \\ \dfrac{-se^{-5s}(e^s+1)}{s^2+9\pi^2} & 4 \leq 5 \\ \\ \dfrac{(s+2)e^{-5s}}{(s+2)^2+25\pi^2} & 5 \leq \infty \end{cases} \tag{20}$$

(b)

Given that

$$X(s) = \frac{s-2}{s(\tau^2 s^2 + 2\zeta\tau s + 1)} \tag{21}$$

Case 1: $\zeta > 1$ Roots would be different, negative and real.

$$X(s) = \frac{s-2}{s(s+\frac{\zeta}{\tau}-\frac{\sqrt{\zeta^2-1}}{\tau})(s+\frac{\zeta}{\tau}+\frac{\sqrt{\zeta^2-1}}{\tau})} \tag{22}$$

The inverse Laplace transform is

$$x(t) = \frac{\tau}{2\sqrt{\zeta^2-1}}\left[\exp\left(\frac{-(\zeta-\sqrt{\zeta^2-1})t}{\tau}\right) - \exp\left(\frac{-(\zeta+\sqrt{\zeta^2-1})t}{\tau}\right)\right]$$

$$-2\tau^2 - \frac{\tau^2}{\sqrt{\zeta^2-1}(\zeta-\sqrt{\zeta^2-1})}\exp\left(\frac{-(\zeta-\sqrt{\zeta^2-1})t}{\tau}\right) - \frac{\tau^2}{\sqrt{\zeta^2-1}(\zeta+\sqrt{\zeta^2-1})}\exp\left(\frac{-(\zeta+\sqrt{\zeta^2-1})t}{\tau}\right)$$

Case 2: $\zeta = 1$ Roots would be identical, negative and real.

$$X(s) = \frac{s-2}{s(s\tau+1)^2}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

$$x(t) = u(t)\left[\frac{1}{\tau^2}t\exp(\frac{-t}{\tau}) - 2 + 2(1-\frac{t}{\tau})\exp(-\frac{t}{\tau})\right]$$

Case 3: $0 \leq \zeta < 1$ Roots would be complex conjugate form.

$$X(s) = \frac{s-2}{\left(s+\frac{\zeta}{\tau}+j\frac{\sqrt{1-\zeta^2}}{\tau}\right)\left(s+\frac{\zeta}{\tau}-j\frac{\sqrt{1-\zeta^2}}{\tau}\right)}$$

$$\begin{aligned} x(t) = {} & \frac{\tau}{\sqrt{1-\zeta^2}}\exp\frac{-\zeta t}{\tau}\sin\frac{\sqrt{1-\zeta^2}}{\tau}t \\ & + \left[\frac{2}{\tau^2} - \frac{2}{\sqrt{1-\zeta^2}\tau}\exp(\frac{-\zeta t}{\tau})\sin\left(\frac{\sqrt{1-\zeta^2}t}{\tau} + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)\right] \end{aligned}$$