2.2 - Exercícios propostos:

7 a) $M = 1,01100000000000000000000_2$

b) $E = 10000011_2 = 2^7 + 2^1 + 2^0 = 131 \quad E^{real} = 131 - 127 = 4 = 00000100_2$

c) $-1,011 \times 2^4 = -10110_2 = -(2^4 + 2^2 + 2^1) = -22,0_{10}$

8 a) Sinal: 0 $\quad 31,25 = 11111,01_2 = 1,111101_2 \times 2^4 \quad E^{real} = 4$

$E^{real} = E - 127 \Leftrightarrow E = 4 + 127 = 131 = 10000011_2$

$31,25 = 0\,\underline{10000011}\,11110100000000000000000_2 =$

$= 41FA0000_H$

b) Sinal: 1 $\quad 0,625 = 0,101_2 = 1,01 \times 2^{-1} \quad E^{real} = -1$

$E^{real} = E - 127 \Leftrightarrow E = -1 + 127 = 126 = 01111110_2$

$0,625 = 1\,\underline{01111110}\,01000000000000000000000_2 =$

$= BF200000_H$

c) 00000000_H

d) Sinal: 0 $\quad 1026,5 = 10000000010,1 = 1,00000000101 \times 2^{10}$

$E^{real} = E - 127 \Leftrightarrow E = 10 + 127 = 137 = 10001001_2$

$1026,5 = 0\,\underline{10001001}\,00000000101000000000000_2 =$

$= 44805000_H$

9 a) $0\,00000001\,00000000000000000000000_2 = 1{,}7549435\times10^{-38}$

b) $0\,11111110\,11111111111111111111111_2 = 3{,}4028235\times10^{38}$

c) $1\,00000001\,00000000000000000000000_2 = -1{,}7549435\times10^{-38}$

d) $1\,11111110\,11111111111111111111111_2 = -3{,}4028235\times10^{38}$

10 a) Sinal: $-$ $\quad E = 1-3 = -2 \quad -1{,}1101\times2^{-2} = -0{,}011101_2 =$

$= -(2^{-2}+2^{-3}+2^{-4}+2^{-6}) = -\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{64}\right) = \left(-\frac{29}{64}\right)_{10}$

b) Sinal: $+$ $\quad E^{real} = 5-3 = 2 \quad 1{,}1001\times2^{2} = 110{,}01 =$

$= 2^{2}+2^{1}+2^{-2} = 6{,}25_{10}$

c) Sinal: 0 $\quad \frac{3}{8} = \frac{2}{8}+\frac{1}{8} = \frac{1}{4}+\frac{1}{8} = 2^{-2}+2^{-3} = 0{,}011 = 1{,}1\times2^{-2}$

$-2 = E-3 \Leftrightarrow E = 1 = 001_2 \quad \frac{3}{8} = 0\,001\,1000_2$

11 a) Sinal A: 0 $\quad 33 = 100001_2 = 1{,}00001\times2^{5} \quad E^{real} = 5$

$E^{real} = E-127 \Leftrightarrow E = 5+127 = 132 = 10000100_2$

$A = 0\,10000100\,00001000000000000000000_2 = 42040000_H$

Sinal B: 1 $\quad 2{,}875 = 10{,}111_2 = 1{,}0111_2\times2^{1} \quad E^{real} = 1$

$E^{real} = E-127 \Leftrightarrow E = 1+127 = 128 = 10000000_2$

$B = 1\,10000000\,01110000000000000000000_2 = C0380000_H$

b) i) sinais diferentes → subtracção $\qquad |A|>|B|$ → valor final é positivo.

$E = 127+5 = 132 = 10000100_2$ ← expoente real do maior nº.

mantissa de $A = 1{,}00001$ $\qquad$ mantissa de $B = 0{,}0010111$

$$\begin{array}{r} 1{,}00001000 \\ -\,0{,}00010111 \\ \hline 0{,}11110001_2 \end{array}$$

$R = 0{,}11110001\times2^{5} = 1{,}1110001\times2^{4}$

$E = 10000100 - 1 = 10000011_2$

$A+B = 0\,10000011\,11100010000000000000000_2 \Leftrightarrow$

$\Leftrightarrow A+B = 41F10000_H$

ii) $B - A = \overset{\ominus}{B} + (\overset{\ominus}{-A}) \to$ sinais iguais $\to$ soma res. final é negativo.

$E = 127 + 5 = 132 = 10000100_2 \to$ (expoente real do maior nº)

mantissa de A: 1,00001 mantissa de B: 0,00010111

$$\begin{array}{r} 1,00001000 \\ +\ 0,00010111 \\ \hline 1,00011111_2 \end{array}$$

$R = 1,00011111_2 \times 2^5$

$B - A = 1\,1000010\,0\,0001111\,1000000000000000_2$

$B - A = C20F8000_H$

iii) $\overset{\oplus}{3} \times \overset{\ominus}{B} \to$ sinais diferentes $\to$ resultado negativo $B = 1,0111_2 \times 2^1$

$3_{10} = 11_2 = 1,1 \times 2^1$ $E_3 = 1 + 127 = 128 = 10000000 = E_B$

$E_{3\times B} = 127 + (E_3^{real} + E_B^{real}) = 127 + 1 + 1 = 129 = 1000\,0001_2$

$R = 10,00101 = 1,000101 \times 2^1$

$$\begin{array}{r} 1,0111 \\ \times\ 1,1 \\ \hline 10111 \\ +\ 10111\ \ \\ \hline 10,00101 \end{array}$$

$E_{3\times B} = 10000001 + 1 = 10000010_2$

$3 \times B = 1\,1000001\,0\,0001010\,0000000000000000_2$

$3 \times B = C10A0000_H$

c) $A + B = 1,1110001 \times 2^4 = 11110,001 = 30,125_{10}$ ✓

$A + B = 33 - 2,875 = 30,125$ ✓

$B - A = 1,00011111 \times 2^5 = -100011,111 = -35,875$ ✓

$B - A = -2,875 - 33 = -35,875$ ✓

$3 \times B = -1,000101 \times 2^3 = -1000,101 = -8,625$ ✓

$3 \times B = 3 \times (-2,875) = -8,625$ ✓

$\underbrace{1100}_{C} \to \underbrace{0100}_{4}$

12) a) $-V_2 = 40000000_H$

b) $V_1 = \underbrace{0100}_{4}\,\underbrace{0010}_{2}\,\underbrace{0001}_{1}\,\underbrace{1101}_{D}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}$

$V_2 = \underbrace{1100}_{C}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}$

$V_1 + V_2 \to$ sinais diferentes $\to$ subtração $|V_1| > |V_2| \to$ resultado tem sinal de V_1 (positivo).

$V_1 = 1{,}0011101 \times 2^5$ $\quad$ $10000100 = 132$ $\quad E^{real} = 132 - 127 = 5$

$V_2 = -1{,}0 \times 2^1$ $\quad$ $10000000 = 128$ $\quad E^{real} = 128 - 127 = 1$

$E = 127 + 5 = 132 = 10000100_2$ ← expoente do maior número.

mantissa de $V_1 = 1{,}0011101$ $\quad$ mantissa de $V_2 = -0{,}00010$

$R = 1{,}0010101$

$$\begin{array}{r} 1{,}0011101 \\ -\,0{,}0001000 \\ \hline 1{,}0010101_2 \end{array}$$

$V_1 + V_2 = \underbrace{0}\,\underbrace{10000100}\,00101010000000000000000_2$

$V_1 + V_2 = 42150000_H$

e) $V_2 - V_1 = \overset{\ominus}{V_2} + (-\overset{\ominus}{V_1})$ → sinais iguais → soma $\quad$ Resultado negativo.

$E = 127 + 5 = 132 = 10000100_2$ ← expoente do maior número

mantissa de $V_1 = 1{,}0011101$ $\quad$ mantissa de $V_2 = 0{,}00010$

$R = 1{,}0100101$

$$\begin{array}{r} 1{,}0011101 \\ +\,0{,}0001000 \\ \hline 1{,}0100101 \end{array}$$

$V_2 - V_1 = \underbrace{1}\,\underbrace{10000100}\,01001010000000000000000_2$

$V_2 - V_1 = C2250000_H$

d) $\overset{\oplus}{V_1} \times \overset{\ominus}{V_2}$ → sinais diferentes → resultado negativo.

$E = 127 + (E^{real}_{V_1} + E^{real}_{V_2}) = 127 + 5 + 1 = 133 = 10000101_2$

$R = 10{,}0111010 = 1{,}00111010 \times 2^1$

$$\begin{array}{r} 1{,}0011101 \\ \times\,1{,}0 \\ \hline 00000000 \\ +\,10011101. \\ \hline 10{,}0111010 \end{array}$$

$E = 10000101 + 1 = 10000110_2$

$V_1 \times V_2 = \underbrace{1}\,10000110\,00111010000000000000000$

$V_1 \times V_2 = C31D0000_H$

13 a) $Y = 11{,}625_{10} = 1011{,}101_2 = 1{,}011101 \times 2^3$ $\quad E^{real} = 3$

$E = 3 + 127 = 130 = 10000010_2$ $\quad Y = \underbrace{0}\,10000010\,01110100000000000\,0000$

b) $\overset{\oplus}{X} + \overset{\oplus}{Y}$ → sinais iguais → soma $\quad$ Resultado positivo

$E_x = 01111110_2 = 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 = 126$ $\quad E^{real}_x = 126 - 127 = -1$

$E_{x+y} = 10000010$ ← expoente do maior número mantissa de X: $1,01\times2^{-1} = 0,000101\times2^{3}$ mantissa de Y: $1,011101\times2^{3}$

$$\begin{array}{r} 1,011101 \\ +\,0,000101 \\ \hline 1,100010 \end{array}$$

$R = 1,100010\times2^{5}$

$X+Y = 0\,1000\,0010\,1000\,1000\,0000\,0000\,0000\,0000_2$

$X+Y = 41440000_H$

(14) a) Sinal: 1 $10,25_{10} = 1010,01_2 = 1,01001\times2^{3}$ $E_B^{real} = 3$

$E = 3+127 = 130 = 10000010_2$

$B = 1\,10000010\,01001000000000000000000_2$

b) $A - B = \overset{\oplus}{A} + (-\overset{\oplus}{B})$ → sinais iguais → soma → resultado final positivo.

$A = 0\,10000000\,10000000000000000000000_2$

$A = 1,1\times2^{1} = 0,011\times2^{3}$ $E_A^{real} = 128-127 = 1$ $B = 1,01001\times2^{3}$

$$\begin{array}{r} 1,01001 \\ +\,0,01100 \\ \hline 1,10101 \end{array}$$

$R = 1,10101\times2^{3}$ $E = 10000010$

$A-B = 0\,1000\,0010\,1010\,1000\,0000\,0000\,0000\,0000_2$

$A-B = +1,10101\times2^{3} = +1101,01_2 = 2^3+2^2+2^0+2^{-2} = 13,25_{10}$

(15) a) $S = 1\,1000\,0010\,1000\,0000\,0000\,0000\,0000\,0000_2$

$S = C1400000_H$

b) $T = 1\,10000011\,01100000000000000000000_2$

$S = 1,1\times2^{3} = 0,11\times2^{4}$ $E_S^{real} = 130-127 = 3$ $E_T^{real} = 131-127 = 4$ $T = 1,011\times2^{4}$

$\overset{\ominus}{S} + \overset{\ominus}{T}$ → sinais iguais → soma → resultado negativo.

$$\begin{array}{r} 1,011 \\ +\,0,110 \\ \hline 10,001 \end{array}$$

$R = 10,001\times2^{4} = 1,0001\times2^{5}$

$E = 127+5 = 132 = 10000100_2$

$S+T = 1\,1000\,0100\,0001\,0000\,0000\,0000\,0000\,0000_2$

$S+T = C2080000_H$

(16) Opção A).