The Normal Distribution

Properties of a Normal Distribution

- A normal curve of normal distribution looks like a bell-shaped frequency .
- The normal distribution is perfectly symmetrical. Meaning if you fold a normal distribution in half the two equal halves will be a mirror image of the other.

Properties of a Normal Distribution

- The middle is also the point at which three measures coincide: the mode, the median, and the mean.
- The normal curve is a theoretical ideal, and real -life distributions never match this model perfectly.
- So, when we say that a distribution is normal, we mean that the distribution closely resembles the idealized curve.

Scores "Normally Distributed?"

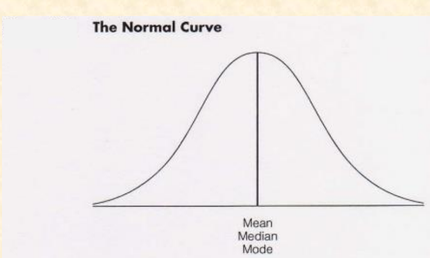
Final Grades in Social Statistics of 44 Studer			
Midpoint Score	Frequency Bar Chart	Freq.	%
50	*****	5	11
60	**********	10	22
70	***************	15	33
80	**********	10	22
90	*****	5	11

- Is this distribution normal?
- There are two things to initially examine: (1) look at the **shape** illustrated by the bar chart, and (2) calculate the **mean**, **median**, and **mode**.

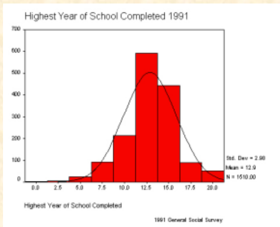
Scores Normally Distributed!

- The Mean = 70
- The Median = 70
- The Mode = 70
- Since all three are essentially **equal**, and this is reflected in the bar graph, we can assume that these data are *normally distributed.*
- Also, since the **median** is **approximately equal** to the **mean**, we know that the distribution is **symmetrical**.

The Shape of a Normal Distribution: The Normal Curve



The Shape of a Normal Distribution



Notice the shape of the normal curve in this graph. Some normal distributions are **tall and thin**, while others are **short and wide**. All normal distributions, though, are **wider in the middle** and symmetrical.

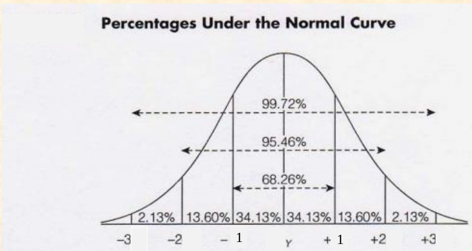
Interpreting the Standard Deviation

- When a distribution is approximately normal and we know the mean and the standard deviation we can determine the percentage of cases that fall between any score and the mean.

Standard (Z) Score

- A Z score is the number of standard deviations that, a given raw score, is above or below the mean.
- If your z score is a positive number that means it is above the mean; if it is a negative number that means it is below the mean.

Areas Under the Normal Curve by Measuring Standard Deviations



Standard (Z) Scores

- A ***standard score*** (also called ***Z score***) is the number of standard deviations that a given raw score is **above** or **below** the mean.

$$Z = \frac{Y - \bar{Y}}{S_y}$$

The Standard Normal Table

- A table showing the **area** (as a proportion, which can be translated into a percentage) under the standard normal curve **corresponding** to any Z score or its fraction

Figure 10.5 **Areas Between Mean and *Z* (B) and Beyond *Z* (C)**

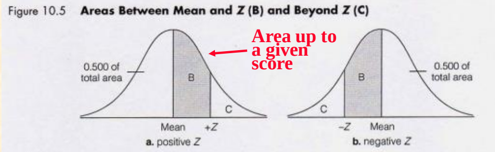


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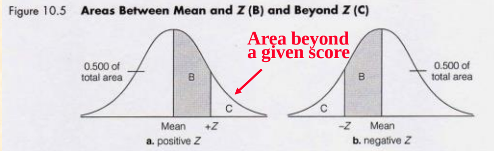
Figure 10.5 **Areas Between Mean and Z (B) and Beyond Z (C)**



Transforming a Z Score into a Raw Score

- We can also reverse the process. If we have the Z score but need to know the raw score we can go backwards.
- The formula for transforming a Z score to a raw score is:

$$Y = \bar{Y} + Z\,(S_y)$$

Finding the Area Between the Mean and a Positive Z Score

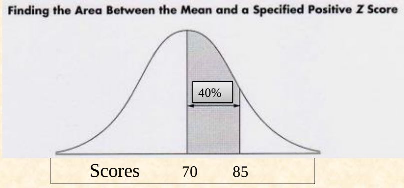
- Using the data presented in the table of stats grades, find the percentage of students whose scores range from the mean (70) to 85.
- (1) Convert 85 to a Z score:

  $Z = (85\text{-}70)/11.68 =$ **1.28**

  (2) Look up the Z score (1.28) in **Column A**, finding the proportion **(.3997)**

Finding the Area Between the Mean and a Positive Z Score

Finding the Area Between the Mean and a Specified Positive Z Score



(3) Convert the proportion (.3997) to a percentage (39.97%); this is the percentage of students scoring between the mean and 85 in the course.

Finding the Area Between the Mean and a Negative Z Score

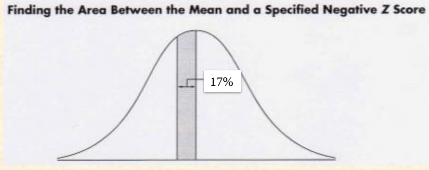
- Find the percentage of students scoring between 65 and the mean (70)
- (1) Convert 65 to a Z score:

  Z = (65-70)/11.68 = **-.43**

•(2) Since the curve is symmetrical and **negative area does not exist**, use .43 to find the area in the standard normal table: **.1664**

Finding the Area Between the Mean and a Negative Z Score

Finding the Area Between the Mean and a Specified Negative Z Score



(3) Convert the proportion (.1664) to a percentage (16.64%); this is the percentage of students scoring between 65 and the mean (70)

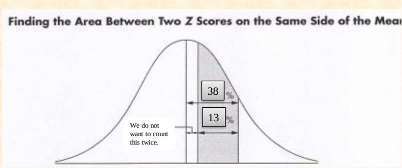
Finding the Area Between 2 Z Scores on the Same Side of the Mean

- Find the percentage of students scoring between 74 and 84.
- (1) Find the Z scores for 74 and 84:

  $Z_{74} = 74 - 70/11.68$

  $Z_{84} = 84 - 70/11.68$
- (2) Look up the corresponding areas for those Z scores in appendix B:

Finding the Area Between 2 Z Scores on the Same Side of the Mean



(3) To find the highlighted area above, **subtract** the smaller area from the larger area (.3830-.1331=**.2499**)
Now, we have the percentage of students scoring between 74 and 84.

Finding the Area Between 2 Z Scores on Opposite Sides of the Mean

- Using the same data, find the percentage of students scoring between 62 and 72.
- (1) Find the Z scores for 62 and 72:

  Z = (72-70)/11.68 = **.17**

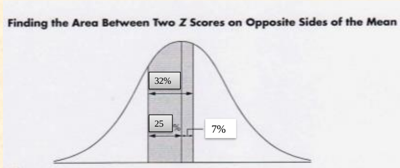
  Z = (62-70)/11.68 = **-.68**

(2) Look up the areas **between** these Z scores and the mean, like in the previous 2 examples:

Z = .17 is .0675 and Z = -.68 is .2517

(3) **Add** the two areas together: .0675 + .2517 = **.3192**

Finding the Area Between 2 Z Scores on Opposite Sides of the Mean



(4) Convert the proportion (.3192) to a percentage (31.92%); this is the percentage of students scoring **between** 62 and 72.

Finding Area Above a Positive Z Score or Below a Negative Z Score

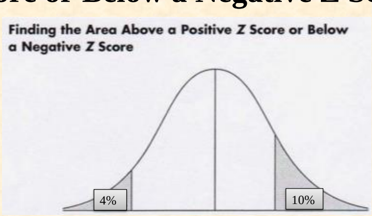
- Find the percentage of students who did (a) very well, scoring above 85, and (b) those students who did poorly, scoring below 50.
- (a) Convert 85 to a Z score, then look up the value in **Column C** of the Standard Normal Table:

  Z = (85-70)/11.68 = **1.28 → 10.03%**

  (b) Convert 50 to a Z score, then look up the value (look for a **<u>positive</u>** Z score!) in **Column C**:

  Z = (50-70)/11.68 = **-1.71 → 4.36%**

Finding Area Above a Positive Z Score or Below a Negative Z Score



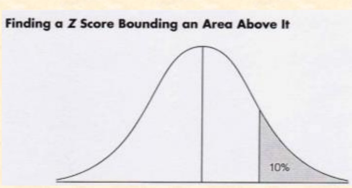
Finding a Z Score Bounding an Area Above It

- Find the raw score that bounds the top 10 percent of the distribution
- (1) 10% = a proportion of .10
- (2) Using the Standard Normal Table, look in **Column C** for .1000, then take the value in **Column A**; this is the Z score **(1.28)**

(3) Finally convert the Z score to a raw score:

Y=70 + 1.28 (11.68) = **84.95**

Finding a Z Score Bounding an Area Above It



(4) 84.95 is the raw score that bounds the upper 10% of the distribution. The Z score associated with 84.95 in this distribution is 1.28

Finding the Percentile Rank of a Score Higher than the Mean

- Suppose your raw score was 85. You want to calculate the percentile (to see where in the class you rank.)
- (1) Convert the raw score to a Z score:

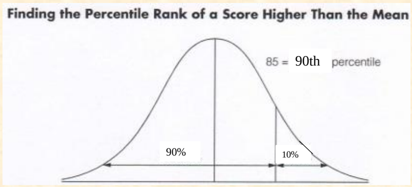
Z = (85-70)/11.68 = **1.28**

(2) Find the area beyond Z in the Standard Normal Table (**Column C**): **.1003**

(3) **Subtract** the area from 1.00 for the percentile, since .1003 is **only** the area **not** below the score:

1.00 - .1003 = **.8997** (proportion of scores below 85)

Finding the Percentile Rank of a Score Higher than the Mean



(4) .8997 represents the proportion of scores less than 85 corresponding to a percentile rank of 90%

Finding the Percentile Rank of a Score Lower than the Mean

- Now, suppose your raw score was 65.
- (1) Convert the raw score to a Z score

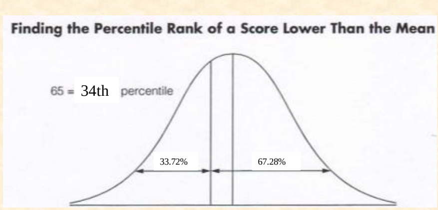
  Z = (65-70)/11.68 = **-.42**

(2) Find the are **beyond** Z in the Standard Normal Table, **Column C**: **.3372**

(3) **Multiply by 100** to obtain the percentile rank:

.3372 x 100 = **33.72**%

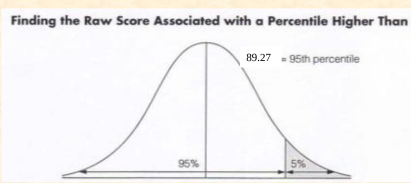
Finding the Percentile Rank of a Score Lower than the Mean



Finding the Raw Score of a Percentile Higher than 50

- Say you need to score in the 95th % to be accepted to a particular grad school program. What's the cutoff for the 95th %?
- (1) Find the area associated with the percentile:
  95/100 = .9500
- (2) **Subtract** the area from **1.00** to find the area above & beyond the percentile rank:
  1.00 - .9500 = .0500
- (3) Find the Z Score by looking in **Column C** of the Standard Normal Table for .0500: Z = 1.65

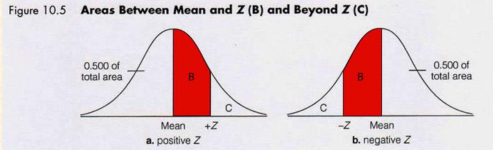
Finding the Raw Score of a Percentile Higher than 50



(4) Convert the Z score to a raw score.
Y= 70+ 1.65(11.68) = **89.27**

Using the Standard Normal Table (Appendix B)

Figure 10.5 **Areas Between Mean and *Z* (B) and Beyond *Z* (C)**



The curve s above shows the area that is given in columns "B" and "C" of the standard normal table.