Chapter 8 - Hypothesis Testing

Intro:

• Since most estimates are based on single samples & different samples may result in different estimates, **sampling results can't be used directly to make statements about a population**

Homework:

• Question 3 & 14 require 2-sample testing

Terms

Definitions:

• **Statistical hypothesis testing** - A procedure that allows us to evaluate hypotheses about population parameter based on sample statistics.

• **Research hypothesis** (H_1) - A statement reflecting the substantive hypothesis,. It's always expressed in terms of popu form varies from test to test. The research hypothesis usually specifies that the **population parameter** is one of the following:

1- Not equal to some specified value: μ doesn't equal some specified value (we use μ since it represents the populat is used when we have some theoretical basis to believe that there is a difference between groups, but we cannot anticipate the direction of that difference, here is where we conduct a two-tailed test & say the research hypothesis is not equal to some specified value.

2- Greater than some specified value: μ > some specified value, this is considered a right-tailed test since the outcom sampling distribution.

3- Less than some specified value μ < some specified value, this is considered a left-tailed test since the outcome will distribution.

• **One-tailed test** - A type of hypothesis test that involves a directional research hypothesis. It specifies that the values of 1 group are either larger or smaller than some specified population value

• **Right-tailed test** - a one-tailed test in which the sample outcome is hypothesized to be at the right tail of the sampling distribution

• **Left-tailed test** - A one-tailed test in which the sample outcome is hypothesized to be at the left tail of the sampling distribution

• **Two-tailed test** - A type of hypothesis test that involves a nondirectional research hypothesis. We're equally interested in whether the values are less than or greater than one another. The sample outcome may be located at both the lower & the higher ends of the sampling distribution.

• **Null Hypothesis** ((H_0)) - A statement of “no difference” that contradicts the research hypothesis & is always expressed in t parameters. Literally saying the research hypothesis is equal to some specified value, think of it as the opposite of the research hypothesis.

■ In hypothesis testing, we hope to reject the null hypothesis to indirectly support the research hypothesis, this will strengthen our belief in the research hypothesis.

• **Z statistic** (obtained) - The test statistic computed by converting a sample statistic (EX: the mean) to a Z-score. The formula for obtaining Z varies from test to test. A negative Z indicates evaluation at the left tail of the distribution, while the opposite for a positive Z.

• **p value** - The probability associated w/the obtained value of Z. Measures how unusual or rare our obtained statistic is compared with what is stated in our null hypothesis. The larger the p value, we can assume that the null hypothesis is true.

• **Alpha** (a) - The level of probability at which the null hypothesis is rejected. It's customary to set alpha at the .05, .01, or .001 level. Null hypothesis is rejected when the p value (p) is less than or equal to alpha (a).

• **Type 1 error** - The probability associated w/rejecting a null hypothesis when it's true.

• **Type 2 error** - The probability associated w/failing to reject a null hypothesis when it's false.

• **t statistic** (obtained) - The test statistic computed to test the null hypothesis about a population mean when the population standard deviation is unknown & is estimated using the sample standard deviation. Represents the number of standard deviation units (or standard error units) that our sample mean is from the hypothesized value of μ (the population mean)

• **t distribution** - A family of curves, each determined by its degrees of freedom (df). Used when the population standard deviation is unknown & the standard error is estimated from the sample standard deviation.

• **Degrees of freedom (df)** - The number of scores that are free to vary in calculating a statistic.

Z Statistic (obtained)

Probability Values & Alpha:

Formula:

• Before calculating, you first need the standard error, the formula for this is:

$$\sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{N}}$$

• Z Statistic (obtained) formula:

$$Z = \frac{\bar{Y} - \mu_{\bar{Y}}}{\sigma / \sqrt{N}}$$

- $\bar{Y}$ = sample mean
- $\mu_{\bar{Y}}$ = population mean
- Replace denominator with standard error (standard deviation)
- After calculation, look into C Column which tells us how improbable or probable (**p value**) it is to pull the sample mean from the population mean

• Converting the sample mean to a Z-score equivalent is called computing the **test statistic**, the Z value obtained is called the Z statistic

- This Z value gives us the number of standard deviations (standard errors) that our sample is from the hypothesized value (μ or $\mu_{\bar{Y}}$), assuming the null hypothesis is true.

• Negative Z statistic would mean the difference would have to be evaluated at the left tail of the distribution

- Positive Z statistics would mean the difference would have to be evaluated at the right tail of the distribution

• **P value** is the probability of pulling that sample from the population, measure of how unusual or rare our obtained statistic is compared w/what's stated in our null hypothesis.

- The smaller the p value, the more evidence we have that the null hypothesis should be rejected in favor of the research hypothesis
- The larger the p value, we can assume null hypothesis is true & fail to reject it

• P value is compared to the **Alpha (a**)

- **Alpha (a)** is the level of probability at which the null hypothesis is rejected, generally set to .05, .01, or .001. If you could reject one alpha level, you can state that your research is statistically significant at the .05 level but not at the .01 or .001 alpha level.
  - → 0.05 -> 5 in 100 chance (most likely to make Type 1 error with, easiest to reject null)
  - → 0.001 -> .1 in 100 chance (most likely to make Type 2 error with, hardest to reject null)
  - → 0.01 -> 1 in 100 chance
- P value equal to or below the Alpha (a) level, we reject the null hypothesis.
- If the P value is above the Alpha (a) level, we fail to reject the null hypothesis.

Z For 2-tailed Test:

• After getting your **p value** that corresponds to the Z score in appendix B, you would multiply it by 2 to obtain the 2-tailed probability

T Statistic

T Statistic:

• Used instead of Z statistic since standard deviation won't be known in most cases
• Aka t-test
• Uses sample standard deviation
• Represents how many standard deviation units (or standard error units) our sample mean is from the hypothesized value of the population mean, assuming our null hypothesis is true.
• Formula for t statistic (obtained):

$$t = \frac{\bar{Y} - \mu_y}{\frac{S_y}{\sqrt{n}}}$$

- $\bar{Y}$ = sample mean
- μ_y = population mean
- Denominator is meant to calculate the estimated standard error

T Distributions & Degrees of Freedom:

• T distributions is bell shaped
• T statistic can have positive & negative values: positive T statistics correspond to the right tail of the distribution while negative corresponds to the left tail.
• When the Degrees of Freedom (**df**) is small, the t distribution is much flatter than the normal curve.
    ■ As the df increases, the shape of the t distribution gets closer to the normal distribution, until the 2 are almost identical when the df is greater than 130
• Which distribution used is determined by degrees of freedom
• Degrees of freedom is calculated by taking sample size & subtracting the amount of restrictions you have
    ■ Amount of restrictions is defined as the number of samples
• Appendix C summarizes the T distribution
• The T table shows:
    ■ degrees of freedom
    ■ probabilities or alpha
    ■ significance levels
• **NOTE**: Since it's estimated from sample data, the denominator of the t statistic is subject to sampling error.
• After knowing your degrees of freedom (df), you go appendix C & grab the number that corresponds to the degrees of freedom & level of significance (aka Alpha), this is called the **t-critical**
    ■ With the **T-critical** & the T statistic (obtained), you can determine if you're going to reject or fail to reject your null hypothesis
    ■ If the value of the T-statistic is greater than the t-critical value, reject the null hypothesis
    ■ If the value of the T-statistic is less than the t-critical value, you fail to reject the null hypothesis

For 2 Samples:

First steps:

1) Find mean
2) Find N (number of samples)
3) Find variance (to get variance from standard deviation, square the standard deviation)

• If one of the variances are more than double of the other, you use this equation to find the estimated standard error:

$$S_{\bar{Y}1} - S_{\bar{Y}2} = \sqrt{\frac{(N_1 - 1)\ S_{y1}^2 + (N_2 - 1)S_{y2}^2}{(N_1 + N_2) - 2}} \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

• To calculate degrees of freedom:

- Take N from 1st sample & add it to 2nd sample
- Subtract the total by 2 (the number of samples there are)

$$df = (N_1 + N_2) - 2$$

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• T-test formula:

$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{S_{\bar{y}_1} - S_{\bar{y}_2}}$$

Difference between the Means

The estimated standard error

Lecture Notes

Assumptions (Requirements) of Statistical Hypothesis Testing

- Sample is a random sample
- dependent variable is measured at interval-ratio level
- population is normally distributed or that the sample size is larger than 50

- We can only at best, estimate the likelihood that the research hypothesis is true or false

Null Hypothesis:

- Rejecting null hypothesis indirectly supports research hypothesis
- We can either **reject** or **fail to reject it**
    - We never accept it
- Rejecting null hypothesis for 2 tail test is more difficult than it is for 1 tail tests

5 Steps to Hypothesis Testing

5 Steps to Hypothesis Testing:

• Statistical hypothesis testing can be organized into 5 basic steps:

1. Making assumptions
2. Stating the research & null hypotheses & selecting alpha
3. Selecting the sampling distribution & specifying the test statistic
4. Computing the test statistic
5. Making a decision & interpreting the results