Probing astrophysical parameters of a precessing binary using harmonics of SpinTaylorF2

Soham Mukherjee, K. Haris, K. Atmjeet, Archana Pai and É Chassande-Mottin
IISER Thiruvananthapuram, APC Paris

Abstract: We evaluate the contribution to the signal-to-noise ratio of the dominating harmonics of *SpinTaylorF2SingleSpin*—a single-spin, frequency domain waveform model that incorporates the effects of spin-induced precession. We show that the observation of specific harmonics can provide direct evidence of precession. We also show that using the information from two of the harmonics, we can infer the binary spin orientation and put bounds on the spin-orbit parameters of the source.

The SpinTaylorF2 waveform model (Lundgren et al. 2014)

- An efficient, single-spin, frequency domain waveform model that incorporates the effects of spin-induced precession. The quadrupolar mode is given by

$$\tilde{h}_+(f) \simeq \sum_{m=-2}^{2} H_m(\theta_J, \psi_J)\, \tilde{h}_m(\eta, \chi_1, \kappa, f)\, e^{i\phi_m(\eta,\chi_1,\kappa,f)} \equiv \sum_{m=-2}^{2} \tilde{h}_{+m}(f) \qquad (1)$$

  - $\tilde{h}_{+m}(f) \rightarrow m^{th}$ sideband
  - $(\theta_J, \psi_J) \rightarrow$ define the orientation of the total angular momentum $\mathbf{J}$ w.r.t to the line of sight $\hat{\mathbf{N}}$ to the binary.
  - $\eta \rightarrow$ symmetric mass ratio , $\chi_1 \rightarrow$ BH spin, and $\kappa = \hat{\mathbf{L}} \cdot \hat{\mathbf{S}}_1 \rightarrow$ spin-alignment parameter.

- Each sideband separates into a precessing and a non-precessing part:
  - $\tilde{h}_2(f)\, e^{i\phi_m}$ encodes the information on precession
  - $H_m(\theta_J, \psi_J)$ encodes the information about the orientation of the binary.

SNR Distribution of SpinTaylorF2

- Signal-to-noise ratio (SNR) $\rho = (h|h)^{1/2}$ and $\text{SNR}_m = (h_m|h_m)^{1/2}$, where SNR_m is the SNR in the m^{th} sideband.
- $|\kappa| \sim 1$ (aligned/anti-aligned spin) $\rightarrow$ non-precessing systems
  - more SNR in face-on ($\theta_J = 0$) or face-off ($\theta_J = \pi$) regions.
- $\kappa \leq 0$ (misaligned spin) and (low η and/or high χ_1) $\rightarrow$ strongly precessing systems
  - more SNR for edge-on ($\theta_J = \pi/2$) and ($\kappa < 0$) region.
  - for higher values of χ_1 and negative values of κ, the amount of precession increases, and so does the SNR for edge-on systems.
- For intermediate κ, SNR results show an interplay between the parameters κ and θ_J.

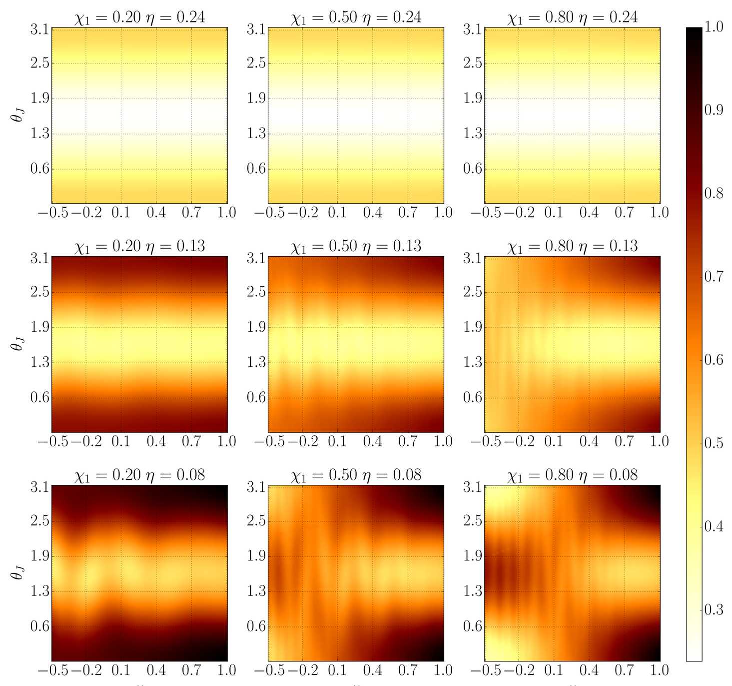

Figure 1: SNR scaled by $\text{SNR}_{max} = 20.78$. D fixed to 400 Mpc.

Overlap Distribution of SpinTaylorF2

- Overlap $\rightarrow$ $\mathcal{O}_m \equiv (h|h_m)/\sqrt{(h|h_m)(h|h_m)}$
- For SNR above the threshold, $m = 0$ and $m = 2$ are the only relevant harmonics.
- The $m = 0$ and $m = 2$ modes dominate in complimentary regions of (θ_J, κ) space.
  - $\mathcal{O}_0$ is dominant in a parabolic region symmetric about $\theta_J = \pi/2$, in the $\kappa < 0$ region $\rightarrow$ coverage maximum for strongly precessing systems.
  - $\mathcal{O}_0$ is dominant in the complimentary region where $\kappa \sim 1$, and $\theta_J = 0, \pi \rightarrow$ coverage is maximum for non-precessing systems.
- Identified 3 regions A ($\mathcal{O}_0 \gg \mathcal{O}_2$), B($\mathcal{O}_0 \ll \mathcal{O}_2$) and C ($\mathcal{O}_0 \sim \mathcal{O}_2$) in the (θ_J, κ) space as a function of η and κ, associated with strongly precessing, moderately precessing and weakly/non-precessing systems.
- Obtained parametric fits for the boundaries of these regions for ($2.4\,M_\odot < m_1 < 20\,M_\odot$), ($0 < \chi_1 < 1$) and ($-0.5 < \kappa < 1$).

- Depending on the relative SNR contribution in the two modes, we can put bounds on the extent of precession in the system.

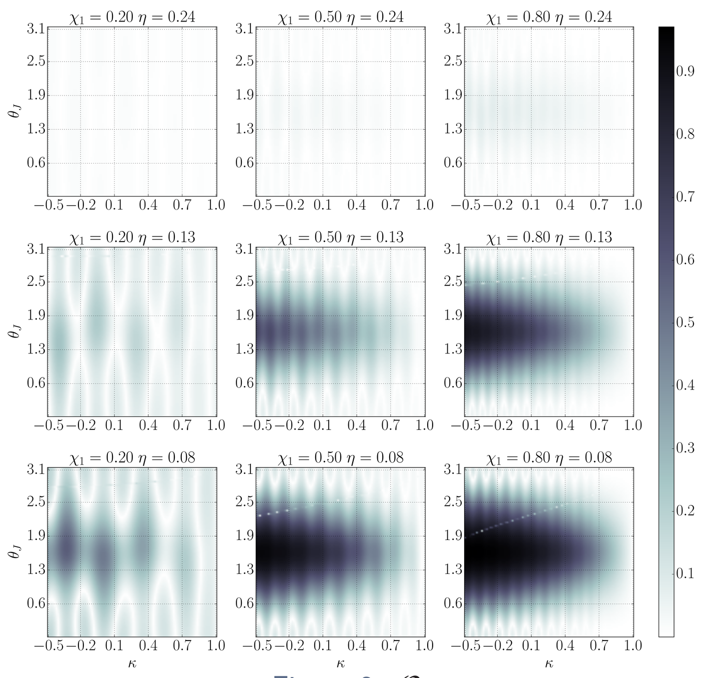

Figure 2: $\mathcal{O}_0$

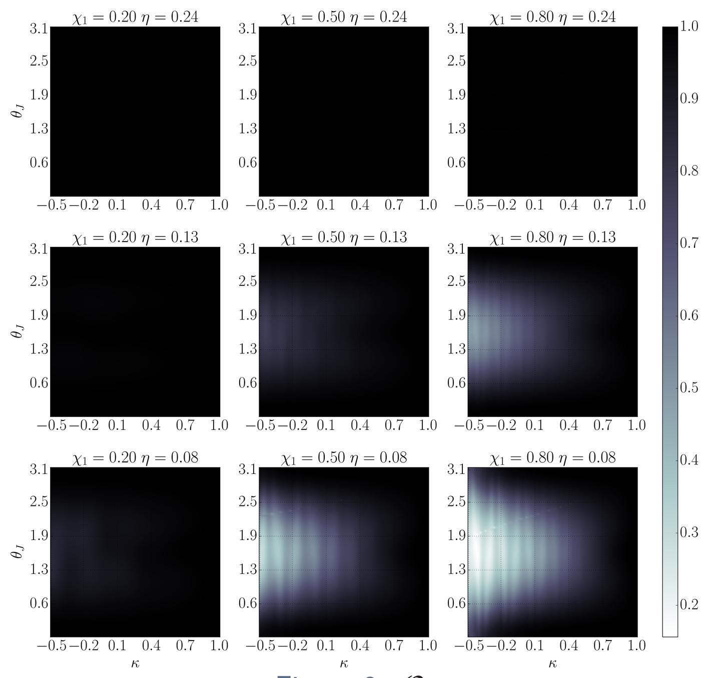

Figure 3: $\mathcal{O}_2$

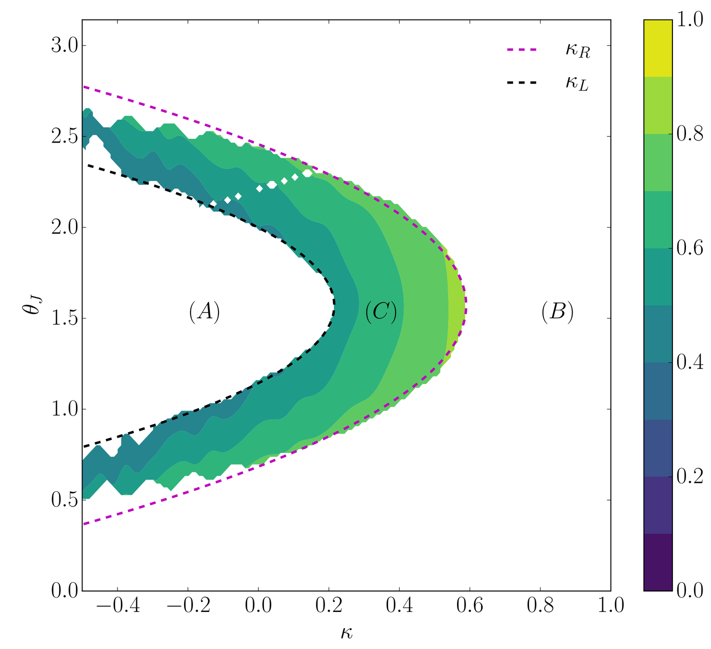

Figure 4: Regions of parameter space A, B and C

Probing astrophysical parameters of a precessing system using information from two harmonics

- Consider, if we observe a GW signal from an NSBH binary system, and are able to determine the component masses of the system. Also, assume that we are able to probe the SNR in the individual $m = 0$ and $m = 2$ spin harmonics. Then, we show below, that we can put bounds on the spin parameters. In such a case, there are 3 possible scenarios:

Scenario I : $\text{SNR}_0 \gg \text{SNR}_2$ $\rightarrow$ region A $\rightarrow$ strongly precessing $\rightarrow$ one can put an upper bound on the maximum possible value of κ.

Scenario II : $\text{SNR}_0 \sim \text{SNR}_2$ $\rightarrow$ region C $\rightarrow$ moderately precessing $\rightarrow$ possible to (1) probe the orientation parameters of the total angular momentum vector $\mathbf{J}$ (2) put bounds on κ.

Scenario III : $\text{SNR}_0 \ll \text{SNR}_2$ $\rightarrow$ region B $\rightarrow$ weakly or non-precessing system.

Scenario I : Upper bound on κ for strongly precessing systems

- Spin modes can reveal crucial information about precession.
- Using fits for the boundary between region A and region C as a function of η, χ_1, we obtain an upper bound on κ as:

$$\kappa_{0A}(\eta, \chi_1) \approx (145.83\chi_1 - 155.92)\,\eta^2 - (1.10\chi_1 + 0.16)\,\eta + 0.08\,\chi_1 + 0.50, \qquad (2)$$

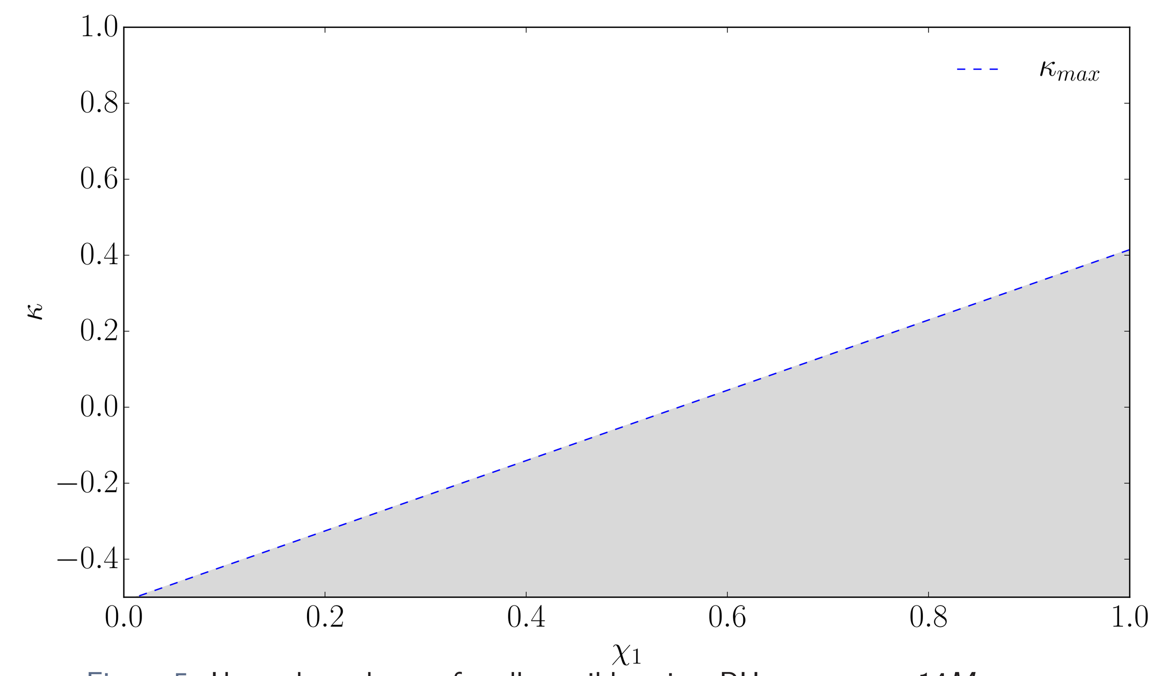

Figure 5: Upper bound on κ for all possible spins; BH mass $m_1 = 14M_\odot$.

- Note that even for $\chi_1 = 1$ (highest possible BH spin), there exists an upper bound on $\kappa \leq 0.414624$.

Scenario II : Upper bound on κ for moderately precessing systems

- Similar to Scenario I, we use fits for the boundary between region C and region B as a function of η, χ_1, to obtain an upper bound on κ as:

$$\kappa_{0B}(\eta, \chi_1) \approx (48.98\chi_1 - 57.13)\,\eta^2 + (1.96\chi_1 - 2.39)\,\eta - 0.09\,\chi_1 + 0.82, \qquad (3)$$

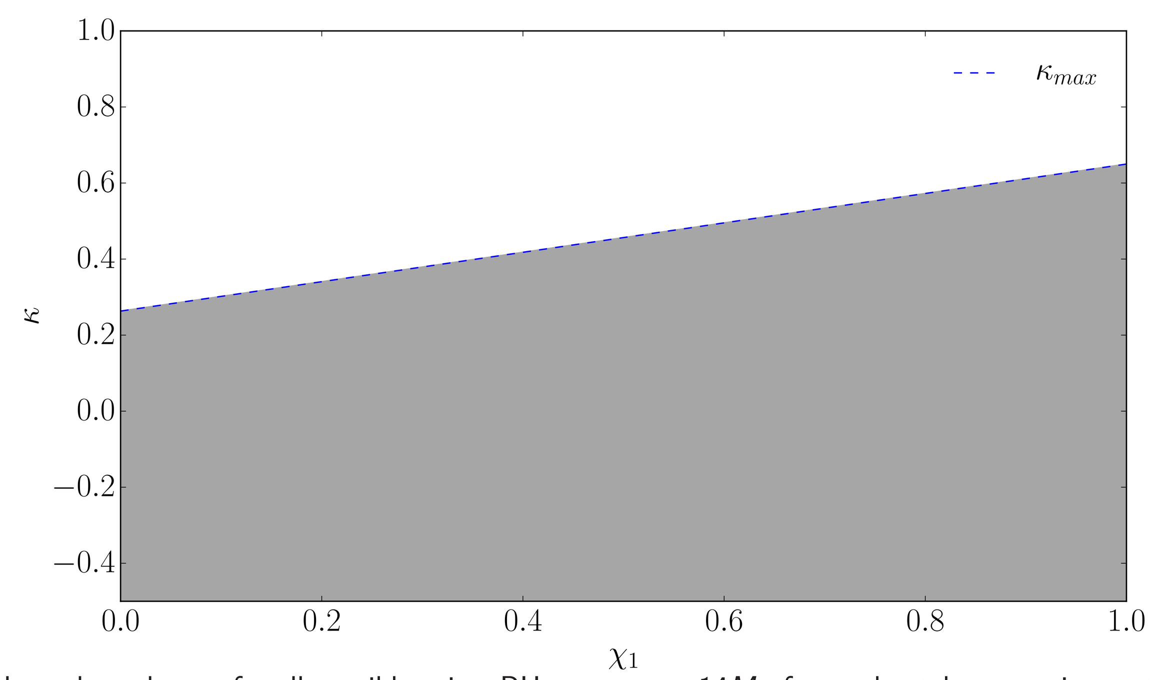

Figure 6: Upper bound on κ for all possible spins; BH mass $m_1 = 14M_\odot$ for moderately precessing systems.

- Even for moderately precessing systems $\kappa \leq 0.64984$ for $\chi_1 = 1$.

Conclusions

- Considered NSBH binary population with NS mass $= 1.4\,M_\odot$ and BH mass varying from ($2.4\,M_\odot - 20\,M_\odot$) and all possible values of BH spins, i.e., ($0 < \chi_1 < 1$) and spin-alignment parameter ($-0.5 < \kappa < 1$).
- Two dominant spin harmonics ($m = 0$ and $m = 2$) capture a large fraction of the SNR across a large region of (θ_J, κ) parameter space.
- For non-precessing as well as weakly precessing systems the spin harmonic $m = 2$ mode contributes to most of the overlap.
- For strongly precessing systems (highly asymmetric, high spins and away from $\kappa = \pm 1$), the spin harmonic $m = 0$ dominates.
- For moderately precessing systems both the harmonics contribute.
- Obtained parametric fits for various regions and proposed bounds on the spin parameters
  - Region A: $\text{SNR}_0 \gg \text{SNR}_2$, i.e, strongly precessing systems $\rightarrow$ bound on the maximum possible value of κ (increases linearly with increasing χ_1).
  - Region C: $\text{SNR}_0 \sim \text{SNR}_2$, i.e, moderately precessing systems $\rightarrow$ also gives bound on the maximum possible value of κ.

  This clearly shows that for moderately precessing systems, the value of the upper bound on κ is higher than that of the highly precessing case for same value of BH spin ($\kappa \leq 0.414624$ for strongly precessing case, whereas $\kappa \leq 0.64984$ for moderately precessing cases.)
- Using $\text{SNR}_0, \text{SNR}_2$, the orientation of total angular momentum vector can be probed.

Scenario II : Estimation of angular parameters θ_J and ψ_J

- Given $(\text{SNR}_2, \text{SNR}_0)$ $\Rightarrow$ $\hat{H}_0, \hat{H}_2$ (maximum likelihood estimates)
- Then, using the amplitude estimates H_0 and H_2, along with the phase difference $\phi_2 - \phi_0$ of the two spin harmonics, we can estimate $\hat{\theta}_J, \hat{\psi'}_J$:

$$\cos\hat{\theta}_J = \left(\frac{3\hat{H}_2\,\cos(\phi_2 - \phi_0) - 2\hat{H}_0}{3\hat{H}_2\,\cos(\phi_2 - \phi_0) + 2\hat{H}_0}\right)^{1/2}, \qquad (4)$$
$$\cos 2\,\hat{\psi'}_J = -\sin(\phi_2 - \phi_0)/\cos(\theta_J),$$

where $\hat{\theta}_J, \hat{\psi'}_J$ define the orientation of the total angular momentum vector $\mathbf{J}$.

Acknowledgments

- Kumar Atmjeet is supported by the DST-MPG Max Planck Partner Group fund at IISER TVM and the CCMS fund of IISER TVM. É Chassande-Mottin acknowledges the support from CNRS, PICS grant respectively.

This document has a LIGO document number LIGO-G1700374.

References

- A. Lundgren and R. O'Shaughnessy, Phys. Rev. D **89**, 044021 (2014), 1304.3332.

soham.m@iisertvm.ac.in