

JAMES STEWART
CALCULUS

EIGHTH EDITION

Early Transcendentals



CALCULUS

EARLY TRANSCENDENTALS

EIGHTH EDITION

JAMES STEWART

McMASTER UNIVERSITY
AND
UNIVERSITY OF TORONTO



Australia • Brazil • Mexico • Singapore • United Kingdom • United States

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Calculus: Early Transcendentals, Eighth Edition

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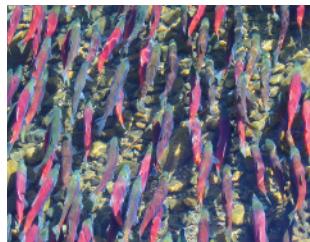
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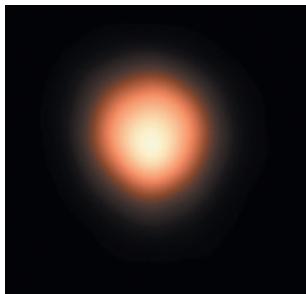


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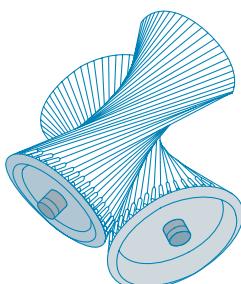
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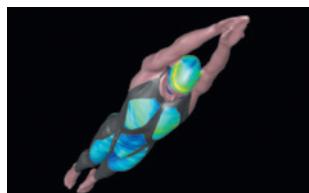
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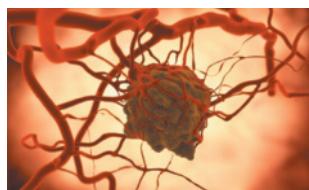
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Preface

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.

GEORGE POLYA

The art of teaching, Mark Van Doren said, is the art of assisting discovery. I have tried to write a book that assists students in discovering calculus—both for its practical power and its surprising beauty. In this edition, as in the first seven editions, I aim to convey to the student a sense of the utility of calculus and develop technical competence, but I also strive to give some appreciation for the intrinsic beauty of the subject. Newton undoubtedly experienced a sense of triumph when he made his great discoveries. I want students to share some of that excitement.

The emphasis is on understanding concepts. I think that nearly everybody agrees that this should be the primary goal of calculus instruction. In fact, the impetus for the current calculus reform movement came from the Tulane Conference in 1986, which formulated as their first recommendation:

Focus on conceptual understanding.

I have tried to implement this goal through the *Rule of Three*: “Topics should be presented geometrically, numerically, and algebraically.” Visualization, numerical and graphical experimentation, and other approaches have changed how we teach conceptual reasoning in fundamental ways. More recently, the Rule of Three has been expanded to become the *Rule of Four* by emphasizing the verbal, or descriptive, point of view as well.

In writing the eighth edition my premise has been that it is possible to achieve conceptual understanding and still retain the best traditions of traditional calculus. The book contains elements of reform, but within the context of a traditional curriculum.

Alternate Versions

I have written several other calculus textbooks that might be preferable for some instructors. Most of them also come in single variable and multivariable versions.

- *Calculus*, Eighth Edition, is similar to the present textbook except that the exponential, logarithmic, and inverse trigonometric functions are covered in the second semester.
- *Essential Calculus*, Second Edition, is a much briefer book (840 pages), though it contains almost all of the topics in *Calculus*, Eighth Edition. The relative brevity is achieved through briefer exposition of some topics and putting some features on the website.
- *Essential Calculus: Early Transcendentals*, Second Edition, resembles *Essential Calculus*, but the exponential, logarithmic, and inverse trigonometric functions are covered in Chapter 3.

- *Calculus: Concepts and Contexts*, Fourth Edition, emphasizes conceptual understanding even more strongly than this book. The coverage of topics is not encyclopedic and the material on transcendental functions and on parametric equations is woven throughout the book instead of being treated in separate chapters.
- *Calculus: Early Vectors* introduces vectors and vector functions in the first semester and integrates them throughout the book. It is suitable for students taking engineering and physics courses concurrently with calculus.
- *Brief Applied Calculus* is intended for students in business, the social sciences, and the life sciences.
- *Biocalculus: Calculus for the Life Sciences* is intended to show students in the life sciences how calculus relates to biology.
- *Biocalculus: Calculus, Probability, and Statistics for the Life Sciences* contains all the content of *Biocalculus: Calculus for the Life Sciences* as well as three additional chapters covering probability and statistics.

What's New in the Eighth Edition?

The changes have resulted from talking with my colleagues and students at the University of Toronto and from reading journals, as well as suggestions from users and reviewers. Here are some of the many improvements that I've incorporated into this edition:

- The data in examples and exercises have been updated to be more timely.
- New examples have been added (see Examples 6.1.5, 11.2.5, and 14.3.3, for instance). And the solutions to some of the existing examples have been amplified.
- Three new projects have been added: The project *Controlling Red Blood Cell Loss During Surgery* (page 244) describes the ANH procedure, in which blood is extracted from the patient before an operation and is replaced by saline solution. This dilutes the patient's blood so that fewer red blood cells are lost during bleeding and the extracted blood is returned to the patient after surgery. The project *Planes and Birds: Minimizing Energy* (page 344) asks how birds can minimize power and energy by flapping their wings versus gliding. In the project *The Speedo LZR Racer* (page 936) it is explained that this suit reduces drag in the water and, as a result, many swimming records were broken. Students are asked why a small decrease in drag can have a big effect on performance.
- I have streamlined Chapter 15 (Multiple Integrals) by combining the first two sections so that iterated integrals are treated earlier.
- More than 20% of the exercises in each chapter are new. Here are some of my favorites: 2.7.61, 2.8.36–38, 3.1.79–80, 3.11.54, 4.1.69, 4.3.34, 4.3.66, 4.4.80, 4.7.39, 4.7.67, 5.1.19–20, 5.2.67–68, 5.4.70, 6.1.51, 8.1.39, 12.5.81, 12.6.29–30, 14.6.65–66. In addition, there are some good new Problems Plus. (See Problems 12–14 on page 272, Problem 13 on page 363, Problems 16–17 on page 426, and Problem 8 on page 986.)

Features

■ Conceptual Exercises

The most important way to foster conceptual understanding is through the problems that we assign. To that end I have devised various types of problems. Some exercise sets begin with requests to explain the meanings of the basic concepts of the section. (See, for instance, the first few exercises in Sections 2.2, 2.5, 11.2, 14.2, and 14.3.) Similarly, all the review sections begin with a Concept Check and a True-False Quiz. Other exercises test conceptual understanding through graphs or tables (see Exercises 2.7.17, 2.8.35–38, 2.8.47–52, 9.1.11–13, 10.1.24–27, 11.10.2, 13.2.1–2, 13.3.33–39, 14.1.1–2, 14.1.32–38, 14.1.41–44, 14.3.3–10, 14.6.1–2, 14.7.3–4, 15.1.6–8, 16.1.11–18, 16.2.17–18, and 16.3.1–2).

Another type of exercise uses verbal description to test conceptual understanding (see Exercises 2.5.10, 2.8.66, 4.3.69–70, and 7.8.67). I particularly value problems that combine and compare graphical, numerical, and algebraic approaches (see Exercises 2.6.45–46, 3.7.27, and 9.4.4).

■ Graded Exercise Sets

Each exercise set is carefully graded, progressing from basic conceptual exercises and skill-development problems to more challenging problems involving applications and proofs.

■ Real-World Data

My assistants and I spent a great deal of time looking in libraries, contacting companies and government agencies, and searching the Internet for interesting real-world data to introduce, motivate, and illustrate the concepts of calculus. As a result, many of the examples and exercises deal with functions defined by such numerical data or graphs. See, for instance, Figure 1 in Section 1.1 (seismograms from the Northridge earthquake), Exercise 2.8.35 (unemployment rates), Exercise 5.1.16 (velocity of the space shuttle *Endeavour*), and Figure 4 in Section 5.4 (San Francisco power consumption). Functions of two variables are illustrated by a table of values of the wind-chill index as a function of air temperature and wind speed (Example 14.1.2). Partial derivatives are introduced in Section 14.3 by examining a column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity. This example is pursued further in connection with linear approximations (Example 14.4.3). Directional derivatives are introduced in Section 14.6 by using a temperature contour map to estimate the rate of change of temperature at Reno in the direction of Las Vegas. Double integrals are used to estimate the average snowfall in Colorado on December 20–21, 2006 (Example 15.1.9). Vector fields are introduced in Section 16.1 by depictions of actual velocity vector fields showing San Francisco Bay wind patterns.

■ Projects

One way of involving students and making them active learners is to have them work (perhaps in groups) on extended projects that give a feeling of substantial accomplishment when completed. I have included four kinds of projects: *Applied Projects* involve applications that are designed to appeal to the imagination of students. The project after Section 9.3 asks whether a ball thrown upward takes longer to reach its maximum height or to fall back to its original height. (The answer might surprise you.) The project after Section 14.8 uses Lagrange multipliers to determine the masses of the three stages of a rocket so as to minimize the total mass while enabling the rocket to reach a desired

velocity. *Laboratory Projects* involve technology; the one following Section 10.2 shows how to use Bézier curves to design shapes that represent letters for a laser printer. *Writing Projects* ask students to compare present-day methods with those of the founders of calculus—Fermat’s method for finding tangents, for instance. Suggested references are supplied. *Discovery Projects* anticipate results to be discussed later or encourage discovery through pattern recognition (see the one following Section 7.6). Others explore aspects of geometry: tetrahedra (after Section 12.4), hyperspheres (after Section 15.6), and intersections of three cylinders (after Section 15.7). Additional projects can be found in the *Instructor’s Guide* (see, for instance, Group Exercise 5.1: Position from Samples).

■ Problem Solving

Students usually have difficulties with problems for which there is no single well-defined procedure for obtaining the answer. I think nobody has improved very much on George Polya’s four-stage problem-solving strategy and, accordingly, I have included a version of his problem-solving principles following Chapter 1. They are applied, both explicitly and implicitly, throughout the book. After the other chapters I have placed sections called *Problems Plus*, which feature examples of how to tackle challenging calculus problems. In selecting the varied problems for these sections I kept in mind the following advice from David Hilbert: “A mathematical problem should be difficult in order to entice us, yet not inaccessible lest it mock our efforts.” When I put these challenging problems on assignments and tests I grade them in a different way. Here I reward a student significantly for ideas toward a solution and for recognizing which problem-solving principles are relevant.

■ Technology

The availability of technology makes it not less important but more important to clearly understand the concepts that underlie the images on the screen. But, when properly used, graphing calculators and computers are powerful tools for discovering and understanding those concepts. This textbook can be used either with or without technology and I use two special symbols to indicate clearly when a particular type of machine is required. The icon indicates an exercise that definitely requires the use of such technology, but that is not to say that it can’t be used on the other exercises as well. The symbol is reserved for problems in which the full resources of a computer algebra system (like Maple, Mathematica, or the TI-89) are required. But technology doesn’t make pencil and paper obsolete. Hand calculation and sketches are often preferable to technology for illustrating and reinforcing some concepts. Both instructors and students need to develop the ability to decide where the hand or the machine is appropriate.

■ Tools for Enriching Calculus

TEC is a companion to the text and is intended to enrich and complement its contents. (It is now accessible in the eBook via CourseMate and Enhanced WebAssign. Selected Visuals and Modules are available at www.stewartcalculus.com.) Developed by Harvey Keynes, Dan Clegg, Hubert Hohn, and myself, TEC uses a discovery and exploratory approach. In sections of the book where technology is particularly appropriate, marginal icons direct students to TEC Modules that provide a laboratory environment in which they can explore the topic in different ways and at different levels. **Visuals are animations of figures in text; Modules are more elaborate activities and include exercises.** Instructors can choose to become involved at several different levels, ranging from simply encouraging students to use the Visuals and Modules for independent exploration, to assigning specific exercises from those included with each Module, or to creating additional exercises, labs, and projects that make use of the Visuals and Modules.

TEC also includes Homework Hints for representative exercises (usually odd-numbered) in every section of the text, indicated by printing the exercise number in red. These hints are usually presented in the form of questions and try to imitate an effective teaching assistant by functioning as a silent tutor. They are constructed so as not to reveal any more of the actual solution than is minimally necessary to make further progress.

■ Enhanced WebAssign

Technology is having an impact on the way homework is assigned to students, particularly in large classes. The use of online homework is growing and its appeal depends on ease of use, grading precision, and reliability. With the Eighth Edition we have been working with the calculus community and WebAssign to develop an online homework system. Up to 70% of the exercises in each section are assignable as online homework, including free response, multiple choice, and multi-part formats.

The system also includes Active Examples, in which students are guided in step-by-step tutorials through text examples, with links to the textbook and to video solutions.

■ Website

Visit CengageBrain.com or stewartcalculus.com for these additional materials:

- Homework Hints
- Algebra Review
- Lies My Calculator and Computer Told Me
- History of Mathematics, with links to the better historical websites
- Additional Topics (complete with exercise sets): Fourier Series, Formulas for the Remainder Term in Taylor Series, Rotation of Axes
- Archived Problems (Drill exercises that appeared in previous editions, together with their solutions)
- Challenge Problems (some from the Problems Plus sections from prior editions)
- Links, for particular topics, to outside Web resources
- Selected Visuals and Modules from Tools for Enriching Calculus (TEC)

Content

Diagnostic Tests

The book begins with four diagnostic tests, in Basic Algebra, Analytic Geometry, Functions, and Trigonometry.

A Preview of Calculus

This is an overview of the subject and includes a list of questions to motivate the study of calculus.

1 Functions and Models

From the beginning, multiple representations of functions are stressed: verbal, numerical, visual, and algebraic. A discussion of mathematical models leads to a review of the standard functions, including exponential and logarithmic functions, from these four points of view.

2 Limits and Derivatives

The material on limits is motivated by a prior discussion of the tangent and velocity problems. Limits are treated from descriptive, graphical, numerical, and algebraic points of view. Section 2.4, on the precise definition of a limit, is an optional section. Sections

2.7 and 2.8 deal with derivatives (especially with functions defined graphically and numerically) before the differentiation rules are covered in Chapter 3. Here the examples and exercises explore the meanings of derivatives in various contexts. Higher derivatives are introduced in Section 2.8.

3 Differentiation Rules

All the basic functions, including exponential, logarithmic, and inverse trigonometric functions, are differentiated here. When derivatives are computed in applied situations, students are asked to explain their meanings. Exponential growth and decay are now covered in this chapter.

4 Applications of Differentiation

The basic facts concerning extreme values and shapes of curves are deduced from the Mean Value Theorem. Graphing with technology emphasizes the interaction between calculus and calculators and the analysis of families of curves. Some substantial optimization problems are provided, including an explanation of why you need to raise your head 42° to see the top of a rainbow.

5 Integrals

The area problem and the distance problem serve to motivate the definite integral, with sigma notation introduced as needed. (Full coverage of sigma notation is provided in Appendix E.) Emphasis is placed on explaining the meanings of integrals in various contexts and on estimating their values from graphs and tables.

6 Applications of Integration

Here I present the applications of integration—area, volume, work, average value—that can reasonably be done without specialized techniques of integration. General methods are emphasized. The goal is for students to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral.

7 Techniques of Integration

All the standard methods are covered but, of course, the real challenge is to be able to recognize which technique is best used in a given situation. Accordingly, in Section 7.5, I present a strategy for integration. The use of computer algebra systems is discussed in Section 7.6.

8 Further Applications of Integration

Here are the applications of integration—arc length and surface area—for which it is useful to have available all the techniques of integration, as well as applications to biology, economics, and physics (hydrostatic force and centers of mass). I have also included a section on probability. There are more applications here than can realistically be covered in a given course. Instructors should select applications suitable for their students and for which they themselves have enthusiasm.

9 Differential Equations

Modeling is the theme that unifies this introductory treatment of differential equations. Direction fields and Euler's method are studied before separable and linear equations are solved explicitly, so that qualitative, numerical, and analytic approaches are given equal consideration. These methods are applied to the exponential, logistic, and other models for population growth. The first four or five sections of this chapter serve as a good introduction to first-order differential equations. An optional final section uses predator-prey models to illustrate systems of differential equations.

10 Parametric Equations and Polar Coordinates

This chapter introduces parametric and polar curves and applies the methods of calculus to them. Parametric curves are well suited to laboratory projects; the two presented here involve families of curves and Bézier curves. A brief treatment of conic sections in polar coordinates prepares the way for Kepler's Laws in Chapter 13.

11 Infinite Sequences and Series

The convergence tests have intuitive justifications (see page 719) as well as formal proofs. Numerical estimates of sums of series are based on which test was used to prove convergence. The emphasis is on Taylor series and polynomials and their applications to physics. Error estimates include those from graphing devices.

12 Vectors and the Geometry of Space

The material on three-dimensional analytic geometry and vectors is divided into two chapters. Chapter 12 deals with vectors, the dot and cross products, lines, planes, and surfaces.

13 Vector Functions

This chapter covers vector-valued functions, their derivatives and integrals, the length and curvature of space curves, and velocity and acceleration along space curves, culminating in Kepler's laws.

14 Partial Derivatives

Functions of two or more variables are studied from verbal, numerical, visual, and algebraic points of view. In particular, I introduce partial derivatives by looking at a specific column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity.

15 Multiple Integrals

Contour maps and the Midpoint Rule are used to estimate the average snowfall and average temperature in given regions. Double and triple integrals are used to compute probabilities, surface areas, and (in projects) volumes of hyperspheres and volumes of intersections of three cylinders. Cylindrical and spherical coordinates are introduced in the context of evaluating triple integrals.

16 Vector Calculus

Vector fields are introduced through pictures of velocity fields showing San Francisco Bay wind patterns. The similarities among the Fundamental Theorem for line integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem are emphasized.

17 Second-Order Differential Equations

Since first-order differential equations are covered in Chapter 9, this final chapter deals with second-order linear differential equations, their application to vibrating springs and electric circuits, and series solutions.

Ancillaries

Calculus, Early Transcendentals, Eighth Edition, is supported by a complete set of ancillaries developed under my direction. Each piece has been designed to enhance student understanding and to facilitate creative instruction. The tables on pages xxi–xxii describe each of these ancillaries.

Acknowledgments

The preparation of this and previous editions has involved much time spent reading the reasoned (but sometimes contradictory) advice from a large number of astute reviewers. I greatly appreciate the time they spent to understand my motivation for the approach taken. I have learned something from each of them.

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To the Student

Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper and calculator at hand to sketch a diagram or make a calculation.

Some students start by trying their homework problems and read the text only if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should look at the definitions to see the exact meanings of the terms. And before you read each example, I suggest that you cover up the solution and try solving the problem yourself. You'll get a lot more from looking at the solution if you do so.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected, step-by-step fashion with explanatory sentences—not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix I. Some exercises ask for a verbal explanation or interpretation or description. In such cases there is no single correct way of expressing the answer, so don't worry that you haven't found the definitive answer. In addition, there are often several different forms in which to express a numerical or algebraic answer, so if your answer differs from mine, don't immediately assume you're wrong. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you're right and rationalizing the denominator will show that the answers are equivalent.

The icon indicates an exercise that definitely requires the use of either a graphing calculator or a computer with graphing software. But that doesn't mean that graphing devices can't be used to check your work on the other exercises as well. The symbol is reserved for problems in

which the full resources of a computer algebra system (like Maple, Mathematica, or the TI-89) are required.

You will also encounter the symbol , which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

Tools for Enriching Calculus, which is a companion to this text, is referred to by means of the symbol and can be accessed in the eBook via Enhanced WebAssign and CourseMate (selected Visuals and Modules are available at www.stewartcalculus.com). It directs you to modules in which you can explore aspects of calculus for which the computer is particularly useful.

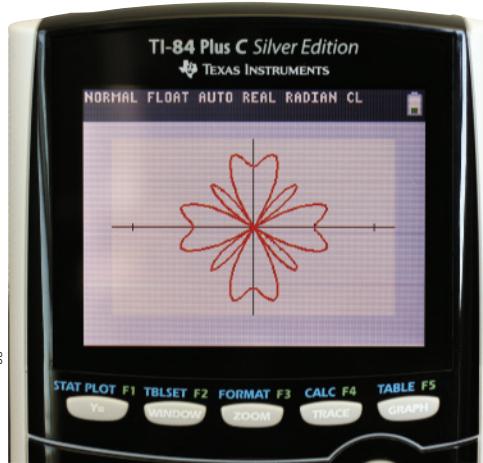
You will notice that some exercise numbers are printed in red: **5**. This indicates that *Homework Hints* are available for the exercise. These hints can be found on stewartcalculus.com as well as Enhanced WebAssign and CourseMate. The homework hints ask you questions that allow you to make progress toward a solution without actually giving you the answer. You need to pursue each hint in an active manner with pencil and paper to work out the details. If a particular hint doesn't enable you to solve the problem, you can click to reveal the next hint.

I recommend that you keep this book for reference purposes after you finish the course. Because you will likely forget some of the specific details of calculus, the book will serve as a useful reminder when you need to use calculus in subsequent courses. And, because this book contains more material than can be covered in any one course, it can also serve as a valuable resource for a working scientist or engineer.

Calculus is an exciting subject, justly considered to be one of the greatest achievements of the human intellect. I hope you will discover that it is not only useful but also intrinsically beautiful.

JAMES STEWART

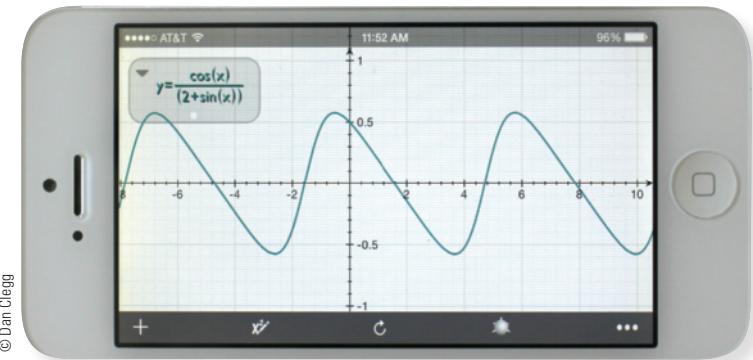
Calculators, Computers, and Other Graphing Devices



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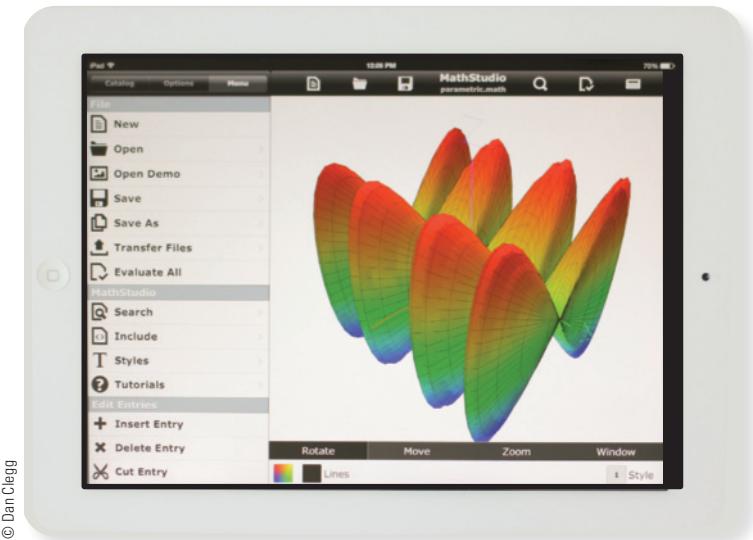
Advances in technology continue to bring a wider variety of tools for doing mathematics. Handheld calculators are becoming more powerful, as are software programs and Internet resources. In addition, many mathematical applications have been released for smartphones and tablets such as the iPad.

Some exercises in this text are marked with a graphing icon , which indicates that the use of some technology is required. Often this means that we intend for a graphing device to be used in drawing the graph of a function or equation. You might also need technology to find the zeros of a graph or the points of intersection of two graphs. In some cases we will use a calculating device to solve an equation or evaluate a definite integral numerically. Many scientific and graphing calculators have these features built in, such as the Texas Instruments TI-84 or TI-Nspire CX. Similar calculators are made by Hewlett Packard, Casio, and Sharp.

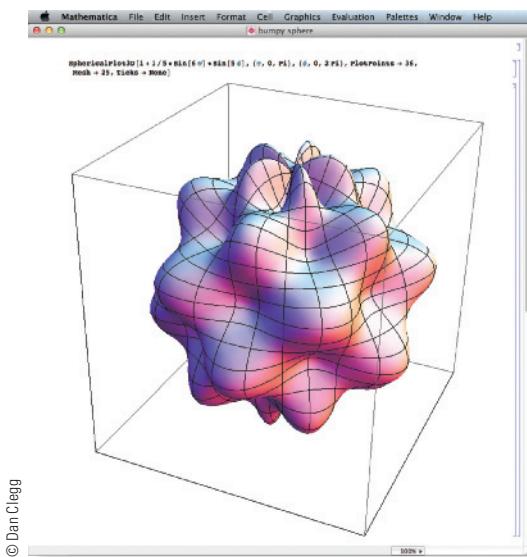


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You can also use computer software such as *Graphing Calculator* by Pacific Tech (www.pacifict.com) to perform many of these functions, as well as apps for phones and tablets, like Quick Graph (Colombiamug) or Math-Studio (Pomegranate Apps). Similar functionality is available using a web interface at WolframAlpha.com.



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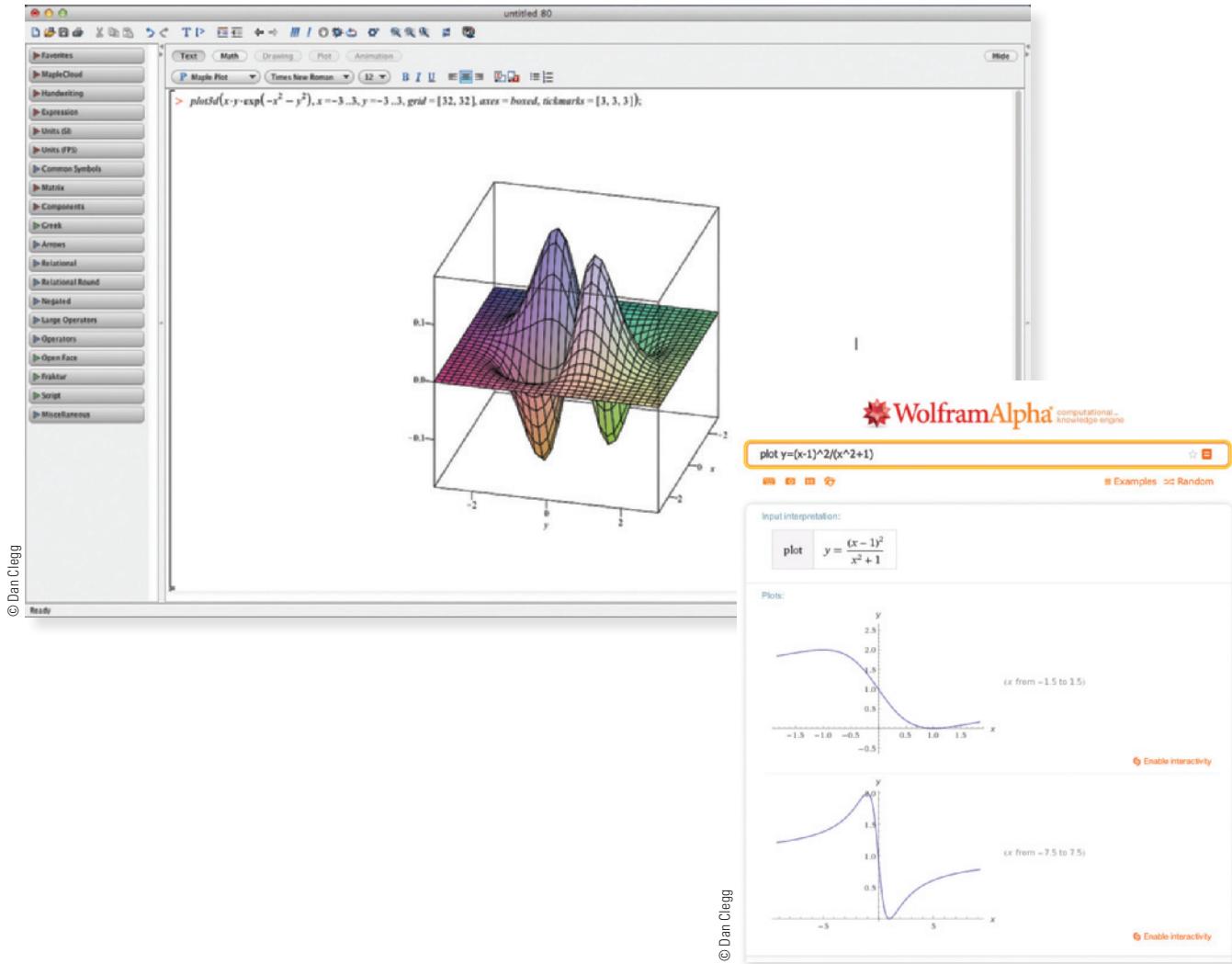


In general, when we use the term “calculator” in this book, we mean the use of any of the resources we have mentioned.

The icon is reserved for problems in which the full resources of a *computer algebra system* (CAS) are required. A CAS is capable of doing mathematics (like solving equations, computing derivatives or integrals) *symbolically* rather than just numerically.

Examples of well-established computer algebra systems are the computer software packages Maple and Mathematica. The WolframAlpha website uses the Mathematica engine to provide CAS functionality via the Web.

Many handheld graphing calculators have CAS capabilities, such as the TI-89 and TI-Nspire CX CAS from Texas Instruments. Some tablet and smartphone apps also provide these capabilities, such as the previously mentioned MathStudio.



Diagnostic Tests

Success in calculus depends to a large extent on knowledge of the mathematics that precedes calculus: algebra, analytic geometry, functions, and trigonometry. The following tests are intended to diagnose weaknesses that you might have in these areas. After taking each test you can check your answers against the given answers and, if necessary, refresh your skills by referring to the review materials that are provided.

A Diagnostic Test: Algebra

1. Evaluate each expression without using a calculator.

(a) $(-3)^4$ (b) -3^4 (c) 3^{-4}
(d) $\frac{5^{23}}{5^{21}}$ (e) $\left(\frac{2}{3}\right)^{-2}$ (f) $16^{-3/4}$

2. Simplify each expression. Write your answer without negative exponents.

(a) $\sqrt{200} - \sqrt{32}$
(b) $(3a^3b^3)(4ab^2)^2$
(c) $\left(\frac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

3. Expand and simplify.

(a) $3(x + 6) + 4(2x - 5)$ (b) $(x + 3)(4x - 5)$
(c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ (d) $(2x + 3)^2$
(e) $(x + 2)^3$

4. Factor each expression.

(a) $4x^2 - 25$ (b) $2x^2 + 5x - 12$
(c) $x^3 - 3x^2 - 4x + 12$ (d) $x^4 + 27x$
(e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ (f) $x^3y - 4xy$

5. Simplify the rational expression.

(a) $\frac{x^2 + 3x + 2}{x^2 - x - 2}$ (b) $\frac{2x^2 - x - 1}{x^2 - 9} \cdot \frac{x + 3}{2x + 1}$
(c) $\frac{x^2}{x^2 - 4} - \frac{x + 1}{x + 2}$ (d) $\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}}$

- 6.** Rationalize the expression and simplify.

(a) $\frac{\sqrt{10}}{\sqrt{5} - 2}$

(b) $\frac{\sqrt{4 + h} - 2}{h}$

- 7.** Rewrite by completing the square.

(a) $x^2 + x + 1$

(b) $2x^2 - 12x + 11$

- 8.** Solve the equation. (Find only the real solutions.)

(a) $x + 5 = 14 - \frac{1}{2}x$

(b) $\frac{2x}{x + 1} = \frac{2x - 1}{x}$

(c) $x^2 - x - 12 = 0$

(d) $2x^2 + 4x + 1 = 0$

(e) $x^4 - 3x^2 + 2 = 0$

(f) $3|x - 4| = 10$

(g) $2x(4 - x)^{-1/2} - 3\sqrt{4 - x} = 0$

- 9.** Solve each inequality. Write your answer using interval notation.

(a) $-4 < 5 - 3x \leq 17$

(b) $x^2 < 2x + 8$

(c) $x(x - 1)(x + 2) > 0$

(d) $|x - 4| < 3$

(e) $\frac{2x - 3}{x + 1} \leq 1$

- 10.** State whether each equation is true or false.

(a) $(p + q)^2 = p^2 + q^2$

(b) $\sqrt{ab} = \sqrt{a}\sqrt{b}$

(c) $\sqrt{a^2 + b^2} = a + b$

(d) $\frac{1 + TC}{C} = 1 + T$

(e) $\frac{1}{x - y} = \frac{1}{x} - \frac{1}{y}$

(f) $\frac{1/x}{a/x - b/x} = \frac{1}{a - b}$

ANSWERS TO DIAGNOSTIC TEST A: ALGEBRA

- | | | |
|--|--|--|
| 1. (a) 81
(b) -81
(c) $\frac{1}{81}$
(d) 25
(e) $\frac{9}{4}$
(f) $\frac{1}{8}$ | 2. (a) $6\sqrt{2}$
(b) $48a^5b^7$
(c) $\frac{x}{9y^7}$ | 6. (a) $5\sqrt{2} + 2\sqrt{10}$
(b) $\frac{1}{\sqrt{4 + h} + 2}$ |
| 3. (a) $11x - 2$
(b) $4x^2 + 7x - 15$
(c) $a - b$
(d) $4x^2 + 12x + 9$
(e) $x^3 + 6x^2 + 12x + 8$ | 7. (a) $\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$
(b) $2(x - 3)^2 - 7$ | 8. (a) 6
(b) 1
(c) $-3, 4$
(d) $-1 \pm \frac{1}{2}\sqrt{2}$
(e) $\pm 1, \pm\sqrt{2}$
(f) $\frac{2}{3}, \frac{22}{3}$
(g) $\frac{12}{5}$ |
| 4. (a) $(2x - 5)(2x + 5)$
(c) $(x - 3)(x - 2)(x + 2)$
(e) $3x^{-1/2}(x - 1)(x - 2)$ | (b) $(2x - 3)(x + 4)$
(d) $x(x + 3)(x^2 - 3x + 9)$
(f) $xy(x - 2)(x + 2)$ | 9. (a) $[-4, 3)$
(c) $(-2, 0) \cup (1, \infty)$
(e) $(-1, 4]$ |
| 5. (a) $\frac{x + 2}{x - 2}$
(c) $\frac{1}{x - 2}$ | (b) $\frac{x - 1}{x - 3}$
(d) $-(x + y)$ | 10. (a) False
(b) True
(c) False
(d) False
(e) False
(f) True |

If you had difficulty with these problems, you may wish to consult the Review of Algebra on the website www.stewartcalculus.com.

B Diagnostic Test: Analytic Geometry

1. Find an equation for the line that passes through the point $(2, -5)$ and
 - has slope -3
 - is parallel to the x -axis
 - is parallel to the y -axis
 - is parallel to the line $2x - 4y = 3$

2. Find an equation for the circle that has center $(-1, 4)$ and passes through the point $(3, -2)$.

3. Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

4. Let $A(-7, 4)$ and $B(5, -12)$ be points in the plane.
 - Find the slope of the line that contains A and B .
 - Find an equation of the line that passes through A and B . What are the intercepts?
 - Find the midpoint of the segment AB .
 - Find the length of the segment AB .
 - Find an equation of the perpendicular bisector of AB .
 - Find an equation of the circle for which AB is a diameter.

5. Sketch the region in the xy -plane defined by the equation or inequalities.

(a) $-1 \leq y \leq 3$ (c) $y < 1 - \frac{1}{2}x$ (e) $x^2 + y^2 < 4$	(b) $ x < 4$ and $ y < 2$ (d) $y \geq x^2 - 1$ (f) $9x^2 + 16y^2 = 144$
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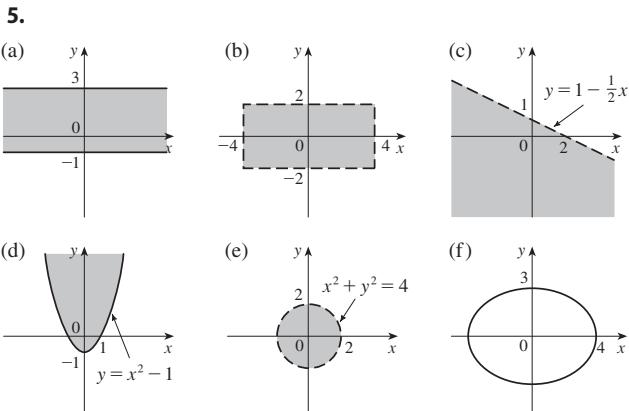
ANSWERS TO DIAGNOSTIC TEST B: ANALYTIC GEOMETRY

1. (a) $y = -3x + 1$ (b) $y = -5$
 (c) $x = 2$ (d) $y = \frac{1}{2}x - 6$

2. $(x + 1)^2 + (y - 4)^2 = 52$

3. Center $(3, -5)$, radius 5

4. (a) $-\frac{4}{3}$
 (b) $4x + 3y + 16 = 0$; x -intercept -4 , y -intercept $-\frac{16}{3}$
 (c) $(-1, -4)$
 (d) 20
 (e) $3x - 4y = 13$
 (f) $(x + 1)^2 + (y + 4)^2 = 100$



If you had difficulty with these problems, you may wish to consult the review of analytic geometry in Appendixes B and C.

C Diagnostic Test: Functions

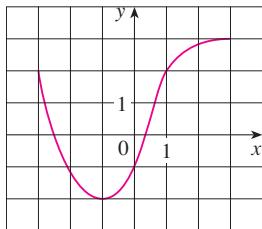


FIGURE FOR PROBLEM 1

1. The graph of a function f is given at the left.
- State the value of $f(-1)$.
 - Estimate the value of $f(2)$.
 - For what values of x is $f(x) = 2$?
 - Estimate the values of x such that $f(x) = 0$.
 - State the domain and range of f .

2. If $f(x) = x^3$, evaluate the difference quotient $\frac{f(2+h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.

$$(a) f(x) = \frac{2x+1}{x^2+x-2} \quad (b) g(x) = \frac{\sqrt[3]{x}}{x^2+1} \quad (c) h(x) = \sqrt{4-x} + \sqrt{x^2-1}$$

4. How are graphs of the functions obtained from the graph of f ?

$$(a) y = -f(x) \quad (b) y = 2f(x) - 1 \quad (c) y = f(x-3) + 2$$

5. Without using a calculator, make a rough sketch of the graph.

$$(a) y = x^3 \quad (b) y = (x+1)^3 \quad (c) y = (x-2)^3 + 3 \\ (d) y = 4 - x^2 \quad (e) y = \sqrt{x} \quad (f) y = 2\sqrt{x} \\ (g) y = -2^x \quad (h) y = 1 + x^{-1}$$

6. Let $f(x) = \begin{cases} 1-x^2 & \text{if } x \leq 0 \\ 2x+1 & \text{if } x > 0 \end{cases}$

- (a) Evaluate $f(-2)$ and $f(1)$. (b) Sketch the graph of f .

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.

$$(a) f \circ g \quad (b) g \circ f \quad (c) g \circ g \circ g$$

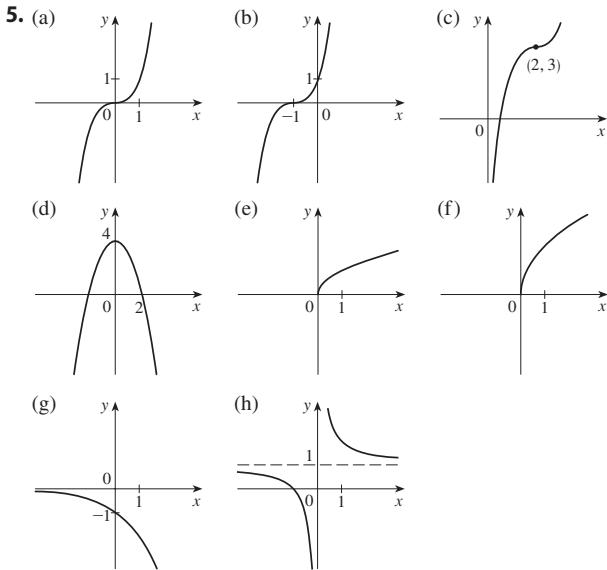
ANSWERS TO DIAGNOSTIC TEST C: FUNCTIONS

1. (a) -2 (b) 2.8
 (c) $-3, 1$ (d) $-2.5, 0.3$
 (e) $[-3, 3], [-2, 3]$

2. $12 + 6h + h^2$

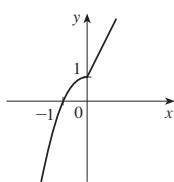
3. (a) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
 (b) $(-\infty, \infty)$
 (c) $(-\infty, -1] \cup [1, 4]$

4. (a) Reflect about the x -axis
 (b) Stretch vertically by a factor of 2, then shift 1 unit downward
 (c) Shift 3 units to the right and 2 units upward



6. (a) $-3, 3$

(b)



7. (a) $(f \circ g)(x) = 4x^2 - 8x + 2$

(b) $(g \circ f)(x) = 2x^2 + 4x - 5$

(c) $(g \circ g \circ g)(x) = 8x - 21$

If you had difficulty with these problems, you should look at sections 1.1–1.3 of this book.

D Diagnostic Test: Trigonometry

- Convert from degrees to radians.
 (a) 300° (b) -18°
- Convert from radians to degrees.
 (a) $5\pi/6$ (b) 2
- Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of 30° .
- Find the exact values.
 (a) $\tan(\pi/3)$ (b) $\sin(7\pi/6)$ (c) $\sec(5\pi/3)$
- Express the lengths a and b in the figure in terms of θ .
- If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.
- Prove the identities.
 (a) $\tan \theta \sin \theta + \cos \theta = \sec \theta$ (b) $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$
- Find all values of x such that $\sin 2x = \sin x$ and $0 \leq x \leq 2\pi$.
- Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.

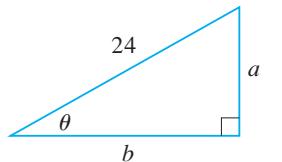


FIGURE FOR PROBLEM 5

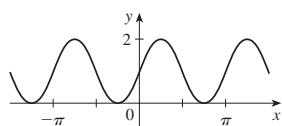
ANSWERS TO DIAGNOSTIC TEST D: TRIGONOMETRY

- (a) $5\pi/3$ (b) $-\pi/10$
- (a) 150° (b) $360^\circ/\pi \approx 114.6^\circ$
- 2π cm
- (a) $\sqrt{3}$ (b) $-\frac{1}{2}$ (c) 2
- (a) $24 \sin \theta$ (b) $24 \cos \theta$

6. $\frac{1}{15}(4 + 6\sqrt{2})$

8. $0, \pi/3, \pi, 5\pi/3, 2\pi$

9.



If you had difficulty with these problems, you should look at Appendix D of this book.