Problem 1:- Suppose that, instead of a frame, a point $P = (3,5,7)^T$ in space was translated a distance of $d = (2,3,4)^T$. Find the new location of the point relative to the reference frame.

⇒ If a frame moves in space without change in its orientation, the transformation is a pure transformation. For this problem,

$$P_{new} = Trans(dx, dy, dz) \times P_{old}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 7 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 8 \\ 11 \\ 1 \end{bmatrix}$$

$\therefore P_{new} = (5, 8, 11)$ Ans.

Problem - 2 :-

The following was moved a distance of $d = (5, 2, 6)^T$:

$$B = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the new location of the frame relative to the reference frame.

⇒ The new location of the frame relative to the reference frame after translation is:

$$B_{new} = Trans(d_x, d_y, d_z) \times B_{old}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 7 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & -1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem-3:- For the following frame, find the values of the missing elements and complete the matrix representation of the frame:

$$F = \begin{bmatrix} ? & 0 & -1 & 5 \\ ? & 0 & 0 & 3 \\ ? & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans:- Representing F in terms of n, o, a the, we have

$$F = \begin{bmatrix} n_x & 0 & -1 & 5 \\ n_y & 0 & 0 & 3 \\ n_z & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We know, the values representing a frame in a matrix must be such that ~~the~~ all equation for the frame is true.

So, $\bar{n} \times \bar{o} = \bar{a}$

$$\Rightarrow \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ n_x & n_y & n_z \\ 0 & 0 & -1 \end{bmatrix} = -\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\Rightarrow -n_y\hat{i} - \hat{j}(-n_x) + \hat{k}(0) = -\hat{i} + 0\hat{j} + 0\hat{k}$$

So, $n_y = 1$

$n_x = 0$

$n_z = 0$

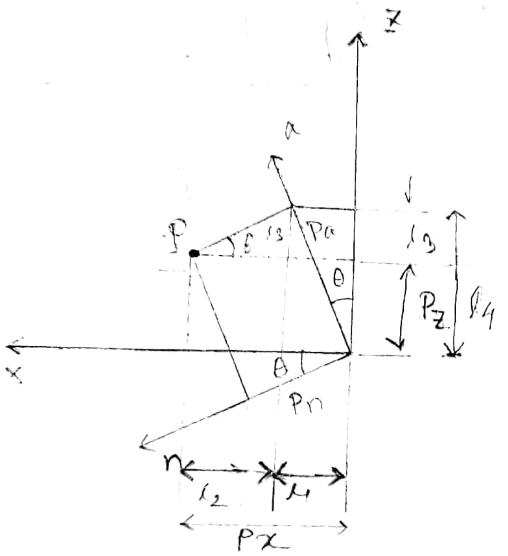
So,

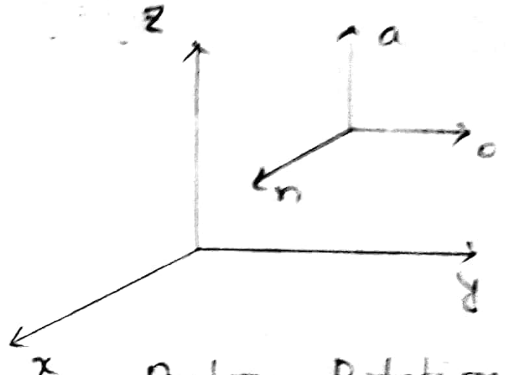
$$F = \begin{bmatrix} 0 & 0 & -1 & 5 \\ 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans.

Problem- 4:- Derive the matrix that represent a pure rotation about the y-axis of the reference frame.

⇒ Assuming a frame $(\bar{n}, \bar{o}, \bar{a})$ located at the origin of the reference frame $(\bar{x}, \bar{y}, \bar{z})$ rotate an angle θ about y-axis





Before Rotation

For the pure rotation about y-axis, if point P is rotate and the new co-ordinate of $P(P_x, P_y, P_z)$.

so, ~~for~~ ~~ro~~ after rotation,

$$P_x = l_1 + l_2$$
$$= P_a \cos(90 - \theta) + P_n \cos\theta$$
$$= P_a \sin\theta + P_n \cos\theta$$

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$P_y = P_o$

$P_z = l_4 - l_3$

$= P_a \cos\theta - P_n \sin\theta$

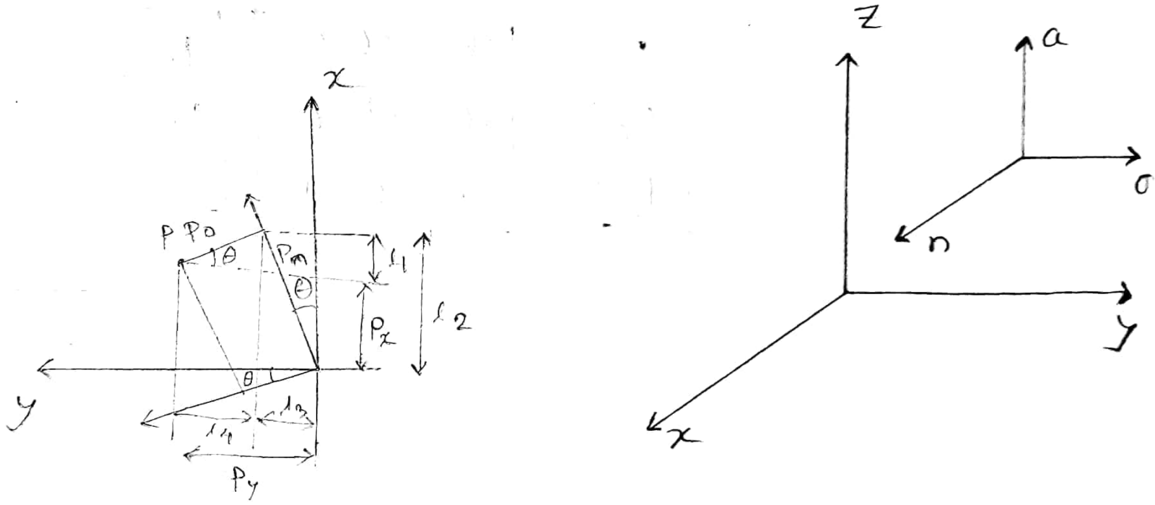
In ~~for~~ the matrix form,

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \begin{bmatrix} P_n \\ P_o \\ P_a \end{bmatrix}$$

Ans.

Problem-5 :- Derive the matrix that represents a pure rotation about the z-axis of the reference frame.

⇒ Assuming a frame $(\bar{n}, \bar{o}, \bar{a})$ located at the origin of the reference frame $(\bar{x}, \bar{y}, \bar{z})$ rotate an angle θ about z-axis.



If point P is rotate and the new co-ordinate of P is $P(P_x, P_y, P_z)$ when rotation is about z-axis.

$$P_x = l_2 - l_1 = P_n \cos\theta - P_o \sin\theta$$

$$P_y = l_3 + l_4 = P_n \cos(90-\theta) + P_o \cos\theta$$

$$P_z = P_a$$

~~In matrix form,~~

$$\cancel{\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} C\theta & -S\theta & C\theta \\ & & \\ & & \end{bmatrix} \begin{bmatrix} P_n \\ P_o \\ P_a \end{bmatrix}}$$

In Matrix form, $P_{xyz} = Rot(z, \theta) \times P_{noa}$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_n \\ P_o \\ P_a \end{bmatrix}$$

<u>Ans.</u>

Problem 6:- Verify that the rotation matrix about the reference frame axes follow the required constraint equations set by orthogonality and length requirements of directional unit vectors.

⇒ Assuming a reference frame $(\bar{n}, \bar{o}, \bar{a})$ located at the origin of the reference frame $(\bar{x}, \bar{y}, \bar{z})$. will rotate an angle θ about x-axis,

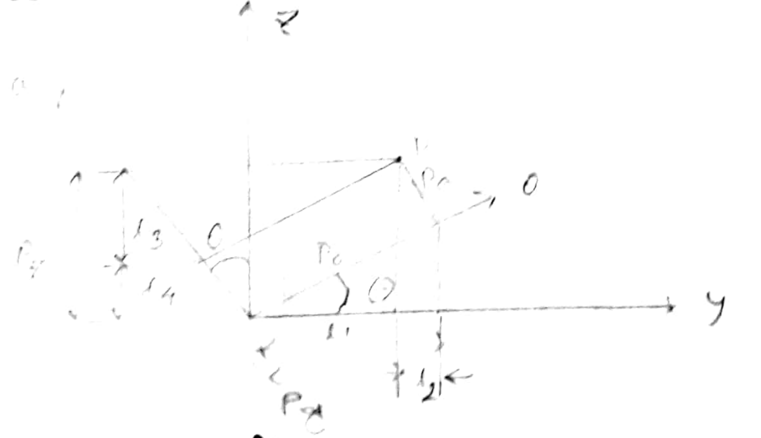


fig. a

From the fig. a after rotation about x-axis the co-ordinate of point P relative to reference frame P_x, P_y, P_z.

Then, $P_x = P_n$

$$P_y = l_1 - l_2 = P_o \cos\theta - P_a \sin\theta$$

$$P_z = l_3 + l_4 = P_o \sin\theta + P_a \cos\theta$$

The matrix form,

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} P_n \\ P_o \\ P_a \end{bmatrix}$$

$$P_{xyz} = Rot(x, \theta) \times P_{noa}$$

$$\therefore \quad Rot(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}$$

So, we have,

$$\bar{n} \cdot \bar{a} = 0$$

$$\bar{n} \cdot \bar{o} = 0$$

$$\bar{a} \cdot \bar{o} = 0$$

$$|n| = 1$$

$$|o| = \sqrt{c^2 + s^2} = 1$$

$$|a| = \sqrt{(-s)^2 + c^2} = 1$$

From Prob-(5) and Prob-(6). We have

$$Rot(y, \theta) = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

$\bar{n}\cdot\bar{o} = 0$

$\bar{n}\cdot\bar{a} = 0$

$\bar{o}\cdot\bar{a} = 0$

$|n| = \sqrt{c^2 + (-s)^2} = 1$

$|a| = \sqrt{s^2 + c^2} = 1$

$|o| = 1$

And
$$Rot(z,\theta) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\bar{n}\cdot\bar{o} = 0$

$\bar{n}\cdot\bar{a} = 0$

$\bar{o}\cdot\bar{a} = 0$

$|n| = \sqrt{c^2 + s^2} = 1$

$|o| = \sqrt{(-s)^2 + c^2} = 1$

$|a| = 1$

Problem - 7 :- Find the co-ordinate of Point $P(2,3,4)^T$ relative to the referencet frame after a rotation of 45 about X-axis.

⇒ After rotation the co-ordinate of point P if (P_x, P_y, P_z), the from ~~equati~~ matrix of pure rotation we have,

$$P_{xyz} = Rot(x, 45) \times P_{noa}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix} \times P_{noa} \quad \text{where } \theta = 45°$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -\frac{1}{\sqrt{2}} \\ \frac{7}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 2 \\ -0.707 \\ 4.95 \end{bmatrix}$$

Ans.

Problem- 5:- Find the co-ordinate of point $P = (3,5,7)^T$ relative to the reference frame after a rotation of 30° about the z-axis.

$\Rightarrow$ After rotation, if the co-ordinate of of Point P is (P_x, P_y, P_z), the from the matrix representation, we have,

$$P_{xyz} = Rot(z,\theta) \cdot \times P_{noa}$$

$$\Rightarrow \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_n \\ P_o \\ P_a \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0.098 \\ 5.83 \\ 7 \end{bmatrix}$$ Ans.

Problem-9:- A point P in space is defined as ${}^{B}P = [5, 3, 4]^T$ relative to frame B and is attached to the origin of the reference frame A, is parallel to it. Apply the following transformation to frame B and find ${}^{A}P$. Using three dimensional grid provided in this chapter. Plot the transformation and the result and verify it. Also verify graphically that you would not get the same result if you apply the transformation relative to the current frame.

1. Rotate 90° about x-axis
2. Then translate 3 units about y-axis, 6-units about the z-axis and 5 units about the x-axis.
3. Then rotate about z-axis.

Solve:-

Applying all the transformation to point P relative to original frame (reference frame) A, if co-ordinate of point P is ${}^{A}P(x,y,z)$ Then,

$${}^{A}P = Rot(z,90)\ Trans(5,3,6) \times Rot(x,90) \times {}^{B}P_{noa}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & -3 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 10 \\ 9 \\ 1 \end{bmatrix}$$ Ans.

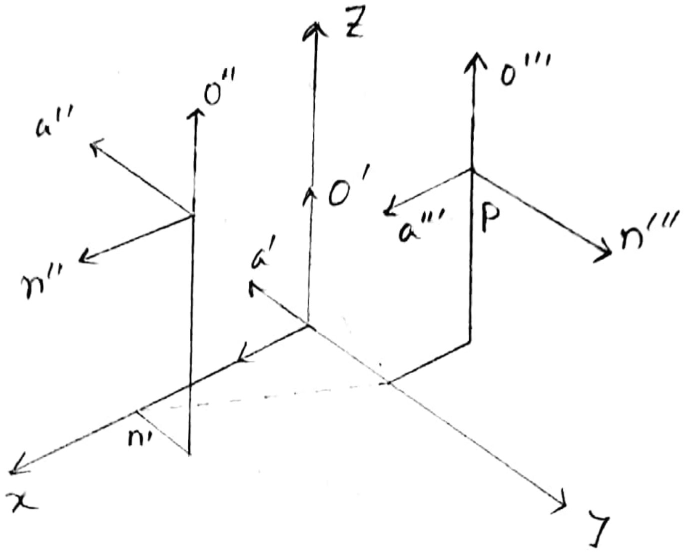


fig-a

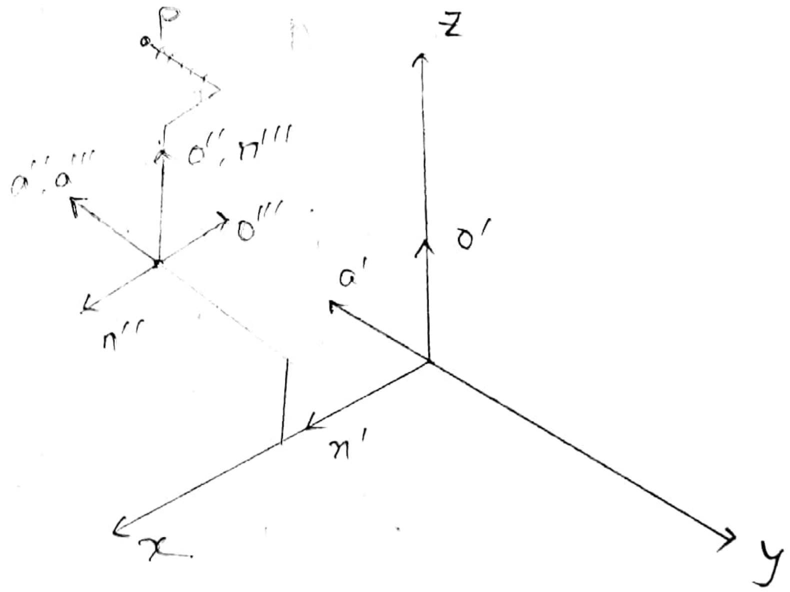


fig-b

Problem-10: A point P in space is defined as ${}^{B}p = (2,3,5)^T$ relative to frame B which is attached to the origin of the reference frame A and is parallel to it. Apply the following transformation to Frame B and find ${}^{A}p$. Using the three-dimentional grid, plot the transformation and the result and verify it.

1. Rotate 90° about the axis.
2. Then Rotate 90° about local a axis
3. Then translate 3 unit about y-axis, 6 units about z-axis, and 5 unit about x-axis.

⇒ The new point ${}^{A}p$ = Trans(5,3,6)· Rot(~~x~~, 90°) Rot(a, 90°) × ${}^{B}p$

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$$= \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -2 \\ 8 \\ 1 \end{bmatrix}$$

Ans.

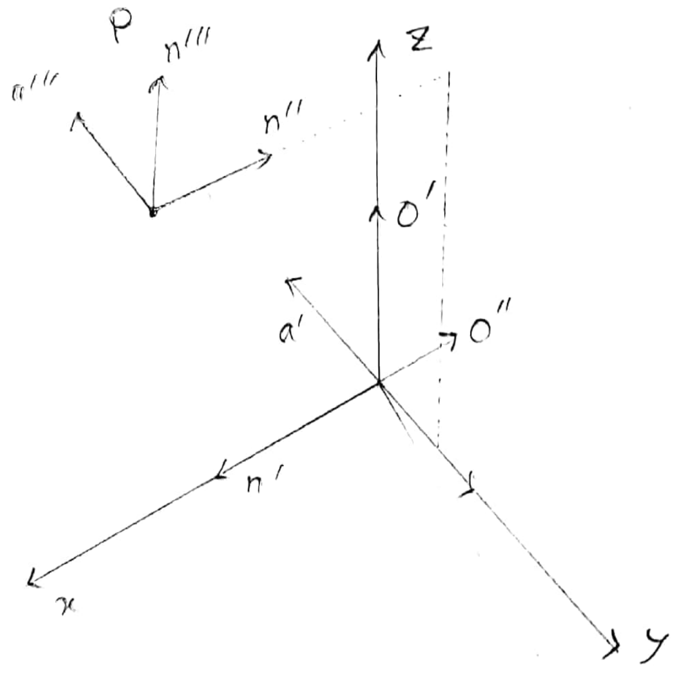

fig. 10

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Problem 11: show that rotation matrix about the x-axis and y-axis are unitary.

$\Rightarrow$ Rotation matrix about y-axis,

$$A = Rot(y,\theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}$$

$$\det(A) = 1\{(C\theta)(C\theta) + (S\theta)(S\theta)\}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

$$A^T = \begin{bmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix}$$

$$\therefore adj\, A = \begin{bmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix}$$

$$\therefore A^{-1} = \frac{adj(A)}{det(A)} = \begin{bmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix} / 1$$

$$= A^T$$

As $A^{-1} = A^T$, so this matrix is unitary.

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rotation matrix about z-axis.

$$B = Rot(z,\theta) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$det(B) = 1\{(c\theta)(c\theta) - (s\theta)(-s\theta)\}$$

$$= \cos^2\theta + \sin^2\theta \Rightarrow 1$$

$$B^T = \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$adj(B) = \begin{bmatrix} c\theta & +s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{adj(B)}{det(B)} = \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = B^T$$

$\therefore$ Rotation matrix about z-axis is unitary.

Problem 12:-

Calculate the inverse of the following transformation matrix.

$$T_1 = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 2 \\ 0.369 & 0.819 & 0.439 & 5 \\ -0.766 & 0 & 0.643 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 0.92 & 0 & 0.39 & 5 \\ 0 & 1 & 0 & 6 \\ -0.39 & 0 & 0.92 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad T_1^T = \begin{bmatrix} 0.527 & 0.369 & -0.766 & 0 \\ -0.574 & 0.819 & 0 & 0 \\ 0.628 & 0.439 & 0.643 & 0 \\ 2 & 5 & 3 & 1 \end{bmatrix}$$

$$\det(T_1) = 1.000718$$

Now, $\operatorname{adj}(T) = T_{1\,minor}^T$

$$= \begin{bmatrix} 0.5266 & 0.36908 & -0.7663 & -0.5996 \\ -0.5735 & 0.8199 & 0 & -2.953 \\ 0.62735 & 0.43968 & 0.6434 & -5.3833 \\ 0 & 0 & 0 & 1.000718 \end{bmatrix}$$

$$\therefore \quad T_1^{-1} = \frac{\operatorname{adj}(T_1)}{\det(T_1)} = \begin{bmatrix} 0.5262 & 0.3688 & -0.766 & -.6 \\ -0.573 & 0.819 & 0 & -2.95 \\ +0.627 & 0.439 & 0.6429 & -5.38 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$T_2^T = \begin{bmatrix} 0.92 & 0 & -.39 & 0 \\ 0 & 1 & 0 & 0 \\ .39 & 0 & 0.92 & 0 \\ 5 & 6 & 0 & 1 \end{bmatrix}$$

$$adj(T_2) = T_{2\,minor}^T$$

$$= \begin{bmatrix} 0.92 & 0 & -0.39 & -3.82 \\ 0 & 0.9985 & 0 & -5.991 \\ 0.39 & 0 & 0.92 & -3.79 \\ 0 & 0 & 0 & 0.9985 \end{bmatrix}$$

$$\therefore T_2^{-1} = \frac{adj(T_2)}{det(T_2)}$$

Now, $det(T_2) = 0.9985$

$$\therefore T_2^{-1} = \begin{bmatrix} 0.921 & 0 & -0.39 & -3.82 \\ 0 & 1 & 0 & -6 \\ 0.3905 & 0 & 0.921 & -3.795 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans

Problem 13: Write the correct sequence of movements that must be made in order to "unrotate" the sperical coordinates and to make it parallel to the reference frame. About what axes are these rotations supposed to be?

⇒ We know,

$$T_{sph}(r, \beta, \gamma) = Rot(z,\gamma)\ Rot(y, \beta)\ Trans(0,0,r)$$

So,

The correct sequence may be written

$$T_{sph}(R, \beta, \gamma)\ Rot(o, -\beta)\ Rot(a, -\gamma)$$

$$= \begin{bmatrix} C\beta \cdot C\gamma & -S\gamma & S\beta \cdot C\gamma & r\,S\beta \cdot C\gamma \\ C\beta S\gamma & C\gamma & S\beta \cdot S\gamma & r\,S\beta \cdot S\gamma \\ -S\beta & 0 & C\beta & r \cdot C\beta \\ 0 & 0 & 0 & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} C(-\beta) & 0 & S(-\beta) & 0 \\ 0 & 1 & 0 & 0 \\ \cdot S\beta & 0 & C(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} C(-\gamma) & -S(-\gamma) & 0 & 0 \\ S(-\gamma) & C(-\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & \pi S\beta C\gamma \\ 0 & 1 & 0 & \pi S\beta S\gamma \\ 0 & 0 & 1 & \pi C\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotation are relative to o- and a- axes actually current axes. This rotations were occured in the current fram not in the reference frame because only the orientations were made different but if this actions were ~~is~~ performed in the reference frame, then the position of the frame ~~will~~ be different but according to the question this is not wanted. so. only for making differen orientation. this action were performed in current frame.

Problem - 14: A spherical co-ordinate system is used to describe the position of the hand of a robot. In a certain situation the hand is later 'unrotated' back to be parallel to the reference frame, and the matrix ~~representation~~ representing it is described as,

$$T_{sph} = \begin{bmatrix} 1 & 0 & 0 & 3.1375 \\ 0 & 1 & 0 & 2.195 \\ 0 & 0 & 1 & 3.214 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find the necessary values of r, β, γ to archieve this location.

(b) Find the components of the original matrix $\bar{n}, \bar{o}, \bar{a}$ vectors for the hand before it was unrotated.

⇒ from the previous problem,

the equations representing position in a spherical co-ordinates ~~may~~ can be set equal to given values.

1. $r \cdot S\beta \cdot C\gamma = 3.1375$

2. $r \cdot S\beta \cdot S\gamma = 2.195$

3. $r \cdot C\beta = 3.214$

1. Assuming $S\beta$ positive,

$$\tan\gamma = \frac{r\, S\beta\, S\gamma}{r\, S\beta\, C\gamma} = \frac{2.195}{3.1375}$$

$$\Rightarrow \gamma = 34.98^\circ$$
$$\simeq 35^\circ$$

Again,

$$\Rightarrow \frac{r\, S\beta\, S\gamma}{r\, C\beta} = \frac{2.195}{3.214}$$

$$\Rightarrow \tan\beta \sin 35^\circ = \frac{2.195}{3.214}$$

$$\Rightarrow \beta = 50^\circ$$

and from (3) $r = 5$

2. Assuming $S\beta$ negative,

from (1) and (2)

$$\Rightarrow \frac{r\, S\beta\, S\gamma}{r\, S\beta\, C\gamma} = \frac{2.195}{3.1375}$$

$$\Rightarrow \cancel{\tan}\gamma = 215^\circ \quad (\pi + \tan^{-1}\theta)$$

Again (3) and (2),

$$\tan\beta\, s\gamma = \frac{2.195}{3.214} \Rightarrow \beta = -50^\circ$$

Again, from (3) $\boxed{r = 5}$ Ans.

(b) We know,

$$T_{sph} = \begin{bmatrix} C\beta \cdot C\gamma & -S\gamma & S\beta \cdot C\gamma & r\,S\beta \cdot C\gamma \\ C\beta \cdot S\gamma & C\gamma & S\beta \cdot S\gamma & r\,S\beta \cdot S\gamma \\ -S\beta & 0 & C\beta & r\,C\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Assuming, $S\beta$ positive

$$T_{sph}(5, 50, 35) = \begin{bmatrix} 0.526 & -0.574 & 0.628 & 3.1375 \\ 0.369 & 0.819 & 0.439 & 2.195 \\ -0.766 & 0 & .643 & 3.214 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Assuming $S\beta$ negative

$$T_{sph}(5, -50, 215) = \begin{bmatrix} -0.526 & 0.574 & 4.628 & 3.1375 \\ -0.369 & -.819 & 0.439 & 2.195 \\ 0.766 & 0 & 0.643 & 3.214 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans.