

RSA (Rivest-Shamir-Adleman)

RSA (Rivest-Shamir-Adleman) is a widely-used public-key cryptosystem that is based on the mathematical concepts of prime numbers and modular arithmetic. It allows for secure communication over an insecure channel by encrypting a message with the intended recipient's public key which only the recipient is able to decrypt using their private key.

The RSA algorithm involves the following steps:

1. Key Generation:

- Choose two large distinct prime numbers 'p' and 'q'
- Calculate $n = pq$
- Choose an integer e, such that $1 < e < \Phi(n)$ and $\gcd(e, \Phi(n)) = 1$
- Calculate d, such that $d \equiv e^{-1} \pmod{\Phi(n)}$

Public key: (e, n)

Private key: (d, n)

2. Encryption:

- Convert the message into a number 'm'
- Apply the following rule:

$$c = m^e \pmod{n}$$

where c is the encrypted message (ciphertext)

3. Decryption:

- Apply the following rule:

$$m = c^d \pmod{n}$$

where m is the original message (plaintext)

The mathematical formulas used in RSA algorithm:

- Prime Number: A prime number is a number that is only divisible by 1 and itself. Example: 2, 3, 5, 7, 11, 13, etc.
- Public key: (e, n) where e is the public exponent
 $n = pq$, p and q are two distinct prime numbers
- Private key: (d, n) where d is the private exponent

$$d \equiv e^{-1} \pmod{\Phi(n)}$$

$$\Phi(n) = (p-1)(q-1)$$

- Encryption: $c = m^e \pmod n$ where m is the plaintext message
 c is the ciphertext message
 e is the public exponent
 n is the product of two primes (i.e. $n = pq$)
- Decryption: $m = c^d \pmod n$ where c is the ciphertext message
 m is the decrypted plaintext message
 d is the private exponent
 n is the product of two primes (i.e. $n = pq$)

Example of JavaScript code for RSA Encryption and Decryption:

```
// RSA Encryption
function rsaEncrypt(publicKey, plaintextMsg) {
  const [e, n] = publicKey;
  const ciphertext = BigInt(plaintextMsg) ** BigInt(e) % BigInt(n);
  return ciphertext.toString();
}

// RSA Decryption
function rsaDecrypt(privateKey, ciphertextMsg) {
  const [d, n] = privateKey;
  const plaintext = BigInt(ciphertextMsg) ** BigInt(d) % BigInt(n);
  return plaintext.toString();
}
```

Note:

Due to the security concerns of using small prime numbers and integer overflows in JavaScript.