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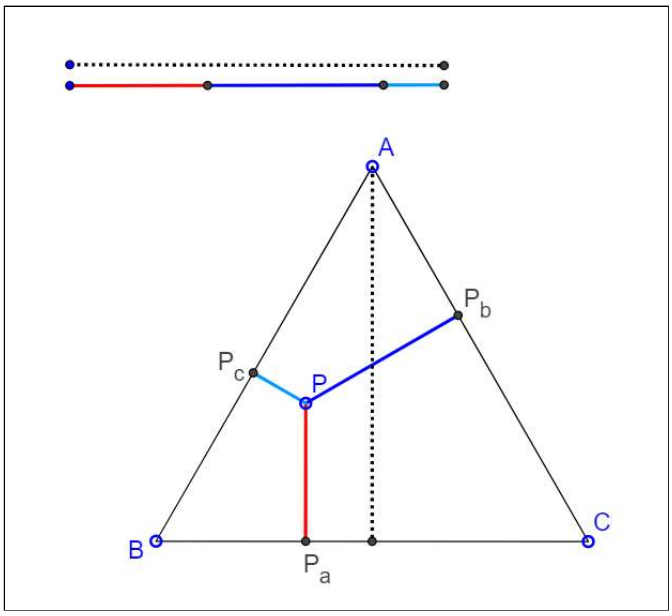
Viviani's Theorem: What is it?
A Mathematical Droodle



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Explanation

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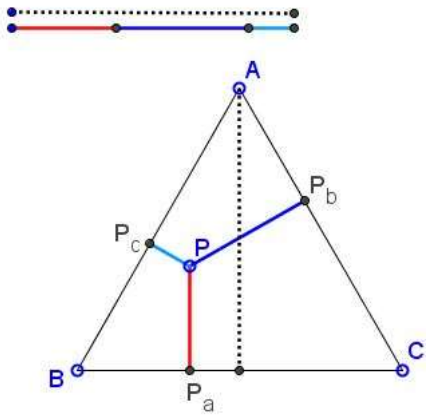
Viviani's Theorem

The applet attempts to illustrate the following theorem:

The sum of distances of a point inside an equilateral triangle or on one of its sides equals the length of its altitude.



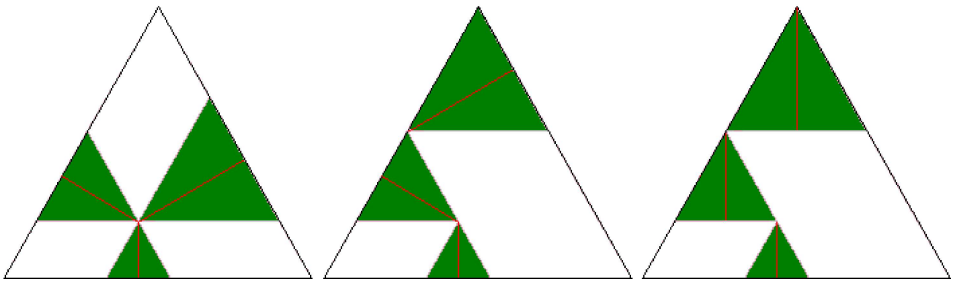
The theorem is named after ***Vincenzo Viviani (https://www-history.mcs.st-and.ac.uk/history/Mathematicians/Viviani.html)*** (1622-1703).

Let P_a, P_b, and P_c be the ***pedal points (https://www.cut-the-knot.org/Curriculum/Geometry/AdamsCircle.shtml#pedal)*** (projections) of a point P on the side lines BC, AC, and AB of ΔABC. Then the theorem claims that

(1) $$h = PP_a + PP_b + PP_c,$$

where h is the length of the altitudes of ΔABC.

To see that this is indeed so, draw the lines through P parallel to the sides of the triangle. I must mention that the current page has been prompted by a ***recent (https://www.cut-the-knot.org/Curriculum/Geometry/VivianiPWW.shtml#Kawasaki) proof without words (https://www.cut-the-knot.org/ctk/pww.shtml)*** by Ken-ichiroh Kawasaki. The diagram below is just a slight modification of Kawasaki's proof:

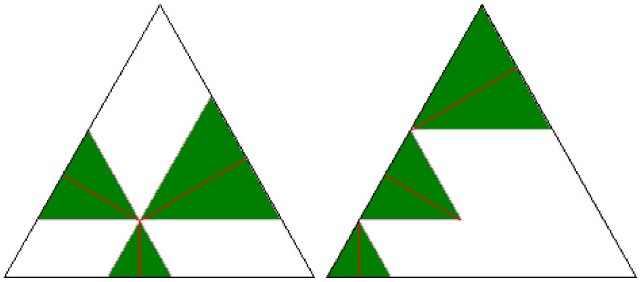


Together with the side lines of the base triangle, the newly drawn lines form three equilateral triangles, which, between them, contain all the pieces in (1) as their altitudes. A one step rearrangement of the triangles followed by a rotation of two of them completes this delightful proof.

The applet above was originally intended to present a dynamic variant of Kawasaki's proof. It ended up as something a little different, hopefully still simple enough:

$$PP_a + PP_b + PP_c = TS + LU + LT$$
$$= VL + LU + TS$$
$$= VS$$
$$= h$$

One can arrive at the same conclusion without rotating the triangles:



The sides of the three small equilateral triangle add up to the side of the big one. Hence, the same holds for their altitudes.

(There is available another proof without words of ***Viviani's theorem (https://www.cut-the-knot.org/Curriculum/Geometry/Viviani.shtml)***. For you to compare. Also, the ***theorem extends (https://www.cut-the-knot.org/Curriculum/Geometry/EquiangularPoly.shtml)*** to equilateral as well as equiangular polygons.)

References

1. Ken-ichiroh Kawasaki, Proof Without Words: Viviani's Theorem, *Mathematics Magazine*, Vol. 78, No. 3 (June 2005), 213.

Proofs Without Words

- ***Proofs Without Words (https://www.cut-the-knot.org/ctk/pww.shtml)***
- ***Sums of Geometric Series - Proofs Without Words (https://www.cut-the-knot.org/Curriculum/Algebra/SequencesSquare.shtml)***
- ***Sine of the Sum Formula (https://www.cut-the-knot.org/Curriculum/Algebra/SineInRhombus.shtml)***
- ***Parallelogram Law: A PWW (https://www.cut-the-knot.org/Curriculum/Geometry/ParallelogramLawIn4.shtml)***
- ***Parallelogram Law (https://www.cut-the-knot.org/Curriculum/Geometry/ParallelogramIdentity.shtml)***
- ***Ceva's Theorem: Proof Without Words (https://www.cut-the-knot.org/Curriculum/Geometry/CevaPWW.shtml)***
- ***Viviani's Theorem (https://www.cut-the-knot.org/Curriculum/Geometry/VivianiPWW.shtml)***
- ***A Property of Rhombi (https://www.cut-the-knot.org/Curriculum/Geometry/RhombiRhombi.shtml)***
- ***Triangular Numbers in a Square (https://www.cut-the-knot.org/Curriculum/Algebra/TriangularNumbers.shtml)***
- ***PWW: How Geometry Helps Algebra (https://www.mathteacherctk.com/blog/2011/06/2100/)***
- ***Varignon's Theorem, Proof Without Words (https://www.cut-the-knot.org/m/Geometry/Varignon.shtml)***

Related material Read more...

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- ***When a Triangle is Equilateral? (https://www.cut-the-knot.org/Curriculum/Geometry/Connes.shtml)***
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- ***Viviani's Theorem (PWW) (https://www.cut-the-knot.org/Curriculum/Geometry/VivianiPWW.shtml)***
- ***Tony Foster's Proof of Viviani's Theorem (https://www.cut-the-knot.org/m/Geometry/TonyFosterViviani.shtml)***
- ***Viviani in Isosceles Triangle (https://www.cut-the-knot.org/Curriculum/Geometry/VivianiIsosceles.shtml)***
- ***Viviani by Vectors (https://www.cut-the-knot.org/triangle/VivianiByVectors.shtml)***
- ***Slanted Viviani (https://www.cut-the-knot.org/m/Geometry/SlantedViviani.shtml)***
- ***Slanted Viviani, PWW (https://www.cut-the-knot.org/m/Geometry/slanted_viviani.shtml)***
- ***Morley's Miracle (https://www.cut-the-knot.org/triangle/Morley/)***
- ***Triangle Classification (https://www.cut-the-knot.org/triangle/Triangles.shtml)***
- ***Napoleon's Theorem (https://www.cut-the-knot.org/proofs/napoleon_intro.shtml)***
- ***Sum of Squares in Equilateral Triangle (https://www.cut-the-knot.org/pythagoras/EquiIn3D.shtml)***
- ***A Property of Equiangular Polygons (https://www.cut-the-knot.org/Curriculum/Geometry/EquiangularPoly.shtml)***
- ***Fixed Point in Isosceles and Equilateral Triangles (https://www.cut-the-knot.org/Curriculum/Geometry/AnonymousVisitor.shtml)***
- ***Parallels through the Vertices of Equilateral Triangle (https://www.cut-the-knot.org/m/Geometry/ThreeParallelsAndEquilateralTriangle.shtml)***

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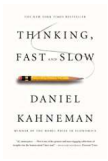

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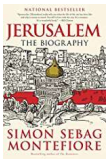

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