EE324 Control Systems Lab

Problem Sheet 6

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Contents

1 Q1

a)

Open Loop Transfer Function is

$$\mathrm{G}(s) = \frac{1}{(s+3)(s+4)(s+12)}$$

$$SSE = 0.489 = \lim_{s\to 0} \frac{sR(s)}{1 + K_p \cdot G(s)} \quad \to \quad K_p = \frac{\frac{1}{0.489} - 1}{\frac{1}{3\cdot 4\cdot 12}} = 150.47852761 \quad \left(R(s) = \frac{1}{s}\right)$$

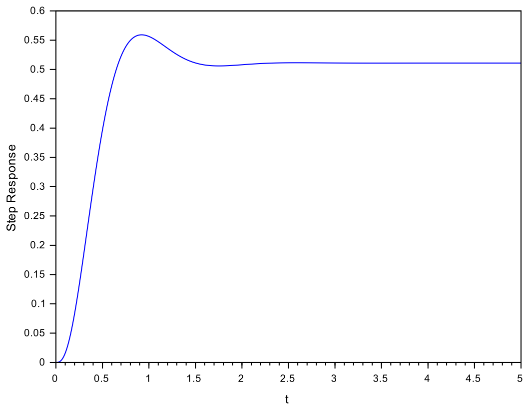


Figure 1.1: Step Response of $G(s)$ for $K = 150.47852761$

```
s = %s;
tMax = 5;
tStep = 0.01;
t = 0:tStep:tMax;
K = (1/0.489 - 1)/(1/(3*4*12));
TF = K/((s+3)*(s+4)*(s+12));
G = TF / (1 + TF);
sys = syslin('c', G);
gp = csim('step', t, sys);
plot(t,gp); xlabel(" t"); ylabel(" Step Response");
xs2pdf(0,'Q1a');
```

b)

To attain $\zeta = 0.35$,

we need to find intersection between the line of slope $= -\frac{\sqrt{1-\zeta^2}}{\zeta} = -2.6764277136$ and the root locus.

Both intersected at $-2.061 + 5.516\iota$, giving $K = 371.9$.

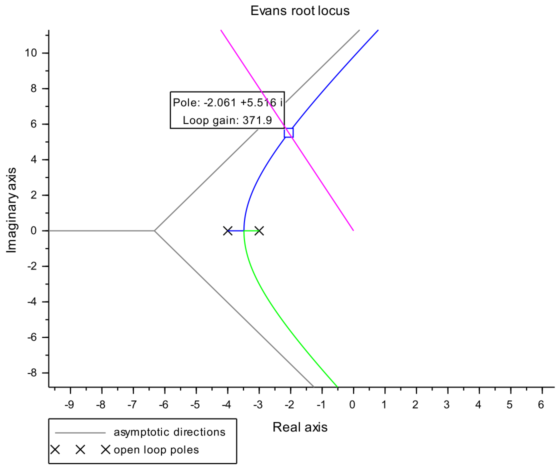


Figure 1.2: Root locus of $G(s)$

Code for plotting

```
s = %s;
TF = 1/((s+3)*(s+4)*(s+12));
evans(TF);
zeta = 0.35;
slope = -sqrt(1-zeta*zeta)/zeta;
x = -10;
plot([0 x], [0 slope*x], 'm');
xs2pdf(0,'Q1b');
```

These values are confirmed by code below

```
kValues = 0:1e-3:1e4;
threshold = 1e-5;
breakLoop = 0;
for i = 1:length(kValues)
    k = kValues(i);
    G = k * TF;
    [zeroes, poles, k] = tf2zp(G/(1+G));
    for j = 1:length(poles)
        p = poles(j);
        if abs(slope - imag(p)/real(p)) < threshold
            disp(k, p);
            breakLoop = 1;
            break;
        end
    end
    if breakLoop == 1
        break;
    end
end
--> 371.879
--> -2.0609593 + 5.5159954i
```

c)

The gain at break away point is 2.127, as can be seen from root locus

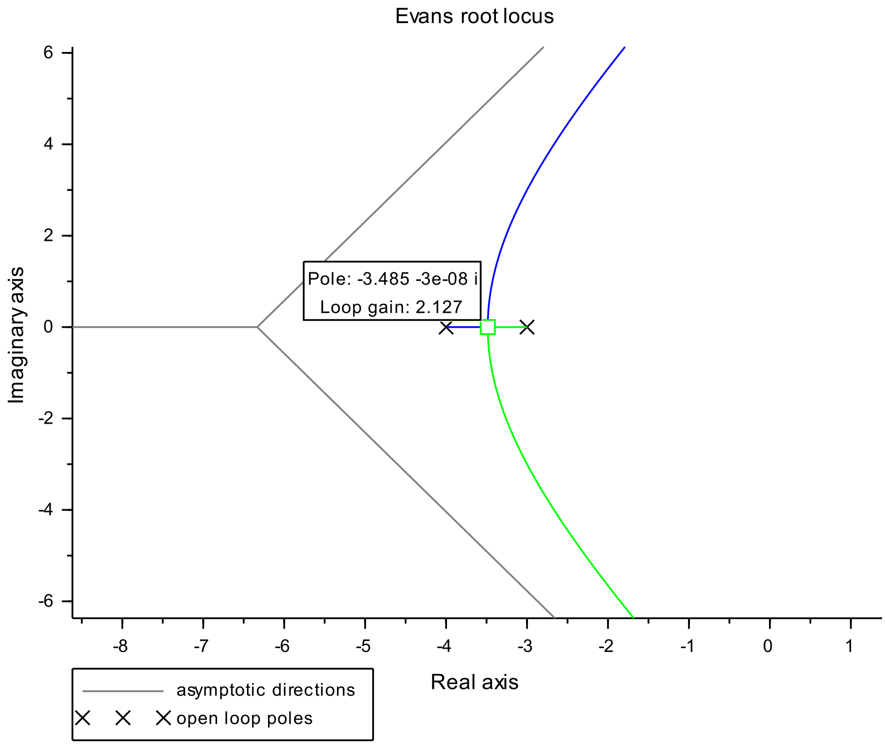
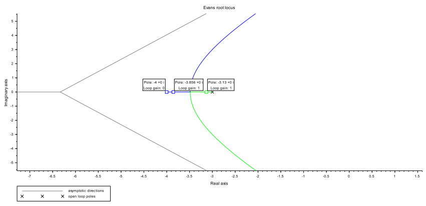


Figure 1.3: Root locus of $G(s)$ with break away point

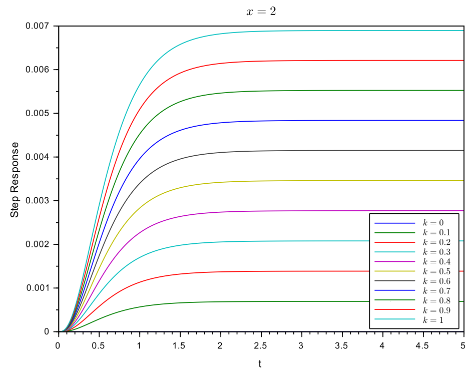
These values are confirmed by code below

```
kValues = 0:1e-3:1e4;
breakLoop = 0;
for i = 1:length(kValues)
    k = kValues(i);
    G = k * TF;
    [zeroes, poles, k] = tf2zp(G/(1+G));
    for j = 1:length(poles)
        p = poles(j);
        if imag(p) > 0
            disp(k);
            breakLoop = 1;
            break;
        end
    end
    disp(k);
    if breakLoop == 1
        break;
    end
end
--> 2.127
```

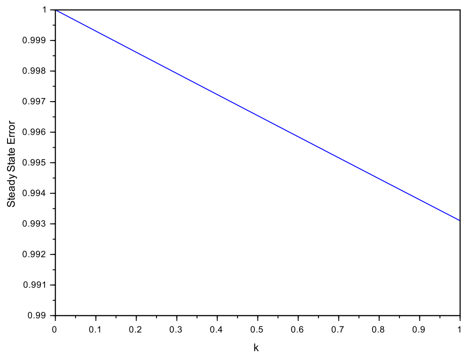
d)



(a) Root Locus of $G(s)$



(b) Step Response of $G(s)$



(c) Steady State Error of $G(s)$

Figure 1.4: Variation of K in range [0,1]

Closed Loop Poles lie between open loop poles on negative x-axis. Hence, the system is stable. In fact, it is overdamped for these range of K.

Steady State Errors decreases linearly (for small K) as K increases.

Code to obtain Figure 1.4a is given below

```
s = %s;
TF = 1/((s+3)*(s+4)*(s+12));
evans(TF);
xs2pdf(0,'Q1d1');
```

Code to obtain Figure 1.4b is given below

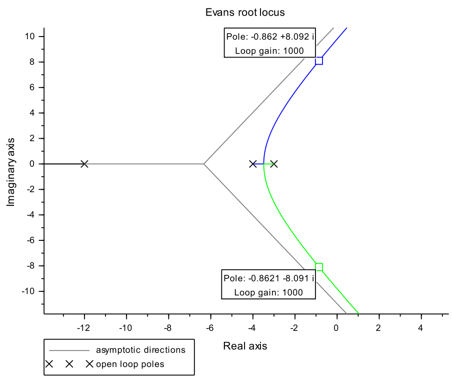
```
s = %s;
tMax = 5;
tStep = 0.01;
t = 0:tStep:tMax;
TF = 1/((s+3)*(s+4)*(s+12));
kValues = 0:0.1:1;
legendValues = []
y = [];
dims = 1;
for x = 1:length(kValues)
    k = kValues(x);
    str = ' $k = ' + string(k) + ' $' ;
    legendValues = cat(dims, legendValues, str);
    G = TF*k;
```

```
    sys = syslin('c',G/(1+G));
    y = cat(dims, y, csim('step', t, sys));
end
plot(t',y'); xlabel("t"); ylabel("Step Response");
legend(legendValues,opt=4);
// legend(['£k=0£','£k=0.1£','£k=0.2£','£k=0.3£','£k=0.4£','£k=0.5£','£k=0.6£','£k=0.7£',
           '$k=0.8$','$k=0.9$','$k=1$'],opt=4);
xs2pdf(0,'Q1d2');
```

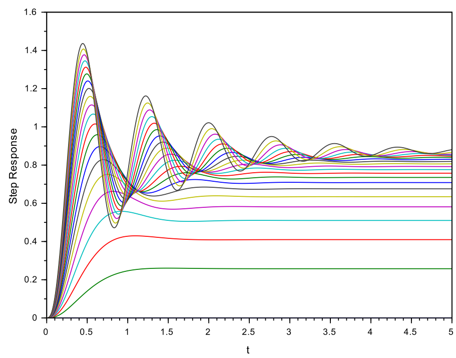
Code to obtain Figure 1.4c is given below

```
s = %s;
TF = 1/((s+3)*(s+4)*(s+12));
kValues = 0:0.1:1;
y = [];
dims = 1;
for x = 1:length(kValues)
    k = kValues(x);
    y = cat(dims, y, 1/(1+k*(1/(3*4*12))));
end
plot(kValues,y'); xlabel("k"); ylabel("Steady State Error");
ax = gca();
ax.data_bounds=[0 0.99;1 1];
xs2pdf(0,'Q1d3');
```

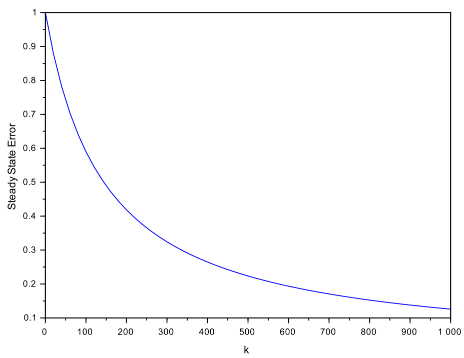
e)



(a) Root Locus of $G(s)$



(b) Step Response of $G(s)$



(c) Steady State Error of $G(s)$

Figure 1.5: Variation of K in range [0,1000]

Closed Loop Poles lie in left half plane. Hence, the system is stable. As K increases, the system goes from overdamped to underdamped. This happens when K passes over break away gain.

Steady State Errors decreases hyperbolically as K increases. It tends to 0 as K tends to ∞.

Settling Times (5%) decreases as K initially increases but then starts to increase for increasing K.

Stability System is stable for all $K \leq 1000$ as, steady state is reached

Code to obtain Figure 1.5a is given below

```
s = %s;
TF = 1/((s+3)*(s+4)*(s+12));
evans(TF);
xs2pdf(0,'Q1e1');
```

Code to obtain Figure 1.5b is given below

```
s = %s;
tMax = 5;
tStep = 0.01;
t = 0:tStep:tMax;
TF = 1/((s+3)*(s+4)*(s+12));
kValues = 0:50:1000;
y = [];
dims = 1;
for x = 1:length(kValues)
    k = kValues(x);
    G = TF*k;
    sys = syslin('c',G/(1+G));
    y = cat(dims, y, csim('step', t, sys));
end
plot(t',y'); xlabel("t"); ylabel("Step Response");
xs2pdf(0,'Q1e2');
```

Code to obtain Figure 1.5c is given below

```
s = %s;
TF = 1/((s+3)*(s+4)*(s+12));
kValues = 0:20:1000;
y = [];
dims = 1;
for x = 1:length(kValues)
    k = kValues(x);
    y = cat(dims, y, 1/(1+k*(1/(3*4*12))));
end
plot(kValues,y'); xlabel("k"); ylabel("Steady State Error");
xs2pdf(0,'Q1e3');
```

2 Q2

Open Loop Transfer Function is

$$\mathrm{G}(s) = \frac{1}{(s+3)(s+4)(s+12)}$$

$$\text{PI Controller} = \frac{K(s+z)}{s}$$

a)

To attain $\zeta = 0.2$, for $z = 0.01$,

we need to find intersection between the line of slope $= -\dfrac{\sqrt{1-\zeta^2}}{\zeta} = -4.89897948557$ and the root locus.

Both intersected at $-1.416 + 6.935\iota$, giving $K = 666.4$.

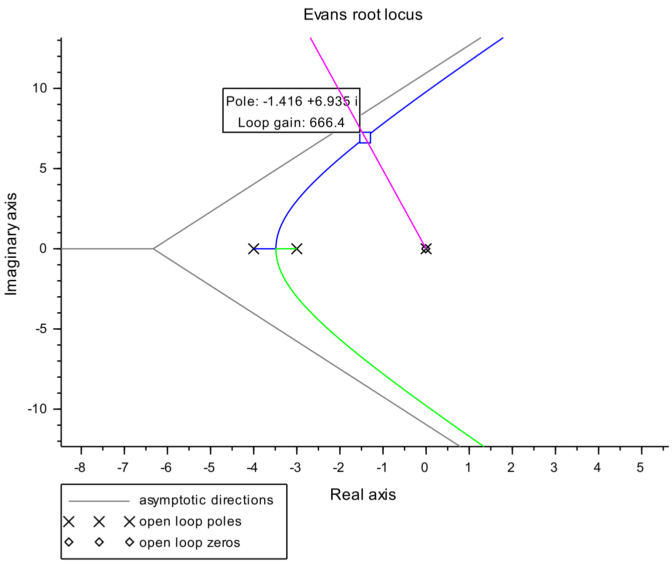


Figure 2.1: Root locus of $G(s)$

So, PI Controller $= \dfrac{666.4(s+0.01)}{s}$
Code for plotting

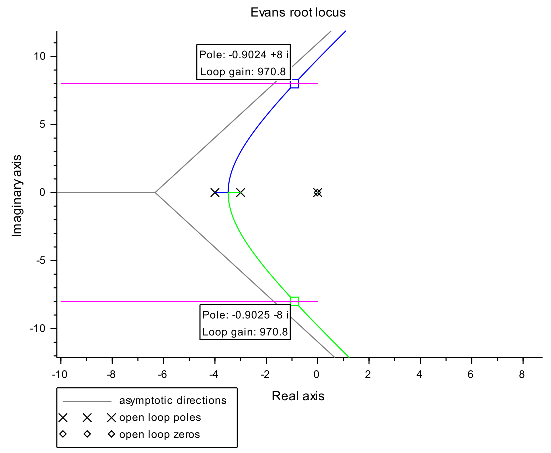
```
s = %s;
G = 1/((s+3)*(s+4)*(s+12));
z = 0.01
PI = (s+z)/s
TF = G * PI
evans(TF);
zeta = 0.2;
slope = -sqrt(1-zeta*zeta)/zeta;
x = -10;
plot([0 x], [0 slope*x], 'm');
xs2pdf(0,'Q2a');
```

These values are confirmed by code below

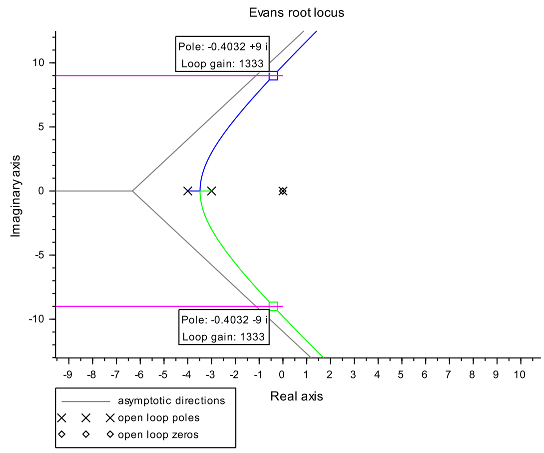
```
kValues = 0:1e-3:1e4;
threshold = 1e-5;
breakLoop = 0;
for i = 1:length(kValues)
    k = kValues(i);
    G = k * TF;
    [zeroes, poles, k] = tf2zp(G/(1+G));
    for j = 1:length(poles)
        p = poles(j);
        if abs(slope - imag(p)/real(p)) < threshold
            disp(k, p);
            breakLoop = 1;
            break;
        end
    end
    if breakLoop == 1
        break;
    end
end
--> 666.314
--> -1.4155168 + 6.9345806i
```

b)

To obtain undamped natural frequencies of 8 and 9 rad/s, we need to find intersection between the horizontal line $y = 8$, $y = 9$ and the root locus respectively.
For $y = 8$, both intersected at $-0.9024 \pm 8\iota$, giving $K = 970.8$.
For $y = 9$, both intersected at $-0.4032 \pm 9\iota$, giving $K = 1333$.



(a) Root Locus of $G(s)$



(b) Root Locus of $G(s)$

So, PI Controllers $= \underbrace{\frac{970.8(s+0.01)}{s}}_{\text{for 8 rad/s}}, \underbrace{\frac{1333(s+0.01)}{s}}_{\text{for 9 rad/s}}$

Code for plotting

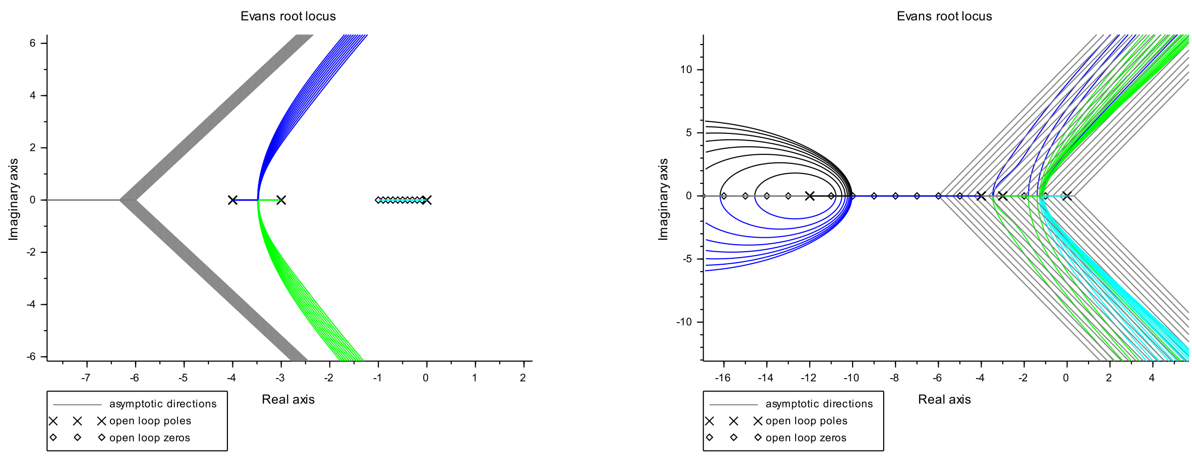
```
s = %s;
G = 1/((s+3)*(s+4)*(s+12));
z = 0.01
PI = (s+z)/s
TF = G * PI
evans(TF);
plot([-10 0], [8 8], 'm');
plot([-10 0], [-8 -8], 'm');
xs2pdf(0,'Q2b1');
evans(TF);
plot([-10 0], [9 9], 'm');
plot([-10 0], [-9 -9], 'm');
xs2pdf(0,'Q2b2');
```

These values are confirmed by code below

```
kValues = 0:1e-3:1e4;
threshold = 1e-5;
breakLoop = 0;
for i = 1:length(kValues)
    k = kValues(i);
    G = k * TF;
    [zeroes, poles, k] = tf2zp(G/(1+G));
    for j = 1:length(poles)
        p = poles(j);
        if abs(8 - imag(p)) < threshold
            disp(k, p);
            breakLoop = 1;
            break;
        end
    end
```

```
    if breakLoop == 1
        break;
    end
end
--> 970.757
--> -0.9024653 + 7.9999921i
kValues = 0:1e-3:1e4;
threshold = 1e-5;
breakLoop = 0;
for i = 1:length(kValues)
    k = kValues(i);
    G = k * TF;
    [zeroes, poles, k] = tf2zp(G/(1+G));
    for j = 1:length(poles)
        p = poles(j);
        if abs(9 - imag(p)) < threshold
            disp(k, p);
            breakLoop = 1;
            break;
        end
    end
    if breakLoop == 1
        break;
    end
end
--> 1332.76
--> -0.4032583 + 8.9999903i
```

c)



(a) From `0:0.1:1` (b) From `1:1:20`

Figure 2.3: Effect of variation in z to Root Locus of $G(s)$

0,1 Increase in z, moves open loop zero leftwards, results in inward bending of branches..

1,20 A further increase in z,introduces new branches, and there are relatively lesser breakaway points in region (-3 to -1).

Code to obtain Figure 2.3

```
s = %s;
G = 1/((s+3)*(s+4)*(s+12));
zValues = 0:0.01:1;
for i = 1:length(zValues)
```

```
    z = zValues(i);
    PI = (s+z)/s;
    TF = G * PI;
    evans(TF);
end
xs2pdf(0,'Q2c1');

s = %s;
G = 1/((s+3)*(s+4)*(s+12));
zValues = 1:1:20;
for i = 1:length(zValues)
    z = zValues(i);
    PI = (s+z)/s;
    TF = G * PI;
    evans(TF);
end
xs2pdf(0,'Q2c2');
```

d)

The pole locations, can be changed slightly for “small” z values, if z values are large then the root locus changes drastically. This can be observed in Figure 2.3.

3 Q3

$$G(s) = \frac{1}{(s^2+5s+6)}$$

a)

Let's consider the frequencies $\omega = 0.5, 1, 2, 5, 10$ rad/s.

Frequency ω (in rad/s)	Gain from Plot	Theoretical Gain	Phase from Plot (in °)	Theoretical Phase (in °)
0.5	0.1594901	0.1594904	-23.491270	-23.498566
1	0.1414203	0.1414214	-44.690708	-45
2	0.0980548	0.0980581	-78.703166	-78.690068
5	0.0318393	0.0318465	-233.01132	52.765166
10	0.0093857	0.0093923	-207.22822	28.009177

Phase from plot (Figure 3.1) doesn't match with Theoretical Phase for higher frequencies, as for higher frequencies we are not sure that closest peaks of input and output are actually close.

```
s = %s;
G = 1/(s^2+5*s+6);
G = syslin('c',G);
w = [0.5, 1, 2, 5, 10];
tMax = [50, 20, 20, 10, 10];
phase = [];
phaseTheoretical = [];
gain = [];
gainTheoretical = [];
tStep = 0.01;

function z = plots(w, t, i)
    t = 0:tStep:tMax(i);
    x = sin(w(i)*t);
    y = csim(x,t,G);
    plot(t,cat(1,x,y));
    legend(['Input';'Output']);
    xlabel("t"); ylabel("Response");
    s = "Q3a"+ string(int(i));
    xs2pdf(0,s);
    gain($+1) = max(y(length(t)/2:$)) / max(x(length(t)/2:$));
    phase($+1) = modulo((find(y == max(y(length(t)/2:$)))
                         - find(x == max(x(length(t)/2:$))))*w(i)*180/%pi, 360);

    jw = complex(0,1)*w(i);
    g = 1/((jw)^2+5*jw+6);
    gainTheoretical($+1) = abs(g);
    phaseTheoretical($+1) = modulo(atan(imag(g)/real(g))*180/%pi,360);
endfunction

plots(w, t, 1);
plots(w, t, 2);
plots(w, t, 3);
plots(w, t, 4);
plots(w, t, 5);
```

b)

The desired relation between phase difference and angle of $G(j\omega)$ is for frequency measured in rad/s.

c)

$$G(s) = \frac{60}{(s^3 + 6s^2 + 11s + 6)}$$

For the frequencies $\omega = 0.5, 1, 2, 5, 10$ rad/s.

Frequency ω (in rad/s)	Gain from Plot	Theoretical Gain	Phase from Plot (in °)	Theoretical Phase (in °)
0.5	8.5591349	8.5591554	-50.133807	-50.063617
1	5.9999639	6	-89.954374	-90
2	2.6312132	2.6311741	-142.09353	37.874984
5	0.3880679	0.3747366	153.76064	-25.924902
10	0.0745715	0.0560739	110.78880	-56.280230

Phase from plot (Figure 3.2) doesn't match with Theoretical Phase for higher frequencies, as for higher frequencies we are not sure that closest peaks of input and output are actually close.

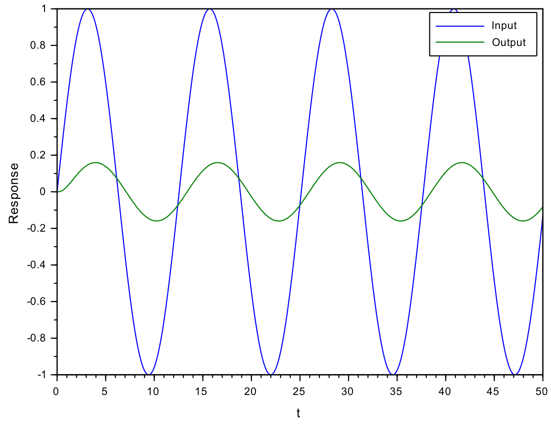
```
s = %s;
G = 60/(s^3+6*s^2+11*s+6);
G = syslin('c',G);
w = [0.5, 1, 2, 5, 10];
tMax = [50, 20, 20, 10, 10];
phase = [];
phaseTheoretical = [];
gain = [];
gainTheoretical = [];
tStep = 0.01;

function z = plots(w, t, i)
    t = 0:tStep:tMax(i);
    x = sin(w(i)*t);
    y = csim(x,t,G);
    plot(t,cat(1,x,y));
    legend(['Input';'Output']);
    xlabel("t"); ylabel("Response");
    s = "Q3a"+ string(int(i));
    xs2pdf(0,s);
    gain($+1) = max(y(length(t)/2:$)) / max(x(length(t)/2:$));
    phase($+1) = modulo((find(y == max(y(length(t)/2:$)))
                        - find(x == max(x(length(t)/2:$))))*w(i)*180/%pi, 360);

    jw = complex(0,1)*w(i);
    g = 60/((jw)^3+6*(jw)^2+11*jw+6);
    gainTheoretical($+1) = abs(g);
    phaseTheoretical($+1) = modulo(atan(imag(g)/real(g))*180/%pi,360);
endfunction

plots(w, t, 1);
plots(w, t, 2);
plots(w, t, 3);
plots(w, t, 4);
plots(w, t, 5);
```

Let the frequency where 180○ phase difference occurs be ω.
Now,

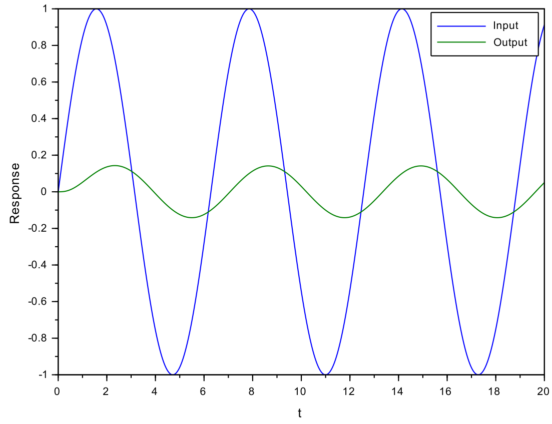
$$\text{Gain} = \frac{60}{(j\omega)^3 + 6(j\omega)^2 + 11(j\omega) + 6} = \frac{60}{-j\omega^3 - 6\omega^2 + 11(j\omega) + 6} = \frac{60}{6 - 6\omega^2 + j(11\omega - \omega^3)}$$

$$\text{Phase Difference} = \arctan\frac{-(11\omega - \omega^3)}{6 - 6\omega^2} \Rightarrow \text{Phase Difference} = 180 \to 11\omega = \omega^3 \to \omega = \sqrt{11}\text{rad/s} \approx 3.316625\text{rad/s}$$

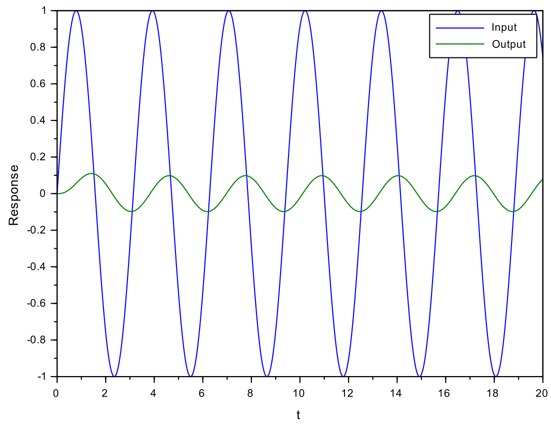
The numerator 60 play no role for finding this frequency.



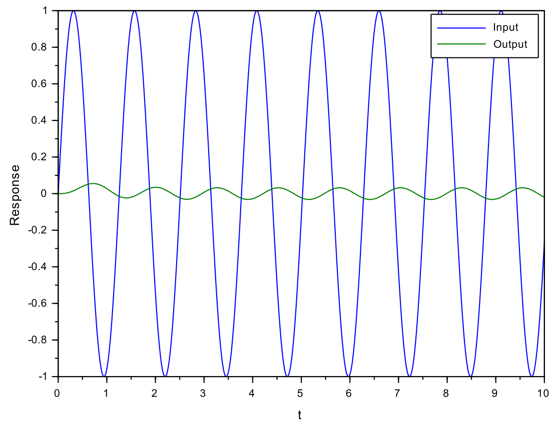
(a) $\omega = 0.5$



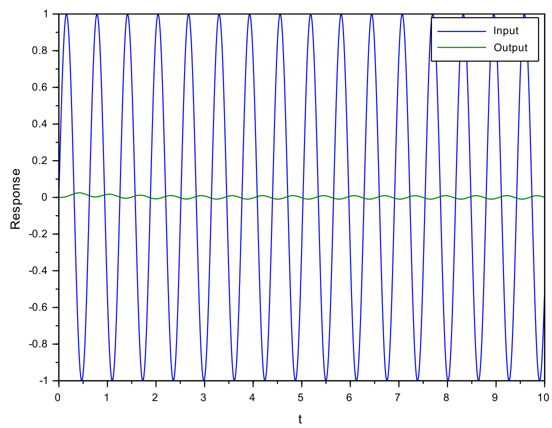
(b) $\omega = 1$



(c) $\omega = 2$

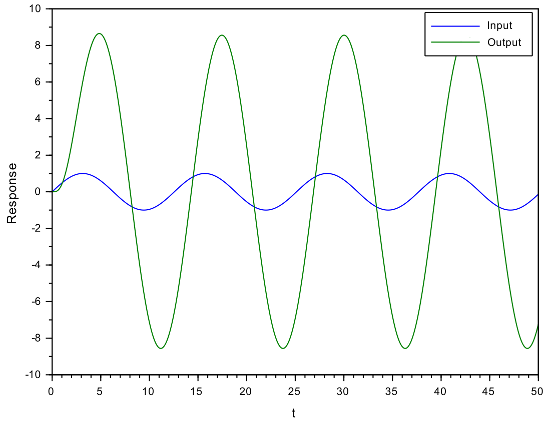


(d) $\omega = 5$

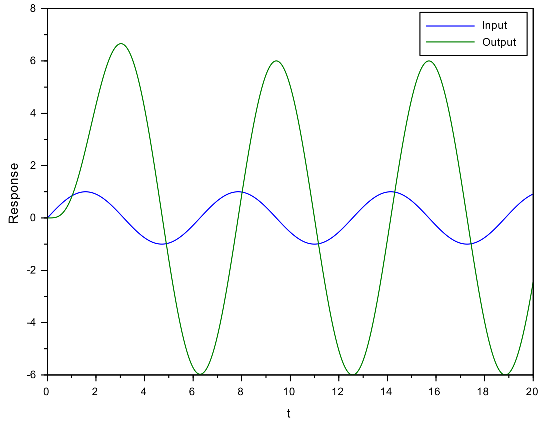


(e) $\omega = 10$

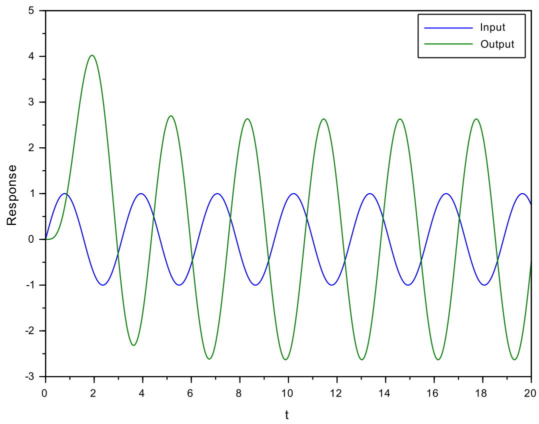
Figure 3.1: Input, Output plots



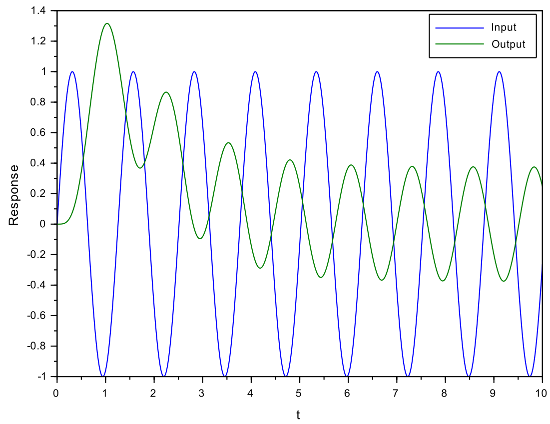
(a) $\omega = 0.5$



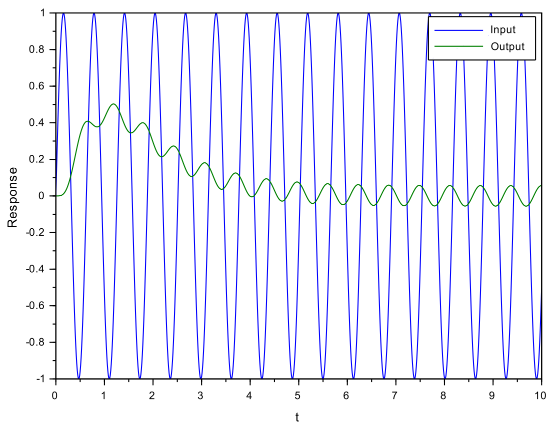
(b) $\omega = 1$



(c) $\omega = 2$



(d) $\omega = 5$



(e) $\omega = 10$

Figure 3.2: Input, Output plots