Visualizing Celestial Bodies in 3D

Tamas Kis | tamas.a.kis@outlook.com | https://tamaskis.github.io

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1 ATTITUDE

To describe the attitude (i.e. orientation) of a planet, we need two angles:

1. ε – obliquity, measures the tilt of the planet's axis with respect to the ecliptic (the orbital plane of the Earth about the Sun)
2. θ – measures the rotation angle of the planet about its 3rd axis

Consider a coordinate system, xyz, where the xy-plane is coplanar with the ecliptic plane. Let's imagine that to start off, the planet's equatorial plane is coplanar to the ecliptic plane. To rotate its equatorial plane to the correct orientation, we rotate first rotate this xyz coordinate system about its x-axis by the obliquity, ε. This creates a new coordinate system, $x'y'z'$, where $x = x'$. This rotation is shown in Fig. 1.

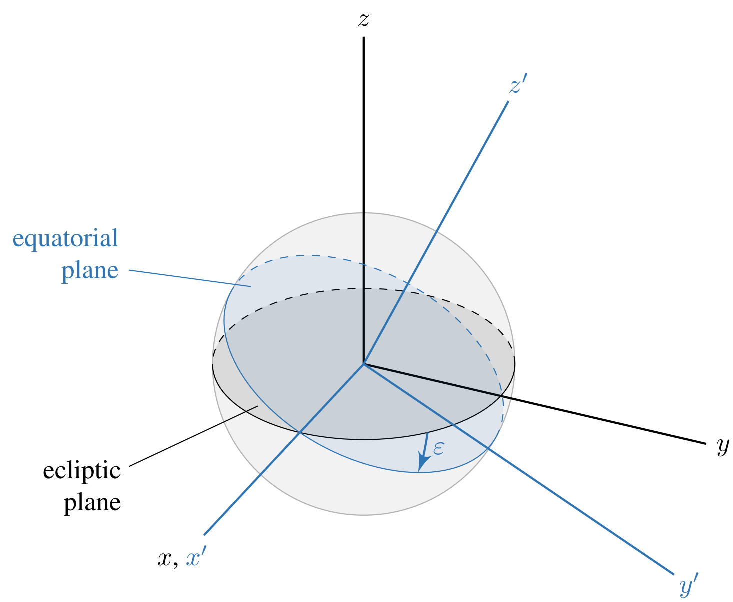

Figure 1: Tilting the planet with respect to the ecliptic plane.

Next, we need to rotate the planet about its z' axis by an angle θ (for Earth, θ would be the Greenwich mean sidereal time). This forms another new coordinate system, $x''y''z''$, where $z' = z''$. This rotation is shown in Fig. 2.

Consider a vector $\mathbf{v}$, resolved in the xyz coordinate system. To resolve it in the $x'y'z'$ coordinate system, we need to apply a rotation matrix that represents a rotation of $-\varepsilon$ about x (the 1st axis of the xyz coordinate system). Thus, we have

$$[\mathbf{v}]_{x'y'z'} = [\mathbf{R}_1(\varepsilon)][\mathbf{v}]_{xyz} \tag{1}$$

where

$$[\mathbf{R}_1(-\varepsilon)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\varepsilon) & \sin(-\varepsilon) \\ 0 & -\sin -\varepsilon) & \cos -\varepsilon) \end{bmatrix} \quad \rightarrow \quad \boxed{[\mathbf{R}_1(-\varepsilon)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & -\sin\varepsilon \\ 0 & \sin\varepsilon & \cos\varepsilon \end{bmatrix}} \tag{2}$$

Next, to resolve $\mathbf{v}$ in the $x''y''z''$ coordinate system, we need to apply a rotation matrix that represents a rotation of θ about z' (the 3rd axis of the $x'y'z'$ coordinate system). Thus, we have

$$[\mathbf{v}]_{x''y''z''} = [\mathbf{R}_3(\varepsilon)][\mathbf{v}]_{x'y'z'} \tag{3}$$

where

$$\boxed{[\mathbf{R}_3(\theta)] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}} \tag{4}$$

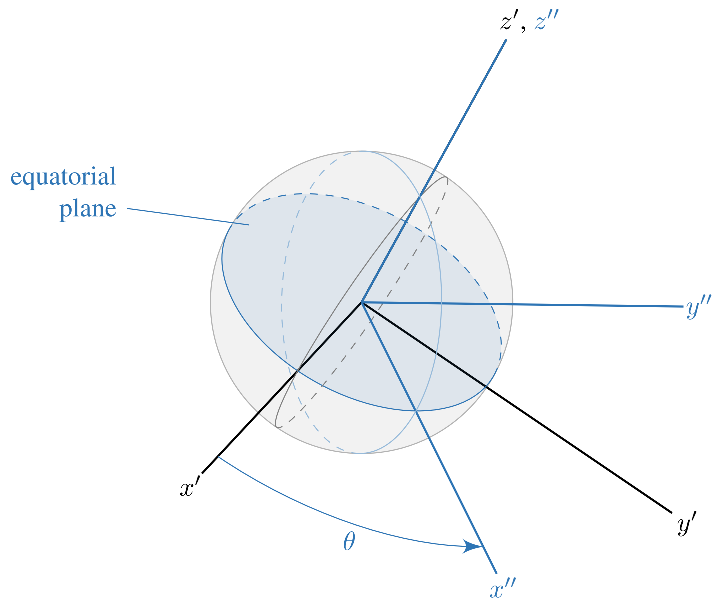

Figure 2: Rotating the planet about its 3rd axis.

Substituting Eq. (1) into Eq. (3),

$$\boxed{[\mathbf{v}]_{x''y''z''} = [\mathbf{R}_3(\theta)][\mathbf{R}_1(\varepsilon)][\mathbf{v}]_{xyz}} \tag{5}$$

To perform these coordinate transformations in MATLAB for the `surface` objects, we can use the `rotate`[1] command to rotate the surface object. However, the `rotate` command requires the axis of rotation, $\boldsymbol{\alpha}$. For the first rotation, this is simply

$$\boxed{\boldsymbol{\alpha}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} \tag{6}$$

since we are rotating about the x-axis. However, for the second rotation, we are rotating about the z'-axis, which is no longer in the direction of $(0,0,1)^T$. Instead, we first need to apply the rotation matrix that would rotate the z-axis to align it with the z'-axis. Since this secon

$$\boxed{\boldsymbol{\alpha}_2 = [\mathbf{R}_1(\varepsilon)] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} \tag{7}$$

[1] Note that this command performs an *active* rotation, so we input $-\varepsilon$ instead of ε

2 DATA AND CONSTANTS

2.1 Astronomical Data

Planet/Body	Equatorial Radius		Flattening		Obliquity	
	Value [km]	Source	Value	Source	Value [°]	Source
Sun	696000	[13]*	0.000 009	[11]	0	-
Moon	1738.0	[13]**	0.0012	[7]	6.68	[13]**
Mercury	2439.0	[13]**	0.0000	[6]	0.0	[13]**
Venus	6052.0	[13]**	0.000	[14]	177.3	[13]**
Earth	6378.1363	[13]**	0.003 352 813 1	[13]**	23.45	[13]**
Mars	3397.2	[13]**	0.006 476 30	[13]**	25.19	[13]**
Jupiter	71492.0	[13]***	0.064 874 4	[13]***	3.12	[13]***
Saturn	60268.0	[13]***	0.097 962 4	[13]***	26.73	[13]***
Uranus	25559.0	[13]***	0.022 927 3	[13]***	97.86	[13]***
Neptune	24764.0	[13]***	0.0171	[13]***	29.56	[13]***
Pluto	1151.0	[13]***	0.0	[13]***	118.0	[13]***

**Table D-5, p. 1043*
***Table D-3, p. 1041*
****Table D-4, p. 1042*

2.2 Semi-Minor Axes

For MATLAB's `ellipsoid` function, we need the semi-minor axis, b, which can be calculated as

$$b = a(1 - f)$$

where a is the semi-major axis (assumed to be the equatorial radius) and f is the flattening [3, p. 7-4 (p. 73 in PDF)].

2.3 Saturn's Rings

Saturn's rings range from $7000\ \mathrm{km}$ to $80000\ \mathrm{km}$ from the surface of the planet [9].

2.4 Unit Conversions

Meters to Astronomical Units [13]:

$$1\ \mathrm{AU} = 149597870\ \mathrm{km} = 149597870000\ \mathrm{m} \quad \rightarrow \quad \boxed{1\ \mathrm{km} = \frac{1}{149597870000}\ \mathrm{AU}}$$

Meters to Kilometers:

$$\boxed{1\ \mathrm{m} = 0.001\ \mathrm{km}}$$

Meters to Feet:

$$1\ \mathrm{m} = (100\ \mathrm{cm})\left(\frac{1\ \mathrm{in}}{2.54\ \mathrm{cm}}\right)\left(\frac{1\ \mathrm{ft}}{12\ \mathrm{in}}\right) \quad \rightarrow \quad \boxed{1\ \mathrm{km} = \frac{100}{30.48}\ \mathrm{ft}}$$

Meters to Miles:

$$1 \text{ m} = \left(\frac{100}{30.48} \text{ ft}\right)\left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right) \quad \rightarrow \quad \boxed{1 \text{ km} = \frac{100}{160934.4} \text{ mi}}$$

Meters to Nautical Miles:

$$1 \text{ nmi} = 1852 \text{ m} \quad \rightarrow \quad \boxed{1 \text{ m} = \frac{1}{1852} \text{ nmi}}$$

3 SOURCES

3.1 Image Sources

Image	File Name	Source	Copyright/License
Sun	`sun.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Moon	`moon.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Mercury	`mercury.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Venus	`venus.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Earth (Day)	`earth.png`	[12]	none [5, 12]
Earth (Night)	`earthnight.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Clouds	`clouds.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Mars	`mars.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Jupiter	`jupiter.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Saturn	`saturn.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Saturn Rings	`saturnrings.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Uranus	`uranus.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Neptune	`neptune.png`	[10]	CC Attribution 4.0 International (CC BY 4.0) [1, 10]
Pluto	`pluto.png`	[8]	none [8]
Milky Way	`milkyway.png`	[10]	CC Attribution 4.0 International (CC BY 4.0)
Stars	`stars.png`	[10]	CC Attribution 4.0 International (CC BY 4.0)

3.2 References for Code

3D Earth Example (`earth_example.m`) [4]:

- Use of `ellipsoid` function to render the Earth.

Earth-sized Sphere with Topography (`earth_sphere`) [2]:

- Handling of unit conversions.

REFERENCES

[1] *Attribution 4.0 International (CC BY 4.0)*. creative commons. https://creativecommons.org/licenses/by/4.0/. (accessed: January 27, 2021).

[2] Will Campbell. *Earth-sized Sphere with Topography*. MATLAB Central File Exchange. https://www.mathworks.com/matlabcentral/fileexchange/27123-earth-sized-sphere-with-topography. (accessed: January 22, 2021).

[3] *Department of Defense World Geodetic System 1984*. Tech. rep. NIMA TR8350.2. https://apps.dtic.mil/sti/pdfs/AD1000581.pdf. National Imagery and Mapping Agency, 2004.

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[5] *Image Use Policy*. NASA visible earth. https://visibleearth.nasa.gov/image-use-policy. (accessed: January 23, 2021).

[6] *Mercury Fact Sheet*. NASA. https://nssdc.gsfc.nasa.gov/planetary/factsheet/mercuryfact.html. (accessed: January 22, 2021).

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[13] David A. Vallado. *Fundamentals of Astrodynamics and Applications*. 4^{th}. Hawthorne, CA: Microcosm Press, 2013.

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