COMP 5630 Fall 2022 Assignment 5

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1. Linear Regression

   (a) Write down an expression for $P(y|x_1, x_2)$

   $$P(y|x_1, x_2) = \frac{1}{\sqrt{2\pi}\sigma} \exp -\frac{(y - w_1 x_1 - w_2 x_2)^2}{2\sigma^2} \tag{1}$$

   (b) Find the conditional log-likelihood

   $$
   \begin{aligned}
   L(\theta) &= \prod_{i=1}^{n} P(y|x_1^i, x_2^i; \theta) \\
   &= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \exp -\frac{(y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2}{2\sigma^2} \\
   \ell(\theta) &= \log\left(\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \exp -\frac{(y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2}{2\sigma^2}\right) \\
   &= \sum_{i=1}^{n}\left[\log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) + \log\left(\exp -\frac{(y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2}{2\sigma^2}\right)\right] \\
   &= \sum_{i=1}^{n} -\frac{(y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2}{2}
   \end{aligned} \tag{2}
   $$

   Dropping all constants that do not depend on $w_0, w_1, or w_2$

   (c) Demonstrate the MLE of the conditional log-likelihood is equivalent to minimizing least squares

   $$
   \begin{aligned}
   MLE &= -\ell(\theta) \\
   &= -\sum_{i=1}^{n} -\frac{(y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2}{2} \\
   &= \frac{1}{2}\sum_{i=1}^{n}(y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2
   \end{aligned} \tag{3}
   $$

2. Regularization

   (a) Find the partial derivative

   $$\frac{\partial}{\partial w_i}\left[\frac{1}{2}\sum_{i=1}^{n}(y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2 + \frac{\lambda}{2}\|w_1, w_2\|_2^2\right] \tag{4}$$

   $$\frac{1}{2}\sum_{i=1}^{n}\frac{\partial}{\partial w_i}(y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2 + \frac{\lambda}{2}\left(\frac{\partial}{\partial w_i}(w_1^2 + w_2^2)\right) \tag{5}$$

1

Where w_0

$$
\begin{aligned}
&= \tfrac{1}{2} \sum_{i=1}^{n} \tfrac{\partial}{\partial w_0} (y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2 \\
&= \tfrac{1}{2} \sum_{i=1}^{n} -2(y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^1 \\
&= -\sum_{i=1}^{n} (y^i - w_0 - w_1 x_1^i - w_2 x_2^i)
\end{aligned}
\tag{6}
$$

Where w_1

$$
\begin{aligned}
&= \tfrac{1}{2} \sum_{i=1}^{n} \tfrac{\partial}{\partial w_1} (y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2 + \tfrac{\lambda}{2} \left(\tfrac{\partial}{\partial w_1} (w_1^2 + w_2^2) \right) \\
&= \tfrac{1}{2} \sum_{i=1}^{n} -2 x_1^i (y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^1 + \tfrac{\lambda}{2} 2 w_1 \\
&= -\sum_{i=1}^{n} x_1^i (y^i - w_0 - w_1 x_1^i - w_2 x_2^i) + \lambda w_1
\end{aligned}
\tag{7}
$$

Where w_2

$$
\begin{aligned}
&= \tfrac{1}{2} \sum_{i=1}^{n} \tfrac{\partial}{\partial w_2} (y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^2 + \tfrac{\lambda}{2} \left(\tfrac{\partial}{\partial w_2} (w_1^2 + w_2^2) \right) \\
&= \tfrac{1}{2} \sum_{i=1}^{n} -2 x_2^i (y^i - w_0 - w_1 x_1^i - w_2 x_2^i)^1 + \tfrac{\lambda}{2} 2 w_2 \\
&= -\sum_{i=1}^{n} x_2^i (y^i - w_0 - w_1 x_1^i - w_2 x_2^i) + \lambda w_2
\end{aligned}
\tag{8}
$$

Gradient Descent Rules

$$
w_i^{t+1} = w_i^t + \frac{\partial E(X, w)}{\partial w_i}
\tag{9}
$$

$$
w_0^{t+1} = w_0^t + \sum_{i=1}^{n} (y^i - w_0 - w_1 x_1^i - w_2 x_2^i)
\tag{10}
$$

$$
w_1^{t+1} = w_1^t + \sum_{i=1}^{n} x_1^i (y^i - w_0 - w_1 x_1^i - w_2 x_2^i) + \lambda w_1
\tag{11}
$$

$$
w_2^{t+1} = w_2^t + \sum_{i=1}^{n} x_2^i (y^i - w_0 - w_1 x_1^i - w_2 x_2^i) + \lambda w_2
\tag{12}
$$

$$
\tag{13}
$$

3. Implementing Linear Regression (code at: github.com/wumphlett)

The learning rate of .01 outperformed the learning rate of .001. This is evidenced by its ability to more effectively minimize loss (17.4 v 34.8).
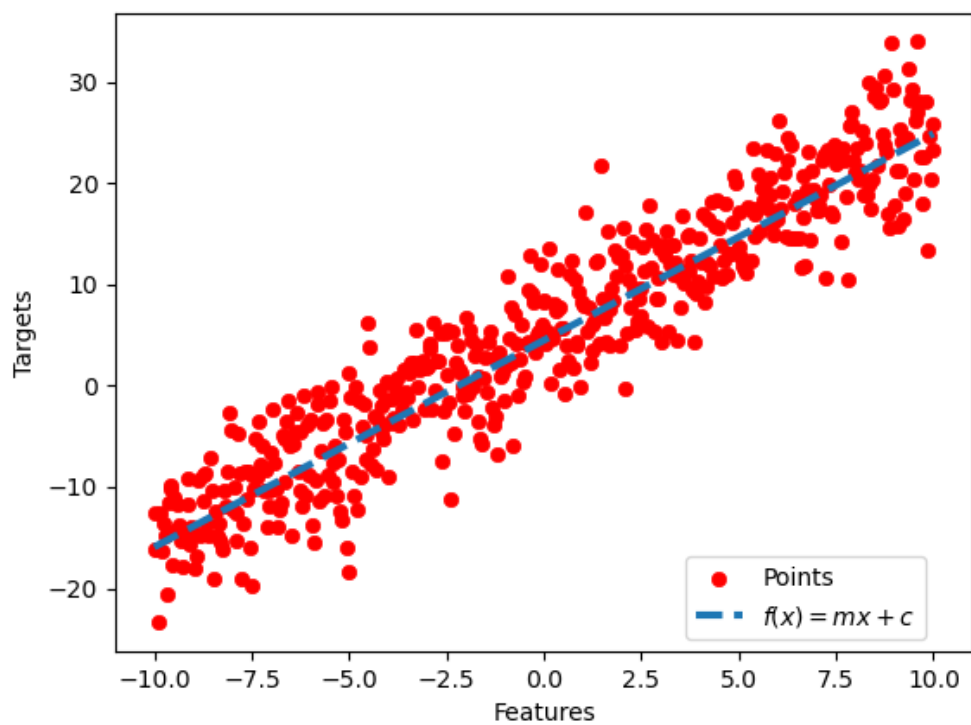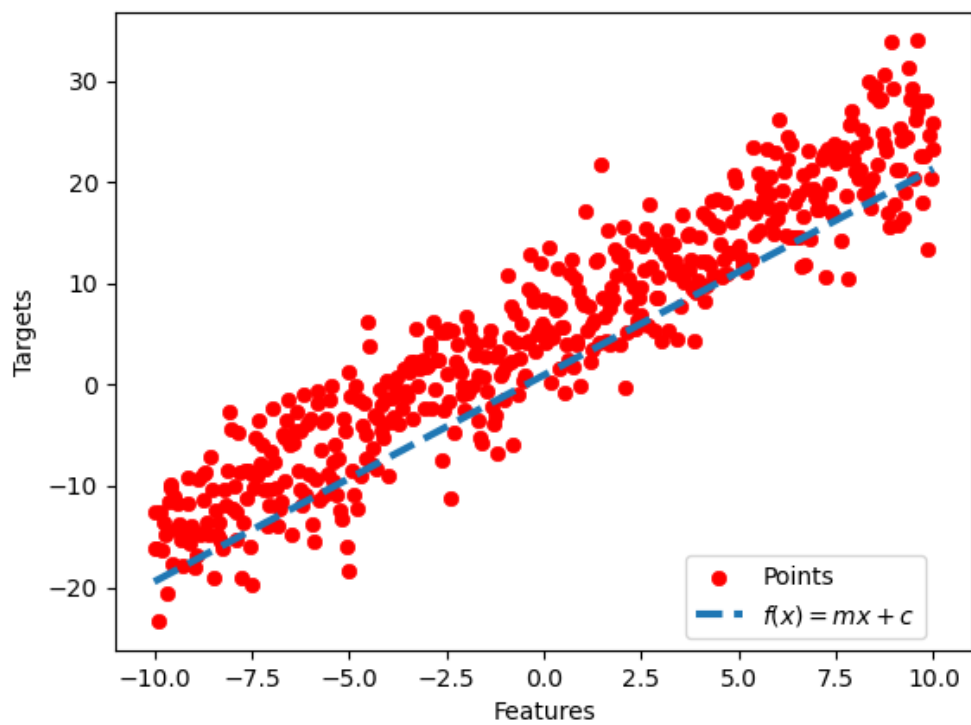
Figure 1: Linear Regression (lr fixed at .01)

Figure 2: Linear Regression (lr fixed at .001)

```
Epoch 1, m: 0.13658349190618496, b:0.010312358814106724, Loss: 164.80508456949963
Epoch 2, m: 0.2640249224302726, b:0.02060409291058523, Loss: 149.00149960714552
Epoch 3, m: 0.3829362050996417, b:0.03087524353887078, Loss: 135.22943429119377
Epoch 4, m: 0.4938882957025641, b:0.04112585186589976, Loss: 123.22608820443448
Epoch 5, m: 0.5974139337480965, b:0.05135595897627468, Loss: 112.76266383116672
Epoch 6, m: 0.6940102004294589, b:0.061565605872428854, Loss: 103.63996700446963
Epoch 7, m: 0.7841409053730308, b:0.07175483347479072, Loss: 95.68457660100776
Epoch 8, m: 0.8682388136330049, b:0.08192368262194787, Loss: 88.74550982980175
Epoch 9, m: 0.9467077236246761, b:0.0920721940708107, Loss: 82.69131899125422
Epoch 10, m: 1.0199244059736183, b:0.10220040849677581, Loss: 77.40756387953363
Epoch 11, m: 1.0882404125901863, b:0.11230836649388898, Loss: 72.79461122472588
Epoch 12, m: 1.1519837646556654, b:0.12239610857500792, Loss: 68.76571885986522
Epoch 13, m: 1.2114605276249781, b:0.1324636751719646, Loss: 65.24536777297833
Epoch 14, m: 1.266956280808369, b:0.1425111066357274, Loss: 62.167809970895874
Epoch 15, m: 1.3187374885883023, b:0.15253844323656268, Loss: 59.47580423146677
Epoch 16, m: 1.3670527798555108, b:0.16254572516419627, Loss: 57.119515433746685
Epoch 17, m: 1.4121341418074391, b:0.1725329925279746, Loss: 55.055556301200994
Epoch 18, m: 1.4541980338411422, b:0.18250028535702537, Loss: 53.246153131444586
Epoch 19, m: 1.4934464268890222, b:0.19244764360041805, Loss: 51.65841947019918
Epoch 20, m: 1.5300677731878056, b:0.20237510712732393, Loss: 50.26372376282578
Epoch 21, m: 1.5642379111371314, b:0.212282715727176, Loss: 49.03713882389985
Epoch 22, m: 1.5961209095924542, b:0.22217050910982836, Loss: 47.95696253858988
Epoch 23, m: 1.6258698556461584, b:0.23203852690571541, Loss: 47.00430057932756
Epoch 24, m: 1.6536275896794341, b:0.2418680866600107, Loss: 46.16270311376041
Epoch 25, m: 1.6795273912142903, b:0.2517153938627854, Loss: 45.417848518184684
Epoch 26, m: 1.7036936188588374, b:0.2615243218891666, Loss: 44.75726801453283
Epoch 27, m: 1.726242307418559, b:0.271313632059495, Loss: 44.170105935915046
Epoch 28, m: 1.747281725040616, b:0.2810833636094827, Loss: 43.64691101082227
Epoch 29, m: 1.766912893066327, b:0.2908335556963705, Loss: 43.17945465256065
Epoch 30, m: 1.785230071087912, b:0.3005642473990845, Loss: 42.76057275977556
Epoch 31, m: 1.8023212095385133, b:0.31027547771839303, Loss: 42.38402798602196
Epoch 32, m: 1.8182683719886135, b:0.319967285577063, Loss: 42.04438982994041
Epoch 33, m: 1.8331481291765226, b:0.3296397098200156, Loss: 41.73693024027347
Epoch 34, m: 1.8470319266648803, b:0.3392927892144823, Loss: 41.45753272829492
Epoch 35, m: 1.8599864278884861, b:0.34892656245016007, Loss: 41.202613239959604
Epoch 36, m: 1.872073834240612, b:0.3585410681393665, Loss: 40.96905126621258
Epoch 37, m: 1.8833521837346998, b:0.3681363448171945, Loss: 40.75412986676654
Epoch 38, m: 1.893875629675476, b:0.37771243094166684, Loss: 40.555483454055086
Epoch 39, m: 1.9036947006775309, b:0.38726936489389024, Loss: 40.37105233329189
Epoch 40, m: 1.912856543279849, b:0.3968071849782092, Loss: 40.199043124479026
Epoch 41, m: 1.9214051483212105, b:0.4063259294223595, Loss: 40.037894305313465
Epoch 42, m: 1.9293815621634105, b:0.41582563637762154, Loss: 39.88624621241109
Epoch 43, m: 1.9368240837764934, b:0.42530634391897304, Loss: 39.74291492399707
Epoch 44, m: 1.943768448632308, b:0.4347680900452418, Loss: 39.606869521849205
Epoch 45, m: 1.9502480002893565, b:0.44421091267925805, Loss: 39.47721229526082
Epoch 46, m: 1.956293850492807, b:0.45363484966800627, Loss: 39.353161506362255
Epoch 47, m: 1.9619350285583912, b:0.463039938782777, Loss: 39.23403638539313
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Epoch 48, m: 1.9671986207574614, b:0.47242621771931814, Loss: 39.1192440673973
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Epoch 50, m: 1.9766924490473072, b:0.49114249546389693, Loss: 38.9006591375846
Epoch 51, m: 1.9809682700152556, b:0.5004725692870758, Loss: 38.796025129435684
Epoch 52, m: 1.9849578937480747, b:0.5097839829626084, Loss: 38.69402500614435
Epoch 53, m: 1.988680476533647, b:0.5190767738107899, Loss: 38.59436154990157
Epoch 54, m: 1.9921538924554154, b:0.528350979077275, Loss: 38.496775825072696
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Epoch 56, m: 1.998418818212071, b:0.5468437814754675, Loss: 38.30696416352858
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Epoch 58, m: 2.0038731403058714, b:0.5652626866352768, Loss: 38.12311135061229
Epoch 59, m: 2.00632965248016, b:0.5744445200761129, Loss: 38.03305709388804
Epoch 60, m: 2.0086217407934845, b:0.5836079898500675, Loss: 37.944094040119595
Epoch 61, m: 2.010760410770807, b:0.592753132684474, Loss: 37.85612320174256
Epoch 62, m: 2.0127559312947376, b:0.6018799852332118, Loss: 37.76905823032188
Epoch 63, m: 2.014617883911856, b:0.6109885840768522, Loss: 37.68282378187898
Epoch 64, m: 2.0163552088387706, b:0.6200789657228052, Loss: 37.59735409372296
Epoch 65, m: 2.0179762478888135, b:0.6291511666054663, Loss: 37.51259174541924
Epoch 66, m: 2.0194887845254867, b:0.6382052230863621, Loss: 37.42848658007095
Epoch 67, m: 2.0209000812349758, b:0.647241171454296, Loss: 37.34499476517041
Epoch 68, m: 2.0222169143971764, b:0.6562590479254942, Loss: 37.26207797496212
Epoch 69, m: 2.0234456068226687, b:0.6652588886437499, Loss: 37.17970267859525
Epoch 70, m: 2.0245920581118653, b:0.6742407296805691, Loss: 37.09783952037769
Epoch 71, m: 2.025661772982106, b:0.6832046070353147, Loss: 37.01646278021492
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Epoch 74, m: 2.02846016579294433, b:0.7099888159216212, Loss: 36.7750389538764
Epoch 75, m: 2.029270973255272, b:0.7188811971038847, Loss: 36.69540817794271
Epoch 76, m: 2.030027510238158, b:0.7277557935237836, Loss: 36.61617528132135
Epoch 77, m: 2.0307334092746947, b:0.7366126407508428, Loss: 36.537328365164825
Epoch 78, m: 2.031392059758485, b:0.7454517742834478, Loss: 36.45885691202005
Epoch 79, m: 2.03200662421791, b:0.7542732295489876, Loss: 36.38075160772502
Epoch 80, m: 2.0325800535010923, b:0.7630770419039963, Loss: 36.30300418634744
Epoch 81, m: 2.0331151009444786, b:0.771863246634295, Loss: 36.22560729518375
Epoch 82, m: 2.033614335593057, b:0.7806318789551331, Loss: 36.14855437722324
Epoch 83, m: 2.034080154535699, b:0.7893829740113295, Loss: 36.07183956881722
Epoch 84, m: 2.034514794414845, b:0.7981165668774136, Loss: 35.99545761058604
Epoch 85, m: 2.034920342165808, b:0.8068326925577655, Loss: 35.919403769851336
Epoch 86, m: 2.0352987450372475, b:0.8155313859867567, Loss: 35.843673773101884
Epoch 87, m: 2.0356518199409392, b:0.8242126820288899, Loss: 35.768263747195235
Epoch 88, m: 2.0359812621757265, b:0.8328766154789388, Loss: 35.693170168164606
Epoch 89, m: 2.0362886535675444, b:0.8415232210620877, Loss: 35.61838981664719
Epoch 90, m: 2.0365754700645993, b:0.8501525334340702, Loss: 35.54391973907687
Epoch 91, m: 2.0368430888241758, b:0.8587645871813088, Loss: 35.46975721389594
Epoch 92, m: 2.0370927948250954, b:0.8673594168210529, Loss: 35.395899722135994
Epoch 93, m: 2.0373257870375765, b:0.8759370568015175, Loss: 35.32234492180301
Epoch 94, m: 2.037543184180124, b:0.8844975415020211, Loss: 35.249090625574254
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Epoch 95, m: 2.0377460300910863, b:0.8930409052331237, Loss: 35.17613478137856
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Epoch 97, m: 2.038111898907503, b:0.9100764066863974, Loss: 35.031110817788445
Epoch 98, m: 2.0382766785421236, b:0.9185686126871314, Loss: 34.95903912896947
Epoch 99, m: 2.038430428838475, b:0.9270438342758639, Loss: 34.8872587293487
Epoch 100, m: 2.038573888032826, b:0.9355021054214189, Loss: 34.81576802915489
2.038573888032826 0.9355021054214189 34.81576802915489
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Epoch 4, m: 2.0161795654332098, b:0.40028369473966147, Loss: 39.63997666358201
Epoch 5, m: 2.032507520858402, b:0.4954016089859355, Loss: 38.726544463098776
Epoch 6, m: 2.037906544195389, b:0.588617164947284, Loss: 37.86438021316991
Epoch 7, m: 2.0396917923929903, b:0.6799684097894055, Loss: 37.0380082214082
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Epoch 9, m: 2.040477298445292, b:0.8572263652810582, Loss: 35.48251486509768
Epoch 10, m: 2.0405418413934595, b:0.9432054261165043, Loss: 34.750667096622365
Epoch 11, m: 2.0405631832500677, b:1.0274649057352414, Loss: 34.0478007356618
Epoch 12, m: 2.0405702401766015, b:1.1100391957616038, Loss: 33.372767908386024
Epoch 13, m: 2.040572573629263, b:1.190961999987439, Loss: 32.72446638389024
Epoch 14, m: 2.0405733452118064, b:1.2702663481287575, Loss: 32.101837600072805
Epoch 15, m: 2.040573600344311, b:1.3479846093072496, Loss: 31.503864916128244
Epoch 16, m: 2.040573684706762, b:1.4241485052621718, Loss: 30.929571950471583
Epoch 17, m: 2.040573712602162, b:1.4987891232979957, Loss: 30.378020986255326
Epoch 18, m: 2.0405737218260915, b:1.571936928973103, Loss: 29.84831144022208
Epoch 19, m: 2.0405737248760882, b:1.6436217785347083, Loss: 29.33957839221175
Epoch 20, m: 2.040573725884604, b:1.7138729311050813, Loss: 28.850991172902635
Epoch 21, m: 2.0405737262180814, b:1.7827190606240468, Loss: 28.38175200747816
Epoch 22, m: 2.0405737263283497, b:1.850188267552633, Loss: 27.93109471300449
Epoch 23, m: 2.040573726364811, b:1.9163080903426477, Loss: 27.49828344739198
Epoch 24, m: 2.0405737263768673, b:1.981105516676862, Loss: 27.082611507897724
Epoch 25, m: 2.040573726380854, b:2.044606994484392, Loss: 26.683400177207442
Epoch 26, m: 2.040573726382172, b:2.1068384427357714, Loss: 26.299997615212494
Epoch 27, m: 2.040573726382608, b:2.1678252620221232, Loss: 25.93177779467255
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Epoch 33, m: 2.040573726382823, b:2.508968656354086, Loss: 24.009242057164986
Epoch 34, m: 2.0405737263828234, b:2.5619128713680714, Loss: 23.73173615672372
Epoch 35, m: 2.0405737263828234, b:2.613798202081777, Loss: 23.46521948993993
Epoch 36, m: 2.0405737263828234, b:2.664645826181209, Loss: 23.209256883160776
Epoch 37, m: 2.0405737263828234, b:2.714476497798652, Loss: 22.96343039561008
Epoch 38, m: 2.0405737263828234, b:2.7633105559837463, Loss: 22.727338636966387
Epoch 39, m: 2.0405737263828234, b:2.8111679330051387, Loss: 22.500596111964985
Epoch 40, m: 2.0405737263828234, b:2.858068162486103, Loss: 22.282832590953635
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Epoch 41, m: 2.0405737263828234, b:2.9040303873774485, Loss: 22.07369250537434
Epoch 42, m: 2.0405737263828234, b:2.949073367770967, Loss: 21.872834367183984
Epoch 43, m: 2.0405737263828234, b:2.993215488556615, Loss: 21.679930211265965
Epoch 44, m: 2.0405737263828234, b:3.03647476692655, Loss: 21.494665059922305
Epoch 45, m: 2.0405737263828234, b:3.078868859729086, Loss: 21.316736408571845
Epoch 46, m: 2.0405737263828234, b:3.1204150706755716, Loss: 21.145853731814867
Epoch 47, m: 2.0405737263828234, b:3.1611303574031275, Loss: 20.981738009057466
Epoch 48, m: 2.0405737263828234, b:3.2010313383961324, Loss: 20.82412126892126
Epoch 49, m: 2.0405737263828234, b:3.240134299769277, Loss: 20.672746151694447
Epoch 50, m: 2.0405737263828234, b:3.2784552019149586, Loss: 20.527365489109812
Epoch 51, m: 2.0405737263828234, b:3.316009686017727, Loss: 20.387741900763533
Epoch 52, m: 2.0405737263828234, b:3.3528130804384397, Loss: 20.25364740651576
Epoch 53, m: 2.0405737263828234, b:3.3888804069707383, Loss: 20.12486305424021
Epoch 54, m: 2.0405737263828234, b:3.424226386972391, Loss: 20.00117856231476
Epoch 55, m: 2.0405737263828234, b:3.4588654473740106, Loss: 19.882391976269563
Epoch 56, m: 2.0405737263828234, b:3.4928117265675978, Loss: 19.768309339031752
Epoch 57, m: 2.0405737263828234, b:3.526079080177313, Loss: 19.65874437422856
Epoch 58, m: 2.0405737263828234, b:3.558681086714834, Loss: 19.55351818203158
Epoch 59, m: 2.0405737263828234, b:3.5906310531216046, Loss: 19.4524589470456
Epoch 60, m: 2.0405737263828234, b:3.62194202020024, Loss: 19.355401657765057
Epoch 61, m: 2.0405737263828234, b:3.6526267679373023, Loss: 19.262187837140033
Epoch 62, m: 2.0405737263828234, b:3.6826978207196235, Loss: 19.172665283811753
Epoch 63, m: 2.0405737263828234, b:3.712167452446298, Loss: 19.086687823595277
Epoch 64, m: 2.0405737263828234, b:3.7410476915384394, Loss: 19.004115070803373
Epoch 65, m: 2.0405737263828234, b:3.769350325848738, Loss: 18.924812199022025
Epoch 66, m: 2.0405737263828234, b:3.7970869074728304, Loss: 18.848649720963223
Epoch 67, m: 2.0405737263828234, b:3.824268757464441, Loss: 18.775503277035547
Epoch 68, m: 2.0405737263828234, b:3.8509069704562195, Loss: 18.705253432287407
Epoch 69, m: 2.0405737263828234, b:3.8770124191881625, Loss: 18.637785481391294
Epoch 70, m: 2.0405737263828234, b:3.9025957589454667, Loss: 18.572989261350664
Epoch 71, m: 2.0405737263828234, b:3.927667431907625, Loss: 18.510758971623645
Epoch 72, m: 2.0405737263828234, b:3.9522376714105394, Loss: 18.450993001369817
Epoch 73, m: 2.0405737263828234, b:3.976316506123396, Loss: 18.39359376353804
Epoch 74, m: 2.0405737263828234, b:3.9999137641419953, Loss: 18.3384675355244
Epoch 75, m: 2.0405737263828234, b:4.023039077000223, Loss: 18.2855243061401
Epoch 76, m: 2.0405737263828234, b:4.045701883601286, Loss: 18.234677628639417
Epoch 77, m: 2.0405737263828234, b:4.067911434070327, Loss: 18.18584447956777
Epoch 78, m: 2.0405737263828234, b:4.089676793529987, Loss: 18.138945123199353
Epoch 79, m: 2.0405737263828234, b:4.111006845800455, Loss: 18.093902981343128
Epoch 80, m: 2.0405737263828234, b:4.131910297025513, Loss: 18.050644508304405
Epoch 81, m: 2.0405737263828234, b:4.15239567922607, Loss: 18.009099070798015
Epoch 82, m: 2.0405737263828234, b:4.172471353782616, Loss: 17.969198832616886
Epoch 83, m: 2.0405737263828234, b:4.192145514848031, Loss: 17.930878643867725
Epoch 84, m: 2.0405737263828234, b:4.211426192692137, Loss: 17.894075934593026
Epoch 85, m: 2.0405737263828234, b:4.230321256979361, Loss: 17.858730612605616
Epoch 86, m: 2.0405737263828234, b:4.248838419980841, Loss: 17.8247849653689
Epoch 87, m: 2.0405737263828234, b:4.266985239722292, Loss: 17.79218356576276
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Epoch 88, m: 2.0405737263828234, b:4.284769123068913, Loss: 17.760873181581026
Epoch 89, m: 2.0405737263828234, b:4.302197328748602, Loss: 17.73080268861288
Epoch 90, m: 2.0405737263828234, b:4.319276970314697, Loss: 17.70192298716628
Epoch 91, m: 2.0405737263828234, b:4.336015019049471, Loss: 17.67418692189696
Epoch 92, m: 2.0405737263828234, b:4.3524183068095486, Loss: 17.64754920481231
Epoch 93, m: 2.0405737263828234, b:4.368493528814425, Loss: 17.62196634132421
Epoch 94, m: 2.0405737263828234, b:4.3842472463792035, Loss: 17.59739655923024
Epoch 95, m: 2.0405737263828234, b:4.3996858895926865, Loss: 17.573799740507187
Epoch 96, m: 2.0405737263828234, b:4.4148157599419, Loss: 17.551137355805572
Epoch 97, m: 2.0405737263828234, b:4.429643032884129, Loss: 17.529372401538136
Epoch 98, m: 2.0405737263828234, b:4.444173760367513, Loss: 17.508469339459698
Epoch 99, m: 2.0405737263828234, b:4.45841387330123, Loss: 17.488394038639562
Epoch 100, m: 2.0405737263828234, b:4.472369183976272, Loss: 17.469113719731904
2.0405737263828234 4.472369183976272 17.469113719731904
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