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Специалност: СИТ

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зад 1

$z = 1 + \sqrt{3}\,i$

$x = 1 \quad y = \sqrt{3}$

$r = |z| = \sqrt{x^2 + y^2} \quad r = 2$

$tg\,\varphi = \frac{y}{x} = \sqrt{3} \quad \varphi = \frac{\pi}{3}$

$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$z^{12} = 2^{12}\left(\underset{1}{\cos\frac{\cancel{12}^{\,4}\pi}{\cancel{3}}} + i\,\underset{0}{\sin\frac{\cancel{12}^{\,4}\pi}{\cancel{3}}}\right) = 4096$

зад 2

$f(x) = x\arccos\frac{x}{2} - \sqrt{4 - x^2}$

$f'(x) = \left(x\arccos\frac{x}{2}\right)' - \left(\sqrt{4 - x^2}\right)' =$

$= (x)'\arccos\frac{x}{2} + x\left(\arccos\frac{x}{2}\right)' - \left(\sqrt{4 - x^2}\right)' =$

$= \arccos\frac{x}{2} + x\left(-\frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}\right) - \left(\sqrt{4 - x^2}\right)' =$

$= \cancel{\arccos\frac{x}{2} - \frac{x \cdot \frac{1}{2}}{\;}}$

$\cancel{\left(\sqrt{4 - x^2}\right)'} = \cancel{(4 - x^2)}$

$$= \arccos\frac{x}{2} - \frac{x\cdot\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}} + \frac{\cancel{2}x}{\cancel{2}\sqrt{4-x^2}} =$$

$$= \arccos\frac{x}{2} - \frac{\cancel{\frac{1}{2}x}}{\cancel{\frac{1}{2}}\sqrt{4-x^2}} + \frac{\cancel{x}}{\sqrt{4-x^2}} = \arccos\frac{x}{2}$$

зад 3

$f(x) = x^3 - 6x^2 + 9x + 1$ $\qquad$ $D: x \in (-\infty, +\infty)$

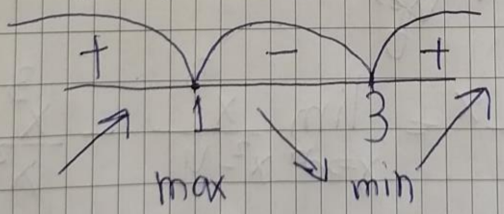
$f'(x) = 3x^2 - 12x + 9 \quad /:3$

$x^2 - 4x + 3$

$D = 4 - 3 = \sqrt{1} = 1$

$x_{12} = 2 \pm 1 <$ 3, 1

$(x-3)(x-1)$



$f(1) = 1 - 6 + 9 + 1 = 5$ − локален максимум f_{max}

$f(3) = \cancel{27} - \cancel{54} + \cancel{27} + 1 = 1$ локален минимум f_{min}

Зад 4

$f(x) = \frac{x^3}{x^2-4}$ D: $x \neq \pm 2$

1) $\lim_{x\to\infty} \frac{x^3}{x^2-4} = \frac{\cancel{x^3}}{\cancel{x^2}(1-\frac{4}{x^2})} \Rightarrow \infty$ няма хор. ас.

2) $\lim_{x\to -2} \frac{x^3}{x^2-4} = \infty$ $\lim_{x\to 2} \frac{x^3}{x^2-4} = \infty$

$\Rightarrow$ има вертикал ас.

3) $k = \lim_{x\to\infty} \frac{x^3}{x(x^2-4)}$

$\lim_{x\to\infty} \frac{x^3}{x^3-4} = \frac{\cancel{x^3}}{\cancel{x^3}(1-\frac{4}{x^3})} = 1$

$\frac{4}{x^3} \to 0$

$n = \lim_{x\to\infty} \frac{x^3}{x^2-4} - x = \lim_{x\to\infty} \frac{x^3}{x^2-4} - \frac{x^3-4x}{x^2-4} =$

$= \lim_{x\to\infty} \frac{x^3-x^3+4x}{x^2-4} = \lim_{x\to\infty} \frac{4x}{x^2(1-\frac{4}{x^2})} = 0$

$\frac{4}{x^2} \to 0$

$\boxed{y=x}$ накл. ас.

зад 5

а) $\int \left(4x^7 + \frac{2}{\cos^2 x} - \frac{5}{x^3}\right) dx = \int 4x^7 dx + \int \frac{2}{\cos^2 x} dx - \int \frac{5}{x^3} dx =$

$= 4 \cdot \frac{x^8}{8_2} + 2\int \frac{1}{\cos^2 x} dx - 5\int x^{-3} dx =$

$= \frac{x^8}{2} + 2\operatorname{tg} x - 5 \cdot \frac{x^{-2}}{-2} = \frac{x^8}{2} + 2\operatorname{tg} x + \frac{5}{2x^2} + c$

б) $\int \left(2\cos 5x - \frac{6}{3x-5}\right) dx = \int 2\cos 5x dx - \int \frac{6}{3x-5} dx =$

$= \frac{2}{5}\int \cos 5x \, d(5x) - 6\int \frac{1}{3x-5} dx =$ $\cancel{6}^{2}\frac{1}{3}$

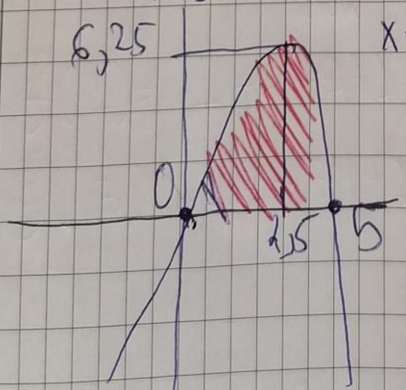
$= \frac{2\sin 5x}{5} - 2\int \frac{1}{3x-5} d(3x-5) =$

$= \frac{2\sin 5x}{5} - 2\ln(|3x-5|) + c$

зад 6 $y = 5x - x^2$ $\quad 5x - x^2 = 0$

$x = 0$ $\quad x(x-5) = 0$

$x = 0 \quad x = 5$



$x_V = \frac{-b}{2a} = \frac{-5}{-2} = 2{,}5$

$y(2{,}5) = 6{,}25$

$$S = \int_0^5 (5x - x^2)\,dx = 5\int x\,dx - \int x^2\,dx =$$

$$= \frac{5x^2}{2} - \frac{x^3}{3}\Bigg|_0^5 = \frac{5\cdot 5^2}{2} - \frac{5^3}{3} - \left(\frac{5\cdot 0^2}{2} - \frac{0^3}{3}\right) =$$

$$\frac{5^3}{2} - \frac{125}{3} = \frac{3\cdot 125 - 2\cdot 125}{6}$$

$$= \boxed{\frac{125}{6}}$$