EE324 Control Systems Lab

Problem Sheet 5

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Contents

1 Q1

a)

Closed Loop Transfer Function is

$$\mathrm{TF}(s) = \frac{50s - 100}{s^3 + s^2 + 150s - 100}$$

$$TF(s) = \frac{G(s)}{1 + G(s)} \quad \rightarrow \quad G(s) = \frac{TF(s)}{1 - TF(s)}$$

So, Open Loop Transfer Function is

$$\mathrm{G}(s) = \frac{10}{s^3 + 4s^2 + 5s}$$

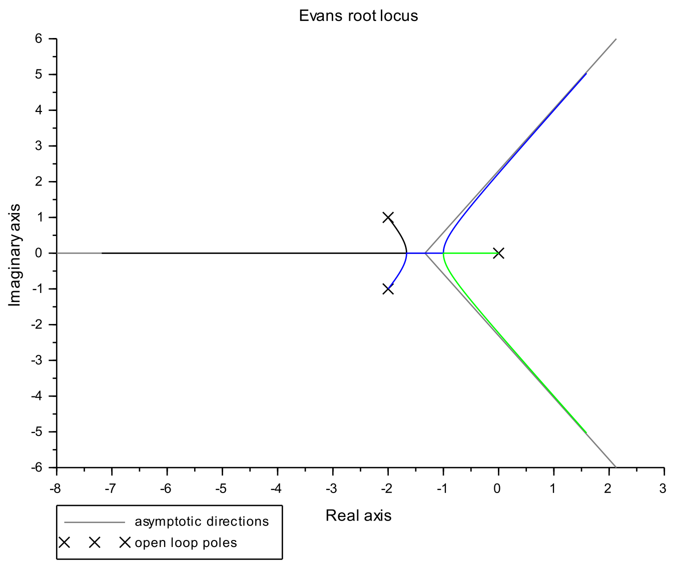


Figure 1.1: Root Locus of $G(s)$

The system has 3 poles, complex conjugate pair and a pole in ORHP. Hence, it is unstable.

As K increases the system CLTF roots goes in ORHP.

Code to obtain Figure 1.1

```
--> s = %s;
--> TF = 10/(s^3+4*s^2+5*s+10);
--> G = TF / (1 - TF)
 G  =
        10
   -----------
   5s +4s² +s³
--> evans(G, 20);
--> xs2pdf(0,'Q1a');
```

b)

Open Loop Transfer Function is

$$G(s) = \frac{(s+1)}{s^2(s+3.6)}$$

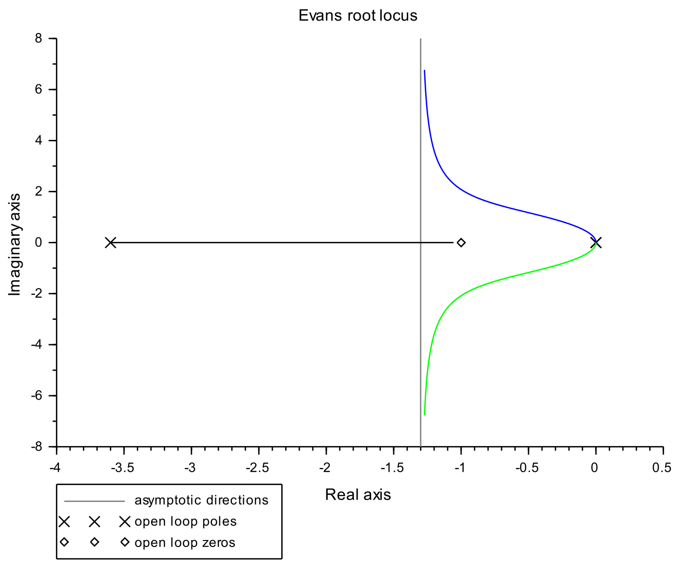


Figure 1.2: Root Locus of $G(s)$

The system has 3 poles, all reals and there are 2 poles (including multiplicity) at 0.

Code to obtain Figure 1.2

```
s = %s;
G =  (s + 1) / (s^2 * (s + 3.6));
evans(G, 50);
xs2pdf(0,'Q1b');
```

c)

Open Loop Transfer Function is

$$G(s) = \frac{(s+0.4)}{s^2(s+3.6)}$$

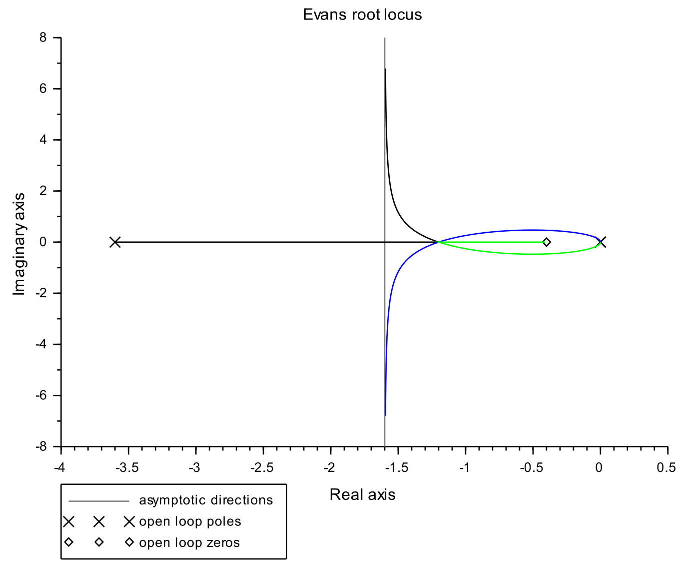


Figure 1.3: Root Locus of $G(s)$

The system has 3 poles, all reals and there are 2 poles (including multiplicity) at 0.

Code to obtain Figure 1.3

```
s = %s;
G =  (s + 0.4) / (s^2 * (s + 3.6));
evans(G, 50);
xs2pdf(0,'Q1c');
```

d)

Open Loop Transfer Function is

$$\mathrm{G}(s) = \frac{(s+p)}{s(s+1)(s+2)}$$

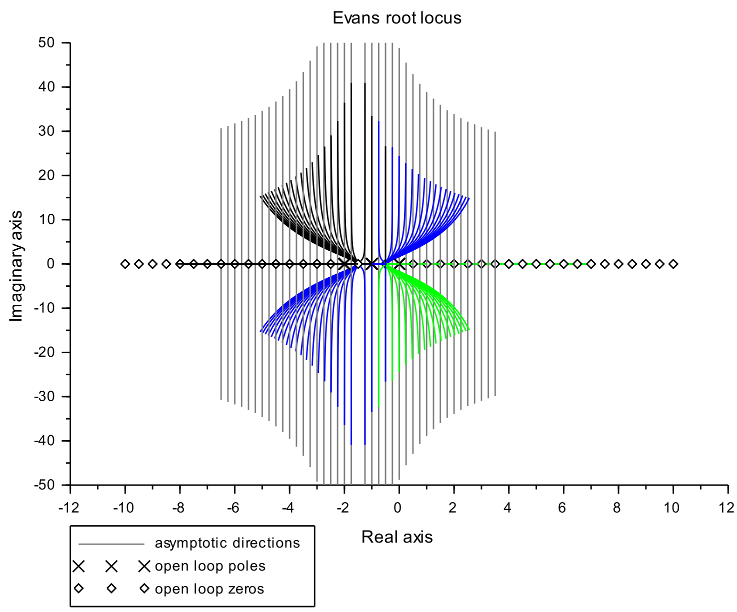
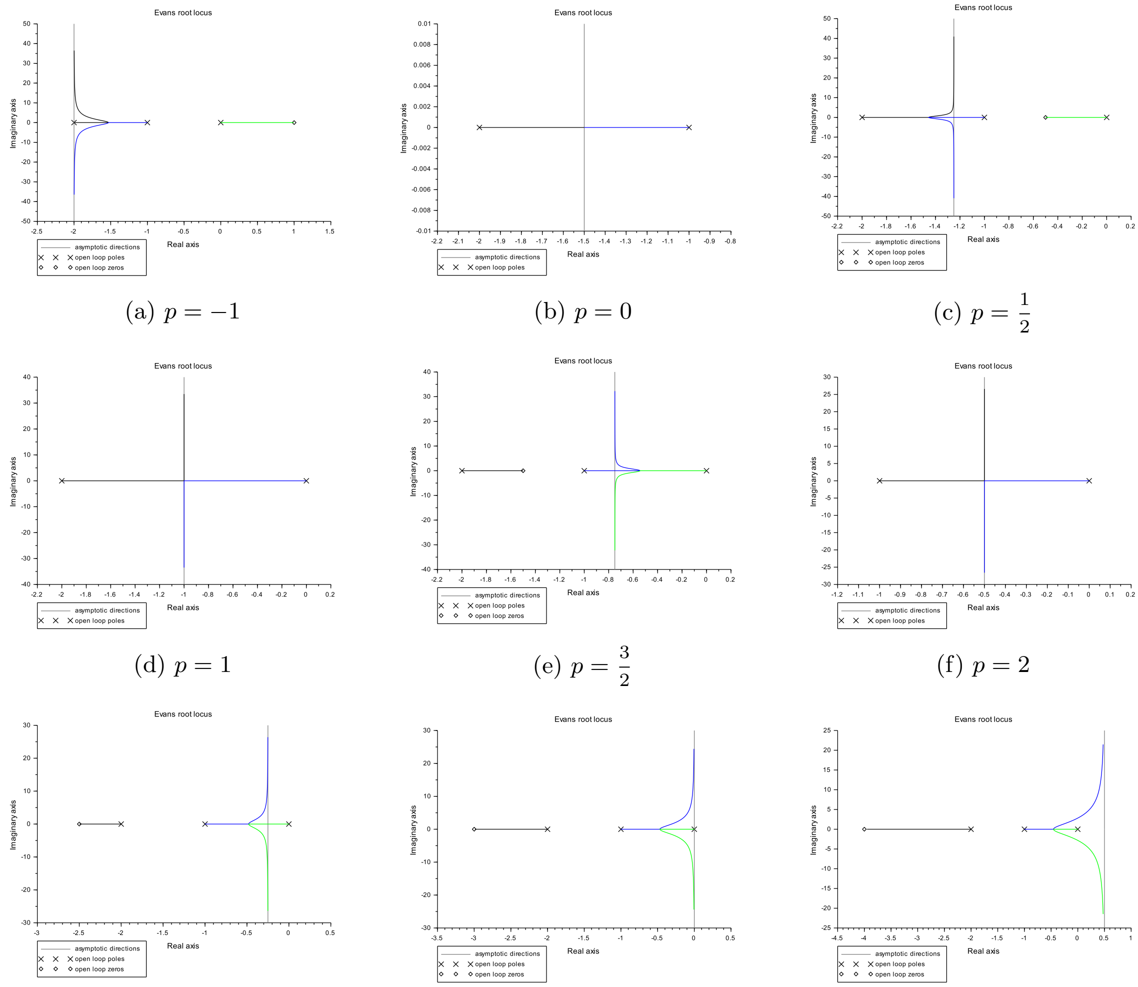


Figure 1.4: Root Locus of $G(s)$ for $p \in \{-10 : 0.5 : 10\}$



(a) $p = -1$ (b) $p = 0$ (c) $p = \frac{1}{2}$

(d) $p = 1$ (e) $p = \frac{3}{2}$ (f) $p = 2$

(g) $p = \frac{5}{2}$ (h) $p = 3$ (i) $p = 4$

Figure 1.5

- For $p < 0$, the system is unstable, as the pole at 0 meets the zero in RHP (as K increases).
- For $p \in (0, 3) \setminus \{1, 2\}$, the system is stable, as complete root locus is in OLHP.
- For $p = 0, 1, 2$, the system is stable, due to pole-zero cancellation which makes this system as 2^{nd} order.
- For $p = 3$, the system is marginally stable, as the $j\omega$ axis is now asymptote of root locus.
- For $p > 3$, the system is unstable (for higher values of K), as a part of root locus is in ORHP.

Code to obtain Figure 1.4

```
s = %s;
Pvalues = -10:0.5:10;
for i = 1:length(Pvalues)
    p = Pvalues(i);
    G =  (s + p) / (s*(s+1)*(s+2));
  evans(G);
end
xs2pdf(0,'Q1d');
```

Example Code to obtain Figure 1.5

```
p = -1;
G =  (s + p) / (s*(s+1)*(s+2));
evans(G);
xs2pdf(0,'Q1d1')
```

2 Q2

a)

When poles and zeroes are placed symmetrically about the origin, the breakaway and breakin points to coincide.

$$G(s) = \frac{s^2+1}{s^2-1}$$

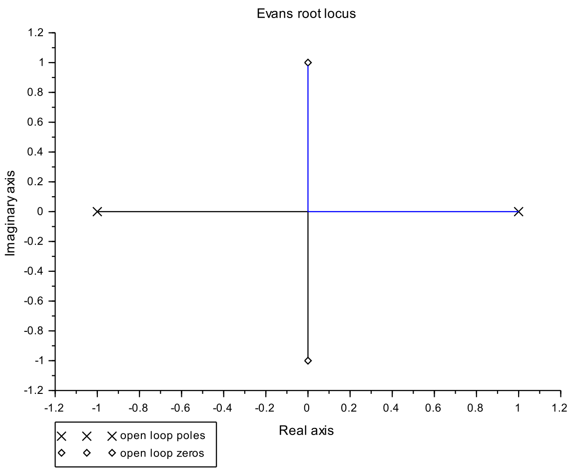


Figure 2.1: Root Locus of $G(s)$

As can be seen in the figure, the breakaway and breakin points coincide (at the origin).

Code to obtain Figure 2.1

```
s = %s;
G =  (s^2 + 1) / (s^2 - 1);
evans(G, 50);
xs2pdf(0,'Q2a');
```

b)

$$G(s) = \frac{s^6+1}{s^6-1}$$

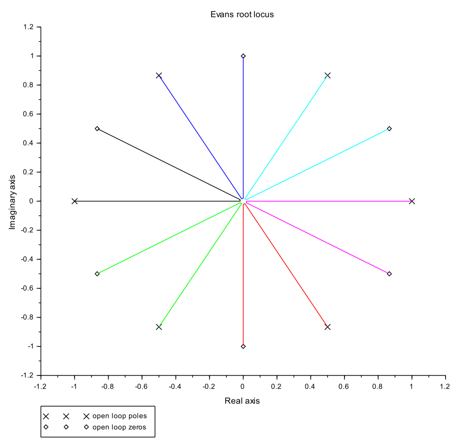


Figure 2.2: Root Locus of $G(s)$

The number of branches at the breakaway or breakin point is $> 4 (= 6)$. Poles (zeroes) are at 6^{th} roots of $1 (-1)$.

Code to obtain Figure 2.2

```
s = %s;
G =  (s^6 + 1) / (s^6 - 1);
evans(G, 50);
xs2pdf(0,'Q2b');
```

c)

$$G(s) = \frac{1}{(s+1)^3}$$

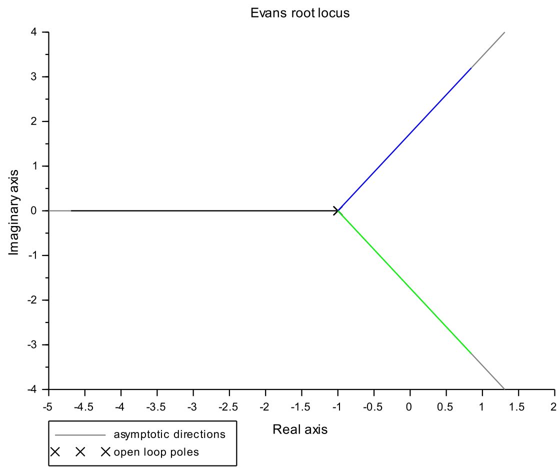


Figure 2.3: Root Locus of $G(s)$

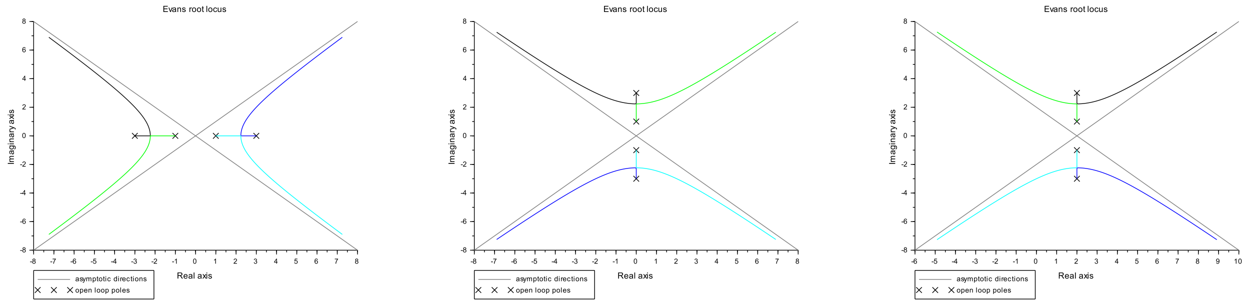
As can be seen in the figure, the branches of the root locus coincide with their asymptotes.

Code to obtain Figure 2.3

```
s = %s;
G =  1 / (s + 1)^3;
evans(G, 50);
xs2pdf(0,'Q2c');
```

d)

$$G_1(s) = \frac{1}{(s^2-1)(s^2-9)} = \frac{1}{s^4 - 10s^2 + 9}$$
$$G_2(s) = \frac{1}{(-s^2-1)(-s^2-9)} = \frac{1}{s^4 + 10s^2 + 9}$$
$$G_3(s) = \frac{1}{(-(s-2)^2-1)(-(s-2)^2-9)} = \frac{1}{s^4 - 8s^3 + 34s^2 - 72s + 65}$$



(a) $G_1(s)$ (b) $G_2(s)$ (c) $G_3(s)$

Figure 2.4: Root Locus

As can be seen in the figure, the breakaway points are complex.

Code to obtain Figure 2.4

```
--> s = %s;

--> G =  1 / ((s^2 - 1) * (s^2 - 9))
 G  =
        1
   -----------
   9 -10s² +s⁴
--> evans(G, 10000);
--> xs2pdf(0,'Q2d1');

--> G =  1 / ((-s^2 - 1) * (-s^2 - 9))
 G  =
        1
   -----------
   9 +10s² +s⁴
--> evans(G, 10000);
--> xs2pdf(0,'Q2d2');

--> G =  1 / ((-(s-2)^2 - 1) * (-(s-2)^2 - 9))
 G  =
             1
   ----------------------
   65 -72s +34s² -8s³ +s⁴
--> evans(G, 10000);
--> xs2pdf(0,'Q2d3');
```

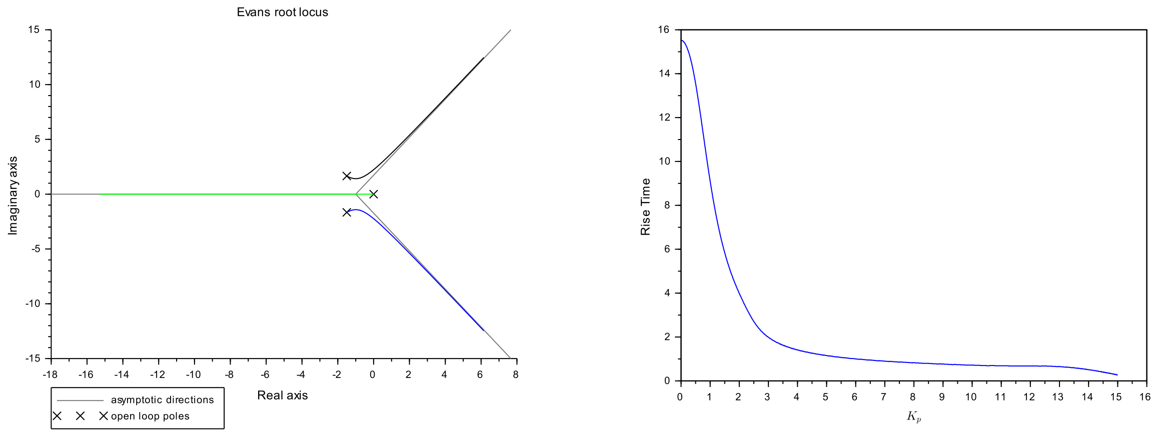
3 Q3

$$G(s) = \frac{1}{s(s^2 + 3s + 5)}$$

$K_p = 3.76$ gives rise time as ≈ 1.5s. (this was found by iterating over many K_p values and finding T_r rise time)

The minimum possible rise time for the given system (maintaining stability) is 0.27 ($K_p \leq 15$ for stability) Code to obtain Figure 3.1a

```
G =  1 / (s*(s^2 + 3*s + 5));
evans(G);
xs2pdf(0,'Q3rl');
```



(a) Root Locus of $G(s)$

(b) Variation of Rise Time with K_p

Figure 3.1

Function to find Rise Time

```
function Tr = riseTime(gp, tStep)
    finalValue = gp($);
    Tr = 0;
    for x = 1:length(gp)
        if gp(x) > 0.9*finalValue
            Tr = Tr + x;
            break
        end
        if gp(x) < 0.1*finalValue
            Tr = -x;
        end
    end
    Tr = Tr * tStep;
endfunction
```

Code to obtain Figure 3.1b

```
s = %s;
tMax = 20;
tStep = 0.01;
t = 0:tStep:tMax;
kValues = 0:0.01:15;
kTimes = [];
for x = 1:length(kValues)
  k = kValues(x);
  G =  k / (s*(s^2 + 3*s + 5));
  TF = G / (1 + G);
  sys = syslin('c', TF);
  gp = csim('step', t, sys);
  kTimes($+1) = riseTime(gp, tStep);
end
plot(kValues(2:$)',kTimes(2:$));
xlabel("$K_p$" ); ylabel("Rise Time" );
xs2pdf(0,'Q3');
```

After $K_p = 15$, the system is unstable.

Code to obtain minimum possible rise time and K_p when rise time is 1.5s

```
minTr = tMax;
minTrK = 0;
reqTr = 1.5;
reqK = [];
for x = 2:length(kValues)
  if kTimes(x) < minTr
    minTrK = kValues(x);
    minTr = kTimes(x);
  end
  if abs(kTimes(x) - reqTr) < 0.001
    reqK($+1) = kValues(x);
  end
end
--> disp(min(reqK));
   3.7600000
--> disp(minTr);
   0.27
```

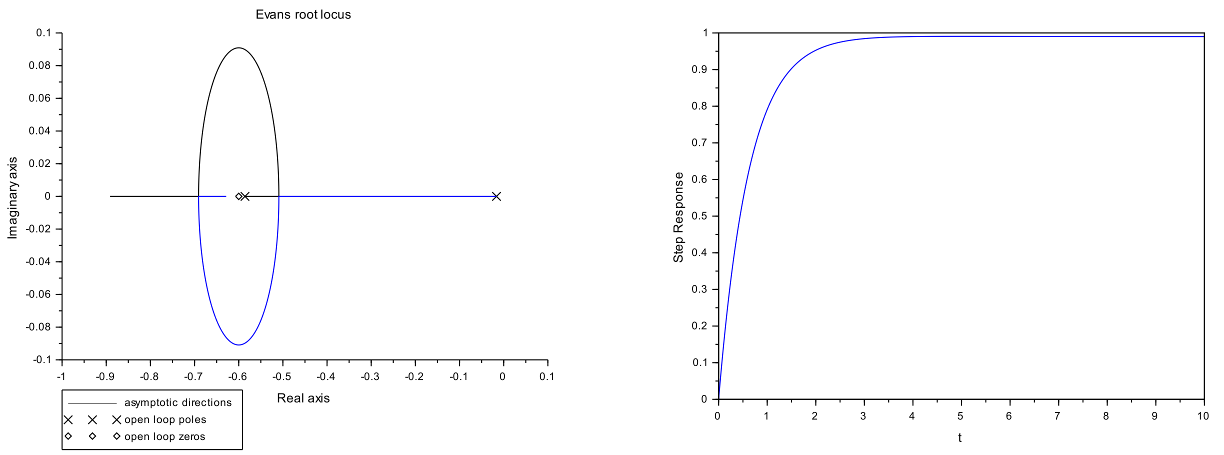
4 Q4

$$G(s) = \frac{0.11 * (s + 0.6)}{6 * s^2 + 3.6127 * s + 0.0572}$$

$$SSE = \frac{1}{100} = \lim_{s \to 0} \frac{1}{1 + K_p \cdot G(s)} \quad \rightarrow \quad K_p = 85.8$$

The system is marginally stable at

$$K_p = -\frac{1}{G(0)} = -\frac{1}{\frac{0.11 \cdot 0.6}{0.0572}} = -0.8\dot{6} \approx -0.87$$



(a) Root Locus of $G(s)$

(b) Step Response of $G(s)$

Figure 4.1

Code to obtain Figure 4.1a

```
s = %s;
G = 0.11*(s+0.6)/(6*s^2+3.6127*s+0.0572);
evans(G, 50);
xs2pdf(0,'Q4rl');
```

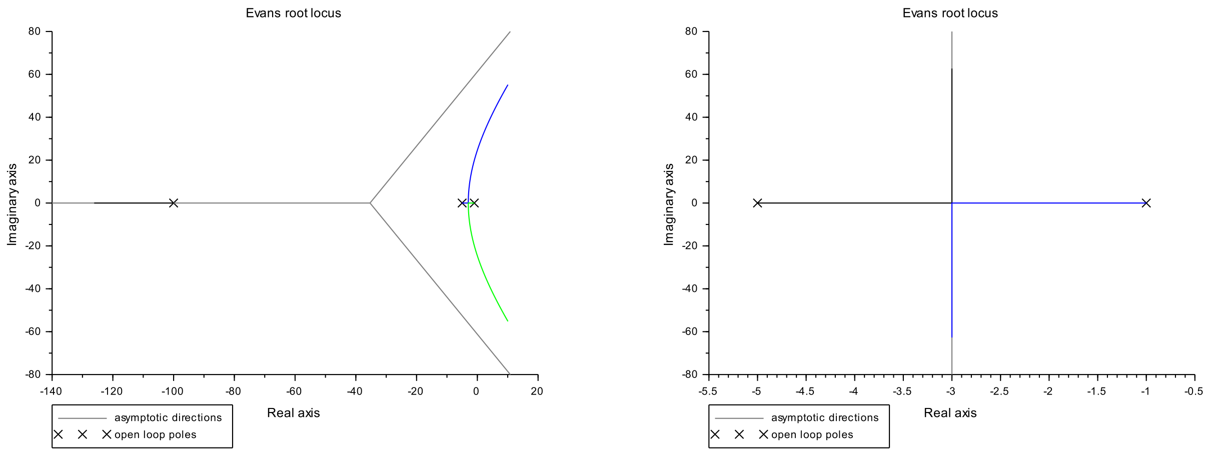
Code to obtain Figure 4.1b

```
tMax = 10;
tStep = 0.01;
t = 0:tStep:tMax;
K_p = (1/0.01 - 1) / (0.11*0.6/0.0572)
K_p_ms = 1 / (0.11*0.6/0.0572)
G = K_p * G;
TF = G / (1+G);
sys = syslin('c',TF);
gp = csim('step', t, sys);
plot(t,gp); xlabel("t" ); ylabel("Step Response" );
xs2pdf(0,'Q4sr');
```

5 Q5

$$G_1(s) = \frac{500}{(s+1)(s+5)(s+100)}$$

$$G_2(s) = \frac{5}{(s+1)(s+5)}$$



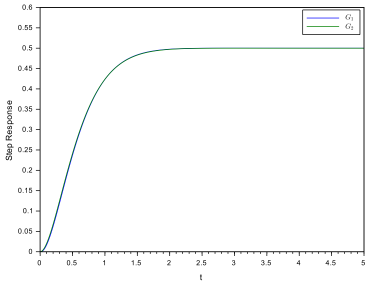
(a) $G_1(s)$ (b) $G_2(s)$

Figure 5.1: Root Locus

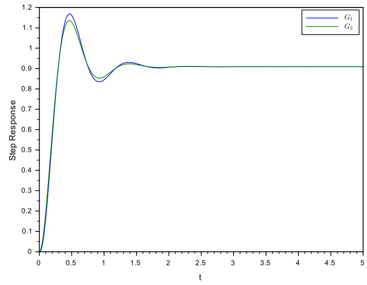
Both responses look very similar till $K = 30$ with major differences only in %OS.

Code to obtain Figure 5.1

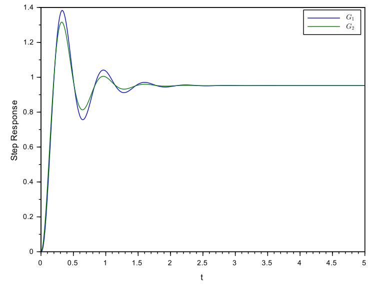
```
s = %s;
G1 = 500/((s+1)*(s+5)*(s+100));
evans(G1);
xs2pdf(0,'Q5rl1');
G2 = 5/((s+1)*(s+5));
evans(G2);
xs2pdf(0,'Q5rl2');
```



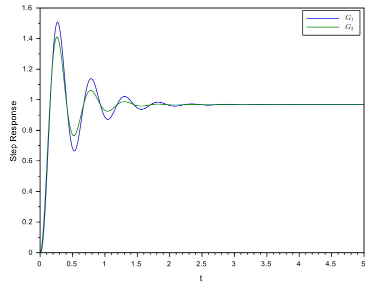
(a) $K = 1$



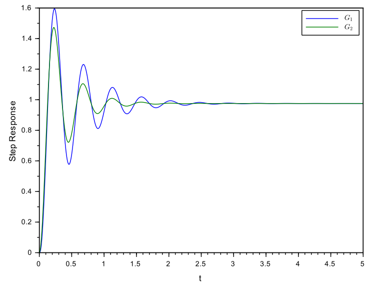
(b) $K = 10$



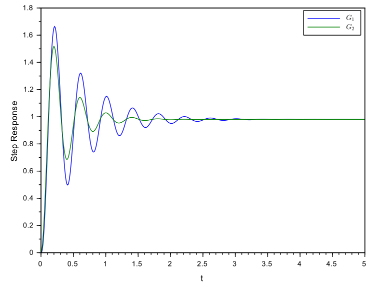
(c) $K = 20$



(d) $K = 30$



(e) $K = 40$



(f) $K = 50$

Figure 5.2: Step response of G_1, G_2 for different K

Code to obtain Figure 5.2

```
tMax = 5;
tStep = 0.01;
t = 0:tStep:tMax;
K = 1;
y = [];
dims = 1;
TF1 = K * G1 / (1 + K * G1);
TF2 = K * G2 / (1 + K * G2);
sys1 = syslin('c',TF1);
sys2 = syslin('c',TF2);
y = cat(dims, y, csim('step', t, sys1));
y = cat(dims, y, csim('step', t, sys2));
plot(t',y'); xlabel("t"); ylabel("Step Response" );
legend(['$G_1$','$G_2$']);
xs2pdf(0,'Q5sr1');
```