Chapter 4

DC Dynamo Torque Relations—DC Motors

4–1 GENERAL

In comparing dc dynamo motor action versus generator action, Section 1–20 concluded with a summary of the fundamental differences between them. This chapter is devoted to the dc dynamo used as a dc motor. It is concerned, therefore, with dc dynamo *torque* relations and the characteristics of the dc motor as a means of producing electromagnetic torque. The summary of Section 1–20 stated that, for *motor* action,

1. The developed electromagnetic torque produces (aids) rotation.
2. The voltage generated in the current-carrying conductors (counter EMF) opposes the armature current (Lenz's law).
3. The counter EMF may be expressed by the equation

$$E_c = V_a - I_a R_a \qquad (1\text{–}8)$$

and is *less than* the applied voltage causing a given armature current flow I_a.

Equation (1–8) may be rewritten in terms of the armature current I_a produced for a given applied voltage and load:

$$I_a = \frac{V_a - E_c}{R_a} \qquad (1\text{–}8)$$

It was also shown in Section 1–17 that the three factors that determine the magnitude and that are required to produce electromagnetic force on a given current-carrying armature conductor (a force orthogonal to B and I) may be expressed by the Biot–Savart law in SI:

$$F = BIl \qquad \text{newtons (N)} \qquad (1\text{–}7b)$$

Finally, the direction of electromagnetic force developed by such a current-carrying conductor in a given magnetic field may be determined by the **left-hand rule** (Section 1–18).

The reader should review the relations just given and Sections 1–16 through 1–20 since they are fundamental and apply to all commercial motor types and the characteristics discussed next.

4–2 RELATION BETWEEN TORQUE AND FORCE

The terms electromagnetic *force* and electromagnetic *torque* have been used in the preceding summary of the motor relations covered in Chapter 1. These terms are *not* the same, but they are related. The relation between the force on a conductor [developed in accordance with the rewritten form of Eq. (1–8)] and the torque produced is shown in **Fig. 4–1**.

A single-turn coil (supported on a structure capable of rotation) is carrying current in a magnetic field, as shown in Fig. 4–1a. In accordance with Eq. (1–8) and the left-hand rule, an orthogonal force f_1 is developed in coil side 1, and a similar force f_2 is developed in coil side 2, as shown in Fig. 4–1b. Forces f_1 and f_2 are developed in such a direction that they tend to produce a clockwise rotation of the structure supporting the conductors about the center of rotation C.

Torque is defined as the *tendency* of a mechanical coupling (of a force and its radial distance to the axis of rotation) to produce rotation. It is expressed in units of force and distance, such as lb·ft, g-cm, ounce-inches, etc.,[1] to distinguish it from *work*,

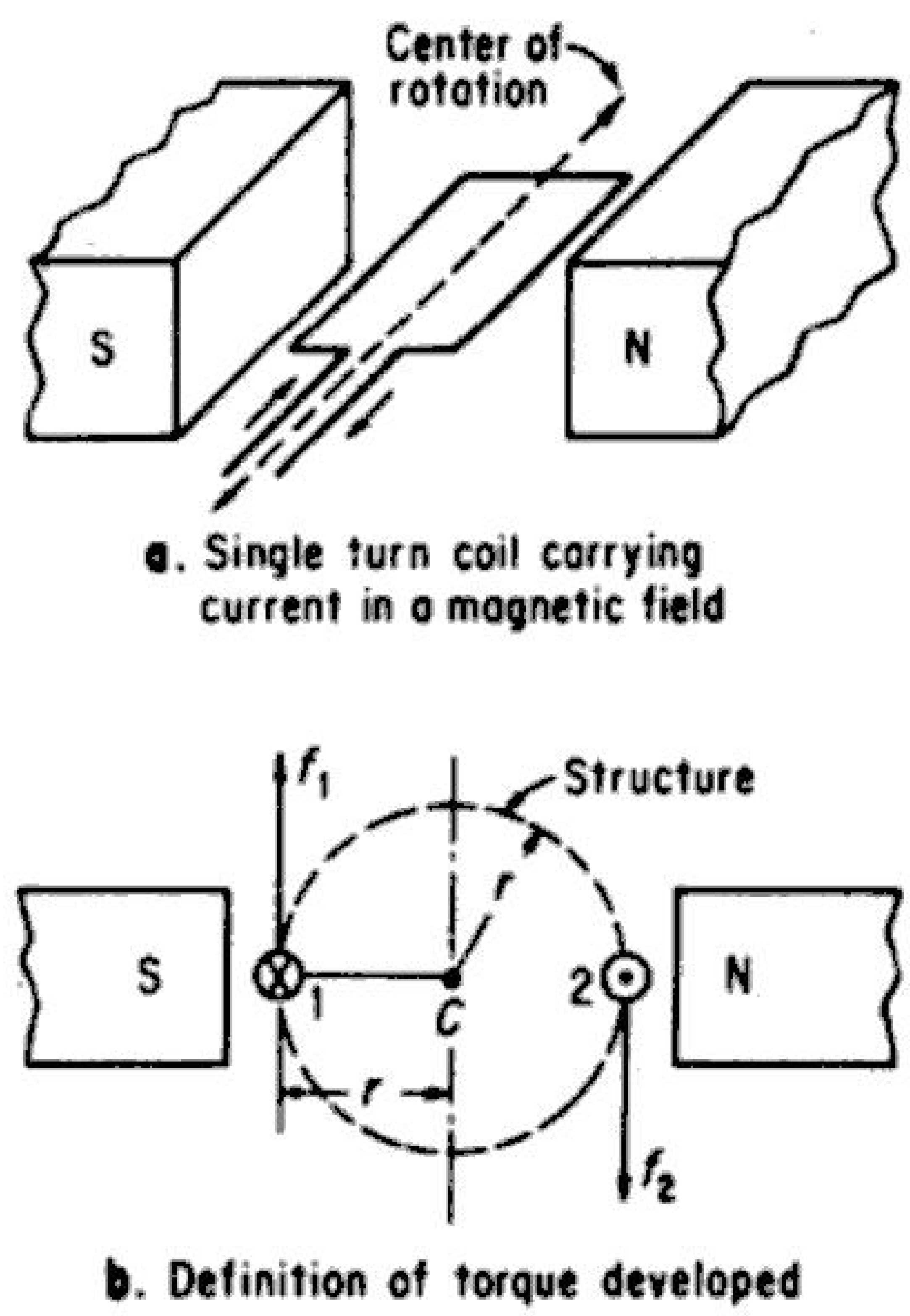


Figure 4–1 Production of torque in a single-turn coil

[1] Torque should not be confused with work. The latter is defined in terms of a force f, acting on a body and causing it to move through a distance d. The work done is the product of that component of force f acting in the same direction in which the body moves (to overcome resistance) for some distance d. If there is a force applied but no motion results, no work is done. Conversely, a force may exist on a body tending to produce rotation (a torque) and, even if the body does not rotate, the torque exists as the product of that force and the radial distance to the center of the axis of rotation.

which is expressed in ft · lb, cm · g, etc. The torque acting on the structure of Fig. 4–1b is the sum of the products f_1r and f_2r, i.e., the total sum of the torques acting on or produced by the individual conductors that tend to produce rotation. It should be noted that forces f_1 and f_2 are equal in magnitude since they lie in a field of the same magnetic strength and carry the same current. This is true for the forces developed by all conductors carrying the same current in a uniform magnetic field; but the torques developed, by definition, are not the same for each of these conductors.

The distinction between the force developed on the various armature conductors and the useful torque developed by these conductors to produce rotation is shown in **Fig. 4–2**. It has been shown in Chapters 1 and 2 that there is essentially no difference in construction between a generator armature and a motor armature. A dc dynamo, as described in Section 1–20, may be considered a motor when it fulfulls the three conditions summarized in Section 4–1.

An armature and a field of a two-pole motor are shown in Fig. 4–2. Note that all the conductors carrying current in the same direction *develop the same force*. This is true because they carry the same current and lie perpendicularly in the same field. But, since torque is defined as the product of a force and its perpendicular distance from the axis, we can see that the *useful component* of the force developed in Eq. (1–7b) is f, or

$$\text{(SI)} \qquad f = F \sin\theta \qquad \text{newtons (N)} \qquad \textbf{(4–1)}$$

where F is the force on each conductor developed in accordance with Eq. (1–7b)
θ is the complement of the angle created by the force developed on the conductor and the useful force f tangential to the armature periphery

Consequently, for the *torque* developed by any conductor T_c on the armature surface, we may write the SI expression

$$\text{(SI)} \qquad T_c = fr = (F \sin\theta)r \qquad \text{newton-meters (N · m)} \qquad \textbf{(4–2b)}$$

where f is the force per conductor in newtons perpendicular to r
r is the radial distance to the axis of rotation measured in meters

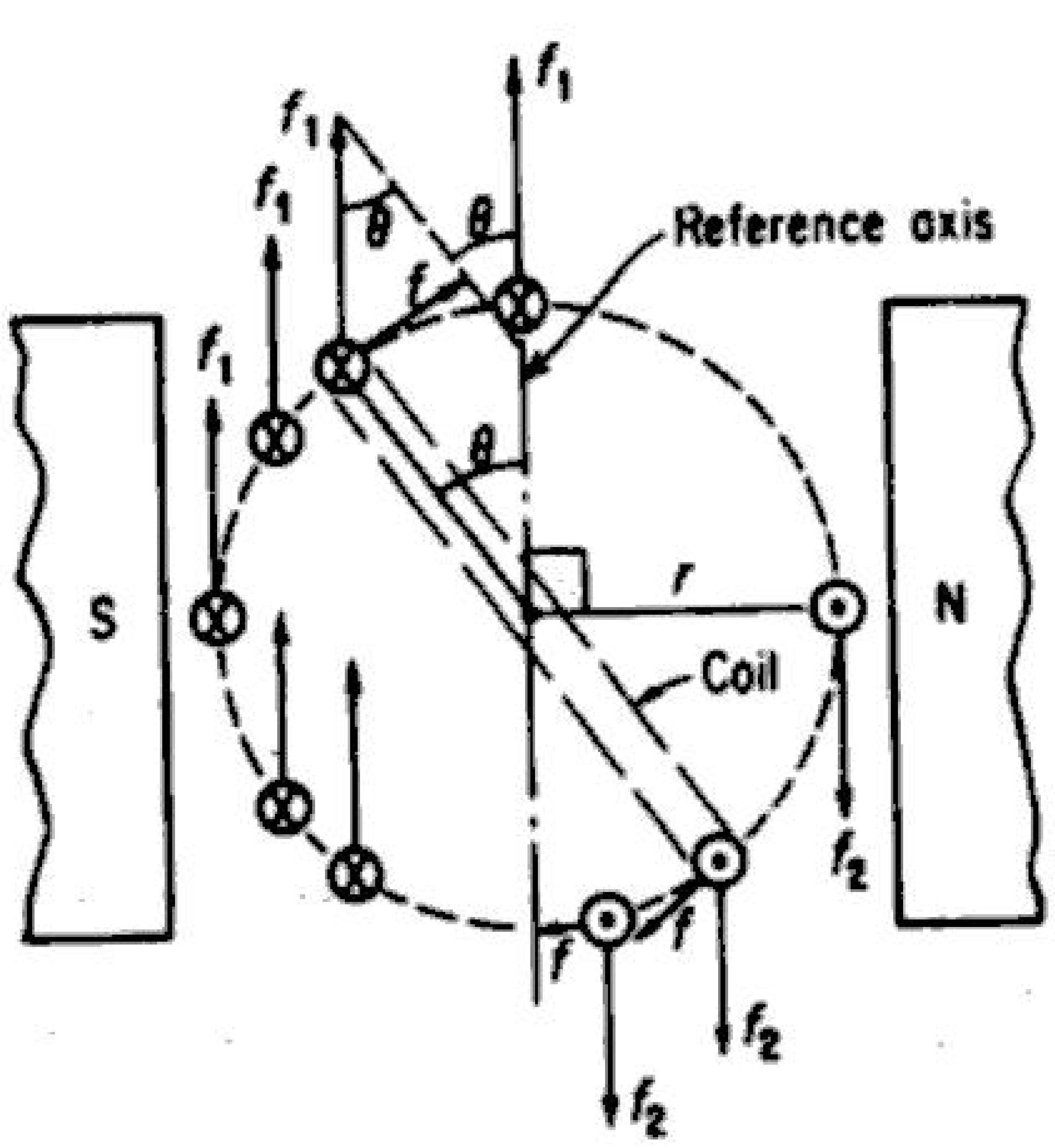


Figure 4–2 Useful torque for rotation

EXAMPLE 4–1 (SI) The single-turn coil in Fig. 4–2 lies on an armature 0.5 m in diameter, having an axial length of 60 cm, in a field with a flux density of 0.4 T. When the coil carries a current of 25 A, calculate **a.** The force developed on each coil side **b.** The useful force at the instant the coil lies at an angle of 60° with respect to the interpolar reference axis in Fig. 4–2 **c.** The torque developed in N·m **d.** The torque developed in lb·ft by two methods.	***Solution*** **a.** $F = BIl = 0.4 \times 25 \times 0.6 = \mathbf{6\ N}$ (1–7b) **b.** $f = F \sin\theta = 6\ \text{N}(\sin 60°) = \mathbf{5.2\ N}$ (4–1) **c.** $T_c = fr = 5.2\ \text{N} \times 0.25\ \text{m} = \mathbf{1.3\ N{\cdot}m}$ (4–2) **d.** $1.3\ \text{N·m} \times 0.2248 \frac{\text{lb}}{\text{N}} \times 3.281 \frac{\text{ft}}{\text{m}} = \mathbf{0.96\ lb{\cdot}ft}$ $1.3\ \text{N·m} \times 0.737562 \frac{\text{lb·ft}}{\text{N·m}} = \mathbf{0.96\ lb{\cdot}ft}$ (from Appendix A–1.3 M)

In order to develop the corresponding English unit equations, we must write

$$\text{(English)} \qquad F = \frac{BIl}{1.13} \times 10^{-7} \qquad \text{pounds (lb)} \qquad (1\text{–}7a)$$

and

$$f = F \sin\theta \qquad (4\text{–}1)$$

where F and f are expressed in units of pounds
B is the flux density in lines/square inch
l is the active length of conductor in inches

Similarly, we may write the expression for the torque developed by any conductor (T_c) on the armature surface in terms of English units as

$$\text{(English)} \qquad T_c = fr = (F\sin\theta)r = \left(\frac{BIl}{1.13}\sin\theta\right)r \times 10^{-7} \qquad \text{pound-feet (lb·ft)} \qquad \mathbf{(4\text{–}2a)}$$

where r is the radial distance to the axis of rotation measured in feet (ft) and all other terms have been previously defined.

EXAMPLE 4–2 The single-turn coil in Fig. 4–2 lies on an armature 18 inches in diameter, having an axial length of 24 inches, in a field with a density of 24 000 lines/in². When the coil carries a current of 26 A, calculate **a.** The force developed on each conductor **b.** The useful force at the instant the coil lies at an angle of 60° with respect to the interpolar reference axis per conductor **c.** The torque developed in lb·ft	***Solution*** **a.** $F = \frac{BIl}{1.13} \times 10^{-7}\ \text{lb}$ $= \frac{24\,000 \times 26 \times 24}{1.13 \times 10^{7}} = \mathbf{1.325\ lb}$ (1–7a) **b.** $f = F\sin\theta = 1.325 \sin 60° = \mathbf{1.15\ lb}$ (4–1) **c.** $T_c = fr = 1.15\ \text{lb}\left(9\ \text{in} \times \frac{1\ \text{ft}}{12\ \text{in}}\right)$ $= \mathbf{0.861\ lb{\cdot}ft/conductor}$ (4–2a)

Note that the conductors lying in the interpolar region of Fig. 4–2 develop (theoretically) just as much force as those lying directly under the pole, but that the

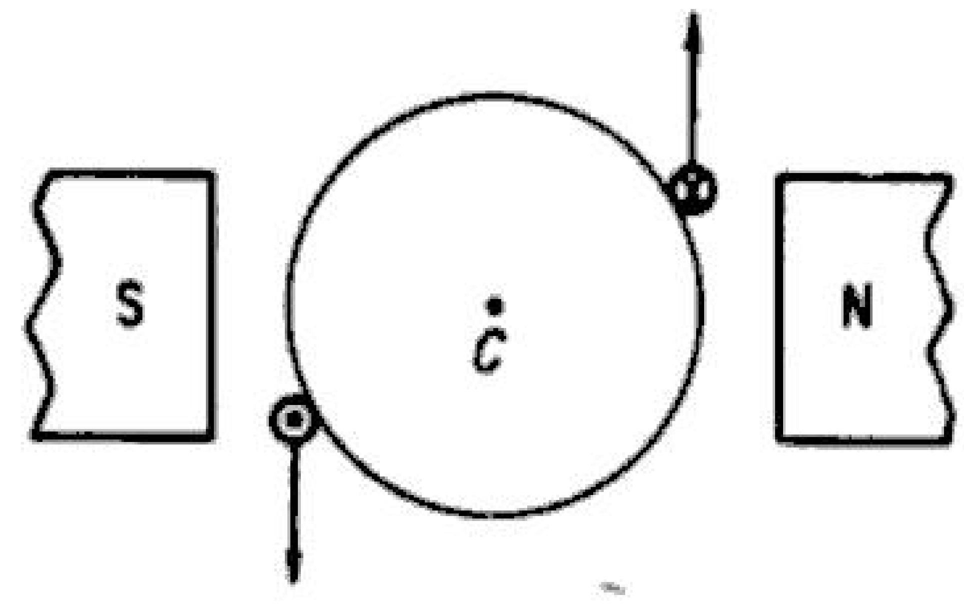


Figure 4–3 Necessity for commutation in a dc motor

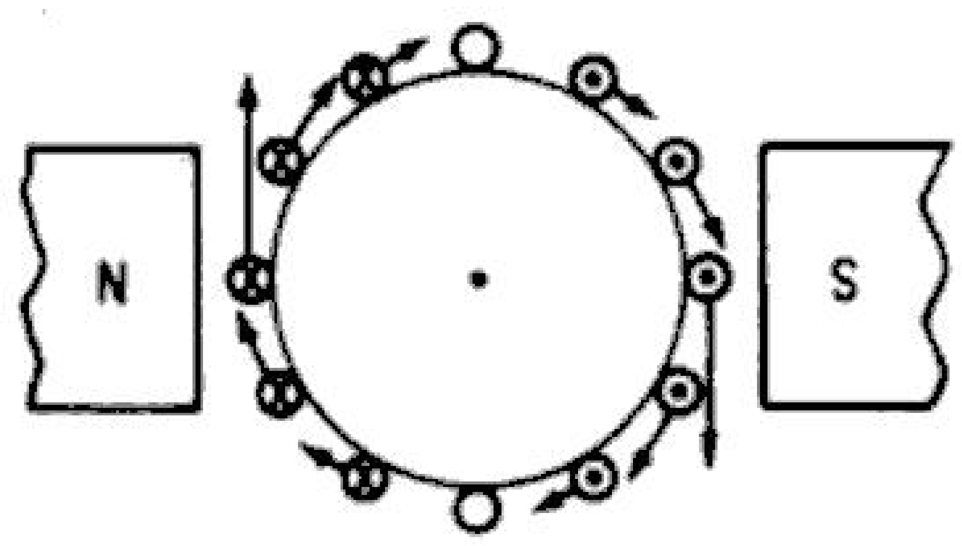


Figure 4–4 Reversal of conductor current required to produce continuous rotation

component of *useful* force *f*, tangential to the armature, is zero. Furthermore, if the coil of Fig. 4–2 is free to rotate in the direction of the developed torque *without* undergoing commutation, the current directions in the conductors would remain *unchanged* but the force developed on them would *reverse*, as shown in **Fig. 4–3**.

The necessity for commutation to *reverse* the current in a conductor as it moves under a pole of *reversed* polarity is just as fundamental for a dc motor as it is for a dc generator. Finally, since no *useful* torque is produced by conductors lying in the interpolar region, very little torque is lost by those conductors undergoing commutation. This is shown in **Fig. 4–4**, where the components of *useful* force and their magnitudes are indicated, as well as the current reversal in the conductor required to produce uniform and continuous rotation.

The preceding relations were developed for an armature having straight field poles and a fairly appreciable neutral interpolar zone. As shown in **Fig. 4–5**, in a commercial armature having many poles, slots, and armature conductors, the difference between the useful force developed directly under the pole and that developed almost at the pole tip is relatively small. It is customary, instead, to consider only that percentage of conductors *directly under the pole* that contribute useful torque and to assume that an average or common torque is produced by each conductor.

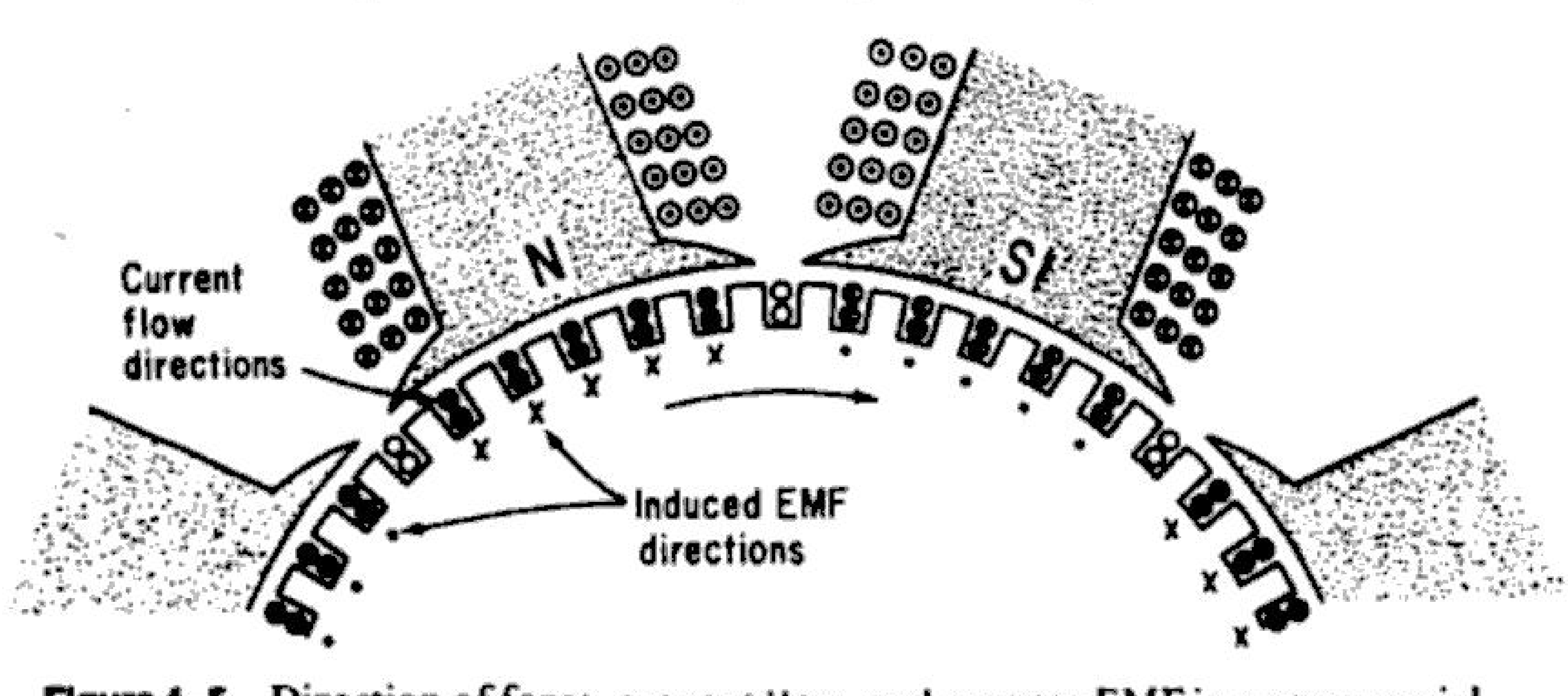


Figure 4–5 Direction of force, current flow, and counter EMF in a commercial dc motor

These assumptions lead to the simple relation

$$\text{(English)} \qquad F_{av} = F_c \times Z_a \qquad \text{pound (lb)} \qquad (4\text{–}1a)$$

where F_{av} is the average total force tending to rotate the armature in pounds
F_c is the average force per conductor directly under the pole [Eq. (1–7)] in pounds
Z_a is the number of active conductors on the armature

This simplifies calculation of the total torque developed by the armature since

$$T_{av} = F_{av} \times r = F_c \times Z_a \times r \qquad \text{pound-feet (lb·ft)} \qquad \mathbf{(4\text{–}2c)}$$

where all terms have been previously defined.

EXAMPLE 4–3
The armature of a dc motor contains 700 conductors and has a diameter of 24 inches and an axial length of 34 inches. If 70 percent of the conductors lie directly under the poles with a flux density of 50 000 lines/in^2 and carry a current of 25 A, calculate
a. The average total force tending to rotate the armature
b. The armature torque in lb·ft.

Solution

a. $F_{av} = F_c \times$ no. of active conductors

$$= \left(\frac{50\,000 \times 25 \times 34}{1.13 \times 10^{+7}}\right)(700 \times 0.7)$$

$$= \mathbf{1843\ lb} \qquad (4\text{–}1a)$$

b. $T_{av} = F_{av} \times r = 1860\ \text{lb} \times 1\ \text{ft}$

$$= \mathbf{1843\ lb{\cdot}ft} \qquad (4\text{–}2c)$$

EXAMPLE 4–4
Find the total developed armature torque of a motor that has the following specifications: 120 armature slots, 6 conductors per slot; 60 000 lines/square inch flux density; 28 inch armature diameter, 14 inch armature axial length; and 4 armature paths in parallel. The pole arcs span 72 percent of the armature surface, and the armature current is 133.5 A.

Solution

$Z_{Ta} = 120$ slots $\times$ 6 conductors/slot $\times$ 0.72
$= 518$ conductors

$$F_t = \left(\frac{BIl}{1.13 \times 10^7}\right) Z_{Ta}$$

$$= \frac{60 \times 10^3 \times 133.5\ \text{A} \times 14 \times 518}{1.13 \times 10^7 \times 4\ \text{paths}} = 1285\ \text{lb}$$

$$T = F_t r = \frac{1285\ \text{lb} \times 28\ \text{in}}{2} \times \frac{1\ \text{ft}}{12\ \text{in}} = \mathbf{1500\ lb{\cdot}ft}$$

4–3 FUNDAMENTAL TORQUE EQUATIONS FOR A DC DYNAMO

The discussion of the preceding sections and examples, particularly Ex. 4–4, would suggest that the total torque developed by the armature of any given dynamo may be computed in terms of the flux density, the number of poles, number of paths, total armature conductors, active length per conductor, and so on. In short, we are seeking an equation, similar to Eq. (1–5), that will solve Ex. 4–4 in a single step. This is easily derived as follows:

$$\text{Torque per conductor } T_c = \frac{BIlr}{1.13 \times 10^7} \qquad (4\text{–}2)$$

$$\text{Armature current per conductor } I = \frac{I_a}{a}$$

For a total armature containing Z conductors, we may now write an equation for the total average torque developed by the magnetically covered conductors working to produce torque as

$$\text{(English)} \qquad T = \left(\frac{BI_a lZr}{a \times 1.13 \times 10^7}\right) \times (\%\ \text{A.S.}) \text{ pound-feet (lb}\cdot\text{ft)} \tag{4–3a}$$

where B is the flux density in lines/square inch
I_a is the total armature current entering (or leaving) the armature in amperes
l is the active length of each armature conductor in inches
Z is the total number of armature conductors
r is the radial distance to the axis of rotation in feet
a is the number of paths in the armature winding
% A.S. is the percentage of armature surface covered by the poles

We may also derive the corresponding equation, Eq. (4–3b), where all units are expressed in SI units as

$$\text{(SI)} \qquad T = \left(\frac{BI_a lZr}{a}\right) \times (\%\ \text{A.S.}) \qquad \text{newton-meters (N}\cdot\text{m)} \tag{4–3b}$$

where B is the flux density in tesla (T) or webers/square meter (Wb/m^2)
l is the active length of each armature conductor in meters
r is the radial distance to the axis of rotation in meters
% A.S. is the percentage of armature surface covered by the poles

We may now test the validity of Eq. (4–3a) by repeating Ex. 4–4.

EXAMPLE 4–5 Given the data of Ex. 4–4, use Eq. (4–3a) to find the total external armature current in one step and to verify the solution of Ex. 4–4.	***Solution*** $I_a = \dfrac{Ta \times 1.13 \times 10^7}{BlZr(\%\ \text{A.S.})}$ (4–3a) $= \dfrac{1500 \times 4 \times 1.13 \times 10^7}{60 \times 10^3 \times 14 \times (120 \times 6)(28/2 \times 12)(0.72)}$ $=$ **133.5 A**

If we examine Eqs. (4–3a) and (4–3b) carefully, we discover that, for a given constructed commercial dynamo, the only possible variables are the armature current I_a and the flux density B. But since the area of the poles is also constant, we may write the Eqs. (4–3a) and (4–3b) for the total developed electromagnetic torque of a dynamo as

$$T = K\phi I_a \qquad \text{lb}\cdot\text{ft} \tag{4–4a}$$

and/or

$$T = K'\phi I_a \qquad \text{N}\cdot\text{m} \tag{4–4b}$$

where K' and K, respectively, are the factors in Eqs. (4–3b) and (4–3a) that are held constant.

The significance of Eqs. (4–4a) and (4–4b) is that there are but two ways to increase the torque of a motor (or a dynamo), i.e., to increase the armature current and/or the field flux, or both.

Note that Eqs. (4–4a) and (4–4b) are but another form of Eqs. (1–7a) and

(1–7b), where the variables B and I for any given dynamo determine the value of the electromagnetic force producing the motor torque.

Also note that this electromagnetic torque *opposes* rotation in a *generator* and *aids* (is in the same direction as) *rotation* in a motor. Since the torque is a function of the flux and the armature current, it is *independent of speed* in the case of either a generator or a motor. It will be seen later that the *speed* of a motor *does*, in fact, *depend on torque* (but not vice versa). The terms *torque* and *speed* should *not*, however, be used synonymously, for a motor that is stalled may tend to develop appreciable torque but no speed.

A change in flux may produce a change in armature current and also produce a change in torque, as shown by Ex. 4–6.

EXAMPLE 4–6

A motor develops a torque of 150 N·m and is subjected to a 10 percent reduction in field flux, which produces a 50 percent increase in armature current. Find the new torque produced as a result of this change in field flux.

Solution

	ϕ	I_a	T
Original condition	1.0	1.0	150 N·m
New condition	0.9	1.5	?

$$T = K'\phi I_a \tag{4–4b}$$

Using the ratio method, the new torque is the product of two new ratio changes:

$$T = 150\left(\frac{0.9\phi}{1.0\phi}\right)\left(\frac{1.5I_a}{1.0I_a}\right) = \mathbf{202.5\ N\cdot m}$$

Example 4–6 shows that an interaction occurs between field flux and armature current, which, in turn, affects the magnitude of electromagnetic torque. We shall later discover that the reason for this lies in the effect of the field flux on the motor counter EMF (CEMF) (see Section 4–5).

Finally, it should be noted that the electromagnetic torque developed by the armature in accordance with Eqs. (4–3) and (4–4) is customarily called the ***developed torque***. The developed torque, developed by the armature conductors, is somewhat analogous to generated EMF (E_g) in that it is developed *internally*, within the armature. The *torque available at the pulley or shaft* of a motor is somewhat *less* than the developed torque because of specific rotational losses that require and consume some portion of the developed torque during motor action (Section 12–3).

4–4 COUNTER EMF OR GENERATED VOLTAGE IN A MOTOR

We are already aware that, when a dc dynamo operates as a motor, generator action simultaneously occurs since the conductors are moving in a magnetic field. The current-carrying conductors that produce a clockwise torque are shown in the armature slots of Fig. 4–5. The opposing direction of induced EMF is shown below the conductors (left-hand versus right-hand rules, respectively) in the figure. The counter EMF generated in the armature conductors is expressed in Eq. (1–5) for a given armature. The current that flows through the armature is limited by (1) the armature resistance and (2) the counter EMF, in accordance with Eq. (1–8) as rewritten in Section 4–1, i.e., $I_a = (V_a - E_c)/R_a$.

It is fairly evident that the counter EMF can never equal the voltage applied across the armature terminals because, as shown in Fig. 4–5, the direction in which the current flow *first* occurs determines the direction of rotation and, in turn, creates the counter EMF. Clearly, then, the counter EMF, like the armature resistance, is a *current-limiting* factor. The nature of the counter EMF in limiting the current may best be understood by Ex. 4–7, which also includes brush volt drop (BD) as a current-limiting factor.

EXAMPLE 4–7

A dc shunt motor having an armature resistance of 0.25 Ω and a brush contact volt drop of 3 V receives an applied voltage across its armature terminals of 120 V. Calculate the armature current when

a. The speed produces a counter EMF of 110 V at a given load
b. The speed drops (due to application of additional load) and the counter EMF is 105 V
c. Compute the percentage of change in counter EMF and in armature current.

Solution

a. $I_a = \dfrac{V - (E_c + \text{BD})}{R_a} = \dfrac{120 - (110 + 3)}{0.25}$

$= \mathbf{28\ A}$ (1–8)

b. At increased load,

$I_a = \dfrac{120 - (105 + 3)}{0.25} = \mathbf{48\ A}$

c. $\delta E_c = \dfrac{110 - 105}{110} \times 100 = \mathbf{4.\overline{54}\ percent}$

$\delta I_a = \dfrac{28 - 48}{28} \times 100 = \mathbf{71.4\ percent}$

Example 4–7 dramatically shows that a small decrease in counter EMF ($4.\overline{54}$ percent) has resulted in a much larger increase in armature current (71.4 percent). Consequently, *small* changes in motor speed (and counter EMF), however slight, are accompanied by correspondingly *large* changes in motor current. For this reason, in some types of servomotor transducer devices, motor current is used as an indication of motor load and motor speed.

4–5 MOTOR SPEED AS A FUNCTION OF COUNTER EMF AND FLUX

The value of counter EMF given in Ex. 4–7 may be computed readily from Eqs. (1–5a) and (1–5b). (See Exs. 1–9 and 1–10, Section 1–15.) For any given dc dynamo, either Eq. (1–5a) or Eq. (1–5b) may be rewritten in terms of its variables, and the counter EMF of a motor may be expressed as

$$\text{(English)} \quad E = K\phi S \quad \text{volts} \qquad \textbf{(4–5a)}$$

$$\text{(SI)} \quad E = K'\phi\omega \quad \text{volts} \qquad \textbf{(4–5b)}$$

where ϕ is the field flux in lines or webers
S is the speed in rev/min
ω is the speed in rad/s

But the counter EMF of a motor, including volt drop across brushes or brush drop (BD), is

$$E_c = V_a - (I_aR_a + \text{BD}) \qquad (1–8)$$

Substituting this expression for counter EMF in Eqs. (4–5a) and (4–5b) yields the following expression for motor speed in both English and SI units:

$$\omega \quad \text{or} \quad S = \frac{V_a - (I_aR_a + \text{BD})}{k\phi} \qquad \textbf{(4–6)}$$

where all terms have been previously defined.

Equation (4–6) has been called the *fundamental dc motor speed equation* since it permits prediction of dc motor performance so readily. For example, if the field flux of a dc motor is weakened considerably, the motor will *run away*. If the denominator of Eq. (4–6) approaches zero, the speed approaches infinity. Similarly, if the load current and flux are held constant, while the voltage impressed across the motor armature is increased, the speed will increase in the same proportion. Finally, if the field flux and the voltage across the armature are fixed, and the armature current is increased because of increased load, the motor speed will drop in the same proportion as the decrease in counter EMF [Eq. (4–5)].

EXAMPLE 4–8

A 120 V dc shunt motor having an armature circuit resistance of 0.2 Ω and a field circuit resistance of 60 Ω, draws a line current of 40 A at full load. The brush volt drop is 3 V and rated full-load speed is 1800 rpm. Calculate

a. The speed at half load

b. The speed at an overload of 125 percent.

Solution

a. At full load,

$$I_a = I_l - I_f = 40\text{ A} - \frac{120\text{ V}}{60\ \Omega} = 38\text{ A} \qquad (1\text{–}8)$$

$$E_c = V_a - (I_aR_a + \text{BD}) = 120 - (38 \times 0.2 + 3) = 109.4\text{ V}$$

At the rated speed of 1800 rpm,

$E_c = 109.4$ V and $I_a = 38$ A (full load)

At half-load speed,

$$I_a = \frac{38\text{ A}}{2} = 19\text{ A}$$

$$E_c = V_a - (I_aR_a + \text{BD}) = 120 - (19 \times 0.2 + 3) = 113.2\text{ V}$$

Using the ratio method, half-load speed is

$$S = S_{\text{orig}}\left(\frac{E_{\text{final}}}{E_{\text{orig}}}\right) = 1800\left(\frac{113.2}{109.4}\right) = \mathbf{1863\ rpm} \qquad (4\text{–}6)$$

b. At $1\frac{1}{4}$ load,

$$I_a = (\tfrac{5}{4})\,38\text{ A} = 47.5\text{ A}$$

$$E_c = V_a - (I_aR_a + \text{BD}) = 120 - (47.5 \times 0.2 + 3) = 107.5\text{ V}$$

$$S_{5/4} = 1800\left(\frac{107.5}{109.4}\right) = \mathbf{1769\ rpm} \qquad (4\text{–}6)$$

These results are tabulated in Ex. 4–10.

EXAMPLE 4–9

The dc motor of Ex. 4–8 is loaded to a line current of 66 A (temporarily), but in order to produce the necessary torque, the field flux is increased by 12 percent by decreasing the field circuit resistance to 50 Ω. Calculate the speed of the motor.

Solution

$$I_a = I_l - I_f = 66 - \frac{120}{50} = 63.6\text{ A}$$

$$E_c = V_a - (I_aR_a + \text{BD}) = 120 - (63.6 \times 0.2 + 3) = 104.3\text{ V}$$

$$S = \frac{KE_c}{\phi} = 1800\left(\frac{104.3}{109.4}\right)\left(\frac{1.0}{1.12}\right) = \mathbf{1532\ rpm} \qquad (4\text{–}6)$$

Note that in Ex. 4–9 the ratio method is used in the solution. The original full-load speed of 1800 rpm is affected by *two* factors, counter EMF and flux. The counter EMF has decreased and, since speed varies *directly* as counter EMF, the speed is multiplied by a *decreasing* ratio. Similarly, the flux has increased, but an increase in ϕ produces a *decrease* in speed. Therefore, the speed is again multiplied by a *decreasing* ratio. This calculation technique is more economical and useful than proportions.

The reader should study it carefully by solving Exs. 4–8 and 4–9 independently, using the ratio method shown.

4–6 COUNTER EMF AND MECHANICAL POWER DEVELOPED BY A MOTOR

In general, it may be noted from the preceding examples that the full-load counter EMF is less than the counter EMF at a lighter value of load. As a function of the armature voltage across the armature circuit, the *full-load* counter EMF will vary from approximately 80 percent in the smaller dynamos to about 95 percent of the armature applied voltage in the larger dc motors.

The counter EMF E_c as a percentage of the armature voltage V_a is an important ratio in determining the relative efficiency and mechanical power developed by a given armature. The mechanical power developed by the armature may be derived in the following way.

The voltage drop across the armature, ignoring brush drop BD, is

$$I_aR_a = V_a - E_c \qquad (1\text{–}8)$$

and the power lost in the armature when V_a is impressed across it and I_a flows is [multiplying both sides of Eq. (1–5) by I_a]

$$(I_aR_a)I_a = I_a(V_a - E_c)$$

or

$$I_a^2R_a = V_aI_a - E_cI_a$$

Solving for E_cI_a, we get

$$E_cI_a = V_aI_a - I_a^2R_a \qquad \text{watts (W)} \qquad (4\text{–}7)$$

The significance of Eq. (4–7) is that when electrical power, V_aI_a, is supplied to the motor armature circuit to produce rotation, a certain amount of power is dissipated in the various components making up the armature circuit resistance; this dissipation is termed an armature copper loss, $I_a^2R_a$. The remaining power, E_cI_a, is required by the armature for the production of developed or internal torque (cf. Fig. 12–1).

The ratio of *power developed by* to *power supplied to* the armature, E_cI_a/V_aI_a, is the same as the ratio E_c/V_a. Thus, the higher the percentage of counter EMF to voltage across the armature, the higher the motor efficiency. Further, for a given load current, it is fairly evident that, when the counter EMF is a maximum, the motor will develop the greatest possible power for that value of armature current I_a.

The last sentence bears some reflection. It would appear from Eq. (4–5) ($E_c = K\phi S$) that, in order to develop the maximum counter EMF possible, it is only necessary to increase the field current and flux to a maximum (without overheating the field winding) and at the same time "operate" the motor at very high speeds.

But Eq. (4–6) shows that when the field flux is *increased*, the speed *decreases* (Ex. 4–9). Furthermore, both the speed and the counter EMF are, in part, determined by the mechanical load on the motor. It is fairly certain, however, that for a *given* mechanical load and resulting line and armature current, there is a particular speed and field rheostat setting that should produce maximum power.

EXAMPLE 4–10

Calculate the armature power developed, P_d, for each of the loads of Exs. 4–8 and 4–9 and tabulate all results for ready reference and comparison.

Solution

Example	I_a	E_c	Speed	P_d or $(E_c I_a)$
4–8a	38	109.4	1800	4157 W at full load
	19	113.2	1863	2151 W at $\frac{1}{2}$ load
4–8b	47.5	107.5	1769	5106 W at $1\frac{1}{4}$ load
4–9	63.6	104.3	1532	6633 W at overload

The conclusions that may be drawn from the tabulated data of Ex. 4–10 show that as the armature current and the load on the motor *increase:*

1. The counter EMF *decreases.*
2. The speed *decreases.*
3. The motor armature power developed *increases.*

The results also show that a small reduction in counter EMF results in a proportionately large increase in armature current, with the result that the power developed increases although the counter EMF decreases with increases in load.

4–7 RELATION BETWEEN TORQUE AND SPEED OF A MOTOR

Let us assume that in the basic speed equation, Eq. (4–6), the brush volt drop BD is zero. In the derivation and discussion of the basic speed equation $S = (V_a - I_a R_a)/K\phi$, the reader may have noticed what may appear to be an obvious inconsistency between this equation and Eq. (4–4), $T = K\phi I_a$. Since torque is defined as a force tending to produce rotation, according to Eq. (4–4), increasing the field flux would tend to increase the torque and (possibly) the speed. On the other hand, increasing the field flux in Eq. (4–6) would reduce the speed. Is there an inconsistency and is it possible to reconcile the two equations?

Actually, there is no inconsistency; and with the help of Eq. (1–8), $I_a = (V_a - E_c)/R_a$, it is possible to give both a qualitative and a quantitative explanation of what happens when the field flux is reduced. Qualitatively, the steps are:

1. The field flux of a shunt motor is reduced by decreasing the field current.
2. The counter EMF, $E_c = K\phi S$, drops instantly (the speed remains constant as a result of the inertia of the large and heavy armature).
3. The decrease in E_c causes an *increase* in the armature current I_a; refer to Eq. (1–8), cited above.
4. But Ex. 4–8 showed that a small reduction in field flux produces a large increase in armature current.
5. In Eq. (4–4), therefore, where $T = K\phi I_a$, the *small decrease* in flux is more than counterbalanced by a *large increase* in armature current. Note that the *torque has increased more than the flux was reduced!*
6. This *increase in torque* produces an *increase in speed.*

In summary, decreasing field current (and field flux) results in an increase in speed!

4–7.1 Effect of Flux on Armature Current

Since the speed of a running machine is determined by the torque developed, the question arises, then, is it possible to increase the field flux and, at the same time, increase the speed? The answer is that it *is* possible, but *only* if the armature current is held *constant* ($T = K\phi I_a$). This is actually done in the dc servomotor, shown in **Fig. 4–6**, in which I_a is constant because the armature is connected to a *constant-current* source (a series or differential compound generator; Section 3–20).

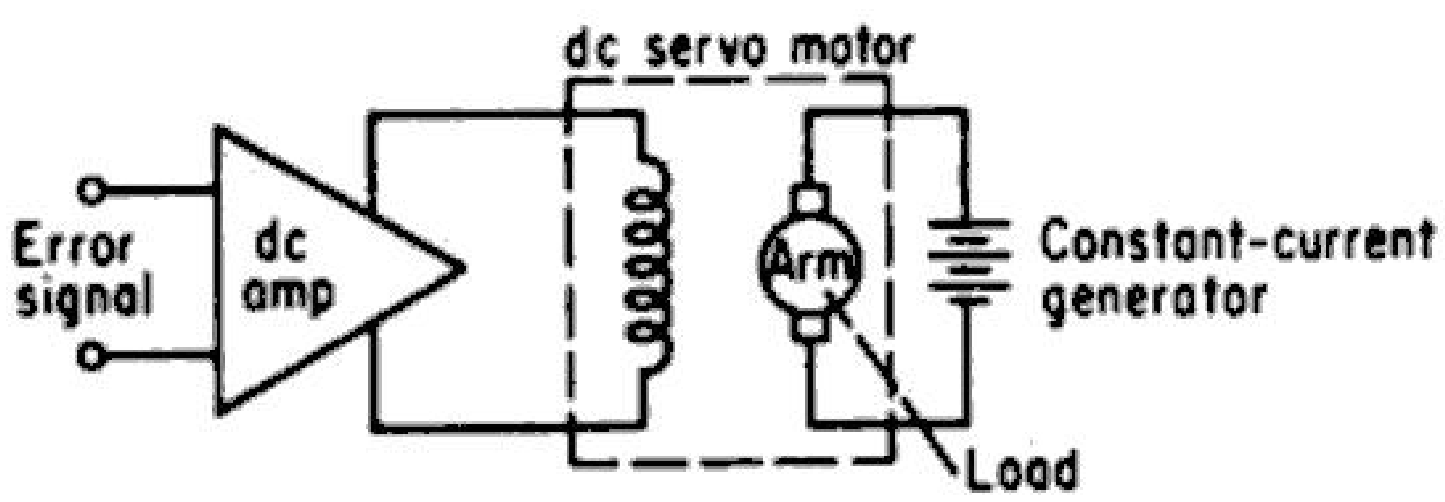


Figure 4–6 Separately excited dc motor

With no dc voltage impressed on the separately excited field, there is no torque [Eq. (4–4)]. When a small dc voltage is applied to the field, a small torque is developed and the armature rotates slowly in accordance with Eq. (4–4). Since the armature current is constant at all times, the torque and speed are therefore proportional only to the field flux. A field flux of zero produces zero, *not infinite*, speed. The separately excited dc servo motor (Section 11–13) does not violate the basic motor equations. On the contrary, it proves them!

A final equation frequently asked (and a tempting one for students in the laboratory) is: What would occur if the field circuit of a loaded shunt motor is suddenly opened? Would the motor, if unprotected by fuses, gain speed to a point where it would destroy itself?

We already know that any *small decrease* in flux produces a *large increase* in armature current and torque. A loaded motor with an open field draws an abnormally high armature current as it races to higher speeds and, in turn, produces higher mechanical loads and centrifugal forces on its armature conductors.

The answer to the question lies in the nature of the source and the lines supplying the armature. Given a source capable of supplying an infinite current, and given feeder lines of zero resistance, an open field will cause higher speed, more load, more armature current, more torque, and, in turn, higher speed. The motor speed will be almost infinite (ultimately), and the motor will, indeed, be destroyed by the centrifugal forces acting on its armature conductors. But, happily for most students in the laboratory, the supply lines have resistance, the voltage supply is limited as to the current it can deliver, and, fortunately, a circuit breaker or fuse opens the circuit before too much damage is done to the motor by excessive armature current and speed.

Summarizing then, in attempting to predict the effect of changes in armature current and flux on either torque or speed, there is no inconsistency between Eqs. (4–4) and (4–6). The reader must bear in mind that when armature current (I_a) is *not* held constant, a *decrease in flux* produces correspondingly *larger increases* in armature current, torque, and speed.

4–7.2 Unit Conversions among Various Units of Torque

Motors are manufactured by many commercial organizations in many countries. Nameplate data for full-load torque are frequently found in a variety of units (cgs, SI, and English), many of which are dated and hardly in current use. For example, torque may be expressed in units of dyne-centimeters (dyne·cm) or gram-centimeters (g·cm) in the cgs system. Correspondingly for smaller motors, torque is expressed in ounce-inches (oz·in) in the English system. Appendix A–1 lists various conversion factors in **ratio form**, and torque conversion factors are found in Appendix A–1.3M.

EXAMPLE 4–11
Convert the following values of torque to units expressed in N·m and lb·ft, respectively:
a. 6.5 dyne-centimeters
b. 10.6 gram-centimeters
c. 12.2 ounce-inches
(Hint: See Appendix A–1.3M.)

Solution

a. $6.5\ \text{dyne}\cdot\text{cm} \times 1.416 \times 10^{-5}\frac{\text{oz}\cdot\text{in}}{\text{dyne}\cdot\text{cm}}$
$= 9.204 \times 10^{-5}\ \text{oz}\cdot\text{in}$
$9.204 \times 10^{-5}\ \text{oz}\cdot\text{in} \times 7.0612 \times 10^{-3}\frac{\text{N}\cdot\text{m}}{\text{oz}\cdot\text{in}}$
$= \mathbf{6.5\times10^{-7}\ N\cdot m}$
$9.204 \times 10^{-5}\ \text{oz}\cdot\text{in} \times 5.208 \times 10^{-3}\frac{\text{lb}\cdot\text{ft}}{\text{oz}\cdot\text{in}}$
$= \mathbf{4.8\times10^{-7}\ lb\cdot ft}$

b. $10.6\ \text{g}\cdot\text{cm} \times \frac{1\ \text{oz}\cdot\text{in}}{72.01\ \text{g}\cdot\text{cm}}$
$= 1.472 \times 10^{-1}\ \text{oz}\cdot\text{in}$
$1.472 \times 10^{-1}\ \text{oz}\cdot\text{in} \times 7.0612 \times 10^{-3}\frac{\text{N}\cdot\text{m}}{\text{oz}\cdot\text{in}} = \mathbf{1.04\times10^{-3}\ N\cdot m}$
$1.472 \times 10^{-1}\ \text{oz}\cdot\text{in} \times 5.208 \times 10^{-3}\frac{\text{lb}\cdot\text{ft}}{\text{oz}\cdot\text{in}} = \mathbf{7.\bar{6}\times10^{-4}\ lb\cdot ft}$

c. $12.2\ \text{oz}\cdot\text{in} \times 7.0612 \times 10^{-3}\frac{\text{N}\cdot\text{m}}{\text{oz}\cdot\text{in}}$
$= \mathbf{8.615\times10^{-2}\ N\cdot m}$
$12.2\ \text{oz}\cdot\text{in} \times 5.208 \times 10^{-3}\frac{\text{lb}\cdot\text{ft}}{\text{oz}\cdot\text{in}}$
$= \mathbf{6.35\times10^{-2}\ lb\cdot ft}$

4–8 STARTERS FOR DC MOTORS

At the instant of applying a voltage V_a across the armature terminals in order to cause a motor to rotate, the motor armature is not producing any counter EMF since the speed is zero [Eq. (4–5)]. The only current-limiting factors are the armature brush volt drop and the resistance of the armature circuit, R_a. Since neither of these, under normal conditions, amounts to more than 10 or 15 percent of applied voltage V_a across the armature (see Section 4–6), the overload is many times the rated armature current, as indicated by Ex. 4–12.

EXAMPLE 4–12
A 120 V dc shunt motor has an armature resistance of 0.2 Ω and a brush volt drop of 2 V. The rated full-load armature current is 75 A. Calculate the current at the instant of starting and the percentage of load current.

Solution
Using Eq. (1–8) to include BD,

$$I_{st} = \frac{V_a - \text{BD}}{R_a} = \frac{120-2}{0.2}$$
$$= \mathbf{590\ A}\ \text{(counter EMF is zero)}$$
$$\text{Percentage at full load} = \frac{590\ \text{A}}{75\ \text{A}} \times 100$$
$$= \mathbf{78\bar{6}\ percent}$$

Example 4–12 shows that the starting current of the motor is approximately 8 times as great as the normal rated full-load armature current. It shows that severe damage might be done to a motor, whenever it is starting, unless the starting current is limited by means of a *commercial starter.*[2]

The current in Ex. 4–12 is excessive because of a lack of counter EMF at the instant of starting. Once rotation has begun, counter EMF is built up in proportion to speed. What is required, then, is a device, usually a tapped or variable resistor, whose purpose is to limit the current during the starting period and whose resistance may be progressively reduced as the motor gains speed.

Given an external resistor R_s in series with the armature, Eq. (1–8) must be modified for computing the armature current to:

$$I_a = \frac{V_a - (E_c + \mathrm{BD})}{R_a + R_s} \quad \text{amperes (A)} \qquad \textbf{(4–8)}$$

where all terms have been previously defined.

The value of the starting resistor at zero speed or any step along the way may be computed from Eq. (4–8), as illustrated by Ex. 4–13.

EXAMPLE 4–13

Calculate the various values (taps) of starting resistance (R_s) to limit the current in the motor of Ex. 4–12 to

a. 150 percent rated load at the instant of starting

b. A counter EMF that is 25 percent of the armature voltage V_a at 150 percent rated load

c. A counter EMF that is 50 percent of the armature voltage at 150 percent rated load

d. Find the counter EMF at full load, without starting resistance.

Solution

From Eq. (4–8),

$$R_s = \frac{V_a - (E_c + \mathrm{BD})}{I_a} - R_a$$

a. At starting, E_c is zero.

$$R_s = \left(\frac{V_a - \mathrm{BD}}{I_a}\right) - R_a$$

$$= \left(\frac{120 - 2}{1.5 \times 75}\right) - 0.2 = 1.05 - 0.2$$

$$= \mathbf{0.85\ \Omega}$$

b. $$R_s = \left[\frac{V_a - (E_c + \mathrm{BD})}{I_a}\right] - R_a$$

$$= \left(\frac{120 - 30 - 2}{1.5 \times 75}\right) - 0.2 = 0.78\overline{2} - 0.2$$

$$= \mathbf{0.58\overline{2}\ \Omega}$$

c. $$R_s = \left[\frac{120 - (60 + 2)}{1.5 \times 75}\right] - 0.2$$

$$= 0.516 - 0.2 = \mathbf{0.316\ \Omega}$$

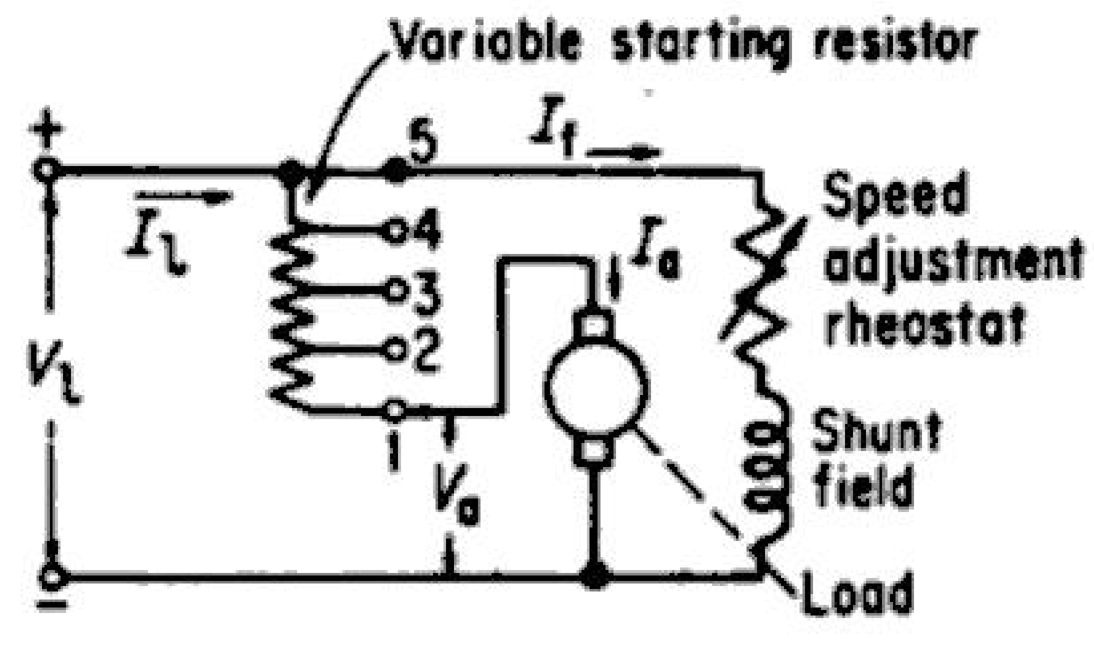
d. $$E_c = V_a - (I_a R_a + \mathrm{BD})$$

$$= 120 - [(75 \times 0.2) + 2] = \mathbf{103\ V}$$

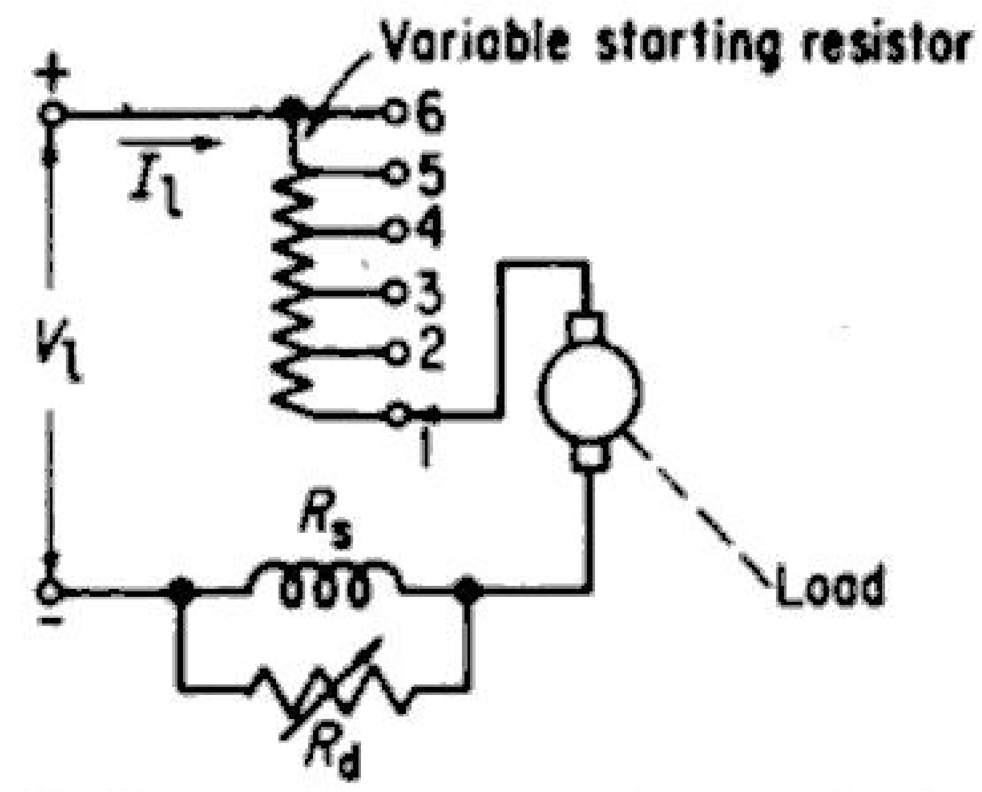
Note that, in Ex. 4–13, a *progressively decreasing* value of starting resistance is required as the motor develops an increased counter EMF owing to acceleration. This is the principle of the *armature resistance motor starter.*

The manner in which a starter is used in conjunction with the three basic types of dc dynamos, used as motors, is shown in **Fig. 4–7.** The techniques shown here for motor starting are *schematic* diagrams only; as stated previously, commercial forms of manual and automatic starters and controllers differ somewhat from these.

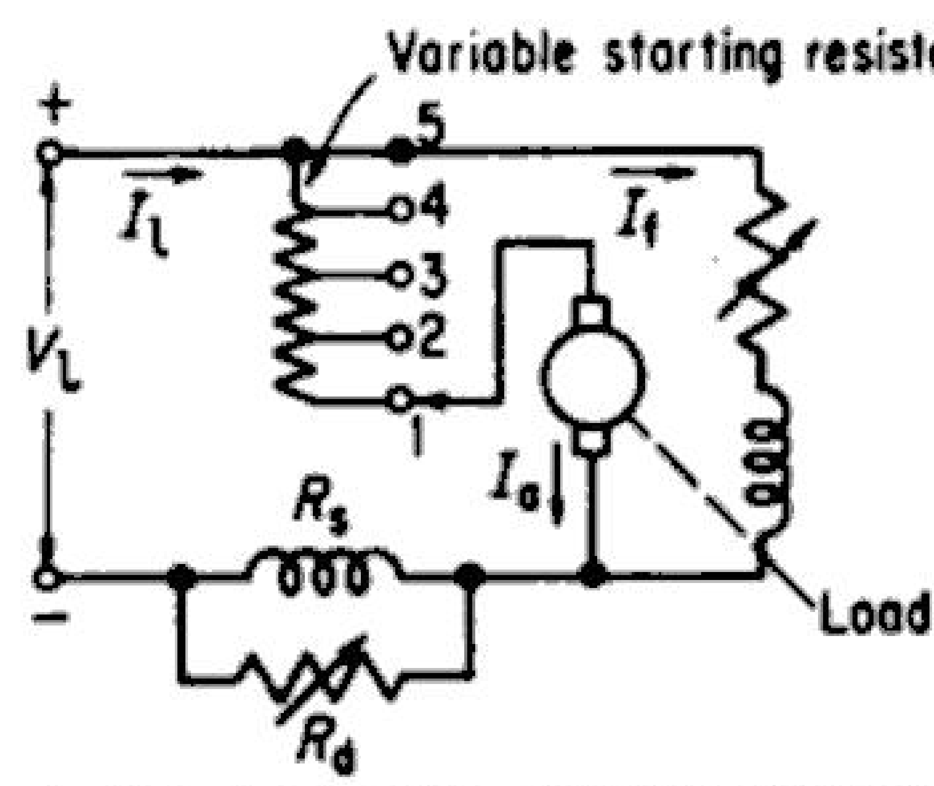
[2] The subject of commercial motor starters, both manual and automatic, is covered in detail in Kosow, *Control of Electric Machines,* (Englewood Cliffs, N.J.: Prentice-Hall, 1973), Chs. 3 and 4.



a. Shunt motor starter (schematic form)



b. Series motor starter (schematic form)



c. Compound motor starter (schematic form)

Figure 4–7 Starter connections for shunt, series, and compound motors in schematic form

The shunt and compound motors are started with *full field excitation* (i.e., the full line voltage is impressed across the field circuit) in order to develop maximum starting torque ($T = K\phi I_a$). In all three types of dynamos, the armature starting current is limited by a high-power series-connected variable starting resistor. In commercial practice, the initial inrush of armature current is generally limited to a higher value than the full-load current, as given in Ex. 4–13, again to develop greater starting torque, particularly in the case of large motors that have great inertia and that come up to speed slowly.

With the starting arm at position 1 in Fig. 4–7a, the maximum series resistance will limit the armature current on starting to about 150 percent of its rated value. As the motor slowly increases its speed, the armature develops counter EMF and the armature current drops to approximately full load. If the starting arm were left at position 1, the armature current would drop somewhat and the speed would stabilize at some value well below the rated speed. In order to accelerate the motor armature once more, it is necessary to move the arm to position 2. Again, there is an inrush of armature current, and the motor rises in speed. This process is continued until the motor armature attains its rated speed, without the need for a series armature resistance, and where the counter EMF at that speed and flux is sufficient to limit the armature current.

It should be noted in Fig. 4–7 that all three types (series, shunt, and compound motors), if started with a mechanical load coupled to the armature, will accelerate more slowly than if started without load. The series motor, particularly, should *never be started without load coupled to its armature* (see Section 4–10). The shunt and compound motors, on the other hand, may be started with or without mechanical load. *Manual* starters (i.e., those operated by the hand of a human operator) require some experience in moving the contact arm through the various steps of resistance to accelerate the motor to rated speed without producing excessive armature current. *Automatic* starters are designed electrically to accelerate the motor to each resistance step, regardless of the degree of motor loading, without damage to the motor.

4–9 ELECTROMAGNETIC TORQUE CHARACTERISTICS OF DC MOTORS

The fundamental torque equation, Eq. (4–4), in which $T = K\phi I_a$, provides a means of predicting how the torque of each of the three types of motors shown in Fig. 4–7 will vary with application of load (i.e., with increased armature current). The torque–load characteristic of each motor type will be taken up in turn. It is now assumed that each motor has been properly started and accelerated so that its armature is connected directly across the line terminals, V_l in Fig. 4–7. What is the effect of *increased* load on the torque of dc motors?

4–9.1 Shunt Motor

During the starting and the running periods, the current in the shunt field circuit, as shown in Fig. 4–7a, is essentially constant for a given setting of the field rheostat, and consequently the flux (for the present) is also essentially constant. As the mechanical load is increased, the motor slows down somewhat, causing decreased counter EMF and increased armature current.[3] In the basic torque equation, therefore, if the flux is essentially constant and if the armature current increases directly with the application of mechanical load, the torque equation for the shunt motor may be expressed as a perfectly linear relation, $T = K'I_a$, shown in **Fig. 4–8** for the shunt motor.

4–9.2 Series Motor

If the shunt field coils were removed from the dc dynamo just mentioned and replaced with a full series field winding, the identical armature and construction would produce the torque curve shown in Fig. 4–8 for the series motor. In a series motor, the armature and series field currents are the same (ignoring the effects of a diverter), and the flux produced by the series field, ϕ, is at all times proportional to the armature current I_a. The basic torque equation for series motor operation, there-

[3] The effect of increased armature current produces an armature MMF called "armature reaction," which, depending on the degree of saturation of the field, will tend to demagnetize and reduce the field flux somewhat. Armature reaction will be covered in detail in Chapter 5.

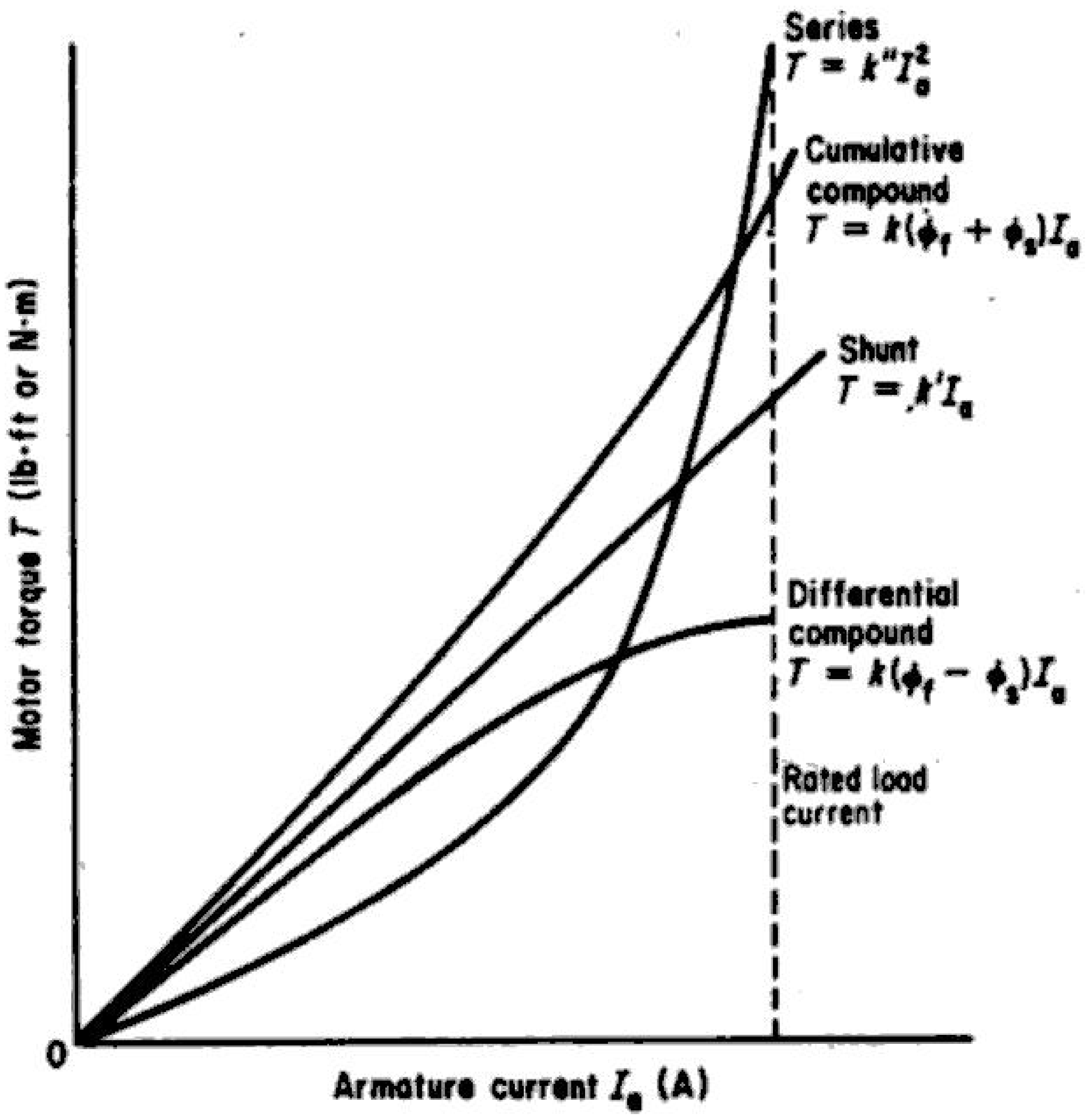


Figure 4–8 Comparison of torque–load characteristics for dc motors

fore, becomes $T = K''I_a^2$. To the extent that the field core is unsaturated (i.e., on the linear portion of its magnetization curve), the relation between series motor torque and load current is exponential, as shown in Fig. 4–8. It should be noted that the series motor torque at extremely light loads (low values of I_a) is less than the shunt motor because it develops less flux. For the same armature current at full load, however, its torque is greater, as evidenced by a comparison of the two equations, respectively, shown in Fig. 4–8.

4–9.3 Compound Motors

When a combined shunt and series field winding is installed on the poles of the same dc dynamo under discussion, the series field may be cumulative or differentially compounded. Regardless of compounding, however, the current in the shunt field circuit and the field flux ϕ_f, during starting or running, is essentially constant. The current in the series field is a function of the load current drawn by the armature.

The basic torque equation for *cumulative compound* motor operation is $T = K(\phi_f + \phi_s)I_a$, where the series field flux ϕ is a function of the armature current I_a. Starting with a flux equal to the shunt field flux at no load and one that increases with armature current, the cumulative compound motor produces a torque curve that is *always* higher than the shunt motor for the *same* armature current as shown in Fig. 4–8.

For the *differential compound* motor, however, the preceding torque equation may be written as $T = K(\phi_f - \phi_s)I_a$, where ϕ_s is still a function of I_a and ϕ_f is (presumably) constant. Starting with a flux equal to the shunt field flux at no load, any value of armature current will produce a series field MMF that *reduces* the total air-gap flux and hence the torque. Thus, the differential compound motor produces a torque curve that is *always less* than that of the shunt motor. (See Sec. 4–10.4.)

EXAMPLE 4–14

A cumulative compound motor is operated as a shunt motor (series field disconnected) and develops a torque of 160 lb·ft when the armature current is 140 A and the field flux is 1.6×10^6 lines. When reconnected as a cumulative compound motor at the same current, it develops a torque of 190 lb·ft. Find

a. The flux increase due to the series field in percent

b. The torque when the compound motor load increases by 10 percent (assume operation on the *linear* portion of the saturation curve).

Solution

The data given for Ex. 4–14 are arranged in tabular form for convenient reference:

	T in lb·ft	I_a in A	ϕ_f in lines
Original	160	140	1.6×10^6
Added flux	190	140	ϕ_f'
Final torque	T_f'	154	$1.1 \times 1.9 \times 10^6$

From the data given, the calculations are worked out as follows:

a. $\phi_f = \phi_{orig}\left(\frac{T_{final}}{T_{orig}}\right) = 1.6 \times 10^6 \times \frac{190}{160}$

$= 1.9 \times 10^6$ lines

Percentage of flux increase

$= \frac{1.9 \times 10^6}{1.6 \times 10^6} \times 100 - 100$

$= 118.8 - 100$

$=$ **18.8 percent**

b. The final field flux is $1.1 \times 1.9 \times 10^6$ lines (due to the 10 percent increase in load). The final torque is

$$T_f = 190 \text{ lb}\cdot\text{ft}\left(\frac{154 \text{ A}}{140 \text{ A}}\right)\left(\frac{1.1 \times 1.9 \times 10^6}{1.0 \times 1.9 \times 10^6}\right)$$

$$= \mathbf{230\ lb\cdot ft} \tag{4–4}$$

Example 4–14a shows that the flux increase of 18.8 percent produced a proportional torque increase (i.e., 190 lb·ft/160 lb·ft) of 18.8 percent in accordance with Eq. (4–4a). Example 4–14b shows the effect of loading in increasing the armature current, which increases the flux and the torque of the cumulative compound motor.

EXAMPLE 4–15

A series motor draws a current of 25 A and develops a torque of 90 lb·ft. Calculate

a. The torque when the current rises to 30 A if the field is *unsaturated*

b. The torque when the current rises to 30 A and the increase in current produces a 10 percent increase in flux.

Solution

a. $T = KI_a^2 = 90 \text{ lb}\cdot\text{ft}\left(\frac{30}{25}\right)^2 =$ **129.6 lb·ft**

b. $T = K\phi I_a = 90 \text{ lb}\cdot\text{ft}\left(\frac{30}{25}\right)\left(\frac{1.1}{1.0}\right)$

$$= \mathbf{118.8\ lb\cdot ft} \tag{4–4}$$

Example 4–15a shows that, if the field is assumed unsaturated, a 20 percent load increase in armature current (i.e., 30 A/25 A) produces an increase in motor torque of 44 percent (i.e., 129.6 lb·ft/90 lb·ft). Example 4–15b shows that, if the field pole is saturated, a 20 percent load increase in armature current now produces an increase in motor torque of 32 percent (i.e., 118.8 lb·ft/90 lb·ft).

4–10 SPEED CHARACTERISTICS OF DC MOTORS

The fundamental speed equation, Eq. (4–6), in which $S = (V_a - I_a R_a)/k\phi$, provides a means of predicting how the speed of each of the motors shown in Fig. 4–7 will vary with application of load. The speed–load characteristic of each motor will be taken up in turn. To simplify the discussion, it is assumed that the brush volt drop BD is zero.

4–10.1 Shunt Motor

Assume that the shunt motor of Fig. 4–7a has been brought up to rated speed and is operating at no load. Since the field flux of the motor (ignoring armature reaction) may be considered *constant*, the speed of the motor may be expressed in terms of the basic speed equation previously shown in Section 4–5:

$$\omega \quad \text{or} \quad S = \frac{E_c}{k'\phi_f} = k\frac{V_a - I_a R_a}{\phi_f} \tag{4–6}$$

As mechanical load is applied to the armature shaft, the counter EMF decreases and the speed decreases proportionately. But since the counter EMF from no load to full load is a change of approximately 20 percent (i.e., from 0.75 V_a at full load to approximately 0.95 V_a to no load), the motor speed is essentially constant, as shown in **Fig. 4–9.**

4–10.2 Series Motor

The basic speed equation, Eq. (4–6), as modified for the circuit of the series motor, is clearly

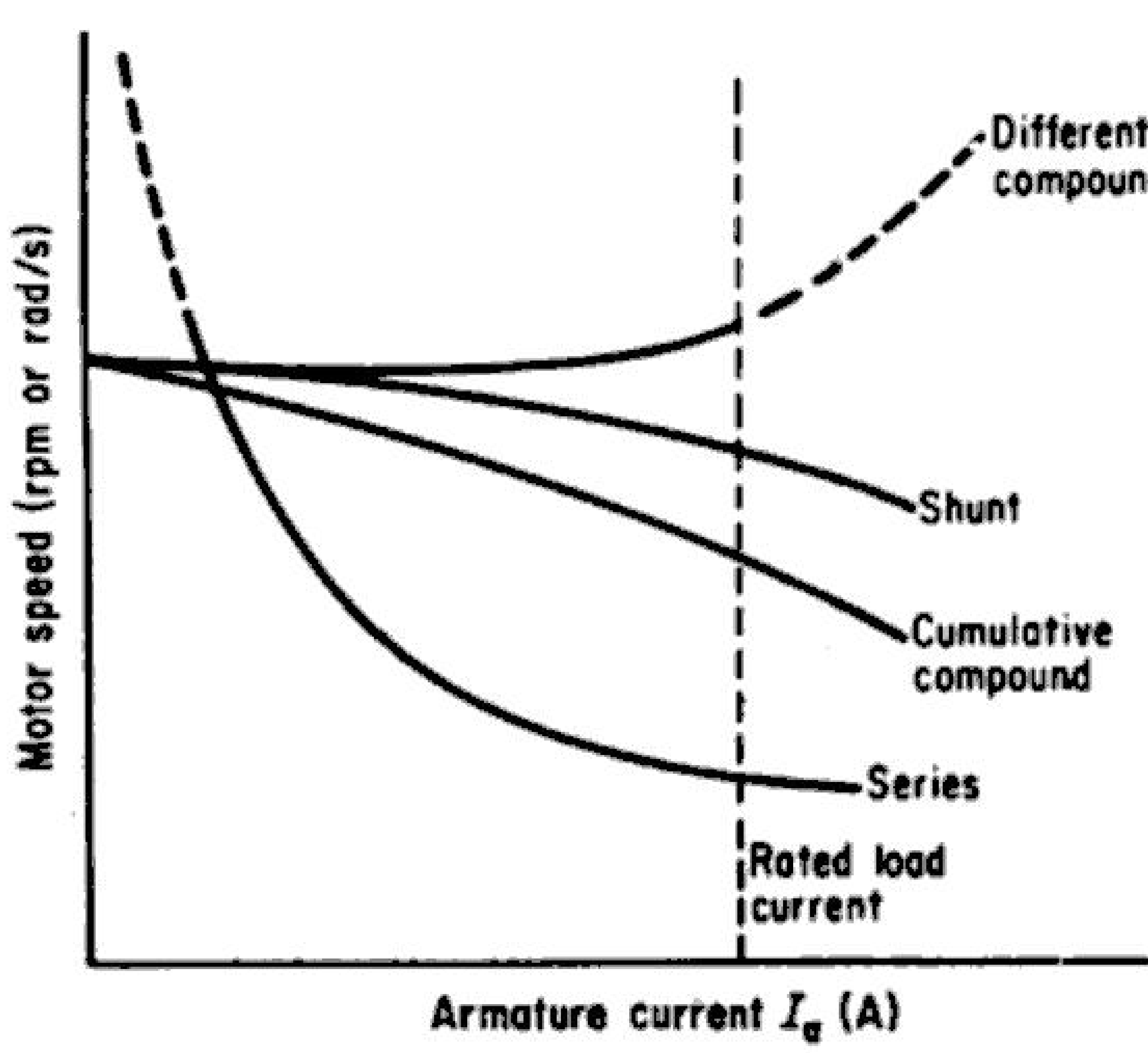


Figure 4–9 Comparison of speed-load characteristics for dc motors

$$\omega \quad \text{or} \quad S = \frac{V_a - I_a(R_a + R_s)}{k\phi} \tag{4-9}$$

where V_a is the voltage applied across the motor terminals. Since the air-gap flux produced by the series field is proportional to the armature current only, the speed may be written as

$$\omega \quad \text{or} \quad S = K' \frac{V_a - I_a(R_a + R_s)}{I_a} \tag{4-10}$$

Equation (4–10) gives us an indication of the speed–load characteristic of a series motor. If a relatively small mechanical load is applied to the shaft of the armature of a series motor, the armature current I_a is small, making the numerator of the fraction in Eq. (4–10) large and its denominator small, resulting in an unusually high speed. At no load, therefore, with little armature current and field flux, the speed is extremely excessive. For this reason, series motors are always operated coupled or geared to a load, as in hoists, cranes, or dc traction (railway) service. As the load increases, however, the *numerator* of the fraction in Eq. (4–10) *decreases faster than the denominator increases* (the numerator decreases by a product of I_a, compared to the denominator, which increases directly with I_a), and the *speed drops rapidly*, as shown in Fig. 4–9. The dashed line represents that lightly loaded portion of the characteristic in which series motors are not operated.

As shown in Fig. 4–9, excessive speed for a series motor does *not* result in a high armature current (as with shunt and compound motors) that will open a fuse or a circuit breaker and disconnect the armature from the line. Some other method of protection against runaway must be used. Series motors are usually equipped with centrifugal switches, normally closed in the operating range, that open at speeds of approximately 150 percent of the rated speed or higher.

4–10.3 Cumulative Compound Motor

The basic speed equation for the cumulative compound motor may be written as

$$\omega \quad \text{or} \quad S = K\frac{V_a - I_a(R_a + R_s)}{\phi_f + \phi_s} \tag{4-11}$$

and further simplified to

$$\omega \quad \text{or} \quad S = K\frac{E_c}{\phi_f + \phi_s} = \frac{KE_c}{\phi_{total}} \tag{4-12}$$

By comparing Eq. (4–12) for the cumulative compound motor with $S = KE/\phi_f$ for the shunt motor, it is evident that, as the load and the armature current increase, the flux produced by the series field also increases, while the counter EMF decreases. The denominator therefore increases while the numerator decreases proportionately more than for a shunt motor. The result is that the speed of the *cumulative* compound motor will drop at a faster rate than the speed of the shunt motor with application of load, as shown in Fig. 4–9.

4–10.4 Differential Compound Motor

Equation (4–12) for the cumulative compound motor may be modified slightly to show the effect of the *opposing* field MMFs, and the speed is

$$\omega \quad \text{or} \quad S = \frac{KE_c}{\phi_f - \phi_s} = K\frac{V_a - I_a(R_a + R_s)}{\phi_f - \phi_s} \tag{4–13}$$

where all terms have been previously defined.

As the load and I_a on a differential compound motor increase, the numerator of the fraction in Eq. (4–13) decreases somewhat but the denominator decreases more rapidly. The speed may drop slightly at light loads; but as the load increases, the speed increases. This condition sets up a dynamic instability.

As the speed increases, most mechanical loads increase automatically (since *more work is done at a higher speed*), causing an increase in current, a decrease in total flux, and a higher speed, producing still more load. *Because of this inherent instability, differential motors are rarely used.*

In a machinery laboratory where differential motors are tested, one may occasionally observe a condition where a differential motor begins to race away and suddenly drops in speed and reverses direction! This may be explained by using Eq. (4–13) and Fig. 4–9. As the counter EMF decreases due to decreased mutual flux, the armature current and torque increase is so excessive that the series field flux *exceeds* the shunt field flux, and the motor reverses direction (in accordance with the left-hand rule).

It is for this reason that, when a differential motor is started, for testing purposes in the laboratory, care should be taken to *short out* the *series* field so that the high starting and armature current will *not* start the motor in the *reverse* direction.

The curves of Figs. 4–8 and 4–9 were developed for the same dc dynamo operating from the same *no-load* point. But, since all electrical machinery is specified in terms of rated (full-load) values, the comparison of torque–load and speed–load characteristics should be made at *rated load*. If one were to compare dc motors of the same voltage, horsepower output, and speed rating, the curves of **Fig. 4–10** would be

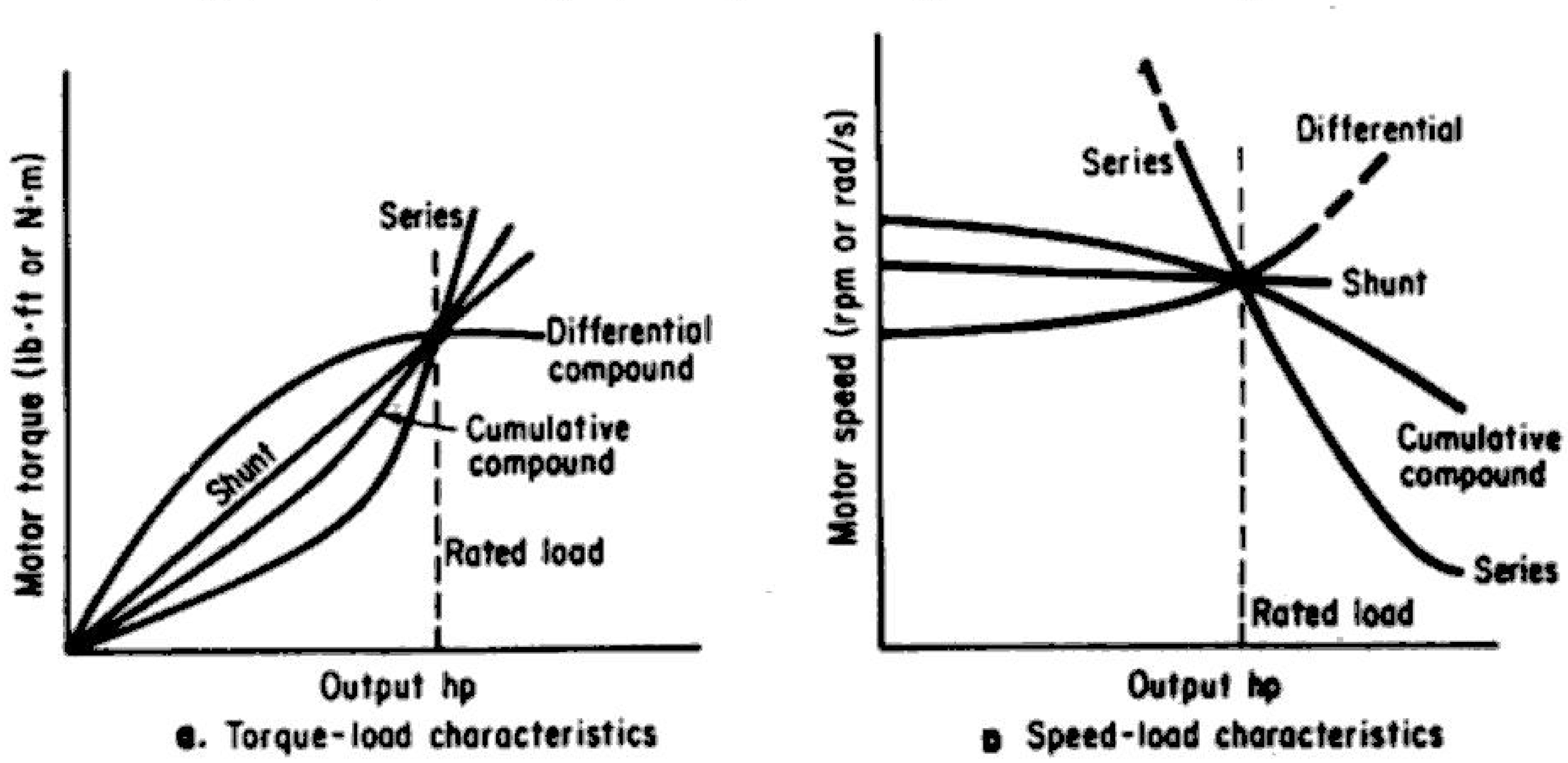


Figure 4–10 Comparison of motor torque and speed–load characteristics at rated load

obtained. The reader should compare the curves of Fig. 4–10 with those of Figs. 4–8 and 4–9 to verify the characteristics.

EXAMPLE 4–16

A 230 V, 10 hp, 1250 rpm cumulative compound motor has an armature resistance of 0.25 Ω, a combined compensating winding and interpole resistance of 0.25, Ω, and a brush volt drop of 5 V. The resistance of the series field is 0.15 Ω, and the shunt field resistance is 230 Ω. When the motor is *shunt* connected, the line current rated load is 55 A and the no-load line current is 4 A. The no-load speed is 1810 rpm. Neglecting armature reaction at rated voltage, calculate

a. Speed at rated load

b. Internal power in watts and internal horsepower developed.

Solution

a. $I_a = I_l - I_f = 4\text{ A} - 1\text{ A} = 3\text{ A}$

No-load $E_c = V_a - (I_a R_a + \text{BD})$
$= 230 - (3 \times 0.5 + 5)$
$= 223.5$ V at a speed of 1810 rpm

Full-load $E_c = V_a - (I_a R_a + \text{BD})$
$= 230 - (54 \times 0.5 + 5)$
$= 198$ V

$$S_r = 1810\left(\frac{198}{223.5}\right) = \mathbf{1603\ rpm} \qquad (4\text{–}5)$$

b. $P_d = E_c I_a = 198\text{ V} \times 54\text{ A} = \mathbf{10\ 700\ W}$

$$\text{hp} = \frac{10\,700\text{ W}}{746\text{ W/hp}} = \mathbf{14.3\ hp}$$

Example 4–16 shows the following:

1. The full-load speed of the shunt motor (1603 rpm) is somewhat higher than the rated (full-load) speed of the cumulative compound motor given (1250 rpm). (See Fig. 4–9.)
2. The internal horsepower developed by the motor (14.3 hp) is somewhat greater than the rated horsepower of the motor (10 hp), available at the output shaft.

EXAMPLE 4–17

The motor of Ex. 4–16 is reconnected as a long-shunt cumulative compound motor. At rated load (55 A), the compound winding increases the flux per pole 25 percent. Calculate

a. The speed at no load (4 A line current)

b. The speed at rated load (55 A line current)

c. Internal torques at full load with and without the series field. Use Eq. (4–15)

d. Internal horsepower of the compound motor based on above flux increase

e. Explain the difference between internal and rated horsepower.

Solution

a. No-load $E_c = V_a - (I_a R_a + I_s R_s + \text{BD})$
$= 230 - [(3 \times 0.5) + (3 \times 0.15) + 5]$
$= 223.05$ V [from Eq. (4–8)]

$$S_{n1} = 1810\left(\frac{223.05}{223.5}\right) = \mathbf{1806\ rpm} \qquad (4\text{–}5)$$

b. Full-load $E_c = 230 - [(54 \times 0.5) + (54 \times 0.15) + 5]$ (4–8)
$= 190$ V

$$S_r = K\left(\frac{\delta E_c}{\delta \phi}\right)$$

$$= 1806\text{ rpm}\left(\frac{190}{223.05}\right)\left(\frac{1.0}{1.25}\right)$$

$$= \mathbf{1231\ rpm} \qquad (4\text{–}12)$$

c. Internal torque of *shunt* motor [Eq. (4–15)] at full load:

$$T_{\text{shunt}} = \frac{\text{hp} \times 5252}{S} = \frac{14.3 \times 5252}{1603}$$

$$= \mathbf{46.85\ lb \cdot ft} \qquad (4\text{–}15)$$

$$T_{\text{comp}} = T_{\text{shunt}}\left(\frac{\phi_2}{\phi_1}\right)\left(\frac{I_{a2}}{I_{a1}}\right)$$

$$= 47.2\left(\frac{1.25}{1.0}\right)\left(\frac{54}{54}\right)$$

$$= \mathbf{59.1\ lb \cdot ft}$$

d. $\text{Horsepower} = \dfrac{E_c I_a}{746} = \dfrac{190 \times 54}{746} = \mathbf{13.8\ hp}$

e. The internal horsepower exceeds the rated horsepower because the power developed in the motor must also overcome the internal mechanical rotational losses (Fig. 12–1).

Example 4–17 verifies the following points:

1. The full-load speed of the compound motor (1231 rpm) is less than the shunt motor full-load speed (1603 rpm, in Ex. 4–16), as shown in Fig. 4–9.
2. The shunt motor develops slightly more horsepower than the compound motor because it is running at a much higher speed. [See Eq. (4–15).]
3. The compound motor torque (59.1 lb · ft) is greater than the shunt motor torque (46.85 lb · ft), as shown in Fig. 4–8. This increased torque is due to the additional flux produced by the series field.
4. It is precisely because of the additional series field flux that the speed of the compound motor drops compared to the shunt motor. [See Eq. (4–12).]

EXAMPLE 4–18

The armature circuit resistance of a 25 hp, 250 V series motor is 0.1 Ω, the brush volt drop is 3 V, and the resistance of the series field is 0.05 Ω. When the series motor takes 85 A, the speed is 600 rpm. Calculate

a. The speed when the current is 100 A
b. The speed when the current is 40 A
   Neglect armature reaction and assume that the machine is operating on the linear portion of its saturation curve at all times
c. Recompute speeds in parts (a) and (b), using a 0.05 Ω diverter at these speeds. (Hint: In computing the counter EMF, the armature resistance must be added to the parallel equivalent of the series field and the diverter resistance R_{sd}.)

Solution

a. $E_{c2} = V_a - I_a\ (R_a + R_s) - \text{BD}$
$= 250 - 100\,(0.15) - 3$
$= 232\ \text{V when } I_a = 100\ \text{A}$ (4–8)
$E_{c1} = 250 - 85\,(0.15) - 3$
$= 234.3$ V at a speed of 600 rpm when $I_a = 85$ A

$S = K\dfrac{E}{\phi}$, assuming ϕ is proportional to I_a (on the linear portion of saturation curve)

$$S_2 = S_1\left(\frac{E_2}{E_1}\right)\left(\frac{\phi_1}{\phi_2}\right) = 600\left(\frac{232}{234.3}\right)\left(\frac{85}{100}\right)$$
$= \mathbf{505\ rpm}$

b. $E_{c3} = V_a - I_a\,(R_a + R_s) - \text{BD}$
$= 250 - 40\,(0.15) - 3 = 241$ V at 40 A

$$S_3 = S_1\left(\frac{E_{c3}}{E_{c1}}\right)\left(\frac{\phi_1}{\phi_3}\right) = 600\left(\frac{241}{234.3}\right)\left(\frac{85}{40}\right)$$
$= \mathbf{1311\ rpm}$

c. The effect of the diverter is to reduce the *series field current* (and *flux*) to *half* their previous values.

$E_{c2} = V_a - I_a\,(R_a + R_{sd}) - \text{BD}$
$= 250 - 100\,(0.125) - 3$
$= 234.5$ V at 100 A
$E_{c3} = V_a - I_a\,(R_a + R_{sd}) - \text{BD}$
$= 250 - 40\,(0.125) - 3$
$= 242$ V at 40 A

$$S_2 = S_1\left(\frac{E_{c2}}{E_{c1}}\right)\left(\frac{\phi_1}{\phi_2}\right)$$
$$= 600\left(\frac{234.5}{234.3}\right)\left(\frac{85\ \text{A}}{\frac{100}{2}\text{A}}\right)$$
$= \mathbf{1021\ rpm}$

$$S_3 = S_1\left(\frac{E_{c3}}{E_{c1}}\right)\left(\frac{\phi_1}{\phi_3}\right)$$
$$= 600\left(\frac{242}{234.3}\right)\left(\frac{85\ \text{A}}{\frac{40}{2}\ \text{A}}\right) = \mathbf{2634\ rpm}$$

The results may be tabulated as follows:

	I_a in A	S_o in rpm	S_d in rpm
1.	85	600	—
2.	100	505	1021
3.	40	1311	2634

Example 4–18 verifies the following points regarding the series motor, without and with the use of a diverter, as shown by the tabulation of calculated results:

1. Without the diverter, as the armature current decreases, the speed S_o rises rapidly, typical of the series motor characteristic shown in Fig. 4–9.
2. The 50 percent reduction in series field current, the effect of diverting current away

from the series field, has resulted in a sharp rise in speed (approximately 200 percent of the original values).

3. In the event of a decrease in mechanical load on the armature, with the series field diverted, there is great danger of excessively high speeds and instability at low armature currents.

4–11 SPEED REGULATION

The speed regulation of a motor is defined as *the change in speed from rated to zero load, expressed in percent of rated load speed.*[4] In equation form, the speed regulation (SR) becomes

$$\text{SR (percent speed regulation)} = \frac{S_{n1} - S_{f1}}{S_{f1}} \times 100 = \frac{\omega_{n1} - \omega_{f1}}{\omega_{f1}} \times 100 \quad \textbf{(4–14)}$$

From an examination of the curves of Fig. 4–10b, it is evident that shunt motors may be classified as motors of fairly constant speed, whose speed regulation is good (a small percentage). The speed regulation of the cumulative compound motor is poorer than that of the shunt motor, and its speed regulation is a higher percentage. The series motor speed regulation is extremely poor (since it has an infinite no-load speed). Both the cumulative and series motors are considered *variable-speed* motors (Section 13–8). The differential compound motor has a negative speed regulation, which may always be associated with load instability.

EXAMPLE 4–19

Compute the percent speed regulation (SR) for the motors of

a. Ex. 4–15

b. Ex. 4–17

c. Ex. 4–18 (assume the 40 A current as no load and the 100 A current as full load, without use of a diverter).

Solution

a. $\text{SR (shunt)} = \frac{S_{n1} - S_{f1}}{S_{f1}} \times 100 = \frac{1810 - 1603}{1603} \times 100 = \textbf{12.9 percent} \quad (4\text{–}14)$

b. $\text{SR (compound)} = \frac{1806 - 1231}{1231} \times 100 = \textbf{46.7 percent}$

c. $\text{SR (series)} = \frac{1311 - 505}{505} \times 100 = \textbf{159.6 percent}$

EXAMPLE 4–20

The percent speed regulation (SR) of a shunt motor is given as 10 percent. If the full-load speed is 60π rad/s, calculate

a. The no-load speed in rad/s

b. The no-load speed in rpm.

Solution

a. $\omega_{n1} = \omega_{f1} + (\omega_{f1} + \text{SR}) \quad (4\text{–}14)$

$= \omega_{f1}(1 + \text{SR})$

$= 60\pi\,(1 + 0.1) = \mathbf{66\pi\ rad/s}$

b. $S = 66\pi \text{ rad/s} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} = \textbf{1980 rpm}$

Example 4–19 verifies the speed regulations of the shunt, compound, and series motors, respectively. Of the three motors, the shunt motor has the best speed regulation, and the series motor has the worst speed regulation. Example 4–20 shows

[4] ASA Standard C50, *Rotating Electrical Machinery*. Note the similarity between this definition and the definition of voltage regulation, Eq. (3–9).

that the speed regulation may be calculated in either English or SI units and also shows the relatively simple conversion from SI to English units, using the conversion factor of $30/\pi$ (see Appendix A–1.3W).

4–12 EXTERNAL TORQUE, RATED HORSEPOWER, AND SPEED

Depending on the size and/or the particular application of the motor, the output may be specified on the nameplate either as *power* or *torque*, along with the *speed*. Given any two of these terms, we may calculate the third since they are related to one another. Generally, the output power is expressed in terms of horsepower (hp), where 746 W equals 1 hp.

Appendix A–2 derives the relations among horsepower, torque, and speed in both English and SI units, respectively, as

$$\text{(English)} \qquad \text{hp} = \frac{TS}{5252}\ \text{(hp)} \qquad \textbf{(4–15a)}$$

where T is the torque in pound-feet (lb · ft)
S is the speed in revolutions per minute (rpm)

$$\text{(SI)} \qquad \text{hp} = \frac{\omega T}{746}\ \text{(hp)} \qquad \textbf{(4–15b)}$$

where T is the torque in newton-meters (N · m)
ω is the speed in radians per second (rad/s)

In the case of extremely small motors, the *output power* is expressed in *watts*. Appendix A–2 also derives the relations among torque, power, and speed in both English and SI units, respectively, as

$$\text{(English)} \qquad T = \frac{7.04P}{S}\ \text{pound-feet (lb · ft)} \qquad \textbf{(4–16a)}$$

$$\text{(SI)} \qquad T_{SI} = \frac{P}{\omega}\ \text{newton-meters (N · m)} \qquad \textbf{(4–16b)}$$

where all terms have been previously defined and P is the power in watts.

Equation (4–16) permits computation of either the internal torque, if one

EXAMPLE 4–21
From the calculated values of rated speed and internal power computed in Ex. 4–16, calculate
a. The internal torque
b. The external torque from rated horsepower and speed given in Ex. 4–16
c. Account for the differences.

Solution

a. $T_{int} = \dfrac{\text{hp} \times 5252}{S} = \dfrac{14.3 \times 5252}{1603}$
$= \mathbf{46.85\ lb \cdot ft}$ (4–15a)

b. $T_{ext} = \dfrac{\text{hp} \times 5252}{S} = \dfrac{10 \times 5252}{1250}$
$= \mathbf{42.0\ lb \cdot ft}$ (4–15a)

c. Internal horsepower is developed as a result of electromagnetic torque produced by energy conversion. Some of the mechanical energy is used internally to overcome mechanical losses of the motor, reducing the torque available at its shaft to perform work.

knows the internal power developed (i.e., $E_c I_a$), or the external torque available at the pulley, if one knows the output power expressed either in horsepower or watts.

EXAMPLE 4–22	*Solution*
A 50 W servo motor runs at a full-load speed of 3000 rpm. Calculate the output torque available at the motor pulley in ounce-inches.	$T = \frac{7.04P}{S} = \frac{7.04 \times 50}{3000}$ (4–16a) $= 0.1173 \text{ lb} \cdot \text{ft} \, (192 \text{ oz} \cdot \text{in/lb} \cdot \text{ft})$ $= \mathbf{22.5 \text{ oz} \cdot \text{in}}$

EXAMPLE 4–23	*Solution*
For the servo motor data given and calculated in Ex. 4–22, calculate	
a. Motor speed in radians per second	**a.** $3000 \frac{\text{rev}}{\text{min}} \times 2\pi \frac{\text{rad}}{\text{rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \mathbf{314.2 \text{ rad/s}}$
b. Output torque in newton-meters	**b.** $T = \frac{P}{\omega} = \frac{50 \text{ W}}{314.2 \text{ rad/s}} = \mathbf{0.1592 \text{ N} \cdot \text{m}}$
c. From part (b), output torque in ounce-inches, using conversion factor given in Appendix A–1.3M	**c.** $T = 0.1592 \text{ N} \cdot \text{m} \times \frac{1 \text{ oz} \cdot \text{in}}{7.0612 \times 10^{-3} \text{ N} \cdot \text{m}}$ $= \mathbf{22.5 \text{ oz} \cdot \text{in}}$
d. Compare answer in part (c) with solution to Ex. 4–22.	**d.** Both answers are the **same.**

4–13 REVERSAL OF DIRECTION OF ROTATION OF A DC MOTOR

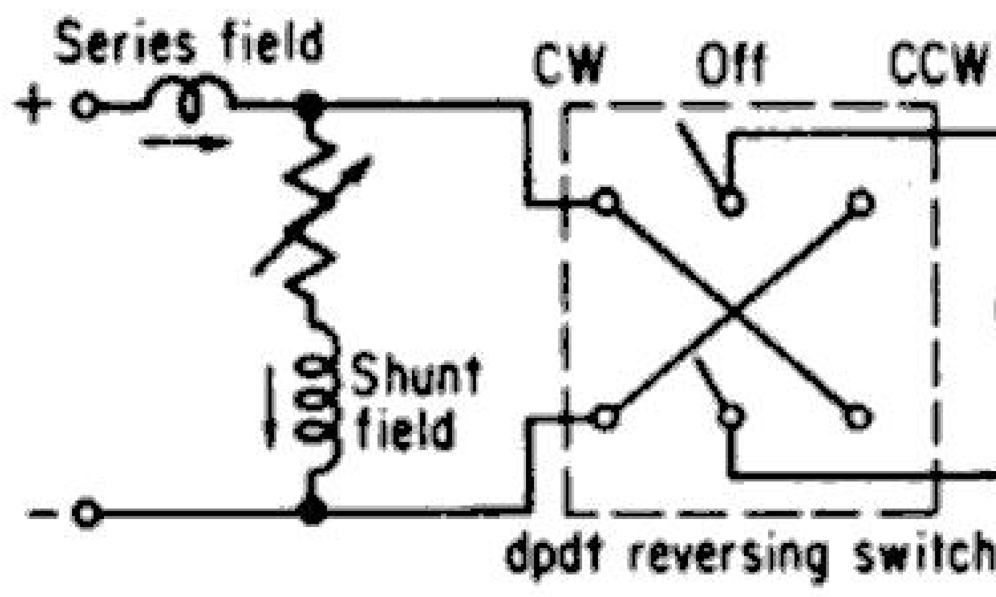
In order to reverse the direction of rotation of any dc motor, it is necessary to reverse the direction of current through the armature with respect to the current of the magnetic field circuit. For either the shunt or series motor, this is done simply by reversing either the armature circuit connections with respect to the field circuit or vice versa. Reversal of *both* circuit connections will produce the *same* direction of rotation.

4–13.1 Choice of Armature Circuit for Reversal of Rotation

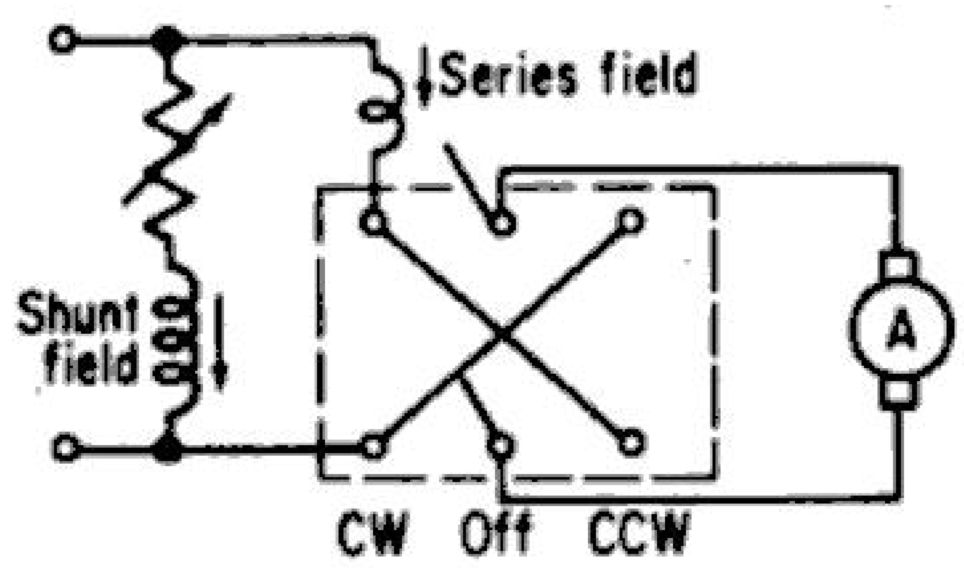
It might appear that since the field circuit carries less current than the armature circuit, the former would be the one selected for reversal. However, in designing automatic starters and control equipment, the *armature* circuit is usually selected for reversal for the following reasons:

1. The field is a highly inductive circuit (Fig. 2–7), and frequent reversals produce high induced EMFs and pitting of the switch contacts that serve to accomplish field circuit reversal.
2. If the shunt field is reversed, the series field must also be reversed; otherwise, a cumulative compound motor will be differentially connected.
3. The armature circuit connections are normally opened for purposes of dynamic, regenerative, or plugging braking; and since these connections are available, they may as well be used for reversal.
4. If the reversing switch is defective and field circuit fails to close, the motor may "run away."

In the case of the compound motor, therefore, reversing *only* the armature connections achieves a reversal of direction of rotation for either the long-shunt or



a. Short shunt connection



b. Long shunt connection

Figure 4–11 Reversal of direction of long- or short-shunt compound motors

short-shunt connections, as shown in **Fig. 4–11,** without changing the direction of the current in the fields.

For the above listed reasons, therefore, reversal of rotation dictates reversal of armature connections *only*, as shown in Figs. 4–11a and b. Care must be taken to ensure that the *entire* armature circuit must be reversed. If the compensating winding or the commutating field windings are separately reversed with respect to the armature winding, their effect is negated and severe sparking occurs at the brushes.

In the case of permanent-magnet (PM) motors, reversal is also accomplished by reversing the armature (or line) connections. In effect, the PM motor is the *only* motor that can be reversed by reversing line connections.

4–13.2 Four-Quadrant Operation of a DC Motor

In many industrial and commercial operations, it is necessary for a motor to operate from standstill to rated speed in one direction, then decelerate to standstill and accelerate to rated speed in the opposite direction. Such motor operation is usually accomplished by means of an electronic *controller* that provides the necessary polarity switching and disconnects, as well as any electrical braking for deceleration.

It was noted earlier that *generator* action and *motor* action occur simultaneously in all dc dynamos, regardless of whether operation is in the *motor mode* or *generator mode*. Since there are *two* modes of *operation* and only *two* possible *directions* of rotation, we must consider four possibilities of dc dynamo operation. This is done graphically by means of a *four-quadrant diagram*, shown in **Fig. 4–12a.**

When the dc dynamo is rotation clockwise (CW) and is operating as a separately excited dc motor, its operation is in the *first quadrant*, as shown in Figs. 4–12a and b. Under these conditions, the armature is drawing current from the supply and the line or applied voltage is greater than the generated counter EMF, E_g.

Let us assume that the motor used is driving a battery-operated electric car,

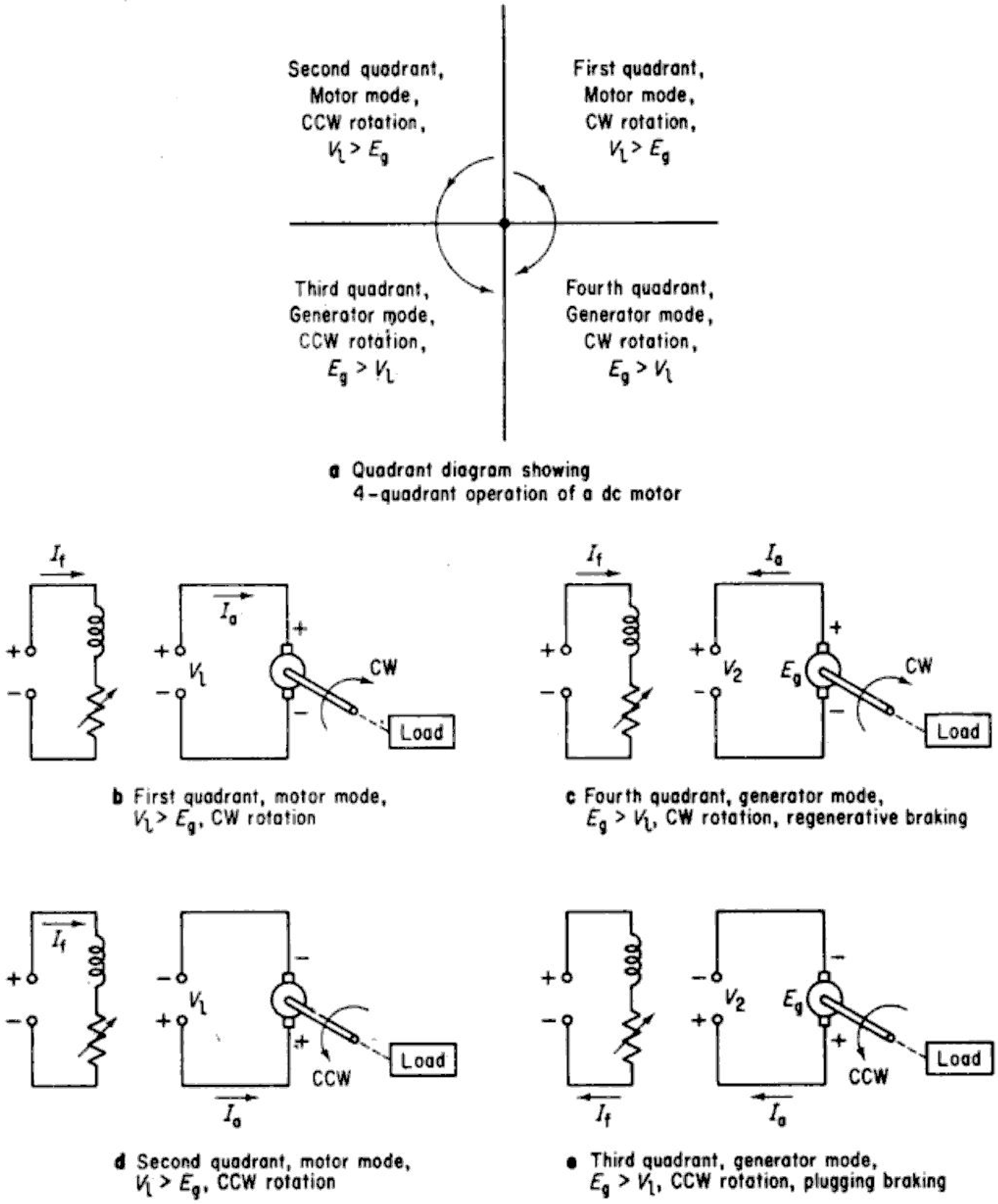


Figure 4–12 Four-quadrant operation of a separately excited dc motor

which has been accelerated to full speed in a forward (CW) direction, up a steep hill. As the car descends downhill, it tends to accelerate and the wheels (normally driven by the dc motor) are now going faster than the driving torque provided by the dc motor. Under these conditions, the system evidences *fourth-quadrant* operation; i.e., the generated EMF E_g exceeds the line voltage and current is being "pumped back" to the battery (tending to recharge it) from the motor armature. In effect, the armature is generating power to the battery, which serves as a load. The generator prime mover is now the car, whose kinetic energy is driving the wheels as it accelerates downhill. Since the battery is loading the generator armature, it tends to slow the armature, thus "braking" the armature by an action known as *regenerative braking.*[5] Regenerative braking is insufficient to bring the motor to a stop, but it has the advantage of using the car's kinetic energy to charge the batteries.

In order to bring the car to a complete standstill, we employ plugging braking, which consists of *reversing* the battery *supply* terminals to the armature, as shown in Fig. 4–12d. Under these conditions, the motor will reverse direction and operate in a CCW direction. Such motor operation is shown in Fig. 4–12a as *second-quadrant* operation.

Once again, whenever the armature is being driven at a speed faster than its driving motor torque can provide, regeneration occurs, as shown in the *third quadrant* of Fig. 4–12a and in Fig. 4–12e.

Four-quadrant diagrams are useful in representing the torque–speed characteristics of motors undergoing braking in both directions of rotation.

4–14 EFFECT OF ARMATURE REACTION ON SPEED REGULATION OF ALL DC MOTORS

Armature reaction (as defined in Section 2–7 and in Chapter 5) is the effect of the MMF produced by the armature conductors (I_aN_a) in reducing and distorting the mutual air-gap flux ϕ_m produced by the field winding (series and shunt fields). The fundamental speed equation, Eq. (4–6), indicates that a reduction of the field flux in the denominator of this equation will cause an increase in speed.

It will be shown in the next chapter that the extent and effect of armature reaction varies directly with the load or with the armature current I_a. As *any* dc motor (regardless of type) is loaded, the effect of armature reaction is to reduce the air-gap flux and (depending on the degree of saturation) tend to *increase the motor speed.*

An examination of the speed–load curves shown in Fig. 4–9 indicates that the speed regulation of *each* of the commercial motor types (shunt, series, and cumulative compound) would be *improved,* somewhat, by this effect (if not too pronounced so as to cause negative speed regulation). In the case of the shunt motor, for example, since armature reaction increases with load, the decrease in flux and increase in speed with

[5] The term implies regeneration of energy back to the supply. There are three forms of *electric* braking: *regenerative* braking, *plugging* braking, and *dynamic* braking. In regenerative braking, the energy produced by the armature is pumped back to an electric supply. Plugging braking consists of applying voltage to the motor of such polarity that it attempts to reverse. In the process of attempting to reverse, it must decelerate and pass through a standstill condition, before reversing in the opposite direction.

The last form, dynamic braking, consists of disconnecting the dc motor armature from the supply and connecting its armature terminals across a suitable resistor. This electrical load on the armature tends to slow down the armature, which is dissipating energy in the load resistor.

For a detailed description of these forms of electrical braking as well as various types of controllers to accomplish such braking, cf., Kosow, *Control of Electric Machines,* (Englewood Cliffs, N.J.: Prentice-Hall, 1973), Ch. 6.

load may increase the load to such an extent that its characteristic may tend to be the same as that of the differential compound motor, shown in Fig. 4–9. A shunt motor operating with a weak field and without some means of compensating for armature reaction (as discussed in Chapter 5) is particularly susceptible to load instability and runaway.

4–15 GLOSSARY OF TERMS USED

Active conductors Those conductors on the armature surface contributing either to the motor torque or the generated EMF of a dynamo.

Armature reaction The magnetomotive force (MMF) due to armature-winding current.

Biot–Savart law A law expressing the force developed on a current-carrying conductor in a given magnetic field, carrying a given current.

Brush volt drop (BD) The voltage drop produced across the brushes of a dynamo that is relatively constant over a wide range of load.

Counter EMF The EMF generated in a motor armature that opposes the voltage applied to the armature.

Cumulative compound motor A motor whose excitation is produced by a shunt field winding and a series field winding, both of which aid in producing flux in the same direction.

Developed torque (See *internal torque*.)

Diverter A resistance shunting the series field of a compound motor to adjust the flux produced by the series field to produce a desired speed regulation of the motor.

Dynamic braking A control function that brakes a drive (or motor) by dissipating its stored energy in a resistor.

External torque (See *output torque*.)

Four-quadrant diagram A graphical representation of dynamo operation as either a motor or a generator during either clockwise or counterclockwise rotation.

Horsepower(hp) A measure of the rate of doing work. (Note: 1 hp = 746 W = 33 000 ft·lb/min = 550 ft·lb/s = 746 N·m/s = 746 J/s.)

Internal torque The torque developed by all active armature conductors as a function of armature current and its interaction with field flux.

Left-hand rule Using conventional current direction, a motor rule that determines the direction of force on a conductor, given the direction of the armature current and the direction of the field flux.

Lenz's law The polarity and direction of an induced EMF (or counter EMF) is always in such a direction as to oppose the developed force that produced rotation. (As applied to motors, this is the armature current.)

Motor starter An electric controller, either manual or automatic, for accelerating a motor from rest to normal speed and for stopping the motor.

Output torque The torque measured at the pulley of a motor.

Permanent-magnet (PM) motor A motor whose field flux is furnished by one or more permanent magnets.

Power The rate of doing work or the time rate of transforming or transferring energy.

Plugging A control function that provides braking by reversing motor line voltage polarity so that the motor develops a countertorque and exerts a retarding force.

Regenerative braking A form of dynamic braking in which the kinetic energy of the motor and its driven machinery is returned to the power supply system.

Series motor A motor whose excitation is produced by a series field connected electrically in series with the armature and whose excitation is a direct function of the motor armature current.

Shunt motor A motor whose excitation is produced by a field connected in parallel with (i.e., shunting) the armature.

Speed regulation The change in speed from no load to full load expressed as a precentage of speed at rated load.

Torque A force tending to produce rotation or a turning moment.

4–16 QUESTIONS

4–1 Using the equation $I_a = (V_a - E_c)/R_a$, explain

a. Why it is impossible for E_c to equal V_a.

b. What proportion of V_a is normally represented by E_c and I_aR_a, respectively, at full load.

4–2 **a.** What is the relation between electromagnetic force and electromagetic torque?

b. What is the relation between torque and work?

4–3 **a.** Distinguish between torque and speed of a motor.

b. What two factors determine motor torque?

c. Distinguish between developed torque and torque available at the pulley. Which is greater and why?

4–4 **a.** Explain why a small change in motor speed and counter EMF will produce correspondingly larger changes in armature current.

b. If the speed is increased, what effect does this have on

1. The counter EMF? (Why?)
2. The armature current?

c. Why is armature current often employed as an indication of motor load and speed?

4–5 Using Eq. (4–6), explain the effect on speed of a shunt motor when

a. Armature current is increased.

b. Counter EMF is decreased.

c. Field flux (field current) is increased.

4–6 Using Fig. 4–7, explain why all dc motors are started

a. With maximum resistance in series with armature.

b. With maximum field excitation.

4–7 Explain why the series motor must be started with a mechanical load coupled to its armature.

4–8 Compare the family of curves shown in Figs. 4–10a and b with those of Figs. 4–8 and 4–9, and

a. Explain the advantages of the former over the latter.

b. Show where starting torque and starting speed should appear on these curves.

4–9 Define

a. Starting torque.

b. Full-load torque.

c. No-load torque.

d. Speed regulation.

e. Internal torque.

f. External torque.

4–10 Give four reasons why the armature connections are selected for reversal of motor direction rather than the field connections of dc motors.

4–11 Given a motor running normally in a CW direction at rated speed, determine the quadrant of operation when it operates

a. As a motor in a CCW direction.

b. As a generator in a CCW direction.
c. As a generator in CW direction.

4–12 What is the effect of armature reaction on the speed regulation of all dc motors?

4–17 PROBLEMS

4–1 The conductors of a dynamo armature have an axial length of 12 inches. When each conductor is carrying a current of 80 A, the field flux density is adjusted to 61 000 lines/in^2. Calculate

a. The force developed by each current-carrying conductor in pounds.
b. The total force developed, given a total of 60 active conductors on the armature, in pounds.
c. The total torque developed, if the armature diameter is 18 inches, in lb · ft.
d. The total torque in newton-meters. (Use 1.356 N·m/lb·ft.)

4–2 Convert the given data of Problem 4–1 to SI units and calculate

a. The force per conductor in newtons.
b. The total force developed on the armature in newtons.
c. The total torque developed in newton-meters.
d. Compare your answer to that found in Problem 4–1d.

4–3 A dc motor armature has 48 slots and a two-layer full-coil simplex lap winding (1 coil/slot) in which each coil has 42 turns. The four field poles span 78 percent of the armature circumference and produce a uniform flux density of 56 000 lines/in^2. The armature core has a diameter of 14 inches and an axial length of 16 inches, but the slots are skewed at an angle of 20° with respect to the shaft. The current per conductor is 20 A. Calculate

a. The number of active conductors.
b. The active length of each conductor.
c. The total electromagnetic force developed by the armature conductors.
d. The torque tending to produce rotation.

4–4 Calculate the torque in Problem 4–3 assuming the flux density is increased by 10 percent and the current is reduced by 20 percent.

4–5 A shunt motor develops a total torque of 250 N · m at rated load. When it is subjected to a 15 percent decrease in field flux, the armature current increases by 40 percent. Calculate the new torque produced as a result of the change in field flux.

4–6 A 220 V dc shunt motor has a 5 V brush drop, an armature resistance of 0.2 Ω, and a rated armature current of 40 A. Calculate

a. The voltage generated in the armature under these conditions of load applied to the armature shaft.
b. Power developed by the armature in watts.
c. Mechanical power developed by the armature in horsepower.

4–7 A 220 V shunt motor develops a torque of 54 N · m at an armature current of 10 A. Find the torque when

a. The armature current is 15 A.
b. The armature current is 20 A.
c. The armature current is 5 A.

4–8 A 120 V shunt motor develops a torque of 75 N · m when its armature current is 30 A. Find the armature current to produce

a. A developed torque of 30 N · m.
b. A developed torque of 60 N · m.
c. A developed torque of 80 N · m.

4–9 A shunt motor running at rated load develops a torque of 50 lb · ft. If the armature current is increased 25 percent and the field flux is reduced 10 percent, calculate

a. The developed torque in lb · ft.
b. The developed torque in N · m.

4–10 A cumulative compound motor is operated as a shunt motor with its series field disconnected, has an armature current of 100 A, and at a shunt field flux of 90 mWb develops a torque of 75 N · m. When the series field is connected, at the same armature current, the motor develops a torque of 90 N · m. Calculate the *increase* is flux produced by the series field.

4–11 A 240 V shunt motor running at 1800 rpm develops a counter EMF of 232 V. Its armature resistance is 0.1 Ω, and the brush volt drop is 3 V. Calculate

a. The armature current at the speed of 1800 rpm.
b. The speed when the armature current is 75 A.
c. The speed when the armature current is 30 A.

4–12 A 10 hp, 1800 rpm shunt motor develops 11 hp internally at a rated armature voltage of 120 V and an armature current of 74 A. Calculate

a. The developed torque at rated speed in lb · ft and in N · m.
b. The developed torque, when the armature current is 82 A, in lb · ft.
c. The speed under the conditions of part (b) if the armature resistance is 0.12 Ω and the brush drop is 5 V, assuming constant field flux.

4–13 A servo motor has an output power of 20 W and rotates at a rated speed of 200 rad/s. Calculate

a. The output torque developed at the pulley in N · m.
b. The output torque developed at the pulley in lb · ft.
c. The output torque developed at the pulley in oz · in.

4–14 A 240 V shunt motor having a field resistance of 120 Ω, an armature resistance of 0.2 Ω, a brush volt drop of 3 V, and a rated armature current of 36 A rotates at a speed of 60π rad/s. If the armature current drawn from the supply drops to 25 A, calculate

a. The motor speed in rad/s.
b. The motor speed in rpm.

4–15 From the given motor data in Problem 4–14, and from Table 430–147 of Appendix A-3, determine

a. Motor horsepower output at rated load.
b. Output torque at rated load in SI units.
c. Developed torque at rated load in SI units.
d. Motor efficiency at rated load.
e. Developed torque, when the armature current is 25 A, in SI units.
f. Motor speed, when the armature current is 25 A, in SI units.
g. Motor efficiency when the armature current is 25 A.

4–16 A 120 V shunt motor has a 60 Ω field resistance, an armature resistance of 0.05 Ω, a brush volt drop of 2 V, and a rated line current of 58 A at a speed of 200 rad/s. Calculate

a. Field and armature currents at rated load.
b. Counter EMF at rated load.
c. Developed power in kW and in hp.
d. Developed torque in N · m and in lb · ft.
e. Motor efficiency at rated load (using Table 430–147 of Appendix A–3).

4–17 A 50 hp, 240 V shunt motor has a brush volt drop of 5 V and an armature resistance of 0.05 Ω. The field circuit resistance is 1 Ω. At *no load*, the motor draws 12 A and has a speed of 1300 rpm. Calculate

a. Rated motor speed and rated armature current (using Appendix A–3, Table 430–147 rated line current accordingly).
b. Speed regulation.
c. Mechanical power developed by the armature at rated load.
d. Compare computed full-load developed hp with rated hp (50 hp) and account for differences.

4–18 Using the developed hp computed at full load in Problem 4–17, calculate

a. The developed torque in lb · ft.
b. The no-load hp and torque using data from Problem 4–17.
c. Tabulate the following parameter for full- and no-load conditions: speed, hp, and torque developed by the armature. Account for differences between these parameters.

4–19 A 10 hp, 240 V *series* motor has a line current of 38 A and a rated speed of 600 rpm. The armature circuit and series field resistances, respectively, are 0.4 Ω and 0.2 Ω. The brush volt drop is 5 V. Assume that the motor is operating on the linear portion of its saturation curve below rated armature current. Calculate

a. Speed when the load current drops to 20 A at half rated load.
b. The no-load speed when the line current is 1 A.
c. The speed at 150 percent rated load when the line current is 60 A and the series field flux is 125 percent of the full-load flux due to saturation.