COMP 5630 Fall 2022 Assignment 4

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1. Logistic Regression

   (a) Explain why logistic regression is a discriminative classifier

   Logistic regression assumes some functional form of the decision boundary to draw classifications. Naive Bayes, by contrast, assumes some functional form of the classification itself. This distinction makes logistic regression discriminative and Naive Bayes generative.

   (b) Decision boundary equation

   $$f(x) = w_0 + w_1 x_1 + w_2 x_2 \tag{1}$$

   Defines the decision plane

   (c) Derivation justification

      i. 2-3: ln properties
      ii. 3-4: Split probability space along all possibilities
      iii. 4-5: ln properties
      iv. 5-6: Substitute probabilities for definitions
      v. 6-7: Simplify and cancel with fractional ln
      vi. 7-8: Cancel ln/exp and ln properties

   (d) Show

$$\begin{aligned}
\frac{\partial \ell(w)}{\partial w_i} &= \frac{\partial}{\partial w_i} \textstyle\sum_{j=1}^{n} [y^j (w_0 + w_1 x_1^j + w_2 x_2^j)] - \frac{\partial}{\partial w_i} \sum_{j=1}^{n} \ln [1 + \exp (w_0 + w_1 x_1^j + w_2 x_2^j)] \\
&= \textstyle\sum_{j=1}^{n} (y^j x_i^j) - \sum_{j=1}^{n} (x_i^j * \frac{\exp (w_0 + w_1 x_1^j + w_2 x_2^j)}{1 + \exp (w_0 + w_1 x_1^j + w_2 x_2^j)}) \\
&= \textstyle\sum_{j=1}^{n} (y^j x_i^j) - \sum_{j=1}^{n} (x_i^j * p(y^j = 1 | x^j; w) \\
&= \textstyle\sum_{j=1}^{n} x_i^j (y^j - p(y^j = 1 | x^j; w))
\end{aligned} \tag{2}$$

2. Implementing Logistic Regression (code at: github.com/wumphlett)

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Optimal Hyperparameters: Learning Rate: 0.05, # of Epochs: 1000
Training Accuracy: %93.331966
Validation Accuracy: %93.292308
Test Accuracy: %91.815385
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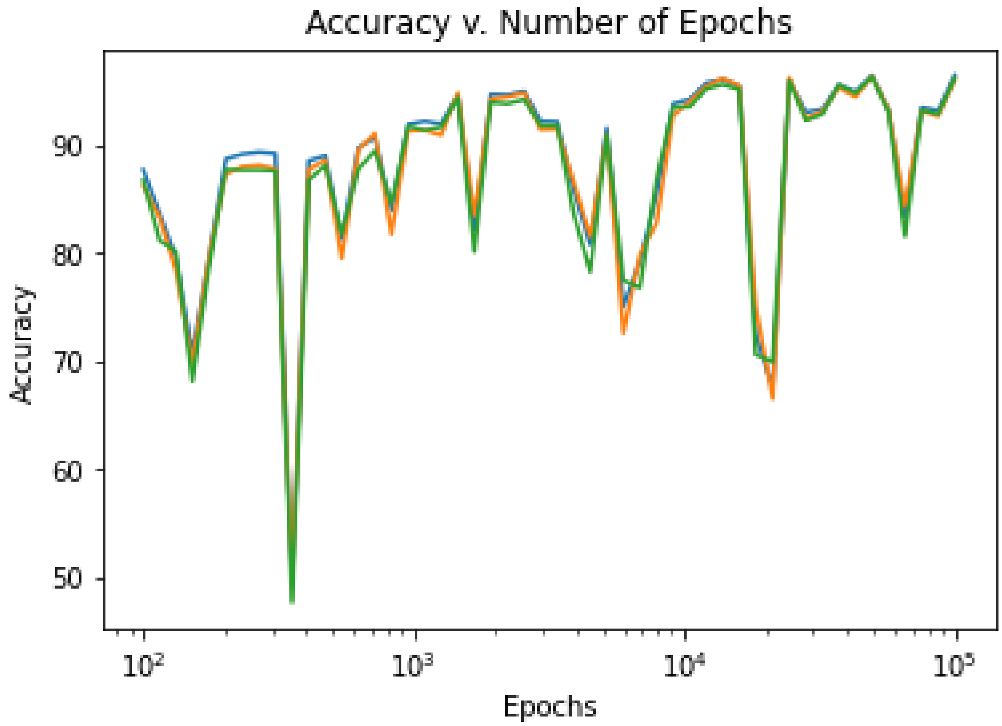

Figure 1: Epochs (lr fixed at .05)

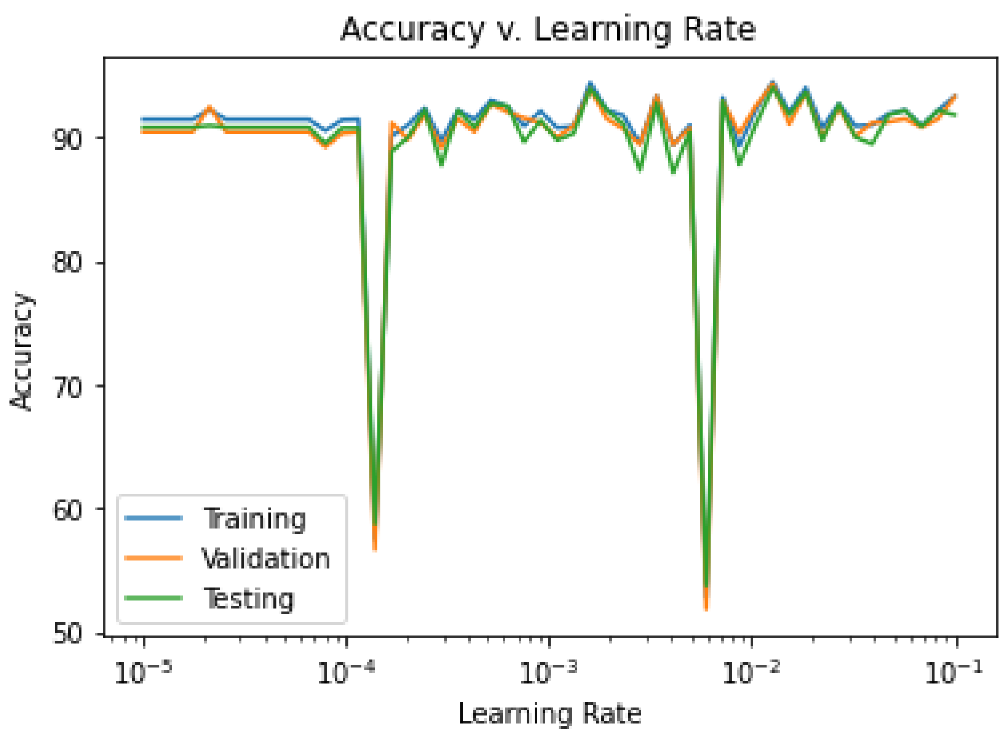

Figure 2: Learning Rate (epochs fixed at 1000)