

production manager : { optimize the available resources to maximize profit
 + Determine quantities of each product to be produced.

optimize the resource utilization so that marginal opportunity cost of each resource unit is equal to its marginal return (also called shadow prices)

$$\text{shadow price} = \frac{\text{change in optimal objective function value}}{\text{Unit change in availability of resource}} \quad +ve \leq \quad -ve \geq$$

$$\text{Max } Z = \sum_{j=1}^n C_j x_j = \sum_{j=1}^n (\text{profit (or return) per unit of variable (activity) } x_j) \times (\text{number of units of variable } x_j)$$

s.t. constraints:

$$\sum_{j=1}^n a_{ij} x_j \leq b_i$$

unit of resource i consumed (required) per unit of variable x_j → maximum unit of resource available

or

$$\sum_{j=1}^n (\text{unit of resource } i \text{ consumed (required) per unit of variable } x_j) (\text{unit of variable } x_j) \leq \text{unit of resource } i \text{ available}$$

$$x_j \geq 0 \text{ for all } j$$

Dual =

$$Z_y = \sum_{i=1}^m b_i y_i =$$

minimum (cost) $Z_y = \sum_{i=1}^m (\text{unit of resource } i) (\text{cost per unit of resource } i)$

s.t.

$$\sum_{i=1}^m a_{ji} y_i \geq c_j$$

or $\sum_{i=1}^m (\text{units of resource } i \text{ consumed per unit of variable } y_i) (\text{cost per unit of resource } i) \geq \text{profit per unit for each variable } x_j$

$$y_i \geq 0 \text{ for all } i$$

Shadow price or y_i in terms of return per unit of resource i

simplex multiplier

$$Z_x \leq Z_y$$

$\Rightarrow \text{profit} \leq \text{worth of resources}$

$$Z_x = Z_y \rightarrow \text{for } 0$$

The optimality (maximum profit or return) is reached only when the resources are completely utilized.

$$\sum_{i=1}^m a_{ji} y_i - c_j \geq 0 \rightarrow c_j \geq \sum_{i=1}^m a_{ji} y_i$$

any non basic variable

$$\max \quad z_j - c_j < 0$$

$$c_j \leq z_j < \dots$$

profit per unit of activity $x_j \geq$

Shadow price of resources used per unit of activity x_j .

Some standard results on duality

1. Dual of the dual LPP will be primal again.
2. If either primal / dual has an unbounded objective function value, then the other problem has no feasible solution.
3. If either primal or dual has finite optimal solution, then the other one also poses the same.
4. ...

$$n = m = m \cdot n$$

Simplex method :- \rightarrow Sub optimal solution
 \rightarrow Entering variable $\min (z_j - c_j < 0)$
 \rightarrow leaving variable
 \rightarrow $x_B \rightarrow 0$

Dual simplex :-
 optimality criterion
 $z_j - c_j \geq 0$
 $x_B < 0$

1. Convert max / - min
2. $\geq \rightarrow \leq$

3. Slack variables

4. $z_j - c_j$

Case 1: $z_j - c_j \geq 0, x_{B_i} > 0$

\rightarrow optimum

Case 2: $z_j - c_j < 0 \rightarrow$ the method fails.

case 3: $z_j - c_j \geq 0$ atleast

one $x_{\beta i} < 0$

~~Step~~ 5. leaving variable

$$\underline{x_{B_k}} = \min \{ \underline{z_{B_i}} \}$$

$$a_i^2 \leq 0$$

$a_{ik} < 0$

$x_{B_i} < 0$

6. Entering variable

$$\max \left\{ \frac{z_j - c_j}{a_{jk}}, a_{jk} < 0 \right\}$$

7.

\downarrow \downarrow \rightarrow ve
 \downarrow \downarrow \rightarrow ve
 pivot $+ve$ $-ve$

element equal to 1

