EE324 Control Systems Lab

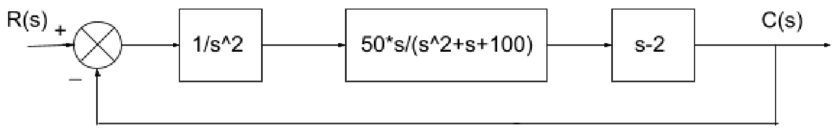
Problem Sheet 4

Param Rathour | 190070049

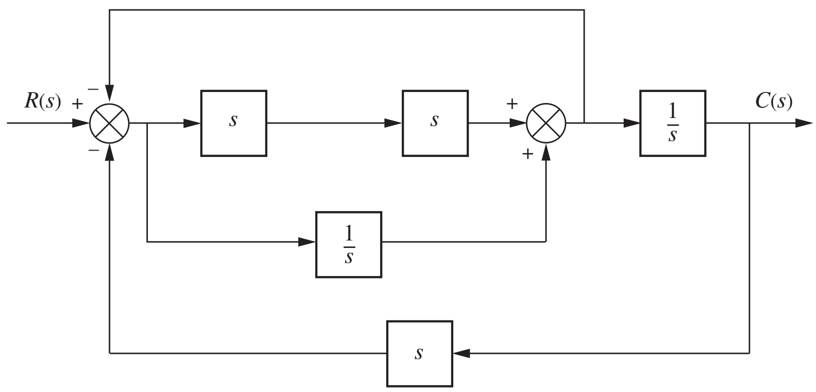
Autumn Semester 2021-22

Contents

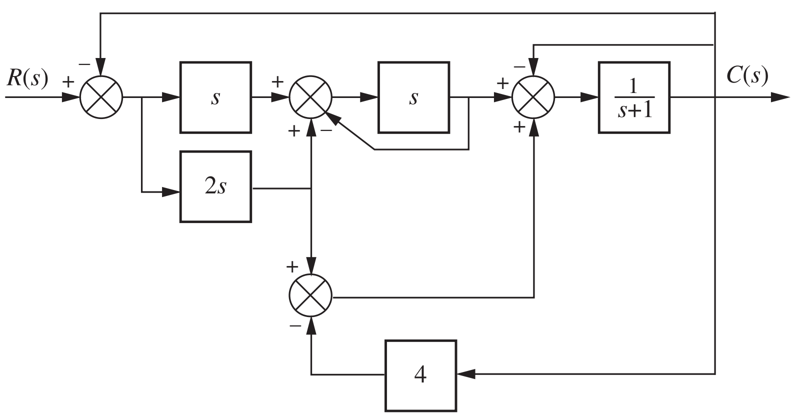
1 Q1



(a)



(b)



(c)

a)

```
 > s = %s;
 > G1 = 1/s^2;
 > G2 = 50*s/(s^2+s+100);
 > G3 = s-2;
 > G = G1*G2*G3;
 > TF = G/(1+G)
TF  =
         -100 +50s
   -------------------
   -100 +150s +1s^2 +s^3
```

Input-Output Transfer Function is

$$\mathrm{TF}(s) = \frac{50s - 100}{s^3 + s^s + 150s - 100}$$

b)

```
 > s = %s;
 > G1 = s;
 > G2 = 1/s;
 > G = G1*G1 ;
 > G = G+G2;
 > G = G*G2;
 > H = G1+G1;
 > TF = G/(1+G*H)
TF  =
       1 +s^3
   -----------
   2s +s^2 +2s^4
```

Input-Output Transfer Function is

$$\mathrm{TF}(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$

c)

```
 > s = %s;
 > G0 = 4;
 > G1 = s;
 > G2 = 2*s;
 > G3 = 1/(s+1);
 > GLA = G1 + G2;
 > GM = G1/(1+G1);
 > GLB = G2;
 > GLA = GLA * GM;
 > GL = GLA + GLB;
 > HR = G0 + 1;
 > GR = G3/(1+G3*HR);
 > G = GL * GR;
 > T = G/(1+G)
T  =
   0.3333333s +0.8333333s^2
   -----------------------
         1 +1.5s +s^2
```

Input-Output Transfer Function is (after simplifying result)

$$\mathrm{TF}(s) = \frac{5s^2 + 2s}{6s^2 + 9s + 6}$$

2 Q2

$$G(s) = \frac{10}{s(s+2)(s+4)}$$

a)

Scilab function to find the closed-loop transfer function for a given value of K

```
--> function TF = CLTFPositiveUnity(G,K)
  >     TF = K*G/(1+K*G);
  > endfunction
```

```
  > s = %s;
  > G = 10/(s*(s+2)*(s+4));
  > K = 4.2;
  > TF = CLTFPositiveUnity(G,K)
 TF  =
          42
    ---------------
    42 +8s +6s^2 +s^3
```

b)

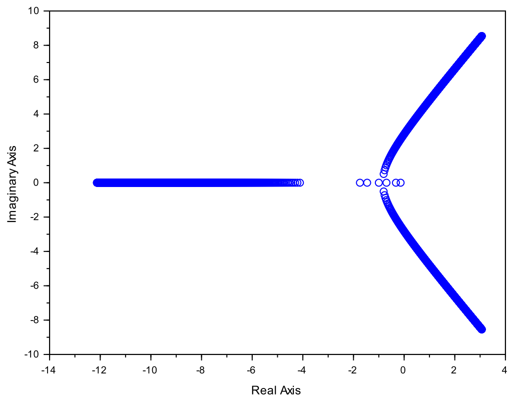


Figure 2.1: $K \in \{0 : 0.1 : 100\}$

Code to obtain Figure 2.1

```
Kvalues = 0:0.1:100;
pR = [];
pI = [];
for i = 1:length(Kvalues)
    K = Kvalues(i);
    TF = CLTFPositiveUnity(G,K);
    [zeroes, poles, k] = tf2zp(TF);
    for  j = 1:length(poles)
        p = poles(j);
        pR($+1) = real(p);
        pI($+1) = imag(p);
    end
end
scatter(pR,pI);
xlabel("Real Axis"); ylabel("Imaginary Axis");
xs2pdf(0,'Q2b');
```

c)

The critical value of K that takes the closed-loop system to the verge of instability is about 4.8

This will happen when a pole has real part 0 but we may not get such pole in simulation. Hence, I have taken the minimum K such that real part of any pole exceeds 0.

Code for finding critical value (if it exists)

```
Kvalues = 0:0.1:100;
Kcritical = -1;
b = 0;
for i = 1:length(Kvalues)
    if b == 1
        break;
    end
    K = Kvalues(i);
    TF = CLTFPositiveUnity(G,K);
    [zeroes, poles, k] = tf2zp(TF);
    for  j = 1:length(poles)
        if real(poles(j)) > 0
            Kcritical = K;
            b = 1
            break
        end
    end
end
  > disp(Kcritical)                         // If Kcritical = -1 then there is no such K in Kvalues
4.8000000
```

d)

It is clear from the following code that the system is unstable as number of sign changes > 0.

A very small negative part demonstrates it's *Criticality*.

For $K = 4.79$ (just less than 4.8) number of sign changes $= 0$, so that system is still stable.

```
  > K = 4.8;
  > TF = CLTFPositiveUnity(G,K);
  > [r, num] = routh_t(TF.den)
 r  =
   1.          8.                         // s^3 row
   6.          48.                        // s^2 row
  -8.882D-15   0.                         // s^1 row
   48.         0.                         // s^0 row
 num  =
   2.

  > K = 4.79;
  > TF = CLTFPositiveUnity(G,K);
  > [r, num] = routh_t(TF.den)
 r  =
   1.          8.                         // s^3 row
   6.          47.9                       // s^2 row
   0.0166667   0.                         // s^1 row
   47.9        0.                         // s^0 row
 num  =
   0.
```

3 Q3

a)

$$P(s) = s^5 + 3 * s^4 + 5 * s^3 + 4 * s^2 + s + 3$$

```
 > P = s^5 + 3*s^4 + 5*s^3 + 4*s^2 + s + 3;
 > [r, num] = routh_t(P)
r  =
  1.          5.    1.                    // s^5 row
  3.          4.    3.                    // s^4 row
  3.6666667   0.    0.                    // s^3 row
  4.          3.    0.                    // s^2 row
 -2.75        0.    0.                    // s^1 row
  3.          0.    0.                    // s^0 row
num  =
  2.
```

b)

$$P(s) = s^5 + 6 * s^3 + 5 * s^2 + 8 * s + 20$$

```
 > P = s^5 + 6*s^3 + 5*s^2 + 8*s + 20;
 > [r, num] = routh_t(P)
r  =
              1                    6       8   // s^5 row
              -                    -       -
              1                    1       1

             eps                   5       20  // s^4 row
             ---                   -       --
              1                    1       1

           -5 +6eps             -20 +8eps  0   // s^3 row
           --------             ---------  -
             eps                   eps     1

      -25 +50eps -8eps^2          20       0   // s^2 row
      -----------------           --       -
          -5 +6eps                1        1

  -2.274D-13 -160eps -64eps^2      0       0   // s^1 row
  --------------------------       -       -
      -25 +50eps -8eps^2           1       1

              20                   0       0   // s^0 row
              --                   -       -
              1                    1       1
num  =
  2.
```

c)

$$P(s) = s^5 - 2*s^4 + 3*s^3 - 6*s^2 + 2*s - 4$$

```
 > P = s^5 - 2*s^4 + 3*s^3- 6*s^2 + 2*s - 4;
 > [r, num] = routh_t(P)
r  =
  1.          3.    2.                           // s5 row
 -2.         -6.   -4.                           // s4 row
 -8.         -12.   0.                           // s3 row
 -3.         -4.    0.                           // s2 row
 -1.3333333   0.    0.                           // s1 row
 -4.          0.    0.                           // s0 row
num  =
  1.
```

d)

$$P(s) = s^6 + s^5 - 6*s^4 + s^2 + s - 6$$

```
 > P = s^6 + s^5 - 6*s^4 + s^2 + s - 6;
 > [r, num] = routh_t(P)
r  =
   1    -6   1   -6                              // s6 row
   -    --   -   --
   1     1   1    1

   1     0   1    0                              // s5 row
   -     -   -    -
   1     1   1    1

   -6    0   -6   0                              // s4 row
   --    -   --   -
   1     1   1    1

  -24    0   0    0                              // s3 row
  ---    -   -    -
   1     1   1    1

  eps   -6   0    0                              // s2 row
  ---   --   -    -
   1     1   1    1

  -144   0   0    0                              // s1 row
  ----   -   -    -
  eps    1   1    1

  864    0   0    0                              // s0 row
  ----   -   -    -
  -144   1   1    1
num  =
  3.
```

4 Q4

a)

To construct a degree 6 polynomial whose R-H table has its entire row corresponding to s^3 to be zero, we need a degree $(3+1=4)$ even polynomial. Say,

$$P(s) = \underbrace{(s^4+16)}_{\text{even}} \cdot (s^4-16)$$

$$P(s) = s^6 + 2s^5 + s^4 + 16s^2 + 32s + 16$$

Note that s^4 and s^5 rows are linearly dependent, hence entire row will be 0 in s^3.

```
 > P = (s^4+16)*(s^2+2*s+1)
P  =
 16 +32s +16s^2 +s^4 +2s^5 +s^6
 > [r, num] = routh_t(P)
r  =
    1    1   16  16                             // s^6 row
    -    -   --  --
    1    1   1   1

    2    0   32  0                              // s^5 row
    -    -   --  -
    1    1   1   1

    1    0   16  0                              // s^4 row
    -    -   --  -
    1    1   1   1

    4    0   0   0                              // s^3 row
    -    -   -   -
    1    1   1   1

   eps   16  0   0                              // s^2 row
   ---   --  -   -
    1    1   1   1

   -64   0   0   0                              // s^1 row
   ---   -   -   -
   eps   1   1   1

  -1024  0   0   0                              // s^0 row
  -----  -   -   -
   -64   1   1   1
num  =
  2.
```

b)

To construct a degree 8 polynomial whose R-H table has its entire row corresponding to s^3 to be zero, we need a degree $(3+1=4)$ even polynomial. Say,

$$P(s) = \underbrace{(s^4+16)}_{\text{even}} \cdot (s^2+2s+1)^2$$

$$P(s) = s^8 + 4s^7 + 6s^6 + 4s^5 + 17s^4 + 64s^3 + 96s^2 + 64s + 16$$

Note that s^4 and s^5 rows are linearly dependent, hence entire row will be 0 in s^3.

```
 > P = (s^4+16)*(s^2+2*s+1)^2
P  =
 s^8 +4s^7 +6s^6 +4s^5 +17s^4 +64s^3 +96s^2 +64s +16
 > [r, num] = routh_t(P)
r  =
    1    6    17    96   16           // s^8 row
    -    -    --    --   --
    1    1    1     1    1

    4    4    64    64   0            // s^7 row
    -    -    --    --   -
    1    1    1     1    1

    5    1    80    16   0            // s^6 row
    -    -    --    --   -
    1    1    1     1    1

   3.2   0    51.2  0    0            // s^5 row
   ---   -    ----  -    -
    1    1    1     1    1

    1    0    16    0    0            // s^4 row
    -    -    --    -    -
    1    1    1     1    1

    4    0    0     0    0            // s^3 row
    -    -    -     -    -
    1    1    1     1    1

   eps   16   0     0    0            // s^2 row
   ---   --   -     -    -
    1    1    1     1    1

   -64   0    0     0    0            // s^1 row
   ---   -    -     -    -
   eps   1    1     1    1

  -1024  0    0     0    0            // s^0 row
  -----  -    -     -    -
   -64   1    1     1    1
num  =
  2.
```

c)

To construct a degree 6 polynomial whose R-H table has its first entry in its row corresponding to s^3 to be zero, first take a general degree 6 polynomial $a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + a_6 s^6$.

$$
\begin{array}{c|ccc}
s^6 & a_6 & a_4 & a_2 \\
s^5 & a_5 & a_3 & a_1 \\
s^4 & (a_6 a_3 - a_5 a_4) & (a_6 a_1 - a_5 a_2) & 0 \\
s^3 & a_5(a_6 a_1 - a_5 a_2) - a_3(a_6 a_3 - a_5 a_4) & \text{x} & 0
\end{array}
$$

Hence, $a_5(a_6 a_1 - a_5 a_2) = a_3(a_6 a_3 - a_5 a_4)$.

Now, take $a_6 = 1, a_5 = 2, a_4 = 3, a_3 = 4, a_2 = 5 \quad \rightarrow \quad a_1 = 6$ take a_0 as 7 (a_0 doesn't effect first entry s^3)

$$P = s^6 + 2s^5 + 3s^4 + 4s^3 + 5s^2 + 6s + 7$$

Note that first 2 elements of s^4 and s^5 rows are linearly dependent, hence first element of s^3 row will be 0

```
 > P = 7+6*s+5*s^2+4*s^3+3*s^4+2*s^5+s^6;
 > [r, num] = routh_t(P)
r  =
         1          3   5  7          // s^6 row
         -          -   -  -
         1          1   1  1

         2          4   6  0          // s^5 row
         -          -   -  -
         1          1   1  1

         1          2   7  0          // s^4 row
         -          -   -  -
         1          1   1  1

        eps         -8  0  0          // s^3 row
        ---         --  -  -
         1          1   1  1

      8 +2eps       7   0  0          // s^2 row
      -------       -   -  -
        eps         1   1  1

   -64 -16eps -7eps^2  0   0  0       // s^1 row
   -----------------   -   -  -
      8 +2eps       1   1  1

         7          0   0  0          // s^0 row
         -          -   -  -
         1          1   1  1
num  =
  2.
```