(c) State Green's theorem in a plane. Verify Green's theorem for $\int_C \left[\left(xy+y^2\right)dx + x^2 dy\right]$, where C is the closed curve of the region bounded by $y=x$ and $y=x^2$. 2+8

8. (a) Define the following : 5x2

(a) Random experiment (ii) Outcome (iii) Sample space (iv) Mutually exclusive events. (v) Equally likely events.

(b) In a group of 20 moles and 5 females, to males and 3 females are service holders. What is the probability that a person selected at random from the group is a service holder, given that the selected person is a male. 5

(c) Find the probability that in the throw of two unbiased dice, the sum of points will be even, or less than 5. 5

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Ex./IEBE/MATH/T/124/2018

BACHELOR OF (I.E.E.) ENGINEERING EXAMINATION, 2018

(1st Year, 2nd Semester)

Mathematics - IV J

Time : Three hours Full Marks : 100

Answer any ***five*** questions.

1. (a) Let $f_1(z)$ and $f_2(z)$ be two complex functions such that $\lim_{z \uparrow z_0} f_1(z) = l_1$ and $\lim_{z \uparrow z_0} f_2(z) = l_2$. Then for constants c_1 and c_2, prove that

$$\lim_{z \uparrow z_0} \left[c_1 f_1(z) + c_2 f_2(z)\right] = c_1 l_1 + c_2 l_2$$ 7

(b) Prove that a necessary and sufficient condition that $\omega = f(z) = u(x, y) + i\nu(x,y)$ tends to a limit $l = m + in$ as $z = x + iy \to a = \alpha + i\beta$ is that

$$y \to \beta$$ 10

(c) When a complex function is called bounded ? Give an example of an unbounded complex function. 3

2. (a) State and prove Cauchy-Ricmann conditions for differentiable complex functions. 6

(b) Let $f(z)=\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ $z\neq 0$, $f(0)=0$. Prove that f(z) is continuous at 0 and satisfies Cauchy-Ricmann conditions but is not differentiable at 0. 8

(c) If $f: C \to C$ is differentiable everywhere and f(z) is real for all $Z \in C$, show that f is a constant function. 6

3. (a) Show that $f(z) = |z|^2$ is differentiable only at the origin. 6

(b) Define a harmonic function. 2

(c) Find analytic function f(z) of which the real part is given by $u = 3x^2 + xy - 3y^2$ 6

(d) Evaluate $\int_c z^2 dz$ where c is given by the straight line $y = 2x$ joining (0,0) and (1,2). 6

4. (a) State Cauchy's integral theorem. 3

(b) If c denotes the circle $|z| = 1$ consider the integral $\int_C \dfrac{dz}{z+2}$ and deduce the value of $\int_C^{\pi} \dfrac{1+2\cos\theta}{5+4\cos\theta}\, d\theta$. 6

(c) Find the following Laplace transforms

(i) $L\left(\dfrac{\cos\sqrt{t}}{\sqrt{t}}\right)$ (ii) $L\left(e^{-t}\sin^2 t\right)$ 6+5

5. (a) Find $L^{-1}\left\{\dfrac{3p-2}{p^2-4p+20}\right\}$ 7

(b) Find $L^{-1}\left(\log\dfrac{p+3}{p+2}\right)$ 4

(c) Solve the differential equation $(D+2)^2 y = 4e^{-2t}$, $y(0) = -1$, $y'(0) = 4$ with the help of Laplace transform. 9

6. (a) Let $\phi(x,y,z) = x^2 + y^2 + xz$. Find the directional derivative of ϕ at the point P(2,–1, 3) in the direction of the vector $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$. 8

(b) State Gauss's Divergence Theorem. Hence evaluate $\iint_S \vec{F}.\,\vec{ds}$ where $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$, and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. 2+10

7. (a) Let $\vec{F} = -3x^2\hat{i} + 5xy\,\hat{j}$ and let C be the curve $y = 2x^2$ in the xy plane. Evaluate the line integral $\int_c \vec{F}.\,d\vec{r}$ from $P_1(0,0)$ to $P_2(1,2)$. 5

(b) Let $\vec{V} = x^2z^2\hat{i} - 2y^2z^2\hat{j} + xy^2zk$. Find curl ($\vec{V}$) at the point (1,–1,1). 5