зад 1. Съставете общо у-ние на р-на α:

$\begin{cases} \ni A(1, 2, -1) \\ \perp \vec{n}(2, 1, -4) \end{cases}$

$\alpha: a(x-1)+b(y-2)+c(z+1)=0$

$2x-2+y-2-4z-4=0$

$\boxed{\alpha: 2x+y-4z-8=0}$

зад 2 $\alpha: \begin{cases} \ni A(1, 2, -1) \\ \parallel \vec{a}(3, 4, -1) \\ \parallel \vec{b}(2, 2, 1) \end{cases}$ $\alpha = ?$

$\alpha: \begin{vmatrix} x-1 & y-2 & z+1 \\ 3 & 4 & -1 \\ 2 & 2 & 1 \end{vmatrix} \begin{matrix} x-1 & y-2 \\ 3 & 4 \\ 2 & 2 \end{matrix}$

$\cdot\ 4(x-1)-2(y-2)+6(z+1)-8(z+1)+2(x-1)-3(y-2)$

$= 6(x-1)-5(y-2)-2(z+1) =$

$= 6x-6-5y+10-2z-2 =$

$= \boxed{6x-5y-2z+2=0}$

зад 3 $\alpha: \begin{cases} \ni A(1,2,-1) \\ \ni B(3,4,5) \\ \ni C(-2,1,3) \end{cases}$

$\alpha: \left|\begin{matrix} x-1 & y-2 & z+1 \\ 2 & 2 & 6 \\ -3 & -1 & 4 \end{matrix}\right| \begin{matrix} x-1 & y-2 \\ 2 & 2 \\ -3 & -1 \end{matrix}$

$8(x-1)-18(y-2)-2(z+1)+6(z+1)+6(x-1)-8(y-2)=$

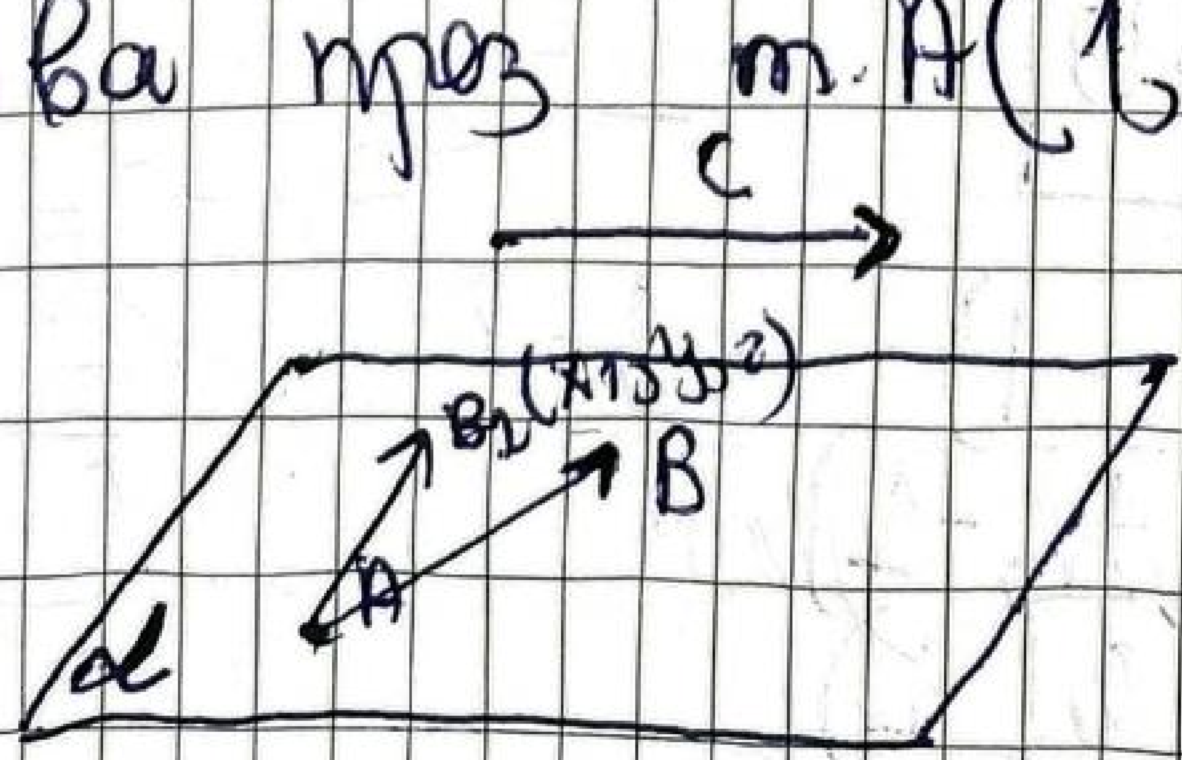
$= 14(x-1)-26(y-2)+4(z+1)=$

$= 14x-\underline{14}-26y+\underline{52}+4z+\underline{4}=$

$= 14x-26y+4z+\overset{42}{\cancel{32}}=0 \ /:2$

$\boxed{\alpha: 7x-13y+2z+\cancel{16}=0 \\ 21}$

зад 4 α?, което е $\parallel \vec{c}(1,2,-3)$ и минава през т. $A(1,2,-1)$ и т. $B(3,4,5)$



$\boxed{\overrightarrow{AB}(2,2,6)}$

$\cancel{6 = x_1+1 = 5 \quad 6 = z_1 - 3}$

$\cancel{2 = y_1 + 2 = y_1 = 0 \quad z_1 = 9}$

$\cancel{\left(\left|\begin{matrix} 1 & 2 & -3 \\ 2 & 2 & 6 \\ x & y & z \end{matrix}\right| \begin{matrix} 1 & 2 \\ 2 & 2 \\ x & y \end{matrix}\right) = 0}$

$$\cancel{= 2z + 12x - 6y + 6x - 6y - 4z = 0}$$
$$\cancel{18x - 12y - 2z = 0 \quad |:2}$$
$$\cancel{9x - 6y - z = 0}$$

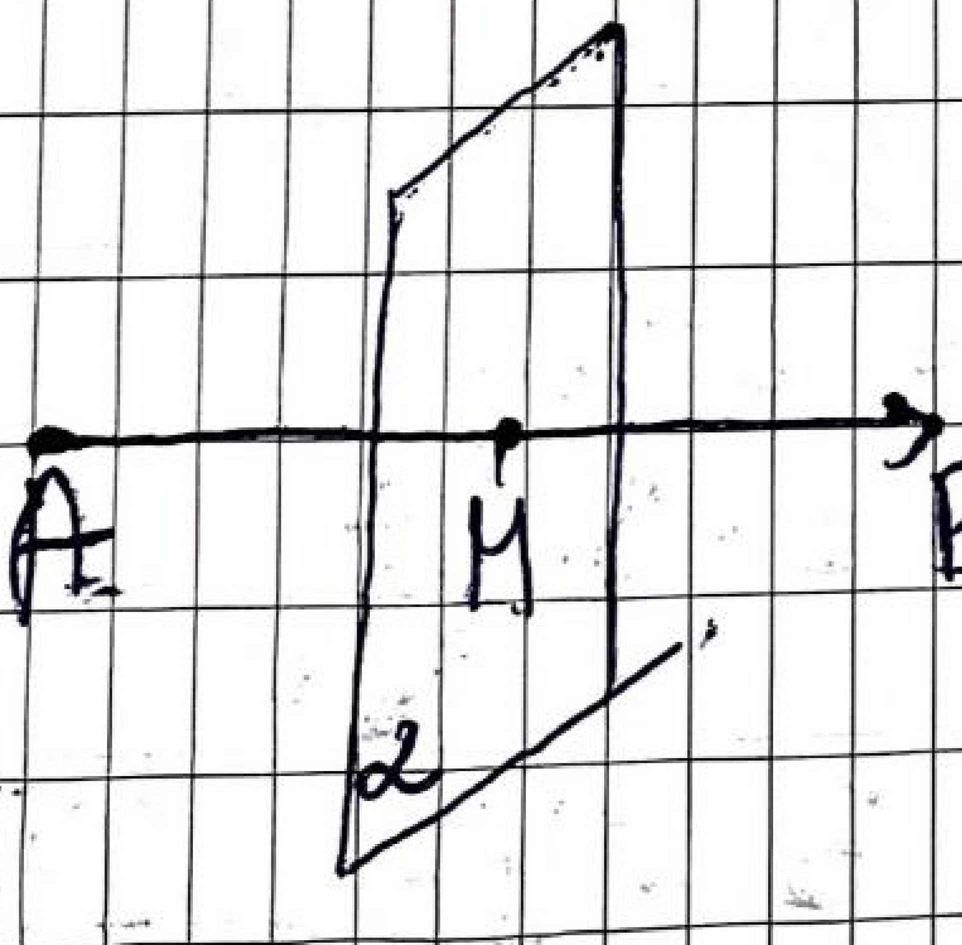
$$\alpha: \begin{vmatrix} x-1 & y-2 & z+1 \\ 1 & 2 & -3 \\ 2 & 2 & 6 \end{vmatrix} \begin{matrix} x-1 & y-2 \\ 1 & 2 \\ 2 & 2 \end{matrix} = 0$$

$$= 12(x-1) - 6(y-2) + 2(z+1) - 4(z+1) + 6(x-1) - 6(y-2) =$$
$$= 18(x-1) - 12(y-2) - 2(z+1) = 0 \quad /:2$$
$$9(x-1) - 6(y-2) - (z+1) = 0$$
$$9x - 9 - 6y + 12 - z - 1 = 0$$
$$\boxed{\alpha: 9x - 6y - z + 2 = 0}$$

Зад 5 т. $A(2,6,-3)$, т. $B(4,8,1)$. Да се състави у-нението на симетралната равнина на отсечката AB.

$\downarrow$

$\boxed{\text{среда и } \perp}$



т. $M(3,7,-1)$

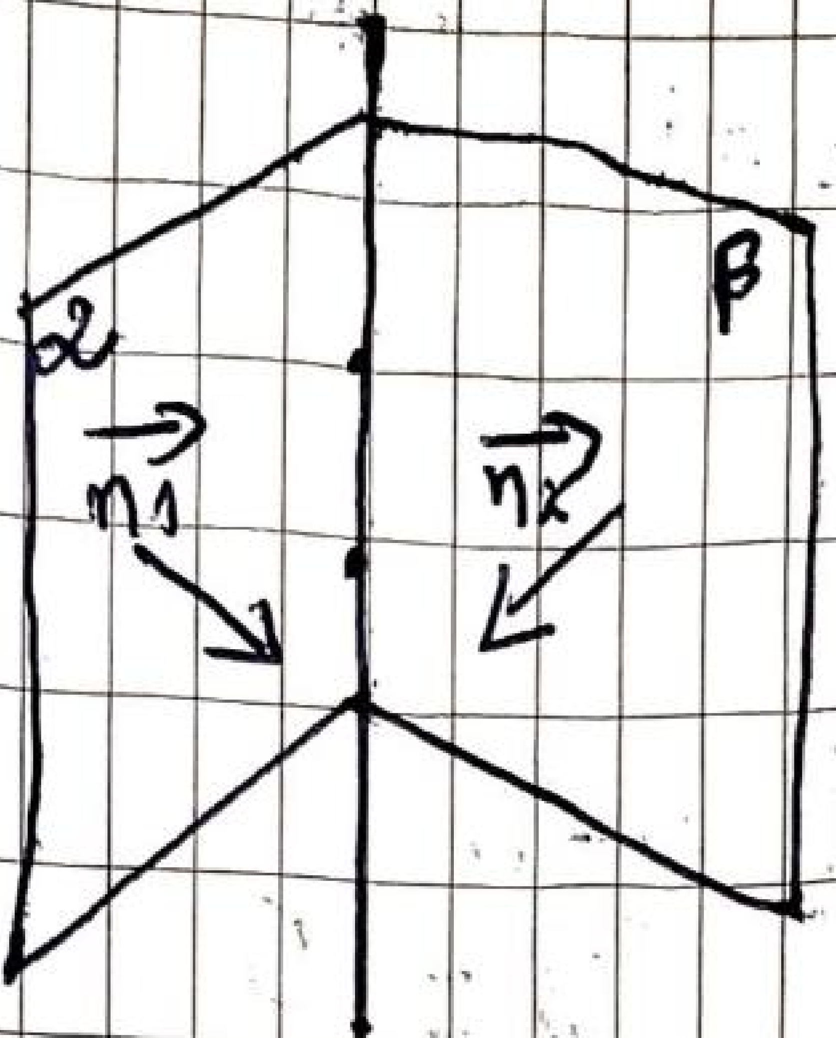
$\overrightarrow{MB}(1,1,2) \quad \overrightarrow{MB} \perp \alpha$

$$\alpha: (x-3)+(y-7)+2(z+1)=0$$
$$x-3+y-7+2z+2=0$$
$$\boxed{\alpha: x+y+2z-8=0}$$

Зад 6 $\alpha: x+2y+2z-3=0$

$\beta: 16x+12y-15z-1=0$

Да се намери ъгълът м/у двете равнини и уравнението на пресечницата им.



$\vec{n_1}(1, 2, 2)$

$\vec{n_2}(16, 12, -15)$

$$\cos(\vec{n_1}\,\hat{}\,\vec{n_2}) = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|} = \frac{16+24-30}{|\vec{n_1}||\vec{n_2}|} = \cancel{\boxed{\cos 0 = 90}}$$

$$|\vec{n_1}| = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$|\vec{n_2}| = \sqrt{256+144+225} = 25$$

$$\cos(\vec{n_1}\,\hat{}\,\vec{n_2}) = \frac{\cancel{10}^{2}}{\cancel{75}_{15}} = \boxed{\frac{2}{15}} \qquad \arccos\frac{2}{15}$$

Пресечницата на двете равнини представлява права

$$\begin{cases} x + 2y + 2z - 3 = 0 \\ 16x + 12y - 15z - 1 = 0 \end{cases}$$

$$z = 0 \quad \begin{cases} x + 2y - 3 = 0 \\ 16x + 12y - 1 = 0 \end{cases}$$

$16(3-2y) + 12y - 1 = 0$

$48 - 32y + 12y - 1 = 0$

$-20y = -47$

$y = \frac{47}{20}$

$x = 3 - 2y$

$x = 3 - \cancel{2}\cdot\frac{47}{\cancel{20}_{10}} = \cancel{\frac{-3.47}{10}}$

$= 3 - \frac{47}{10} = -\frac{17}{10}$

$$M_1\left(-\frac{17}{10}, \frac{47}{20}, 0\right)$$

$z = 1$

$$\begin{cases} x + 2y - 1 = 0 \\ 16x + 12y - 16 = 0 \end{cases} \quad x = 1 - 2y \quad x = 1$$

$16(1 - 2y) + 12y = 16$

$16 - 32y + 12y = 16 \quad m.\ M_2(1, 0, 1)$

$-20y = 0$

$y = 0$

$$\boxed{e: \frac{x + \frac{17}{10}}{1 + \frac{17}{10}} = \frac{y - \frac{47}{20}}{0 - \frac{47}{20}} = \frac{z - 0}{1}}$$

зад 1 m. $A(1, 2, -4)$ $g: \frac{x-1}{2} = \frac{y+1}{4} = \frac{z-2}{3}$

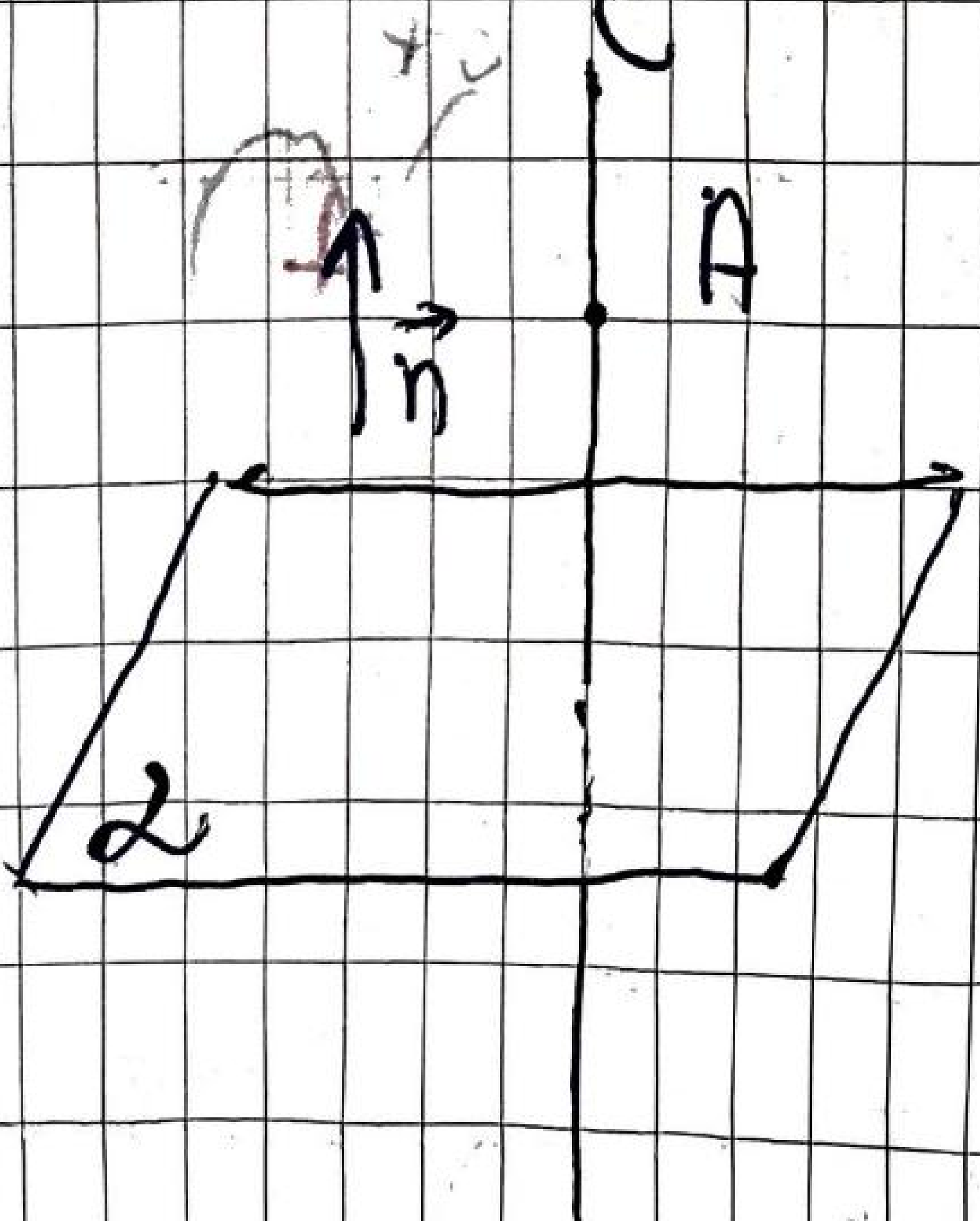
$e = ?$ $e: \begin{cases} \ni A \\ \parallel g \end{cases}$

$\vec{g}(2, 4, 3)$

$e: \begin{cases} x = 1 + 2t \\ y = 2 + 4t \\ z = -4 + 3t \end{cases}$ $\qquad e: \boxed{\frac{x-1}{2} = \frac{y-2}{4} = \frac{z+4}{3}}$

зад 2 m $A(2, 1, -3)$ $\alpha: x + 2y - 3z + 4 = 0$

$e = ?$ $e: \begin{cases} \ni A \\ \perp \alpha \end{cases}$



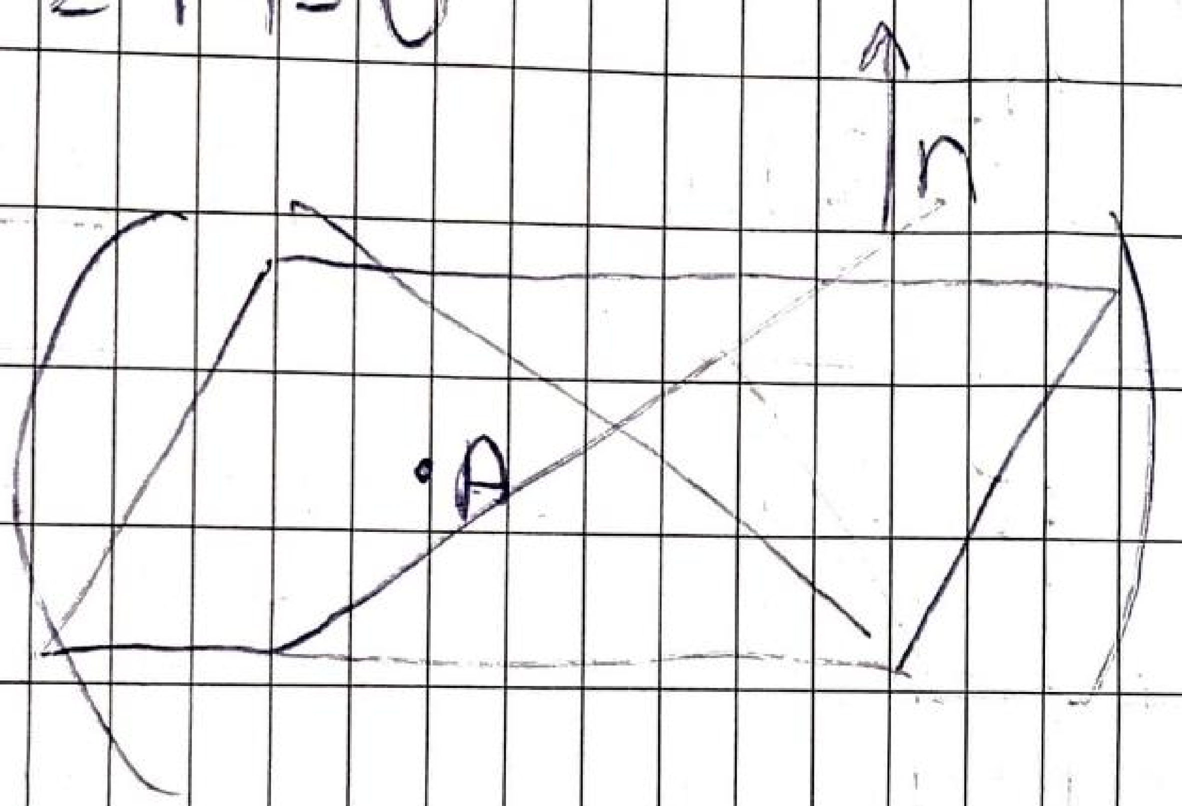
$\vec{n} - e$:

$\vec{n}(1, 2, -3)$ $\qquad \boxed{e: \frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{3}}$

зад 1. Определете взаимното положение на ℓ и α.

а) $\ell: \frac{x+1}{2} = \frac{y+1}{1} = \frac{z-3}{5} = t$

$\alpha: 3x - y - z + 4 = 0$



~~$\ell: \begin{cases} \ni A(-1,-1,3) \\ \vec{p}(2,1,5) \end{cases}$~~

~~$3(-1) - (-1) - 3 + 4 \neq 0$~~

~~$\Rightarrow$ няма обща точка~~

~~$\ell \parallel \alpha$~~

$\ell: \begin{cases} \frac{x+1}{2} = t \\ \frac{y+1}{1} = t \\ \frac{z-3}{5} = t \end{cases}$

$\ell: \begin{cases} x = -1+2t \\ y = 1+t \\ z = 3+5t \end{cases}$

$\alpha: 3(2t-1) - 1 - t - 3 - 5t + 4 = 0$

$-3 \neq 0 \Rightarrow \boxed{\ell \parallel \alpha}$

б) $\ell: \frac{x+1}{2} = \frac{y-4}{-1} = \frac{z-2}{1} = t$

$\alpha: 3x+y-z-3=0$

$$\left.\begin{array}{l} \frac{x+1}{2} = t \\ \frac{y-4}{-1} = t \\ z-2=t \end{array}\right\} \quad \begin{array}{l} x+1=2t \\ y-4=-t \\ z-2=t \end{array} \quad \begin{array}{l} x=-1+2t \\ y=4-t \\ z=2+t \end{array}$$

Скаларно: - парам. у-я ги заместваме в у-ята за равнината

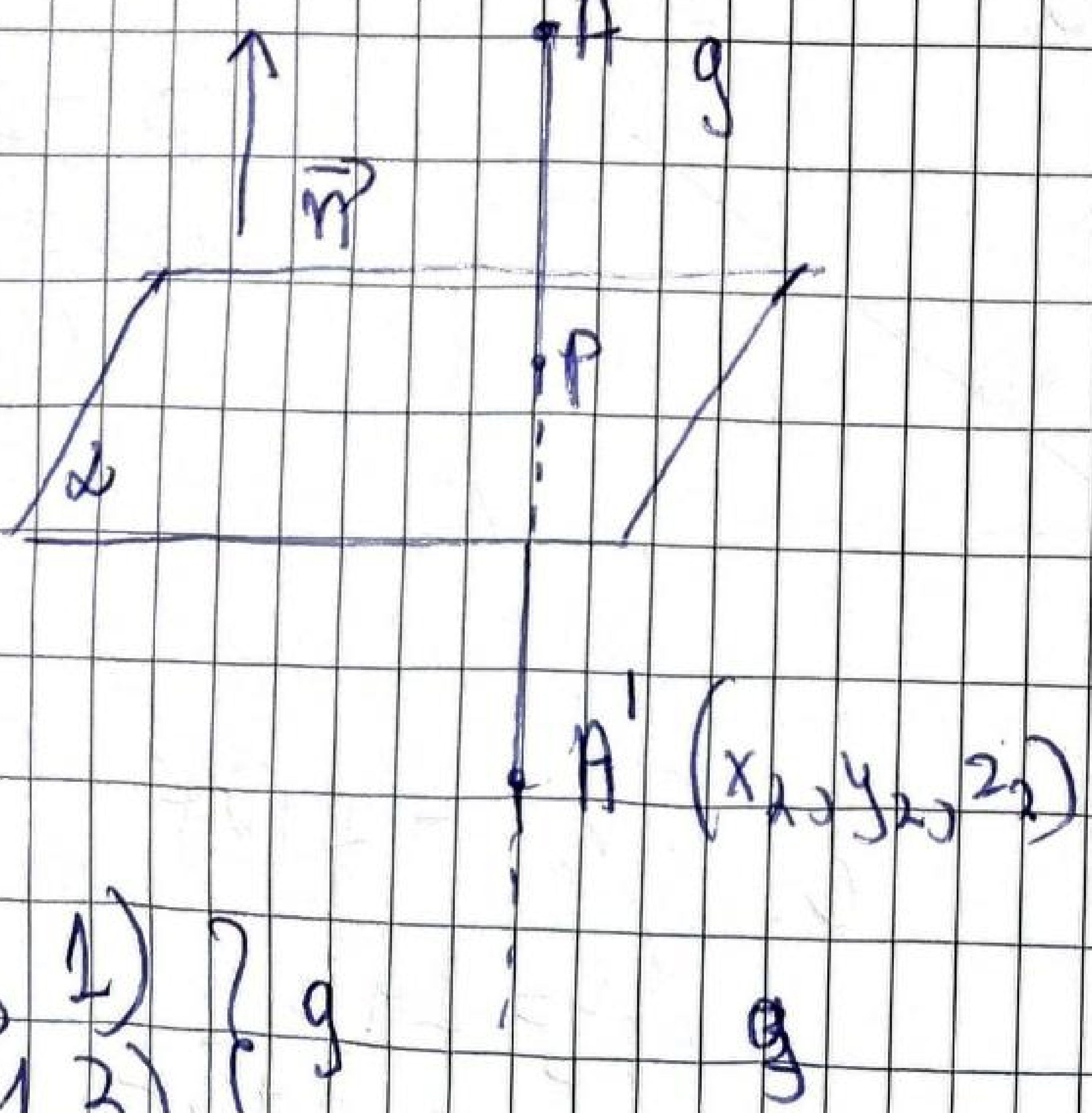
$3(2t-1)+4-t-2-t-3=0$

$4t=4$

$t=1$

$x=1 \quad y=3 \quad z=3 \quad P(1,3,3)$ пробод коорд.

зад 2 Намерете симетричната на т. $A(2,1,3)$ относно равнината $\alpha: x+y+z-9=0$



$g: \frac{x-2}{1} = y-1 = z-3$

$g: \begin{cases} x-2=t \\ y-1=t \\ z-3=t \end{cases}$

$$\left.\begin{array}{l} \vec{n}(1,1,1) \\ m \ni A(2,1,3) \end{array}\right\} g \qquad g$$

$x=t+2$

$y=t+1$

$z=t+3$

$t+2+t+1+t+3-9=0$

$3t=3$

$t=1 \Rightarrow P(3,2,4)$

$$P\left(\frac{2+x_2}{2}, \frac{1+y_2}{2}, \frac{3+z_2}{2}\right)$$

$6=2+x_2$ $\quad$ $4=1+y_2$ $\quad$ $8=3+z_2$

$x_2=4$ $\quad$ $y_2=3$ $\quad$ $z_2=5$

$\boxed{A'(4,3,5)}$