If temporal term is discretized with three-time-level method, we get

$$\frac{V}{2\Delta t}(3q^{n+1} - 4q^{n} + q^{n-1}) = -\sum R(q^{n+1}). \tag{1}$$

Denote new time level as $s+1$ instead of $n+1$ to obtain

$$\frac{V}{2\Delta t}(3q^{s+1} - 4q^{n} + q^{n-1}) = -\sum R(q^{s+1}). \tag{2}$$

Add pseudo temporal term, $\frac{V}{\Delta \tau}(q^{s+1} - q^{s})$ to the LHS to obtain

$$\frac{V}{\Delta \tau}(q^{s+1} - q^{s}) + \frac{V}{2\Delta t}(3q^{s+1} - 4q^{n} + q^{n-1}) = -\sum R(q^{s+1}). \tag{3}$$

When steady state is reached in pseudo time, $q^{s+1} = q^{s}$, therefore, Equation (2) and in turn Equation (1) are recovered. Substract $\frac{3V}{2\Delta t}q^{s}$ from both sides:

$$\frac{V}{\Delta \tau}(q^{s+1} - q^{s}) + \frac{V}{2\Delta t}(3q^{s+1} - 4q^{n} + q^{n-1}) - \frac{3V}{2\Delta t}q^{s} = -\sum R(q^{s+1}) - \frac{3V}{2\Delta t}q^{s}. \tag{4}$$

Change positions of q^{s} with q^{n} and q^{n-1} in second and third terms on RHS:

$$\frac{V}{\Delta \tau}(q^{s+1} - q^{s}) + \frac{3V}{2\Delta t}(q^{s+1} - q^{s}) + \frac{V}{2\Delta t}(-4q^{n} + q^{n-1}) = -\sum R(q^{s+1}) - \frac{3V}{2\Delta t}q^{s}. \tag{5}$$

Put the last term on LHS to RHS so that

$$\frac{V}{\Delta \tau}(q^{s+1} - q^{s}) + \frac{3V}{2\Delta t}(q^{s+1} - q^{s}) = -\sum R(q^{s+1}) - \frac{V}{2\Delta t}(3q^{s} - 4q^{n} + q^{n-1}). \tag{6}$$

Simplify the LHS to obtain

$$V\left(\frac{1}{\Delta \tau} + \frac{3}{2\Delta t}\right)(q^{s+1} - q^{s}) = -\sum R(q^{s+1}) - \frac{V}{2\Delta t}(3q^{s} - 4q^{n} + q^{n-1}). \tag{7}$$