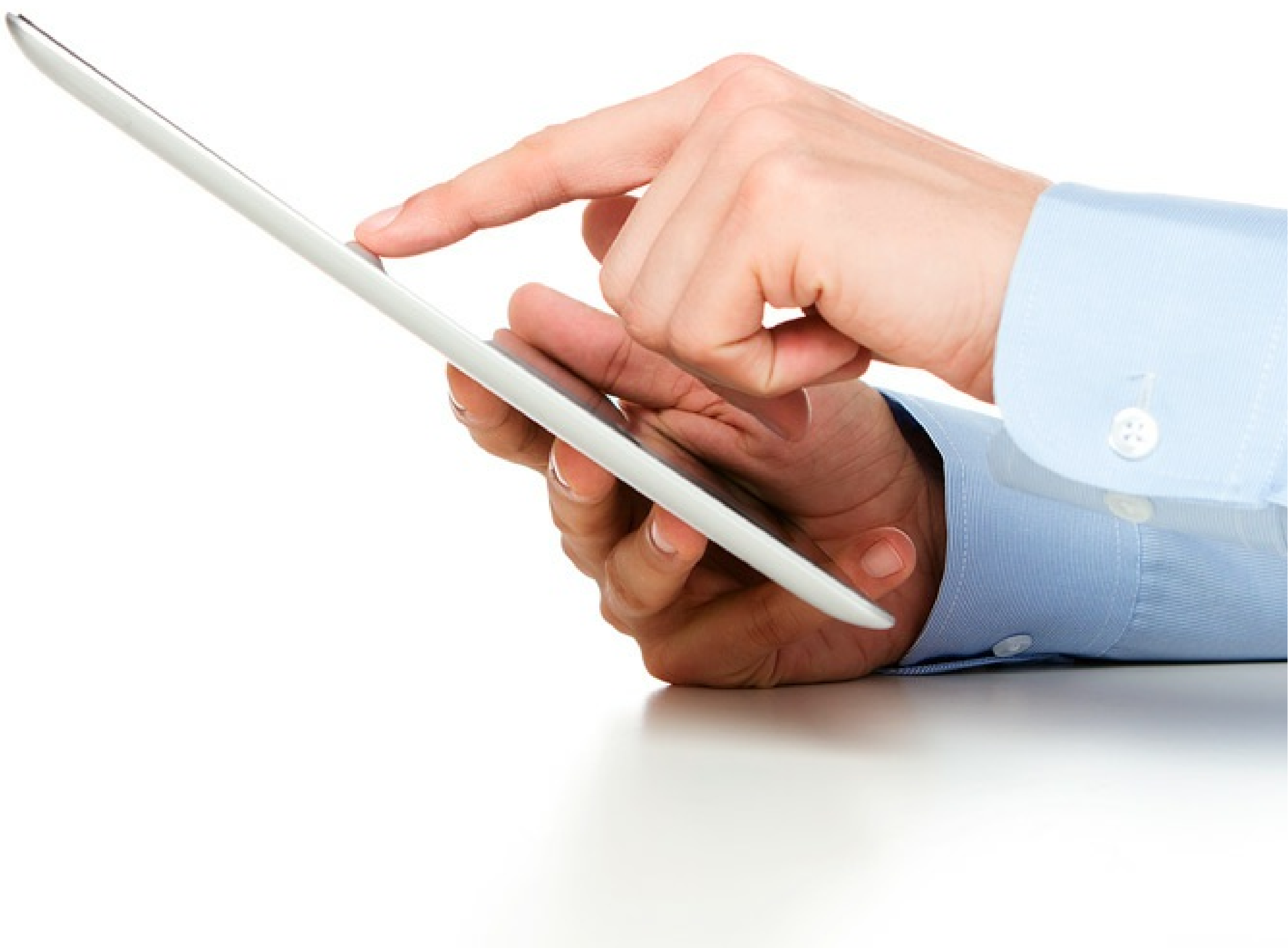
Learn Calculus 2 on Your Mobile Device

Live-streamed YouTube Classes with Dr Chris Tisdell

Christopher C. Tisdell



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CHRISTOPHER C. TISDELL

LEARN CALCULUS 2 ON YOUR MOBILE DEVICE

LIVE-STREAMED YOUTUBE CLASSES WITH DR CHRIS TISDELL

Learn Calculus 2 on Your Mobile Device: Live-streamed YouTube Classes with Dr Chris Tisdell
1st edition

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Thanks for Reading my Book

Thanks for choosing to download this book. You're now part of a learning community of over 10 million readers around the world who engage with my books.

I really hope that you will find this book to be useful. I'm always keen to get feedback from you about how to improve my books and your learning process.

Please feel free to get in touch via the following platforms:

YouTube `http://www.youtube.com/DrChrisTisdell`
Facebook `http://www.facebook.com/DrChrisTisdell.Edu`
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What Makes This Book Different?

Thousands of books have been written about calculus. I recently performed a book search on Amazon.com that returned over 57,000 results for the term "calculus". Do we really need another calculus textbook?

So, what makes this book different? The way I see it, some points of distinction between this book and others include:

- Open learning design
- Multimodal learning format
- Live–streaming presentations
- Active learning spaces
- Optimization for mobile devices.

Open learning design

My tagline of "everyone deserves access to learning on a level playing field" is grounded in the belief that open access to education is a public right and a public good. The design of this book follows these beliefs in the sense that the book is absolutely free; and does not require any special software to function. The book can be printed out or used purely in electronic form.

Multimodal learning format

Traditional textbooks feature, well, text. In recent years, graphics have played a more common role within textbooks, especially with the move away from black and white texts to full colour. However, the traditional textbook is still percieved as being static and unimodal in the sense that you can read the text. While some texts have attempted to use video as an "added extra", the current textbook aims to fully integrate online video into the learning experience. When the rich and expressive format of video is integrated with simple text it leads to what I call a multimodal learning experience (sight, sound, movement etc), going way beyond what a traditional textbook can offer.

Live–streaming presentations

The video tutorials that are integrated into this textbook are all "live–streamed". This means that the presentations go out live, with no editing or postproduction. They have a distinctly low budget feel. It's my view that the live element makes the presentations feel more engaging, dynamic and real. The use of live–streamed video is one of the aspects that makes this book unique.

Active Learning Spaces

It's far too easy to sit back and just passively "watch" – whether it's a lecture, a tutorial, a TV show or an online video. However, learning is not a spectator sport. I believe in the power of active learning: that is, in learners doing things and thinking about what they are doing and what they have done.

To encourage active learning, this book features blank spaces where learners are required to actively engage by taking notes, making annotations, drawing diagrams and the like. I call these blank spaces "active learning spaces".

Optimization for mobile devices

The final dimension of this book that makes it unique is in its optimization for mobile devices. By this, I mean that all of the associated online videos have been designed with small screens in mind. The aim is to enable learning anywhere, anytime on smart phones, tablets and laptops.

How to Use This Workbook

This workbook is designed to be used in conjunction with my free online video tutorials. Inside this workbook each chapter is divided into learning modules (subsections), each having its own dedicated video tutorial.

View the online video via the hyperlink located at the top of the page of each learning module, with workbook and paper or tablet at the ready. Or click on the *Learn Calculus 2 on Your Mobile Device* playlist where all the videos for the workbook are located in chronological order:

Learn Calculus 2 on Your Mobile Device.
https://www.youtube.com/watch?v=KLL6jd5AfI8&list=PLGCj8f6sgswnaZq6z5W7DnsLV8YRktq3_.

While watching each video, fill in the spaces provided after each example in the workbook and annotate to the associated text.

You can also access the above via my YouTube channel

Dr Chris Tisdell's YouTube Channel
http://www.youtube.com/DrChrisTisdell

Please feel free to look around my YouTube channel, where you'll find educational and fun videos about mathematics. Enjoy!

Acknowledgement

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Chapter 1

Functions of Two Variables

One of the aims of mathematics is to act as a scientific framework from which we can model and understand our world. In our quest to better–understand more complicated phenomena, we require more sophisticated mathematics that is up to the task.

In this section we look at functions that depend on two variables. In doing so, we extend our capability of basic modeling through functions such as $y = f(x)$ where there is one (dependent) variable, to the case of $z = f(x, y)$, where now there are two independent variables x and y.

In particular, we will learn how to extend and apply basic calculus in the wider setting of functions of two variables. In a nutshell, calculus is concerned with rates of change, and the ideas form an important part of applied mathematics. In this chapter we will look at problems concerning: partial derivatives; second–order partial derivatives (there are four!); chain rule(s); error estimation; and some geometrical concepts, such as normal vector and tangent plane to a surface.

The ideas herein generalise to the case when a function has more than two variables in a standard way.

1.1 Partial Derivatives

View this lesson on YouTube [1]

Example.

Let

$$z = e^{x^2 y}$$

Calculate $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

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1.2 Second Order Partial Derivatives

View this lesson on YouTube [2]

Example.

Let

$$z = \cos(x^2 y)$$

Calculate $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial^2 z}{\partial y \partial x}$.

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1.3 Chain Rule for Partial Derivatives

View this lesson on YouTube [3]

Example.

Let f be a differentiable function and consider

$$F(x, y) := f(2x + y^2).$$

Show that F satisfies the partial differential equation

$$y\frac{\partial F}{\partial x} - \frac{\partial F}{\partial y} = 0.$$

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1.4 Error Estimation

View this lesson on YouTube [4]

Example.

We measure the dimensions of a cylinder with each measurement having an error of 1%. Obtain an estimate on the maximum percentage error in the volume $V = \pi r^2 h$.

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1.5 Normal Vector and Tangent Plane

View this lesson on YouTube [5]

Example.

Let $A(2, -1, 6)$ and consider the surface associated with

$$z = x^2 + 2y^2.$$

Determine a normal vector and the equation of the tangent plane to our surface at A.

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Chapter 2

Techniques of Integration

If differentiation is the "yin" of calculus, then integration is the "yang". In fact, the two are reverse processes and one cannot really get a good understanding of calculus without mastery of both parts and comprehension of the connection between them.

In this section, we explore various techniques that are used in integration processes. While the different techniques may seem rather random in nature at times, the common principle throughout is to turn a complicated integral into something that is simpler and more managable. Possible techniques involve: using a substitution; applying trigonometric formulae; employing a reduction formula; completing the square in the denominator; or exerting the method of partial fractions. We shall meet all of these ideas in this chapter.

The examples in this chapter will assume the reader can recall basic identities, such as

$$\begin{gathered}(a+b)^2 = a^2 + 2ab + b^2 \\ \sin^2 x + \cos^2 x = 1 \\ \tan^2 x + 1 = \sec^2 x \\ \cos 2x = 2\cos^2 x - 1.\end{gathered}$$

The approach is reasonably theoretical in this chapter, but we'll see some nice applications of integration in the final chapter of this book.

2.1 Integration by Substitution

View this lesson on YouTube [6]

Example.

Use a trigonometric substitution to calculate

$$I := \int_0^3 \sqrt{9 - x^2}\, dx.$$

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2.2 Integrals of Trigonometric Powers

View this lesson on YouTube [7]

Example.

Determine

$$I := \int_{\pi/2}^{\pi} \sin^3\theta \cos^2\theta \; d\theta.$$

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2.3 Integral of an Odd Powers of Cosine

View this lesson on YouTube [8]

Example.

Determine

$$I := \int \cos^5 \theta \; d\theta.$$

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2.4 Reduction Formula for Integrals

View this lesson on YouTube [9]

Example.

Let

$$I_n := \int_0^{\pi/4} \tan^n x \, dx.$$

Construct the reduction formula

$$I_n = \frac{1}{n-1} - I_{n-2}, \quad n \geq 2.$$

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2.5 Integration With Irreducible Denominators 1

View this lesson on YouTube [10]

Example.

Calculate

$$I := \int \frac{1}{x^2 + 4x + 13}\, dx.$$

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2.6 Integration With Irreducible Denominators 2

View this lesson on YouTube [11]

Example.

Calculate

$$I := \int \frac{x}{x^2 + 6x + 10}\, dx.$$

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2.7 Integration by Partial Fractions

View this lesson on YouTube [12]

Example.

Calculate

$$I := \int \frac{5x-4}{(x+1)(x-2)^2}\,dx.$$

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Chapter 3

First Order Ordinary Differential Equations

An "ordinary differential equation" (ODE) involves at least two things:

1. the derivative(s) of a function of one variable;
2. an equals sign.

A general (first order) form of an ODE is

$$\frac{dy}{dx} = f(x, y)$$

where f is a known function of two variables and $y = y(x)$ is the unknown function.

The motivation for the study of differential equations lies in their use in applications. By solving differential equations we can gain a deeper understanding of the physical processes that the equations are describing.

Like most equations arising in mathematics and its applications, we want to “solve” these kinds of equations. Thus, we try to find a *function* $y = y(x)$ that satisfies the differential equation for all values of x in some interval I. Rather than taking an arbitrary guess at what the solution might be, we will build up a collection of solution methods, basing our choice of method on the form of the differential equation under consideration.

The use of differential equations may empower us to make precise predictions about the future behaviour of our models. Even if we can’t completely solve a differential equation, we may still be able to determine useful properties about its solution (so–called qualitative information).

In this chapter we discuss some basic first order differential equations that can be explicitly solved.

3.1 Separable Equations 1

View this lesson on YouTube [13]

Example.

Solve the problem

$$\frac{dy}{dx} = 2x\sqrt{y}, \quad y(0) = 1.$$

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3.2 Separable Equations 2

View this lesson on YouTube [14]

Example.

Solve the problem

$$\frac{dy}{dx} = e^{x-y}, \quad y(0) = \ln 2.$$

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3.3 Linear First Order Equations 1

View this lesson on YouTube [15]

Example.

Solve the problem

$$\frac{dy}{dx} - y = e^{3x}, \quad y(0) = 0.$$

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3.4 Linear First Order Equations 2

View this lesson on YouTube [16]

Example.

Solve

$$\frac{dy}{dx} - \frac{2}{x+1}y = 3, \quad y(0) = 2.$$

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3.5 Exact First Order Equations

View this lesson on YouTube [17]

Example.

Solve the problem

$$(2xy + 1) + (x^2 + 3y^2)\frac{dy}{dx} = 0.$$

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Chapter 4

Second Order Ordinary Differential Equations

Of all the different types of differential equations, the form

$$ay'' + by' + cy = 0 \tag{4.0.1}$$

is perhaps the most important due to its simplicity and ability to model a wide range of phenomena. Above: a, b and c are given constants.

We will concentrate our analysis on solving a quadratic equation that is related to (4.0.1). This special polynomial equation is called the "characteristic equation" (or auxiliary equation) of (4.0.1) and is

$$a\lambda^2 + b\lambda + c = 0. \tag{4.0.2}$$

We normally go straight to the characteristic equation (4.0.2), solve it and simply write down the "general solution", which is formed by taking all linear combinations of any two (linearly independent) solutions to (4.0.1).

In this chapter, we'll see several examples involving the above process. We'll also see a connection between solutions to (4.0.1) and solutions to a more general problem when the right hand side of (4.0.1) has "0" replaced by a known function of x.

By the way, (4.0.2) arises from the assumption that solutions y to (4.0.1) "don't change much when differentiated", as the derivatives in the left hand side of (4.0.1) need to add up to zero. Such an assumed form is something like $y = Ae^{\lambda x}$ where A is a constant and λ is to be determined.

4.1 Real and Unequal Roots

View this lesson on YouTube [18]

Example.

Solve the problem

$$y'' + 3y' - 10y = 0.$$

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4.2 Real and Equal Roots

View this lesson on YouTube [19]

Example.

Solve the problem

$$y'' - 8y' + 16y = 0.$$

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4.3 Complex Roots

View this lesson on YouTube [20]

Example.

Solve the problem

$$y'' + 2y' + 17y = 0.$$

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4.4 Inhomogenous Problem

View this lesson on YouTube [21]

Example.

Solve

$$y'' - y = 2x + 1.$$

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Chapter 5

Sequences and Series of Constants

The first part of this chapter explores sequences. Sequences are like functions, where the domain is restricted to whole numbers.

Sequences occur in nature all around us and a good understanding enables accurate modelling of many "discrete" phenomena. For example, you might have heard of a Fibonacci sequence which is seen in describing population models, such as in the breeding of rabbits; and the reproduction of honey bees.

Sequences are also a very useful tool in approximating solutions to complicated equations. For example, you may have come across the Newton–Raphson method for approximating the solutions of equations. The method employs a basic sequence where a solution to the problem is obtained via a limiting process.

Sequences are also one of the basic building blocks in the fascinating area of "mathematical analysis".

In this section we will see how we can apply various methods from calculus to calculate the limit of a sequence. That is, if a_n is a sequence of numbers (with domain, say, $n = 1, 2, 3, \ldots$) then what is

$$\lim_{n\to\infty} a_n?$$

We will use basic identities, such as

$$e^{\ln x} = x$$

and apply the squeeze theorem (also called the sandwich theorem and the pinching theorem).

In the second part of this chapter, we investigate infinite series. Infinite series are a fundamental pillar of integration and integral calculus.

An infinite series is the sum of an infinite sequence of numbers:

$$a_1 + a_2 + a_3 + \cdots = \sum_{i=1}^{\infty} a_i$$

How to add together infinitely many numbers is not so clear.

Infinite series sometimes have a finite sum. For example, consider

$$1/2 + 1/4 + 1/8 + \cdots = 1$$

which may be verified by adding up the areas of the repeatedly halved unit square.

Other series do not have a finite value. Consider

$$1 + 2 + 3 + 4 + \cdots$$

It is not obvious whether the following infinite series has a finite value or not

$$1/2 + 1/3 + 1/4 + \cdots$$

We will explore different approaches to answer the question, does a given series "converge" in the sense that the following limit exists (and is finite)

$$\lim_{N\to\infty} (a_1 + a_2 + a_3 + \cdots + a_N) = \sum_{i=1}^{\infty} a_i.$$

The tools that we shall consider include: telescoping sums; the integral test; the comparison test; the ratio test; and the alternating series test.

5.1 Basic Limits of Sequences

View this lesson on YouTube [22]

Example.

Compute

$$\lim_{n\to\infty} \frac{\ln n}{n}.$$

Hence compute

$$\lim_{n\to\infty} \sqrt[n]{n}.$$

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5.2 Limits via the Squeeze Theorem

View this lesson on YouTube [23]

Example.

Compute

$$\lim_{n\to\infty} \frac{\cos^2 n}{n}.$$

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5.3 Telescoping Series

View this lesson on YouTube [24]

Example.

Compute

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}.$$

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5.4 The Integral Test for Series

View this lesson on YouTube [25]

Example.

Which series converge / diverge?

$$\sum_{n=1}^{\infty} \frac{1}{n^2}, \quad \sum_{n=1}^{\infty} \frac{1}{n}.$$

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5.5 The Comparison Test for Series

View this lesson on YouTube [26]

Example.

Which series converge / diverge?

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^3}, \quad \sum_{n=3}^{\infty} \frac{\ln n}{n}.$$

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5.6 The Ratio Test for Series

View this lesson on YouTube [27]

Example.

Which series converge / diverge?

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}, \quad \sum_{n=1}^{\infty} \frac{n^n}{n!}.$$

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5.7 The Alternating Series Test

View this lesson on YouTube [28]

Example.

Does the following converge or diverge?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+2}.$$

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Chapter 6

Power Series

Power series aim to express a function $f = f(x)$ in terms of an infinite sum involving powers of x, namely in the form

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots \qquad (6.0.1)$$

where the numbers a_i are either given, or are to be determined.

From a calculus point of view, one of the advantages of writing a suitable function f in the form (6.0.1) is due to powers of x being easy to integrate and differentiate.

In this chapter, we'll look at: how to determine the domain of a given power series (the so–called interval of convergence); how to determine a special power series (known as a Maclaurin series) of a given function; gaining some idea of how to use power series to obtain other interesting results. We'll also look at how polynomials can approximate given functions, which is important from an approximation point of view: try to approximate complicated functions with simple polynomials.

In our work with Maclaurin series, we will use the identity

$$n! := n(n-1)\cdots 3\cdot 2\cdot 1.$$

6.1 Power Series and the Interval of Convergence

View this lesson on YouTube [29]

Example.

Determine the interval of convergence for

$$f(x) := \sum_{k=1}^{\infty} \frac{x^k}{3^k k^2}.$$

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6.2 Computing Maclaurin Polynomials

View this lesson on YouTube [30]

Example.

If

$$f(x) := \sqrt{1-x}$$

then compute the first four non–zero terms of the Maclaurin polynomial for f.

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6.3 Applications of Maclaurin Series

View this lesson on YouTube [31]

Example.

Compute the (convergent) Maclaurin series for $f(x) := e^x$. Write down the Maclaurin series for $g(x) := xe^x$ and differentiate to compute

$$\sum_{n=1}^{\infty} \frac{n}{(n-1)!}.$$

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Chapter 7

Applications of Integration

In this chapter, we look at some of the geometric uses for integration. In a first course in calculus, you see integration as a means for computing area under a curve. We now move to the next phase, illustrating how integration can be used for obtaining: the length of a curve; surface area; and the volume of a certain solids.

Although not explicitly mentioned in this chapter, the idea above are of more than just geometrical interest. If we wish to compute the total mass of a spring, for example, and the spring is in the shape of a known function and has a constant density, then being able to calculate the length of the spring is key to the total mass calculation.

There is much more about these kinds of applications in [35].

The identity

$$\cos 2x = 1 - 2\sin^2 x$$

will be required in the final example on volume.

7.1 Computing Lengths of Curves

View this lesson on YouTube [32]

Example.

Compute the arc length of the curve with parametric equations

$$\begin{aligned} x(t) &= 3t^2 - 9 \\ y(t) &= t^3 - 3t, \quad 0 \leq t \leq 3. \end{aligned}$$

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7.2 Finding Surface Areas by Integration

View this lesson on YouTube [33]

Example.

Rotate the curve

$$f(x) := \sqrt{x}, \quad 0 \leq x \leq 4$$

about the x–axis. Compute the surface area of the resulting solid.

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7.3 Finding Volumes by Integration

View this lesson on YouTube [34]

Example.

Rotate the region bounded by the curves:

$$f(x) := 2 - \sin x;$$
$$x = 0;\ \ x = 2\pi;\ \ y = 0;$$

about the x–axis. Compute the volume of the resulting solid.

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