EE324 Control Systems Lab

Problem Sheet 1

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Contents

1 Q1

$$G_1 = \frac{10}{s^2 + 2s + 10}$$

$$G_2 = \frac{5}{s + 5}$$

Initialising code

1. For the given Cascade System, the tranfer function is given by $G_1(s) \cdot G_2(s) = \frac{50}{50 + 20s + 7s^2 + s^3}$

```
--> s = poly(0, 's');                // define the symbolic variable 's'.
--> G1 = 10 / (s^2 + 2 * s + 10);    // define G1(s)
--> G2 =  5 / (s + 5);               // define G2(s)
// here the input is given to G1 and that output of G1 is used as input in G2 to get output
--> G = G1 * G2
 G  =
          50
   ----------------
   50 +20s +7s^2 +s^3
```

2. For the given Parallel System, the tranfer function is given by $G_1(s) + G_2(s) = \frac{100 + 20s + 5s^2}{50 + 20s + 7s^2 + s^3}$

```
--> s = poly(0, 's');                // define the symbolic variable 's'.
--> G1 = 10 / (s^2 + 2 * s + 10);    // define G1(s)
--> G2 =  5 / (s + 5);               // define G2(s)
// here G1 and G2 has same input and their output is added to get system's output
--> G = G1 + G2
 G  =
    100 +20s +5s^2
   ----------------
   50 +20s +7s^2 +s^3
```

3. For the given Feedback(closed loop) system, the tranfer function is given by $\frac{G_1(s)}{1 + G_1(s) \cdot G_2(s)} = \frac{50 + 10s}{100 + 20s + 7s^2 + s^3}$

```
--> s = poly(0, 's');                // define the symbolic variable 's'.
--> G1 = 10 / (s^2 + 2 * s + 10);    // define G1(s)
```

```
--> G2 =  5 / (s + 5);                  // define G2(s)
// here addition of input and (output of G1) (after applying G2) is G1's input
--> G = G1 / (1 + G1 * G2)
 G  =
         50 +10s
    -----------------
    100 +20s +7s^2 +s^3
```

4. Unit step response to the system with the tranfer function is given by $G_1(s)$ in the below figure

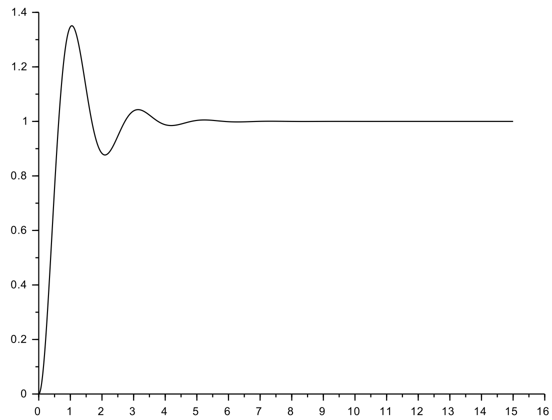


Figure 1.1: Unit Step response to the system with TF $G_1(s)$

```
--> s = poly(0, 's');                   // define the symbolic variable 's'.
--> G1 = 10 / (s^2 + 2 * s + 10);       // define G1(s)
--> t = 0:0.01:15;                      // simulate for first 15 seconds with time step = 0.01s
--> gp1 = csim('step',t,G1);            // G1's response to step input
--> plot2d(t,gp1)                       // plot response
```

2 Q2

1. For the given Cascade System, poles of the system are $-5, -1 \pm 3j$

```
--> s = poly(0, 's');                   // define the symbolic variable 's'.
--> G1 = 10 / (s^2 + 2 * s + 10);       // define G1(s)
--> G2 =  5 / (s + 5);                  // define G2(s)
--> G = G1 * G2;
--> [z, p, k] = tf2zp(G)                // tf2zp gives zeroes and poles
 z  =
     []
 p  =
   -5. + 0.i
   -1. + 3.i
   -1. - 3.i
 k  =
    50.
```

2. For the given Parallel System, poles of the system are $-5, -1 \pm 3j$, zeroes $-2 \pm 4j$

```
--> s = poly(0, 's');                   // define the symbolic variable 's'.
--> G1 = 10 / (s^2 + 2 * s + 10);       // define G1(s)
--> G2 =  5 / (s + 5);                  // define G2(s)
--> G = G1 + G2
```

```
 G  =
    100 +20s +5s^2
   ----------------
   50 +20s +7s^2 +s^3
--> [z, p, k] = tf2zp(G)            // tf2zp gives zeroes and poles
 z  =
  -2. + 4.i
  -2. - 4.i
 p  =
  -5. + 0.i
  -1. + 3.i
  -1. - 3.i
 k  =
   5.
```

3. For the given Feedback system, poles of the system are -6.3347665, $-0.3326167 \pm 3.9592004j$ and zero is at -5

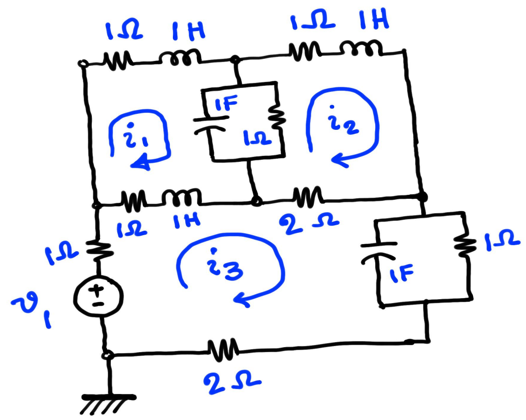
```
--> s = poly(0, 's');               // define the symbolic variable 's'.
--> G1 = 10 / (s^2 + 2 * s + 10);   // define G1(s)
--> G2 =  5 / (s + 5);              // define G2(s)
--> G = G1 / (1 + G1 * G2)
 G  =
        50 +10s
   -----------------
   100 +20s +7s^2 +s^3
--> [z, p, k] = tf2zp(G)            // tf2zp gives zeroes and poles
 z  =
  -5.0000000
 p  =
  -6.3347665 + 0.i
  -0.3326167 + 3.9592004i
  -0.3326167 - 3.9592004i
 k  =
   10.000000
```

3 Polynomial Matrices Computation

```
--> s = poly(0, 's');

--> A = [s 1+s; 1+s+2*s^2 1+s+2*s^2+3*s^3]
 A  =
  s          1 +s

  1 +s +2s^2  1 +s +2s^2 +3s^3

--> B = [2+s 3*s+5; 8*s^2+13*s 21]
 B  =
  2 +s       5 +3s

  13s +8s^2  21

--> A+B
 ans  =
  2 +2s          6 +4s

  1 +14s +10s^2  22 +s +2s^2 +3s^3

--> A*B
 ans  =
```

```
   15s +22s^2 +8s^3                21 +26s +3s^2

   2 +16s +26s^2 +36s^3 +55s^4 +24s^5   26 +29s +55s^2 +69s^3

--> det(A)
 ans  =
   -1 -s -2s^2 +3s^4

--> inv(A)
 ans  =
     1 +s +2s^2 +3s^3          -1 -s
     ---------------   ---------------
     -1 -s -2s^2 +3s^4   -1 -s -2s^2 +3s^4

       -1 -s -2s^2              s
     ---------------   ---------------
     -1 -s -2s^2 +3s^4   -1 -s -2s^2 +3s^4
```

4 Q3



Replace Inductances and Capacitances by their Impedances ($L \to sL$ and $C \to \frac{1}{sC}$) in the above circuit

Now, we can write equation obtained via mesh analysis, in matrix-vector form $Z(s)I(s) = V(s)$.

$$\begin{bmatrix}\text{Sum of applied}\\ \text{voltages}\\ \text{around Mesh 1}\end{bmatrix} = \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 1}\end{bmatrix} I_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to Mesh 1 and 2}\end{bmatrix}(-I_2(s)) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to Mesh 1 and 3}\end{bmatrix}(-I_3(s))$$

$$\begin{bmatrix}\text{Sum of applied}\\ \text{voltages}\\ \text{around Mesh 2}\end{bmatrix} = \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to Mesh 2 and 1}\end{bmatrix}(-I_1(s)) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 2}\end{bmatrix} I_2(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to Mesh 2 and 3}\end{bmatrix}(-I_3(s))$$

$$\begin{bmatrix}\text{Sum of applied}\\ \text{voltages}\\ \text{around Mesh 3}\end{bmatrix} = \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to Mesh 3 and 1}\end{bmatrix}(-I_1(s)) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to Mesh 3 and 2}\end{bmatrix}(-I_2(s)) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 3}\end{bmatrix} I_3(s)$$

$$Z(s) = \begin{bmatrix} 2+2s+\frac{1}{(s+1)} & -\frac{1}{(s+1)} & -(1+s) \\ -\frac{1}{(s+1)} & 3+s+\frac{1}{(s+1)} & -2 \\ -(1+s) & -2 & 6+s+\frac{1}{(s+1)} \end{bmatrix} \quad I = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \quad V = \begin{bmatrix} 0 \\ 0 \\ V_1 \end{bmatrix}$$

Now,

$$i(s) = [Z(s)]^{-1}V(s)$$

and,

$$Z^{-1}(s) = \begin{bmatrix} \frac{24+48s+35s^2+11s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{9+13s+7s^2+2s^3}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{9+13s+7s^2+2s^3}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{20+45s+39s^2+14s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \end{bmatrix}$$

So, the i^{th} row of 3rd column of Z^{-1} gives $\frac{I_i(s)}{V_1(s)}$

$$\begin{bmatrix} i_1(s) \\ i_2(s) \\ i_3(s) \end{bmatrix} = \begin{bmatrix} \frac{24+48s+35s^2+11s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{9+13s+7s^2+2s^3}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{9+13s+7s^2+2s^3}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{20+45s+39s^2+14s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ V_1(s) \end{bmatrix}$$

$$\begin{bmatrix} i_1(s) \\ i_2(s) \\ i_3(s) \end{bmatrix} = \begin{bmatrix} \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \cdot V_1(s) \\ \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} \cdot V_1(s) \\ \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \cdot V_1(s) \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} T_1(s) \\ T_2(s) \\ T_3(s) \end{bmatrix} = \begin{bmatrix} i_1(s)/V_1(s) \\ i_2(s)/V_1(s) \\ i_3(s)/V_1(s) \end{bmatrix} = \begin{bmatrix} \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \end{bmatrix}$$

The code for finding transfer functions is given below

```
--> Z = [2+2*s+1/(s+1) -1/(s+1) -(1+s);-1/(s+1) 3+s+1/(s+1) -2;-(1+s) -2 6+s+1/(s+1)]
 Z  =
   3 +4s +2s^2     -1          -1 -s
   ----------     ----         -----
      1 +s        1 +s           1

      -1       4 +4s  +s^2      -2
     ----     ---------         --
     1 +s       1 +s            1

    -1 -s          -2       7 +7s  +s^2
    -----          --       ---------
      1            1          1 +s

--> Zinv = inv(Z)
 Zinv  =
      24 +48s +35s^2 +11s^3 +1s^4            9 +13s +7s^2 +2s^3             6 +14s +13s^2 +6s^3 +1s^4
   ------------------------------   ------------------------------   ------------------------------
   57 +144s +147s^2 +74s^3 +17s^4 +s^5  57 +144s +147s^2 +74s^3 +17s^4 +s^5  57 +144s +147s^2 +74s^3 +17s^4 +s^5

          9 +13s +7s^2 +2s^3            20 +45s +39s^2 +14s^3 +1s^4           7 +16s +13s^2 +4s^3
   ------------------------------   ------------------------------   ------------------------------
   57 +144s +147s^2 +74s^3 +17s^4 +s^5  57 +144s +147s^2 +74s^3 +17s^4 +s^5  57 +144s +147s^2 +74s^3 +17s^4 +s^5

       6 +14s +13s^2 +6s^3 +1s^4             7 +16s +13s^2 +4s^3            11 +28s +27s^2 +12s^3 +2s^4
   ------------------------------   ------------------------------   ------------------------------
   57 +144s +147s^2 +74s^3 +17s^4 +s^5  57 +144s +147s^2 +74s^3 +17s^4 +s^5  57 +144s +147s^2 +74s^3 +17s^4 +s^5

--> TF = Zinv * [0; 0; 1]
 TF  =
       6 +14s +13s^2 +6s^3 +1s^4
   ------------------------------
   57 +144s +147s^2 +74s^3 +17s^4 +s^5

          7 +16s +13s^2 +4s^3
   ------------------------------
   57 +144s +147s^2 +74s^3 +17s^4 +s^5

      11 +28s +27s^2 +12s^3 +2s^4
   ------------------------------
   57 +144s +147s^2 +74s^3 +17s^4 +s^5
```