B.E. Production Engineering 1st year 2nd Sem.Examination 2018
MATHEMATICS – III

Time: Three hours

Full Marks: 100

Use a separate Answer-Script for each part

(Notations/ Symbols have their usual meanings.)

Part –I

Answer any 5 questions.

1 a) If $\bar{\alpha} = t^2\hat{\imath} - t\hat{\jmath} + (2t+1)\hat{k}$ and $\bar{\beta} = (2t-3)\hat{\imath} + \hat{\jmath} - t\,\hat{k}$, then find $\frac{d}{dt}(\bar{\alpha} \times \frac{d\bar{\beta}}{dt})$ at $t = 2$. 2

b. Evaluate $\int_2^3 (\bar{r} \times \frac{d^2\bar{r}}{dt^2})\,dt$, where $\bar{r} = t^3\hat{\imath} + t^2\hat{\jmath} + t\,\hat{k}$ 3

c. If $\bar{r} = x\,\hat{\imath} + y\hat{\jmath} + z\,\hat{k}$, then find $\bar{\nabla} r^n$. 5

2 a) Find the constants p and q so that the surface $px^2 - qyz = (p+2)x$ is orthogonal to the surface $4x^2y + z^3$ at the point (1,2,-1). 5

b. Evaluate $\nabla^2(log r)$. 5

3 a) Show that the vector $\bar{v} = 2xyz^3\,\hat{\imath} + x^2z^3\,\hat{\jmath} + 3x^2yz^2\,\hat{k}$ is irrotational. Find the scalar potential φ such that $\bar{v} = grad\ \varphi$. 5

b. If $\bar{A}$ is a vector and φ is a scalar, show that
$\bar{\nabla} \mathrm{X} (\varphi\bar{A}) = \varphi(\bar{\nabla}\ \mathrm{X}\ \bar{A}) + (\bar{\nabla}\varphi)\,\mathrm{X}\,\bar{A}$ 5

[Turn over

4 State Green's theorem.

Verify Green's theorem in a plane for $\oint_C \{(xy+y^2)dx + x^2\,dy\}$,

Where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 10

5 a) If $F(s)$ is Fourier transform of $f(x)$, then show that $F[f'(x)] = -isF(s)$, if $f(x) \to 0$ as $x \to \infty$ 5

b. Show that $F[x^n f(x)] = (-i)^n \frac{d^n F(s)}{ds^n}$ 5

6. Find Fourier transform of

$$f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| \geq a \end{cases}$$

Where "a" is a positive real number. Hence deduce that $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$ 10.

7.a Prove that $Z(z^p) = -z\frac{d}{dz}Z(n^{p-1})$, where p is a positive integer. Hence deduce that $Z(n^2) = \frac{z^2+2}{(z-1)^3}$ 5

b. Solve $y_{n+2} + 6y_{n+1} + 9\,y_n = 2^n$, given $y_0 = y_1 = 0$ 5

Part – II

Answer any 5 questions.

8. Expand the function $f(x) = x\sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$. Deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots = \frac{\pi-2}{4}$. 8+2

9. Obtain a Fourier series for the function

$$f(x) = \pi x, \quad 0 \leq x \leq 1,$$
$$= \pi(2-x), \quad 1 \leq x \leq 2.$$ 10

10. If the Fourier series of f(x) over an interval $(c < x < c+2L)$ is given as

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right)$$

then show that

$$\frac{1}{2L}\int_c^{c+2L}[f(x)]^2dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$ 10

11. The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent times and show that the midpoint of the string always remains constant. 10

12. A tightly stretched string with fixed end points x = 0 and x = a is initially in a position given by $y = y_0 \, sin^3\left(\frac{\pi x}{a}\right)$. If it is released from rest from this position, find the displacement y(x,t). **10**

13. An insulated rod of length l has its ends A and B maintained at $0^0 C$ and $100^0 C$ respectively until steady state conditions prevail. If B is suddenly reduced to $0^0 C$ and maintained at $0^0 C$, find the temperature at a distance x from A at time t. 10

14. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, *where* $0 \le x \le a, 0 \le y \le b$, given that $u(0,y) = u(a,y) = u(x,b) = 0 \; and \; u(x,0) = x(a-x)$. 10