

ASSIGNMENT 7: MATH. PHYS. II

Due May 13h

- **Arfken** (7th edition)
- Exercises: 6.2.9, 6.2.12, 6.2.15.
- A system of N degrees of freedoms is subject to the following potential

$$V = \frac{1}{2}V_{ij}x_ix_j, \quad i, j = 1, \dots, N.$$

- (i) Derive Newton's second law for the system.
- (ii) Assume we are looking for solutions of the form

$$X(t) = X(0) \sin \omega t, \quad X(t) = \begin{pmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{pmatrix}.$$

Show that $X(0)$ satisfies an eigenvalue problem.

- (ii) What are the conditions on the eigenvalues of V_{ij} in order to have normal modes (i.e., oscillatory SH motion with a fixed frequency).
- Consider the previous example with the following potential

$$V = x_1x_2 + x_1x_3 + x_2x_3.$$

Find the normal modes.