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# ECON 301 - INTERMEDIATE MICROECONOMIC ANALYSIS

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## 进阶微观经济学分析

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# 1 Consumer Behaviour (消费者行为)

Consumer theory is an economic theory that describes consumer behaviour. This theory focuses on the **demand side** of the market, **explains situational decision-making** which includes *budget allocation, savings and investment, and decisions when facing uncertainties*. It may further **predict individuals' responses** to changes in, for example, prices, taxes, income, and the interest rate.

We make the assumption that the consumer always choose the **best** bundle of good and service that can be afforded. This covers both the consumer's *preference* (偏好) and *budget constraint* (预算限制) .

## 1.1 Consumer Preferences (消费者偏好)

We first define a commodity bundle as a **particular combination of goods and services**, and the preference of a consumer means the **ability to compare and rank commodity bundles**. In this case, we represent bundles in uppercase, and the quantity of each commodity in the bundle in lowercase.

A commodity space describes the combinations of commodity bundles, where we also assume that consumers can consume in partial units, as we can simply adjust the time frame during which the bundle is consumed.

**Definition 1.1.** Consider 2 commodity bundles  $A$  and  $B$ . We define:

1.  $A$  is preferred to  $B$ :  $A \succ B$
2.  $B$  is preferred to  $A$ :  $B \succ A$
3.  $A$  and  $B$  are equally preferred, or indifferent:  $A \sim B$

Furthermore, we make 4 assumptions about consumer preferences:

1. Completeness and rankability (完全性与可排序性) : The consumer can make one of the three statements about two bundles  $A$  and  $B$  as in **Definition 1.1**.

2. Monotonicity (单调性) :

Ceteris paribus

1. Weak monotonicity: the more the consumer consumes a good, the consumer cannot be worse off.

2. Strict monotonicity: the more the consumer consumes a good, the consumer is better off.

If unwanted units can be discarded at no cost, assume weak monotonicity;

for most goods over realistic ranges of consumption, assume strict monotonicity.

3. Transitivity (传递性) :

Given bundles  $A$ ,  $B$  and  $C$ ,

1. If  $A \succ B, B \succ C$ , then  $A \succ C$
2. If  $A \sim B, B \sim C$ , then  $A \sim C$

4. Strict convexity (严格外凸性) : ceteris paribus, the more one has of Good A, the less one is willing to give up of Good B in exchange for more Good A. This is because people value variety in consumption.

In some cases, the assumptions of monotonicity and strict convexity may not apply.

## 1.2 Describing Preferences (描述偏好)

There are three methods to describe a consumer's preference: ranking every possible pair of consumption bundles (impractical), using a *utility function* (效用函数), or using *indifference curves* (无差异曲线).

### 1.2.1 Utility Function (效用函数)

We want the utility function to have these properties:

1. The utility of a more preferred bundle is larger than a less preferred bundle.
2. When two bundles are equally preferred, the utility should be equal.
3. The utility number should reflect the level of satisfaction.

In modern economic theory, we assume utility numbers to be **ordinal** not **cardinal**, that is reflection of ranking, NOT reflection of size.

Mathematically,

**Definition 1.2.** Consider a function  $U(\cdot)$ , where

1.  $U(A) > U(B) \iff A \succ B$
2.  $U(A) = U(B) \iff A \sim B$

If there are only two goods, whose quantities are represented by  $t, c$ , then a general utility function is  $U = U(t, c)$ , and a specific utility function can be  $U(t, c) = t^{0.6}c^{0.4}$ . There are several types of utility functions which will be discussed later with indifference curves.

For one particular set of preferences, there can be many utility functions, thus

**Definition 1.3.** Assume a utility function for two goods with quantities  $q_1, q_2$  to be  $U = f(q_1, q_2)$ , if we have another function  $V = g(U)$  such that  $\frac{dV}{dU} > 0$ , then we say  $g$  is a *positive monotonic transform* (正方向单调变换).

This thus allows us to describe the *marginal utility* (边际效用) of good  $i$  to be the rate-of-change of the total utility as  $q_i$  increase by a small amount. Mathematically,

we have

$$MU_i = \frac{\partial U}{\partial q_i}$$

### 1.2.2 Indifference Curves (无差异曲线)

The indifference curves are formed in the commodity space by constructing a set of bundles equally preferred to some reference bundle.

The strict monotonicity assumption indicates such curves must have a negative slope, and the strict convexity assumption indicates such curves must be smooth and bow in toward the origin.

We can view them as contours on a map, and since we allow consumers to consume fractional units, there are  $\infty$  indifferent curves in a 2D commodity space, that is we can always draw another IC between any two ICs.

A key property of indifference curves is that they **cannot cross each other**, as that would contradict the assumptions we made for them.

We can thus introduce the *marginal rate of substitution* (边际替换速率), which is defined from the negative slope of ICs:

$$MRS = -\frac{dq_2}{dq_1}$$

This represents the consumer's marginal willingness to pay. With the strict convexity assumption, the *MRS* will always be diminishing.

For a particular utility function  $U = U(q_1, q_2)$ , the marginal rate of substitution can always be expressed as

$$MRS = \frac{MU_1}{MU_2}$$

where positive monotonic transforms do not impact this relationship.

### 1.2.3 Types of Preferences (偏好类型)

There are four major types of preferences.

The first one is *linear preferences* (线性偏好), they represent two goods that are **perfect substitutes** of each other (e.g. Pepsi and Coca Cola). The *MRS* of two goods will be a constant, and in general, it is represented by the utility function:

$$U(q_1, q_2) = mq_1 + nq_2, m > 0, n > 0$$

The *MRS* would thus be  $\frac{m}{n}$ .

The second one is *Leontief preferences* (里昂提夫偏好), they represent two goods that are **perfect complements** of each other (e.g. meat patty and bun). In general, it is represented by the utility function:

$$U(q_1, q_2) = \min \{aq_1, bq_2\}, a > 0, b > 0$$

The corner points are perfect combinations, and  $a q_1 = b q_2$ .

The third one is **Cobb-Douglas preferences** (柯布——道格拉斯偏好), they can represent both substitutability and complementarity, as they capture changes in preferences as consumption changes. In general, it is represented by the utility function:

$$U(q_1, q_2) = q_1^\alpha q_2^\beta, \alpha > 0, \beta > 0$$

This is usually written in a way such that  $\alpha + \beta = 1$ , where a positive monotonic transform can be  $V = U^{\frac{1}{\alpha+\beta}}$ .

The last one is **quasilinear preferences** (准线性偏好), they represent a consumer's preference where one will spend all the income on good 1 when the income is low, and as income increases, there will be a cap on this good. The ICs is merely a vertical translation, so that for the same  $q_1$ , the  $MRS$  on each IC will equal to each other. In general, it is represented by the utility function:

$$U(q_1, q_2) = \alpha f(q_1) + \beta q_2, \alpha > 0, \beta > 0, \frac{df}{dq_1} > 0, \frac{d^2 f}{dq_1^2} < 0$$

### 1.3 Well-behaved Preferences (良态偏好)

These preferences are complete and rankable, transitive, strictly monotonic, and **convex** (not strictly convex).

Formally,

**Definition 1.4.** Consider two bundles  $A$  and  $B$ ,  
if  $A \sim B$  and the preference is convex, given  $0 \leq t \leq 1$ , we have

$$tA + (1-t)B \succeq A$$

that is a combination of  $A$  and  $B$  will be preferred or equally preferred to  $A$  itself

### 1.4 Budget Constraints (预算限制)

With a budget of  $m$  and unit price  $p_1, p_2$ , the budget constraint is described as

$$p_1 q_1 + p_2 q_2 \leq m$$

A **budget set** (预算集) is the set of all consumption bundles that satisfy the budget constraint, where the **budget line** is the set of all bundles that **exhaust** the budget. Therefore, the budget line can be described as:

$$p_1 q_1 + p_2 q_2 = m \rightarrow q_2 = -\frac{p_1}{p_2} q_1 + \frac{m}{p_2}$$

The slope  $-\frac{p_1}{p_2}$  represents the opportunity cost.

The budget constraint can also appear in the form of an endowment.

To analyze one commodity, one may use the budget constraint with composite good, where the composite good is all goods except for the good of interest (aggregate good), which is measured in dollars with unit of price to be \$1.

The budget constraint can also be non-linear, some typical types include:

1. Quantity discounts: a discount is applied after the consumer consumes more than a certain amount of goods.
2. Quantity limit: consumers cannot buy more than a certain amount.
3. Food stamps: a type of stamp that can only exchange for a certain type of good for a certain amount.

## 1.5 Rational Constrained Choice (理性的受限选择)

Since we assume the consumers are utility maximizers, then the consumer's problem can thus be described as

$$\max_{q_1, q_2} U(q_1, q_2) \text{ s.t. } p_1 q_1 + p_2 q_2 = m$$

Assume the consumer's optimal bundle consists of two quantities  $q_1^*, q_2^*$ , and these two quantities are referred to as the **consumer's ordinary demand**. This optimal bundle is **interior**, i.e.  $q_1^* > 0, q_2^* > 0$

**Definition 1.5.** In economic models, *exogenous variables* are variables determined outside the model, where *endogenous variables* are variables determined within the model.

At the optimal bundle, the slope of IC = the slope of BL (budget line), that is:

$$\begin{aligned} \frac{dq_2}{dq_1} &= -\frac{p_1}{p_2} \\ MRS &= \frac{p_1}{p_2} \end{aligned}$$

So this indicates, given the budget constraint and the tangency condition, the optimal bundle satisfies

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

where the ordinary demand functions are expressed as

$$q_1^* = q_1^*(p_1, p_2, m), q_2^* = q_2^*(p_1, p_2, m)$$

### 1.5.1 Computing Ordinary Demands (计算常规需求)

Two methods can be used to compute ordinary demands. The first one involves the budget constraint and tangency condition, and the second one involves the Lagrangian.



Consider the Cobb-Douglas preference, using either methods, we have

$$q_1^* = \frac{\alpha}{\alpha + \beta} \cdot \frac{m}{p_1}$$

$$q_2^* = \frac{\beta}{\alpha + \beta} \cdot \frac{m}{p_2}$$

Consider the Leontief preference, we have

$$q_1^* = \frac{m}{p_1 + \frac{a}{b}p_2}$$

$$q_2^* = \frac{\frac{a}{b}m}{p_1 + \frac{a}{b}p_2}$$

We can think the denominator as the price of 1 unit of combination of the good.

Consider the linear preference, the optimal bundle is usually a corner solution, where either  $q_1^* = 0$  or  $q_2^* = 0$ , it depends on whichever is larger  $\frac{MU_1}{p_1}, \frac{MU_2}{p_2}$ .

For quasilinear preferences, it depends on the income of the consumer. Consider a threshold income  $\bar{m}$ , if  $m \leq \bar{m}$ , it would be corner solution where all money will be spent on good 1; if  $m > \bar{m}$ , it would be an interior solution where  $q_1^* = \frac{\bar{m}}{p_1}, q_2^* = \frac{m - \bar{m}}{p_2}$ . For the interior solution, notice that  $MRS = \frac{p_1}{p_2}$  still holds.

## 1.6 Indirect Utility Function (非直接效用函数)

These functions represent the maximum utility as a function of exogenous parameters only, that is substituting

$$q_1^*, q_2^*$$

for  $q_1, q_2$  in the utility function. We denote this function as  $V$ :

$$V = V(q_1^*, q_2^*)$$

## 1.7 Expenditure Minimization (花费最小化)

This problem is choosing the bundle that minimizes the expenditure while reaching a certain utility. Mathematically, this is described as:

$$\min_{q_1, q_2} p_1 q_1 + p_2 q_2 \text{ s.t. } U(q_1, q_2) = \bar{U}$$

This can be solved using the Lagrangian.

## 2 Demand (需求)

### 2.1 Properties of Demand Functions (需求函数的性质)

#### 2.1.1 Change in Income (收入变化)

**Definition 2.1.** The *income expansion path* (收入扩充曲线) is the locus of optimal bundles traced out as  $m$  changes while keeping  $p_1, p_2$  constant.

**Definition 2.2.** The *Engel curves* (恩格尔曲线) represent the relationship between  $m$  and  $q_1^*, q_2^*$  respectively.

For Cobb-Douglas preference, we have

$$m = p_1 q_1^* \cdot \frac{\alpha + \beta}{\alpha}$$

$$m = p_2 q_2^* \cdot \frac{\alpha + \beta}{\beta}$$

where they are linear Engel curves. We also have

$$q_2^* = \frac{\beta}{\alpha} \cdot \frac{p_1}{p_2} \cdot q_1^*$$

as the IEP. Notice that the slope of IEP represents the  $MRS$  is constant.

The Leontief preference has the IEP connecting all the corner bundles.

The linear preference has the IEP aligned either along the  $x$ -axis, or the  $y$ -axis, such that the Engel curves would be combination of vertical line along the  $y$ -axis, and a linear line.

The Cobb-Douglas, Leontief, and linear preferences are considered *homothetic preferences* (同源偏好), where they all have

1. Linear Engel curves (through the origin)
2. Fixed proportion of income spent on each good
3. Same  $MRS$  along a ray through the origin, that is  $MRS = f(\frac{q_2}{q_1})$

For normal goods, if  $q^*$  grows more rapidly than  $m$ , then it is a luxury good; if  $q^*$  grows less rapidly than  $m$ , then it is a necessary good.

For quasilinear preference, the IEP would be a vertical line at  $\bar{q}_1$ , the threshold quantity.

#### 2.1.2 Change in Price (价格变化)

**Definition 2.3.** The *price consumption curve* (价格消费曲线) is the locus of optimal bundles traced out as  $p$  changes while keeping  $m$  constant.

In the case of a Cobb-Douglas preference, given a fixed  $\alpha, \beta, m, p_2$ , we would yield the demand curve for good 1, where the ordinary demand function is already calculated

to be  $q_1^* = f(p_1) = \frac{\alpha}{\alpha+\beta} \cdot \frac{m}{p_1}$

## 2.2 Income and Substitution Effects (收入效应和替换效应)

When price changes, we consider it having two effects: the *income effect* and the *substitution effect*.

For example, if the price of good 1 drops:

1. The relative price of good 1 drops (opportunity cost drops), so that consumers buy more of good 1. This is the substitution effect (*SE*).
2. The budget set expands such that the real income increases, so that consumer buy more of good 1. This is the income effect (*IE*).

Assume  $m, p_2$  to be constant, as  $p_1$  drops,  $U^O \rightarrow U^F, q_1^O \rightarrow q_1^F, q_2^O \rightarrow q_2^F$ , where  $O$  represents "original", and  $F$  represents "final", then the total effect (*TE*) is

$$TE = SE + IE = q_1^F - q_1^O$$

The substitution effect is described as: how would  $q_1$  change if  $m$  drops to just attain original utility after  $p_1$  drops. This forms the *compensated budget line* (补偿预算线). This is the Hick's decomposition where we attain the original utility. If we label  $C$  as the cost minimization point on the compensated budget line, then

$$SE = q_1^C - q_1^O$$

then

$$IE = q_1^F - q_1^C = TE - SE$$

For normal goods, *SE* and *IE* reinforce each other, that is they are always the same sign.

For inferior goods, *SE* and *IE* oppose each other, where  $|SE| > |IE| : TE > 0$  if  $p$  drops.

For Giffen goods (extremely inferior), we have  $|SE| < |IE| : TE < 0$  if  $p$  drops.

For Leontief preference,  $SE = 0, TE = IE$ .

For linear preference, it can be the case that  $SE = IE = TE = 0$ , or the case that  $SE = TE, IE = 0$ , or  $TE = SE + IE$ .

For quasilinear preference, we usually analyze from one interior bundle to another interior bundle, where  $SE = TE, IE = 0$ .

## 2.3 Welfare Effect of a Price Change (价格变化的福利效应)

There are three *money measures* of a price change: compensating variation, equivalent variation, and change in consumer surplus.

### 2.3.1 Compensating Variation (补偿变化)

The compensating variation is the change in  $m$ , where if there is a change in  $p$ , the compensating variation will restore consumer's original utility level.

This is often considered when there is a price increase, we can think this as a compensation for a price increase, or a take-away for a price decrease.

### 2.3.2 Equivalent Variation (等效变化)

The equivalent variation is the change in  $m$ , where if there is a change in  $p$ , the equivalent variation will reach the consumer's final utility level.

### 2.3.3 Change in Consumer Surplus (消费者效用盈余变化)

The change in consumer surplus will be the area under the **Marshallian demand curve**, use integral to solve for the change.

## 2.4 Relationship between $CV, EV, \Delta CS$

If the  $IE = 0$ , then  $|CV| = |\Delta CS| = |EV|$  If the  $IE \approx 0$ , then  $|CV| \approx |\Delta CS| \approx |EV|$  The larger the income effect of the price change, the larger the difference between the three welfare measures.

The income effect is typically small for goods that account for only a small fraction of total expenditure.

### 3 Production (生产)

**Definition 3.1.** A *firm* (企业) is an organization that transforms inputs (resources) into outputs (goods and services).

**Production** is the process of transforming inputs into outputs, including other ancillary activities.

Production theory analyzes how much output should be produced, and what combination of inputs should be used to produce.

We make the following assumptions about production:

1. Produces single good.
2. Has already selected the good to be produced.
3. Uses only two inputs, labour ( $L$ ) and capital ( $K$ ).
4. Can buy unlimited quantities of both inputs at fixed prices.
5. Can access unlimited financial resources through bankers, investors.

Thus, we can consider a **production function (生产函数)**, such that it **shows the maximum amount of output given some input, can be represented by an equation, a graph, or a table.**

The inputs and outputs are measured in physical units per period of time, so if we assume two inputs, the general production can be

$$Q = f(L, K)$$

Notice that **positive monotonic transformation** does not apply to production functions as the output is cardinal, not ordinal.

Firms can vary some inputs more easily and more quickly, so inputs can be classified as either fixed or variable, where

1. Fixed input: an input that cannot be varied over the given time frame.
2. Variable input: an input that can be varied over the given time frame.

Thus, we can define

**Definition 3.2.** The **long run** is the time frame in which all inputs are variable; the **short run** is any time frame short enough that at least 1 input will not vary.

#### 3.1 Production with One Variable Input (单变量生产输入)

This is a short run production.

The **total product (所有产出)** curve shows the how  $Q$  changes by holding 1 input constant, and 1 input variable.

For most short-run TP curves:

1.  $TP = 0$  when  $L = 0$
2. Initial: increasing at an increasing rate
3. Then: increasing at a decreasing rate
4. Finally: decreasing

The **marginal product** (边际产出) is defined to be

$$MP_L = \frac{dTP_L}{dL}$$

Notice that marginal product initially increase (labour specialization), then starts to decline at some point, finally becoming negative (crowding). This is the **Law of Diminishing Marginal Returns** (边际产出递减), which only applies in short run.

The **average product** (平均产出) is defined to be

$$AP_L = \frac{TP_L}{L}$$

The relationship between  $AP$  and  $MP$  is as follows:

1. when  $MP > AP$ ,  $AP$  is increasing
2. when  $MP = AP$ ,  $AP$  obtains its maximum
3. when  $MP < AP$ ,  $AP$  is decreasing

### 3.2 Production with Two Variable Inputs (双变量生产输入)

This could be a short-run or long-run production depending on the number of inputs required.

We use the **production isoquant** (等产出线) to refer to all the combinations of inputs for some given output. We mainly focus on negative-sloped isoquants.

This introduces the concept of **marginal rate of technical substitution** (技术替换边际速率), that is the rate at which a small quantity of one input can be substituted by the other given a constant output. Mathematically,

$$MRTS = -\frac{dK}{dL}$$

$$MRTS = \frac{MP_L}{MP_K}$$

#### 3.2.1 Substitutability of Inputs (生产输入的可替换性)

Consider a Leontief production function,  $Q = \min\{ax_1, bx_2\}$ , the  $MRTS$  is undefined.

Consider a linear production function,  $Q = \alpha x_1 + \beta x_2$ , the  $MRTS$  is  $\frac{\alpha}{\beta}$ .

Consider a Cobb-Douglas production function,  $Q = Ax_1^\alpha x_2^\beta$ , where  $A$  measures the productive efficiency, the  $MRTS$  will be a function of  $x_1, x_2$ .

### 3.2.2 Isocost Lines (等成本线)

Given two inputs  $L, K$  with their unit price to be  $w, r$ , where  $w$  represents wage and  $r$  represents rent, then the firm's total cost is

$$C = wL + rK$$

If we consider  $K = f(L)$ , then from the above cost function,  $\frac{C}{r}$  is the intercept and  $-\frac{w}{r}$  is the slope.

For a fixed output target, if the firm wishes to maximize profit, it must minimize the costs. Given that

$$\pi = TR - TC$$

Cost-minimization occurs when the slope of the isoquant equals the slope of the isocost, that is

$$\frac{dK}{dL} = \frac{w}{r} \rightarrow \frac{dK}{dL} = \frac{dL}{r} \rightarrow \frac{MP_L}{w} = \frac{MP_K}{r}$$

When the input prices change then there will be a change in the input mix and a change in the **optimal mix of input**.

### 3.2.3 Firm's Long Run Problem (企业长期运营问题)

Consider a firm's production function  $Q = f(L, K)$ , the firm aims to minimize the cost, in the form

$$\min_{L, K} wL + rK \text{ s.t. } Q = \bar{Q}$$

We use the Lagrangian to solve for  $L^*, K^*$ , and then we can substitute back into the cost function to find the minimum cost.

If the production function is linear, the minimum cost depends on which input has a larger marginal product per unit.

If the production function is Leontief, the minimum cost occurs when  $\alpha x_1^* = \beta x_2^* = Q$ .

### 3.2.4 Returns to Scale (规模报酬)

This describes what happens to the output when all inputs are scaled by the same factor.

If the inputs are scaled by a factor of  $k$ , then

1. Constant returns to scale: output is also scaled by a factor of  $k$ .
2. Increasing returns to scale: output is scaled by a factor of  $> k$ .
3. Decreasing returns to scale: output is scaled by a factor of  $< k$ .

The *long-run expansion path* (长期扩大生产曲线) is the input combinations the firm would use at different output levels in the long run.

## 4 Costs (生产成本)

### 4.1 Costs in the Short Run (短期生产成本)

In the short run, the total costs can be broken down into fixed costs and variable costs, where  $TC = FC + VC$ .

Consider a production function,  $Q = f(L, \bar{K})$ , when  $Q = 0$ , we have  $TC = FC$ , and we further have  $MC = \frac{dTC}{dQ} = \frac{dVC}{dQ}$ .

Other relevant parameters and relationships include:

$$AFC = \frac{FC}{Q}, AVC = \frac{VC}{Q}, ATC = \frac{TC}{Q}$$

$$VC = \int_0^Q MC dQ$$

At  $Q = 1$ , we also have  $MC = AVC$ . If we label  $Q_V$  to be the quantity where  $AVC$  reaches a minimum and  $Q_T$  to be the quantity where  $ATC$  reaches a minimum, then when  $FC > 0$ ,  $Q_T > Q_V$ .  $MC$  curve always intersects  $AVC$  and  $ATC$  at their minimums.

### 4.2 Short-run and Long-run Costs (短期与长期生产成本关系)

Consider a firm with only  $L, K$  as its inputs, we can have one long-run expansion path, and many short-run expansion path given a certain level of  $K$ .

So for a fixed quantity of  $K$ , there is a unique output level where  $TC^{SR} = TC^{LR}$ , but for other output levels,  $TC^{SR} > TC^{LR}$ . This can be considered to be a cost penalty in SR compared to LR, except for that unique level of output.

Given these relationships, we also know  $ATC^{SR} \geq ATC^{LR}$ , and the equal sign is obtained at that unique output level. In the long run, a profit-maximizing firm would install the amount of capital that would minimize  $ATC$  of production at its target output level, thus, the  $ATC^{LR}$  is the **lower envelope** of the  $ATC^{SR}$  curves. The  $ATC^{LR}$  curves will be smooth if firm can use any quantity of capital it wants in the long run.

#### 4.2.1 Long-Run Average Costs

If  $ATC^{LR}$  decreases as  $Q$  increases, the firm is experiencing *economies of scale* (规模经济);

If  $ATC^{LR}$  remains the same as  $Q$  increases, the firm is experiencing *constant economies of scale*;

If  $ATC^{LR}$  increases as  $Q$  increases, the firm is experiencing *diseconomies of scale*.

A firm either experiences EOS and DOS or EOS + CEOS + DOS, as output expands. When the firm is only experiencing EOS + DOS, the quantity at which  $ATC^{LR}$  obtains its minimum is *efficient scale of operation*, where if there is also CEOS, the lowest quantity at which  $ATC^{LR}$  obtains its minimum is *minimum efficient scale*.



*of operation.*

The relationship between returns to scale and economies of scale is as follows:

1. If the firm is experiencing decreasing returns to scale: the firm can have either EOS, CEOS, or DOS.
2. If the firm is experiencing constant returns to scale: the firm can have EOS, or CEOS.
3. If the firm is experiencing increasing returns to scale: the firm can only have EOS.

## 5 Perfect Competition (完全市场竞争)

### 5.1 Firm Supply and Market Structure (企业供给与市场结构)

A firm's supply not only depends on its goals (assume profit maximization), the technology it can use (product / cost function), but also the structure of the market. The two structures we often use to analyze the market are: perfect competition, and monopoly.

### 5.2 Perfectly Competitive Market (完全竞争的市场)

In a perfectly competitive market, its characteristics include:

1. There are many independent buyers and sellers
2. Goods offered by sellers are identical
3. Each firm's output is only small fraction of industry output
4. Buyers know everything about products and price charged by each firm
5. In the long run, firms are free to enter or exit the market

These characteristics determine that firms are **price takers** in a perfectly competitive market.

### 5.3 Firm Demand and Marginal Revenue (企业需求与边际营收)

We use uppercase letter to represent the industry side, and lowercase letter to represent the firm side. In the industry side, the Law of Demand is still in effect, where there will be an equilibrium price at which the market clears. For individual firms, they have only one option: matching the equilibrium price at the industry level.

Each firm is only a small proportion compared to the market, and each one's ability to produce is limited by their cost structures.

Note that  $P = p = f(Q) \neq f(q)$ , given the total revenue to be  $TR = pq$  for individual firms, we have  $MR = \frac{dTR}{dq} = p$ .

### 5.4 Profit Maximization (利润最大化)

We are given  $\pi = TR - TC$ , and firms are price takers.

So a two-step process is involved:

1. First, determine which positive output level maximizes profit;
2. Then, check if  $q = 0$  will generate more profit

Since if  $MR > MC$ , then  $\pi$  will increase as  $q$  increases; if  $MR < MC$ , then  $\pi$  increases as  $q$  decreases. Thus, to maximize profit, we must have  $p = MR = MC$ . That means  $\pi_{\max}$  requires setting  $q$  when  $MC = MR$  and  $\frac{dMC}{dq} > 0$

### 5.5 Firm Supply in the Short Run (企业短期供给)

In the short run, the firm either produces  $q > 0$  s.t.  $p = MC, \frac{dMC}{dq} > 0$ , or produces  $q = 0$ .

Therefore, we are essentially solving one of these two problems

$$\max_q \pi = pq - (VC + FC)$$

$$\max_L \pi = pq - (wL + r\bar{K})$$

Both of which yields that  $p = MC^{SR} = \frac{w}{MP_L}$ .

In the short run, if a firm **shuts down** and produces  $q = 0$ , the firm must cover its fixed costs, this is not the same as **exiting the market**.

If the firm is producing at a positive quantity, then  $\pi^{SR} = TR - (VC + FC)$ ; if the firm is producing at  $q = 0$ , then  $\pi^{SR} = -FC$ .

A firm should shut down if  $-FC > TR - (VC + FC) \iff TR < VC \iff p < AVC_{\min}$ .

### 5.6 Firm Profit in the Short Run (企业短期利润)

Since the profit is determined by  $\pi = TR - (VC + FC)$ , and the producer surplus is  $TR - VC$ , we thus have

$$\begin{aligned}\pi &= TR - TC \\ &= pq - TC \\ &= q(p - ATC)\end{aligned}$$

Thus, if  $p > ATC$ , then  $\pi > 0$ ; if  $p = ATC$ , then  $\pi = 0$ ; if  $p < ATC$ , then  $\pi < 0$ . And when  $p = AVC_{\min}$ , the firm should shut down.

### 5.7 Industry Supply in the Short Run (产业短期供给)

In the short run, the number of firms in a market is fixed with no exiting or entering. Thus the aggregate quantity is the sum of quantities supplied by each firm. Assume each firm's supply function to be  $q = f(P)$ , and there are  $n$  firms in the market, then the industry supply function will be  $Q = nq = nf(P)$ .

For example, if one firm has the supply function  $q = \frac{1}{2}P$ , and there are 50 firms in the industry, then the industry has a supply function of  $Q = 50 \times \frac{1}{2}P = 25P$ .

### 5.8 Firm Supply in the Long Run (企业长期供给)

In the long run, the firm is essentially solving one of these two problems:

$$\max_q \pi = pq - C$$

$$\max_{L,K} \pi = pq - (wL + rK)$$

Both of which yields that profit is maximized when  $p = MC^{LR} = \frac{w}{MP_L} = \frac{r}{MP_K}$ .

Thus, a firm should either produce  $q > 0$  where  $p = MC^{LR}$ ,  $\frac{dMC}{dq} > 0$ , or produce at  $q = 0$ , where the firm ***exits the market***.

If the production is at  $q > 0$ , then  $\pi = TR - TC$ ; if  $q = 0$ , then  $\pi = 0$ , as no  $L, K$  is employed. A firm should exit the market when  $TR - TC < 0 \iff pq < TC \iff p < ATC^{LR}$ .

In the long run, there is no barrier of entry.

## 5.9 Equilibrium in the Long Run (长期均衡)

In the long run, in a perfectly competitive market, new firms can enter and incumbent firms can leave.

If  $\pi > 0$ , 1+ new firms will enter the industry:

1. increase supply
2. drive down the market price
3. decrease the profit of firms in the industry
4. positive economic profit: not in LR equilibrium

If  $\pi < 0$ , 1+ incumbent firms will exit the industry:

1. decrease supply
2. drive up the market price
3. increase the profit of firms in the industry
4. negative economic profit: not in LR equilibrium

If  $\pi = 0$ ,

1. no new firms will enter the industry
2. no incumbent firms will exit
3. this is in LR equilibrium

This mean  $\pi = q(p - ATC^{LR}) = 0 \iff p = ATC^{LR}$ , that is firms must operate at efficient scale of production at long run equilibrium.

## 5.10 Industry Supply in the Long Run (产业长期供给)

In the long run, we can assume the following scenario, the industry starts with

$$Q = 600, q = 10, n = 60, P = 34$$

Then there is an increase in demand, then the industry becomes

$$Q = 660, q = 11, n = 60, P = 48$$

More firms will enter the market, then the industry becomes

$$Q = 700, q = 10, n = 70, P = 34$$

## 6 Monopoly (完全市场垄断)

A monopoly is a market served by only single seller of a product with no close substitutes.

There are barriers to entry in a monopoly, these include:

1. extreme economies of scale
2. switching costs
3. product differentiation
4. control of key inputs
5. absolute cost advantages
6. government regulation

This means, the firm is a *price setter* in the market, and the industry is the firm.

### 6.1 Monopoly and Demand (垄断与需求)

We first assume *uniform pricing* (单一定价) for the firm, and the firm faces the market demand curve.

If the price increases, the firm sells less; if the price decreases, the firm sells more.

### 6.2 Marginal Revenue and the Price Elasticity of Demand (边际营收与需求价格弹性)

We know that  $TR = P \times Q$ , so  $MR = \frac{dTR}{dQ}$ , and by assuming that  $P = f(Q)$ , by knowing the price elasticity of demand is

$$\epsilon_D = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

then, the marginal revenue is

$$MR = P(1 - \frac{1}{|\epsilon_D|})$$

We take the absolute value of price elasticity of demand, as it is negative.

Note that:

1. When demand is elastic  $\rightarrow |\epsilon_D| > 1$ , we have  $MR > 0$
2. When demand is inelastic  $\rightarrow |\epsilon_D| < 1$ , we have  $MR < 0$
2. When demand is unielastic  $\rightarrow |\epsilon_D| = 1$ , we have  $MR = 0$

### 6.3 Marginal Revenue with Linear Demand (线性需求的边际营收)

Assume the linear demand function is characterized by

$$Q = \alpha - \beta P$$

Then, the inverse demand function is  $P = \frac{\alpha}{\beta} - \frac{1}{\beta}Q$  The marginal revenue function is thus  $MR = \frac{\alpha}{\beta} - \frac{2}{\beta}Q$

## 6.4 Monopoly and Profit Maximization (垄断与其利润最大化)

The goal of the firm is to

$$\max_Q \pi = TR(Q) - TC(Q)$$

This also indicates that profit maximization is achieved when  $MR = MC$  and  $\frac{dMC}{dQ} > 0$ . The market thus clears at some  $(Q_M, P_M)$ .

## 6.5 The Lerner Index (勒尔纳指数)

The Lerner index is measures of a firm's markup and indicates the amount of market power the firm enjoys. It is defined as

$$LI = \frac{P - MC}{P}$$

And we have its relationship with the price elasticity of demand to be

$$LI = \frac{1}{|\epsilon_D|}$$

This means, the more inelastic a product, the larger the market power the firm has. This equation is obtained at profit maximization level.

## 6.6 Profit under Monopoly (垄断的利润)

As usual,  $\pi = TR - TC = (P - ATC)Q$ .

The producer surplus is thus  $PS = TR - TVC = TR - \int MC dQ$ . Here we have  $PS > \pi$ , since  $FC > 0$ .

## 6.7 The Welfare Cost of Monopoly (垄断的福利成本)

Since monopoly charges  $P > MC$ , thus, the  $CS$  is lower under monopoly and the  $PS$  is higher compared to a perfectly competitive market.

Therefore, monopoly generates a deadweight loss.

## 7 Pricing Strategies (定价策略)

The pricing strategies describe how firm sets price(s) based on objectives and characteristics of the product and the market.

For advanced pricing strategies to be possible and profitable, the firm must

1. possess some degree of market power.
2. be able to cost-efficiently prevent resale and arbitrage.

**Definition 7.1.** *Price discrimination, or differential pricing* involves charging different consumers different prices for the same good, when price differences do not reflect differences in the cost of providing the good to the different consumer.

### 7.1 First-Degree Price Discrimination (一级价格分歧)

Beyond the definition of price discrimination, the first-degree price discrimination has two more conditions:

1. firm's customers have different willingness to pay (WTP) for the firm's product
2. firm can identify customers' WTP before transaction occurs

The final phenomena will be

1. each unit is sold at a different price (varies across buyers)
2. each unit is sold for consumer's maximum WTP

In this case,  $MR = D$ , with  $CS = 0$ ,  $DWL = 0$ .

### 7.2 Third-Degree Price Discrimination (三级价格分歧)

The additional conditions that a third-degree price discrimination must satisfy include:

1. firm has several **types** of consumers with different WTP
2. firm can identify customer's type before transaction happens

The final phenomena will be

1. different types of consumers will be charged different price
2. buyers of the same type share the same price

Assume two markets, with  $p_1, q_1; p_2, q_2$ , uniform pricing within each market, to maximize profit, we must have

$$MR_1 = MR_2 = MC$$

And for the more inelastic market, say market 1, the clearing price across markets will have the relationship that  $p_1^* > p_2^*$ , even when the marginal cost is the same for both markets.



### 7.3 Second-Degree Price Discrimination (二级价格分歧)

Second-degree price discrimination does not classify customers, thus it can be **possible** if the following condition is satisfied:

1. firm has different types of customers with different WTP

This is a type of *indirect price discrimination*, which involves offering **a menu of options**, where the customer is **free to choose preferred option**. Thus, the menu of options needs to be *incentive compatible*.

Assume the firm offers 2 options and has two types of customer ( $A$  and  $B$ ), the firm wishes type- $A$  customers buy the first option and type- $B$  customers buy the second option. Then, to be incentive compatible, we must have

$$CS_A(1) \geq CS_A(2), CS_B(2) \geq CS_B(1)$$

The four common forms of second-degree price discrimination are:

1. Quantity discounts
2. Versioning
3. Discount coupons
4. Bundling

#### 7.3.1 Quantity Discounts

This type offers lower per-unit price if a particular threshold quantity is met. Take the example of two types of customers: occasional bakers, and serious bakers. The firm that produces flour has two options: either \$2.7/kg and buy however much flour, or \$1.2/kg if buy 20+ kg flour. The firm wants occasional bakers to choose option 1, and serious bakers to choose option 2.

#### 7.3.2 Versioning

This type offers a range of products that all varieties is of the same core product. Each version of the product will have a **different markup** and a **different marginal cost**.

Take the example of NVIDIA producing RTX4060 and RTX4070, where there are two types of consumers, casual gamers, and serious gamers.

NVIDIA wishes the casual gamers to buy RTX4060, where the serious gamers to buy RTX4070.

#### 7.3.3 Discount coupons

This type involves charging customers with a less elastic demand more. Since the smaller the fraction of income spent on an item, the less elastic the demand; thus wealthier consumers generally have less elastic demand.

Wealthier consumers also tend to place a higher value on their time and so are less likely to search for low prices or coupons.

We want the coupons to minimally cover the opportunity cost of finding the coupons for consumers with less income.

#### **7.3.4 Bundling**

This involves firm selling 2+ products together as a package. The firm can adopt standalone pricing as usual, or use either pure-bundling (sell products only as part of a bundle) or mixed bundling (sell products both individually and as a bundle).

If there is a negative correlation for the same product for different consumers across products, that is if  $A$  values good 1 more than 2, but  $B$  values good 2 more than 1, then bundling can be considered.

Specifically, mixed bundling can be applied when the marginal cost of some goods is high enough that it makes sense to let some consumers to opt out the bundle. Thus, the firm needs to adjust the price to make sure the firm is not compensating for consumer, and the price change will be incentive compatible.