# Corrigendum to: Bayesian inverse kinematics vs. least-squares inverse kinematics in estimates of planar postures and rotations in the absence of soft tissue artifact

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Keywords: *biomechanics; motion capture; human movement analysis; Bayesian analysis; Markov-Chain Monte Carlo simulations*

## 1. Introduction

Pohl et al. (2021) show that the main result of Pataky et al. (2019) — that Bayesian inverse kinematics (IK) can yield order-of-magnitude more accurate results than traditional least-squares IK (LS-IK) — is unexpected when non-informative prior distributions are used for Bayesian estimates. This finding has prompted us to revisit the results we reported in Pataky et al. (2019).

We discovered that our original code contained a major error: it incorrectly propagated known pose truths to Bayesian calculations. This error stemmed from a subclass method overriding error within our object-oriented programming code. Specifically, a method `get_pose` was used to inform the Bayesian priors when constructing the Bayesian model. We also had two mechanism classes: (i) `Mechanism`, which contained true pose details and true marker positions, and for which `get_pose` returned the true pose, and (ii) `NoisyMechanism`, which contained noisy marker positions and which was intended to provide access only to LS-estimated and not true poses. Due to a programming error `get_pose` inadvertently returned the true mechanism pose for our `NoisyMechanism` class. As suspected by Pohl et al. (2021) this resulted in a highly informative prior.

We have corrected our code, and we have prepared this Corregendum to report corrected versions of the relevant results from Pataky et al. (2019). All corrected source code is provided in this project's repository (https://github.com/0todd0000/BayesIK).

## 2. Methods

To correct our errors we have done the following:

- Simplified our code and underlying classes to ensure that no true pose details were propagated to Bayesian calculations.
- Reconducted all analyses from Pataky et al. (2019)
- Revised our previous Appendix D (see repository)
- Corrected our previous results including: Table 1, Fig.5 and Fig.6 (see Results below)
- Revised our main conclusions (see Discussion below)

## 3. Results

Corrected versions of Table 1, Fig.5 and Fig.6 are provided below. These results agree closely with those of Pohl et al. (2021): when using non-informative priors, Bayesian IK does not yield systematically more accurate results than LS-IK (Table 1, Fig.5, Fig.6) and Bayesian IK accuracy, like LS-IK accuracy, worsens as marker noise increases (Fig.6).

The main exception to this general trend was observed for 1-link rotations, where the Bayesian approach yielded greater accuracy than Halvorsen et al. (1999) in approximately 78% of simulation cases (Table 1). However, Bayesian IK generally had lower accuracy than Söderkvist and Wedin (1993).

Another minor exception to the aforementioned general trend was that, unlike the LS-IK methods, Bayesian IK error decreased on average in cases of large noise (Fig.6e,f; Noise=1.8 mm). This was also observed in Serrien et al. (2020), where Bayesian and LS-IK methods tended to diverge for large errors. However, as shown in Pohl et al. (2021), this divergence is also likely to the information content of priors, and more specifically the informativeness and inappropriateness of prior distribution ranges with high marker noise (see Pohl et al., 2021, Appendix D).

## 4. Discussion

Our corrected results suggest that Bayesian IK offers no or at-best marginally improved accuracy over LS-IK. We concur that the order-of-magnitude improvements originally reported in Pataky et al. (2019) are possible to achieve only when using highly informative priors (Pohl et al., 2021). As reported in both Pataky et al. (2019) and Serrien et al. (2020), more substantial differences between Bayesian IK and LS-IK emerge for greater marker noise of approximately 2 cm, but this noise magnitude is beyond what is expected in modern motion capture systems. More importantly, this improvement is secondary to the information content of priors, where even vague priors can become informative as error magnitude increases — due largely to inappropriate prior ranges — and thereby inappropriately improve Bayesian IK estimates (Pohl et al., 2021, Appendix D).

The relatively small differences amongst Halvorsen et al. (1999), Söderkvist and Wedin (1993) and the current Bayesian approach may have been caused by different assumptions regarding kinematic reality. For example, Söderkvist and Wedin (1993) employs a Procrustes technique, which allows for shape deformation including overall size change. The Bayesian approach similarly allows size changes through random error, but less flexibly than Procrustes modeling. In contrast, Halvorsen et al. (1999) does not account for size changes. Since these three approaches make different assumptions regarding kinematic reality, the current results emphasize the idea that the Bayesian IK model embodies more information regarding kinematic truth than Halvorsen et al. (1999) but not more information than Söderkvist and Wedin (1993).

Overall, Bayesian IK appears to offer little-to-no practical improvements over LS-IK unless informative priors are used and/or if the forward kinematic model used for Bayesian inference embodies more information about kinematic truth than the LS technique. The latter point is moot because a forward model is necessary for Bayesian inference, and LS techniques can be readily applied to any forward model. Thus, in the absence of informative priors, the estimates from LS and Bayesian estimates are similar.

Use of highly informative priors (i.e., priors which contain true kinematic details) in Bayesian IK is unjustifiable because kinematic truth is unknown in noninvasive laboratory experiments. Vaguely informative priors (e.g. priors that are informed by LS-IK solutions) are not expected to yield a large improvement in IK estimates. We conclude that Bayesian IK offers marginal-to-no practical accuracy improvements over LS-IK.

We are grateful to Pohl et al. for identifying the problems in Pataky et al. (2019) and for taking the time to publish their results and thereby correct the literature. We sincerely apologize to readers who may have been misled by our original, erroneous results.

## Conflict of Interest Statement

The authors report no conflict of interest, financial or otherwise.

## References

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Table 1: Percentage of simulations in which Bayesian errors were smaller than least-squares errors (overall median percentage = 95.3%). Results are shown for the proximal joint center ($r_x$ and $r_y$) and rotation angles ($\phi$). For the rotation results (bottom two rows) $\phi_1$ represents angular displacement. Each percent value was derived from 1000 simulations.

| Model | Reference | $r_x$ | $r_y$ | $\phi_1$ | $\phi_2$ | $\phi_3$ |
|---|---|---|---|---|---|---|
| 1-link posture | - | 48.5 | 46.4 | 47.1 | - | - |
| 2-link posture | - | 47.5 | 47.8 | 48.0 | 48.3 | - |
| 3-link posture | - | 50.8 | 48.8 | 49.8 | 50.0 | 47.3 |
| 1-link rotation | Halvorsen et al. (1999) | 85.2 | 90.5 | 78.4 | - | - |
| 1-link rotation | Söderkvist et al. (1993) | 48.2 | 50.7 | 51.4 | - | - |

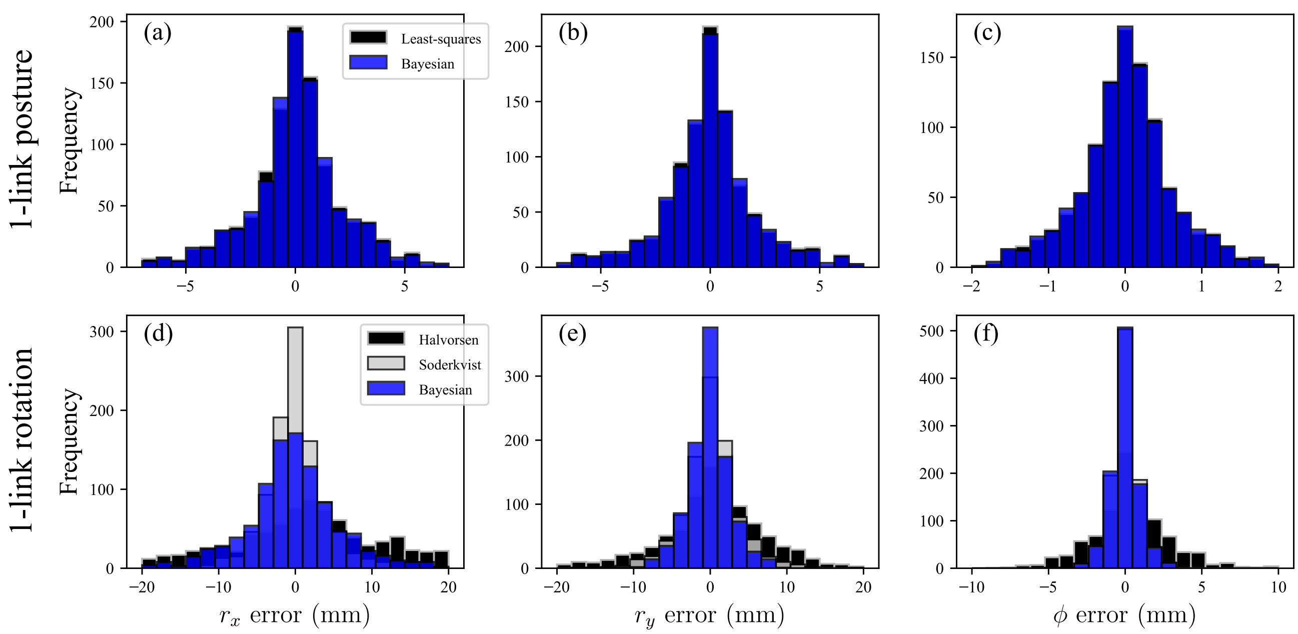


Figure 5: Error distributions for estimates of 1-link posture (a–c) and rotation (d–f). The three columns represent the three estimated parameters (x position, y position, angular position/rotation). A total of 1000 simulations was conducted for each panel in which both marker noise amplitude and angular posture / displacement were randomly varied. Similar results were obtained for 2-link and 3-link posture estimates (see Supplementary Material)

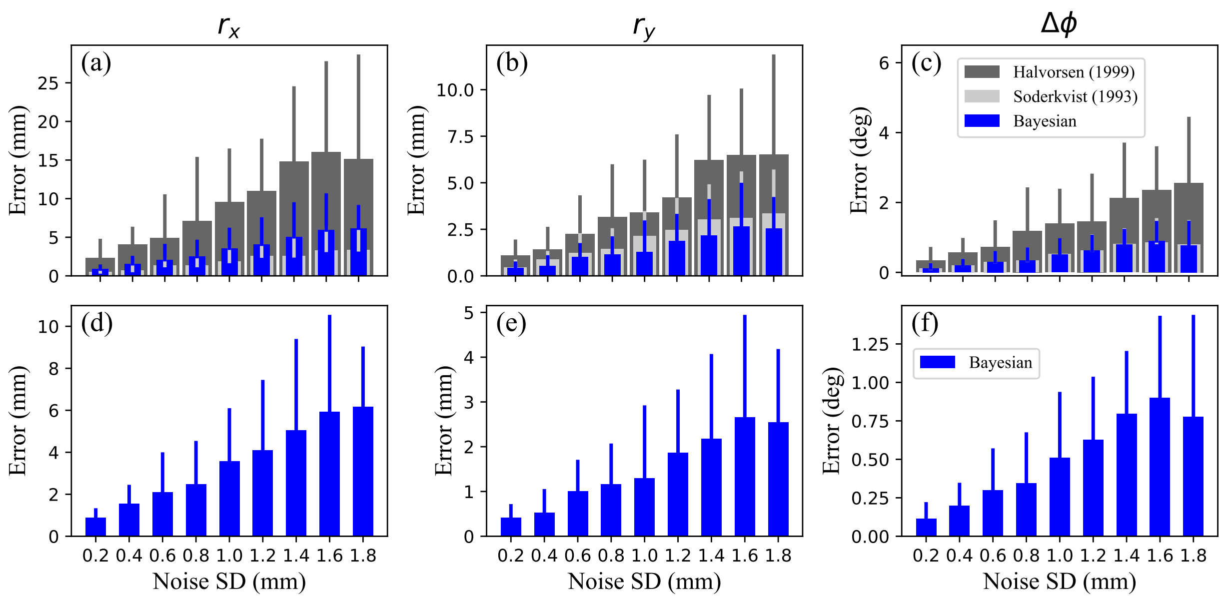


Figure 6: IK errors vs. marker noise standard deviation (SD). Vertical bars indicate inter-quartile ranges. (a–c) Errors from all three methods. (d–f) Bayesian errors only (magnified for clarity).