Solutions to Self Test

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1 (0 pts) Bayes' Rule

Solution:
Consider the following events:

1. R: Rainfall
2. $\bar{R}$: No Rainfall
3. W: Weatherly predicting rain

From the problem statement, the following probabilities can be inferred

1. $P(R) = 73/365 = 0.2$
2. $P(\bar{R}) = 0.8$
3. $P(W/R) = 0.7$
4. $P(W/\bar{R}) = 0.3$

We need to compute $P(R/W)$.
Using Bayes Theorem,

$$\begin{aligned} P(R/W) &= \frac{P(R) \cdot P(W/R)}{P(W)} \\ &= \frac{P(R) \cdot P(W/R)}{P(R) \cdot P(W/R) + P(\bar{R}) \cdot P(W/\bar{R})} \\ &= \frac{0.2 \times 0.7}{0.2 \times 0.7 + 0.8 \times 0.3} \\ &= 0.37 \end{aligned}$$

2 (0 pts) Discrete Expectation

Solution:

$$E[X] = (1 \cdot 1/2) + (2 \cdot 1/10) + (3 \cdot 1/10) + (4 \cdot 1/10) + (5 \cdot 1/10) + (6 \cdot 1/10) = 2.5$$

3 (0 pts) Matrices/Linear Equations

Solution:

a.

$$Ax = b$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ -2 \end{bmatrix}$$

b. Consider the above equation with x replaced by s

$$As = b$$

Multiply by A^{-1} on both sides

$$AA^{-1}s = A^{-1}b$$

$$Is = A^{-1}b \quad \text{where } I \text{ is the Identity Matrix}$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} -1 & 1/2 & 1/2 \\ 1 & -1/2 & 0 \\ 2 & -1/2 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

Verification: $A \cdot s = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ -3 \end{bmatrix} = b$

c. $b = c_1 \cdot x_1 + c_2 \cdot x_2 + c_3 \cdot x_3$, where,

$$c_1 = \left[2//4//2\right] c_2 \qquad = \left[1//0//2\right] c_3 = \left[1//2//0\right]$$

4 (0 pts) Eigenvalues/Eigenvectors

Solution:
a. $Det(A) = 2$
b. Since $Det(A)$ is non-zero, A is invertible
c. $Rank(A) = 3$

5 (0 pts) Norms

Solution:
0-norm = number of non-zero elements in $x = 4$
1-norm $= 2 + 1 + |-4| + |-2| = 9$
∞-norm $= max(2, 1, |-4|, |-2|) = 4$