

## Phi ( $\Phi$ ) function - Euler's totient

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Phi ( $\Phi$ ) function, also known as Euler's totient function, is an arithmetic function that counts the positive integers up to a given integer  $n$  that are relatively prime to  $n$ . In other words, the function returns the number of integers from 1 to  $n-1$  that have no common factor with  $n$  other than 1.

The phi function is written as  $\Phi(n)$  or  $\phi(n)$ . Here are the mathematical formulas to calculate phi function:

1. If  $p$  is a prime number, then  $\Phi(p) = p - 1$
2. If  $p$  and  $q$  are distinct prime numbers, then  $\Phi(pq) = (p - 1)(q - 1)$
3. For a general  $n$ , let  $p_1, p_2, \dots, p_m$  be the distinct prime factors of  $n$ . Then,

$$\Phi(n) = n * (1-1/p_1) * (1-1/p_2) * \dots * (1-1/p_m)$$

**Example of JavaScript code to calculate phi function  $\Phi(n)$ :**

```
// Compute phi function  $\Phi(n)$ 
function phi(n) {
    let result = n; // Initialize result with n

    // Check for all prime factors smaller or equal to sqrt(n)
    for (let i = 2; i*i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0) {
                n /= i;
            }
            result -= (result / i);
        }
    }

    // If n has a prime factor greater than sqrt(n)
    if (n > 1) {
        result -= (result / n);
    }

    return result;
}
```

### Note:

This implementation of phi function uses a well-known algorithm called Euler's Totient Function Formula.

## Funny note:

Christian Goldbach in a letter to the Euler make a discussion. The "Goldbach Conjecture" [read more](#)

- $2m = p + q$
- $p \geq m$
- $q \leq m$

He calculate the chance of  $q$  &  $p$  can be prime number ...

Calculation power speed = (  $\ln n \ll \sqrt{n}$  ) {i think:  $n \log n^2$ , not sure}

