EE324 Control Systems Lab

Problem Sheet 10

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Autumn Semester 2021-22

Contents

1 Q1

a)

The following state space realisation was used

$$A = \begin{bmatrix} 4 & 3 & 5 \\ 2 & 8 & 2 \\ 3 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 7 \\ 6 \end{bmatrix} \quad C = \begin{bmatrix} 5 & 7 & 8 \end{bmatrix} \quad D = 6$$

Now, similarity transformation was applied with

$$T = \begin{bmatrix} 6 & 9 & 0 \\ 3 & 4 & 6 \\ 10 & 7 & 5 \end{bmatrix} \quad \text{so} \quad A' = T^{-1}AT, \quad B' = T^{-1}B, \quad C' = CT, \quad D' = D$$

In both cases, the transfer function is the same

$$G(s) = \frac{354 - 332s + 17s^2 + 6s^3}{24 + 47s - 15s^2 + s^3}$$

```
s = %s;
n = 3;
intMin = 0;
intMax = 10;
T = grand(n,n,"uin",intMin,intMax);
--> 6.    9.   0.
    3.    4.   6.
    10.   7.   5.
A = grand(n,n,"uin",intMin,intMax);
--> 4.   3.   5.
    2.   8.   2.
    3.   0.   3.
B = grand(n,1,"uin",intMin,intMax);
--> 2.
    7.
    6.
C = grand(1,n,"uin",intMin,intMax);
```

```
--> 5. 7. 8.
D = grand(1,1,"uin",intMin,intMax);
--> 6
G1 = D + C*inv(s*eye(n,n) - A)*B;
--> 354 -332s +17s^2 +6s^3
    --------------------
     24 +47s -15s^2 +s^3

Anew = inv(T)*A*T;
Bnew = inv(T)*B;
Cnew = C*T;
G2 = D + Cnew*inv(s*eye(n,n) - Anew)*Bnew;
--> 354 -332s +17s^2 +6s^3
    --------------------
     24 +47s -15s^2 +s^3
```

b)

Eigenvalues of A are $-0.4454348, 10.122807, 5.3226279$ which is same as the poles of $G(s)$

```
[z, p, k] = tf2zp(G1);
disp(p);
--> 10.122807 + 0.i
    5.3226279 + 0.i
   -0.4454348 + 0.i
disp(spec(A));
--> -0.4454348 + 0.i
    10.122807 + 0.i
    5.3226279 + 0.i
```

c)

I took 2 transfer functions as

$$G_1(s) = \frac{(s+4)\cdot(s+7)}{(s+5)\cdot(s+3)} \quad \rightarrow \quad \text{Proper} \tag{1.1}$$

$$G_2(s) = \frac{(s+7)}{(s+5)\cdot(s+3)} \quad \rightarrow \quad \text{Strictly Proper} \tag{1.2}$$

The corresponding state space realisations are

$$A_1 = \begin{bmatrix} -3.5559105 & 0.1022364 \\ 7.8522364 & -4.4440895 \end{bmatrix} \quad B_1 = \begin{bmatrix} -2.7131204 \\ 2.9392138 \end{bmatrix} \quad C_1 = \begin{bmatrix} -1.1057379 & -2.220D-16 \end{bmatrix} \quad D_1 = 1$$

$$A_2 = \begin{bmatrix} -1.8615385 & -0.4923077 \\ 7.2576923 & -6.1384615 \end{bmatrix} \quad B_2 = \begin{bmatrix} -1.4032928 \\ 2.4557625 \end{bmatrix} \quad C_2 = \begin{bmatrix} -0.7126096 & 0 \end{bmatrix} \quad D_2 = 0$$

Hence, D is non zero for when degree of numerator and denominator are same and, $D = 0$ for strictly proper transfer functions (As $\lim_{s\to\infty} G(s) = D\ (= 0)$)

```
s = %s;
G1 = (s+4)*(s+7)/((s+5)*(s+3));
G2 = (s+7)/((s+5)*(s+3));
SS1 = tf2ss(G1);
SS2 = tf2ss(G2);
disp(SS1("D"), SS2("D"))
--> 1, 0
```

2 Q2

$$G_1(s) = \frac{(s+3)}{(s^2+5s+4)}$$

$$G_2(s) = \frac{(s+1)}{(s^2+5s+4)}$$

The corresponding state space realisations are

$$A_1 = \begin{bmatrix} -1.5384615 & 0.3076923 \\ 4.3076923 & -3.4615385 \end{bmatrix} \quad B_1 = \begin{bmatrix} -1.1094004 \\ 1.6641006 \end{bmatrix} \quad C_1 = \begin{bmatrix} -0.9013878 & 5.551D-17 \end{bmatrix} \quad D_1 = 0$$

$$A_2 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} \quad B_2 = \begin{bmatrix} -1.2649111 \\ 0 \end{bmatrix} \quad C_2 = \begin{bmatrix} -0.7905694 & 0 \end{bmatrix} \quad D_2 = 0$$

In $G_2(s)$ there is pole zero cancellation. Hence, I was getting a 1×1 state space realisation.

To convert it into 2×2 state space realisation, I made a similarity transformation with $T = I_2$ and appended 0 to both B and C for preserving dimensions.

Also I turned off scilab simplication to get better results using `simp_mode(%F)`.

```
s = %s;
n = 2;
G1 = (s+3)/(s^2+5*s+4);
SSc1 = tf2des(G1);
simp_mode(%F);
G2 = (s+1)/(s^2+5*s+4);
SSc2 = tf2des(G2);
T = eye(2,2);
A = SSc2("A"); B = SSc2("B"); C = SSc2("C"); D = SSc2("D");
Anew = inv(T)*A*T
--> -4.   0.
     0.  -4.
Bnew = inv(T)*[SSc2("B");0]
-->  -1.2649111
     0.
Cnew = [SSc2("C"), 0]*T
-->  -0.7905694 0
Dnew =  D
-->  0
simp_mode(%T);
G3 = Dnew + Cnew*inv(s*eye(n,n) - Anew)*Bnew;
-->  1
    ----
    4 +s
```

3 Q3

Case	A	B	C	D	Eigenvalues of A	Poles of $G(s)$
i)	$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$	$\begin{bmatrix} 9 \\ 4 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 10 & 5 & 8 \end{bmatrix}$	4	1,4,7	1,4,7
ii)	$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$	$\begin{bmatrix} 9 \\ 0 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 10 & 5 & 8 \end{bmatrix}$	4	1,4,7	4,7
iii)	$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$	$\begin{bmatrix} 9 \\ 4 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 0 & 5 & 8 \end{bmatrix}$	4	1,4,7	1,4

We can see the effect of pole zero cancellation from the above table.
When a entry of B is zero, we lose a pole with corresponding entry of A.

When B_2 was made zero, we lost pole $A_{22} = 1$.
When C_1 was made zero, we lost pole $A_{11} = 7$.

```
s = %s;
n = 3;
A = [[grand("uin",intMin,intMax) 0 0] ;[0 grand("uin",intMin,intMax) 0] ;
     [0 0 grand("uin",intMin,intMax)]];
--> 7.   0.   0.
    0.   1.   0.
    0.   0.   4.
B = grand(n,1,"uin",intMin+1,intMax);
--> 9.
    4.
    4.
C = grand(1,n,"uin",intMin+1,intMax);
--> 10.   5.   8.
D = grand("uin",intMin,intMax);
--> 4.
G1 = D + C*inv(s*eye(n,n) - A)*B;
--> 1032 -770s +94s^2 +4s^3
    ---------------------
     -28 +39s -12s^2 +s^3
disp(spec(A));
[z, p, k] = tf2zp(G1);
disp(p);
Bnew = B;
Bnew(2) = 0;
G2 = D + C*inv(s*eye(n,n) - A)*Bnew;
--> -472 +78s +4s^2
    --------------
     28 -11s +s^2
[z, p, k] = tf2zp(G2);
disp(p);
Cnew = C;
Cnew(1) = 0;
G3 = D + Cnew*inv(s*eye(n,n) - A)*B;
--> -96 +32s +4s^2
    -------------
      4 -5s +s^2
[z, p, k] = tf2zp(G3);
disp(p);
```

4 Q4

Case	A	B	C	D	Eigenvalues of A	Poles of $G(s)$
i)	$\begin{bmatrix} 1 & 8 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 5 & 7 & 10 \end{bmatrix}$	4	1,2,2	1,2,2
ii)	$\begin{bmatrix} 1 & -1.4 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 5 & 7 & 10 \end{bmatrix}$	4	1,2,2	1,2

As we can see, there can be pole zero cancellation even when all elements in B and C are nonzero.

```
s = %s;
n = 3;
a11 = grand("uin",intMin,intMax);
a12 = grand("uin",intMin,intMax);
a22 = grand("uin",intMin,intMax);
a23 = grand("uin",intMin,intMax);
a33 = a22;
A = [[a11 a12 0] ; [0 a22 a23] ; [0 0 a33]];
--> 1.   8.   0.
    0.   2.   5.
    0.   0.   2.
B = grand(n,1,"uin",intMin+1,intMax);
--> 1.
    7.
    3.
C = grand(1,n,"uin",intMin+1,intMax);
--> 5.   7.   10.
D = grand("uin",intMin,intMax);
--> 5.
G1 = D + C*inv(s*eye(n,n) - A)*B;
--> 93 +168s +59s^2 +5s^3
   -------------------
     -4 +8s -5s^2 +s^3
disp(spec(A));
[z, p, k] = tf2zp(G1);
disp(p);
A(1,2) = C(2) * (A(1,1) - A(2,2)) / C(1);
--> 1.   -1.4   0.
    0.   2.    5.
    0.   0.    2.
G2 = D + C*inv(s*eye(n,n) - A)*B;
--> -23 +69s +5s^2
    -------------
      2 -3s +s^2
disp(spec(A));
[z, p, k] = tf2zp(G2);
disp(p);
```