If temporal term is discretized with forward Euler method, we get

$$\frac{V}{\Delta t}(q^{n+1} - q^n) = -\sum R(q^{n+1}). \tag{1}$$

Denote new time level as $s+1$ instead of $n+1$ to obtain

$$\frac{V}{\Delta t}(q^{s+1} - q^n) = -\sum R(q^{s+1}). \tag{2}$$

Add pseudo temporal term, $\frac{V}{\Delta \tau}(q^{s+1} - q^s)$ to the LHS to obtain

$$\frac{V}{\Delta \tau}(q^{s+1} - q^s) + \frac{V}{\Delta t}(q^{s+1} - q^n) = -\sum R(q^{s+1}). \tag{3}$$

When steady state is reached in pseudo time, $q^{s+1} = q^s$, therefore, Equation (2) and in turn Equation (1) are recovered. Substract $\frac{V}{\Delta t} q^s$ from both sides:

$$\frac{V}{\Delta \tau}(q^{s+1} - q^s) + \frac{V}{\Delta t}(q^{s+1} - q^n) - \frac{V}{\Delta t} q^s = -\sum R(q^{s+1}) - \frac{V}{\Delta t} q^s. \tag{4}$$

Change positions of $q^s$ and $q^n$ in second and third terms on RHS:

$$\frac{V}{\Delta \tau}(q^{s+1} - q^s) + \frac{V}{\Delta t}(q^{s+1} - q^s) - \frac{V}{\Delta t} q^n = -\sum R(q^{s+1}) - \frac{V}{\Delta t} q^s. \tag{5}$$

Put the last term on LHS to RHS so that

$$\frac{V}{\Delta \tau}(q^{s+1} - q^s) + \frac{V}{\Delta t}(q^{s+1} - q^s) = -\sum R(q^{s+1}) - \frac{V}{\Delta t}(q^s - q^n). \tag{6}$$

Simplify the LHS to obtain

$$\boxed{V\left(\frac{1}{\Delta \tau} + \frac{1}{\Delta t}\right)(q^{s+1} - q^s) = -\sum R(q^{s+1}) - \frac{V}{\Delta t}(q^s - q^n).} \tag{7}$$