Magnetic Analogue of the Haldane Honeycomb Model

Christina Lee, Judit Romhanyi, Nic Shannon

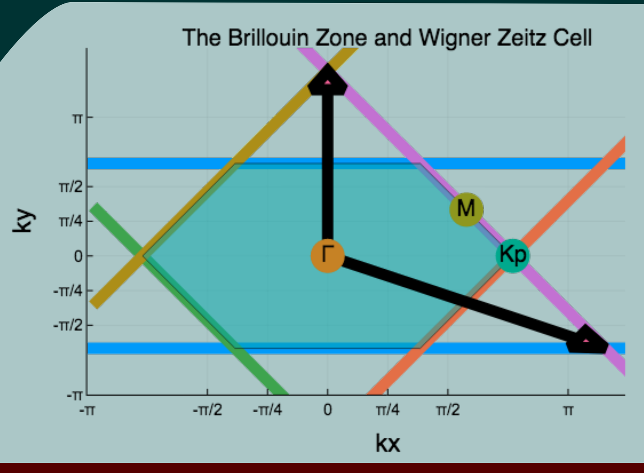
christina.lee@oist.jp Github: @albi3ro Twitter: @PyrochloreLee

Dzyaloshinskii-Moriya Interaction

Heisenberg Interaction

Github @ oist/Haldane_Analog Replicate Yourself!

The Model



$$\mathcal{H} = t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j + \sum_{\langle\langle i,j \rangle\rangle} e^{-i\nu_{i,j}\phi} c_i^\dagger c_j - M \sum_i \varepsilon_i c_i^\dagger c_i$$

How?

Spin-Orbit Coupling

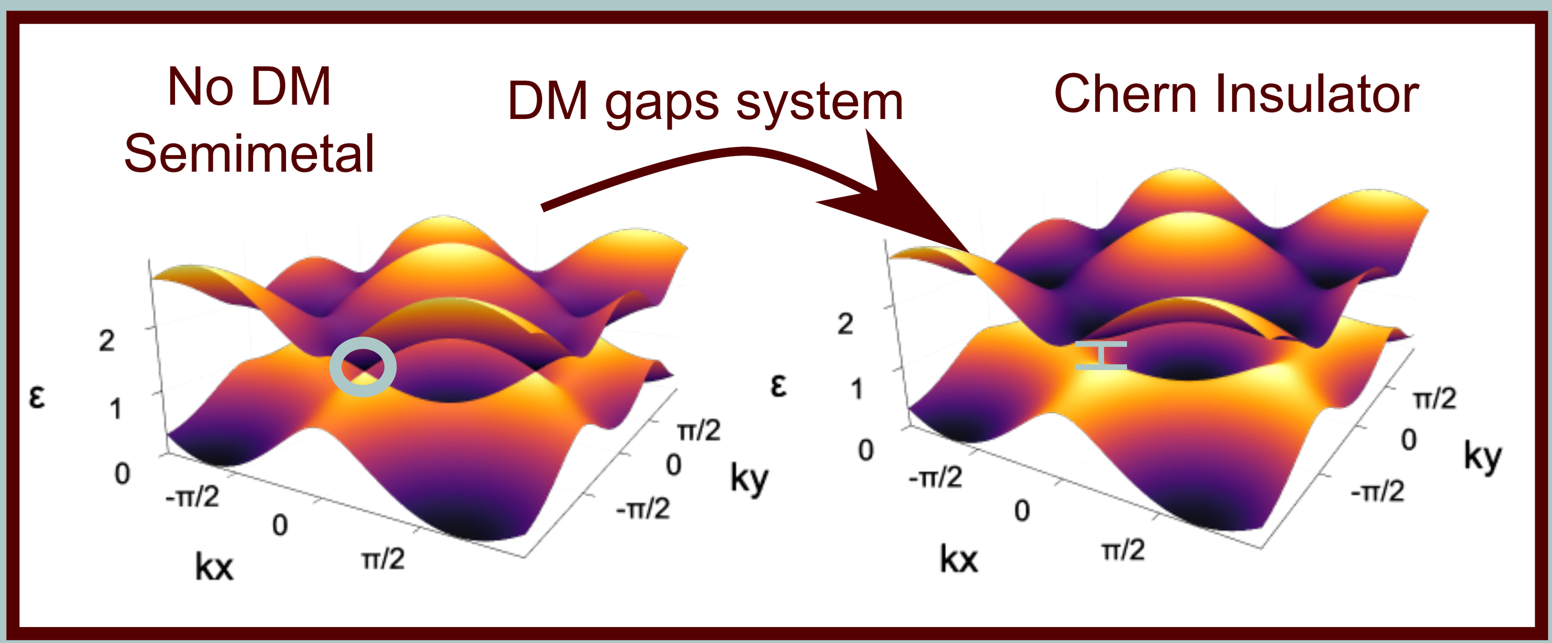
$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{\langle\langle i,j \rangle\rangle} \mathbf{D}_{i,j} \cdot \mathbf{S}_i \times \mathbf{S}_j$$

Holstein-Pirmakoff Transform

Fourier Transform

$$\tilde{\mathcal{H}} = J \begin{bmatrix} a_i^\dagger \\ b_i^\dagger \end{bmatrix}^T \begin{bmatrix} -\frac{3}{2} + D'g(k) & f(k) \\ f(k)^* & -\frac{3}{2} - D'g(k) \end{bmatrix} \begin{bmatrix} a_i \\ b_i \end{bmatrix}$$

$$= R_0(k)\mathbf{I} + \vec{R}(k) \cdot \vec{\sigma}$$



$$\varepsilon(k) = \pm\sqrt{f(k)^2 + D'^2 g(k)^2}$$

Haldane, F. D. M. Model for a quantum hall effect without landau levels: Condensed-matter realization of the 'parity anomaly'. Phys. Rev. Lett. 61, 2015–2018 (1988)
Kim, H.-S. & Kee, H.-Y. Realizing Haldane Model in Fe-based Honeycomb Ferromagnetic Insulators. npj Quantum Mater. 1–4 (2016). doi:10.1038/s41535-017-0021-z
Owerre, S. A. Topological honeycomb magnon Hall effect: A calculation of thermal Hall conductivity of magnetic spin excitations. J. Appl. Phys. 120, (2016)..

Topology

Lower Band $|\phi\rangle$

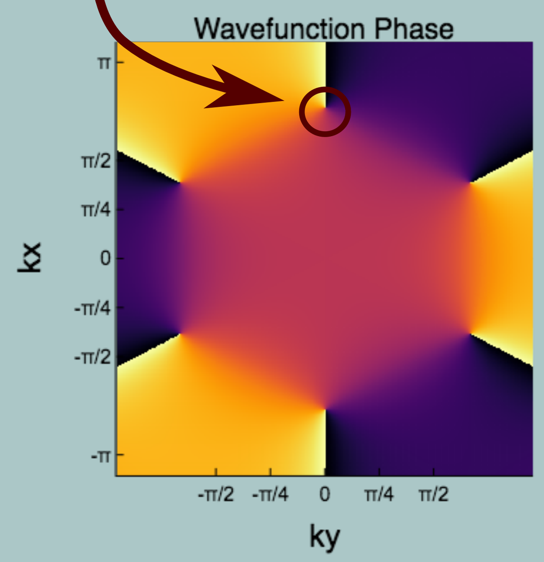
Phase Undefined

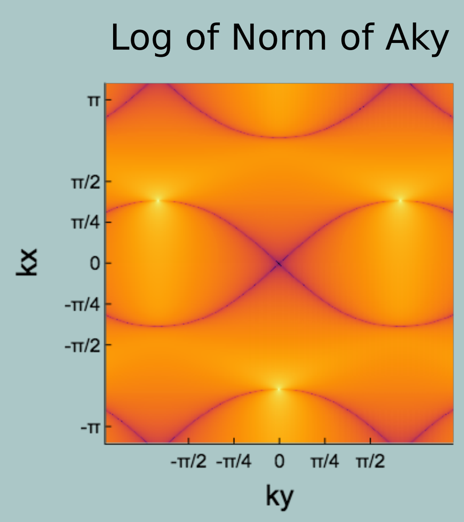
Connection

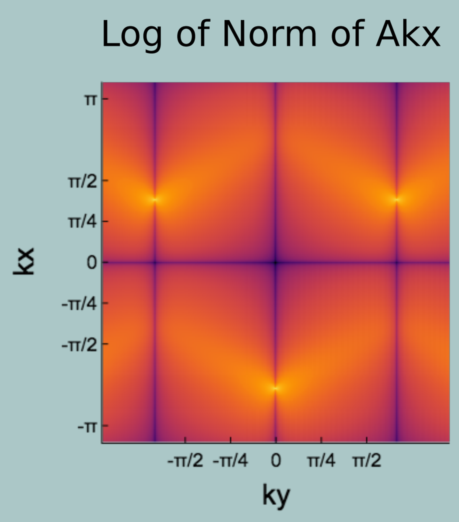
$$A^i = \langle \phi | \partial_i \phi \rangle$$

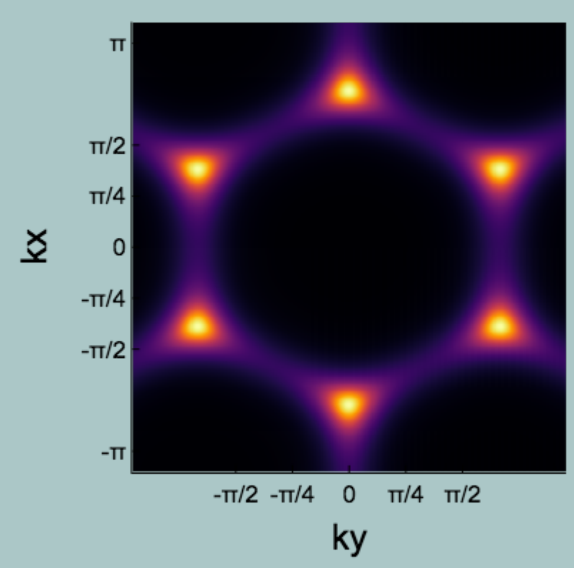
Curvature

$$F = \partial_{k_x} A^{k_y} - \partial_{k_y} A^{k_x}$$









Chern Number

$$Ch_1 = \frac{1}{2\pi i} \iint F d^2k$$

1

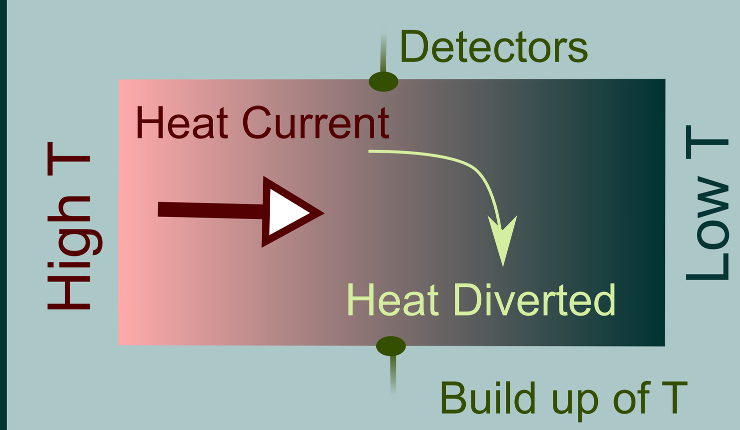
Stone, M. & Goldbart, P. Mathematics For Physics. Cambridge University Press (Cambridge University Press, 2011). doi:10.1017/CBO9780511627040

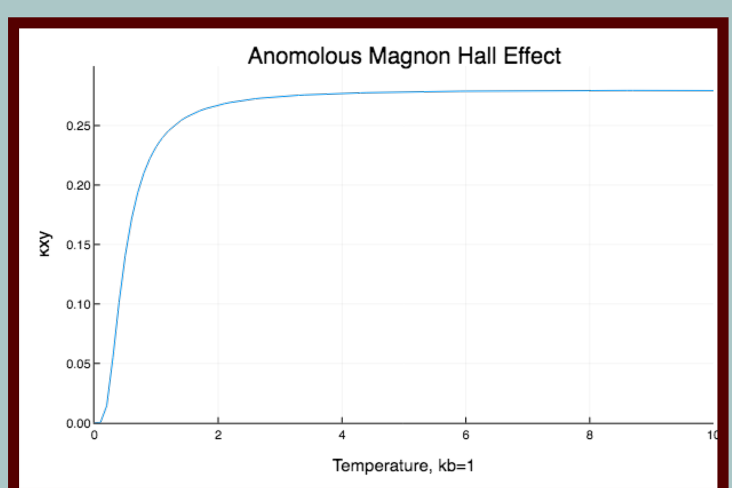
Thermal Conductivity

Analogue of QHE conductivity

Wavefunction Curvature bends magnons

$$\frac{q_x}{\nabla_y T} = -\frac{k_b T}{(2\pi)^2} \sum_\lambda \int_{BZ} \mathrm{d}^2 k c_2(n_\lambda) F_\lambda(k)$$



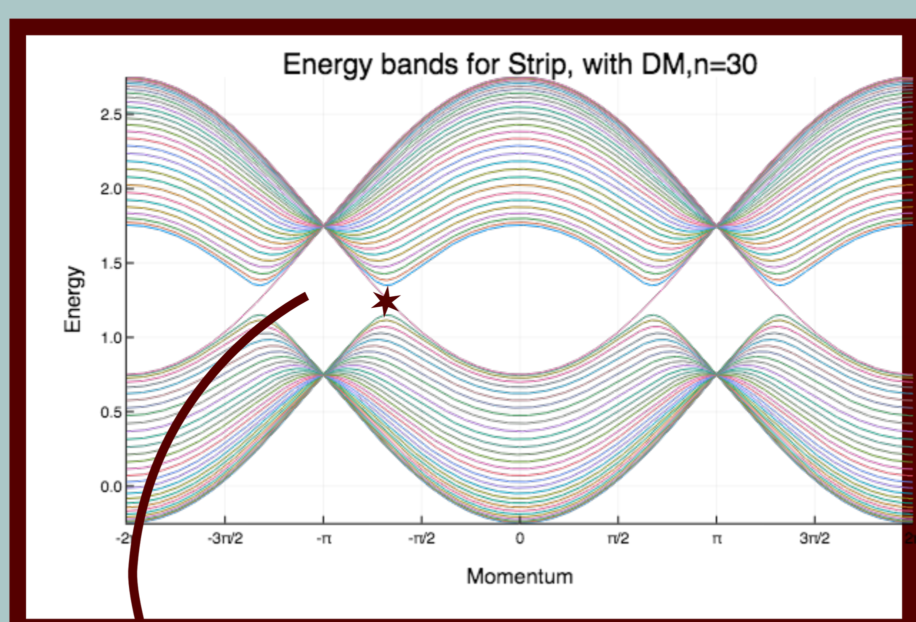


Onose, Y. et al. Observation of Magnon Hall Effect. 297, 297–300 (2010).
Murakami, S. & Okamoto, A. Thermal hall effect of magnons. J. Phys. Soc. Japan 86, 1–11 (2017).
Katsura, H., Nagaosa, N. & Lee, P. A. Theory of the thermal hall effect in quantum magnets. Phys. Rev. Lett. 104, 1–4 (2010).

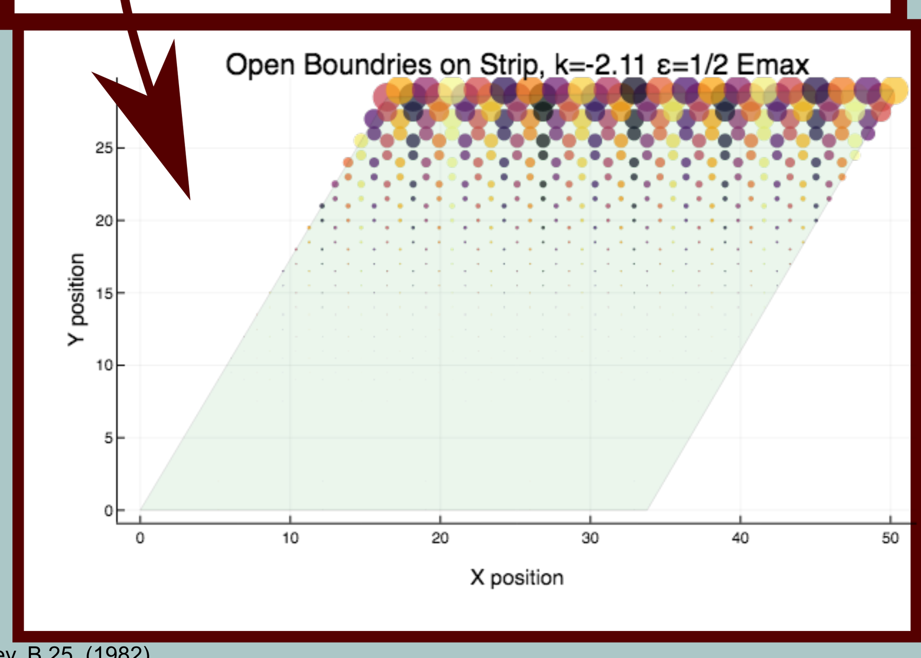
Edge Modes

Bulk-Boundary Correspondance

Conducting localized surface



At boundary, transitions from topological phase to trivial phase



To do so, must close gap

Halperin, B. I. Quantized Hall Conductance. Phys. Rev. B 25, (1982)
Mong, R. S. K. & Shivamoggi, V. Edge states and the bulk-boundary correspondence in Dirac Hamiltonians. Phys. Rev. B - Condens. Matter Mater. Phys. 83, 1–15 (2011)..

Thanks to the Okinawa Institute of Science and Technology Graduate University and the Theory of Quantum Matter Unit.

OIST