3. Consider the problem of localizing a robot in a grid world using noisy sensor observations. The robot's environment is a grid of size (X, Y) discretized with grid cells of size 1×1, see figure below. The dark-shaded grid cells represent walls and hence represent infeasible regions. The feasible grid cells are shown in white.

   The robot's action space is discrete with four actions: *MoveNorth*, *MoveSouth*, *MoveEast* or *MoveWest* that move the robot to an adjacent (feasible) grid cell. Assume that the probability of action selection is uniformly distributed among the set of actions that lead the robot to a feasible grid cell. Actions that lead to infeasible grid cells have zero likelihood. It is assumed that there is at least one feasible action for any grid cell, i.e., the a grid cell is not enclosed on all sides with an infeasible grid cells and that there is an enclosing wall in the grid.

   The robot has four noisy sensors pointing in the $(N)orth$, $(S)outh$, $(E)ast$ or $(W)est$ directions. The sensors report a discrete binary observation that a wall is *Close* or *Far*. The robot's observation at any time instance is a four tuple $\{d_N, d_S, d_E, d_W\}$ where each d_i is a binary discrete output as *Close* or *Far* measured independently in each direction. The probability of $p(Close|Distance)$ drops linearly from one to zero till a distance of R_{max} for each sensor. The inter-grid distances are measured from the centre of a grid cell. For instance, if $R_{max} = 5$ then the likelihood $p(Close|Distance)$ at discrete distances 1, 2, 3 and 4 decreases as 1, 0.75, 0.5, 0.25 and 0 (for a distance of 5 and beyond). The likelihood $p(Far|distance)$ are complementary and hence the corresponding likelihoods are 0, 0.25, 0.5, 0.75 and 1.0 (for distances ≥ 5).

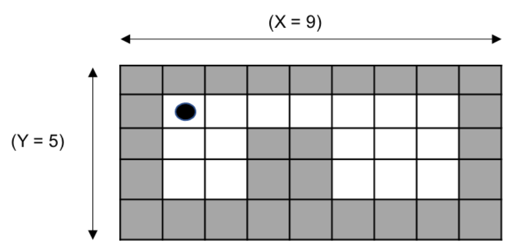

Figure 1: A robot (shown as a black dot) is moving in a grid world of size (X=9, Y=5) with four noisy sensors along the N, S, E and W directions. The white-colored and gray-colored cells are feasible and infeasible grid cells respectively. There is a surrounding wall in the grid.

(a) Simulate the robot's motion in the grid ($X = 20$, $Y = 20$) by adding obstacles appropriately. The initial position can be randomly picked. Simulate and record the sequence of sensor observations for $T = 25$ time steps. Estimate (i) the robot's current position at time t and (ii) the robot's *most-likely* path, given the sequence of observations generated by the simulation above till time T.

(b) Visualize and plot the log-likelihood for the robot's estimated position at time t as a spatial distribution over the grid. Observe how the belief updates with the arrival of new observations. Record the changing belief as a video.

(c) Compare the trajectory obtained by estimating the robot's current position with the actual location using the Manhattan distance metric. Similarly, compare the robot's *most-likely* path given observations with the actual trajectory. Perform at least $N = 50$ runs and report the average and variance in your results.

(d) Increase and decrease the parameter R_{max} as $\{1, 10\}$ and evaluate its effect on the current state estimate of the robot in relation to the ground truth.

(e) Modify the grid by changing the layout of obstacles. Report a grid layout where localization is challenging (takes many observations to converge) and a grid layout where localization occurs very rapidly.