Automatica 45 (2009) 2312–2318

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive neural network tracking control for manipulators with uncertain kinematics, dynamics and actuator model☆

Long Cheng, Zeng-Guang Hou*, Min Tan

Key Laboratory of Complex Systems and Intelligence Science, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China

ARTICLE INFO

Article history:
Received 14 May 2008
Received in revised form
30 March 2009
Accepted 9 June 2009
Available online 24 July 2009

Keywords:
Manipulators
Tracking
Uncertainty
Neural networks

ABSTRACT

A neural-network-based adaptive controller is proposed for the tracking problem of manipulators with uncertain kinematics, dynamics and actuator model. The adaptive Jacobian scheme is used to estimate the unknown kinematics parameters. Uncertainties in the manipulator dynamics and actuator model are compensated by three-layer neural networks. External disturbances and approximation errors are counteracted by robust signals. The actuator controller is designed based on the backstepping scheme. Compared with the existing work, the proposed method considers the manipulator kinematics uncertainty, does not need the "linearity-in-parameters" assumption for the uncertain terms in the dynamics of manipulator and actuator, and guarantees the tracking error to be as small as desired. Finally, the performance of the proposed approach is illustrated by the simulation example.

1. Introduction

Recently, several adaptive controllers have been proposed to deal with the manipulator trajectory tracking problem in the presence of dynamics uncertainties (see the survey (Ortega & Spong, 1989)). However, a critical assumption in these controllers is that the uncertain term should satisfy the "linearity-in-parameters" condition. Moreover, tedious analysis and computations have to be done to determine the regressor matrix. To overcome these drawbacks, a class of neural-network-based adaptive approaches has been proposed for the manipulator tracking problem (Kwan, Lewis, & Dawson, 1998; Lewis, Jagannathan, & Yesildirek, 1998). For the general framework of this neural-network-based method, the readers are referred to Farrell and Polycarpou (2006).

It is noted that most existing controllers are designed for the joint trajectory tracking (Kwan et al., 1998; Lewis et al., 1998; Ortega & Spong, 1989). However, on many occasions, it is more convenient to drive the end-effector to follow a given trajectory in the Cartesian space. In this case, the manipulator kinematics should be considered. Due to the imprecise measurement of physical parameters and the interaction between manipulator and different environments, the kinematics parameters may not be known *a priori*. As reported in Arimoto (1999), the research on the control problem with uncertain kinematics is just a beginning. To deal with the kinematics uncertainty, some results have been published which are based on the approximate Jacobian technique (Cheah, Kawamura, & Arimoto, 2003; Dixon, 2007). However, these methods focus on the setpoint control of a robot. As to the tracking control, Cheah, Liu, and Slotine (2004) suggested an adaptive Jacobian approach for the non-redundant robot with uncertain kinematics and dynamics. Extensions to the redundant robots and unknown actuator parameters were made in Cheah, Liu, and Slotine (2006). Braganza, Dixon, Dawson, and Xian (2008) also presented a tracking controller for manipulators with uncertain kinematics and dynamics; the unit quaternion was used to represent the orientation of manipulator end-effector. It is noted that controllers proposed in Braganza et al. (2008), Cheah et al. (2003), Cheah et al. (2004), Cheah et al. (2006), and Dixon (2007) employed the traditional adaptive control scheme to deal with the uncertain dynamics of manipulator and actuator. Therefore, they suffer from the "linearity-in-parameters" assumption and other aforementioned drawbacks. In addition, external disturbances in manipulator dynamics have been neglected in the controller design.

This paper addresses the manipulator tracking problem in the presence of uncertain kinematics, dynamics, and actuator model. Adaptive Jacobin method, neural network approximation, and the backstepping method are employed to design the tracking controller. The contributions of this paper are: (1) the manipulator kinematics uncertainty is considered in the controller design; (2) compared with the previous work (Braganza et al., 2008; Cheah et al., 2003, 2004, 2006; Dixon, 2007), the "linearity-in-parameters" assumption for the uncertain dynamics of manip-

☆ The material in this paper was partially presented at the 17th IFAC World Congress, 2008, Seoul, Korea. This work was supported in part by the National Natural Science Foundation of China (Grants 60775043, 60725309, and 60621001) and the National Hi-Tech R&D Program (863) of China (Grant 2009AA04Z201). This paper was recommended for publication in revised form by Associate Editor Raul Ordóñez under the direction of Editor Miroslav Krstic.

* Corresponding address: Institute of Automation, Chinese Academy of Science, No. 95, Zhongguancun East Road, P.O. Box 2728, Beijing, 100190, China. Tel.: +86 10 62565502; fax: +86 10 62565502.

E-mail addresses: hou@compsys.ia.ac.cn, zengguang.hou@ia.ac.cn (Z.-G. Hou).

0005-1098/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2009.06.007

ulator and actuator is not necessary, and external disturbances in the dynamics of manipulator and actuator are taken into account; (3) the tracking error can be reduced as small as desired by choosing appropriate controller parameters. Therefore, the proposed method contributes to the current literature. This work is an extension to the conference papers (Cheng, Hou, & Tan, 2008; Cheng, Hou, Tan, & Wang, 2008), which considers the uncertain actuator model and further analyzes the tracking performance.

Notations. For a given vector, $\|\cdot\|$ denotes the vector Euclidean norm; for a given matrix, $\|\cdot\|_F$ denotes the matrix *Frobenius* norm; I_n denotes the n-dimensional unity matrix; $(\cdot)_i$ denotes the ith element of a given vector; $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues of a given matrix, respectively; $\mathrm{Tr}(\cdot)$ denotes the trace operator.

2. Problem formulation and preliminaries

2.1. Manipulator-plus-actuator system description

The dynamics model for a rigid n-link, serially connected manipulator can be expressed as (Lewis et al., 1998)

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + \tau_{ed} = \tau, \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ denote the joint position, velocity, and acceleration vectors, respectively; $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix; $V(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the centripetal-Coriolis matrix; $G(q) \in \mathbb{R}^n$ is the gravitational vector; $\tau_{ed} \in \mathbb{R}^n$ denotes the bounded unknown disturbance vector including unstructured unmodeled dynamics, and it is assumed that $\|\tau_{ed}\| \le \Delta_{\tau ed}$; $\tau \in \mathbb{R}^n$ represents the torque input vector. Two important properties of the dynamics equation described by (1) are given as follows (Lewis et al., 1998).

Property 1. *The inertia matrix $M(q)$ is symmetric and positive definite, and satisfies: $m_1\|y\|^2 \le y^{\mathrm{T}}M(q)y \le m_2\|y\|^2$, $\forall y \in \mathbb{R}^n$, where m_1 and m_2 are known positive constants.*

Property 2. *The time derivative of the inertia matrix and the centripetal-Coriolis matrix satisfy the skew symmetric relation; that is, $y^{\mathrm{T}}\left(\dot{M}(q) - 2V(q,\dot{q})\right)y = 0$, $\forall y \in \mathbb{R}^n$.*

For simplicity, it is assumed that the manipulator is driven by armature-controlled DC motors with voltages being input to amplifiers. The dynamics of this type of motor can be described as follows (Cheah et al., 2006)

$$\tau = K_T I, \tag{2a}$$

$$L\dot{I} + RI + K_e\dot{q} + u_{ed} = u, \tag{2b}$$

where $I \in \mathbb{R}^n$ is the armature current vector; $u \in \mathbb{R}^n$ is the armature voltage vector; $u_{ed} \in \mathbb{R}^n$ is the additive bounded voltage disturbance vector, and it is assumed that $\|u_{ed}\| \le \Delta_{ued}$; $K_T \in \mathbb{R}^{n\times n}$ is the positive definite constant diagonal matrix which characterizes the electro-mechanical conversion between current and torque; $R, L, K_e \in \mathbb{R}^{n\times n}$ are the positive definite constant diagonal matrices denoting the circuit resistance, circuit inductance, and voltage constant of the motor, respectively. And it is assumed that the following bounded condition holds

$$k_1\|x\|^2 \le x^{\mathrm{T}}K_T x \le k_2\|x\|^2, \quad \forall x \in \mathbb{R}^n, \tag{3}$$

where k_1 and k_2 are known positive constants.

By (2a) and (2b), it follows that

$$L\dot{\tau} + R\tau + K_T K_e\dot{q} + K_T u_{ed} = K_T u. \tag{4}$$

Let $x \in \mathbb{R}^m$ $(m \le n)$ represent the Cartesian space position vector which is related to the manipulator joint vector as $x = h(q)$, where $h(q) \in \mathbb{R}^m$ is the differentiable forward kinematics of the manipulator. The Cartesian space velocity $\dot{x}$ is related to joint velocity $\dot{q}$ as

$$\dot{x} = J(q,\phi_J)\dot{q}, \tag{5}$$

where $\phi_J \in \mathbb{R}^p$ represents the kinematics parameters, such as link lengths and joint offsets; $J(q,\phi_J) \overset{\text{def}}{=} (\partial h/\partial q) \in \mathbb{R}^{m\times n}$ denotes the manipulator Jacobian matrix which has the following property.

Property 3. *The product of the manipulator Jacobian matrix with the joint velocity vector can be linearly parameterized as*

$$J(q,\phi_J)\dot{q} = Y_J(q,\dot{q})\phi_J, \tag{6}$$

where $Y_J(q,\dot{q}) \in \mathbb{R}^{m\times p}$ is called the kinematics regressor matrix which can be computed directly by the measurable joint position and velocity vectors q and $\dot{q}$.

2.2. Multi-layer neural networks

The three-layer neural network, shown in Fig. 1, is usually used for the function approximation. The output of neural network can be determined as follows

$$y_i = \sum_{j=1}^{N_h}\left[w_{ij}\bar{\sigma}\left(\sum_{k=1}^{N_i} v_{jk}z_k + \theta_{vj}\right) + \theta_{wi}\right], \quad i = 1,\ldots,N_o, \tag{7}$$

where N_i, N_h and N_o denote the numbers of input-layer neurons, hidden-layer neurons and output-layer neurons, respectively; w_{ij} and v_{jk} are the adjustable synaptic weights, respectively. The threshold offsets are denoted by θ_{wi} and θ_{vj}; $\bar{\sigma}(\cdot)$ is the sigmoid activation function

$$\bar{\sigma}(s) = \frac{1}{1+\mathrm{e}^{-s}}. \tag{8}$$

For convenience, Eq. (7) can be rewritten in the following compact form

$$y = W\sigma\left(V\bar{z}\right), \tag{9}$$

where $W \in \mathbb{R}^{N_o\times(N_h+1)}$, $V \in \mathbb{R}^{N_h\times(N_i+1)}$ are augmented weight matrices; $\bar{z} = [1, z_1, z_2, \ldots, z_{N_i}]^{\mathrm{T}} \in \mathbb{R}^{N_i+1}$; $y = [y_1, y_2, \ldots, y_{N_o}]^{\mathrm{T}} \in \mathbb{R}^{N_o}$; $\sigma\left(V\bar{z}\right) = \left[1, \bar{\sigma}\left(V_{r_1}\bar{z}\right), \bar{\sigma}\left(V_{r_2}\bar{z}\right), \ldots, \bar{\sigma}\left(V_{r_{N_h}}\bar{z}\right)\right]^{\mathrm{T}} \in \mathbb{R}^{N_h+1}$ (V_{r_i} represents the ith row of matrix V). It is emphasized that $\sigma(\cdot)$ is a map from $\mathbb{R}^{N_h}$ to $\mathbb{R}^{N_h+1}$. By this augmented expression, θ_{wi} and θ_{vj} are included as the first columns of W and V, respectively. Therefore, any tuning of W and V will include tuning of the thresholds as well.

Let S be a compact simply connected set of $\mathbb{R}^{N_i}$, and $g(z)$ be a continuous function from S to $\mathbb{R}^{N_o}$. Then, for any given positive constant ε_N, there exist ideal parameters W^*, V^*, N_h such that

$$g(z) = W^*\sigma(V^*\bar{z}) + \varepsilon, \tag{10}$$

where ε is the bounded function approximation error with $\|\varepsilon\| < \varepsilon_N$ in S.

Assumption 1. The ideal neural network parameters are bounded by some positive values. That is $\|V^*\|_F \le V_M$ and $\|W^*\|_F \le W_M$.

It should be noted that W^* and V^* are only quantities required for analytical purpose. In real control applications, their estimations $\hat{W}$ and $\hat{V}$ are used for the function approximation. Then the estimation of $g(z)$ is given by

$$\hat{g}(z) = \hat{W}\sigma(\hat{V}\bar{z}). \tag{11}$$

Lemma 1. *For the neural network defined by (11), the function approximation error is, $\hat{g}(z) - g(z) = \tilde{W}\left(\sigma(\hat{V}\bar{z}) - \hat{\sigma}'(\hat{V}\bar{z})\hat{V}\bar{z}\right) + \hat{W}\hat{\sigma}'(\hat{V}\bar{z})\tilde{V}\bar{z} + d_u$, where $\hat{\sigma}'(\hat{V}\bar{z}) = [\mathbf{0}, \mathrm{diag}\{\hat{\sigma}_1', \hat{\sigma}_2', \ldots, \hat{\sigma}_{N_h}'\}]^{\mathrm{T}} \in \mathbb{R}^{(N_h+1)\times N_h}$ with $\hat{\sigma}_i' = \mathrm{d}\bar{\sigma}(s)/\mathrm{d}s|_{s=\hat{V}_{ri}\bar{z}}$ and $\mathbf{0} = (0,0,\ldots,0)^{\mathrm{T}} \in \mathbb{R}^{N_h}$; It is emphasized that $\hat{\sigma}'(\cdot)$ is a map from $\mathbb{R}^{N_h}$ to $\mathbb{R}^{(N_h+1)\times N_h}$; the weight estimation errors are $\tilde{W} = \hat{W} - W^*$ and $\tilde{V} = \hat{V} - V^*$; and the residual term is $d_u = \tilde{W}\hat{\sigma}'(\hat{V}\bar{z})V^*\bar{z} + W^*O(\tilde{V}\bar{z})^2 - \varepsilon$, which is*

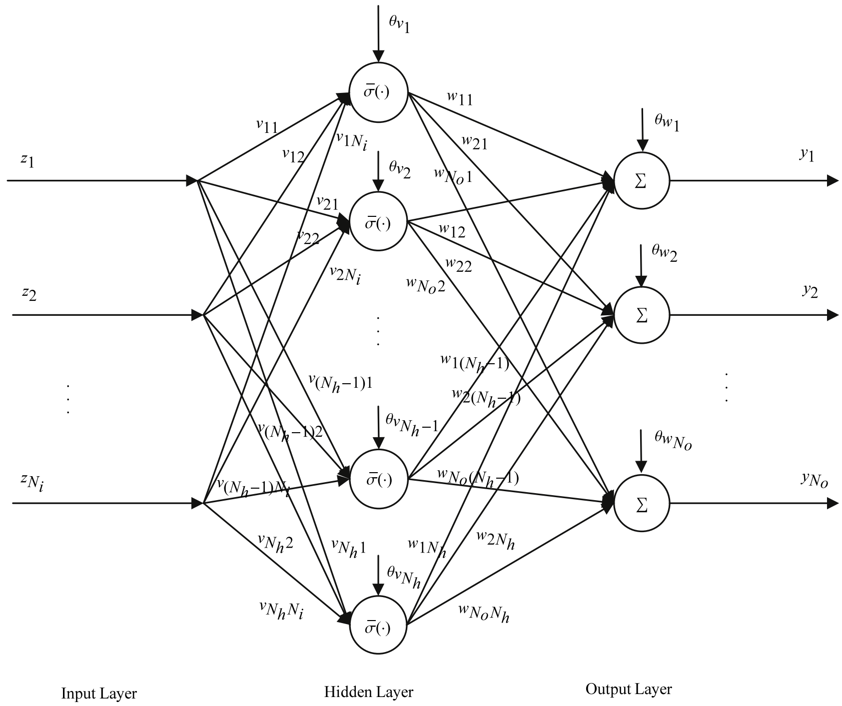

Fig. 1. The structure of the three-layer neural network.

bounded by

$$\|d_u\| \le c_0 + c_1\|\bar{z}\| + c_2\|\tilde{W}\|_F\|\bar{z}\|, \tag{12}$$

where c_0, c_1 and c_2 are positive constants related with V_M, W_M, ε_N and N_h.

Proof. The proof is similar with the proof of Lemma 3.1 in Ge, Huang, and Zhang (1999), and is omitted here. □

Lemma 2. *If the function $E(t) > 0$ and $\dot{E}(t) \le -\gamma E(t) + \varepsilon$, where γ is a positive constant, then*

$$E(t) \le E(0)\mathrm{e}^{-\gamma t} + \frac{\varepsilon}{\gamma}(1 - \mathrm{e}^{-\gamma t}).$$

3. Controller design using backstepping technique and neural networks

Based on the backstepping method, a trajectory tracking controller is designed for the manipulator-plus-actuator system with uncertainties and external disturbances. First, a mild assumption for the desired tracking trajectory $x_d(t)$, which always holds in practical applications, is stated.

Assumption 2. $x_d(t)$ and its derivatives up to the third order are bounded in the sense that

$$\left\| \left(x_d^{\mathrm{T}}(t), \dot{x}_d^{\mathrm{T}}(t), \ddot{x}_d^{\mathrm{T}}(t), \dddot{x}_d^{\mathrm{T}}(t)\right)^{\mathrm{T}} \right\| \le X_M, \tag{13}$$

where X_M is a known constant.

Step 1: Define the tracking error $e_1(t) = x(t) - x_d(t)$. Then, differentiating $e_1(t)$ with respect to time obtains

$$\dot{e}_1 = \dot{x} - \dot{x}_d = J(q, \phi_J)\dot{q} - \dot{x}_d. \tag{14}$$

The auxiliary controller for $\dot{q}$ is designed as follows

$$\dot{q}_d = \left(I_n - J^+\left(q, \hat{\phi}_J\right) J\left(q, \hat{\phi}_J\right)\right)\lambda + J^+\left(q, \hat{\phi}_J\right) \times (\dot{x}_d - \eta_1 e_1), \tag{15}$$

where $J^+\left(q, \hat{\phi}_J\right) = J^T\left(q, \hat{\phi}_J\right)\left(J\left(q, \hat{\phi}_J\right)J^T\left(q, \hat{\phi}_J\right)\right)^{-1}$ is the generalized inverse matrix of the approximate Jacobian matrix $J\left(q, \hat{\phi}_J\right)$; $\hat{\phi}_J \in \mathbb{R}^p$ is the estimation of uncertain kinematics parameters; η_1 is the designed control gain; $\lambda \in \mathbb{R}^n$ is an auxiliary term which can be used for optimization purposes. It is assumed that the manipulator is operating in a finite task space such that the approximate Jacobian matrix is of full rank. This assumption is commonly adopted to deal with the manipulator kinematics uncertainties in the existing literature (Braganza et al., 2008; Cheah et al., 2006; Dixon, 2007).

Substituting (15) into (14) obtains

$$\dot{e}_1 = -\eta_1 e_1 + J(q, \phi_J)e_2 - Y_J(q, \dot{q}_d)\tilde{\phi}_J, \tag{16}$$

where $e_2 = \dot{q} - \dot{q}_d$ and $\tilde{\phi}_J = \phi_J - \hat{\phi}_J$.

Step 2: Design an auxiliary controller for the torque τ to make e_2 as small as possible. By (1), the dynamics of e_2 can be derived as follows

$$\begin{aligned} M(q)\dot{e}_2 + V(q, \dot{q})e_2 &= \tau - M(q)\ddot{q}_d - V(q, \dot{q})\dot{q}_d - G(q) - \tau_{ed} \\ &= \tau - f_1 - \tau_{ed}. \end{aligned} \tag{17}$$

Then the auxiliary controller τ_d is designed as follows

$$\tau_d = \hat{F}_1 - \eta_2 e_2 - \gamma_1, \tag{18}$$

where η_2 is the designed control gain; γ_1 is a robust signal whose form is stated in the following part of this section; $\hat{F}_1 = \hat{W}_1\sigma(\hat{V}_1\bar{Z}_1)$ is the estimation of $F_1 = f_1 - J^{\mathrm{T}}(q, \phi_J)e_1$. It is noted that, on a given compact set Ω_1, F_1 can be approximated by $F_1 = W_1^*\sigma(V_1^*\bar{Z}_1) + \varepsilon_1$ with the approximation error $\|\varepsilon_1\| \le \varepsilon_{N1}$ and neural network input $\bar{Z}_1 = [1, e_1^{\mathrm{T}}, e_2^{\mathrm{T}}, x_d^{\mathrm{T}}, \dot{x}_d^{\mathrm{T}}, \ddot{x}_d^{\mathrm{T}}, \hat{\phi}_J^{\mathrm{T}}]^{\mathrm{T}} \in \Omega_1$. According to Assumption 1, it is assumed that $\|W_1^*\|_F \le W_{M1}$ and $\|V_1^*\|_F \le V_{M1}$.

By Lemma 1, substituting (18) into (17) obtains

$$\begin{aligned} M(q)\dot{e}_2 + V(q, \dot{q})e_2 &= \hat{F}_1 - K_2 e_2 - \gamma_1 - f_1 - \tau_{ed} + e_3 \\ &= -\eta_2 e_2 - J^{\mathrm{T}}(q, \phi_J)e_1 + \tilde{W}_1\left(\hat{\sigma}_{n1} - \hat{\sigma}'_{n1}\hat{V}_1\bar{Z}_1\right) \\ &\quad + \hat{W}_1\hat{\sigma}'_{n1}\tilde{V}_1\bar{Z}_1 - \gamma_1 + \delta_1 + e_3, \end{aligned} \tag{19}$$

where $\hat{\sigma}_{n1} = \sigma(\hat{V}_1\bar{Z}_1)$; $\hat{\sigma}'_{n1} = \hat{\sigma}'(\hat{V}_1\bar{Z}_1)$; $\delta_1 = -\tau_{ed} - \varepsilon_1 - \tilde{W}_1\hat{\sigma}'_{n1}V_1^*\bar{Z}_1 - W_1^*O(\tilde{V}_1\bar{Z}_1)^2$; and $e_3 = \tau - \tau_d$.

Step 3: Design the actuator voltage controller u which makes e_3 as small as possible. By (4), the dynamics of e_3 is given as follows

$$\begin{aligned} L\dot{e}_3 &= K_T u - K_T u_{ed} - K_T K_e \dot{q} - R\tau - L\dot{\tau}_d \\ &= K_T u - f_2 - K_T u_{ed}. \end{aligned} \tag{20}$$

Then the controller u can be designed as follows

$$u = \frac{1}{k_1}(\hat{F}_2 - \eta_3 e_3 - \gamma_2), \tag{21}$$

where η_3 is the designed control gain; γ_2 is a robust signal. $\hat{F}_2 = \hat{W}_2\sigma(\hat{V}_2\bar{Z}_2)$ is the estimation of $F_2 = k_1K_T^{-1}f_2 - e_2$. By the neural network approximation, on the compact set Ω_2, $F_2 = W_2^*\sigma(V_2^*\bar{Z}_2) + \varepsilon_2$, where $\|\varepsilon_2\| \le \varepsilon_{N2}$ is the approximation error and $\bar{Z}_2 = [1, e_1^{\mathrm{T}}, e_2^{\mathrm{T}}, e_3^{\mathrm{T}}, x_d^{\mathrm{T}}, \dot{x}_d^{\mathrm{T}}, \ddot{x}_d^{\mathrm{T}}, \dddot{x}_d^{\mathrm{T}}, \mathrm{vec}(\hat{W}_1), \mathrm{vec}(\hat{V}_1)]^{\mathrm{T}} \in \Omega_2$ is the neural network input vector. Here, for $H = (h_{ij}) \in \mathbb{R}^{m\times n}$, $\mathrm{vec}(H) = (h_{11}, \dots, h_{1n}, h_{21}, \dots, h_{2n}, \dots\dots, h_{m1}, \dots, h_{mn})$. According to Assumption 1, it is assumed that $\|W_2^*\|_F \le W_{M2}$ and $\|V_2^*\|_F \le V_{M2}$.

By Lemma 1, substituting (21) into (20) obtains

$$\begin{aligned} k_1K_T^{-1}L\dot{e}_3 &= -\eta_3 e_3 + \hat{F}_2 - F_2 - e_2 - k_1u_{ed} - \gamma_2 \\ &= -\eta_3 e_3 - e_2 + \tilde{W}_2\left(\hat{\sigma}_{n2} - \hat{\sigma}'_{n2}\hat{V}_2\bar{Z}_2\right) \\ &\quad + \hat{W}_2\hat{\sigma}'_{n2}\tilde{V}_2\bar{Z}_2 - \gamma_2 + \delta_2, \end{aligned} \tag{22}$$

where $\hat{\sigma}_{n2} = \sigma(\hat{V}_2\bar{Z}_2)$; $\hat{\sigma}'_{n2} = \hat{\sigma}'(\hat{V}_2\bar{Z}_2)$ and $\delta_2 = -k_1u_{ed} - \varepsilon_2 - \tilde{W}_2\hat{\sigma}'_{n2}V_2^*\bar{Z}_2 - W_2^*O(\tilde{V}_2\bar{Z}_2)^2$.

By the projection algorithm, the updating laws for $\hat{\phi}_J$ and $\hat{W}_i$, $\hat{V}_i$ $(i = 1, 2)$ are derived as follows.

$$\left(\dot{\hat{\phi}}_J\right)_j = \begin{cases} \beta\left(Y_J^{\mathrm{T}}(q,\dot{q}_d)e_1\right)_j, & \text{if } (\phi_J^-)_j < (\hat{\phi}_J)_j < (\phi_J^+)_j \\ \quad \text{or if } (\hat{\phi}_J)_j = (\phi_J^-)_j \text{ and } \left(Y_J^{\mathrm{T}}(q,\dot{q}_d)e_1\right)_j > 0, \\ \quad \text{or if } (\hat{\phi}_J)_j = (\phi_J^+)_j \text{ and } \left(Y_J^{\mathrm{T}}(q,\dot{q}_d)e_1\right)_j \le 0; \\ 0, \quad \text{if } (\hat{\phi}_J)_j = (\phi_J^-)_j \text{ and } \left(Y_J^{\mathrm{T}}(q,\dot{q}_d)e_1\right)_j \le 0, \\ \quad \text{or if } (\hat{\phi}_J)_j = (\phi_J^+)_j \text{ and } \left(Y_J^{\mathrm{T}}(q,\dot{q}_d)e_1\right)_j > 0; \end{cases}$$
$$j = 1, 2, \dots, p, \tag{23}$$

where $\beta > 0$ is the adaption gain; ϕ_J^- and ϕ_J^+ are the lower and upper bounds of the real kinematics parameters ϕ_J, respectively.

$$\dot{\hat{W}}_i = \begin{cases} -\chi_{wi}e_{i+1}a_i^{\mathrm{T}}, \quad \text{if } \mathrm{Tr}\left(\hat{W}_i\hat{W}_i^{\mathrm{T}}\right) < W_{mi}, \\ \quad \text{or if } \mathrm{Tr}\left(\hat{W}_i\hat{W}_i^{\mathrm{T}}\right) = W_{mi} \text{ and } e_{i+1}^{\mathrm{T}}\hat{W}_ia_i > 0; \\ -\chi_{wi}e_{i+1}a_i^{\mathrm{T}} + \chi_{wi}\dfrac{e_{i+1}^{\mathrm{T}}\hat{W}_ia_i}{\mathrm{Tr}\left(\hat{W}_i\hat{W}_i^{\mathrm{T}}\right)}\hat{W}_i, \\ \quad \text{if } \mathrm{Tr}\left(\hat{W}_i\hat{W}_i^{\mathrm{T}}\right) = W_{mi} \text{ and } e_{i+1}^{\mathrm{T}}\hat{W}_ia_i \le 0; \end{cases} \tag{24}$$

$$\dot{\hat{V}}_i = \begin{cases} -\chi_{vi}b_i^{\mathrm{T}}\bar{Z}_i^{\mathrm{T}}, \quad \text{if } \mathrm{Tr}\left(\hat{V}_i\hat{V}_i^{\mathrm{T}}\right) < V_{mi}, \\ \quad \text{or if } \mathrm{Tr}\left(\hat{V}_i\hat{V}_i^{\mathrm{T}}\right) = V_{mi} \text{ and } b_i\hat{V}_i\bar{Z}_i > 0; \\ -\chi_{vi}b_i^{\mathrm{T}}\bar{Z}_i^{\mathrm{T}} + \chi_{vi}\dfrac{b_i\hat{V}_i\bar{Z}_i}{\mathrm{Tr}\left(\hat{V}_i^{\mathrm{T}}\hat{V}_i\right)}\hat{V}_i, \\ \quad \text{if } \mathrm{Tr}\left(\hat{V}_i\hat{V}_i^{\mathrm{T}}\right) = V_{mi} \text{ and } b_i\hat{V}_i\bar{Z}_i \le 0; \end{cases} \tag{25}$$

where $a_i = \hat{\sigma}_{ni} - \hat{\sigma}'_{ni}\hat{V}_i\bar{Z}_i$; $b_i = e_{i+1}^{\mathrm{T}}\hat{W}_i\hat{\sigma}'_{ni}$; χ_{wi} and χ_{vi} are the given positive adaption gains; W_{mi} and V_{mi} are the given positive constants for limiting the estimated neural network weight matrices, which satisfy that $W_{mi} \ge W_{Mi}^2$ and $V_{mi} \ge V_{Mi}^2$.

It is noted that the initial kinematics parameters $\hat{\phi}_J(0)$ should satisfy that $(\phi_J^-)_j \le (\hat{\phi}_J(0))_j \le (\phi_J^+)_j$, and the initial neural network weight matrices $\hat{W}_i(0)$ and $\hat{V}_i(0)$ should satisfy that $\mathrm{Tr}\left(\hat{W}_i(0)\hat{W}_i^{\mathrm{T}}(0)\right) \le W_{mi}$ and $\mathrm{Tr}\left(\hat{V}_i(0)\hat{V}_i^{\mathrm{T}}(0)\right) \le V_{mi}$.

According to Lemma 1, there exist positive constants $c_{01}, c_{11}, c_{21}, c_{02}, c_{12}, c_{22}$ such that

$$\begin{aligned} \|\delta_1\| \le\ & \Delta_{\tau ed} + c_{01} \\ & + (c_{11} + c_{21}\|\tilde{W}_1\|_F)(1 + X_M + \|\hat{\phi}_J\| + \|e_1\| + \|e_2\|), \\ \|\delta_2\| \le\ & k_1\Delta_{ued} + c_{02} + (c_{12} + c_{22}\|\tilde{W}_2\|_F) \\ & \times (1 + X_M + \|\hat{W}_1\|_F + \|\hat{V}_1\|_F + \|e_1\| + \|e_2\| + \|e_3\|). \end{aligned}$$

Then the robust signals γ_1 and γ_2 in (18) and (21) are defined as follows

$$(\gamma_i)_j = \delta_{Mi}\tanh\left(\frac{nk_u\delta_{Mi}(e_{i+1})_j}{\epsilon_i}\right), \quad j = 1, \dots, n;\ i = 1, 2 \tag{26}$$

where $k_u = 0.2785$, ϵ_i is any given positive scalar, and δ_{Mi} satisfies the following conditions

$$\begin{aligned} \delta_{M1} \ge\ & \Delta_{\tau ed} + c_{01} + \left(c_{11} + c_{21}\left(\sqrt{W_{m1}} + W_{M1}\right)\right) \\ & \times \left(1 + X_M + \sqrt{\sum_{j=1}^{p}\max((\phi_J^-)_j^2, (\phi_J^+)_j^2)}\right), \\ \delta_{M2} \ge\ & k_1\Delta_{ued} + c_{02} + \left(c_{12} + c_{22}\left(\sqrt{W_{m2}} + W_{M2}\right)\right) \\ & \times \left(1 + X_M + \sqrt{W_{m1}} + \sqrt{V_{m1}}\right). \end{aligned} \tag{27}$$

It is easy to verify that γ_i satisfies the following conditions

$$e_{i+1}^{\mathrm{T}}\gamma_i \ge 0, \quad \delta_{Mi}\|e_{i+1}\| - e_{i+1}^{\mathrm{T}}\gamma_i \le \epsilon_i. \tag{28}$$

4. Stability analysis

Theorem 1. *Given the manipulator-plus-actuator system defined by (1), (2) and (5), if the controller is constructed by (15), (18) and (21), the parameters updating laws are (23)–(25), and the estimated parameters satisfy the initial conditions, then the trajectory tracking error $e_1 = x - x_d$ can be reduced to an arbitrary small neighborhood around zero by choosing appropriate parameters.*

Proof. According to the projection algorithm, it is easy to check that $\hat{\phi}_J(t)$ is bounded by its upper and lower limitations.

To prove $\mathrm{Tr}(\hat{W}_i(t)\hat{W}_i^{\mathrm{T}}(t)) \le W_{mi}$, let $L_{wi} = \mathrm{Tr}(\hat{W}_i\hat{W}_i^{\mathrm{T}})$, $i = 1, 2$. By (24), it follows that

1. When $L_{wi} < W_{mi}$, the conclusion has already held;
2. When $L_{wi} = W_{mi}$ and $e_{i+1}^{\mathrm{T}}\hat{W}_ia_i > 0$,

$$\frac{\mathrm{d}L_{wi}}{\mathrm{d}t} = -2\,\mathrm{Tr}\left(\chi_{wi}\hat{W}_ia_ie_{i+1}^{\mathrm{T}}\right) = -2\chi_{wi}e_{i+1}^{\mathrm{T}}\hat{W}_ia_i \le 0.$$

3. When $L_{wi} = W_{mi}$ and $e_{i+1}^{\mathrm{T}}\hat{W}_ia_i \le 0$,

$$\begin{aligned} \frac{\mathrm{d}L_{wi}}{\mathrm{d}t} &= 2\,\mathrm{Tr}\left(\hat{W}_i\frac{\chi_{wi}e_{i+1}^{\mathrm{T}}\hat{W}_ia_i}{\mathrm{Tr}\left(\hat{W}_i\hat{W}_i^{\mathrm{T}}\right)}\hat{W}_i^{\mathrm{T}}\right) - 2\,\mathrm{Tr}\left(\chi_{wi}\hat{W}_ia_ie_{i+1}^{\mathrm{T}}\right) \\ &= 2\chi_{wi}e_{i+1}^{\mathrm{T}}\hat{W}_ia_i - 2\chi_{wi}e_{i+1}^{\mathrm{T}}\hat{W}_ia_i = 0. \end{aligned}$$

Hence, if the initial condition for $\hat{W}_i(0)$ holds, then $\mathrm{Tr}(\hat{W}_i(t)\hat{W}_i^{\mathrm{T}}(t)) \le W_{mi}$ $(i = 1, 2)$ always holds, which means that $\|\hat{W}_i\|_F \le \sqrt{W_{mi}}$. Therefore, $\|\tilde{W}_i\|_F = \|\hat{W}_i - W_i^*\|_F \le \|\hat{W}_i\|_F + \|W_i^*\|_F = \sqrt{W_{mi}} + W_{Mi} = \tilde{W}_{Mi}$ is also bounded.

By the similar way, it can be proved that $\|\tilde{V}_i\|_F \le \|\hat{V}_i\|_F + \|V_i^*\|_F = \sqrt{V_{mi}} + V_{Mi} = \tilde{V}_{Mi}$, $i = 1, 2$.

Then consider the following Lyapunov function

$$E = E_1 + E_2 + E_3, \tag{29}$$

where

$$E_1 = \frac{1}{2}e_1^{\mathrm{T}}e_1 + \frac{1}{2\beta}\tilde{\phi}_J^{\mathrm{T}}\tilde{\phi}_J,$$

$$E_2 = \frac{e_2^{\mathrm{T}}M(q)e_2}{2} + \mathrm{Tr}\left(\frac{\tilde{W}_1\tilde{W}_1^{\mathrm{T}}}{2\chi_{w1}}\right) + \mathrm{Tr}\left(\frac{\tilde{V}_1\tilde{V}_1^{\mathrm{T}}}{2\chi_{v1}}\right),$$

$$E_3 = \frac{e_3^{\mathrm{T}}k_1K_T^{-1}Le_3}{2} + \mathrm{Tr}\left(\frac{\tilde{W}_2\tilde{W}_2^{\mathrm{T}}}{2\chi_{w2}}\right) + \mathrm{Tr}\left(\frac{\tilde{V}_2\tilde{V}_2^{\mathrm{T}}}{2\chi_{v2}}\right).$$

By (16) and (23), differentiating E_1 with respect to time obtains

$$\begin{aligned}\dot{E}_1 &= e_1^{\mathrm{T}}J(q,\phi_J)e_2 - \eta_1 e_1^{\mathrm{T}}e_1 - \tilde{\phi}_J^{\mathrm{T}}\left(Y_J^{\mathrm{T}}(q,\dot{q}_d)e_1 - \frac{1}{\beta}\dot{\hat{\phi}}_J\right)\\ &\le -\eta_1 e_1^{\mathrm{T}}e_1 + e_1^{\mathrm{T}}J(q,\phi_J)e_2.\end{aligned} \tag{30}$$

By (19), (24) and (25), the time derivative of E_2 is

$$\begin{aligned}\dot{E}_2 &= -\eta_2 e_2^{\mathrm{T}}e_2 - e_2^{\mathrm{T}}J^{\mathrm{T}}(q,\phi_J)e_1 + e_2^{\mathrm{T}}\tilde{W}_1 a_1 + b_1\tilde{V}_1\bar{Z}_1\\ &\quad + e_2^{\mathrm{T}}e_3 + \mathrm{Tr}\left(\frac{1}{\chi_{w1}}\tilde{W}_1\dot{\hat{W}}_1^{\mathrm{T}}\right) + \mathrm{Tr}\left(\frac{1}{\chi_{v1}}\tilde{V}_1\dot{\hat{V}}_1^{\mathrm{T}}\right)\\ &\quad + e_2^{\mathrm{T}}\left(\frac{1}{2}\dot{M}(q) - V(q,\dot{q})\right)e_2 + e_2^{\mathrm{T}}\delta_1 - e_2^{\mathrm{T}}\gamma_1\\ &\le -(\eta_2 - 3(c_{11} + c_{21}\tilde{W}_{M1})/2)e_2^{\mathrm{T}}e_2\\ &\quad + \mathrm{Tr}\left(\tilde{W}_1\left(a_1e_2^{\mathrm{T}} + \frac{1}{\chi_{w1}}\dot{\hat{W}}_1^{\mathrm{T}}\right)\right)\\ &\quad + \mathrm{Tr}\left(\tilde{V}_1\left(\bar{Z}_1b_1 + \frac{1}{\chi_{v1}}\dot{\hat{V}}_1^{\mathrm{T}}\right)\right) + e_2^{\mathrm{T}}e_3 + \epsilon_1\\ &\quad - e_2^{\mathrm{T}}J^{\mathrm{T}}(q,\phi_J)e_1 + ((c_{11} + c_{21}\tilde{W}_{M1})/2)e_1^{\mathrm{T}}e_1,\end{aligned} \tag{31}$$

where the following facts have been used

- By Property 2, $e_2^{\mathrm{T}}\left(\frac{1}{2}\dot{M}(q) - V(q,\dot{q})\right)e_2 = 0$.
- $\|e_2\|(\Delta_{\tau ed} + c_{01} + (c_{11} + c_{21}\|\tilde{W}_1\|_F)(1 + X_M + \|\hat{\phi}_J\|)) - e_2^{\mathrm{T}}\gamma_1 \le \delta_{M1}\|e_2\| - e_2^{\mathrm{T}}\gamma_1 \le \epsilon_1$.

By (24), it follows that

1. If $\dot{\hat{W}}_1 = -\chi_{w1}e_2a_1^{\mathrm{T}}$, $\mathrm{Tr}\left(\tilde{W}_1\left(a_1e_2^{\mathrm{T}} + \frac{1}{\chi_{w1}}\dot{\hat{W}}_1^{\mathrm{T}}\right)\right) = 0$.
2. If $\dot{\hat{W}}_1 = -\chi_{w1}e_2a_1^{\mathrm{T}} + \chi_{w1}\frac{e_2^{\mathrm{T}}\hat{W}_1a_1}{\mathrm{Tr}\left(\hat{W}_1\hat{W}_1^{\mathrm{T}}\right)}\hat{W}_1$, then $\mathrm{Tr}\left(\hat{W}_1\hat{W}_1^{\mathrm{T}}\right) = W_{m1}$, $e_2^{\mathrm{T}}\hat{W}_1a_1 \le 0$, and

$$\mathrm{Tr}\left(\tilde{W}_1\left(a_1e_2^{\mathrm{T}} + \frac{1}{\chi_{w1}}\dot{\hat{W}}_1^{\mathrm{T}}\right)\right) = \frac{e_2^{\mathrm{T}}\hat{W}_1a_1\,\mathrm{Tr}\left(\tilde{W}_1\hat{W}_1^{\mathrm{T}}\right)}{\mathrm{Tr}\left(\hat{W}_1\hat{W}_1^{\mathrm{T}}\right)} \le 0.$$

This is because $\mathrm{Tr}\left(\tilde{W}_1\hat{W}_1^{\mathrm{T}}\right) = \mathrm{Tr}\left(\tilde{W}_1\tilde{W}_1^{\mathrm{T}}\right) + \mathrm{Tr}\left(\tilde{W}_1W_1^{*\mathrm{T}}\right) = \frac{1}{2}\mathrm{Tr}\left(\tilde{W}_1\tilde{W}_1^{\mathrm{T}}\right) + \frac{1}{2}\mathrm{Tr}\left(\hat{W}_1\hat{W}_1^{\mathrm{T}}\right) - \frac{1}{2}\mathrm{Tr}\left(W_1^{*}W_1^{*\mathrm{T}}\right) \ge 0$, where the facts that $\mathrm{Tr}\left(\hat{W}_1\hat{W}_1^{\mathrm{T}}\right) = W_{m1} \ge \mathrm{Tr}\left(W_1^{*}W_1^{*\mathrm{T}}\right)$ and $\mathrm{Tr}\left(\tilde{W}_1\tilde{W}_1^{\mathrm{T}}\right) \ge 0$ have been used.

From the above two cases, it is obtained that $\mathrm{Tr}\left(\tilde{W}_1\left(a_1e_2^{\mathrm{T}} + \frac{1}{\chi_{w1}}\dot{\hat{W}}_1^{\mathrm{T}}\right)\right) \le 0$. In the similar way, it is proved that $\mathrm{Tr}\left(\tilde{V}_1\left(\bar{Z}_1b_1 + \frac{1}{\chi_{w1}}\dot{\hat{V}}_1^{\mathrm{T}}\right)\right) \le 0$.

Hence, according to (31), it can be obtained that

$$\begin{aligned}\dot{E}_2 &\le -(\eta_2 - 3(c_{11} + c_{21}\tilde{W}_{M1})/2)e_2^{\mathrm{T}}e_2 - e_2^{\mathrm{T}}J^{\mathrm{T}}(q,\phi_J)e_1\\ &\quad + e_2^{\mathrm{T}}e_3 + \epsilon_1 + ((c_{11} + c_{21}\tilde{W}_{M1})/2)e_1^{\mathrm{T}}e_1.\end{aligned} \tag{32}$$

By the similar analysis, differentiating E_3 along (22) yields

$$\begin{aligned}\dot{E}_3 &= -\eta_3 e_3^{\mathrm{T}}e_3 - e_2^{\mathrm{T}}e_3 + e_3^{\mathrm{T}}\tilde{W}_2a_2 + b_2\tilde{V}_2\bar{Z}_2 - e_3^{\mathrm{T}}\gamma_2 + e_3^{\mathrm{T}}\delta_2\\ &\quad + \mathrm{Tr}\left(\frac{1}{\chi_{w2}}\tilde{W}_2\dot{\hat{W}}_2^{\mathrm{T}}\right) + \mathrm{Tr}\left(\frac{1}{\chi_{v2}}\tilde{V}_2\dot{\hat{V}}_2^{\mathrm{T}}\right)\\ &\le -(\eta_3 - 2(c_{12} + c_{22}\tilde{W}_{M2}))e_3^{\mathrm{T}}e_3 - e_2^{\mathrm{T}}e_3 + \epsilon_2\\ &\quad + ((c_{12} + c_{22}\tilde{W}_{M2})/2)e_2^{\mathrm{T}}e_2\\ &\quad + ((c_{12} + c_{22}\tilde{W}_{M2})/2)e_1^{\mathrm{T}}e_1.\end{aligned} \tag{33}$$

Thus, by (30), (32) and (33), the time derivative of E is

$$\begin{aligned}\dot{E} &= \dot{E}_1 + \dot{E}_2 + \dot{E}_3\\ &\le -(\eta_1 - (c_{11} + c_{12} + c_{21}\tilde{W}_{M1} + c_{22}\tilde{W}_{M2})/2)e_1^{\mathrm{T}}e_1\\ &\quad - (\eta_2 - (3(c_{11} + c_{21}\tilde{W}_{M1}) + c_{12} + c_{22}\tilde{W}_{M2})/2)e_2^{\mathrm{T}}e_2\\ &\quad - (\eta_3 - 2(c_{12} + c_{22}\tilde{W}_{M2}))e_3^{\mathrm{T}}e_3 + \epsilon\\ &\le -2\eta\left(\frac{1}{2}e_1^{\mathrm{T}}e_1 + \frac{1}{2}e_2^{\mathrm{T}}M(q)e_2 + \frac{1}{2}e_3^{\mathrm{T}}k_1K_T^{-1}Le_3\right.\\ &\quad \left. + \sum_{i=1}^{2}\frac{\mathrm{Tr}(\tilde{W}_i\tilde{W}_i^{\mathrm{T}})}{2\chi_{wi}} + \sum_{i=1}^{2}\frac{\mathrm{Tr}(\tilde{V}_i\tilde{V}_i^{\mathrm{T}})}{2\chi_{vi}} + \frac{\tilde{\phi}_J^{\mathrm{T}}\tilde{\phi}_J}{2\beta}\right)\\ &\quad + \eta\sum_{i=1}^{2}\frac{\tilde{W}_{Mi}^2}{\chi_{wi}} + \eta\sum_{i=1}^{2}\frac{\tilde{V}_{Mi}^2}{\chi_{vi}} + \frac{\eta\|\phi_J^{+} - \phi_J^{-}\|^2}{\beta} + \epsilon\\ &= -2\eta E(t) + \rho,\end{aligned} \tag{34}$$

where $\eta = \min((\eta_1 - (c_{11} + c_{12} + c_{21}\tilde{W}_{M1} + c_{22}\tilde{W}_{M2})/2), (\eta_2 - (3(c_{11} + c_{21}\tilde{W}_{M1}) + c_{12} + c_{22}\tilde{W}_{M2})/2)/(m_2), (\eta_3 - 2(c_{12} + c_{22}\tilde{W}_{M2}))/(\lambda_{\max}(k_1K_T^{-1}L)))$, and $\epsilon = \epsilon_1 + \epsilon_2$.

The controller gains η_1, η_2, η_3 are designed to satisfy the condition $\eta > 0$.

According to Lemma 2, it follows that

$$E(t) \le \frac{\rho}{2\eta}(1 - \mathrm{e}^{-2\eta t}) + E(0)\mathrm{e}^{-2\eta t}. \tag{35}$$

From (35), using the boundedness theorem (e.g., Qu (1998), Th. 2.14), it can be obtained that e_1, e_2, and e_3 are all uniformly ultimately bounded signals.

Let $\mu = \min\left(1, m_1, \lambda_{\min}(k_1K_T^{-1}L)\right)$. By (35), it follows that,

$$\frac{\mu}{2}e_i^{\mathrm{T}}e_i \le V(t) \le E(0)\mathrm{e}^{-2\eta t} + \frac{\rho}{2\eta}(1 - \mathrm{e}^{-2\eta t}), \quad i = 1, 2, 3. \tag{36}$$

For any given small value $\varsigma > 0$, by choosing $\chi_{wi} = (12\tilde{W}_{Mi}^2)/(\mu\varsigma)$, $\chi_{vi} = (12\tilde{V}_{Mi}^2)/(\mu\varsigma)$, $\beta = (12\|\phi_J^{+} - \phi_J^{-}\|^2)/(\mu\varsigma)$, and $\epsilon/\eta = (\mu\varsigma)/12$, there exists $T = (1/(2\eta))\ln((4E(0))/(\varsigma\mu))$ such that

$$\forall t \ge T, \quad \|e_i(t)\|^2 \le \varsigma. \tag{37}$$

Therefore, the trajectory tracking error $e_1 = x - x_d$ can be reduced to an arbitrary small neighborhood around zero by choosing appropriate parameters. It should be noted that the above parameter selection method is very conservative. In practice, there is no need to set the parameter such a large value. □

Remark 1. Because $\bar{Z}_1 = [1, e_1^{\mathrm{T}}, e_2^{\mathrm{T}}, x_d^{\mathrm{T}}, \dot{x}_d^{\mathrm{T}}, \ddot{x}_d^{\mathrm{T}}, \hat{\phi}_J^{\mathrm{T}}]^{\mathrm{T}}$, it can be obtained that $\|\bar{Z}_1\| \le 1 + \|e_1\| + \|e_2\| + \|x_d\| + \|\dot{x}_d\| +$

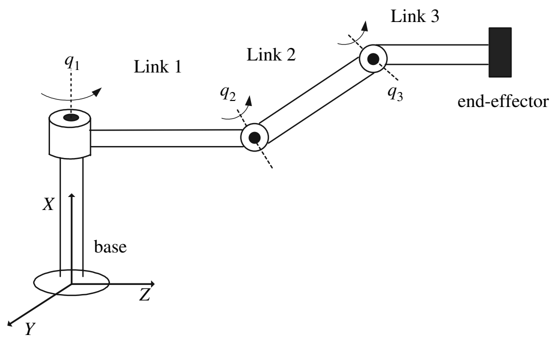

Fig. 2. A three-link revolute manipulator.

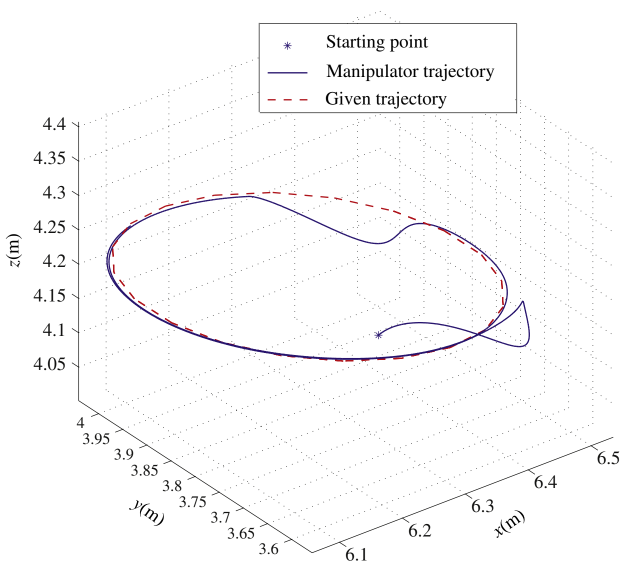

Fig. 3. The circular trajectory tracking performance of the three-link manipulator.

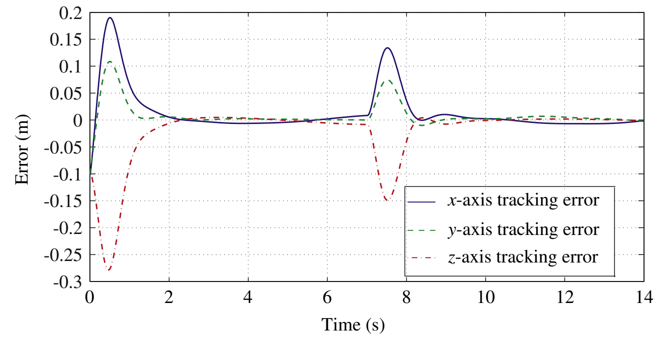

Fig. 4. The tracking errors in three coordinate axes with the circular trajectory.

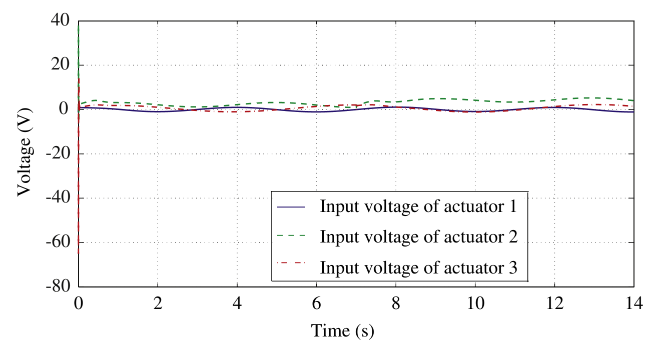

Fig. 5. The profile of actuator input voltage with the circular trajectory.

$\|\ddot{x}_d\| + \|\hat{\phi}_J\|$. According to Assumption 1, $\|x_d\| + \|\dot{x}_d\| + \|\ddot{x}_d\|$ is bounded by X_M. According to the proof of Theorem 1, $\|\hat{\phi}_J\| \leq \sqrt{\sum_{j=1}^{p} \max((\phi_J^-)_j^2, (\phi_J^+)_j^2)}$. By (29) and (35), for all $t \geq 0$, $\|e_1(t)\|^2 + m_1\|e_2(t)\|^2 + \lambda_{\min}(k_1 K_T^{-1} L)\|e_3(t)\|^2 \leq \|e_1(0)\|^2 + m_2\|e_2(0)\|^2 + \lambda_{\max}(k_1 K_T^{-1} L)\|e_3(0)\|^2 + \frac{2\rho}{\eta}$. Therefore, there exists a compact set Ω_1 from which the neural network input $\bar{Z}_1$ can never escape. Similar result can also be obtained for $\bar{Z}_2$.

Remark 2. According to the above analysis, the tracking error can be reduced as small as desired like the H_∞ tracking design cases in Chen, Chang, and Lee (1997) and Chang and Chen (1997). However, in this paper, the uncertain manipulator kinematics and actuator model are taken into account, and there is no need to solve the algebraic Riccati-like equation.

5. Simulation examples

A simulation example based on a three-link revolute manipulator is conducted to demonstrate the effectiveness of the proposed controller.

The three-link revolute manipulator is shown in Fig. 2. The parameters of this manipulator are set as follows: all links are modeled as thin uniform rods; the link lengths are $l_1 = 3.2$ m, $l_2 = 2.5$ m and $l_3 = 3.5$ m; the link masses are $m_1 = m_2 = m_3 = 1$ kg; the initial joint configuration of the manipulator is $q(0) = [\pi/6, \pi/6, \pi/6]^{\mathrm{T}}$ rad and $\dot{q}(0) = [0, 0, 0]^{\mathrm{T}}$ rad/s. The parameters in (1) and (2) are set as: $K_T = \mathrm{diag}\{100, 100, 100\}$, $L = \mathrm{diag}\{0.2, 0.2, 0.2\}$, $R = \mathrm{diag}\{5, 5, 5\}$, $K_e = \mathrm{diag}\{0.02, 0.02, 0.02\}$, $\tau_{ed} = (\cos(\pi t/2), \sin(\pi t/2) + \mathrm{e}^{-t}, \cos(t) + \sin(\pi t/3))^{\mathrm{T}}$, and $u_{ed} = (1.5\mathrm{e}^{-2t}, \sin(t), \cos(t/2)\mathrm{e}^{-t})^{\mathrm{T}}$. The manipulator end-effector is commanded to follow a circular trajectory specified in the Cartesian space as $x_d(1) = (1/4)\sqrt{(2/3)}\sin(\pi t/4) + 6.2621$, $x_d(2) = -(1/8)(\sqrt{2/3}\sin(\pi t/4) + \sqrt{2}\cos(\pi t/4)) + 3.8342$, $x_d(3) = -(1/8)(\sqrt{2/3}\sin(\pi t/4) - \sqrt{2}\cos(\pi t/4)) + 4.2041$.

In the proposed controller, the initial link lengths are estimated as $\hat{l}_1(0) = \hat{l}_2(0) = \hat{l}_3(0) = 3$ m. The upper and lower limits are $l^+ = (3.9, 3.8, 4.0)^{\mathrm{T}}$ and $l^- = (2.5, 2.1, 2.0)^{\mathrm{T}}$. β in (23) is set to be 2. The controller gains η_1, η_2 and η_3 in (15), (18) and (21) are $\eta_1 = 3$, $\eta_2 = 80$ and $\eta_3 = 80$. The numbers of hidden neurons of the two neural networks are chosen to be 60. The neural network initial weight matrices $\hat{W}_1(0) \in \mathbb{R}^{3\times 61}$, $\hat{W}_2(0) \in \mathbb{R}^{3\times 61}$, $\hat{V}_1(0) \in \mathbb{R}^{60\times 19}$, and $\hat{V}_2(0) \in \mathbb{R}^{60\times 1345}$ are set to be zero matrices. The scaling constants χ_{wi} and χ_{vi} in (24) and (25) are set as $\chi_{w1} = \chi_{v1} = 10$, $\chi_{w2} = \chi_{v2} = 50$. W_{mi} and V_{mi} are chosen to be $W_{mi} = V_{mi} = 10\,000$. The parameters in robust terms defined by (26) are $\delta_{M1} = 20$, $\delta_{M2} = 5$, $\epsilon_1 = 1.5$ and $\epsilon_2 = 1.2$.

In the simulation, an external load (2 kg) is exerted on the end-effector after $t = 7$ s. Fig. 3 shows the trajectory tracking performance of the three-link manipulator by the proposed controller. The tracking error is provided in Fig. 4. The profile of actuator voltage is shown in Fig. 5. The simulation results verify the good tracking performance of the proposed controller.

6. Concluding remarks

A neural-network-based tracking controller is proposed for the manipulator with uncertain kinematics, dynamics and the actuator model. The "linearity-in-parameters" assumption for the uncertain terms in the dynamics of manipulator and actuator is no longer necessary, and external disturbances are considered. According to the theoretical analysis, the tracking error can be reduced as small as desired. At last, it is noted that the proposed design procedure can also be applied to the nonholonomic mobile robot tracking control (Hou, Zou, Cheng, & Tan, 2009).

References

Arimoto, S. (1999). Robotics research toward explication of everyday physics. *International Journal of Robotics Research*, *18*(11), 1056–1063.

Braganza, D., Dixon, W., Dawson, D., & Xian, B. (2008). Tracking control for robot manipulators with kinematic and dynamic uncertainty. *International Journal of Robotics and Automation*, *23*(2), 3102–3122.

Chang, Y.-C., & Chen, B.-S. (1997). A nonlinear adaptive h_∞ tracking control design in robotic systems via neural networks. *IEEE Transactions on Control Systems Technology*, *5*(1), 13–29.

Cheah, C., Kawamura, S., & Arimoto, S. (2003). Stability of hybrid position and force control for robotic manipulator with kinematics and dynamics uncertainties. *Automatica*, *39*(5), 847–855.

Cheah, C., Liu, C., & Slotine, J. 2004. Approximate jacobian adaptive control for robot manipulators. In *Proceedings of IEEE international conference on robotics and automation* (pp. 3075–3080).

Cheah, C., Liu, C., & Slotine, J. (2006). Adaptive jacobian tracking control of robots with uncertainties in kinematic, dynamic and actuator models. *IEEE Transactions on Automatic Control*, *51*(6), 1024–1029.

Chen, B.-S., Chang, Y.-C., & Lee, T.-C. (1997). Adaptive control in robotic systems with h_∞ tracking performance. *Automatica*, *33*(2), 227–234.

Cheng, L., Hou, Z.-G., & Tan, M. 2008. Adaptive neural network tracking control of manipulators using quaternion feedback. In *Proceedings of IEEE international conference on robotics and automation* (pp. 3371–3376).

Cheng, L., Hou, Z.-G., Tan, M., & Wang, H. 2008. Adaptive neural network tracking control for manipulators with uncertainties. In *Proceedings of the 17th IFAC world congress* (pp. 2382–2387).

Dixon, W. (2007). Adaptive regulation of amplitude limited robot manipulators with uncertain kinematics and dynamics. *IEEE Transactions on Automatic Control*, *52*(3), 488–493.

Farrell, J., & Polycarpou, M. (2006). *Adaptive approximation based control: Unifying neural, fuzzy and traditional adaptive approximation approaches*. Hoboken, NJ: Wiley-Interscience.

Ge, S., Huang, C., & Zhang, T. (1999). Nonlinear adaptive control using neural networks and its application to cstr systems. *Journal of Process Control*, *9*(4), 313–323.

Hou, Z.-G., Zou, A.-M., Cheng, L., & Tan, M. (2009). Adaptive control of an electrically driven nonholonomic mobile robot via backstepping and fuzzy approach. *IEEE Transactions on Control Systems Technology*, *17*(4), 803–815.

Kwan, C., Lewis, F., & Dawson, D. (1998). Robust neural-network control of rigid-link electrically driven robots. *IEEE Transactions on Neural Networks*, *9*(4), 581–588.

Lewis, F., Jagannathan, S., & Yesildirek, A. (1998). *Neural network control of robot manipulators and nonlinear systems*. NY: Taylor & Francis.

Ortega, R., & Spong, M. (1989). Adaptive motion control of rigid robots: A tutorial. *Automatica*, *25*(6), 877–888.

Qu, Z. (1998). *Robust control of nonlinear uncertain systems*. NY: Wiley.

Long Cheng received the B.E. degree (with honors) in Control Engineering from Nankai University, Tianjin, China, in 2004 and the Ph.D. degree in Control Theory and Control Engineering from Institute of Automation, Chinese Academy of Sciences, Beijing, in 2009. Currently, he is an Assistant Professor at the Laboratory of Complex Systems and Intelligent Science, Institute of Automation, Chinese Academy of Sciences. His current research interests include neural networks, optimization, nonlinear control, and their applications to robotics.

Zeng-Guang Hou received the B.E. and M.E. degrees in Electrical Engineering from Yanshan University (formerly North-East Heavy Machinery Institute), Qinhuangdao, China, in 1991 and 1993, respectively, and the Ph.D. degree in Electrical Engineering from Beijing Institute of Technology, Beijing, China, in 1997. He is now a Professor at the Laboratory of Complex Systems and Intelligent Science, Institute of Automation, Chinese Academy of Sciences. His current research interests include neural networks, optimization algorithms, robotics, and intelligent control systems. He currently serves as the Associate Editor/Editorial Board Member of several leading journals including IEEE Transactions on Neural Networks.

Min Tan received the B.S. degree in Control Engineering from Tsinghua University, Beijing, China, in 1986 and the Ph.D. degree in control theory and Control Engineering from Institute of Automation, Chinese Academy of Sciences, Beijing, in 1990. He is a Professor at the Laboratory of Complex Systems and Intelligent Science, Institute of Automation, Chinese Academy of Sciences. His research interests include advanced robot control, multirobot systems, biomimetic robots, and manufacturing systems.