3.107 compound propositions

negation $\neg$

conjunction $\wedge$

~~disco~~ disjunction $\vee$

exclusive-or $\oplus$

Precedence:

$\neg$	1
$\wedge$	2
$\vee$	3

3.202 logical implication

$p \to q$, conditional statement/implication

p: hypothesis / antecedent / premise

q: conclusion / consequence

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

A: the conditional statement $p \to q$

$q \to p$: converse of A

$\neg q \to \neg p$: contrapositive of A

$\neg p \to \neg q$: inverse of A

3.204 logical equivalence

~~bio~~ biconditional $p \leftrightarrow q$

$p \equiv q$: logically equivalent

: $p \leftrightarrow q$ is a always true

operator	precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

4.103

definition of a predicate

The statement "x squared is equal to 4" has two parts:

— The variable x, which is subject of the statement

— The predicate "square is equal to 4", which is the property that the subject of the statement can have

This statement can be formalised as $P(x)$ where P is the predicate "squared — 4" and X is the variable.

• P is said to be the propositional function

Once a value is assigned to the variable X, the statement $P(X)$ becomes a proposition and has a truth value.

eg: $P(2)$ is True

$P(3)$ is False

4.105 Quantifiers

The universal quantification of a predicate $P(x)$ is the proposition: "$P(x)$ is true for all values of x in the universal of discourse"

notation: $\forall x \; P(x) \Rightarrow$ "for all x"

$\forall x \; P(x) \Leftrightarrow P(n_1) \land P(n_2) \land \cdots$

eg: S: "For every x and every y, $x+y>10$"

$\Rightarrow \forall x, y \; P(x, y)$

Existential quantifier

"There exists ~~a value~~ x in the universe"

$\exists x \; P(x) \Rightarrow$ "there exists x"

$\exists x \; P(x) \Leftrightarrow P(n_1) \lor P(n_2) \lor \cdots$

Uniqueness quantifier

"There exists a unique value X in the universe of discourse such that P(X) is true."

notation: $\exists! x \; P(x)$.

There exists a unique X

eg. $P(x)$ "$x^2=4$"

E: $\exists! x \; P(x)$

If the universe of discourse is positive integers, E is True ($x=2$)

If ___ integers, E is FALSE ($x=2, -2$)

4.107 Nested ~~quat~~ quantifiers

$\forall x \exists y \; P(x,y)$, for every x, there is a y for which $P(x,y)$ is True

$\exists x \forall y \; P(x,y)$, there is an x for which $P(x,y)$ is True for every y.

eg. $\exists x \; P(x,y)$

x is bound

y is free

logical operations:

eg. $P(x)$ denotes "$x>3$"

$Q(x)$ denotes "x squared is even"

$\exists x (P(x) \land Q(x)) \equiv T$ (ex. $x=4$)

$\forall x (P(x) \rightarrow Q(x)) \equiv F$ (ex. $x=5$)

order of operations:

eg. $\forall x \forall y \; P(x,y) \equiv \forall y \forall x \; P(x,y)$

$\exists x \exists y \equiv \exists y \exists x$

$\forall x \exists y \not\equiv \exists y \forall x$

precedence:
$\forall$ $\exists$ have a higher precedence priority than all logical operators

4.201 De Morgan's law for quantifiers
for negating quantifiers:

$$\neg\forall x\, P(x) \equiv \exists x\ \neg P(x)$$
$$\neg\exists x P(x) \equiv \forall x\ \neg P(x)$$

$$\exists x \neg\exists y \forall z P(x,y,z)$$
$$\equiv \exists x \forall y \neg\forall z\ P(x,y,z)$$
$$\equiv \exists x \forall y \exists z\ \neg P(x,y,z)$$

4.203 Rules of inference
Tautology: $(p \wedge (p \to q)) \to q$
The rule of inference:

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Tautology: $(\neg q \wedge (p \to q)) \to \neg p$

$$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore \neg p \end{array}$$

$((p) \wedge (q)) \to (p \wedge q)$

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

$(p \wedge q) \to p$

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$$

$$\frac{p}{\therefore p \vee q} \qquad p \to (p \vee q)$$

$$((p \to q) \wedge (q \to r)) \to (p \to r)$$

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

$$((p \vee q) \wedge \neg p) \to q$$

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

$$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$$

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

4.205 Rules of inference with quantifiers

Universal instantiation, UI

$$\begin{array}{l} \forall x P(x) \\ \hline \therefore P(c) \end{array}$$

Universal generalization, UG

$$\begin{array}{l} P(c) \text{ for an arbitrary element of the domain} \\ \hline \therefore \forall P(x) \end{array}$$

Existential instantiation, EI

$$\begin{array}{l} \exists x P(x) \\ \hline \therefore P(c) \text{ for some element of the domain} \end{array}$$

Existential generalization (EG)

$$\begin{array}{l} P(c) \text{ for some element of the domain} \\ \hline \therefore \exists x P(x) \end{array}$$

Universal modus ponens

$\forall x P(x) \to Q(x)$

$P(a)$ for some element of the domain

$\therefore Q(a)$

Universal modus tollens

$\forall x P(x) \to Q(x)$

$\neg Q(a)$ for some element of the domain

$\neg P(a)$

~~Expressing complex statement~~

5.101

AND: $x \cdot y$, $x \cap y$, $x \wedge y$

OR: $x + y$, $x \cup y$, $x \vee y$

NOT: x', $\overline{x}$, $\neg x$

NOT > AND > OR

5.103 Postulates of Bool algebra

axioms:

$x(y+z) = (x \cdot y) + (x \cdot z)$

$x + (y \cdot z) = (x+y) \cdot (x+z)$

Basic theorems:

$x + (x \cdot y) = x$

$x \cdot (x+y) = x$

if $y + x = 1$, $y \cdot x = 0 \Rightarrow x = y'$

(SEE NEXT PAGE)

De Morgan's theorems

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$
$$\overline{x + y} = \overline{x} \cdot \overline{y}$$

Principle of duality

OR → AND, $+ \rightarrow \cdot$

AND → OR, $\cdot \rightarrow +$

$0 \rightarrow 1$, $1 \rightarrow 0$

Huntington's postulates

closure: any result $\in \{0,1\}$

identity:

$$x+0=x, \quad x \cdot 1 = x$$

commutativity

$$x+y=y+x, \quad x \cdot y = y \cdot x$$

distributivity

$$x(y+z) = (x \cdot y) + (x \cdot z)$$
$$x + (y \cdot z) = (x+y) \cdot (x+z)$$

complements

$$x + x' = 1, \quad x \cdot x' = 0$$

distinct elements: $0 \neq 1$

Basic theorems:

idempotent laws

$$x+x=x, \quad x \cdot x = x$$

tautology and contradiction

$$x+1=1, \quad x \cdot 0 = 0$$

involution

$$(x')' = x$$

associative laws

$$(x+y)+z = x+(y+z), \quad (x \cdot y) \cdot z = x \cdot (y \cdot z)$$

absorption laws

$x+(x \cdot y)=x$

$x \cdot (x+y)=x$

uniqueness of complement

if $y+x=1$, and $y \cdot x=0$

then $x=y'$

~~m~~

$1'=0$, $0'=1$

5.105 Boolean functions

$x \oplus y = x'y + xy'$

$x \to y = x' + y$

sum-of-products:

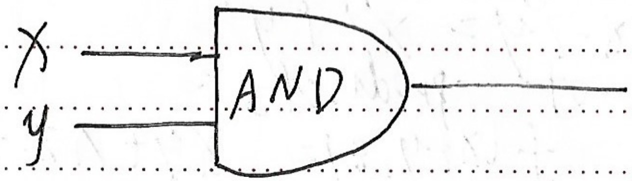
$f(x,y,z) = xy + xz + yz$

product-of-sums:

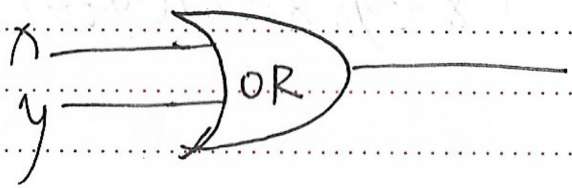
$(x+y)(x+z)(y+z)$

5.201 Logic gates

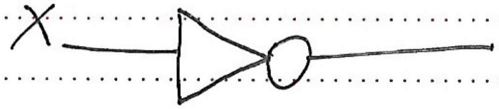
AND



OR

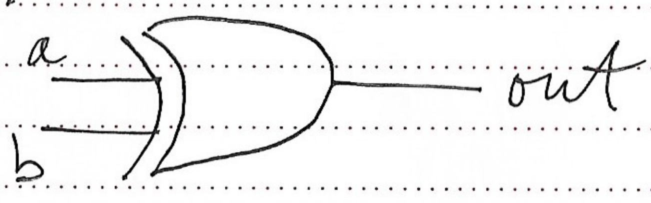


NOT/inverter gate

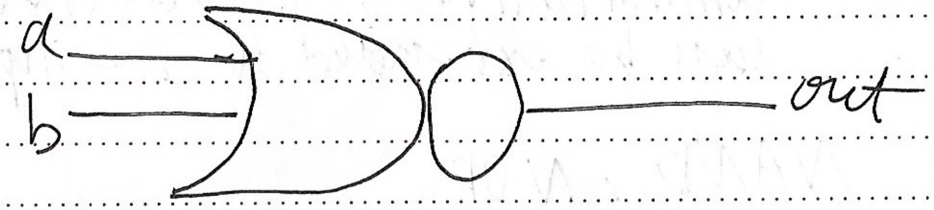


$f = \overline{x}$

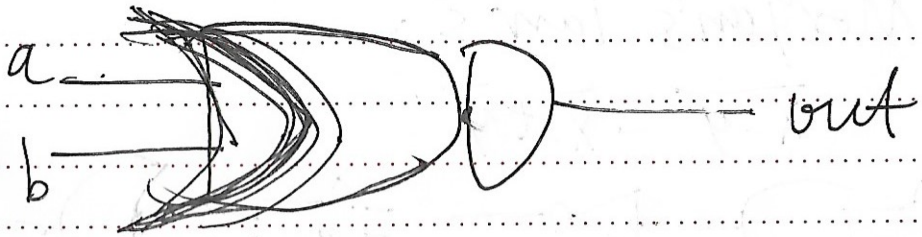
XOR/~~other~~



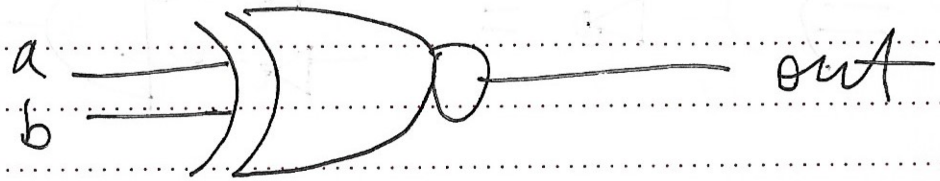
NAND (not AND)



NOR (not OR)



XNOR (not XOR)



AND, OR, XOR, XNOR:

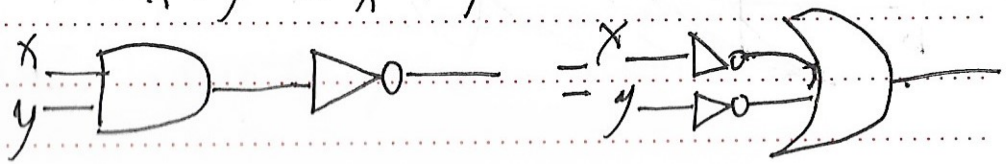
commutative, associative,
can be extended to ≥ 2 inputs

NAND, NOR:

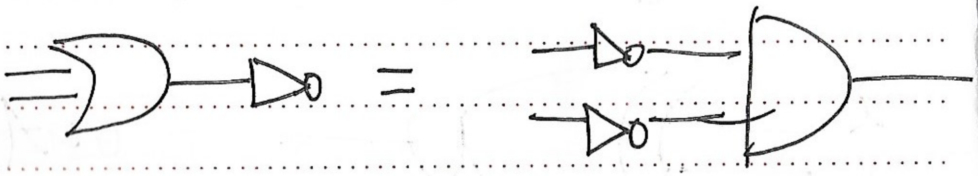
only commutative

De Morgan's laws:

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$



$$\overline{x + y} = \overline{x} \cdot \overline{y}$$



6.101 Intro to proofs

theorem 定理
axiom 公理
lemma 引理
corollary 推论

6.206 Solving recurrence Relations

eg: Fibonacci

$f_n = f_{n-1} + f_{n-2}$, $f_0 = 0, f_1 = 1$

Solution: $r^2 - r - 1 = 0$

$r_1 = \frac{1+\sqrt{5}}{2}$

$r_2 = \frac{1-\sqrt{5}}{2}$

$f_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

$f_0 = \alpha_1 + \alpha_2 = 0$

$f_1 = \frac{1+\sqrt{5}}{2}\alpha_1 + \frac{1-\sqrt{5}}{2}\alpha_2 = 1$

$\Rightarrow \alpha_1 = \frac{1}{\sqrt{5}}, \alpha_2 = -\frac{1}{\sqrt{5}}$

eg. $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$

$a_0 = 1, a_1 = -2, a_2 = -1$

$r^3 + 3r^2 + 3r + 1 = 0$

$\Rightarrow r_1 = -1$

$a_n = (\alpha + \alpha_1 n + \alpha n^2) r_1^n$

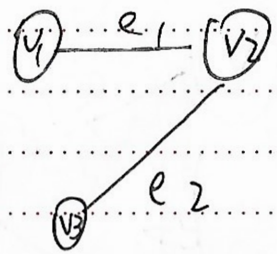
$a_0 = \alpha_0 = 1$

$a_1 = -(\alpha_0 + \alpha_1 + \alpha_2) = -2$

$a_2 = -(\alpha_0 + 2\alpha_1 + 4\alpha_2) = -1$

7.103 Definition of a graph

Vertex



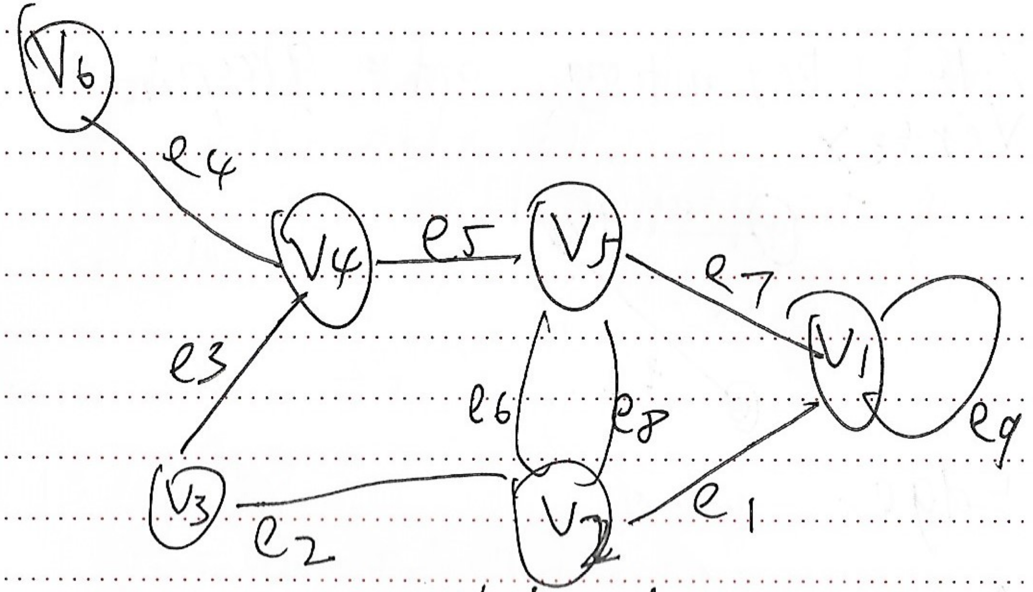
Edge

Adjacency:

v_1, v_2

e_1, e_2

e_1 and v_1 are incident



e_6, e_8: parallel edges
e_9: a loop

Directed graph: Digraph

7.10 5

Walk

$V_1 \to V_6$:
$V_1V_2, V_2V_3, V_3V_4, V_4V_6$
$= e_1, e_2, e_3, e_4$
$= V_1V_2V_3V_4V_6$

length $= 4$

or: $V_1V_2V_3V_2V_5$
(pass twice througe e_2)

Trail: a walk, no edge is repeated

Path: a trail, neither vertices nor edges are repeated.

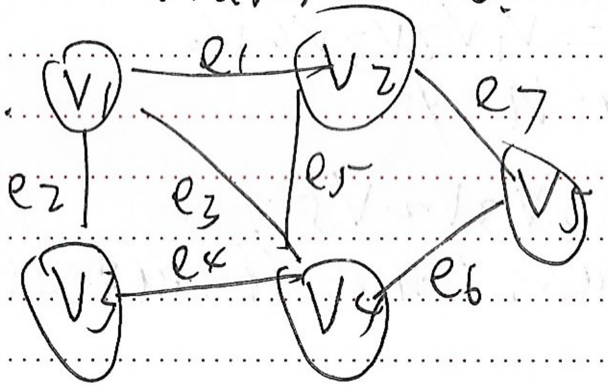
Cycle: $V_1V_2V_5V_1$

Euler path:

A Eulerian path: a path that uses each edge precisely once.

If such path exists => traversable.

eg.



$V_1 V_3 V_4 V_5 V_2 V_4 V_1 V_2$

Hamiltonian path:

(a traceable path): a path that ~~exis~~ visits each vertex exactly once.

A graph that contains Hamiltonian path: a traceable graph.

Hamiltonian cycle:

a cycle that visits each vertex exactly once, except for the starting vertex.

Hamiltonian graph:

a graph contains Hamiltonian cycle.

Connectivity: (undirected graph)

any two nodes are connected by a path

Strong connectivity:

a directed graph is strongly connected if there is a directed path from any node to any other node.

Transitive Closure:

G^* has the same vertices as G

If G has a directed path from u to v,
G^* has a directed edge from u to v.
$(u \neq v)$

7.107 The degree sequence of a graph

$deg(v)$: number of edges incident on V

$In\text{-}deg(v)$: v is the terminal vertex

$Out\text{-}deg(v)$: v is initial vertex

Degree sequence of a graph:

an undirected graph G: a monotonic nonincreasing sequence of the vertex degrees of all the vertices of G

Property:

1. The sum of the degree sequence of a graph is always even.
2. ~~number~~ $|E| = \frac{1}{2}\sum deg(v)$

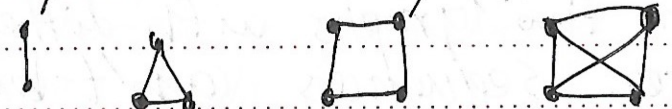
7.109 Special graphs

Simple graphs: without loops and parallel edges

property: $deg(v) \leq n-1$, $n = |V|$

Regular graphs: all local degrees are the same number

r-regular, $deg(v) = r$

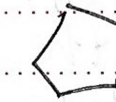


degree sequence of G: $r, r, \cdots, r$

$\sum deg(v) = r \times n$

$|E| = \frac{1}{2} rn$

Special regular graphs: cycles

C_3  C_4  C_5 

2-regular

complete graphs: every pair of vertices are adjacent

K_n

properties: $deg(v) = n-1$

$\sum deg(v) = n(n-1)$

$|E| = \frac{1}{2} n(n-1)$

7.201 Isomorphic graphs

Definition of isomorphism:

there is a bijection (invertible function)

$f: G_1 \to G_2$

that preserves adjacency and non-adjacency.

If uv is in $E(G_1)$, then

$f(u)f(v)$ is in $E(G_2)$

properties: two graphs with different degree sequences can't be isomorphic.

7.203 Bipartile graphs

Definition: A graph $G(V, E)$:

if the set of V can be partitioned into two no-empty disjoint sets V_1 and V_2 in such a way that each edge e in G has one endpoint in V_1 and another endpoint in V_2.

— The graph is 2 colorable

— No odd-length cycle

Matching: a matching is a set of pairwise non-adjacent edges, none of which are loops; that is, no two edges share a common endpoint.

A vertex is matched / saturated if it is an endpoint of one of the edges in the matching. Otherwise the vertex is unmatched.

The Hopcroft-Karp algorithm

: solving the maximum matching problem in a bipartite graph

augmenting path:

breadth first search

depth-first search

1. Initialize $M = \{ \}$

2. while there exists an augmenting path P

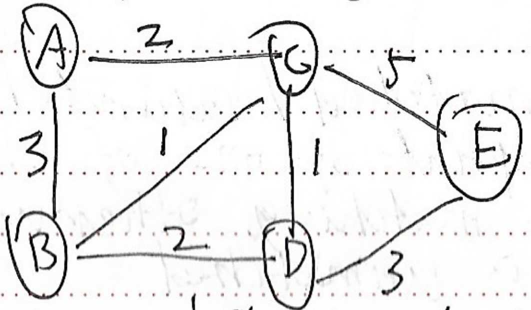
1) use BFS to build layers that terminate at free vertices.

2) start at the free vertices in C, use DFS

3、Return M

7.207 Dijkstra's algorithm

eg:



Vertex	Shortest distane from A	Previous Vertex
A	0	
B	∞	undefined
C	∞	~~un~~
D	∞	un
E	∞	un

Initialisation

unvisited = [ ] ABCDE

for each v in G :

shortest_dis[A] ← 0

shortest_dis[v] ← ∞

previous_vertex[v] ← undefined

add v to unvisited

Interation 1

while unvisited is not empty:

u ← vertex in unvisited with min shortest_dis[u]

remove u from unvisited

for each neighbor v of u

alt ← shortest_dis[u] + length(u,v)

if alt < shortest_dis[v]

shortest_dis[v] ← alt

previous_vertex[v] ← u

A	0	
B	3	A
C	2	A
D	inf	un
E	inf	un

unvisited [B,C,D,E]

A	0	A
B	3	A
C	2	A
D	3	C
E	7	C

unvisited = [B,D,E]

8.103 Definition of a tree

acyclic graph: if and only if G has no cycles

A tree is a connected acyclic undirected graph.

A disconnected graph containing no cycles is called a forest

Theorem 1: An undirected graph is a tree if and only if there is unique simple path between any two of its vertices

Theorem 2: A tree with n vertices has n-1 edges.

Rooted trees: a rooted tree is when one vertex has been designated as the root and every edge is directed away from root.

8.105 Spanning trees of a graph

Definition: a spanning tree of a graph G is a connected subgraph of G which contains all vertices of G, but with no cycles.

Constructing a spanning tree

1. keep all vertices
2. break all the cycles but keep the tree connected

Non-isomorphic spanning trees.

Two spanning trees are said isomorphic if there is a bijection preserving adjacency between two trees.

8.107 Minimum spanning tree

A minimum-cost spanning tree is a spanning tree that has the lowest weight / cost

Kruskal's algorithm

Start with the cheapest edges in the spanning tree

Repeatedly add the cheapest edge that does not creat a cycle

Prim's algorithm

Start with any one node in the spanning tree.

Repeatedly add the cheapest edge, and the node it leads to, for which the node is not already in the spanning tree

8.201 Rooted trees

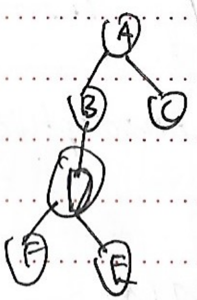
A rooted tree is a directed tree having one distinguished vertex r, called a root, such that for every vertex v there is a directed path from r to v.

Theorem: a directed tree is represented as a rooted tree if and only if one vertex has in-degree 0 whereas all other vertices have in-degree 1

8.201 Rooted trees

A rooted tree is a directed tree having one distinguished vertex r, called a root, such that for every vertex v, there is a directed path from r to v.

Theorem: A directed tree is represente as a rooted tree if and only if one vertex has in-degree 0 whereas all other vertices have in-degree 1.



A: root

B: parent of D

E, F: children of D

B, A: ancestors of E, F

E, F are siblings

B, D: internal nodes
C, E, F: external nodes

The depth or path length of a node in a tree is the number of edges from the root to that node.

The height of a node in a tree is the longest path from that node to a leaf.

Special trees:

Binary Tree

Ternary Tree

m-ary Tree

Regular ~~ne~~ rooted trees.

An m-ary tree is regular if every one of its internal nodes has exactly m children

Properties:

An m-ary tree has at most m^h vertices at level h.

Isomorphic trees:

Two trees T_1, T_2 are isomorphic if there is a bijection $f: V(T_1) \to V(T_2)$ which preserves adjacency and non-adjacency.

That is, if uv is in $E(T_1)$ and $f(u) f(v)$ is in $E(T_2)$

Notation: $T_1 \cong T_2$

Properties:

Two trees with different degree sequences are not isomorphic

Isomorphic rooted trees:

Two isomorphic trees are ~~isomorphic~~ isomorphic as rooted trees if and only if there is a bijection that maps the root of one tree to the root of the other.

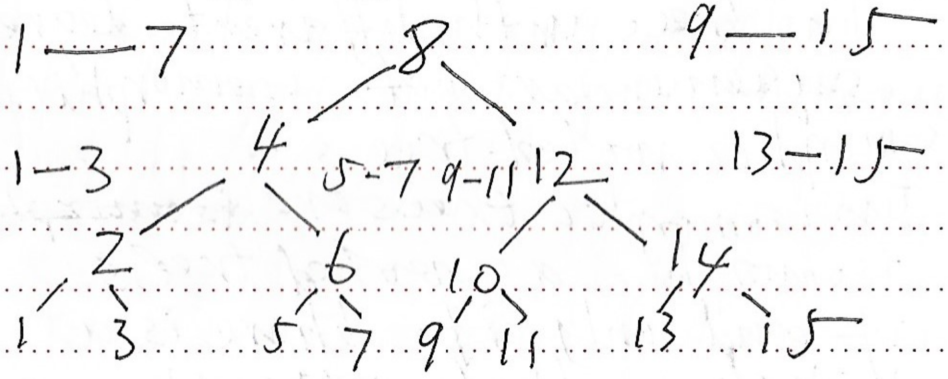
8.203 Binary search trees

A binary search tree is a binary tree in which the vertices are labelled with items so that a lable of a vertex is greater than the labels of all vertices in the left subtree of this vertex and is less than the labels of all vertices in the right subtree of this vertex.

Application:

min 1, 2, 3, ______ 15 max

root = $\lfloor \frac{1+15}{2} \rfloor = 8$



Height of the tree

① $2^{h-1} < 1+N \le 2^h$

$h-1 < \log_2(1+N) \le h$

② $h = \lceil \log_2(N+1) \rceil$

eg. $N=15$, $h=4$

$2^{4-1} < 1+15 \le 2^4$

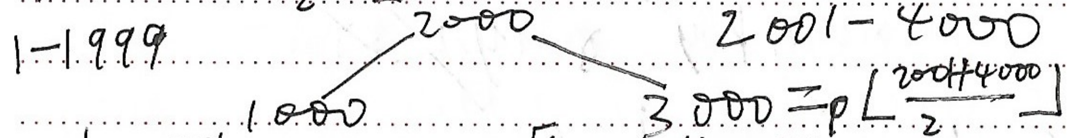
$h = \lceil \log_2(15+1) \rceil = 4$

eg. find the first 3 level of a binary search tree to store 4000 records

Find the height of the tree

1 4000

root = $\lfloor \frac{1+4000}{2} \rfloor = 2000$



$h = \lceil \log_2(N+1) \rceil = \lceil \log_2 4001 \rceil = 12$

Binary search algorithm

root - sub - subsub - ......

9 103

What is a relation?

Let A, B be sets.

Let R be a relation linking elements of A to elements to B

Let $x \in A$ and $y \in B$

We say that x is related to y with respect to the relation R, and we write xRy

A relation is a link between two elements of a set.

eg: x is a 'SON OF' y,

$R =$ 'SON OF'

xRy, $\cancel{yRx}$

Cartesian product

Let A and B be two sets.
The Cartesian product $A \times B$ is defined by a set of pairs (x, y) such that $x \in A$ and $y \in B$

$$A \times B = \{(x,y) : x \in A \text{ and } y \in B\}$$

A binary relation from A to B is a subset of a Cartesian product $A \times B$

- $R \subseteq A \times B$ means R is a set of ordered pairs of the form (x,y) where $x \in A$ and $y \in B$
- $(x,y) \in R$ means xRy

eg. $A = \{a, b, c\}$, $B = \{1, 2, 3\}$
$R = \{(a,1), (b,2), (c,3)\}$
$aR1$, $bR2$, $cR3$

When $A = B$, $R \subseteq A \times A$

eg. $A = \{1, 2, 3, 4\}$
Let R be a relation on set A
$x, y \in A$, xRy if and only if $x < y$

: $1R2$, $1R3$, $1R4$, $2R3$, $2R4$, $3R$
$R = \{(1,2), \ldots, (3,4)\}$

9.105: representations of a relation

$n_a = |A|$, $n_b = |B|$
The matrix of R is $n_a \times n_b$ matri
$M_R = [m_{ij}]\ n_a \times n_b$

$$m_{ij} = \begin{cases} 1, & \text{if } (a_i, b_j) \in R \\ 0, & \text{if } (a_i, b_j) \notin R \end{cases}$$

combining relations

Union: $R \cup S$, R or S
Intersection: $R \cap S$, R and S
$R \cap S = \{(a,b) : (a,b) \in R \text{ and } (a,b) \in S\}$

Via Boolean operators

Join: $M_{R \cup S} = M_R \vee M_S$
Meet: $M_{R \cap S} = M_R \wedge M_S$

using directed graphs

$(a,b) \in R$ arrows are drawn $a \to b$

eg. $A = \{1, 2, 3, 4\}$
$R = \{(x,y) \mid x \text{ divides } y\}$

9.107
reflexivity: A relation R in a set S is said to be reflexive, if and only if $xRx, \forall x \in S$
$(x,x) \in R, \forall x \in S$
eg: $R = \{(a,b) \in \mathbb{Z}^2 \mid a \le b\}$
Digraph of a reflexive relation on elements in a set S contains a loop on every element of S.
Matrix of reflexive relation: M_R
all the values of the diagonal of M_R are equal to 1

Symmetry: $\forall a,b \in S$, if aRb then bRa
eg. $R = \{(a,b) \in \mathbb{Z}^2 \mid a \bmod 2 = b \bmod 2\}$
Digraph: no single connection between any two different vertices.
Matrix: M_R is symmetric with respect to the diagonal

Anti-symmetric: $\forall a,b \in S$, if aRb and bRa, then $a=b$
eg. $R = \{(a,b) \in \mathbb{Z}^2 \mid a \le b\}$

Let $a,b \in \mathbb{Z}$, aRb and bRa
- $a \le b$ and $b \le a$
- $a = b$

• R is anti-symmetric
Digraph: contains no parallel edges between any two difference vertices
Matrix: if $i \ne j$ and $m_{ij} \ne 0$, then $m_{ji} = 0$

9.109 transivity
A relation R on set S is called transitive if and only if $\forall a,b,c \in S$, if (aRb and bRc) then aRc
Transitive closure of a relation
$R = \{(a,b), (b,c), (c,d)\}$
R enhanced $= \{(a,b), (b,c), (a,c), (c,d), (b,d), (a,d)\}$
(transitive closure of R)

9.201 Equivalence relations / classes
R is an equivalence relation if and only if R is reflexive, symmetric, and transitive

eg: $R = \{(a,b) \in Z^2 \mid a \bmod 2 = b \bmod 2\}$

Let R be an equivalence relation on a set S. Then the equivalence class of $a \in S$ is: the subset of S containing all the elements related to a through R.

$[a] = \{x : x \in S \text{ and } xRa\}$

eg. Let $S = \{1,2,3,4\}$ and R be a relation on elements in S:

$R = \{(a,b) \in S^2 \mid a \bmod 2 = b \bmod 2\}$

R is an equivalence relation with 2 equivalence classes:

$[1] = [3] = \{1,3\}$

$[2] = [4] = \{2,4\}$

9.203

R is a partial order, iff R is reflexive, anti-symmetric, and transitive.

eg. $R = \{(a,b) \in Z^2 \mid a \leq b\}$

R is total order, iff R is partial order, and for all a,b elements in S, we have either aRb, or bRa

eg. $R = \{(a,b) \in Z^2 \mid a \leq b\}$

$\forall a,b \in Z$, $a \leq b$ or $b \leq a$ is true

10.103 The basics of counting

Product rule:

A and B are disjoint

$|A \times B| = |A| \cdot |B|$

Addition rule

A and B are disjoint

$|A \cup B| = |A| + |B|$

Subtraction rule

principle of inclusion-exclusion

$|A \cup B| = |A| + |B| - |A \cap B|$

Division rule

Suppose a task can be done using a procedure that can be carried out in n ways, and for every way w, exactly d of the n ways correspond to w, Then this task can done in n/d ways.

In term of sets: if the finite set A is the union of n pairwise disjoint subsets each with d elements, then $n = |A|/d$

10.105 Pigeonhole principle

10.107 Permutations and combinations
$P(n,r)$
$C(n,r)$

10.201
$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$
$2^n = \sum_{k=0}^{n} \binom{n}{k}$
Pascal's identity: $n \geq k \geq 1$: $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$

10.204
Permutations with repetition
The number of r-permutations of a set of n objects with repetition allowed is n^r.

Combination with repetition $\binom{k+n-1}{k}$
= the total number of ways to put k identical balls into n distinct boxes / the total number of functions from a set of k identical elements to a set of n distinct elements

	order matters	order not matters
repetition ~~perm~~ not permitted	$\frac{n!}{(n-k)!}$ $P(n,k)$	$\frac{n!}{k!(n-k)!}$ $\binom{n}{k}$
repetition permitted	n^k	$\binom{k+n-1}{k}$

10.206 Distinguishable objects and boxes with exclusion:
distribute k balls, numbered from 1 to k, into ~~a~~ n boxes, numbered from 1 to n, in such a way that no box receives more than one ball
$P(n,k)$

without exclusion:
distribute k balls $(1-k)$, into n boxes $(1-n)$ without restrictions on the number of ~~box~~ balls in each box
n^k

indistinguishable objects and distinguishable boxes with exclusion,
distribute k balls into n boxes

numbered (1–n), no box receives more than one ball

$\binom{n}{k}$

without exclusion: without restrictions on the number of balls in each box

$\binom{n+k-1}{k}$

eg. place 8 indistinguishable balls into 6 distinguishable boxes

$\binom{8+6-1}{8}$