## Phi (Φ) function - Euler's totient

Phi  $(\Phi)$  function, also known as Euler's totient function, is an arithmetic function that counts the positive integers up to a given integer n that are relatively prime to n. In other words, the function returns the number of integers from 1 to n-1 that have no common factor with n other than 1.

The phi function is written as  $\Phi(n)$  or  $\phi(n)$ . Here are the mathematical formulas to calculate phi function:

- 1. If p is a prime number, then  $\Phi(p) = p 1$
- 2. If p and q are distinct prime numbers, then  $\Phi(pq) = (p 1)(q 1)$
- 3. For a general n, let p1, p2, ..., pm be the distinct prime factors of n. Then,

```
\Phi(n) = n * (1-1/p1) * (1-1/p2) * ... * (1-1/pm)
```

Example of JavaScript code to calculate phi function  $\Phi(n)$ :

```
// Compute phi function \Phi(n)
function phi(n) {
    let result = n; // Initialize result with n
    // Check for all prime factors smaller or equal to sqrt(n)
    for (let i = 2; i*i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0) {
                n /= i;
            result -= (result / i);
        }
    }
    // If n has a prime factor greater than sqrt(n)
    if (n > 1) {
        result -= (result / n);
    }
    return result;
}
```

## Note:

This implementation of phi function uses a well-known algorithm called Euler's Totient Function Formula.

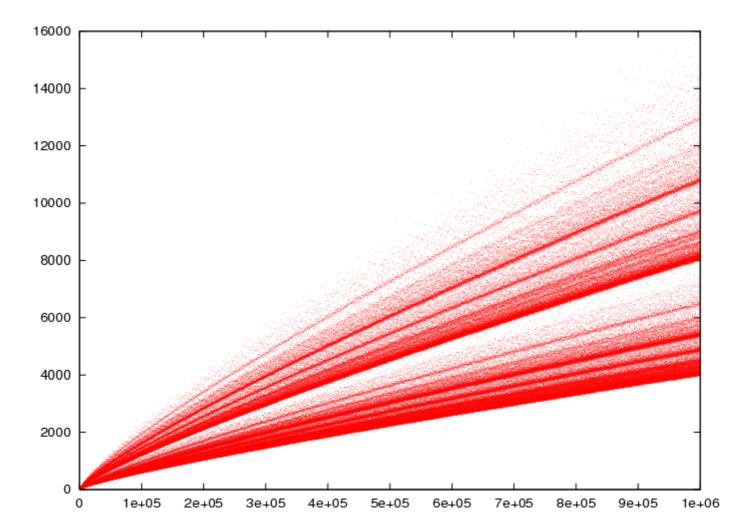
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## Funny note:

Christian Goldbach in a letter to the Euler make a discussion. The "Goldbach Conjecture" read more

- $\bullet$  2m = p + q
- $p \ge m$
- q ≤ m

He calculate the chance of q & p can be prime number ... Calculation power speed = (  $ln n \ll ln$  ) {i think: n log n^2, not sure}



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