(b) Determine the shortest distance between the two show lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{1}$ and $\frac{x+1}{5}=\frac{y-2}{1}$; $z=2$. 5

11. (a) A sphase of constant radius 'r' passes through the origin 'O' and cuts the axes in A, B and C. Prove that the locus of the foot of the perpendicular from 'O' to the plane ABC is $(x^2+y^2+z^2)(x^{-2}+y^{-2}+z^{-2})=4r^2$.

(b) Prove that

$$J_{5/2}(x)=\sqrt{\frac{2}{\pi x}}\left[\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right]$$ 5+5

12. Solve the heat equation : 10

$$\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$$

by using separation of variables, subject to the following conditions :

(i) u is not infinite for $t \to \alpha$.

(ii) $\frac{\partial u}{\partial x}=0$ for $x=0$ and $x=l$.

(iii) $u=lx-x^2$ for $t=0$ between $x=0$ and $x=l$.

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Ex./PE/MATH/T/122/2018

BACHELOR OF POWER ENGINEERING EXAMINATION, 2018

(1st Year, 2nd Semester)

Mathematics - III Q

Time : Three hours | Full Marks : 100

Answer any ***ten*** questions.

1. (a) Find the curvature and torsion to a space curve given by $\vec{r}=\hat{i}e^t\sin t+\hat{j}e^t\cos t+\hat{k}e^t$ at t=0.

(b) Check whether the vector field defined by $\vec{V}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ is irrotational or rwf. Is it solenoidal ? Justify. 5+5

2. (a) If $\varphi=2z^2y-xy^2$, find $\nabla\varphi$ and the directional derivative of φ at (2,1,1) in the direction of $3\hat{i}+b\hat{j}+2\hat{k}$.

(b) Prove that $\nabla r^n=n\,r^{n-2}\,\vec{r}$, where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$. 5+5

3. (a) If $\vec{F}=z\hat{i}-x\hat{j}+y\hat{k}$, find the work done by $\vec{F}$ along the curve $x=\cos t$, $y=\sin t$, $z=t$, from $t=0$ to $t=2\pi$.

(b) Solve the differential equation $y=px+\sqrt{1+p^2}$ where $p=\frac{dy}{dx}$. 5+5

(Turn Over)

4. State Stokes' theorem. What is the name of this theorem on xy-plane ? Obtain its expression on this plane. Verify this theorem for $\vec{F} = (x^2 + y^2)i - 2xy\hat{j}$ along the rectangle $x = \pm a$ $y = 0$, $y = b$. 10

5. Solve the following differential equations : 5+5

(a) $\frac{dy}{dx} - \frac{x}{1+x^2} y = x\sqrt{y}$

(b) $(6x - 5y + 4)dy + (y - 2x - 1)dx = 0$.

6. Solve the followings : 5+5

(a) $(D^2 - 1)y = 2 + 5x$

(b) $(D^2 - 5D + 6)y = e^{3x}$

7. (a) Show that the length of the portion of the tangent to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at any point of it, intercepted between the coordinate axes is constant.

(b) If the straight line $lx + my = n$ is a normal to the hyperbita $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, show that $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2 + b^2)^2}{n^2}$. 5+5

8. (a) A straight line l makes angles α, β, γ and δ with the four diagonals of a cube; prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = \frac{8}{3}$.

(b) Prove that two lines whose direction cosines are connected by the two relatives $al + bm + cn = 0$ & $ul^2 + \nu m^2 + \omega n^2 = 0$ are perpendicular if $a^2(\nu + \omega) + b^2(\omega + u) + c^2(u + \nu) = 0$. 5+5

9. (a) A variable plane passes through a fixed point (a,b,c) and meets the axes of reference in A, B and C. Show that the locus of the point of intersection of the planes through A, B and C parallel to the coordinate planes is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$.

(b) A variable plane which is at a constant distance 3p from the origin 'O' cuts the axes at A, B and C. Show that the locus of the controid of the traingle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$. 5+5

10. (a) Find the cartesian equation of the straight line which is perpendicular to the lines $\frac{x}{2} = \frac{y}{1} = \frac{z}{3}$ and $\frac{x-3}{-1} = \frac{y-2}{3} = \frac{z+5}{5}$ and which passes through the point (1,2,3). 5