B E Mechanical Engineering 1ˢᵗ year 2ⁿᵈ Sem. Exam. 2018(old).
MATHEMATICS – IIIM

Time: Three hours   Full Marks: 100

Answer any *ten* questions.
(Notations have their usual meanings)

1. Test the convergence of the following series:

   a) $1 + \frac{1}{2^2} + \frac{2^2}{3^2} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + - - - - \infty$

   b) $(\frac{2^2}{1^2} - \frac{2}{1})^{-1} + (\frac{3^3}{2^3} - \frac{3}{2})^{-2} + (\frac{4^4}{3^4} - \frac{4}{3})^{-3} - - - - - - - - - \infty$ 10

2. State D'Alembert's ratio test for convergence of an infinite series. Examine the convergence or divergence of the series

   $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + - - - - - - \infty$ 10

3. a) Show that the following sequences $\{x_n\}$ are divergent:

   i) $x_n = \sqrt{n}$ ii) $x_n = -3^n$ iii) $x_n = \log\frac{1}{n}$ 6

   b) Show that the following sequence $\{x_n\}$, where

   $x_n = \frac{(3n+1)(n-2)}{n(n+3)}$ 4

4. Find the Laplace transform of the of the functions:

   a) $(Sin^2 t)e^t$ b) $t \cos at$ 10

5. If a function $F(t)$ is continuous and has a Laplace transform $f(s)$, then show that

   a) $L[F'(t)] = sL[F(t)] - F(0)$ b) $L\int_0^t F(\tau)d\tau = \frac{1}{s}f(s)$

   Where $F(0) = $ value of $F(t)$ for $t = 0$.   10

6. Solve the differential equation by using Laplace transform

   (a) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 0$ given $y(0) = 1$ and $\dot{y}(0) = 0$.

   (b) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$ given $y(0) = 1$ and $\dot{y}(0) = -1$.   10

7. (a) A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.

   (b) Obtain the median for the following frequency distribution:

x:	1	2	3	4	5	6	7	8	9
f:	8	10	11	16	20	25	15	9	6

   5+5

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8. A random varable X has the following probability function:

Values of X,	x :	0	1	2	3	4	5	6	7
	p(x):	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Find K, (ii) Evaluate P(X<6), P(X≥6), p(3<X≤ 6) and (iii) Find the minimum value of x so that P(X≤ x) > 1/2.   10

7. (a) A problem in mechanics is given to three students A, B, C whose chances of solving it are 1/2, 1/3, 1/4 respectively. What is the probability that the problem will be solved?

(b) Compute the arithmetic mean for the following data:

Height (in cm):	219	216	213	210	207	204	201	198	195
No. Of persons:	2	4	6	10	11	7	5	4	1

5+5

8. (a) A bag contains 6 white, 8 red and 4 black balls. Two balls are drawn at random. Find the probability that they will both be white.

b) Find the Mode of the following data relating to weight of 120 articles:

Weight (in gm):	0-10	10-20	20-30	30-40	40-50	50-60
No. of articles	14	17	22	26	23	18

5+5

10. (a) (Evaluate $L^{-}\left\{ \frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9} \right\}$

(b) Use the convolution theorem to evaluate
$$L^{-}\left\{ \frac{1}{(s-1)(s+2)} \right\}$$ 5+5

11. (a) Solve the following differential equation by using Laplace transformation

$$\frac{d^2y}{dt^2} + y = 8cost, \text{ given } y(0) = -1, \dot{y}(0) = 1.$$ 5+5

(b) Solve the simultaneous differential equations:

$$\frac{dx}{dt} - 2y = cos2t$$

$$\frac{dy}{dt} + 2x = sin2t \text{ subject to the conditions } x(0) = 1 \text{ and } y(0) = 0.$$ 5+5

12. (a) Calculate the standard deviation for the following data:

Size of item	6	7	8	9	10	11	12
Frequency	3	6	9	13	8	5	4

(b) Find the correlation coefficient between x and y from the following data:

x:	78	89	97	69	59	79	68	57
Y:	125	137	156	112	107	138	123	108

5+5