Robotics 2

Kinematic calibration

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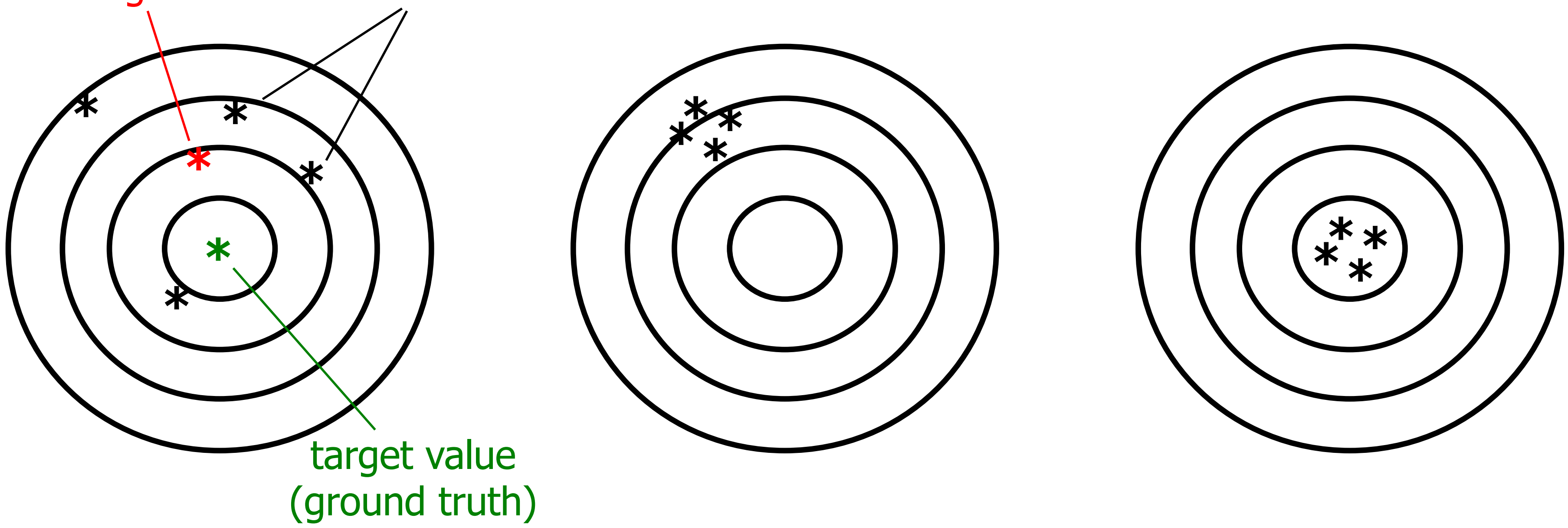
Accuracy and Repeatability

robot as a measuring device
through direct kinematics

from Robotics 1



low accuracy
low repeatability

low accuracy
high repeatability

high accuracy
high repeatability

better components!

calibration!

Direct kinematics

- nominal set of Denavit-Hartenberg (D-H) parameters

given by the manufacturer

$$\boldsymbol{\alpha} = \overbrace{\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}}^{\text{constants}} \quad \boldsymbol{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \quad \boldsymbol{d} = \begin{pmatrix} d_1 \\ \vdots \\ d_n \end{pmatrix}$$

distances from adjacent frames

for simplicity, suppose an all-revolute joints manipulator

- nominal direct kinematics

$$\boldsymbol{r}_{nom} = f(\boldsymbol{\alpha}, \boldsymbol{a}, \boldsymbol{d}, \boldsymbol{\theta}) \qquad \boldsymbol{\theta} = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix}$$

Variables

θ are typically measured by encoders

The nominal direct kinematics may not be the true one =>

various reasons

Need for calibration

- tolerances in mechanical construction and in assembly of links/joints imply small errors in actual end-effector pose (real ≠ nominal parameters)

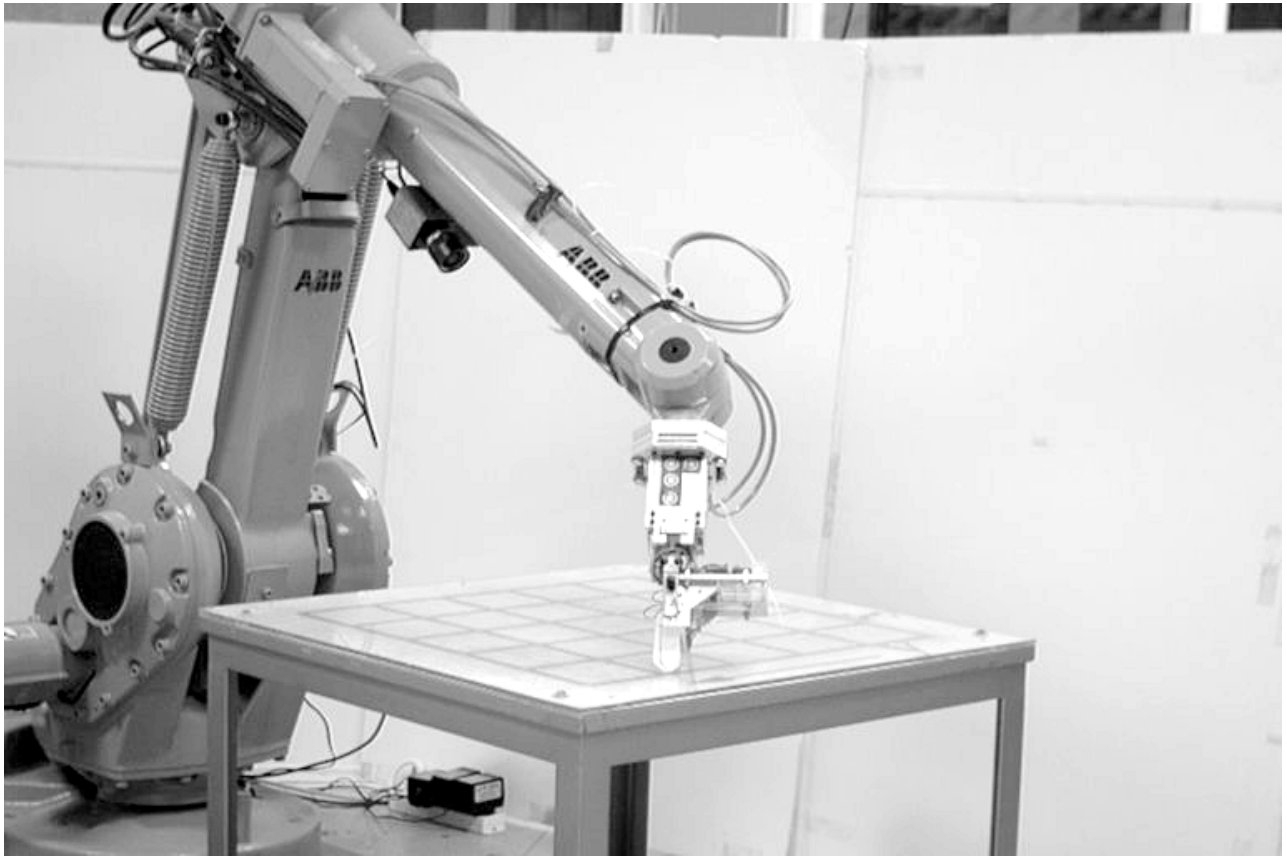
two axes which are not really parallel, or a link not exactly that long

- encoder mounting on motor axes may not be consistent with the "zero reference" of the robot direct kinematics (joint angle measures are constantly biased)
- errors distributed "along" the arm are amplified, due to the open chain kinematic structure of most robots
- calibration goal: recover as much as possible E-E pose errors by correcting the nominal set of D-H parameters, based on independent external (accurate!) measurements

We need a device to tell us ground truth

- experiments to be done once for each robot, before starting operation... (and maybe repeated from time to time)

Cartesian measurement systems - 1

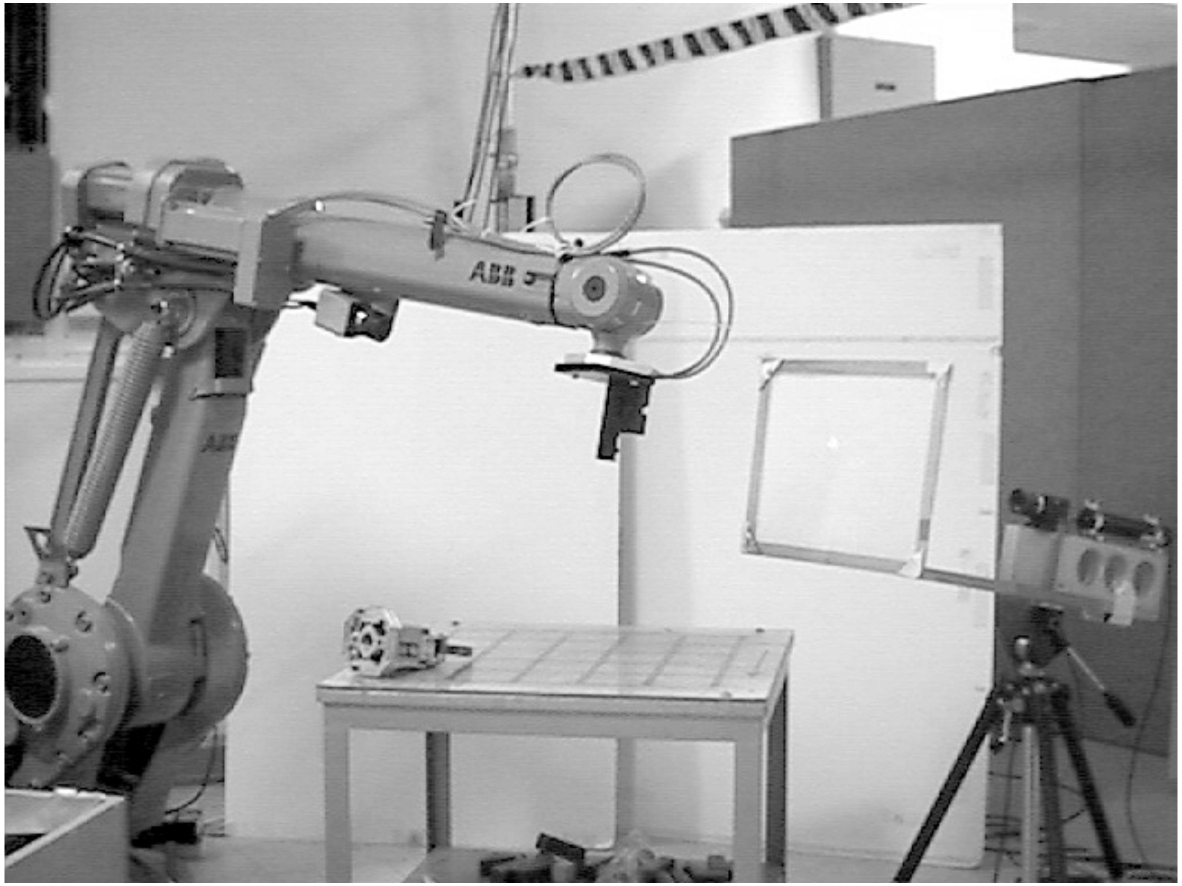


- External
  Video camera
  with triangulation
  ⇓
  we compare what
  the camera sees
  with the direct
  kinematics on
  nominal data

- Target on the
  structure to
  localize the
  end-effector

calibration table

Cartesian measurement systems - 2



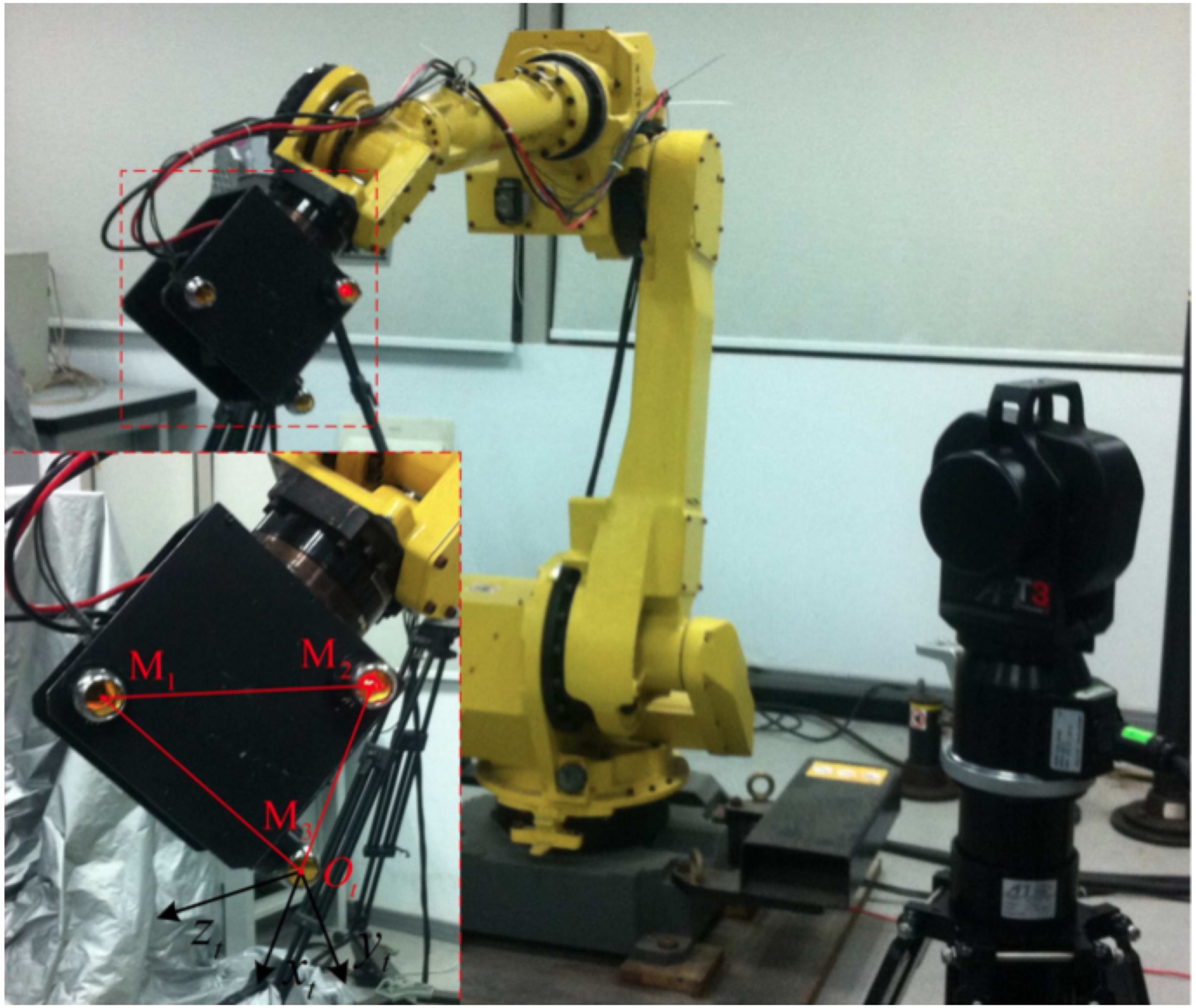
- Not a table, but an object mounted on the end effector

laser/camera system + triangulation

Cartesian measurement systems - 3

FANUC 6R robot
M-710iC/50

3 SMRs
(Spherically-
Mounted
Reflectors)



API
laser tracker III
www.apisensor.com

laser tracker + targets on end-effector

Acquiring data for calibration

FARO ION
laser tracker

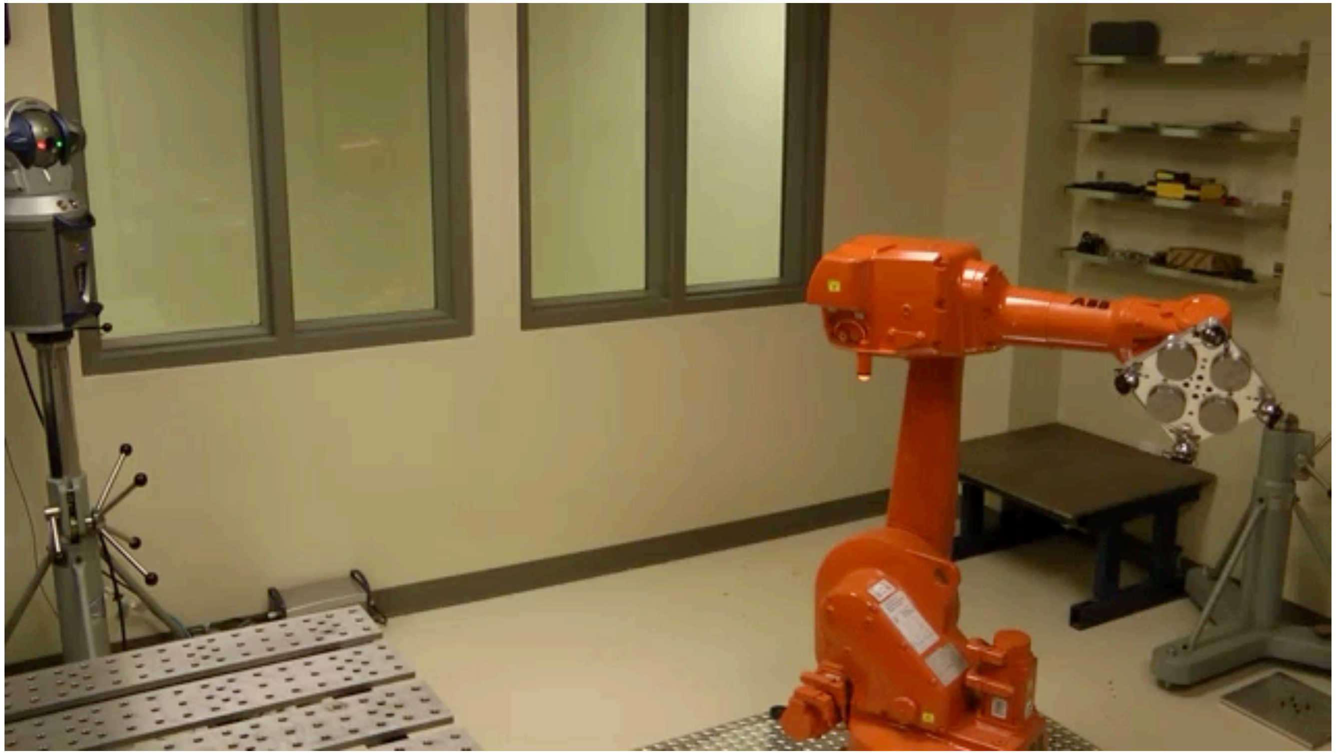
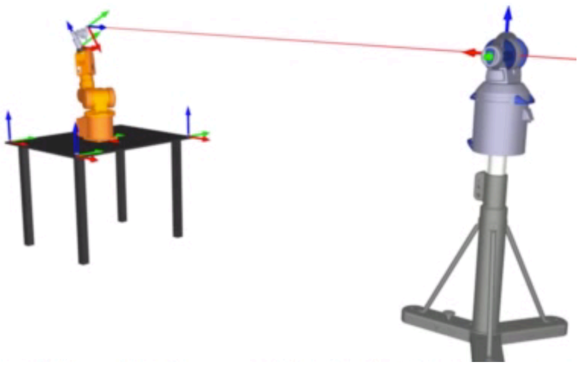


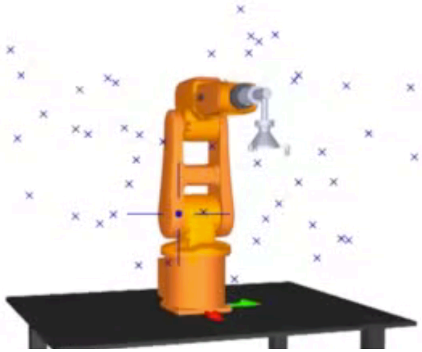
ABB
IRB 1600
robot

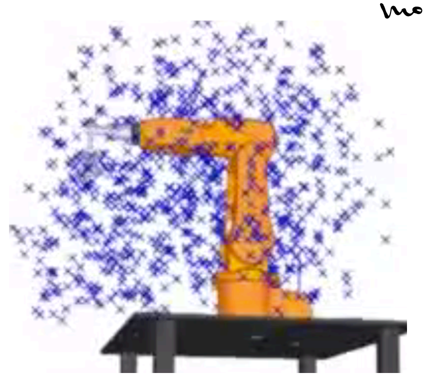
4 SMRs

video
@CoRo Lab
ETS Montréal

Errors on the boundary of the workspace are more amplified







I take r_{nom}, perturbate it and take the first order taylor expansion, so that we obtain a linear expression $\rightarrow r = r_{nom} + \Delta r$

non linear function of parameters and joint variables

Linearization of direct kinematics

partial Jacobians evaluated in nominal conditions

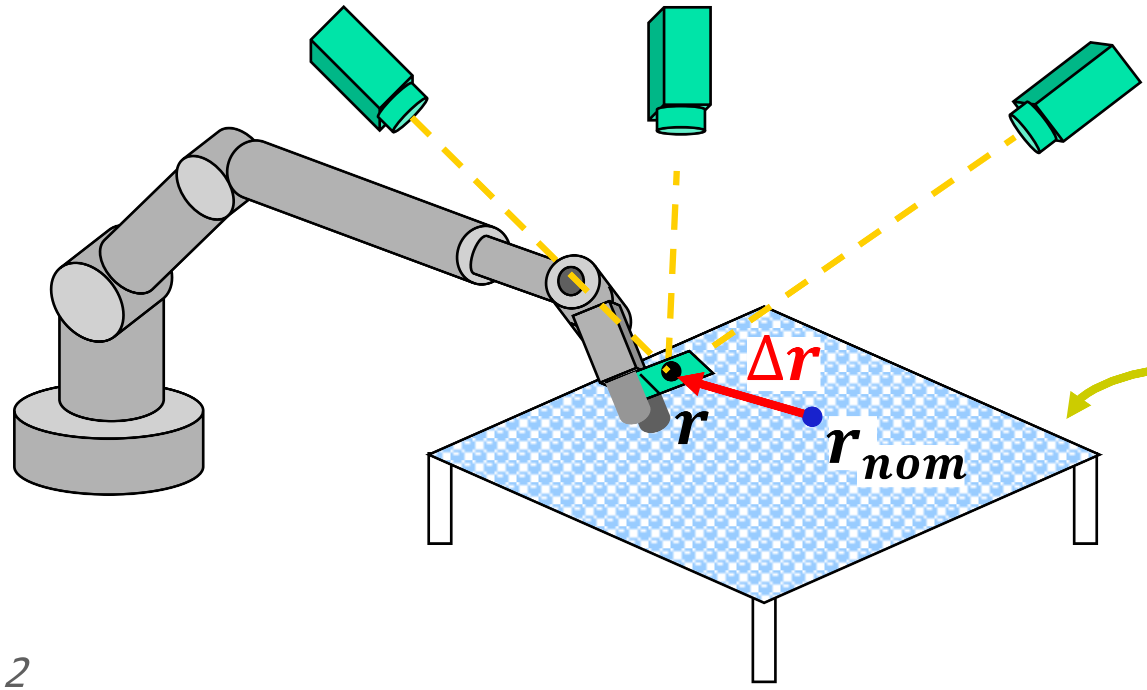
ground truth

$$\Delta r = r - r_{nom} = \frac{\partial f}{\partial \alpha} \cdot \Delta\alpha + \frac{\partial f}{\partial a} \cdot \Delta a + \frac{\partial f}{\partial d} \cdot \Delta d + \frac{\partial f}{\partial \theta} \cdot \Delta\theta$$

"small" errors

obtained by external measurement system

first-order variations



cameras + triangulation

calibration table

$$r = f(\alpha, a, d, \theta) \cong \overbrace{f(\alpha_{nom}, a_{nom}, d_{nom}, \theta_{nom})}^{r_{nom}} + \left.\frac{\partial f}{\partial X}\right|_{X=X_{nom}} \Delta X$$

current measure

measured

$$X = \begin{pmatrix} \alpha \\ a \\ d \\ \theta \end{pmatrix} \qquad \Delta X = X - X_{nom}$$

$$r - r_{nom} = \Delta r = \left.\frac{\partial f}{\partial X}\right|_{X=X_{nom}} \Delta X$$

$\Delta\theta$

↓

bias in the encoder reading

↓

difference between what you have and what you measure n the number of joints

Calibration equation

$4n \times 1$

$$\Delta\varphi = \begin{pmatrix} \Delta\alpha \\ \Delta a \\ \Delta d \\ \Delta\theta \end{pmatrix} \qquad \Phi = \begin{pmatrix} \frac{\partial f}{\partial \alpha} & \frac{\partial f}{\partial a} & \frac{\partial f}{\partial d} & \frac{\partial f}{\partial \theta} \end{pmatrix}$$

analytic Jacobian

min. 13

$\boxed{6 \times 4n}$

if the robot is 6dof we have 24 unknown variables

if the task is six dimensional

$$\boxed{\Delta r = \Phi \cdot \Delta\varphi}$$

evaluated at nominal conditions

6dim vector

ℓ experiments ($\ell \gg$ n)

$6\ell \times 1$ $\qquad$ $6\ell \times 4n$

$$\Delta\bar{r} = \begin{pmatrix} \Delta r_1 \\ \Delta r_2 \\ \vdots \\ \Delta r_\ell \end{pmatrix} \qquad \bar{\Phi} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_\ell \end{pmatrix}$$

theese matrices change for each measured configuration

$$\Delta\bar{r} = \bar{\Phi} \cdot \Delta\varphi$$

measures

unknowns

since I have much more equations it's difficult to find a set of vars that satisfy all of them

regressor matrix evaluated at nominal parameters

I have to guarantee this in order to find a minnorm sol

full column rank

(for sufficiently large ℓ)

because I will have much more rows (equations) than columns (unknowns)

Under- and over-determined systems of linear equations

only case in which A is non-singular

full rank

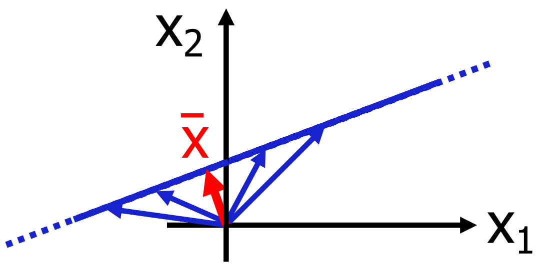
A → large matrix → full row rank

$$A x = b$$

full column rank ← tall matrix ← full rank

the columns are independent

A



1 equation → m
2 unknowns → n

$A \in \mathbb{R}^{1\times 2}$
$x \in \mathbb{R}^{2\times 1}$
$b \in \mathbb{R}^{1\times 1}$

$\boxed{n > m}$

minimum norm solution

$$\min \frac{1}{2}\|x\|^2, \text{ among } x:\ A x = b$$

→ minimum distance from the origin

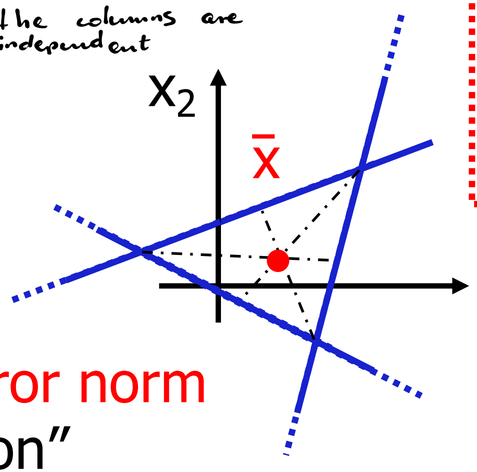
square full row rank

$$A^{\#} = A^T\left(A A^T\right)^{-1}$$

3 equations
2 unknowns

$A \in \mathbb{R}^{3\times 2}$
$x \in \mathbb{R}^{2\times 1}$
$b \in \mathbb{R}^{3\times 1}$

$\boxed{m > n}$



min. 26:30

We want a minimum norm solution because

minimum error norm "solution"

$$\min \frac{1}{2}\|Ax - b\|^2$$

$$A^{\#} = \underbrace{\left(A^T A\right)^{-1}}_{2\times 2 \text{ nonsing full column rank}}\ \underbrace{A^T}_{2\times 3} \in \mathbb{R}^{2\times 3}$$

$$\bar{x} = A^{\#} b$$

pseudoinverse !!

REDUNDANCY

if multiple solutions x exist (having the same minimum **error** norm), the pseudoinverse provides the one having **minimum norm**

KINEMATIC CALIBRATION

Calibration algorithm

$$\Delta\bar{r} = \bar{\Phi} \cdot \Delta\varphi$$

I select the DH parameters that minimize the distance between nominal direct kinematics and measured pose

$$\Delta\varphi = \bar{\Phi}^{\#}\Delta\bar{r} = (\bar{\Phi}^T\bar{\Phi})^{-1}\bar{\Phi}^T \cdot \Delta\bar{r}$$

minimizes $\frac{1}{2}\|\bar{\Phi}\,\Delta\varphi - \Delta\bar{r}\|^2$

$$\varphi_{nom} + \Delta\varphi = \varphi'$$

new set of DH parameters
+ "bias" on measures of θ

$$\Delta\bar{r}' = \bar{\Phi}' \cdot \Delta\varphi$$

evaluated with new values φ'

... ITERATE!

because we have taken a nonlin function and linearized it with Taylor → So it was only an approximation → around the initial guess

Improvement by kinematic calibration

- ABB IRB 120 6R industrial robot
- 1000 random configurations (collision-free by simulation)
- 50 arbitrary configurations used for measurement in calibration
- 950 configurations used for validation
- Cartesian position errors

	before calibration	after calibration
Average	1.746 mm	0.193 mm
Median	1.567 mm	0.180 mm
Standard Deviation	1.043 mm	0.085 mm
Min	0.050 mm	0.010 mm
Max	4.423 mm	0.516 mm

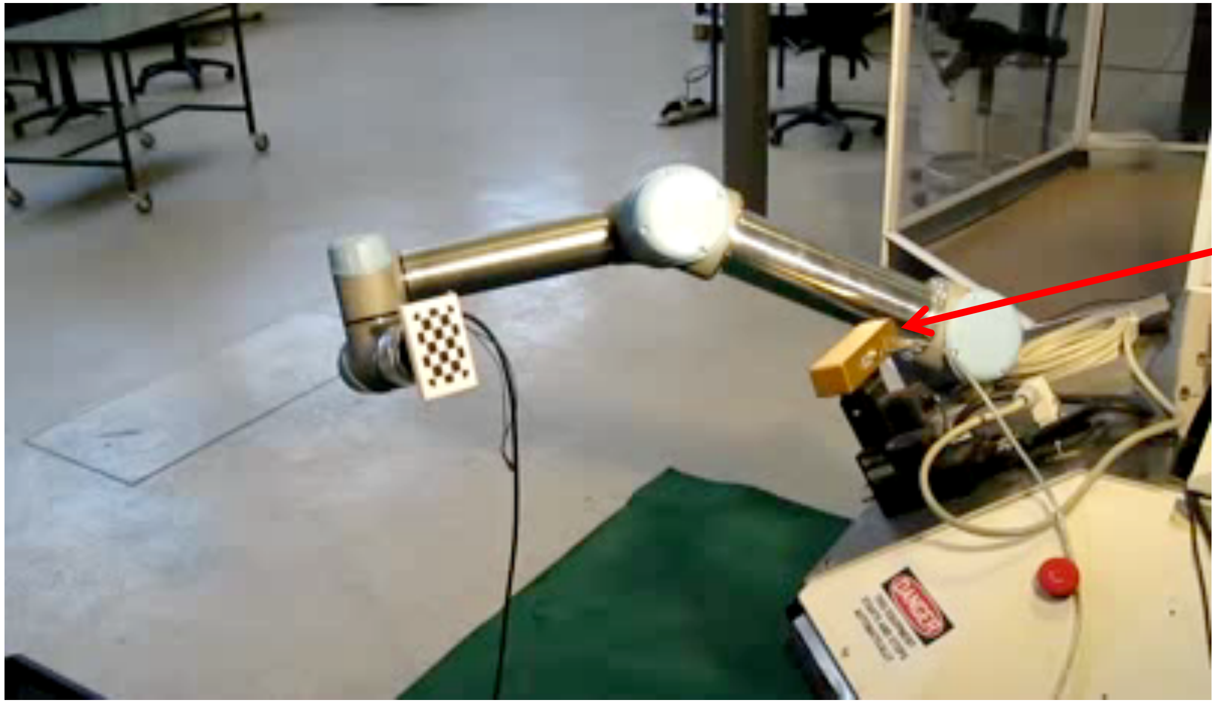
- Improvement by **a factor 8÷10**

Final comments

- an iterative least squares method
  - original problem is nonlinear in the unknowns, then linearized using first-order Taylor expansion
- it is useful to calibrate first and separately those quantities that are less accurate (typically, the encoder bias) 15.04.20
  - keeping the remaining ones at their nominal values min 50.00 k2. 2
- alternative kinematic descriptions can be used
  - more complex than D-H parameters, but leading to a better numerical conditioning of the regressor matrix in calibration algorithm
  - one such description uses the POE (Product Of Exponential) formula
- more in general, 6 base parameters should also be included
  - to locate 0-th robot frame w.r.t. world coordinate frame (of external sensor)
- accurate calibration/estimation of real parameters is a general problem in robotics (and beyond...)
  - for sensors (e.g., camera calibration)
  - for models (identification of dynamic parameters of a manipulator)

Calibration experiment
in a research environment

video



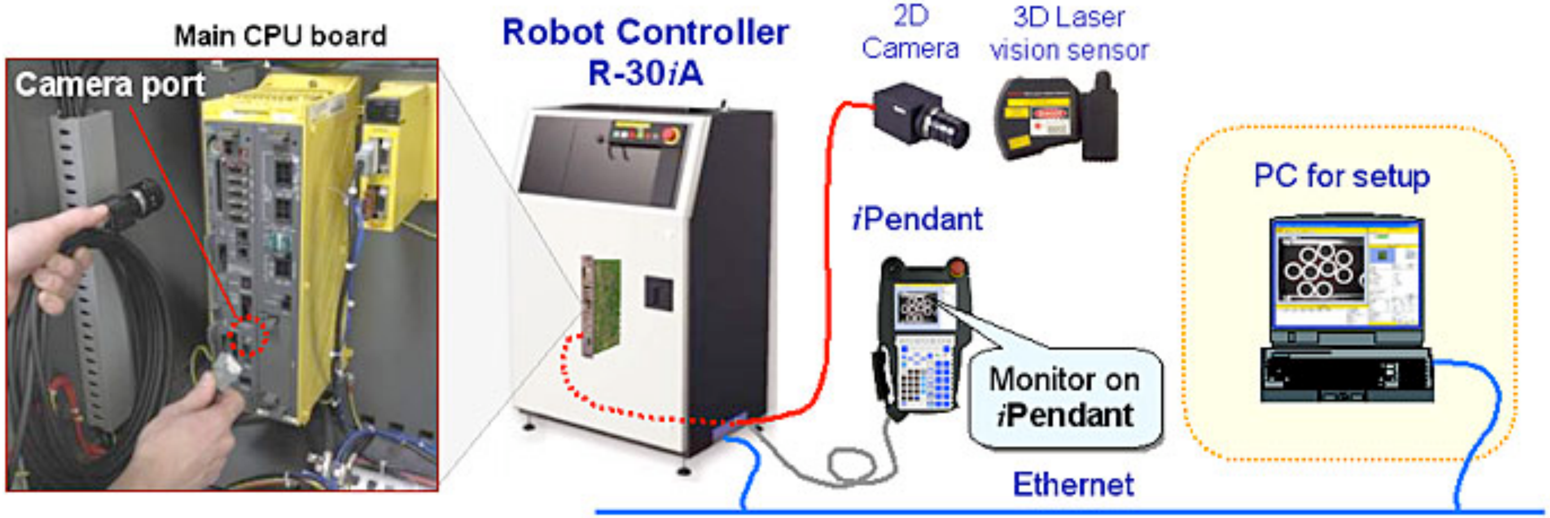
Videre Design
stereovision
camera

- automatic data acquisition for simultaneous calibration of
  - robot-camera transformation
  - DH parameters of the manipulator

Calibration experiment
in an industrial setting

FANUC
3D Laser
calibration
(with iR Vision)

video



i	α	a	d	θ
1	0	L_1	0	θ_1
2	0	L_2	0	θ_2
	ok		ok	

we consider only a and θ for calibration

Regressor matrix for single experiment ?

$$\Phi \in \mathbb{R}^{2\times 4}$$

$$\Delta q = \begin{bmatrix} \Delta a_1 \\ \Delta a_2 \\ \Delta \theta_1 \\ \Delta \theta_2 \end{bmatrix}$$

2x1 2x4 4x1

$$\Delta p = \Phi \, \Delta \varphi$$

$$\Delta r = \frac{\partial f}{\partial a} \Delta a + \frac{\partial f}{\partial \theta} \Delta \theta$$

$$\Phi = \begin{bmatrix} \frac{\partial f}{\partial a} & \frac{\partial f}{\partial \theta} \end{bmatrix}$$

2x2

Now direct kinematics

$$p = \begin{bmatrix} L_1 c_1 + L_2 c_{12} \\ L_1 s_1 + L_2 s_{12} \end{bmatrix} = f(a, \theta)$$

$$\frac{\partial f}{\partial \theta} = \begin{bmatrix} -L_1 s_1 - L_2 s_{12} & -L_2 s_{12} \\ L_1 c_1 + L_2 c_{12} & L_2 c_{12} \end{bmatrix}$$

$$\frac{\partial f}{\partial a} = \begin{bmatrix} c_1 & c_{12} \\ s_1 & s_{12} \end{bmatrix}$$

min 57.00