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PHY0064 Advanced Experiments
Experiment 4

The Compton-Effect: Determining the Rest Mass of an Electron

Abstract

In this lab, modern equipment is used to replicate the experiment showing the compton effect - the result of photons colliding with and scattering off of electrons in atoms of a target material. When the energy of the scattered photons is measured, a relationship is seen where the energy is dependent on the angle of scattering. This relationship is detailed in the Introduction section. In this experiment, data was recorded for scattering angles in five degree increments from zero degrees to 130 degrees. When analyzed, a determination of the rest mass of an electron was found. This value was determined to be 475.17 keV. When compared to the current accepted value of 511 keV, a 7.01% discrepancy was found. Additionally, the ratio between the energy of the compton edge and the photopeak energy was experimentally determined to be .7023 - compared to a theoretical value of .7215 this gives a percent discrepancy of 2.66%.

Introduction

The goal of this experiment is to determine the rest mass of an electron by observing the dependence of the scattered photon energy on the scattering angle during the compton effect. To begin, the relativistic energy for a particle - determined by Einstein - is used. The equation is shown below.

$$E^2 = p^2c^2 + m^2c^4 \quad (1)$$

Because this experiment relies on the use of photons (x-rays of energy 662 keV), the particles inducing the collision are massless, therefore equation 1 becomes:

$$E = pc$$

During the compton effect - the interaction observed in this experiment - a photon collides with and scatters off of an electron. As a result of this, conservation of energy can be applied to this collision between the photon and the electron.

$$E_i = E_\gamma + E_{e_{initial}} = E_\gamma + m_e c^2 \quad (2)$$

$$E_f = E_\gamma' + E_{e_{final}} = E_\gamma' + \sqrt{p_e{}^2c^2 + m_e{}^2c^4} \quad (3)$$

$$E_i = E_f = E_\gamma + m_e c^2 = E_\gamma' + \sqrt{p_e{}^2c^2 + m_e{}^2c^4} \quad (4)$$

Rearranging equation 4 yields:

$$(E_\gamma + m_e c^2 - E_\gamma')^2 - m_e{}^2c^4 = p_e{}^2c^2 \quad (5)$$

Applying conservation of momentum to this same collision allows for the following analysis.

$$p_e = p - p' \quad (6)$$

$$p_e{}^2 = p^2 - 2p * p' + p'^2 \quad (7)$$

$$E = pc \quad (8)$$

Combining equations 7 and 8 yields:

$$p_e{}^2c^2 = E^2 + E'^2 - 2EE'cos(\theta) \quad (9)$$

As a result of equations 5 and 9, the following relationship can be found.

$$(E_\gamma + m_ec^2 - E_\gamma{}')^2 - m_e{}^2c^4 = E^2 + E'^2 - 2EE'cos(\theta) \quad (10)$$

$$\frac{1}{E'} = \frac{1}{E} + (1 - cos(\theta))\frac{1}{m_ec^2} \quad (11)$$

$$E' = \frac{E}{1+E\frac{(1-cos(\theta))}{m_ec^2}} \quad (12)$$

Equation 12 shows the relationship that will be attempted to be discovered in this lab. By scattering x-rays at various angles and measuring the resulting energies, it is expected that this relationship - scattered photon energy dependent on $(1 - cos(\theta))$ where θ is the scattering angle - will be seen. In this lab, scattering is measured at 5 degree increments from 0 degrees to 130 degrees. The resulting shifts in photopeaks will then be converted to shifts in energy as a result of the angle of scattering. This conversion is done by measuring the values of peaks with known energies and determining the relationship. By plotting $\frac{E}{E'}$ against $(1 - cos(\theta))$ a linear relationship is expected, with slope equal to $\frac{E}{m_ec^2}$.

When equation 12 is evaluated at the minimum value - resulting from the scattering angle - the following is seen.

$$E'_{min} = \frac{E}{1+\frac{2E}{m_ec^2}} \quad (13)$$

This results from the photon scattering directly backwards ($\theta = 180^\circ$). This then leads to a maximum energy transferred to the particle, with energy equal to the value shown in equation 14 below.

$$T_{max} = \frac{E}{1+\frac{m_ec^2}{2E}} \quad (14)$$

Experimental Methods

A detailed description of the instructions and equipment used to conduct this experiment can be found in the Physics 0064 “Modern Physics Laboratory” Booklet, created by the Tufts University Department of Physics and Astronomy, Spring 2018.

Data and Results

Figure 1. Calibration Data - Spectrum of Known Energies Without Scattering Source

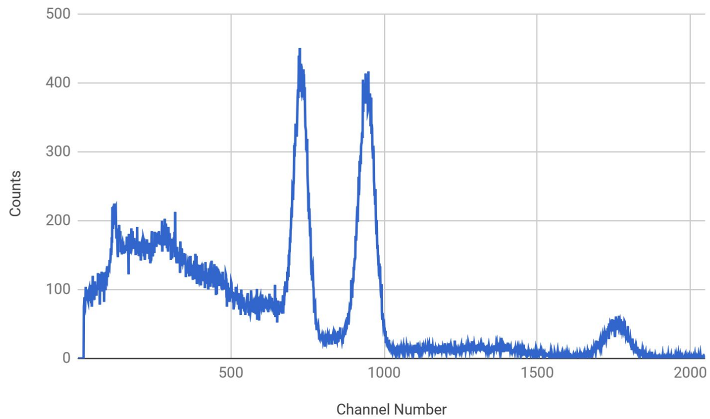


Figure 1 allows for the determination of the correlation between Channel Number and Energy. Based on the sources of these peaks, we know the associated energies to be 511 keV, 622 keV, and 1274 keV for the first, second, and third peaks respectively. This data is shown in Table 1 below. In the Results and Calculations section, this correlation will be determined.

Table 1. Channel Number and Respective Energy for Each Peak in Figure 1

	Peak 1	Peak 2	Peak 3
Channel Number	730	945	1760
Energy	511 keV	662 keV	1274 keV

Figure 2. Compton Scattering with Scattering Angle of Zero - Peak at Channel 940

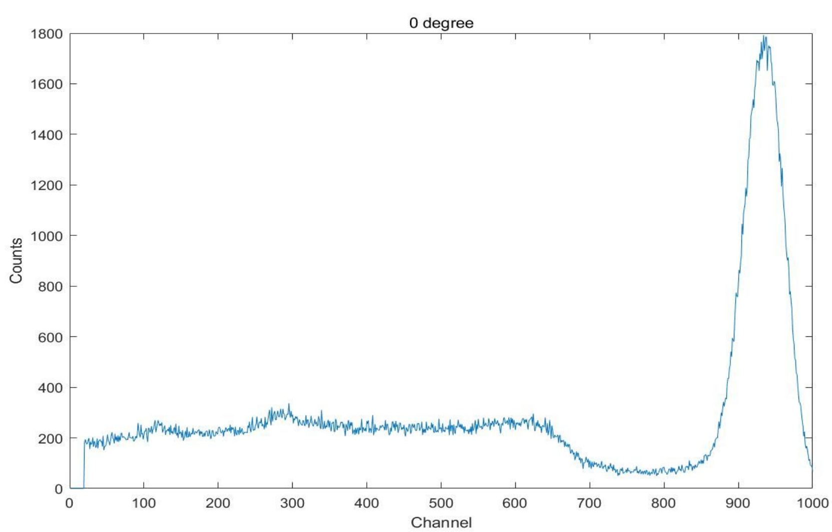
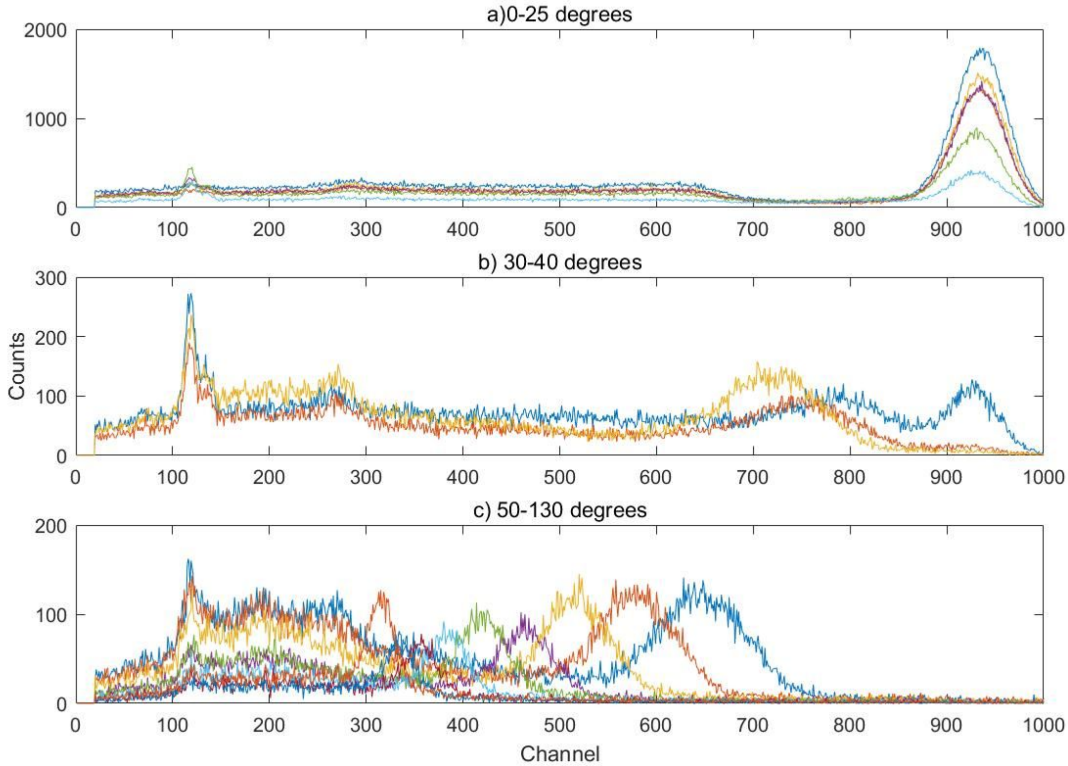


Figure 3. Compton scattering at a) 0-25 degrees , b) 30-40 degrees , c) 50-130 degrees



Note: The colors in Figure 3 (above) are in the order of Blue-Orange-Yellow-Purple-Green-Sky blue-etc, as the angle of scattering increases.

A repercussion of the experimental setup is shown in Figure 3. The blue plot in Figure 3b shows two separate photopeaks and the plots in Figure 3a show photopeaks at the same Channel Number - independent of the scattering angle. The cause for this is noted in the Discussion and Conclusion section of this paper, however, for the calculations displayed below, the data shown in Figure 3a is ignored. Reasoning for determining these points as invalid outliers is outlined in the Discussion and Conclusions section.

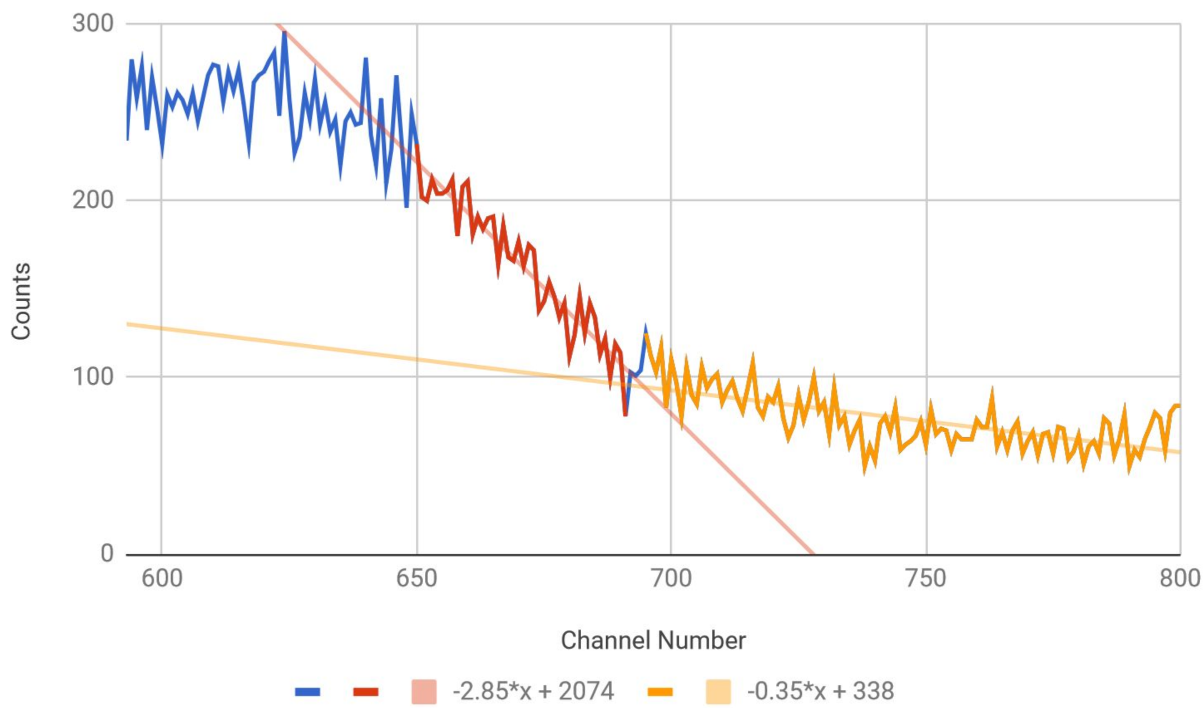
Results and Calculations

The first step in this lab is to determine the relationship between Channel Number and Energy based on the data given in Figure 1. Table 1 above shows the Channel Number and associated energy for each peak. Assuming a linear relationship, plotting the data in Table 1 gives the linear fit shown below where Energy is in units keV.

$$Energy \ = \ (.744)(Channel\ Number) \ - \ 35.7$$

Next, based on the scattering shown when the angle is zero (data represented in Figure 2), the location of the compton edge is determined. Figure 4 below shows a smaller sample of the data in Figure 2, where only the points near the compton edge are included. The data points colored yellow are assumed to be points past the compton edge and points colored red are assumed to be in the compton edge. Fitting linear trend-lines to these two sections of data should result in an intersection point where the compton edge ends.

Figure 4. Plot of Data Points from Figure 2 Near the Compton Edge



Finding the intersection of the two trend-lines shown in Figure 4 above can be done by simply setting them equal and solving for x. This is shown below.

$$-2.85x + 2074 = -0.35x + 388$$

$$x = 674.4$$

Using the relationship between Channel Number and Energy Yields:

$$Compton\ Edge\ Energy = 466.1\ keV$$

From Figure 2, the position of the photopeak is at Channel Number 940, or 663.7 keV. The ratio of energies between the photopeak and the compton edge is shown below.

$$\frac{Compton\ Energy}{Photopeak\ Energy} = \frac{466.1}{663.7} = .7023$$

Based on equation 14 shown in the Introduction section, this ratio is expected to be:

$$\frac{Compton\ Energy}{Photopeak\ Energy} = \frac{T_{max}}{E} = \frac{1}{1+\frac{mc^2}{2E}} = \frac{1}{1+\frac{511}{2(662)}} = .7215$$

This gives a percent discrepancy of 2.66%.

Next, an examination of the shift in the photopeak with respect to the scattering angle is performed - again, the data represented in Figure 3a is ignored for reasons outlined in the Discussion and Conclusion section of this paper. Table 2 below shows the data used to find the relationship between the scattering angle and the photopeak energy.

Table 2. Measurements and Calculations Determined from Data Shown in Figure 3

$\theta/^\circ$	$1 - cos\theta$	E'/Channel	E'/MeV	0.662/E'
30	**0.133975**	**790**	**0.553372**	**1.196301**
35	**0.180848**	**750**	**0.523679**	**1.264134**
40	**0.233956**	**720**	**0.501408**	**1.320281**
45	**0.292893**	**690**	**0.479138**	**1.381648**
50	**0.357212**	**650**	**0.449444**	**1.47293**
55	**0.426424**	**600**	**0.412327**	**1.605521**
60	**0.5**	**580**	**0.39748**	**1.665491**
65	**0.577382**	**550**	**0.37521**	**1.764345**
70	**0.65798**	**520**	**0.35294**	**1.875674**
75	**0.741181**	**500**	**0.338093**	**1.958041**
80	**0.826352**	**470**	**0.315823**	**2.096113**
85	**0.912844**	**440**	**0.293552**	**2.255134**
90	**1**	**430**	**0.286129**	**2.313642**
95	**1.087156**	**405**	**0.26757**	**2.474115**
100	**1.173648**	**390**	**0.256435**	**2.581549**
105	**1.258819**	**370**	**0.241588**	**2.740198**
110	**1.34202**	**355**	**0.230453**	**2.8726**
115	**1.422618**	**342**	**0.220803**	**2.998151**
120	**1.5**	**335**	**0.215606**	**3.07041**
125	**1.573576**	**325**	**0.208183**	**3.179896**
130	**1.642788**	**315**	**0.20076**	**3.297478**

Figure 5. Compton scattering at 50-100 degrees(Colors in opposite order) for 300-800 channels

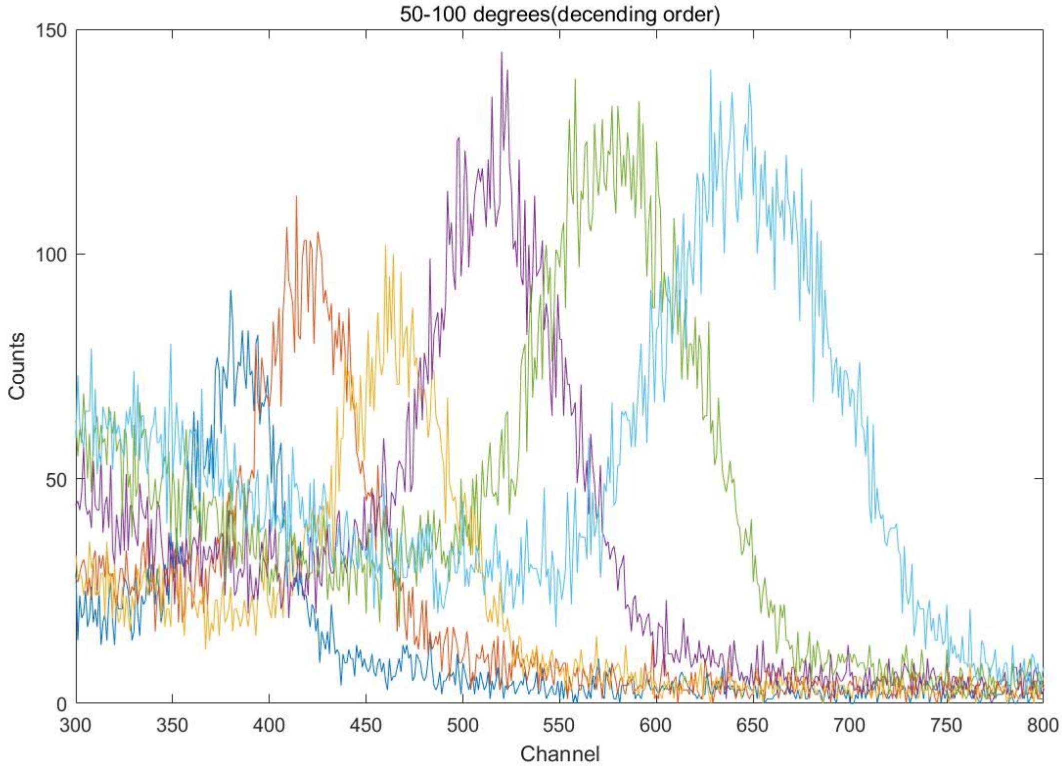
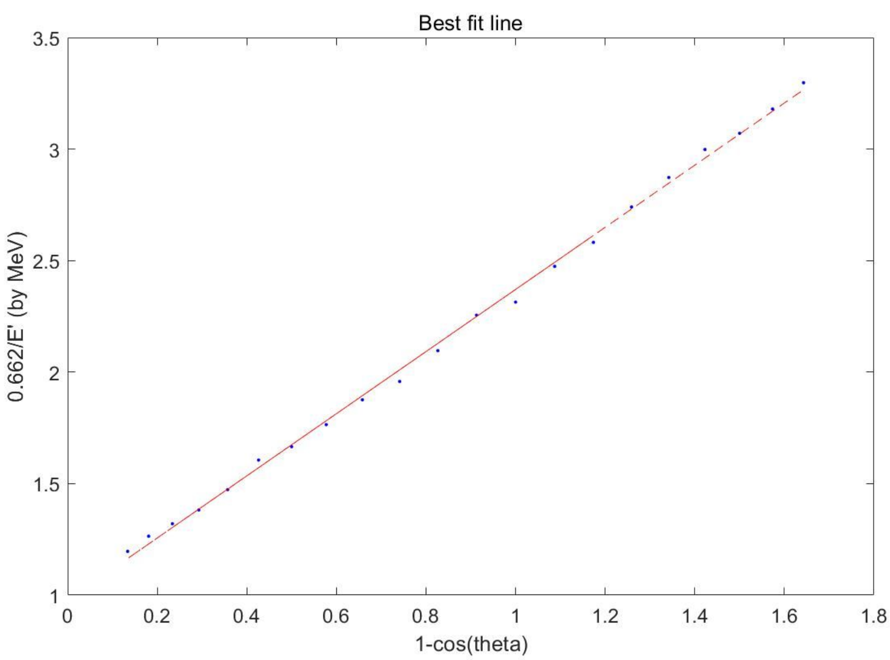


Figure 6. Best Fit Line of E/E' vs cos(θ) for Angles From 35 to 130 Degrees



Equation for the line of the best fit: $y = (1.3932)x + 0.9775$

Figure 6 above shows the plot of the ratio of photopeak energies against $1 - cos(\theta)$ where θ is the scattering angle. Based on the equations shown in the Introduction section, the slope of this line can then be used to find the rest-mass of an electron. The slope of the linear trend line in Figure 6 (above) is 1.3932.

$$E/E' = 1 + E(1 - cos(\theta))/mc^2$$
$$\Rightarrow slope = E/mc^2$$

The incident x-ray ray energy, E, is 662 keV. Therefore the mass of the target (electron) is

$$mc^2 = \frac{E}{slope} = \frac{662}{1.3932} = 475.17\ keV$$

The current accepted value for the mass of an electron is 511 keV. Therefore the percent discrepancy is 7.01%, as shown below.

$$\text{Percent discrepancy} = \frac{511-475.17}{511} \times 100 = 7.01\%$$

Discussion and conclusion

The two objectives of this lab were to experimentally determine the ratio between the observed compton edge and photopeak and then to experimentally determine the rest mass of an electron. Observing the spectrum recorded when the scattering angle was set at zero degrees allowed for a clear determination of the ratio between the compton edge and the photopeak - this ratio gets disturbed when the scattering angle is increased above zero degrees so was only measured at zero degrees. This ratio was determined to be 0.7023. Theoretically, a ratio of 0.7215 was expected, giving a relatively small percent discrepancy of -2.66%. Based on this good agreement, it can be assumed that this goal of the experiment was successful.

The next goal of the experiment was, again, to determine the rest mass of an electron. By measuring the energies of the scattered photons and finding the relationship between these energies and the scattering angle, the rest mass was able to be found. Experimentally, this value was found to be 475.17 keV. The current accepted value of the rest mass of an electron is 511 keV - a discrepancy of -7.01%. This small negative discrepancy was expected based on the buoyant-like force that is thought to negatively decrease the measured rest mass of the electron.

It is noted above that unexpected outliers are seen in the data represented in Figures 3a and 3b. Possible reasoning for this unusual behavior is dependent on both the experimental setup and the realistic rates of observed collisions as the scattering angle is increased. First, it is known that as the scattering angle is increased, the rate of observed collisions decreases dramatically. This means that collisions with smaller scattering angles are much more probable than collisions with larger scattering angles and will therefore dominate measurements. Additionally, the experimental setup used in this lab was not precise enough to filter observed collisions to exactly one angle of scattering. Based on the aperture of the lead cylinders used to collect the scattered photons, collisions of multiple different angles will be observed regardless of where the lead cylinders are placed. In conjunction with the fact that smaller scattering angles produce many more collisions, it can be reasoned that smaller scattering angles than what the setup was meant to collect can dominate the readings of the photopeaks. A clear example of this is shown in the blue plot of Figure 3b. Two peaks are seen, one around channel number 940 - an indication of no scattering angle - and another peak around channel number 800 - an indication of a scattering angle of 30 degrees. This is clear evidence that collisions of no scattering angle are present even when the setup is tailored to only accept photons scattering at a larger angle. Dominance of these smaller scattering angle collisions is seen in the plots of Figure 3a. Regardless of the scattering angle that the setup was tailored to accept, no shifts in the photopeaks are seen. This would indicate that the collisions without scattering are still being observed and dominating measurements over the photons scattering with any angle. As a result of this, the data represented in the plots in Figure 3a was deemed to be unreasonable outliers for the purpose of this experiment and was therefore ignored in the calculations. In order to increase the precision of this experiment in the future, lead blocks with smaller apertures should be used. It should be noted though that decreasing the aperture will lead to a decrease in the rate of observed collisions. This will mean each scattering angle will need to be observed for much longer in order to get a large enough sample size.