Northeastern University, Data Mining Techniques - CS6220
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Solutions to Homework 3, Part 1

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1. PCA Eigenvector Orthogonality

Solution:

Given,

$$A\vec{x} = \lambda_1 \vec{x}$$
$$A\vec{y} = \lambda_2 \vec{y}$$

Multiply each equation above with the transpose of the other eigenvector

$$\begin{aligned} & \vec{y}^t A\vec{x} = \lambda_1 \vec{y}^t \vec{x} \\ \implies & \vec{x}^t A\vec{y} = \lambda_1 \vec{x}^t \vec{y} \\ & \vec{x}^t A\vec{y} = \lambda_2 \vec{x}^t \vec{y} \end{aligned}$$

Let's subtract the above two equations. We get,

$$\begin{aligned} (\lambda_2 - \lambda_1) \cdot \vec{x}^t \vec{y} &= 0 \\ \vec{x}^t \vec{y} &= 0 \end{aligned}$$

Hence since $\vec{x}^t \vec{y} = 0$, x and y are orthogonal.