Study of Tungsten Blackbody Source's Radiance Spectrum, The Planck Model and Wien's Displacement Law

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Abstract

By utilizing a setup consisting of a tungsten blackbody source being heated up using a heater being passed through two sets of mirrors with 98.5% reflectivity in focused configuration, a circular beam chopper, and an equilateral barium fluoride prism, and the deflected beam passing through a thermopile detector, we were able to plot the radiance spectrum of our blackbody source over temperatures of $700^{\circ}C$, $800^{\circ}C$, $900^{\circ}C$, $1000^{\circ}C$, and $1100^{\circ}C$, and determine the maximum/peak wavelength λ_{max} having an inverse relationship with the temperature of the blackbody source, abiding by Wien's displacement law $\lambda_{max} = \frac{b}{T}$ at higher temperatures, where our fit derived the Wien's displacement constant b of $2.714 \cdot 10^{-3}\ m\ K$, a 6.349% error from the National Institute of Standards and Technology's documented Wien's displacement constant of $2.898 \cdot 10^{-3}\ m\ K$ [1]. We also concluded that Planck's model was the most accurate in representing the blackbody radiance spectra when compared against the Rayleigh-Jeans model, and utilized the Planck model to fit our data to a representation of the blackbody spectrum over five different temperature levels.

1 Introduction

1.1 Physics Motivation

Every object with a properly defined surface and geometry absorbs light at certain frequencies and reflects back some others. The majority of objects on Earth "has color", which means they absorb all other visible light wavelengths and reflect back that one color, or multiple colors.

An ideal blackbody is an object that theoretically absorbs all frequencies of light emitting towards it, and due to the laws of thermodynamics, this ideal blackbody needs to be able to re-emit all of that light back, as much as it absorbed. The study of such materials is crucial to the employment of thermal imaging and calibration, as well as aiding the study and understanding of stars and galaxies in the Observable Universe, as stars are often modeled in simulations and renders as blackbodies. More than that, a study by Wilson and Penzias (who were both Nobel prize winnners for heir discovery) showed that the Cosmic Microwave Background radiation, the electromagnetic radiation remnant from the Big Bang, possesses a radiation spectrum extremely close to a blackbody curve with a temperature of 2.275 K, and this discovery was interpreted as the Universe's expansion and cooling for about 13.7 billion years [2]. This proves that studies of blackbody radiation have great potential into taking humankind further back into the origins of our Universe, Ionization, and the first formation of galaxies and stars.

1.2 Historical context

The history of blackbody radiation dates back all the way to the 1800s, with German-British astronomer William Herschel. Herschel experimented with a light spectrum created by a prism and measured the temperature at different points of the spectrum, which led to the discovery that the temperature was highest below the visible red part of the light spectrum, which we now know is infrared light. Infrared light, for the majority of bodies on Earth, is the primary source of thermal radiation [3].
After that, in the 1850s, Scottish physicist Balfour Stewart discovered that lampblack surfaces had the greatest radiation absorption and self-radiation intensity compared to other non-black surfaces, since black surfaces tend to absorb the greatest light and thus the greatest radiation as well [4]. One year after Stewart's discovery, German physicist Gustav Kirchhoff also independently made the same discovery, and expanded on such findings with his own theory of thermal emission, stating that a surface that is in thermal equilibrium has an equal capacity for thermal radiation absorption and emission. Thus, any given body that is capable of absorbing all thermal radiation, in other words, a blackbody, will have the most thermal radiation emission capability. Moreover, Kirchoff's law of thermal radiation also helped confirm that the ratio between absorption and emission of a perfect blackbody is a function of temperature and nothing else, at least not yet with Kirchhoff's findings [5].

About 40 years later, another German physicist Max Planck refined the concept and discovered the function which gives a blackbody's ratio between emission and absorption, showing that radiation increases as temperature increases, and the wavelength with the highest radiation decreases as temperature increases [5].

2 Theoretical background

The first precise measurements of the blackbody radiance spectrum were conducted by William Wein, Otto Lummer, Ferdinand Krulbaum, and Ernst Pringsheim at the Physikalisch-Technische Reichsantalt (PTR) in Berlin. Their observations demonstrated the radiance spectrum of a perfect blackbody has a maximum wavelength inversely proportional to the absolute temperature of the blackbody, that is

$$\lambda_{max} = \frac{b}{T}$$

where b is called the Wien's displacement constant, and possesses a value of $2.898 \cdot 10^{-3}\ m\ K$ [1]. This states that the hotter an object is, the shorter wavelength its radiation is. In 1896, Wien developed a formula for the blackbody radiance spectrum with dependences upon the wavelength *lambda* and temperature T as

$$B_{Wien} = \frac{A}{\lambda^5} \cdot e^{-\frac{a}{\lambda T}}$$

and A and a are constants that can only be obtained during case-by-case experimentation. In 1900, scientists Rayleigh and Jeans worked both theoretically and experimentally to figure out the radiance spectrum of a perfect blackbody as

$$B_{Rayleigh-Jeans} = \frac{2ckT}{\lambda^4}$$

In 1900, German physicist Max Planck wrote another theory also on the radiance spectrum of an ideal blackbody as

$$B_{Planck} = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

where h is known as *Planck's constant*, approximately equal to $6.626 \cdot 10^{-34}\ J \cdot s$. Experimentation throughout the years have confirmed that Planck's theory is the most accurate one, however, if the wavelength is calculated when the Planck spectrum is at its maximum, the result would end up at Wien's theory [6].

3 Experimental setup

3.1 Apparatus

Our experiment utilizes a setup consisting of a tungsten blackbody source with an adjustable temperature controller, whose radiation is passed through a narrow slit and reflected off of a parabolic mirror to be redirected about 90° and into an equilateral barium fluoride prism. This prism refracts the radiation into a parallel output beam that is intercepted with another parabolic mirror that focuses the parallel outgoing beam from the prism into a narrow slit and finally a thermopile detector. The second parabolic mirror, the final slit and the detector are mounted on top of a motorized-rotatable platform that can be automatically be controlled by LabView to conduct deflection angle sweeps and record intensity of radiation over increments to determine the radiance spectrum. We set the blackbody radiation source to be at five different temperature increments - $700°C$, $800°C$, $900°C$, $1000°C$, and $1100°C$ and used LabView as mentioned above to initially measure and plot the radiance spectrum over an angular range of $32 - 36°$. We then used a Python inverse module with deflection-angle-vs-wavelength relation equation to convert the spectrum plot from intensity vs. deflection angle to intensity vs. wavelength.

3.2 Data Collection

Initially, we placed a 635mm wavelength LED light in place of the blackbody source in order to be able to calibrate the proper angles for the focusing mirrors and the barium fluoride prism. We adjusted the angles of each component until the full radiation beam is fully reflected into one side of the prism, refracted out of another and reflected off of the final mirror to be focused as a narrow slit focused on the thermopile detector. Then, we started an initial sweep of the platform angles to determine the proper minimum deflection angle of the laser to calibrate the platform motor. As a result, the peak, or maximum intensity of the laser signal, occurs when the angle of the platform is at $34.894°(see Appendix)$. We used this exact angle to calibrate the proper real-life angle vs. LabView motor controller angle indication.

After having the setup properly calibrated, we switched the LED out for the blackbody radiation source, and started by configuring the temperature of the blackbody to $700°C$, and measure the radiance spectrum of the blackbody by rotating the platform by $0.01°$ increments from $32°$ to $36°$, then afterwards increasing our temperature by $100°C$ increments and recording the same spectra until $1100°C$. Our resulting data was a cumulative intensity vs incident angle plot showing the signal peaking at different intensities for different temperatures as such:

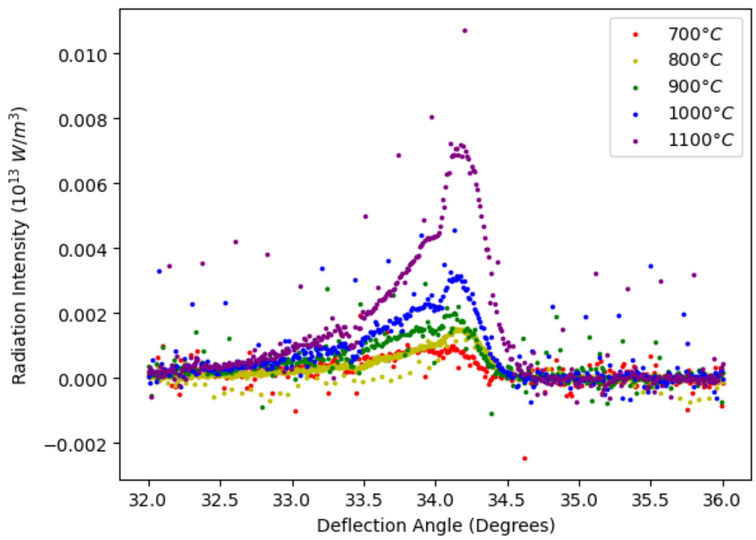

Figure 1: Radiation intensity through deflection angles at 5 different temperatures.

It is important to note that the radiance spectra we obtained are across an angle range of 32 to 36°, instead of the light wavelength spectrum. Thus, we used a combination of the equation of the *deflection angle of the light beam by a prism vs. refraction index*, the equation of the *refraction index of barium fluoride vs. wavelength of source*, and a function inverse Python module to produce an equation of direct relationship between the deflection angle produced by the prism and the wavelength (see Appendix). We then used this equation to transform our existing dataset of all 5 temperatures into a sweep across wavelengths in search of the peak wavelength of the blackbody source at 5 temperatures to see where the intensity of radiation is the greatest.

The resulting data was then plotted again, with the new plot representing the true radiance spectrum of radiation intensity through different wavelengths, as such:

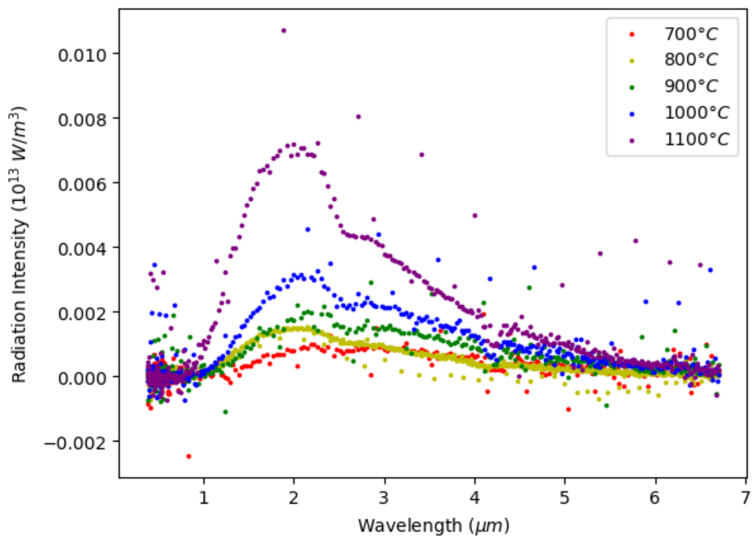

Figure 2: Recorded Radiance Spectra - Radiation intensity through wavelengths at 5 different temperatures.

A common trend was shared among our recorded data for all 5 temperatures was that absorptions lines were spotted at around wavelengths of $\lambda_1 \approxeq 2.7\mu m$ and $\lambda_2 \approxeq 4.2\mu m$. This is highly likely due to the chemical makeup within the Earth's atmosphere, specifically mostly water (vapor) particles absorbing light signals of wavelengths of around 2.7 to $2.9\mu m$, and carbon dioxide particles absorbing light signals of wavelengths of around 4 to $4.2\mu m$ [7].

We know that at higher temperatures, radiation data would be more prevalent across the entire spectrum against our environmental light noise, so we've picked out the temperature of $1100°C$ to experiment with the most efficient within the two theories - the Planck theory and the Rayleigh-Jeans theory. The recorded data was fitted against the two equations, and represented them in the plots below:

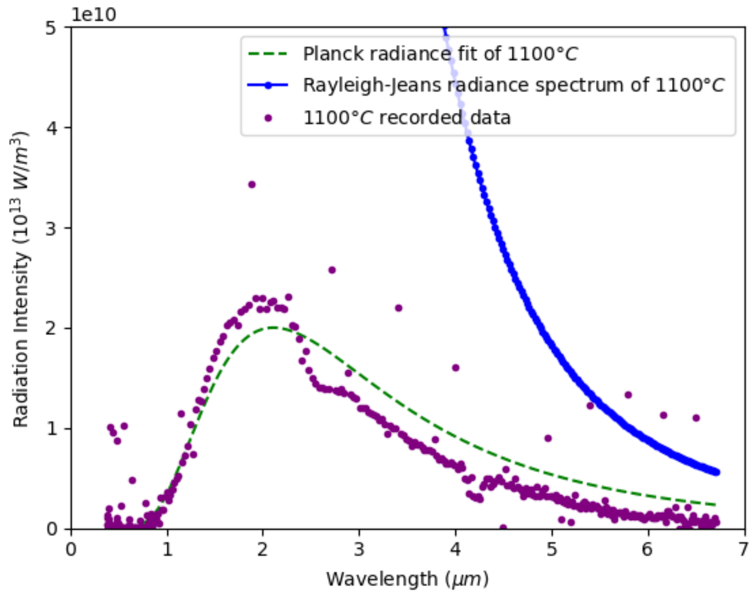

Figure 3: Comparison of 1100°C data fitted against Planck and Rayleigh-Jeans models.

It is evident from this fit comparison that the Planck model fit was a significantly better fit compared to the Rayleigh-Jeans fit, and thus we would be using the Planck fit for all data from all of our temperatures for a demonstration of Wien's displacement law.

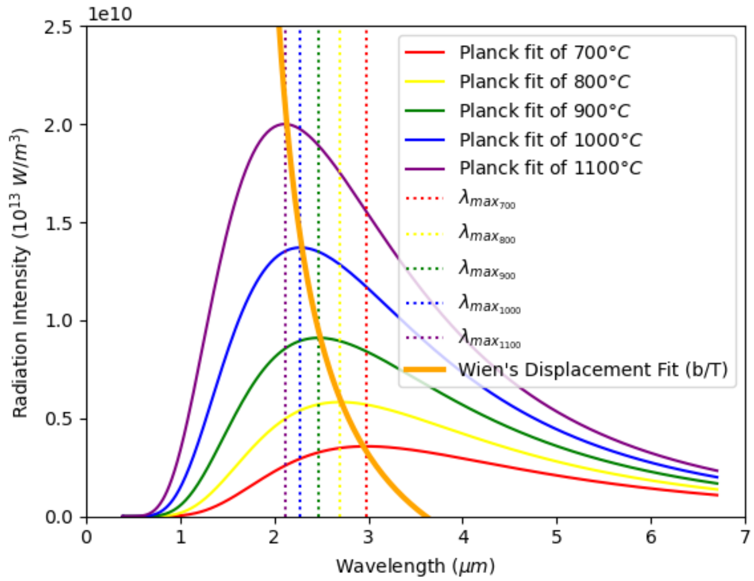

Figure 4: All 5 Planck-fitted temperatures, with peak wavelength fitted using Planck's displacement law of $\lambda_{peak} = b/T$.

After fitting all of our temperatures with the Planck model for blackbody radiation, we determined the peak wavelength of each fit to be - $2.873\mu m$ for $700°C$, $2.703\mu m$ for $800°C$, $2.473\mu m$ for $900°C$, $2.274\mu m$ for $1000°C$, and $2.013\mu m$ for $1100°C$. A fit to $\alpha T + \beta$ was also determined for Wien's displacement constant to be around $2.714 \cdot 10^{-3}$ m K, an approximate 6.349% error from the commonly-used Wien's displacement constant of $2.898 \cdot 10^{-3}$ m K documented by NIST [1].

4 Results

Through the process of using our setup to measure the radiance spectrum and deriving the wavelength component from a deflection angle/wavelength relation equation, we were able to extract a radiance spectrum of our tungsten blackbody source through five different temperature increments - $700°C$, $800°C$, $900°C$, $1000°C$, and $1100°C$. Moreover, we were also able to determine that Max Planck's theory and formula on blackbody radiation is a much better representative of our recorded data than Rayleigh-Jeans' theory and formula. By further fitting all the data into the Planck model, we were able to determine that the wavelength with the greatest radiation intensity of each temperature does follow Wien's displacement law, and by fitting our peak wavelengths into an inverse temperature plot, our value of Wien's displacement constant was $2.714 \cdot 10^{-3}$ m K, an approximate 6.349% error from NIST's documented Wien's displacement constant of $2.898 \cdot 10^{-3}$ m K.

References

[1] National Institute of Standards and Technology, **Wien wavelength displacement law constant b**.

[2] COSMOS - The SAO Encyclopedia of Astronomy, **Blackbody Radiation**, Section B.

[3] Glenn Elert, 2004, **The Physics Hypertextbook**.

[4] Swineburne University, 1998, **Blackbody Radiation, COSMOS**.

[5] Pierre Marie-Robitaille, 2012, **Kirschoff's Law of Thermal Emission: 150 Years**.

[6] Daniel J. Heinzen, 2018, **Blackbody Lab Manual Phys 353L**, pg. 4-6.

[7] MKS Instruments, **IR Absorption Spectroscopy**, Figure 1 - IT Transmission Spectrum of various gases and vapors.

Appendices

4.1 Laser minimum deflection angle

With $n = 1.4733$ being the refraction index of barium fluoride, we have

$$\sigma_{min} = 2sin^{-1}\left(\frac{n}{2}\right) - 60° = 34.894°$$

4.2 Wavelength and deflection angle relation

$$\sigma = 47.447° + sin^{-1}\left(nsin\left(60° - sin^{-1}\left(\frac{0.73665}{n}\right)\right)\right) - 60°$$

$$n = \sqrt{1.33973 + \frac{0.8107\lambda^2}{\lambda^2 - 0.01013} + \frac{0.19652\lambda^2}{\lambda^2 - 892.2169} + \frac{4.52469\lambda^2}{\lambda^2 - 2896.5924}}$$