Ref. No.: **Ex/PG/CSE/T/127D/2018**

M.E. COMPUTER SCIENCE AND ENGINEERING
1^{st} Year, 2^{nd} Semester Examination, 2018
Computational Geometry

Full Marks: 100 Time : 3 Hr

Answer Five Questions. Write answers to the point and state all the assumptions (wherever required).
ALL PARTS OF THE QUESTION SHOULD BE ANSWERED TOGETHER

Q 1) (a) Given 4 points a, b, c, and d in convex position in the plane such that d is in the interior of the circumcircle of abc. What are the triangles of the Delaunay triangulation of $\{a, b, c, d\}$?

Note *Given a finite number of points $x_1, x_2, \ldots, x_n$, in a real vector space, a convex combination of these points is a point of the form $\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n$ where the real numbers α_i, satisfy $\alpha_i \geq 0$ and $\alpha_1 + \alpha_2 + \cdots + \alpha_n = 1$*

(b) Given 5 points in the plane, is it possible that every pair of points forms an edge of the Delaunay triangulation? Just give a yes or no answer with a one sentence justification.

(c) Given 6 points in convex position, how many different ways are there to triangulate the points.

(d) What are necessary and sufficient conditions on a set of points to guarantee that the Delaunay triangulation is unique.

Q 2) (a) Describe the structure of the Voronoi diagram for the vertices of a regular polygon.

(b) Detail the geometric properties of a one-dimensional Voronoi diagram: n sites on a line. Design an algorithm to compute it and analyze its computational complexity.

Q 3) (a) In the road map G_{road} that was constructed on the trapezoidal decomposition of the free space we added a node in the center of each trapezoid and on each vertical wall. It is possible to avoid the nodes in the center of each trapezoid. Show how the graph can be changed such that only nodes on the vertical walls are required. (Avoid an increase in the number of edges in the graph.)

(b) Let P_1 and P_2 be two convex polygons. Let S_1 be the collection of vertices of P_1 and S_2 be the collection of vertices of P_2. Prove that

$$P_1 \oplus P_2 = ConvexHull(S_1 \oplus S_2)$$

Q 4) Assume a triangulation of S is available. A triangulation contains information on how points of S are located with respect to each other. This information might help in locating all points of S in A. Show that in spite of knowing a triangulation of S it still takes $\Omega(N \log N)$ time to locate all points of S in A.

Q 5) Given a sequence of N points in the plane, $p_1, \ldots, p_N$ find their convex hull in such a way that after p_i is processed we have $ConvexHull(p_1, \ldots, p_i)$.

Q 6) Suppose we are given the Delaunay triangulation of a point set S with n points. Design an algorithm that constructs the Delaunay triangulation of the remaining $n-1$ sites if a site from S is deleted. How does this algorithm change if the deleted site was on the hull of S. Give simple examples to demonstrate your algorithm.

Q 7) Draw the Minkowski sum $P_1 \oplus P_2$ for the case where

(i) both P_1 and P_2 are unit discs;

(ii) both P_1 and P_2 are unit squares;

(iii) P_1 is a unit disc and P_2 is a unit square;

(iv) P_1 is a unit square and P_2 is a triangle with vertices (0, 0), (1, 0), (0, 1).