Комплексни числа

$\mathbb{N} = \{1, 2, 3 \dots, n, \dots\}$ ест. числа

$\mathbb{Z}$ - цели числа $\{-n, \dots -1, 0, 1, \dots n\}$

$\mathbb{Q}$ - рац. числа - {всички десетични, крайни и безкрайни, но периодични дес. дроби}

$\mathbb{I}$ - ирационални числа {всички безкр. непериод. десетични дроби}

$\mathbb{R} = \{\mathbb{Q}, \mathbb{I}\}$ реални числа

$Z = (x, y) \quad x, y \in \mathbb{R}$ Комплексни числа

$x = \operatorname{Re} z, \quad y = \operatorname{Im} z$

$\boxed{z = x + iy}$ - алгебричен вид на z

i - имагинерна единица $i = (0, 1)$

$\boxed{i^2 = -1}$

$i^3 = i^2 \cdot i = -i$

$i^{14} = (i^2)^7 = -1^7 = -1$

$i^{21} = i^{20} \cdot i^1 = \cancel{i^{20}} \, (i^2)^{10} \cdot i = i$

$z_1 = x_1 + iy_1 \qquad z_2 = x + iy_2$

$z_1 = z_2 \Leftrightarrow \begin{cases} x_1 = x_2 \\ y_1 = y_2 \end{cases}$

$z_1 + z_2 = x_1 + iy_1 + x_2 + iy_2 = (x_1 + x_2) + i(y_1 + y_2)$

събиране

събират се реалните | събират се имагинерните

$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

изваждане

$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1 x_2 + x_1 i y_2 + i y_1 x_2 + \overset{-1}{i^2} y_1 y_2 =$

$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2)$

умножение

$\boxed{\bar{z} = x - iy \text{ - комплексно спрегнато на } z}$

$z \bar{z} = x^2 + y^2$

компл. спрегнатото на знаменателя

$\frac{z_1}{z_2} \cdot \frac{\bar{z}_2}{\bar{z}_2} = \boxed{\frac{(x_1 + iy_1)(x_2 - iy_2)}{x_2^2 + y_2^2}} =$

отделяме реалното от имагинерната част

$= \frac{x_1 x_2 - x_1 i y_2 + i y_1 x_2 - \overset{-1}{i^2} y_1 y_2}{x_2^2 + y_2^2} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y^2}$

$+ i \frac{y_1 x_2 - x_1 y_2}{x_2^2 + y_2^2}$ Делене с комплексно спрегнато

$z + \bar{z} = 2x = 2\operatorname{Re} z$; $z - \bar{z} = iy2 = i2\operatorname{Im} z$

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$; $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$;

2 пъти имаг. част на z

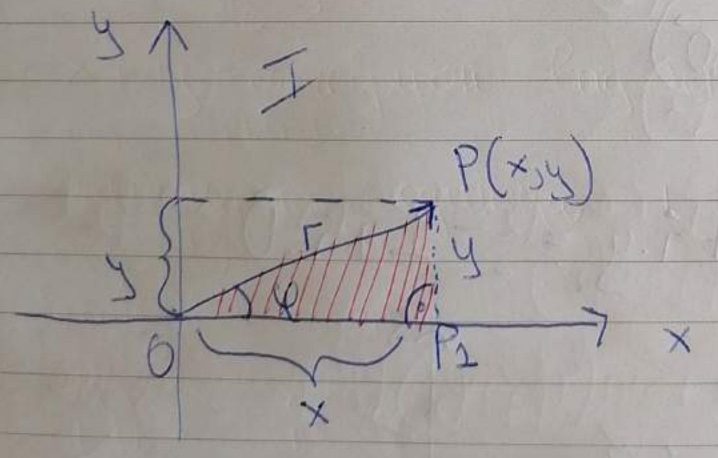
$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$ два пъти реалната част на Z

Събиране, изваждане с компл. спр

Компл. спрегнато. Операции

Тригонометричен вид на комплексните числа

$z = x + iy$, $z \to P(x, y)$ т. от равнината



r - радиус вектор

$\triangle OP_1P$, $\angle OP_1P = 90^\circ$

$\frac{y}{r} = \sin\varphi \Rightarrow y = r\sin\varphi$

$\frac{x}{r} = \cos\varphi \Rightarrow x = r\cos\varphi$

$r = |z|$ - модул на z

φ - аргумент на z

$z = r(\cos\varphi + i\sin\varphi)$ - тригонометричен вид на z

умножение, деление, степенуване, коренуване

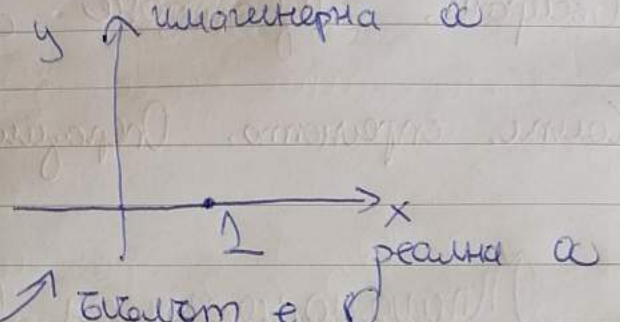
$r = |z| = \sqrt{x^2+y^2}$; $tg\,\varphi = \frac{y}{x}$ $\Rightarrow$ $Arg\,z = \varphi + 2k\pi$

Изобразете с точки в равнината и представете в тр. вид числата:

а) $z = 1 = 1 + i.0^{\to y}$

$r = |z| = \sqrt{1^2+0} = 1$

$tg\,\varphi = \frac{0}{1} = 0 \Rightarrow \varphi = 0$



~~Arg z = ...~~

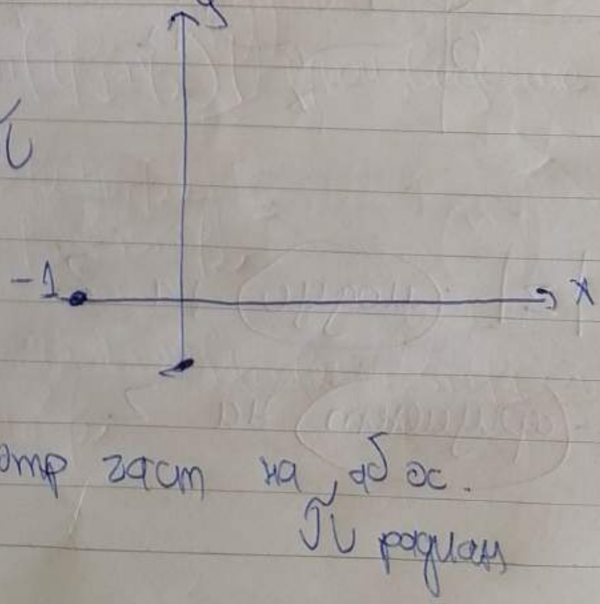
$z = 1 = 1.(\cos 0 + i\sin 0^\circ)$

б) $z = -1 = -1 + i.y = -1 + i.0$

$r = |z| = \sqrt{x^2+y^2} = 1$

$tg\,\varphi = \frac{0}{-1} = -0$ $\quad \varphi = 180^\circ = \pi$

$z = 1(\cos 180 + i\sin 180)$



b) $z = -i$

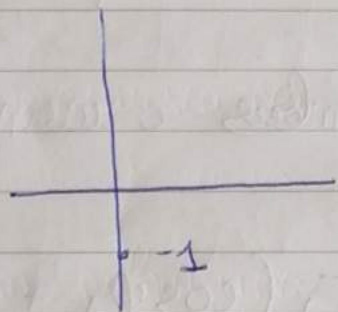
$z = x + iy$
$z = 0 + i(-1)$
$y = -1$
$x = 0$

$r = |z| = \sqrt{x^2+y^2} = \sqrt{0+1} = 1$

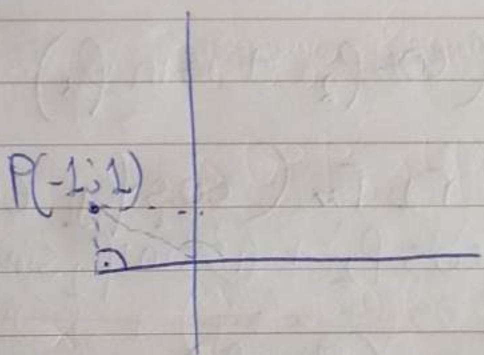
$\operatorname{tg}\varphi = \frac{-1}{0} \not\exists \to -\infty \Rightarrow \varphi = -\frac{\pi}{2}$

$z = r(\cos\varphi + i\sin\varphi)$
$z = (\cos\varphi + i\sin\varphi)$

$\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)$



v) $z = -1 + i$
$z = x + iy$
$z = -1 + i \cdot 1$
$x = -1$
$y = 1$



$r = |z| = \sqrt{x^2+y^2} = \sqrt{1+1} = \sqrt{2}$

$\operatorname{tg}\varphi = -1 \Rightarrow \varphi = \frac{3\pi}{4}$

$z = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

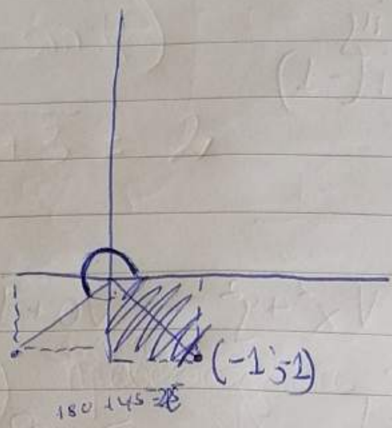
g) $z = -1 - i$

$z = x + iy$

$z = -1 + i(-1)$

$x = -1$

$y = -1$



$r = |z| = \sqrt{x^2+y^2} = \sqrt{2}$

$tg\varphi = 1 \quad \varphi \frac{5\pi}{4}$

$z = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

Действия с комплексни числа в тригонометричен вид

$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1)$

$z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$

$z_1 \cdot z_2 = r_1 r_2 (\cos\varphi_1 \cos\varphi_2 + \cos\varphi_1 \cdot i\sin\varphi_2 + i\sin\varphi_1 \cdot \cos\varphi_2 + \overset{-1}{i^2}\sin\varphi_1 \sin\varphi_2) =$

$r_1 r_2 [\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)]$ произведение

$\frac{z_1}{z_2} = \frac{r_1}{r_2} \frac{\cos\varphi_1 + i\sin\varphi_1}{\cos\varphi_2 + i\sin\varphi_2} \cdot \frac{\cos\varphi_2 - i\sin\varphi_2}{\cos\varphi_2 - i\sin\varphi_2} =$

$\frac{r_1}{r_2} \frac{\cos\varphi_1 \cos\varphi_2 - \cos\varphi_1 \cdot i\sin\varphi_2 + i\sin\varphi_1 \cos\varphi_2 - \overset{-1}{i^2}\sin\varphi_1 \cdot \sin\varphi_2}{\cos^2\varphi_2 - i^2\sin^2\varphi_2}$

$= \frac{r_1}{r_2}\left[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)\right]$ делене (с комплексно спрегнато)

Ако $z = r(\cos\varphi + i\sin\varphi)$

z^n, $n \in \mathbb{N}$, $z^n = r^n(\cos n\varphi + i\sin n\varphi)$ степенуване

$\sqrt[n]{z} = r_1(\cos\varphi_1 + i\sin\varphi_1) \Rightarrow$

$z = r_1^n(\cos n\varphi_1 + i\sin n\varphi_1)$

$z = r(\cos\varphi + i\sin\varphi)$ $\Big| \Rightarrow r_1^n = r \Rightarrow$

$r_1 = \sqrt[n]{r}$

$\cos n\varphi_1 = \varphi$ и $\sin n\varphi_1 = \sin\varphi$

$n\varphi_1 = \varphi + 2k\pi$ $\quad k = 0, 1, 2 \dots$

$\varphi_1 = \frac{\varphi + 2k\pi}{n}$

арг.

$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right)$ коренуване

$k = 0, 1, 2 \dots n-1$

многозначна ф-я

Действия с компл. числа в тригонометр. вид

$A = (1 + i\sqrt{3})(1+i)(\cos\varphi + \sin\varphi)$

1) $1 + i\sqrt{3}$, $r = |z| = \sqrt{1+3} = 2$

$tg\,\varphi = \sqrt{3}$ $\quad \varphi = \frac{\pi}{3}$

$1 + i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ превеждане в тригонометричен вид

$1 + i$, $r = |z| = \sqrt{2}$, $tg\,\varphi = 1 \Rightarrow \frac{\pi}{4}$

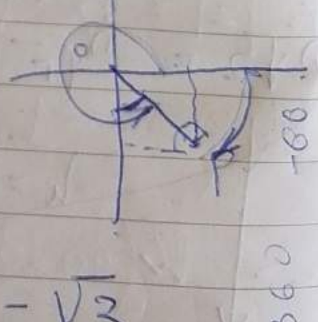
$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\cdot\sin\frac{\pi}{4}\right)$

$A = 2\left(\cos\frac{\pi}{3} + i\cdot\sin\frac{\pi}{3}\right)\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)(\cos\varphi + i\sin\varphi) =$

$= 2\sqrt{2}\left(\cos\left(\frac{\pi}{3} + \frac{\pi}{4} + \varphi\right) + i.\sin\left(\frac{\pi}{3} + \frac{\pi}{4} + \varphi\right)\right) =$

$= 2\sqrt{2}\left(\cos\left(\frac{7\pi}{12} + \varphi\right) + i.\sin\left(\frac{7\pi}{12} + \varphi\right)\right)$

$B = \frac{(1 - i\sqrt{3})(\cos\varphi + i\sin\varphi)}{2(1-i)(\cos\varphi - i\sin\varphi)}$



$1 - i\sqrt{3}$, $r = |z| = \sqrt{1+3} = 2$ $\quad tg\,\varphi = -\sqrt{3}$

$x = 1$, $y = -\sqrt{3}$

$\varphi = \cancel{\frac{8\pi}{3}}\ \left(-\frac{\pi}{3}\right)\ \sqrt{3}$

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$-\frac{\pi}{3}$; $\frac{5\pi}{3}$

-60 , 300

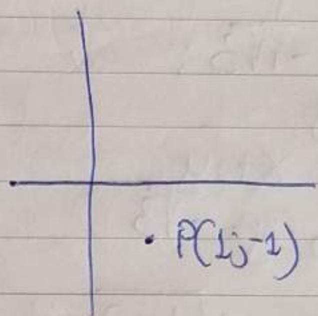
$$1 - i\sqrt{3} = 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

$1 - i$

$z = 1 - i = x + iy$

$x = 1 \quad y = -1$

$r = |z| = \sqrt{1+1} = \sqrt{2}$



$tg\,\varphi = -1$

$\varphi = \frac{7\pi}{4}$

$$1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

B =

$$B = \frac{2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\left(\cos\varphi + i\sin\varphi\right)}{2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\left(\cos\varphi - i\sin\varphi\right)}$$

$\cos\varphi - i\sin\varphi \rightarrow \cos(-\varphi) + i\sin(-\varphi)$

mp beug

$\frac{\sqrt{2}}{\sqrt{2}}$

$$B = \frac{\cancel{2}\left[\cos\left(\frac{5\pi}{3} + \varphi\right) + i\sin\left(\frac{5\pi}{3} + \varphi\right)\right]}{\cancel{2}\sqrt{2}\left[\cos\left(\frac{7\pi}{4} + \varphi\right) + i\sin\left(\frac{7\pi}{4} + \varphi\right)\right]}$$

$$B = \frac{\sqrt{2}\left[\cos\left(\frac{5\pi}{3} + \varphi\right) + i\sin\left(\frac{5\pi}{3} + \varphi\right)\right]}{2\left[\cos\left(\frac{7\pi}{4} + \varphi\right) + i\sin\left(\frac{7\pi}{4} + \varphi\right)\right]}$$

След деление:

Зад 2.

$$B = \frac{(1 - i\sqrt{3})(\cos\varphi + i\sin\varphi)}{2(1-i)(\cos\varphi - i\sin\varphi)}$$

1. $1 - i\sqrt{3}$;

$z = x + iy$

$1 - i\sqrt{3} = x + iy$

$x = 1$; $y = -\sqrt{3}$

$r = |z| = \sqrt{1+3} = 2$

$tg\varphi = -\sqrt{3} \Rightarrow \varphi = \frac{5\pi}{3}$

$$1 - i\sqrt{3} = 2\left[\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right]$$

2. $1 - i$;

$z = x + iy$

$1 - i = x + iy$

$x = 1$ $y = -1$

$r = \sqrt{1+1} = \sqrt{2}$

$tg\varphi = -1 = \frac{7\pi}{4}$

$$1 - i = \sqrt{2}\left[\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right]$$

$$B=\frac{2\left[\cos\left(\frac{5\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}\right)\right](\cos\varphi+i\sin\varphi)}{2\sqrt{2}\left[\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right](\cos(-\varphi)+i\sin(-\varphi))}=$$

$$=\frac{\cancel{2}\left[\cos\left(\frac{5\pi}{3}+\varphi\right)+i\sin\left(\frac{5\pi}{3}+\varphi\right)\right]}{\cancel{2}\sqrt{2}\left[\cos\left(\frac{7\pi}{4}-\varphi\right)+i\sin\left(\frac{7\pi}{4}-\varphi\right)\right]}=$$

$$=\frac{\sqrt{2}}{2}\left[\cos\left(\frac{5\pi}{3}+\varphi-\frac{7\pi}{4}+\varphi\right)+i\sin\left(\frac{5\pi}{3}+\varphi-\frac{7\pi}{4}+\varphi\right)\right]$$

зад 3

$$C=\left(\frac{1+i\sqrt{3}}{1-i}\right)^{30}=\left[\frac{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}{\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)}\right]^{30}=$$

$$=\sqrt[2]{2}^{30}\left[\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)\right]^{30}$$

$$2^{15}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)^{30}=2^{15}\left(\cos\frac{\cancel{30}^{15}\cdot 7\pi}{\cancel{12}_{2}}+i\sin\frac{\cancel{30}^{5}\cdot 7\pi}{\cancel{12}_{2}}\right)$$

$$=2^{15}\left(\cos\frac{35\pi}{2}+i\sin\frac{35\pi}{2}\right)$$

Да се намерят всички корени на уравненията:

а) $z^3 - i = 0$; б) $z^3 - 2 + 2i = 0$

а) $z^3 = i = 0 + iy$, $r = |z| = 1$

$tg\varphi = 1$ $\varphi = \frac{\pi}{2}$

$z^3 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$z^3 = x$
$z = \sqrt[3]{x}$

$z = \cos\left(\frac{\pi/2 + 2k\pi}{3}\right) + i\sin\left(\frac{\pi/2 + 2k\pi}{3}\right)$ $k = 0, 1, 2$

$k=0$; $z_1 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{1}{2}\left(\sqrt{3} + i\right)$

$k=1$; $z_2 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = \frac{1}{2}\left(-\sqrt{3} + i\right)$

$k=2$; $z_3 = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$

б) $z^3 = 2 - 2i$; $r = \sqrt{8}$ $tg\varphi = -1$ $\varphi = \frac{7\pi}{4}$

$z^3 = \sqrt{8}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$\sqrt[6]{8}\left[\cos\left(\frac{7\pi/4 + 2k\pi}{3}\right) + i\sin\left(\frac{7\pi/4 + 2k\pi}{3}\right)\right]$ $k = 0, 1, 2 \ldots$

Логаритми и степен

$z = x + iy$

$\boxed{\operatorname{Log} z = \ln|z| + i(\arg z + 2k\pi),\ k = 0, \pm 1, \pm 2 \ldots}$

а) $\operatorname{Log} 4$, $4 = x + iy = 4 + i0$, $r = 4 = |z|$

$\operatorname{tg}\varphi = \frac{0}{4} = 0 \Rightarrow$ ~~arg~~ $\arg 4 = 0$, $4 = 4(\cos 0 + i \sin 0)$

$\operatorname{Log} 4 = \ln 4 + i(0 + 2k\pi) = \ln 4 + 2k\pi i$, $k = 0, \pm 1, \pm 2 \ldots$

б) $\operatorname{Log} i$, $z = i$

$z = x + iy$

$z = i \Rightarrow x = 0\ \ y = 1 \quad r = |z| = \sqrt{1} = 1$

$\operatorname{tg}\varphi = \frac{1}{0} = \varphi = \frac{\pi}{2}$

$\operatorname{Log} i = \ln 1 + i\left(\frac{\pi}{2} + 2k\pi\right) = i\left(\frac{\pi}{2} + 2k\pi\right)$

в) $\operatorname{Log} \frac{1+i}{\sqrt{2}}$

$z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

$z = \frac{1}{\sqrt{2}} + i \cdot \frac{1}{\sqrt{2}} \qquad |z| = 1$

$\operatorname{tg}\varphi = 1 \Rightarrow \varphi = \frac{\pi}{4} = \operatorname{Log} \frac{1+i}{\sqrt{2}} = i\left(\frac{\pi}{4} + 2k\pi\right) \quad k \in \mathbb{Z}$

зад 2

$\mathrm{Log}_0(-i)$

$\downarrow$

главна стойност на $\mathrm{Log}\,Z$ (без $+2k\pi i$)

$z=-i$

$x=0,\ y=-1\quad |z|=1,\ \mathrm{tg}\,\varphi=0;\ \varphi=-\frac{\pi}{2}$

$\mathrm{Log}_0(-i)=\ln 1+i\left(\frac{-\pi}{2}\right)$

зад 3

z^m, $z, m \in \mathbb{C}$ - множество на компл. числа

а) $z^m=e^{m\,\mathrm{Log}\,z}$ показателен вид

$1^{\sqrt{2}}=e^{\sqrt{2}\,\mathrm{Log}\,1}\qquad |1|=1\ \ \mathrm{tg}\,\varphi=0,\ \varphi=0$

$\mathrm{Log}\,1=\ln 1+i(0+2k\pi)$

$=e^{\sqrt{2}\cdot 2k\pi i}=\cos 2\sqrt{2}k\pi+i\sin 2\sqrt{2}k\pi$ Ойлер ф-ла

б) $2^i=e^{i\,\mathrm{Log}\,2}=e^{i(\ln 2+i(0+2k\pi))}=$

$=e^{i\ln 2-2k\pi}=e^{-2k\pi}\cdot e^{i\ln 2}=e^{-2k\pi}(\cos\ln 2+i\sin\ln 2)$

$i^2 = -1$

в) $1^{-i} = e^{-i\,\mathrm{Log}\,1} = e^{-i(\ln 1 + i(0+2k\pi))} = e^{2k\pi}$ $k \in \mathbb{Z}$

г) $i^i = e^{i\,\mathrm{Log}\,i} = e^{i(\ln 1 + i(\frac{\pi}{2}+2k\pi))} = e^{-(\frac{\pi}{2}+2k\pi)}$

д) $\left(\frac{1-i}{\sqrt{2}}\right)^{1+i} = e^{(1+i)\,\mathrm{Log}\,\frac{1-i}{\sqrt{2}}} =$

арг. на компл. число

$* \; z = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$, $|z| = 1$, $\tg\varphi = -1$

$\varphi(\text{арг. доп}) = \frac{-\pi}{4}$

$= e^{(1+i)\,i\left(-\frac{\pi}{2}+2k\pi\right)} =$

$= e^{(i-1)(-\pi/4+2k\pi)} =$

$= e^{i(-\frac{\pi}{4}+2k\pi) - (-\frac{\pi}{4}+2k\pi)} =$

$= e^{\frac{\pi}{4}-2k\pi}\left(\cos\left(-\frac{\pi}{4}+2k\pi\right)+i\sin\left(-\frac{\pi}{4}+2k\pi\right)\right) =$

$= e^{\frac{\pi}{4}-2k\pi}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right) = \frac{\sqrt{2}}{2}(1-i)e^{\frac{\pi}{4}-2k\pi}$

$k = 0, \pm 1, \pm 2 \ldots$

r – модул на компл. число

φ – аргумент на z

Ако r и φ са дадени $\Rightarrow$ $x = r\cos\varphi$
$y = r\sin\varphi$

$\cos\varphi = \frac{x}{r}$

$\sin\varphi = \frac{y}{r}$

z – алгебричен вид $x + iy$

тригонометричен вид $r(\cos\varphi + i\sin\varphi)$

Повдигане на цяла положителна степен n

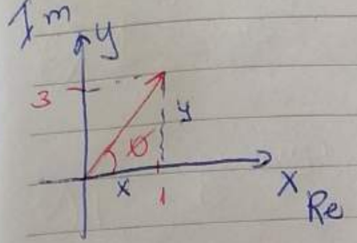
при $r = 1$ $(\cos\varphi + i\cdot\sin\varphi)^n = \cos n\varphi + i\cdot\sin n\varphi$ - ф-ла на Моавър

Зад
$z_1 = -1 + i$
$z_2 = 1 - i\sqrt{3}$
$z_3 = 1$

а) $z_1 \cdot z_2$
ал. вид: $(-1+i)(1-i\sqrt{3}) = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$
$= (-1+\sqrt{3}) + i(\sqrt{3}+1)$

$\sqrt{3} - 1 + i(\sqrt{3}+1)$

тригонометричен вид:



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$y = r \cdot \sin\theta$
$x = r \cdot \cos\theta$

$z = r\cos\theta + ir\sin\theta$

$z = r(\cos\theta + i\sin\theta)$

$$\frac{-1+2i}{3-4i} = \frac{(-1+2i)(3+4i)}{(3-4i)(3+4i)} = \frac{-3-4i+6i+8i^2}{9-16i^2}$$

$\boxed{i^2 = -1}$ $\frac{-11+2i}{25} = -\frac{11}{25} + \frac{2i}{25}$

Re Im

при степенуване използваме същите ф-ли, които знаем

можем да намираме радианите

експоненциален вид

$z = x + iy$ $z = r \cdot e^{i\theta}$

$r = \sqrt{x^2+y^2}$ $-\pi < \theta \leq \pi$

$2 + \sqrt{3}\, i$
$r = \sqrt{7}$
$\theta = \arctg\left(\frac{y}{x}\right) = \frac{\sqrt{3}}{2} = 0{,}71\ rad \approx 41°$ $= \sqrt{7} \cdot e^{i\,0{,}71}$

Wolfram Alpha

Задачи

(1) $e^{i\pi} = 1(\cos\pi + i\sin\pi)$

$z = -1 + i.0$

$y = r\cdot\sin\theta$ $\qquad x = 1\cdot\cos\pi = 1\cdot(-1) = -1$

$x = r\cdot\cos\theta$ $\qquad y = 0$

2 зад от експ. в тригонометр. в алг. вид

$$z = e^{i\frac{\pi}{4}} = 1\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$

$y\cancel{x} = r\cdot\sin\frac{\pi}{4}$ $\qquad y = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cancel{x} = x =$ $\qquad x = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$z = x + iy$

$z = \frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}$

$$z = e^{-2 + \pi/2\, i} = \underbrace{e^{-2}}_{r}\cdot e^{\frac{\pi}{2}i} = r\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

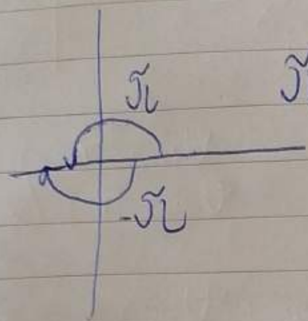
$x = 0$

$y = e^{-2}$

$z = 0 + e^{-2}i$

$$e^{1-\pi i} = \underset{r}{e^{1}} \cdot e^{-\pi i} = e\left(\cos(-\pi) + i\sin(-\pi)\right)$$

$$e\left[\cos\pi + i\sin\pi\right) =$$



$\pi = -\pi$

$$= y = r \cdot \sin\pi$$

$$y = e \cdot \sin 180 = 0$$

$$x = e \cdot \cos\pi = -e$$

$$z = -e$$

$$\cos(z) = \frac{1}{2}\left(e^{iz} + e^{-iz}\right)$$

$$\sin(z) = \frac{1}{2i}\left(e^{iz} - e^{-iz}\right)$$

$$\cos\left(\frac{\pi}{2} - i\pi\right) = \frac{1}{2}\left[e^{i\left(\frac{\pi}{2} - i\pi\right)} + e^{-i\left(\frac{\pi}{2} - i\pi\right)}\right] =$$

$$= \frac{1}{2}\left[e^{i\frac{\pi}{2} + \pi} + e^{-i\frac{\pi}{2} - \pi}\right] =$$

$$= \frac{1}{2}\left(e^{\pi} \cdot e^{i\frac{\pi}{2}} + e^{-\pi} \cdot e^{-i\frac{\pi}{2}}\right) =$$

$$= \frac{1}{2}\left(e^{\pi}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) + e^{-\pi}\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)\right) =$$

$$= \frac{1}{2}\left[\left(e^{\pi} \cdot 0 + e^{\pi}\, 1i\right) + \left(e^{-\pi} \cdot 0 + e^{-\pi}\, i \cdot (-1)\right)\right) =$$

$$= \frac{1}{2}\left[i \cdot e^{\pi} - i e^{-\pi}\right] = i\,\underline{e^{\pi} - e^{-\pi}}$$

само има четврта там

$$\operatorname{Ln} z = \ln|z| + i(\arg z + 2k\pi)$$

$$\arg z = \theta = \operatorname{arctg}\left(\frac{y}{x}\right)$$

$$-\pi < \theta \leq \pi$$

$\operatorname{tg}\theta$.

$$\operatorname{Log}(1+i) = \ln\left|\sqrt{x^2+y^2}\right| + i(\arg(1+i) + 2k\pi) =$$

$$= \ln\sqrt{2} + i\left(\frac{\pi}{4} + 2k\pi\right)$$

$$\operatorname{Arccos}(z) = \frac{1}{i}\operatorname{Log}\left(z \pm \sqrt{z^2-1}\right)$$

$$\operatorname{Arcsin}(z) = \frac{1}{i}\operatorname{Log}\left(iz \pm \sqrt{1-z^2}\right)$$

$$\operatorname{Arctg}(z) = \frac{1}{2i}\operatorname{Log}\frac{1+iz}{1-iz}$$

$$\operatorname{Arccotg}(z) = -\frac{1}{2i}\operatorname{Log}\frac{z-i}{z+i}$$

$$(1+i)^{-i}$$

$$W = z_1^{z_2} = e^{z_2 \operatorname{Ln} z_1}$$

$$e^{-i\operatorname{Ln}(1+i)} = e^{-i\left(\ln\left|\sqrt{1^2+1^2}\right| + i\left(\frac{\pi}{4} + 2k\pi\right)\right)} =$$

$$e^{-i\left(\ln\sqrt{2}+i\left(\frac{\pi}{4}+2k\pi\right)\right)} = e^{\overset{Re}{\frac{\pi}{4}+2k\pi} - \overset{Im}{i\ln\sqrt{2}}} =$$

$$= e^{\frac{\pi}{4}+2k\pi} \cdot e^{-i\ln\sqrt{2}} = e^{\frac{\pi}{4}+2k\pi}\left(\cos(-\ln\sqrt{2}) + i\sin(-\ln\sqrt{2})\right)$$

$$\boxed{\begin{aligned} x &= e^{\frac{\pi}{4}+2k\pi} \cdot \cos(-\ln\sqrt{2}) \\ y &= e^{\frac{\pi}{4}+2k\pi} \cdot \sin(-\ln\sqrt{2}) \end{aligned}}$$

$k = 0, \pm 1, \pm 2 \ldots$

Полиноми

1. Дефиниция

$\underline{fn(x)} = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n,$

$(a_0 \neq 0)$, $a_0, a_1 \cdots a_{n-1}$, $a_n \in \mathbb{R}$, се нарича полином на x от степен n;

Числата $a_0, a_1 \ldots a_{n-1}$ се наричат коефициенти

a_n - свободен член

Означаване $fn(x)$ $P_n(x)$, $Q_n(x)$...

$f_0(x) = a_0$ - полином от нулева степен

$f_1(x) = a_0x^1 + a_1$ - полином от първа степен

$f_2(x) = a_0x^2 + a_1x + a_2$ - полином от втора степен и т.н.