

Question #1

## Written Problem (a3)

①

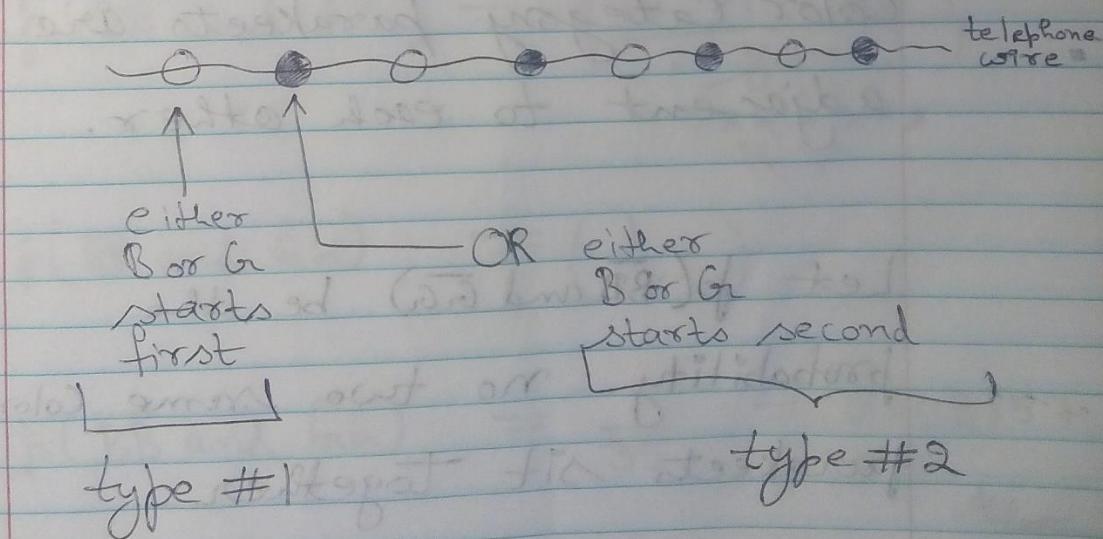
Assumption: All parakeets are unique;

Let  $B$  represents blue parakeets  
Category:

Let  $G$  represents green parakeets  
category;

We place  $B$  &  $G$  category parakeets  
in two patterns:

- (a)  $B$  starts first,  $G$  starts 2<sup>nd</sup>;
- (b)  $G$  starts first,  $B$  starts 2<sup>nd</sup>;



(80) without restrictions

① Let B chooses either type #1 or type #2.

$$\text{No. of ways for this} = 2 \times 1 = 2$$

② No. of ways of landing on type #1 position for 4 parakeets of a B or G category = 4!

So, either B parakeets lands at type #1 or type #2 positions

such that No two same

Color Category parakeets are adjacent to each other.

Let  $p(\overline{BB} \text{ and } \overline{GG})$  be the probability no two same color parakeets sit together.

$$P(\overline{BB} \text{ and } \overline{GG}) = \frac{\text{favourable case}}{\text{total case}}$$

(3)  $\boxed{\text{favourable case} = 2^4 * 4! * 4!}$   
 $= (2 * 4! * 4!)$

(4)  $\boxed{\text{Total case} = \left( \begin{array}{l} \text{No. of ways 8 parakeets} \\ \text{land at 8 positions} \\ \text{on the telephone wire} \end{array} \right)}$   
 $= 8!$

$$P(\overline{BB} \text{ and } \overline{GG}) = \frac{(2 * 4! * 4!)^2}{8!}$$

$5 \times 4 \times 7 \times 8$   
 $2,1$

$$P(\overline{BB} \text{ and } \overline{GG}) = \left( \frac{1}{5 \times 7} \right)$$

$$P(\overline{BB} \text{ and } \overline{GG}) = \frac{1}{35} = 0.02857$$

Ans

Question #2

## written Problem (a3)

Q-2)

(a) Probability a computing core is defective = 0.3  
 $P(\text{core} = \text{defective}) = 0.3$

Probability a computing core is correct =  $(1 - 0.3)$   
 $P(\text{core} = \text{correct}) = 0.7$

Assumption: All computing cores are conditionally independent from each other.

$$P(8 \text{ cores of CPU are correct}) = (P(\text{core} = \text{correct}))^8$$
$$\text{or, } P(8 \text{ cores correct}) = (P(\text{core} = \text{correct}))^8$$
$$= (0.7)^8$$
$$= 0.0576$$

(Ex) model of software

(Q-2)

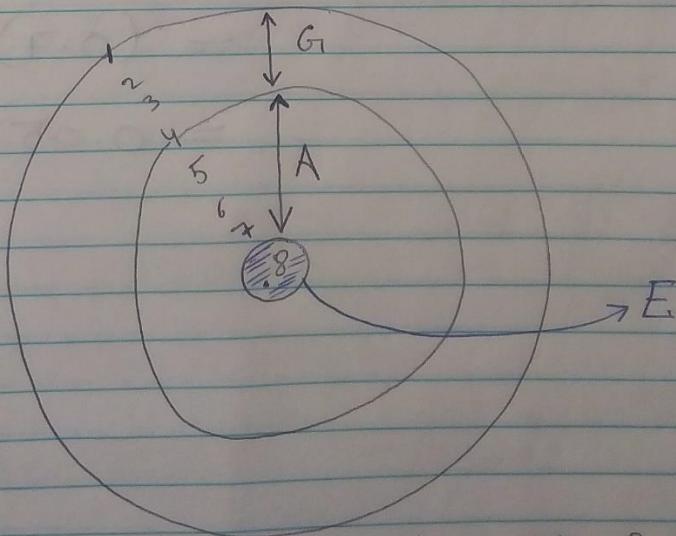
- (b) Let A = Advance models  
G<sub>1</sub> = Great models  
E = Extreme models

G = at least 1 functioning core

A = at least 4 functioning cores

E = exactly 8 functioning cores

Assumption: Manufacturer knows during CPU core testing that how many cores of a CPU are correct.



"Business Model diagram based on No. of Working Cores"

$$\begin{aligned}
 P(1 \text{ CPU core working}) &= {}^8C_1 \times (0.7) \times (0.3)^7 \\
 &= 8 \times (0.7) \times (0.3)^7 \\
 &= 0.00122472
 \end{aligned}$$

$$\begin{aligned}
 P(2 \text{ CPU cores working}) &= {}^8C_2 \times (0.7)^2 \times (0.3)^6 \\
 &= \frac{8!}{2! \times 6!} \times (0.7)^2 \times (0.3)^6 \\
 &= 28 \times (0.7)^2 \times (0.3)^6 \\
 &= 0.01000188
 \end{aligned}$$

$$\begin{aligned}
 P(3 \text{ CPU cores working}) &= {}^8C_3 \times (0.7)^3 \times (0.3)^5 \\
 &= \frac{8!}{3! \times 5!} \times (0.7)^3 \times (0.3)^5 \\
 &= 56 \times (0.7)^3 \times (0.3)^5 \\
 &= 0.04667544
 \end{aligned}$$

$$P(\text{Great CPU}) = P(1 \text{ CPU core working}) + P(2 \text{ CPU cores working}) + P(3 \text{ CPU cores working})$$

$$P(\text{Great CPU}) = 0.00122472 + 0.01000188 + 0.04667544$$

$$P(\text{Great CPU}) = 0.05790204$$

$$P(4 \text{ CPU cores working}) = {}^8C_4 \times (0.7)^4 \times (0.3)^4$$

$$= \frac{8!}{4! \times 4!} \times (0.7)^4 \times (0.3)^4$$

$$= 70 \times (0.7)^4 \times (0.3)^4$$

$$= 0.1361367$$

$$P(5 \text{ CPU cores working}) = {}^8C_5 \times (0.7)^5 \times (0.3)^3$$

$$= 56 \times (0.7)^5 \times (0.3)^3$$

$$= 0.25412184$$

$$P(6 \text{ CPU cores working}) = {}^8C_6 \times (0.7)^6 \times (0.3)^2$$

$$= \frac{8!}{6! \times 2!} \times (0.7)^6 \times (0.3)^2$$

$$P(6 \text{ CPU cores working}) = 28 \times (0.7)^6 \times (0.3)^2$$

$$= 0.29647548$$

$$P(7 \text{ CPU cores working}) = {}^8C_7 \times (0.7)^7 \times (0.3)^1$$

$$= 8 \times (0.7)^7 \times (0.3)^1$$

$$= 0.19765032$$

$$P(\text{Advance CPU}) = P(4 \text{ cores working}) +$$

$$P(5 \text{ cores working}) +$$

$$P(6 \text{ cores working}) +$$

$$P(7 \text{ cores working})$$

$$= (0.1361367 +$$

$$0.25412184 +$$

$$0.29647548 +$$

$$0.19765032)$$

$$= 0.88438434$$

~~$$P(\text{Extreme CPU}) = P(8 \text{ cores working})$$~~

~~$$= {}^8C_8 \times (0.7)^8 \times (0.3)^0$$~~

~~$$= (0.7)^8 = 0.05764801$$~~

If 1000 CPUs are manufactured,

$$\text{Great CPUs Expected} = 1000 \times P(\text{Great CPU})$$

$$= 1000 \times 0.05790204$$

$$\approx 57.90$$

$$\approx 58 \text{ (approx.)}$$

$$\text{Advanced CPUs Expected} = 1000 \times P(\text{Advanced CPU})$$

$$= 1000 \times 0.88438434$$

$$\approx 884.38$$

$$\approx 884 \text{ (approx.)}$$

$$\text{Extreme CPUs Expected} = 1000 \times P(\text{Extreme CPU})$$

$$= 1000 \times 0.05764801$$

$$\approx 57.64$$

$$\approx 58 \text{ (approx.)}$$

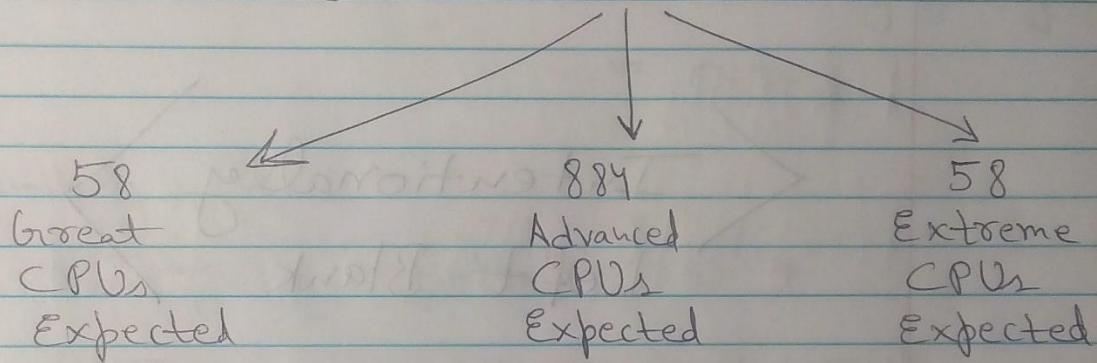
(Q-2)

(c) Price of Great Model = \$50

Price of Advanced Model = \$100

Price of Extreme Model = \$1000

$$\text{Total CPUs} = 1000$$



$$\begin{aligned}\text{Revenue for Great CPUs} &= 58 \times \$50 \\ &= \$2900\end{aligned}$$

$$\begin{aligned}\text{Revenue for Advanced CPUs} &= \$100 \times 884 \\ &= \$88400\end{aligned}$$

$$\begin{aligned}\text{Revenue for Extreme CPUs} &= \$1000 \times 58 \\ &= \$58000\end{aligned}$$

$$\begin{aligned}\text{Total revenue expected} &= (\$2900 + \\ &\quad \$88400 + \\ &\quad \$58000) \\ &= \$149,300\end{aligned}$$

Question #3

(Q-3)

$P(J_1 = \text{guilty} | A = \text{guilty})$  = probability Judge #1  
votes guilty when  
Accused is given  
actually guilty

$$= 0.7$$

Similarly,

$$P(J_2 = \text{guilty} | A = \text{guilty}) = 0.7$$

$$P(J_3 = \text{guilty} | A = \text{guilty}) = 0.7$$

Now,

$P(J_1 = \text{guilty} | A = \text{! guilty})$  = probability Judge #1  
votes guilty when  
Accused is actually  
not guilty (or say  
innocent)

$$= 0.2$$

$$P(J_2 = \text{guilty} | A = \text{! guilty}) = 0.2$$

$$P(J_3 = \text{guilty} | A = \text{! guilty}) = 0.2$$

drawing the event diagram for finding other probability data for joint distribution table:

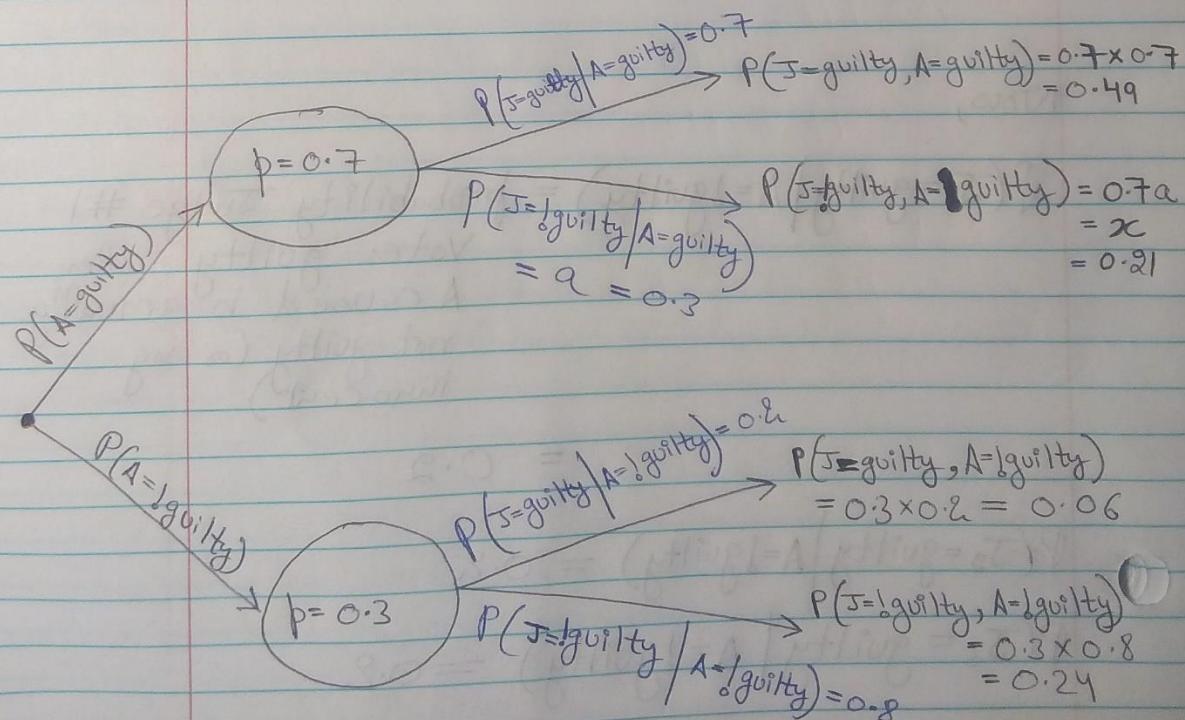
$$P(A=\text{guilty}) = 70\% \text{ of accused people are actually guilty}$$

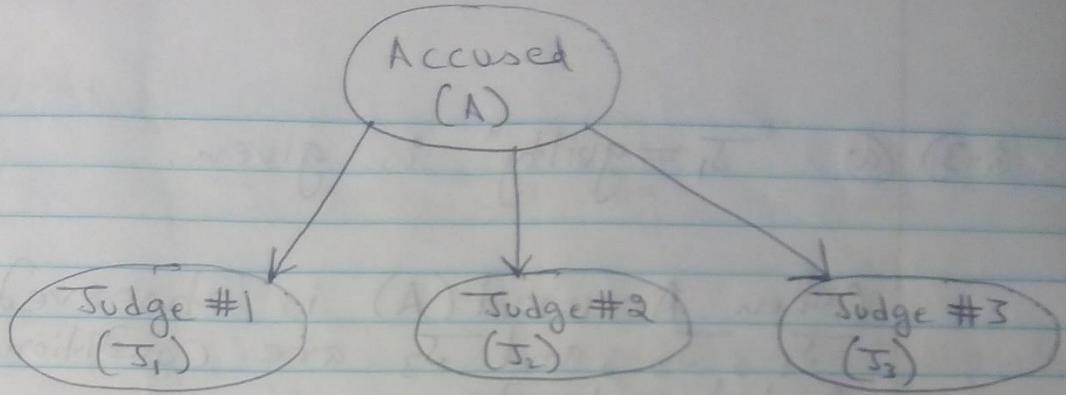
$$= 0.7$$

$$P(A=\text{!guilty}) = 0.3$$

$$P(J=\text{guilty} | A=\text{guilty}) = 0.7 \text{ (for any judge)}$$

$$P(J=\text{guilty} | A=\text{!guilty}) = 0.2 \text{ (for any judge)}$$





< Bayes Net >

↳ Accused (A) : Probability distribution

guilty	!guilty
0.7	0.3

↳ (Judge Vote) - (Accused) Joint Distribution:

		← Accused →	
		guilty	!guilty
↑ Judge Vote (J)	guilty	0.49	0.06
	!guilty	0.21	0.24

(Q-3) @ " $J_1 = \text{guilty}$ " is given.

When Accused (A) is observed,  
 $J_1, J_2$ , and  $J_3$  are conditionally  
independent.

So,

$$P(A=\text{guilty} | J_1=\text{guilty}) = \frac{P(A=\text{guilty}, J_1=\text{guilty})}{P(J_1=\text{guilty})}$$
$$= \frac{0.49}{0.49 + 0.06}$$

$$= \frac{0.49}{0.55}$$

$$= 0.89$$

Ans

Q-3) b

$(J_1 = \text{guilty}, J_2 = \text{guilty}, J_3 = \text{guilty})$  is given.

$J_1, J_2$ , and  $J_3$  are conditionally independent when 'A' is observed.

$$P(A = \text{guilty} \mid J_1 = \text{guilty}, J_2 = \text{guilty}, J_3 = \text{guilty}) \\ = \frac{(P(J_1 = \text{guilty}, J_2 = \text{guilty}, J_3 = \text{guilty} \mid A = \text{guilty}) * P(A = \text{guilty}))}{P(J_1 = \text{guilty}, J_2 = \text{guilty}, J_3 = \text{guilty})}$$

Short hand term,  
Let  $g = \text{guilty}$ ;  $\neg g = \text{not guilty}$

$$P(A=g \mid J_1=g, J_2=g, J_3=g) = \frac{(P(J_1=g, J_2=g, J_3=g \mid A=g) * P(A=g))}{P(J_1=g, J_2=g, J_3=g)}$$

Solving denominator,

$$P(J_1=g, J_2=g, J_3=g) = \{P(J_1=g \mid A=g) * P(J_2=g \mid A=g) * P(J_3=g \mid A=g)\} * P(A=g) + \{P(J_1=g \mid A=\neg g) * P(J_2=g \mid A=\neg g) * P(J_3=g \mid A=\neg g)\} * P(A=\neg g)$$

$$P(J_1=g, J_2=g, J_3=g) = \frac{(0.7 \times 0.7 \times 0.7 \times 0.7) + (0.2 \times 0.2 \times 0.2 \times 0.3)}{7^4 \times 10^4 + 2^3 \times 3^1 \times 10^4}$$

put this <sup>↑</sup> to equation ①,

$$\begin{aligned} P(A=g | J_1=g, J_2=g, J_3=g) &= \frac{(0.7 \times 0.7 \times 0.7) \times (0.7)}{(7^4 \times 10^4 + 2^3 \times 3^1 \times 10^4)} \\ &= \frac{7^4 \times 10^4}{7^4 \times 10^4 + 2^3 \times 3^1 \times 10^4} \\ &= \frac{2401}{2401 + 24} \\ &= \left( \frac{2401}{2425} \right) \end{aligned}$$

$$= 0.9901$$

(Q-3) C

$P(J_3=g \mid J_1=\text{!g}, J_2=\text{!g})$  = given that Judge 1 & Judge 2 have voted innocent (or say not guilty), what is the probability that Judge 3 votes guilty?

For this probability, Accused (A) is not observed. It cannot be the case that  $J_1, J_2$  &  $J_3$  are conditionally independent.

Assume:  $J_1, J_2, J_3$  are NOT ~~independent~~ entities in absence of accused A.

$$P(J_3=g \mid J_1=\text{!g}, J_2=\text{!g}) = \left\{ \frac{P(J_2=\text{!g}, J_1=\text{!g} \mid J_3=g) \cdot P(J_3=g)}{P(J_2=\text{!g}, J_1=\text{!g})} \right\}$$

using,

$$P(J_2=\text{!g} \mid J_1=\text{!g} \mid J_3=g) = \frac{P(J_2=\text{!g}, J_1=\text{!g}, J_3=g)}{P(J_3=g)}$$

we get...

$$P(J_3=g \mid J_1=\text{!g}, J_2=\text{!g}) = \left( \frac{P(J_2=\text{!g}, J_1=\text{!g}, J_3=g)}{P(J_2=\text{!g}, J_1=\text{!g})} \right) \quad \text{--- ①}$$

We know that  $J_1, J_2$  &  $J_3$  are  
NOT independent when A is not observed.

↳ We consider sum over missing variable  
values for each:

(a)  $P(J_2=1g, J_1=1g, J_3=g)$ , and

(b)  $P(J_2=1g, J_1=1g)$

Expanding (a),

$$P(J_2=1g, J_1=1g, J_3=g) = \underbrace{P(J_2=1g | A=g) \times P(J_3=g | A=g) \times P(J_1=1g | A=g)}_{\text{and}}.$$

$$\left\{ P(J_2=1g | A=g) \times P(J_3=g | A=g) \times P(J_1=1g | A=g) \right\} \times \\ P(A=g) +$$

$$\left\{ P(J_2=1g | A=1g) \times P(J_3=g | A=1g) \times P(J_1=1g | A=1g) \right\} \times \\ P(A=1g)$$

$$= \underbrace{\left\{ (0.3)(0.7)(0.3)(0.7) + (0.8)(0.2)(0.8)(0.3) \right\}}$$

$$= (0.0441 + 0.0384) = 0.0825$$

Expanding (b),

$$P(J_2 = \text{!g}, J_1 = \text{!g})$$

$$= \left\{ P(J_2 = \text{!g} | A = g) \times P(J_1 = \text{!g} | A = g) \times \left( P(J_3 = \text{!g} | A = g) + P(J_3 = g | A = g) \right) \times P(A = g) \right\} +$$

$$\left\{ P(J_2 = \text{!g} | A = \text{!g}) \times P(J_1 = \text{!g} | A = \text{!g}) \times \left( P(J_3 = \text{!g} | A = \text{!g}) + P(J_3 = g | A = \text{!g}) \right) \times P(A = \text{!g}) \right\}$$

$$= \left\{ (0.3) \times (0.3) \times ((0.3) + (0.7)) \times (0.7) \right.$$
  
$$\quad \quad \quad + \left. (0.8) \times (0.8) \times ((0.8) + (0.2)) \times (0.3) \right\}$$

$$= (0.3 \times 0.3 \times 0.7 + 0.8 \times 0.8 \times 0.3)$$

$$= (0.063 + 0.192)$$

$$= 0.255$$

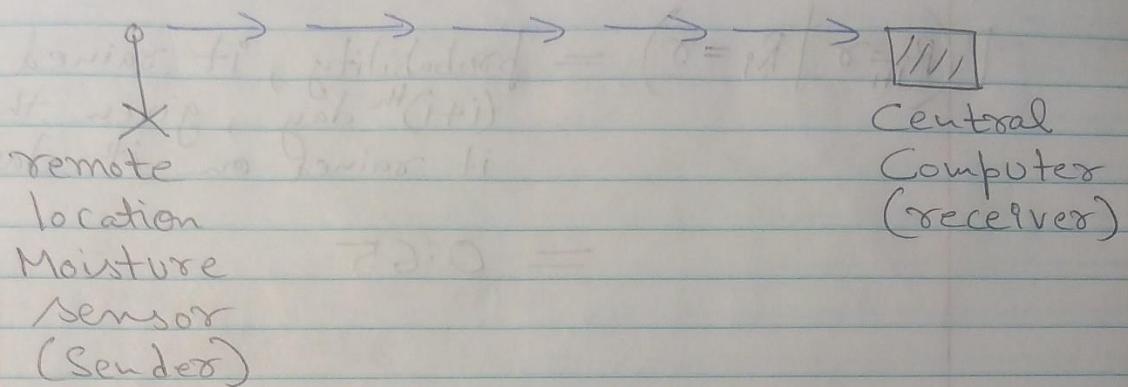
Using Expansions (a) & (b) in  
equation ①,

$$P(J_3 = g \mid J_1 = !g, J_2 = !g) = \frac{0.0825}{0.255}$$

$$= 0.3235$$

Question #4

(Q-4)



Conventions:

$R_i \rightarrow$  "rain" variable on  $i^{\text{th}}$  day;

$\text{Rec}_i \rightarrow$  "sensor received data" variable  
on  $i^{\text{th}}$  day.

$R_i = \gamma \rightarrow$  it rained on  $i^{\text{th}}$  day; (Yes);

$R_i = !\gamma \rightarrow$  it did NOT rain on  $i^{\text{th}}$  day; (No);

$\text{Rec}_i = \gamma \rightarrow$  Received value says it rained  
on  $i^{\text{th}}$  day; (or say Yes);

$\text{Rec}_i = !\gamma \rightarrow$  Received value says it did NOT  
rain on  $i^{\text{th}}$  day; (or say No);

Listing down given probabilities:

$$P(R_{i+1} = \text{rain} \mid R_i = \text{rain}) = \text{probability, it rained on } (i+1)^{\text{th}} \text{ day, given that it rained on } i^{\text{th}} \text{ day}$$
$$= 0.65$$

$$P(R_{i+1} = \text{rain} \mid R_i = \text{no rain}) = \text{probability, it rained on } (i+1)^{\text{th}} \text{ day, given that it did NOT rain on } i^{\text{th}} \text{ day.}$$
$$= 0.25$$

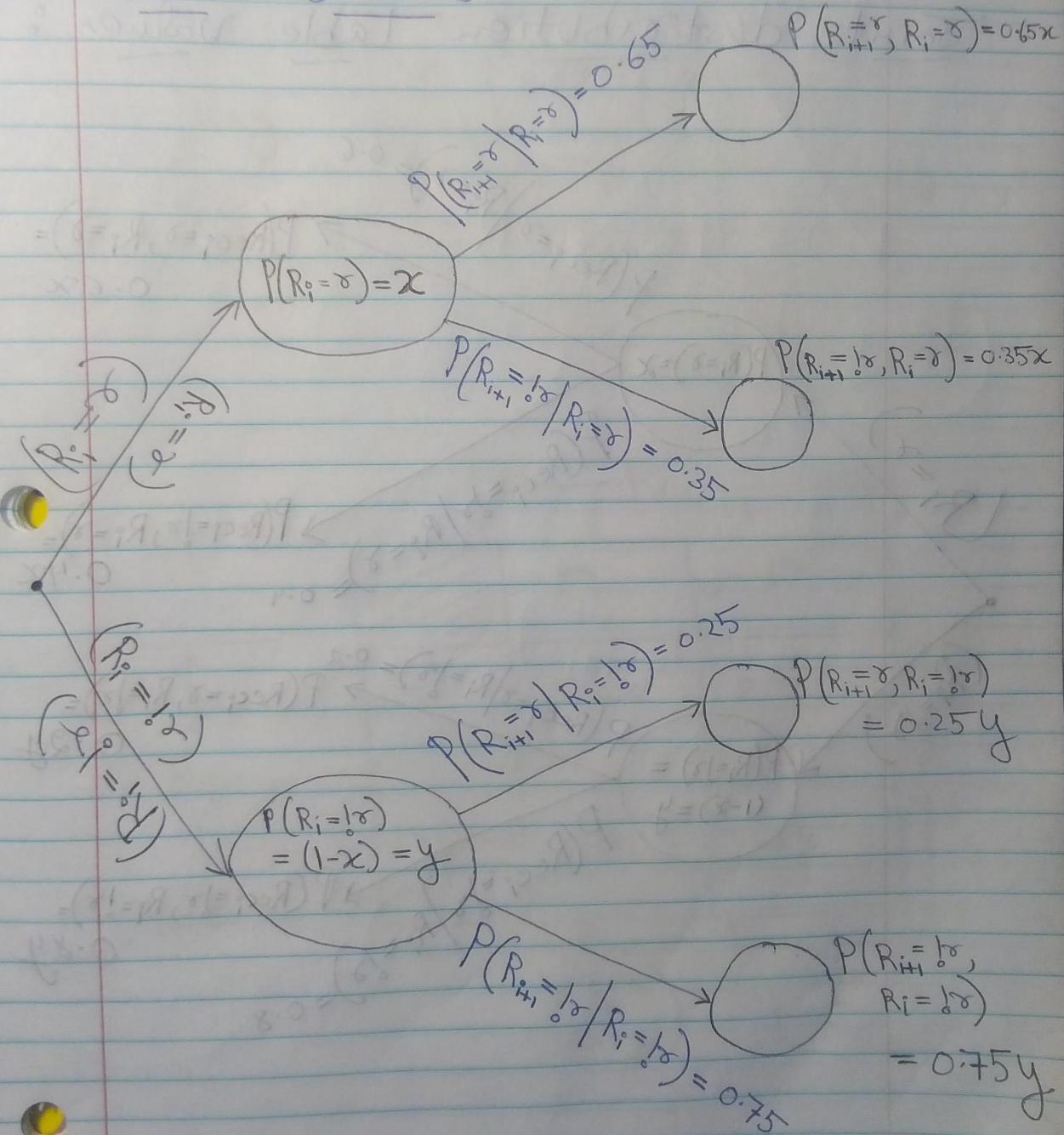
$$P(\text{Rec}_i = \text{no rain} \mid R_i = \text{rain}) = 0.4 = \text{probability of receiving no rain signal when it actually rained on } i^{\text{th}} \text{ day}$$

$$P(\text{Rec}_i = \text{yes} \mid R_i = \text{no rain}) = 0.2 = \text{probability of receiving YES rain signal when it actually did NOT rain on } i^{\text{th}} \text{ day.}$$

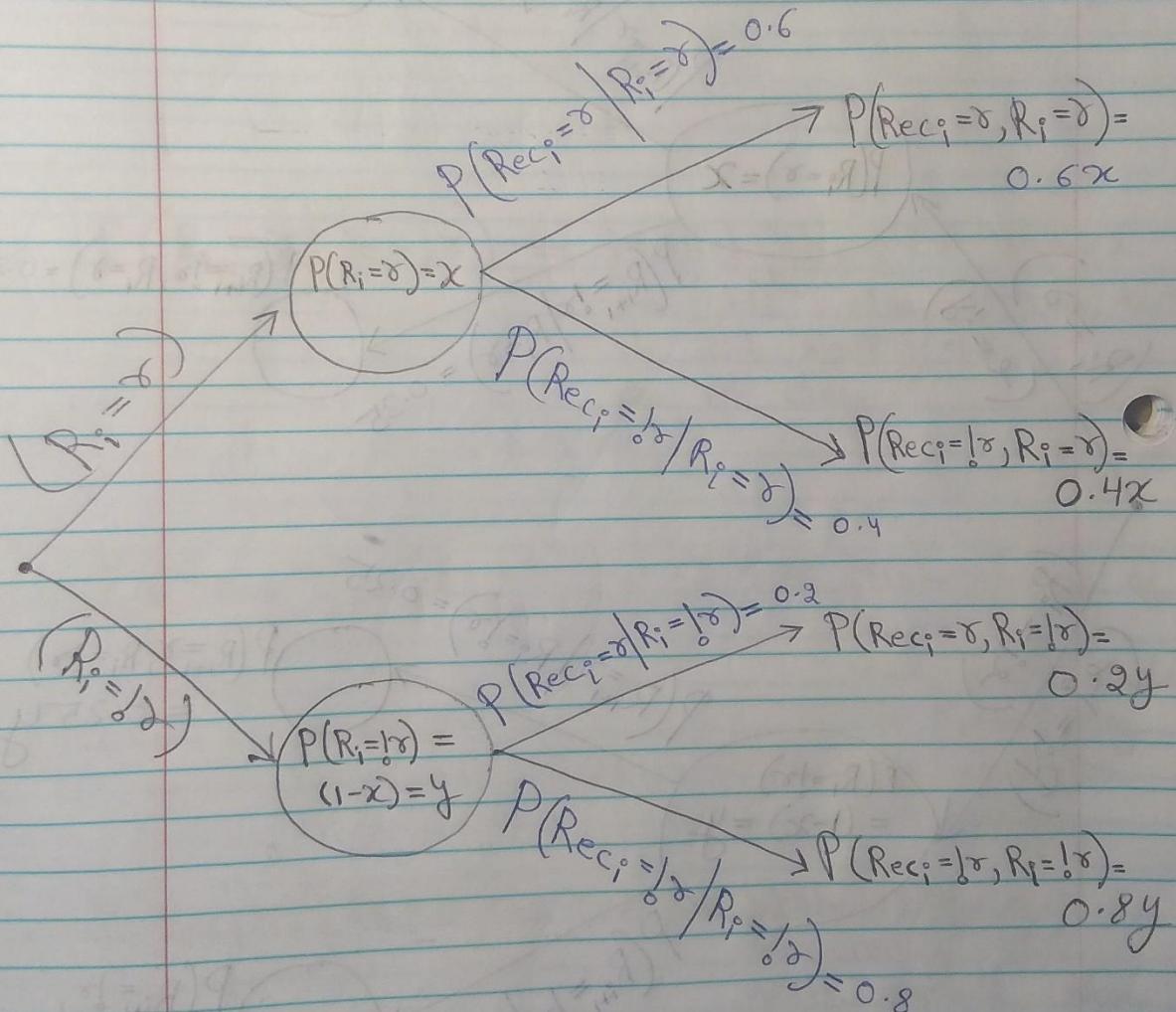
→ Making Rain Event Diagram

to calculate Joint distribution

Table Values:



$\Rightarrow$  Making Sensor Received  
data Event diagram to Calculate  
Joint distribution Table Values:



## Joint distribution Tables:

$\leftarrow R_{i+1} \text{ day} \rightarrow$

	$\gamma$	$! \gamma$
$R_i \text{ day}$	$0.65x$	$0.35x$
	$0.25y$	$0.75y$

$\left. \begin{matrix} 0.65x \\ + \\ 0.25y \end{matrix} \right\} \rightarrow x$        $\left. \begin{matrix} 0.35x \\ + \\ 0.75y \end{matrix} \right\} \rightarrow y$

$| \leftarrow Rec_i \rightarrow |$

	$\gamma$	$! \gamma$
$R_i$	$0.6x$	$0.4x$
	$0.2y$	$0.8y$

$\left. \begin{matrix} 0.6x \\ + \\ 0.2y \end{matrix} \right\} x$        $\left. \begin{matrix} 0.4x \\ + \\ 0.8y \end{matrix} \right\} y$

(Q-4)

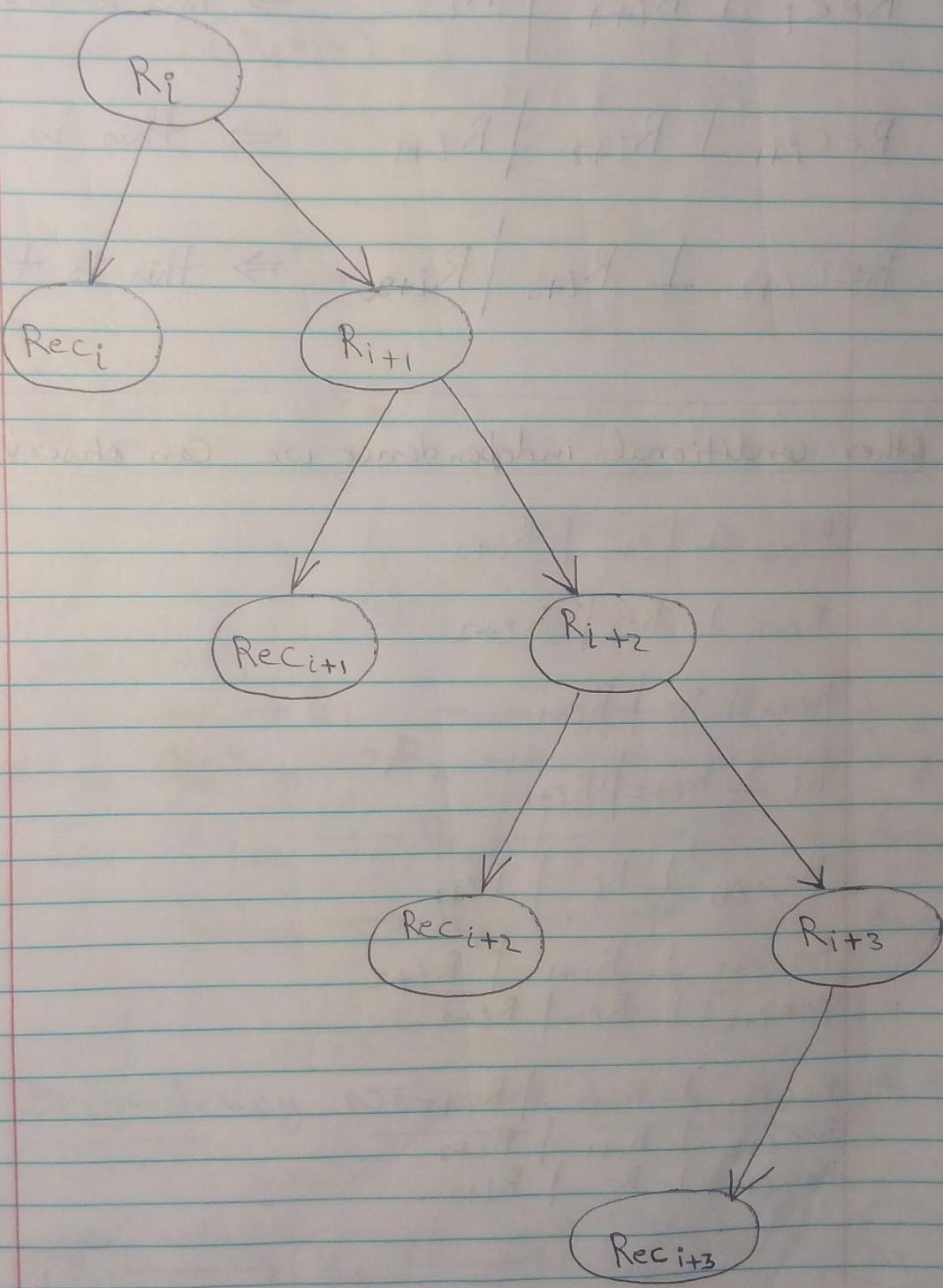
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Draw bayes Network to model the system in terms of

↳ 4 unobserved }  $\rightarrow (R_i, R_{i+1}, R_{i+2}, R_{i+3})$   
Variables }

↳ 4 observed }  $\rightarrow (Rec_i, Rec_{i+1}, Rec_{i+2},$   
Variables }  $Rec_{i+3})$

## Bayes Network



## ↳ Conditional Independence assumptions from the bayes Net :

①  $\text{Rec}_{i_1} \perp R_{i_2} \mid R_i$

this is true for all  $i$  under following conditions :

$i_1 < i_2$	$i_1 : i, (i+1), (i+2)$
$i_1 \leq i$	$i_2 : (i+1), (i+2), (i+3)$

$i \in \text{Natural Number}$

②  $R_{i_1} \perp R_{i_2} \mid R_{i_3}$

this is true for all  $i$  under following conditions :

$$i \leq i_1 < i_2 < i_3$$

$$i_1 : i, (i+1)$$

$$i_2 : (i+1), (i+2)$$

$$i_3 : (i+2), (i+3)$$

$i \in \text{Natural Number}$

③

$$Rec_i_1 \perp Rec_{i_2} \mid R_{i_1}$$

this is true under following conditions for  $i$ :

$$\boxed{i \leq i_1 < i_2}$$

$$i_1: i, i+1, i+2$$

$$i_2: i+1, i+2, i+3$$

$$i \in \text{Natural Number}$$

④

~~$$Rec_{i_1} \perp Rec_{i_2} \mid R_{i_1}$$~~

this is true for all  $i$  under following conditions:

$$\boxed{i \leq i_1 < i_2 < i_3}$$

$$i_1: i, i+1, i+2$$

$$i_2: i+1, i+2, i+3$$

$$i_3: i+1, i+2, i+3$$

$$i \in \text{Natural Number}$$

↳ Example  
Conditional Independence Assumptions:

①  $\text{Rec}_i \perp R_{i+1} \mid R_i \Rightarrow$  this is true

②  $\text{Rec}_{i+1} \perp R_{i+2} \mid R_{i+1} \Rightarrow$  this is true

③  $\text{Rec}_{i+2} \perp R_{i+3} \mid R_{i+2} \Rightarrow$  this is true

↳ Other conditional independence we can observe:

④  $R_{i+3} \perp R_{i+1} \mid R_{i+2}$

⑤  $R_{i+3} \perp R_i \mid R_{i+2}$

⑥  $R_{i+3} \perp R_i \mid R_{i+1}$

⑦  $R_i \perp R_{i+2} \mid R_{i+1}$

⑧  $\text{Rec}_{i+1} \perp R_i \mid R_{i+1}$

⑨  $\text{Rec}_{i+2} \perp R_{i+1} \mid R_{i+2}$

⑩  $\text{Rec}_{i+2} \perp R_i \mid R_{i+2}$

⑪  $\text{Rec}_{i+3} \perp R_{i+2} \mid R_{i+3}$

⑫  $\text{Rec}_{i+3} \perp R_{i+1} \mid R_{i+3}$

⑬  $\text{Rec}_{i+3} \perp R_i \mid R_{i+3}$

Just for  
examples ...

and many more...

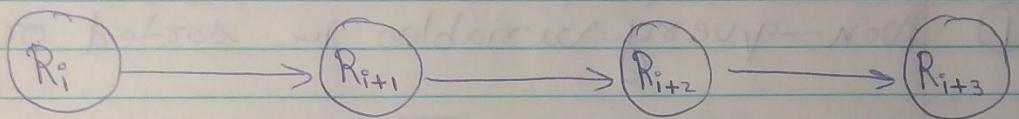
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(Q-4)

(b)

Given  $P(R_i = \text{!rain}) = 1$

ASSUMPTION: We know it did NOT rain on first day ( $R_i = \text{!rain}$ ). We are NOT at the computer center where we may have used sensor data. Ignoring sensor data.



(Dependency Bayes Net)

Let  $i=1$ ,

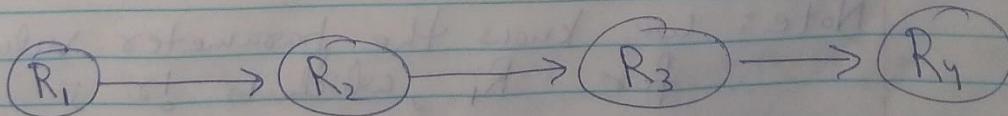
$\therefore R_i \rightarrow R_1 \longrightarrow$  first day

~~$R_{i+1} \rightarrow R_2 \longrightarrow$~~  Second day

$R_{i+2} \rightarrow R_3 \longrightarrow$  third day

$R_{i+3} \rightarrow R_4 \longrightarrow$  fourth day

↳ Dependency Network for  $R_1, R_2, R_3$  &  $R_4$



we have to find  $P(R_4 = \sigma | R_1 = !\sigma)$

We're given  $P(R_1 = !\sigma) = 1$

↳ Using Variable Elimination Method:

① Non-query variables in sorted order:

$$Z = [R_2, R_3] ; n=2$$

② Set of factors  $F$  of the form

$$P(z_j | p_a(z_j))$$

$$F = \{P(R_2 | R_1 = !\sigma), P(R_3 | R_2), P(R_4 | R_3)\}$$

Note: we know the parameter value  
for  $R_1$ , which is  $!\sigma$ ;

③ For each  $Z_1$  in  $Z$ :

$Z_1 = R_2$

a)  $F' = \{ P(R_2 | R_1 = \text{!}x), P(R_3 | R_2) \}$

here  $V = \{ R_2, R_3 \}$  as parameters in  $F'$ ;

b) Take product of  $F'$  elements:

$$\text{pdt } Z_1 = P(R_3 | R_2) * P(R_2 | R_1 = \text{!}x)$$

c) Sum  $\text{pdt } Z_1$  over all values

of  $(Z_1 = R_2)$ , producing a new element  $T(R_3)$  (let's call it this), parameterized by  $[R_2, R_3]$ . We do this, so we can eliminate  $Z_1 = R_2$  non-query variable.

$$\text{Sum } Z_1 = \sum_{R_2} P(R_3 | R_2) * P(R_2 | R_1 = \text{!}x)$$

What we know,

$$P(R_2 | R_1 = \text{!}x)$$

$$P(R_2 = x | R_1 = \text{!}x) = 0.25$$

$$P(R_2 = \text{!}x | R_1 = \text{!}x) = 0.75$$

$$P(R_3 | R_2) \xrightarrow{R_2 = \gamma} P(R_3 | R_2 = \gamma)$$

$$\xrightarrow{R_2 = 1 - \gamma} P(R_3 | R_2 = 1 - \gamma)$$

$$\text{Sum } Z_1 = \{ P(R_3 | R_2 = \gamma) \times (0.25) + \\ P(R_3 | R_2 = 1 - \gamma) \times (0.75) \}$$

↳ When  $(R_3 = \gamma)$  :

$$\text{Sum } Z_1(R_3 = \gamma) = \{ (0.65)(0.25) + (0.25)(0.75) \}$$

$$= 0.35$$

$$SF.0 = (0.65)(0.25)$$

↳ When  $(R_3 = 1\text{yr})$  :

$$\text{Sum } Z_1 (R_3 = 1\text{yr}) = \{(0.35)(0.25) + (0.75)(0.75)\} \\ = 0.65$$

$T(R_3)$  Table :

← Rain on day 3 → (given  $R_1 = 1\text{yr}$ )

$R_3 = \text{yr}$ (Yes)	$R_3 = 1\text{yr}$ (No)
0.35	0.65

- ④ Remove elements of  $F'$  from  $F$ , then add  $f = T(R_3)$  to  $F$ .

$$F = \left\{ P(R_1 | R_3), T(R_3) \right\}$$

Now, eliminate  $(Z_2 = R_3)$  :

$$@ F' = \{ P(R_4 | R_3), T(R_3) \}$$

Here  $v = \{R_4, R_3\}$  as parameters in  $F'$ .

(b) Take product  $F'$  elements :

$$PdtZ_2 = P(R_4 | R_3) * T(R_3)$$

(c) Sum  $PdtZ_2$  over all values

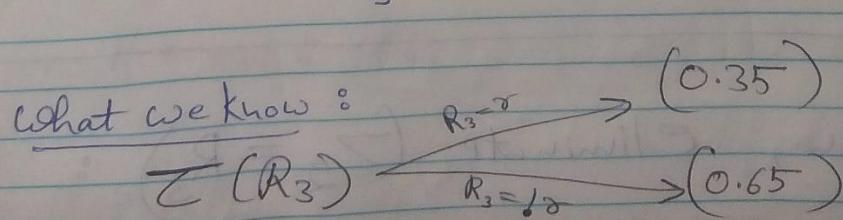
of ( $Z_2 = R_3$ ), producing a new element, let's say,  $T(R_4)$ ,

parameterized by  $[R_4]$ . We do

this so we can eliminate  $Z_2 = R_3$

non-query variable.

$$\text{Sum} Z_2 = \sum_{R_3} P(R_4 | R_3) * T(R_3)$$



$$P(R_4 | R_3)$$

$$P(R_4 | R_3 = \gamma)$$

$$P(R_4 | R_3 = !\gamma)$$

~~Sum Z<sub>2</sub> = { P(R<sub>4</sub> | R<sub>3</sub> = γ) × (0.35) + P(R<sub>4</sub> | R<sub>3</sub> = !γ) × (0.65) }~~

$$\text{Sum } Z_2 = \{ P(R_4 | R_3 = \gamma) \times (0.35) + P(R_4 | R_3 = !\gamma) \times (0.65) \}$$

↪ when  $(R_4 = \gamma)$  :

$$\text{Sum } Z_2 (R_4 = \gamma) = \{ (0.65)(0.35) + (0.25)(0.65) \}$$

$$= (0.2275 + 0.1625) = 0.39$$

↪ when  $(R_4 = !\gamma)$  :

$$\text{Sum } Z_2 (R_4 = !\gamma) = \{ (0.35)(0.35) + (0.75)(0.65) \}$$

$$= (0.1225 + 0.4875) = 0.61$$

$\mathcal{T}(R_y)$  Table :

Given  
 $R_1 = !\gamma$

↓ rain on day 4 →

$R_y = \gamma$	$R_y = !\gamma$
0.39	0.61

- ③ Remove elements of  $F'$  from  $F$ ,  
 Then add  $t = \mathcal{T}(R_y)$  to  $F$ .

$$F = \{ \mathcal{T}(R_y) \}$$

↳ Variable elimination algo terminates here.

Output =  $[\mathcal{T}(R_y)]$  table.

↳ Now query  $R_y = \gamma$  into  $\mathcal{T}(R_y)$  table,

We get  $\boxed{\mathcal{T}(R_y = \gamma) = 0.39}$

Ans

In case any text is not readable, and if allowed, please ask me questions. I can also provide the original notebook that has these solutions.

Regards,  
Mayank