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A robust novel method for Face recognition from 2D Depth Images using DWT and DCT Fusion

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Abstract

Face recognition has been an area of interest in pattern recognition for past few decades. This paper focuses on the problems of person identification using 2D Face Depth data. The use of unregistered 2D Face depth data increases operational speed of the system with huge database enrollment. Here the unregistered 2D Face Depth data is fed to classifier in multiple spectral representations. Discrete Wavelet Transform (DWT) and Discrete Cosine Transform (DCT) are used for the spectral representations. Fusion of scores improves the recognition accuracy. Analysis which covers the FAR (False Acceptance Rate) and TRR (True Rejection Rate) is also done.

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Keywords: Rotation Invariance; Pose Correction, Depth Map; Spectral Transformations; and Principal Component Analysis

1. Introduction

3D Face recognition has been an active area of research in the past few decades. The complications encountered in the enrollment phase and the huge computational requirements in the implementation phase have been the major hindrance in this area of research. More over the use of costly 3D face scanners restricts the use of 3D face geometry completely for face recognition. So as an alternative, low cost 2D depth acquisition devices may be used to acquire the geometry of face as 2D images and apply the conventional 2D face recognition algorithms in such systems. Though poor resolution is a major drawback encountered in 2D face depth images, the geometrical information present in 2D depth facial database can be exploited to overcome the challenges in 3D face recognition systems like pose variations, bad illumination, ageing etc.

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In this work, focus is made on an identification problem based on 2D face depth data using fusion schemes. Identification corresponds to the person recognition without requiring any additional information other than with the 2D facial scan. The system arrives at an identity from among the enrolled faces in the database.

[2]Alexander M. Bronstein et al. proposed an idea of face recognition using geometric invariants using Geodesic distances. [3]C. Beumier utilized parallel planar cuts of the facial surfaces for comparison. [4]Gang Pan et al extracted ROI of facial surface by considering bilateral symmetry of facial plane. [5]Xue Yuan et al proposed a face recognition system using PCA, Fuzzy clustering and Parallel Neural networks. [6]Trina Russ et al proposed a method in which correspondence of facial points is obtained by registering 3D Face information to a scaled generic 3D reference face. [7]Ajmal Mian et al used Spherical Face Representation for identification. [8]Ondrej Smirg et al used DCT for gender classification since the DCT best describes the features after de-correlation. [9]Hua Gao et al used Active Appearance model for fitting faces with pose variations. [10]Naser et al used 2D-PCA for getting the feature matrix vectors and used Euclidean distance for classification. , [11]Omid Gervei et al proposed an approach for 3D Face recognition based on finding the principal components of range images by using modified PCA approaches namely 2D-PCA and bidirectional 2D-PCA.

A typical 3D Face is shown in Fig.1 with its axis level representation in Fig.2

Fig. 1. 3D Face Model

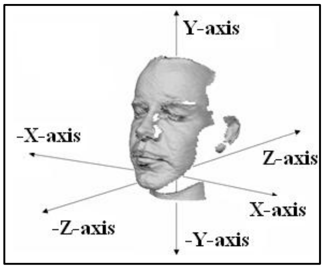

Fig. 2. 3D Face in Space

Fusion of the representative transformations from 2D Depth Spatial domain to 2D Spectral domain using 2D-DWT and 2D-DCT is discussed here. Since, only a sparse set of points in the 2D depth data are available, it is necessary to increase the data density by using multiple data representations. For this the data is transformed into spectral domain using 2D-DWT and 2D-DCT. This sparse set of data with occlusion can be effectively countered by invoking multiple score fusion schemes which can effectively improve the feature data density.

2. Proposed scheme for face recognition

The proposed scheme makes use of the 2D Spatial to 2D spectral domain transformation. The system aims at extracting the feature from the input data through multiple feature extraction tools and fuses the error scores to get a system with better recognition accuracy. The main feature extraction principle used in this system is the spectral transformation. The spectral transformation tools used here are 2D-DWT and 2D-DCT. These spectral transformations convert the data to be more consistent and more or less free from artifacts which increase the accuracy of the recognition systems.

The most important part of this work lies in the pattern classification problem. A pattern of data points is available. This pattern is not sufficient for the recognition system to work since the data will be highly occluded due to pose variations in the X, Y and Z axis.

The 2D Face recognition scheme is affected by pose variations of the subject. There are methods available in which the correction to this effect of pose variations also is included. One such method is the Iterative Closest Point (ICP) algorithm. But the main disadvantage of these methods is that a reference face is to be used as a model for other rotated faces to be corrected. Since we are having only 2D face depth data, for applying ICP, 2D to 3D space projection is necessary. Further, the reliability of this result depends on the accuracy in selection of the reference face model used. Therefore, in this work, the correction to the effect of pose variation is not considered. The method aims at recognizing the subject without much computationally complex mathematical procedure. Also, the results prove that the efficiency of the system is comparable with a system with pose correction. The idea behind spectral representation of data is that, when data is in spatial domain, comparison will be done as one to one pixel level or voxel level. So the rotation and translation of data will highly affect the result. When spectral transformation is used the distributed data will be concentrated or it may be represented in a more uniform way, i.e. the input data will be concentrated and represented uniformly in spectral domain. The translation and rotation invariance properties of the transformations used will aid to improve the accuracy of system significantly.

The data available for the analysis and testing will be in Depth Map format which is a matrix array of size M x N. For each face depth data input the number of depth points in 2D plane can be different.

The proposed method involves the following steps given below in sequence.

1) The 2D face depth data is first normalized with the maximum intensity value. From this 2D depth map nose tip is detected using Maximum Intensity Method and the area around the nose (ROI-Region of Interest) is extracted (Fig.3 and Fig.4).

2) On this ROI data, 2D-DWT and 2D-DCT are applied. The detailed explanations are given on following sections.

3) Once spectral representations are obtained, Principal Component Analysis (PCA) is applied on that data to get the corresponding weight vectors. These weight vectors are fed to a classifier which uses Euclidean distance for classification.

2.1. Depth Map Normalization

Depth map obtained is normalized with the maximum intensity value to make the depth data more visible. Here the depth values are normalized between the range 0 and 255.

$$\text{Normalized Depthmap}=\frac{\text{Original Depthmap}*255}{\text{Max}\,Intensity(\text{Original Depthmap})}$$

2.2. Nose tip Localization and face area extraction

For localizing the nose tip, Maximum Intensity Method is used. In this method, assumption is made that the nose tip will be the point with maximum pixel intensity. Once the nose tip is found, the circular area (ROI) around the nose tip is extracted using an optimum radius of 55. Now, the depth map will contain the face area only, all other unwanted portions are cropped away. Next the face area is centralized by making the nose tip as the center pixel of the image as otherwise the matching process will result in a lower accuracy. The face area is also normalized by the maximum intensity. The centralized face image is as shown in Fig.3 and Fig.4.

Fig. 3. Depth Map

Fig. 4. ROI from Depth Map

2.3. 2D Discrete Wavelet Transform

The main advantage of Wavelet Transform is the availability of both spatial and spectral information simultaneously. Here the 2D Depth data is decomposed into four subband using 'Haar' wavelet. It may be noted that approximation coefficients and horizontal detail coefficients alone are taken for forming the feature vector since the use of diagonal and vertical detail coefficients do not seem to improve the efficiency of the system. The scaling and wavelet functions are as given in equation 1, 2, 3 and 4.

$$\varphi\ n1, n2\ = \varphi\ n1\ * \varphi\ n2 \quad (1)$$

$$\psi^{H}\ n1, n2\ = \psi\ n1\ * \varphi\ n2 \quad (2)$$

$$\psi^{V}\ n1, n2\ = \varphi\ n1\ * \psi\ n2 \quad (3)$$

$$\psi^{D}\ n1, n2\ = \psi\ n1\ * \psi(n2) \quad (4)$$

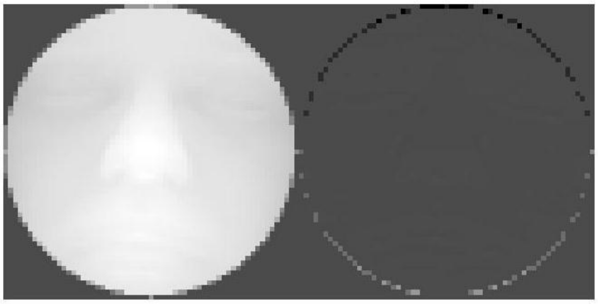

Fig. 5. 2D-DWT of Depth Data with Approximation Coefficients and Horizontal Detail Coefficients

The analysis equations are given below.

$$W_{\phi}\ j_{0,}m, n\ = \frac{1}{MN}\ {}_{x=0}^{M-1}\quad {}_{y=0}^{N-1} f\ x, y\ \phi_{j_{0,m,n}}\ (x, y) \quad (5)$$

$$W_{\psi}^{H}\ J, m, n\ = \frac{1}{MN}\ {}_{x=0}^{M-1}\quad {}_{y=0}^{N-1} f\ x, y\ \phi_{j_{0,m,n}}^{H}\quad x, y \quad (6)$$

The above equations are used for obtaining the coefficients of LL and HL bands. Also the feature values formed by the horizontal concatenation of the approximation and horizontal detail coefficients are used. These feature values are in turn projected to the PCA Space using 2D PCA.

2.4. 2D Discrete Cosine Transform

The Depth image is transformed to spectral domain using 2D-DCT. The energy compaction will take place and the result will be again an M x N matrix. The result is shown in Fig.6. Here, the global feature extraction by DCT is made used of. DCT has the property of de-correlation which enables the data structure to loose spatial pixel dependency. The low frequency components which mainly form the facial features will be prominent in the transformed data which makes the pattern classification more reliable, as the human eyes are more sensitive to information in low frequency spectrum.
Transformation to spectral domain using 2D-DCT can be done using equation (7) and the transformed image is shown in Fig.6.

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\cos(\frac{(2x+1)\pi u}{2M})\cos(\frac{(2y+1)\pi v}{2N}), \text{ for a } M \times N \text{ depth image.} \quad (7)$$

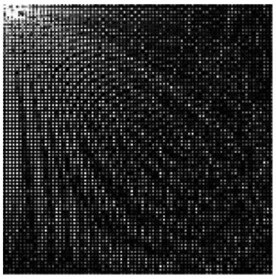

Fig. 6. DCT Representation of ROI Depth Map

2.5. Principal Component analysis

Use of spectral transformations will make the data samples almost spatially uncorrelated. Even then, some spatial dependency may exist. So [11]Principal Component Analysis (PCA) which uses the orthogonal transformations to get linear uncorrelated data sets called Principal Components is employed. Conventional covariance method is used for the above. To start with feature extraction using 1D-PCA is discussed for better understanding.

Let X_i be the spectral transformed 1D data which representations i^{th} person, it is grouped as a M' x N matrix $X=[X_1 X_2 \ldots X_N]$, where N is the number of face samples under consideration and M' is the length of each feature vector.

Mean vector is calculated as follows

$$X_m = \frac{1}{N}\sum_{i=1}^{N} X_i \tag{8}$$

Standard deviation is calculated as

$$X_{SD} = \frac{1}{N}\sum_{i=1}^{N}(X_i - X_m) \tag{9}$$

Covariance matrix is calculated as

$$X_{COV} = X_{SD^*} X^T_{SD} \tag{10}$$

Here, the covariance matrix is of size M' x M', which is of very large dimension. Also it gives M' Eigen values and M' Eigen vectors which are too large in number to process. Therefore, dimensional reduction is adopted by altering the construction of covariance matrix as follows.

$$X_{COV} = X^T_{SD^*} X_{SD} \tag{11}$$

The result is a matrix having dimension size N x N, where N is the number of subjects under consideration. It gives N Eigen values and N Eigen vectors. The Eigen values are sorted in descending order and the first N' largest Eigen values and corresponding Eigen vectors are selected as others are insignificant. Eigen vectors in N' dimension is transformed to the higher dimension of M' by multiplying with Standard deviation Matrix. The test data is projected to this lower dimension space to get the corresponding weight vectors.

Now feature extraction using 2D-PCA is considered using spectral representation of depth map. The only difference with 1D-PCA in calculating the Covariance matrix is that here a 2D matrix is used when compared to 1D Matrix in 1D-PCA. After determining the Eigen values and Eigen vectors, a 2D weight vector matrix is obtained which is then converted to a column matrix.

2.6. Score Fusion

Next the error score is estimated using all the multiple representations separately. For processing 2D DWT and 2D DCT representation, 2D-PCA is used. 1D-PCA was also experimented for the 2D representations but 2D-PCA gave better result for 2D representations. Error values are calculated for each data representations and all this error values are combined as a single error value using the linear expression as given in equation 12.

$$\text{Error} = \text{W*Error\_DWT\_Norm} + (1\text{-W})\text{*Error\_DCT\_Norm} \tag{12}$$

$$\text{Error\_DWT\_Norm} = \frac{\text{ErrorDWT} - \min(\text{ErrorDWT})}{\max(\text{ErrorDWT}) - \min(\text{ErrorDWT})} \tag{13}$$

$$\text{Error\_DCT\_Norm} = \frac{\text{ErrorDCT} - \min(\text{ErrorDCT})}{\max(\text{ErrorDCT}) - \min(\text{ErrorDCT})} \tag{14}$$

By trial and error, the optimum value for W can be approximately obtained in the range -1 and +1.

3. Performance Analysis

3.1. Results of Individual representation accuracy analysis

For analysis and testing, FRAV3D database excluding point cloud data is used, so that it is analogues to 2D depth images acquired using low cost scanners. It contains 106 subjects. Of these 100 subjects were taken into consideration. Testing was done on the input data with rotation on X, Y and Z axis separately. For each degree of rotation 200 samples were tested and for Y and Z axis 1600 samples at $+5^o$,-5^o, $+10^o$,-10^o, $+15^o$,-15^o, $+20^o$ and -20^o pose variations were tested. Comparison of the fusion scheme along with the direct 2D-PCA on depth data is also shown here (Table 1). A substantial change in the recognition accuracy is observed. Table 1 show the recognition accuracy obtained with different data representations in spatial domain when the pose variation is along X, Y and Z axis. Rotation in degree means the combined pose variation tested towards right and left direction. As the orientation increases the accuracy falls to a lower value when used as a single data representation. Table 2 show the recognition accuracy obtained when 2D DWT and 2D-DCT are applied on face depth data, whose pose variation is along Z axis. Similarly Table 3 and Table4 denote the pose variations along X and Y axis respectively.

Table 1: Recognition accuracy achieved without Spectral Transformation

Rotation in Degree	X-axis %	Y-axis %	Z-axis %
+5,-5	55	61.5	92
+10,-10	20.5	22	87.5
+15,-15	6	12	69
+20,-20	4	7	53

Table 2: For Rotation in Z Axis with 10 to 40 Degree Variations

Rotation in Degree	2D-DWT FRA %	2D-DCT FRA %
+5,-5	95.50	94.50
+10,-10	92.50	94.50
+15,-15	80.00	82.00
+20,-20	63.50	59.00

Table 3: For Rotation in X Axis with 10 to 40 Degree Variations

Rotation in Degree	2D-DWT FRA %	2D-DCT FRA %
+5,-5	81.00	77.50

Table 4: For Rotation in Y Axis with 10 to 40 Degree Variations

Rotation in Degree	2D-DWT FRA %	2D-DCT FRA %
+5,-5	81.00	94.50
+10,-10	57.00	89.50
+15,-15	56.50	80.00
+20,-20	33.50	49.50

Where FRA % - Face Recognition Accuracy in percentage

Here the recognition accuracy using 2D DWT maintains the minimum accuracy up to 63.5% for Z axis (Ref Table 2), 33.5% for Y axis (Ref Table 3). For 10^{o} variations about Z axis, the accuracy is above 90%, while for other axis it is above 80% only. Rotation in X axis is the worst affected case, since it will result in partial/complete occlusion of data in the depth map. So 10 degree pose variation in X axis alone is considered while testing the algorithm.
For Z axis, 2D DWT seems to have improved recognition accuracy when pose variation increases while for Y axis 2D DCT maintains higher accuracy throughout the pose variations. So both DWT and DCT is used together for improving the accuracy.

3.2. Results of Fusion Scheme analysis

The Tables 5, 6 and 7 shown summarize the results obtained by using fusion scheme. In Z axis, the data points are completely redistributed, so that the spatial domain comparison gives lower accuracy for higher pose variations. Accuracy of Z axis pose variations ranges from 65% to 96% as shown in Table 5. With 40^{o} pose variation along Y axis, nearly half of the face area is occluded. So the data set will be a sparse dataset and the accuracy ranging from 63% to 94.5% (Ref Table 7) is obtained. In X axis the accuracy falls tremendously since some face data near the jaw points are missing there in addition to the reduction in the projected area of nose tip and other geometric features. Table 8 and 9 shows the results tested over 800 samples in Z and Y axis.

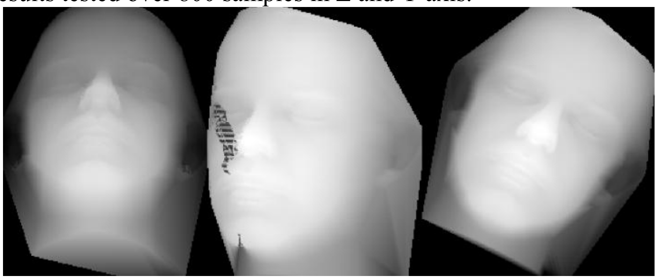

Fig. 7. Depth Map with different pose variations along X, Y and Z axis.

Table 5: For Rotation in Z Axis with 10 to 40 Degree Variations

Rotation in Degree	Using 2D-DWT FRA %	Using 2D-DCT FRA %	Using FUSION FRA %
+5,-5	95.50	94.50	96.00

+10,-10	92.50	94.50	95.00
+15,-15	80.00	82.00	84.00
+20,-20	63.50	59.00	65.00

Table 6: For Rotation in X Axis with 10 to 40 Degree Variations

Rotation in Degree	Using 2D-DWT FRA %	Using 2D-DCT FRA %	Using FUSION FRA %
+5,-5	81.00	77.50	83.00

Table 7: For Rotation in Y Axis with 10 to 40 Degree Variations

Rotation in Degree	Using 2D-DWT FRA %	Using 2D-DCT FRA %	Using FUSION FRA %
+5,-5	81.00	94.50	94.50
+10,-10	57.00	89.50	92.50
+15,-15	56.50	80.00	86.50
+20,-20	33.50	49.50	63.00

Table 8: Recognition Accuracy for Rotation along X Axis

Axis	Samples	2D-DWT	2D-DCT	FUSION
Z Axis	80	86.25	81.25	87.50
	160	84.38	81.88	85.63
	240	86.25	85.42	88.75
	320	86.25	85.63	88.44
	480	83.33	84.38	86.46
	560	81.25	82.32	84.11
	640	81.88	82.34	84.06
	800	82.38	82.38	84.25

Table 9: Recognition Accuracy for Rotation along X Axis

Axis	Samples	2D-DWT	2D-DCT	FUSION
Y Axis	80	63.75	75.00	78.75
	160	60.63	80.00	83.75
	240	62.92	81.67	85.83
	320	61.88	83.13	87.81
	480	60.21	80.42	86.25
	560	58.57	80.18	85.71
	640	59.53	79.84	84.84
	800	57.00	78.38	84.13

3.3. Improvement in accuracy rates by Fusion Scheme

When the accuracy obtained in the fusion scheme is analyzed, the improvement in maximum accuracy for the Z-axis is about 1.23 times for representation of data in spectral domain as shown in Table 10. But for Y axis, it reaches a maximum of 9 times and for X axis it is about 1.51 times.
The highest rate of improvement using multiple/fusion spectral representation is almost 9 times the existing accuracy for rotation in Y axis using individual representation.

Table 10: Improvement in recognition accuracy over the accuracy achieved Without spectral transformation

Rotation in Degree	X-axis	Y-axis	Z-axis
+5,-5	1.51	1.54	1.04
+10,-10	X	4.20	1.09
+15,-15	X	7.21	1.22
+20,-20	X	9.00	1.23

An average an improvement (FRRI-Face Recognition Rate Improvement) of 1.51, 5.49 and 1.14 times in accuracy along for X, Y and Z axis respectively is obtained when the depth data is used without pose correction with multiple spectral representations as input to PCA to get the weight vectors. The improvement is tabulated by comparing the FRA obtained when testing done using a spatial domain data. X denotes insignificant results for the higher pose variations along X axis.
This result is comparable with the existing systems using pose correction. Table 5, 6 and 7 shows the improvement in recognition with the fusion scheme invoked, but without pose correction. Graphical representation of the table is also shown in Fig.8 and Fig.9 with the pose angle along the X axis and Face recognition rate along the Y axis.

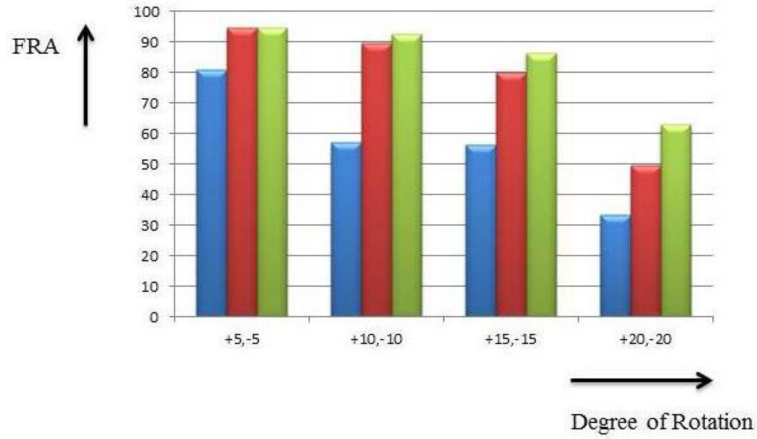

Fig. 8. Accuracy Comparison for Y axis Pose Variation

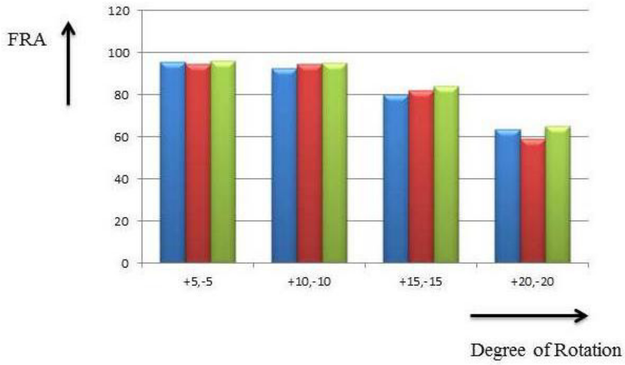

Fig. 9. Accuracy Comparison for Z axis Pose Variation

3.4. Reliability Test of the Face recognition system with Fusion scheme

The algorithm is tested for robustness against the chances of wrong user authentication (FAR) and the denial of authentication to enrolled users (TRR). The fusion scheme improves the rejection of unauthorized access of a user not enrolled in the database by a factor of 2, i.e in this case the false rejection rate which was near 51.50% was made 100% with the invoke of fusion scheme. Table 11 and 12 summarizes the results of TAR, TRR, FRR and FAR analysis done along Z axis oriented and Y axis oriented data.

Table 11: Reliability test done along Z axis

Scheme	True Acceptance Rate	True Rejection Rate	False Rejection Rate	False Acceptance Rate
	TAR %	TRR %	FRR %	FAR %
2D-DWT	94.00	6.00	51.50	48.50
2D-DCT	96.50	3.50	51.50	48.50
FUSION	96.00	4.00	100.00	0.00

Table 12: Reliability test done along Y axis

Scheme	True Acceptance Rate	True Rejection Rate	False Rejection Rate	False Acceptance Rate
	TAR %	TRR %	FRR %	FAR %
2D-DWT	96.00	4.00	100.00	0.00
2D-DCT	96.67	3.33	100.00	0.00
FUSION	97.33	2.67	100.00	0.00

It seems that the fusion scheme has reduced the true rejection rate of individual scheme and has improved the false rejection rate by double the factor.

3.5. Computation time

Testing of algorithm is done on 3GHz, Core I-5 processor; the average testing time for a single sample is approximately 206.73 milliseconds. That is 206.73ms for testing and validating a single user. This will again increase as the number of subjects increase. By using down sampling and abstracting the data representations computational time can be further reduced. But with a dedicated system, the testing time can be further reduced to microseconds.

4. Conclusions

The fusion algorithm is tested on unregistered 2D Depth Faces with orientations starting from 10^{o} to 40^{o}. The algorithm gives a maximum accuracy of 96% for pose variations along Z axis, 94.50% along Y axis and 83% along X axis for 10 Degree pose variations and decreases as rotation angle increases. On an average, the algorithm gives an average accuracy of 84.12% and 84.25% FRA, when tested together over 800 samples each along Y axis and Z axis respectively. The algorithm provides a low FRA when tested with pose variations above 10 degree along X axis. The experimental results show that the features can be effectively extracted from 2D depth data using spectral transformation. Here fusion experiments were conducted at score level. There is ample scope for further improvement using more fusion schemes at the representation level and at spectral level. The method can also be implemented in real time systems since the processing time required is less.

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