EE324 Control Systems Lab

Problem Sheet 8

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Contents

1 Q1

Lag Compensator is

$$\mathrm{G}(s) = \frac{(s+K_1)}{(s+K_2)} = \frac{(s+5K)}{(s+K)}$$

a)

As shown in Figure 1, as K increases, rise time as well as 2% settling time decreases. So, by moving the pole away from (towards) the origin, the transient response grows faster (slower). As a lag compensator is used for steady steady response, moving poles and zeros further away change the transient response. Hence, not ideal.

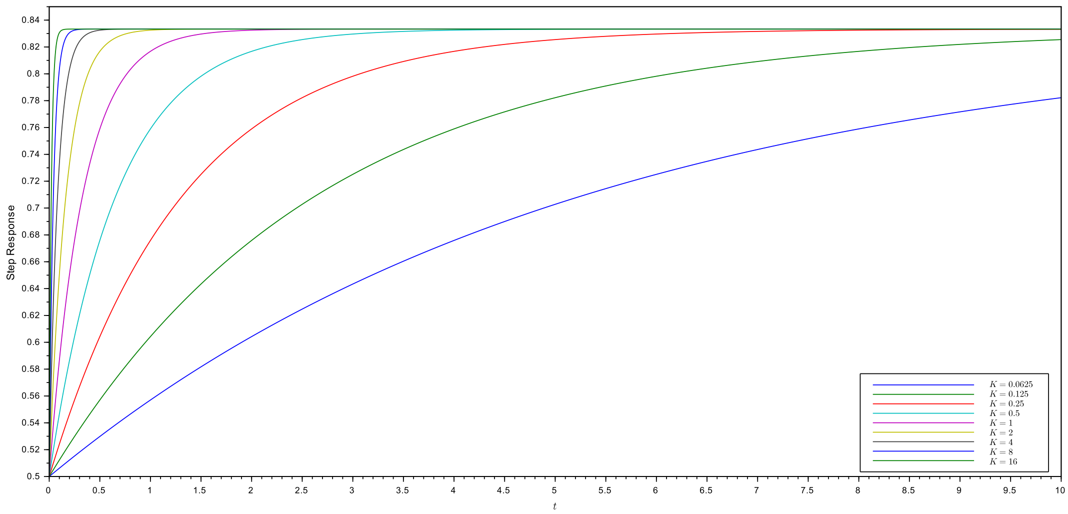


Figure 1: Variation in Step Response with K

Code to obtain K and Figure 1 is given below

```
s = %s;
tMax = 10;
tStep = 0.001;
t = 0:tStep:tMax;
ratio = 5;
y = [];
kValues = [1/16 1/8 1/4 1/2 1 2 4 8 16];
legendValues = [];
dims = 1;
for i = 1:length(kValues)
    k = kValues(i);
    K1 = ratio * k;
    K2 = k;
    lagCompensator = (s+K1)/(s+K2);
    str = "$K = " + string(k) + "$";
    legendValues = cat(dims, legendValues, str);
    sys = syslin('c', lagCompensator/(1+lagCompensator));
    gp = csim('step', t, sys);
    y = cat(dims, y, gp);
end
plot(t',y'); xlabel("$t$"); ylabel("Step Response");
legend(legendValues,opt=4);
xs2pdf(0,'Q1a');
```

b)

As shown in Figure 2, as K increases, i.e. moving the pole away from (towards) the origin, the transient response decays faster (slower). Again, not desired from a lag compensator. Also, the initial value, is higher for larger K.

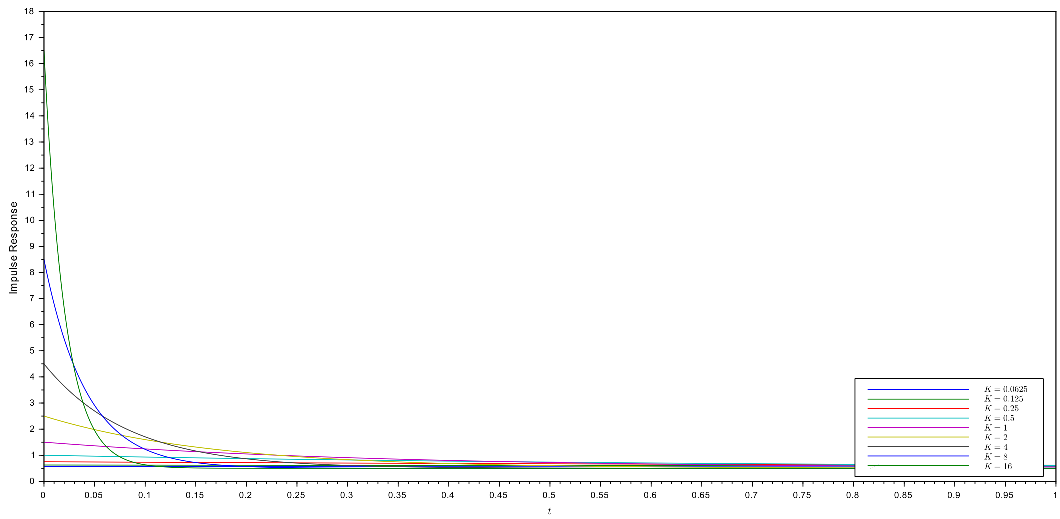


Figure 2: Variation in Impulse Response with K

Code to obtain K and Figure 2 is given below

```
s = %s;
tMax = 1;
tStep = 0.001;
t = 0:tStep:tMax;
ratio = 5;
y = [];
kValues = [1/16 1/8 1/4 1/2 1 2 4 8 16];
legendValues = [];
dims = 1;
for i = 1:length(kValues)
    k = kValues(i);
    K1 = ratio * k;
    K2 = k;
    lagCompensator = (s+K1)/(s+K2);
    str = "$K = " + string(k) + "$";
    legendValues = cat(dims, legendValues, str);
    sys = syslin('c', lagCompensator/(1+lagCompensator));
    gp = csim('impuls', t, sys);
    y = cat(dims, y, gp);
end
plot(t',y'); xlabel("$t$"); ylabel("Impulse Response");
legend(legendValues,opt=4);
xs2pdf(0,'Q1b');
```

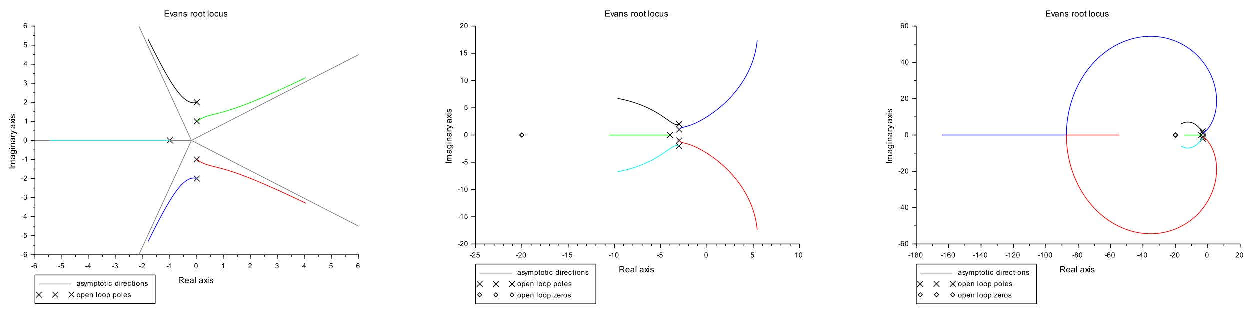
2 Q2

a)

Open loop transfer function is

$$G_1(s) = \frac{1}{(s+1)\cdot(s+\imath)(s-\imath)\cdot(s+2\imath)(s-2\imath)} = \frac{1}{(s+1)\cdot(s^2+1)\cdot(s^2+4)}$$

Root Locus of $G_1(s)$ is shown in Figure 3a



(a) $G_1(s)$ (b) $G_3(s)$ (c) $G_3(s)$ zoomed out

Code to obtain Figure 3a is given below

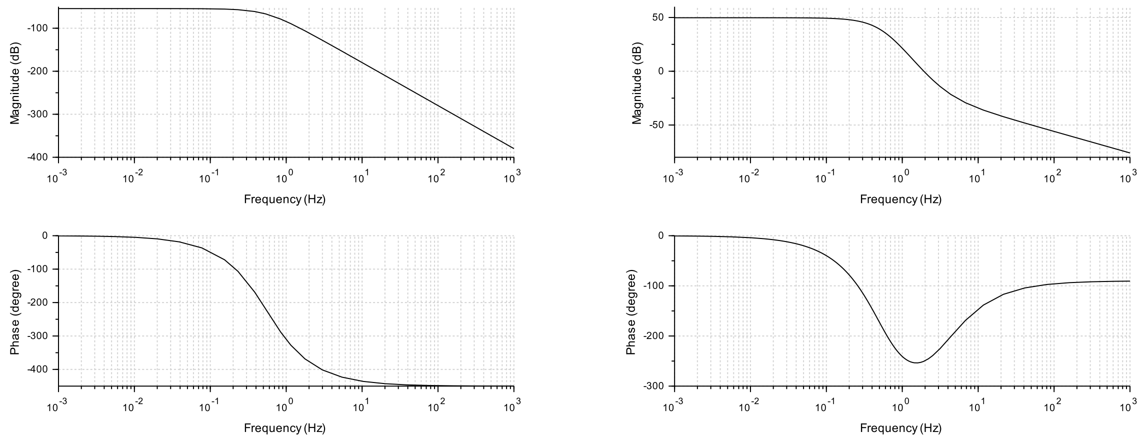
```
s = %s;
G1 = 1/((s+1)*(s+%i)*(s-%i)*(s+2*%i)*(s-2*%i))
--> G1  =
              1
    ----------------------
    4 +4s +5s^2 +5s^3 +s^4 +s^5
evans(G1);
xs2pdf(0,'Q2a');
```

b)

Let $k = 3$, now origin is shifted to $-k$,

$$G_2(s) = \frac{1}{((s+k)+1)\cdot((s+k)^2+1)\cdot((s+k)^2+4)} = \frac{1}{((s+3)+1)\cdot((s+3)^2+1)\cdot((s+3)^2+4)}$$

Bode Plot of $G_2(s)$ is shown in Figure 4a



(a) $G_2(s)$ (b) $G_3(s)$

Code to obtain Figure 4a is given below

```
k = 3;
G2 = 1/(((s+k)+1)*((s+k)+%i)*((s+k)-%i)*((s+k)+2*%i)*((s+k)-2*%i))
--> G2  =
                    1
   ----------------------------------
   520 +682s +374s^2 +107s^3 +16s^4 +s^5
G2 = syslin('c', G2);
bode(G2);
xs2pdf(0,'Q2b');
```

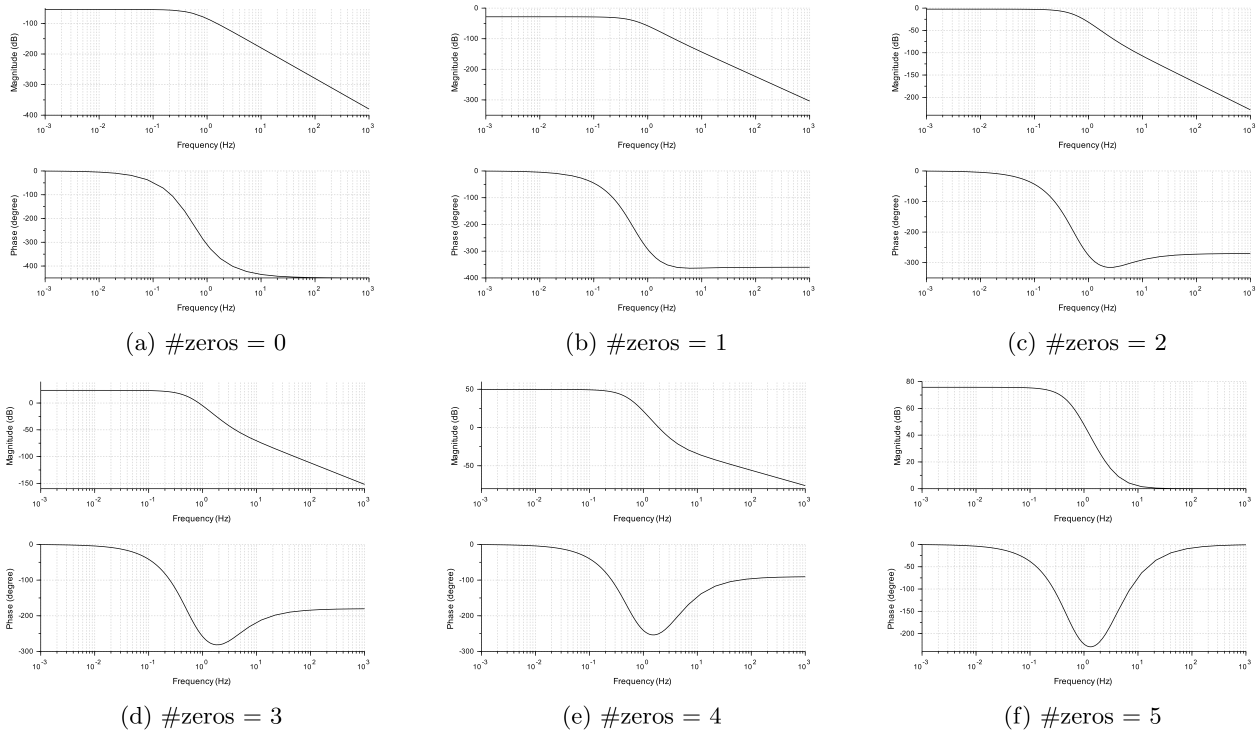
c)

$G_2(s)$ already has a phase crossover. So, we add zero(s) such that phase response again increases once it goes below $-180°$ for the 2^{nd} crossover. These zero(s) should be sufficiently away from the poles. It turns out that by adding just 2 zeros, the phase response increases back till $-270°$ only. A 3^{rd} zero makes that phase *tend to* $-180°$. And the 4^{th} will make sure the phase goes over $-180°$. So, we should add atleast 4 zeros. Also, adding a 5^{th} zero makes the phase tend to $0°$, hence it is not a good choice. (Shown in Figure ??)
Hence, I have kept all 4 zeros at -20.

$$G_3(s) = \frac{(s+20)^4}{((s+3)+1)\cdot((s+3)^2+1)\cdot((s+3)^2+4)}$$

Bode Plot of $G_3(s)$ is shown in Figure 4b
Code to obtain Figure 4b is given below

```
G3 = (s+20)^4/(((s+k)+1)*((s+k)+i)*((s+k)-%i)*((s+k)+2*%i)*((s+k)-2*%i))
G3 = syslin('c', G3);
--> G3  =
   160000 +32000s +2400s^2 +80s^3 +s^4
   ----------------------------------
   520 +682s +374s^2 +107s^3 +16s^4 +s^5
bode(G3);
xs2pdf(0,'Q2c');
```



(a) #zeros = 0 (b) #zeros = 1 (c) #zeros = 2

(d) #zeros = 3 (e) #zeros = 4 (f) #zeros = 5

Figure 5: Variation of Bode Plot with number of zeros

Here,

$$G'(s) = \frac{(s+20)^{\#\text{zeros}}}{((s+3)+1)\cdot((s+3)^2+1)\cdot((s+3)^2+4)}$$

d)

Code to obtain Figure 3b & 3c is given below

```
evans(G3);
xs2pdf(0,'Q2d');
evans(G3, 250);
xs2pdf(0,'Q2de');
```

Root Locus of $G_3(s)$ is shown in Figure 3b & 3c. It does intersect 2 times with imaginary axis.

3 Q3

There are 4 (poles + zeros) as shown in the asymptotic approximation plot (magnitude). Poles are at $-5, -10, -100$ (magnitude response increasing). Zero at -1 (magnitude response decreasing). So,

$$G(s) = \frac{K(s+1)}{(s+5)\cdot(s+10)\cdot(s+100)} \quad \text{where} \quad \lim_{s\to 0} G(s) = K \cdot \frac{1}{5\cdot 10\cdot 100} \Rightarrow K = 10^{-\frac{75}{20}} \cdot 5\cdot 10\cdot 100 = 0.8891397$$

Initial value of magnitude response ≈ -75dB.

$$G(s) = \frac{0.8891397(s+1)}{(s+5)\cdot(s+10)\cdot(s+100)}$$

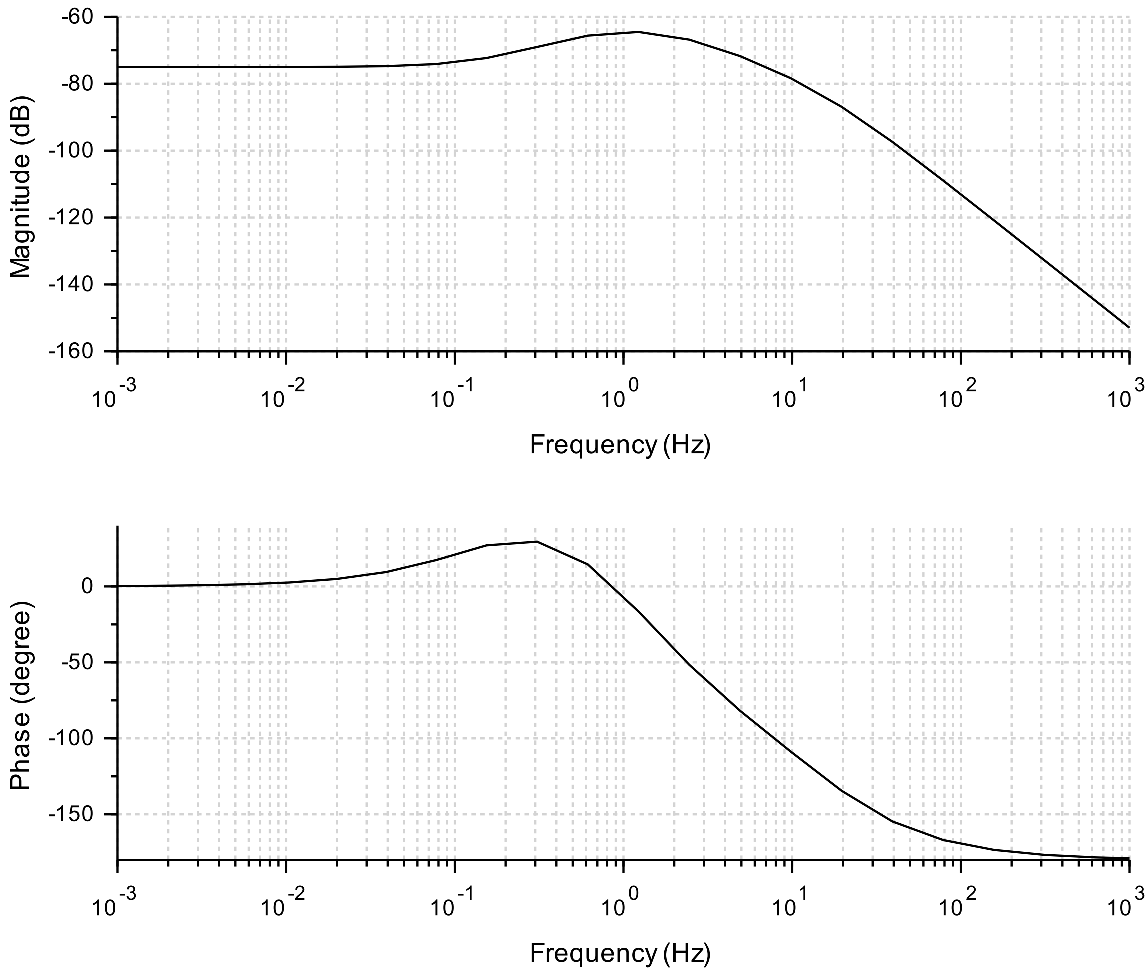


Figure 6: Bode Plot of $G(s)$

Code to obtain Figure 6 is given below

```
initialMagnitude = -75
K = 10^(initialMagnitude/20)*5000
G = K*(s+1)/((s+5)*(s+10)*(s+100));
G = syslin('c', G);
bode(G)
xs2pdf(0,'Q3');
```