TRANSPORTATION AND ASSIGNMENT PROBLEMS

- The Transportation Model
- Solution of a Transportation Problem
- The Assignment Model
- Solution of the Assignment Model

TRANSPORTATION AND ASSIGNMENT PROBLEMS

OVERVIEW

- Part of a larger class of linear programming problems known as network flow models.

- Possess special mathematical features that enabled development of very efficient, unique solution methods.

- Methods are variations of traditional simplex procedure.

THE TRANSPORTATION MODEL CHARACTERISTICS

- A product is transported from a number of sources to a number of destinations at the minimum possible cost.
- Each source is able to supply a fixed number of units of the product, and each destination has a fixed demand for the product.
- The linear programming model has constraints for supply at each source and demand at each destination.
- All constraints are equalities in a balanced transportation model where supply equals demand.
- Constraints contain inequalities in unbalanced models where supply does not equal demand.

TRANSPORTATION MODEL EXAMPLE
PROBLEM DEFINITION AND DATA

- Problem: How many tons of wheat to transport from each grain elevator to each mill on a monthly basis in order to minimize the total cost of transportation ?

- Data:

Grain Elevator	Supply	Mill	Demand
1. Kansas City	150	A. Chicago	200
2. Omaha	175	B. St.Louis	100
3. Des Moines	275	C. Cincinnati	300
Total	600 tons	Total	600 tons

	Transport cost from Grain Elevator to Mill ($/ton)		
Grain Elevator	A. Chicago	B. St. Louis	C. Cincinnati
1. Kansas City	$6	8	10
2. Omaha	7	11	11
3. Des Moines	4	5	12

Transportation Model Example
Model Formulation

minimize $Z = \$6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$

subject to $x_{1A} + x_{1B} + x_{1C} = 150$

$x_{2A} + x_{2B} + x_{2C} = 175$

$x_{3A} + x_{3B} + x_{3C} = 275$

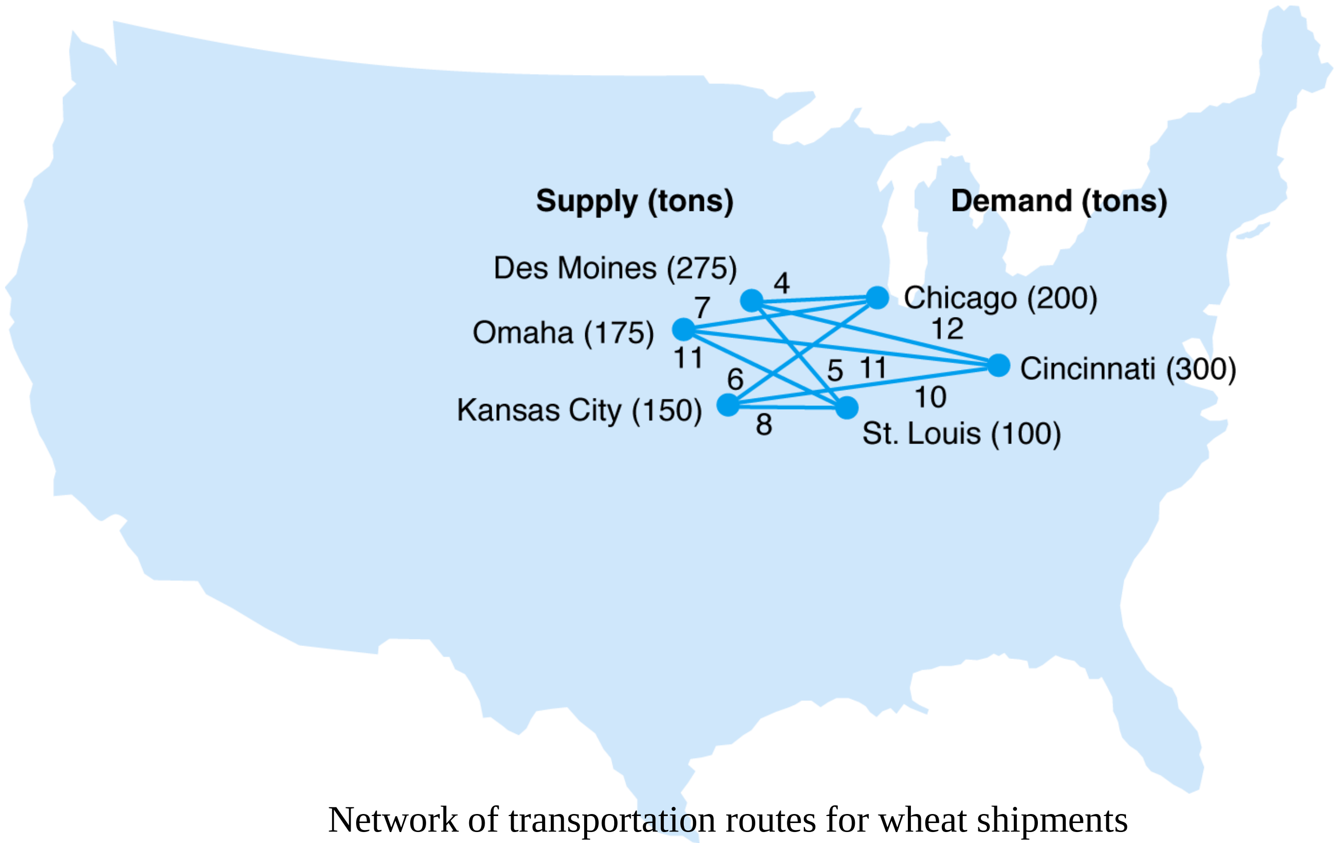
$x_{1A} + x_{2A} + x_{3A} = 200$

$x_{1B} + x_{2B} + x_{3B} = 100$

$x_{1C} + x_{2C} + x_{3C} = 300$

$xij \geq 0$

where x_{ij} = tons of wheat from each grain elevator, i, i = 1, 2, 3, to each mill j, j = A,B,C



Network of transportation routes for wheat shipments

SOLUTION OF THE TRANSPORTATION MODEL
TABLEAU FORMAT

• Transportation problems are solved manually within a *tableau* format.

• Each cell in a transportation tableau is analogous to a decision variable that indicates the amount allocated from a source to a destination.

• The supply and demand values along the outside rim of a tableau are called *rim values.*

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	300	600

The Transportation Tableau

SOLUTION OF THE TRANSPORTATION MODEL

SOLUTION METHODS

• Transportation models do not start at the origin where all decision values are zero; they must instead be given an *initial feasible solution*.

• Initial feasible solution determination methods include:

- northwest corner method
- minimum cell cost method
- Vogel's Approximation Method

• Methods for solving the transportation problem itself include:

- stepping-stone method and
- modified distribution method.

THE NORTHWEST CORNER METHOD

- In the northwest corner method the largest possible allocation is made to the cell in the upper left-hand corner of the tableau , followed by allocations to adjacent feasible cells.

The Initial NW Corner Solution

From \ To	A	B	C	Supply
1	[6] 150	[8]	[10]	150
2	[7] 50	[11] 100	[11] 25	175
3	[4]	[5]	[12] 275	275
Demand	200	100	300	600

- The initial solution is complete when all rim requirements are satisfied.

- Transportation cost is computed by evaluating the objective function:

$$Z = \$6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$$

$$= 6(150) + 8(0) + 10(0) + 7(50) + 11(100) + 11(25) + 4(0) + 5(0) + !2(275)$$

$$= \$5,925$$

THE NORTHWEST CORNER METHOD

SUMMARY OF STEPS

1. Allocate as much as possible to the cell in the upper left-hand corner, subject to the supply and demand conditions.
2. Allocate as much as possible to the next adjacent feasible cell.
3. Repeat step 2 until all rim requirements are met.

THE MINIMUM CELL COST METHOD (1 OF 3)

- In the minimum cell cost method as much as possible is allocated to the cell with the minimum cost followed by allocation to the feasible cell with minimum cost.

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4 200	5	12	275
Demand	200	100	300	600

The Initial Minimum Cell Cost Allocation

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4 200	5 75	12	275
Demand	200	100	300	600

The Second Minimum Cell Cost Allocation

THE MINIMUM CELL COST METHOD (2 OF 3)

- The complete initial minimum cell cost solution; total cost = $4,550.

- The minimum cell cost method will provide a solution with a lower cost than the northwest corner solution because it considers cost in the allocation process.

The Initial Solution

From \ To	A	B	C	Supply
1	[6]	[8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 200	[5] 75	[12]	275
Demand	200	100	300	600

THE MINIMUM CELL COST METHOD SUMMARY OF STEPS (3 OF 3)

1. Allocate as much as possible to the feasible cell with the minimum transportation cost, and adjust the rim requirements.
2. Repeat step 1 until all rim requirements have been met.

VOGEL'S APPROXIMATION METHOD (VAM) (1 OF 5)

- Method is based on the concept of *penalty cost* or *regret*.

- A penalty cost is the difference between the largest and the next largest cell cost in a row (or column).

- In VAM the first step is to develop a penalty cost for each source and destination.

- Penalty cost is calculated by subtracting the minimum cell cost from the next higher cell cost in each row and column.

The VAM Penalty Costs

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7	11	11	175	4
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	1		

VOGEL'S APPROXIMATION METHOD (VAM) (2 OF 5)

- VAM allocates as much as possible to the minimum cost cell in the row or column with the largest penalty cost.

The Initial VAM Allocation

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7 175	11	11	175	
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	2		

VOGEL'S APPROXIMATION METHOD (VAM) (3 OF 5)

- After each VAM cell allocation, all row and column penalty costs are recomputed.

The Second
M Allocation

From \ To	A	B	C	Supply	
1	6	8	10	150	4
2	7 175	11	11	175	
3	4	5 100	12	275	8
Demand	200	100	300	600	
	2		2		

VOGEL'S APPROXIMATION METHOD (VAM) (4 OF 5)

- Recomputed penalty costs after the third allocation.

From \ To	A	B	C	Supply
1	6	8	10 150	150
2	7 175	11	11	175
3	4 25	5 100	12 150	275
Demand	200	100	300	600

The Third VAM Allocation

VOGEL'S APPROXIMATION METHOD (VAM) (5 OF 5)

- The initial VAM solution; total cost = $5,125

- VAM and minimum cell cost methods both provide better initial solutions than does the northwest corner method.

From \ To	A	B	C	Supply
1	6	8	10 150	150
2	7 175	11	11	175
3	4 25	5 100	12 150	275
Demand	200	100	300	600

The Initial VAM Solution

VOGEL'S APPROXIMATION METHOD (VAM) SUMMARY OF STEPS

1. Determine the penalty cost for each row and column.
2. Select the row or column with the highest penalty cost.
3. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.
4. Repeat steps 1, 2, and 3 until all rim requirements have been met.

- Once an initial solution is derived, the problem must be solved using either the stepping-stone method or the modified distribution method (MODI).

- The initial solution used as a starting point in this problem is the minimum cell cost method solution because it had the minimum total cost of the three methods used.

From \ To	A	B	C	Supply
1	(6)	(8) 25	(10) 125	150
2	(7)	(11)	(11) 175	175
3	(4) 200	(5) 75	(12)	275
Demand	200	100	300	600

The Minimum Cell Cost Solution

THE STEPPING-STONE SOLUTION METHOD (2 OF 12)

- The stepping-stone method determines if there is a cell with no allocation that would reduce cost if used.

From \ To	A	B	C	Supply	
1	+1 $\boxed{6}$	$\boxed{8}$ 25	$\boxed{10}$ 125	150	151
2	$\boxed{7}$	$\boxed{11}$	$\boxed{11}$ 175	175	
3	$\boxed{4}$ 200	$\boxed{5}$ 75	$\boxed{12}$	275	
Demand	200	100	300	600	

The Allocation of One Ton to Cell 1A

THE STEPPING-STONE SOLUTION METHOD (3 OF 12)

- Must subtract one ton from another allocation along that row.

From \ To	A	B	C	Supply
1	+1 [6]	−1 [8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 200	[5] 75	[12]	275
Demand	200	100	300	600

The Subtraction of One Ton from Cell 1B

99

THE STEPPING-STONE SOLUTION METHOD (4 OF 12)

- A requirement of this solution method is that units can only be added to and subtracted from cells that already have allocations, thus one ton must be added to a cell as shown.

The Addition of One Ton to Cell 3B and the Subtraction of One Ton from Cell 3A

From \ To	A	B	C	Supply
1	+1 (6)	−1 (8) 25	(10) 125	150
2	(7)	(11)	(11) 175	175
3	−1 (4) 200	+1 (5) 75	(12)	275
Demand	200	100	300	600

THE STEPPING-STONE SOLUTION METHOD (5 OF 12)

- An empty cell that will reduce cost is a potential entering variable.

- To evaluate the cost reduction potential of an empty cell, a closed path connecting used cells to the empty cells is identified.

The Stepping-Stone Path for Cell 2A

From \ To	A	B	C	Supply
1	6	− 8 25	+ 10 125	150
2	+ 7	11	− 11 175	175
3	− 4 200	+ 5 75	12	275
Demand	200	100	300	600

2A → 2C → 1C → 1B → 3B → 3A

+ \$7 – 11 + 10 – 8 + 5 – 4 = –\$1

THE STEPPING-STONE SOLUTION METHOD (6 OF 12)

- The remaining stepping-stone paths and resulting computations for cells 2B and 3C.

From \ To	A	B	C	Supply
1	6	8 25	10 125	150
2	7	11	11 175	175
3	4 200	5 75	12	275
Demand	200	100	300	600

2B → 2C → 1C → 1B
+ \$11 – 11 + 10 – 8 = +\$2

The Stepping-Stone Path for Cell 2B

From \ To	A	B	C	Supply
1	6	8 25	10 125	150
2	7	11	11 175	175
3	4 200	5 75	12	275
Demand	200	100	300	600

3C → 1C → 1B → 3B
+ \$12 – 10 + 8 – 5 = +\$5

The Stepping-Stone Path for Cell 3C

The Stepping-Stone Solution Method (7 of 12)

- After all empty cells are evaluated, the one with the greatest cost reduction potential is the entering variable.

- A tie can be broken arbitrarily.

From \ To	A	B	C	Supply
1	+ [6]	− [8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	− [4] 200	+ [5] 75	[12]	275
Demand	200	100	300	600

The Stepping-Stone Path for Cell 1A

THE STEPPING-STONE SOLUTION METHOD (8 OF 12)

- When reallocating units to the entering variable (cell), the amount is the minimum amount subtracted on the stepping-stone path.

- At each iteration one variable enters and one leaves (just as in the simplex method).

From \ To	A	B	C	Supply
1	[6] 25	[8]	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 175	[5] 100	[12]	275
Demand	200	100	300	600

The Second Iteration of the Stepping-Stone Method

THE STEPPING-STONE SOLUTION METHOD (9 OF 12)

- Check to see if the solution is optimal.

From \ To	A	B	C	Supply
1	6 − 25	8	+ 10 125	150
2	+ 7	11	− 11 175	175
3	4 175	5 100	12	275
Demand	200	100	300	600

2A → 2C → 1C → 1A

+ $7 – 11 + 10 – 6 = $0

The Stepping-Stone Path for Cell 2A

From \ To	A	B	C	Supply
1	− 6 25	+ 8	10 125	150
2	7	11	11 175	175
3	+ 4 175	− 5 100	12	275
Demand	200	100	300	600

1B → 3B → 3A → 1A

+ $8 – 5 + 4 – 6 = +$1

The Stepping-Stone Path for Cell 1B

THE STEPPING-STONE SOLUTION METHOD (10 OF 12)

- Continuing check for optimality.

From \ To	A	B	C	Supply
1	− [6] 25	[8]	+ [10] 125	150
2	[7]	+ [11]	− [11] 175	175
3	+ [4] 175	− [5] 100	[12]	275
Demand	200	100	300	600

2B → 3B → 3A → 1A → 1C → 2C
+ \$11 – 5 + 4 – 6 + 10 – 11 = +\$3

The Stepping-Stone
Path for Cell 2B

From \ To	A	B	C	Supply
1	+ [6] 25	[8]	− [10] 125	150
2	[7]	[11]	[11] 175	175
3	− [4] 175	[5] 100	+ [12]	275
Demand	200	100	300	600

The Stepping-Stone
Path for Cell 3C

3C → 3A → 1A → 1C
+ \$12 – 4 + 6 – 10 = +\$4

THE STEPPING-STONE SOLUTION METHOD (11 OF 12)

- The stepping-stone process is repeated until none of the empty cells will reduce costs (i.e., an optimal solution).

- In example, evaluation of four paths indicates no cost reductions, therefore Table 19 solution is optimal.

- Solution and total minimum cost :

x_{1A} = 25 tons, x_{2C} = 175 tons, x_{3A} = 175 tons, x_{1C} = 125 tons, x_{3B} = 100 tons

$Z = \$6(25) + 8(0) + 10(125) + 7(0) + 11(0) + 11(175) + 4(175) + 5(100) + 12(0)$

$= \$4{,}525$

THE STEPPING-STONE SOLUTION METHOD (12 OF 12)

- A multiple optimal solution occurs when an empty cell has a cost change of zero and all other empty cells are positive.

- An alternate optimal solution is determined by allocating to the empty cell with a zero cost change.

- Alternate optimal total minimum cost also equals $4,525.

From \ To	A	B	C	Supply
1	6	8	10 150	150
2	7 25	11	11 150	175
3	4 175	5 100	12	275
Demand	200	100	300	600

The Alternative Optimal Solution

THE STEPPING-STONE SOLUTION METHOD
SUMMARY OF STEPS

1. Determine the stepping-stone paths and cost changes for each empty cell in the tableau.
2. Allocate as much as possible to the empty cell with the greatest net decrease in cost.
3. Repeat steps 1 and 2 until all empty cells have positive cost changes that indicate an optimal solution.

THE MODIFIED DISTRIBUTION METHOD (MODI) (1 OF 6)

- MODI is a modified version of the stepping-stone method in which math equations replace the stepping-stone paths.

- In the table, the extra left-hand column with the u_i symbols and the extra top row with the v_j symbols represent values that must be computed.

- Computed for all cells with allocations :

$$\mathbf{u_i + v_j = c_{ij} = \text{unit transportation cost for cell }_{ij}.}$$

	v_j	$v_A =$	$v_B =$	$v_C =$	
u_i	From \ To	A	B	C	Supply
$u_1 =$	1	6	8 / 25	10 / 125	150
$u_2 =$	2	7	11	11 / 175	175
$u_3 =$	3	4 / 200	5 / 75	12	275
	Demand	200	100	300	600

The Minimum Cell Cost Initial Solution

THE MODIFIED DISTRIBUTION METHOD (MODI) (2 OF 6)

- Formulas for cells containing allocations:

x_{1B}: $u_1 + v_B = 8$

x_{1C}: $u_1 + v_C = 10$

x_{2C}: $u_2 + v_C = 11$

x_{3A}: $u_3 + v_A = 4$

x_{3B}: $u_3 + v_B = 5$

	v_j	$v_A = 7$	$v_B = 8$	$v_C = 10$	
u_i	From \ To	**A**	**B**	**C**	**Supply**
$u_1 = 0$	**1**	6	8 25	10 125	150
$u_2 = 1$	**2**	7	11	11 175	175
$u_3 = -3$	**3**	4 200	5 75	12	275
	Demand	200	100	300	600

The Initial Solution with All ui and vj Values

- Five equations with 6 unknowns, therefore let $u_1 = 0$ and solve to obtain:

$v_B = 8$, $v_C = 10$, $u_2 = 1$, $u_3 = -3$, $v_A = 7$

THE MODIFIED DISTRIBUTION METHOD (MODI) (3 OF 6)

- Each MODI allocation replicates the stepping-stone allocation.
- Use following to evaluate all empty cells:

$$c_{ij} - u_i - v_j = k_{ij}$$

where k_{ij} equals the cost increase or decrease that would occur by allocating to a cell.

- For the empty cells in Table 26:

x_{1A}: $k_{1A} = c_{1A} - u_1 - v_A = 6 - 0 - 7 = -1$

x_{2A}: $k_{2A} = c_{2A} - u_2 - v_A = 7 - 1 - 7 = -1$

x_{2B}: $k_{2B} = c_{2B} - u_2 - v_B = 11 - 1 - 8 = +2$

x_{3C}: $k_{3C} = c_{3C} - u_3 - v_C = 12 - (-3) - 10 = +5$

THE MODIFIED DISTRIBUTION METHOD (MODI) (4 OF 6)

- After each allocation to an empty cell, the u_i and v_j values must be recomputed.

	v_j	$v_A =$	$v_B =$	$v_C =$	
u_i	From \ To	A	B	C	Supply
$u_1 =$	1	6 25	8	10 125	150
$u_2 =$	2	7	11	11 175	175
$u_3 =$	3	4 175	5 100	12	275
	Demand	200	100	300	600

The Second Iteration of the MODI Solution Method

THE MODIFIED DISTRIBUTION METHOD (MODI) (5 OF 6)

- Recomputing u_i and v_j values:

x_{1A}: $u_1 + v_A = 6, v_A = 6$ x_{1C}: $u_1 + v_C = 10, v_C = 10$ x_{2C}: $u_2 + v_C = 11, u_2 = 1$

x_{3A}: $u_3 + v_A = 4, u_3 = -2$ x_{3B}: $u_3 + v_B = 5, v_B = 7$

	v_j	$v_A = 6$	$v_B = 7$	$v_C = 10$	
u_i	From \ To	A	B	C	Supply
$u_1 = 0$	1	[6] 25	[8]	[10] 125	150
$u_2 = 1$	2	[7]	[11]	[11] 175	175
$u_3 = -2$	3	[4] 175	[5] 100	[12]	275
	Demand	200	100	300	600

The New ui and vj Values for the Second Iteration

THE MODIFIED DISTRIBUTION METHOD (MODI) (6 OF 6)

- Cost changes for the empty cells, $c_{ij} - u_i - v_j = k_{ij}$;

x_{1B}: $k_{1B} = c_{1B} - u_1 - v_B = 8 - 0 - 7 = +1$

x_{2A}: $k_{2A} = c_{2A} - u_2 - v_A = 7 - 1 - 6 = 0$

x_{2B}: $k_{2B} = c_{2B} - u_2 - v_B = 11 - 1 -7 = +3$

x_{3C}: $k_{2B} = c_{2B} - u_3 - v_C = 12 - (-2) - 10 = +4$

- Since none of the values are negative, solution obtained is optimal.

- Cell 2A with a zero cost change indicates a multiple optimal solution.

THE MODIFIED DISTRIBUTION METHOD (MODI)
SUMMARY OF STEPS

1. Develop an initial solution.
2. Compute the u_i and v_j values for each row and column.
3. Compute the cost change, k_{ij}, for each empty cell.
4. Allocate as much as possible to the empty cell that will result in the greatest net decrease in cost (most negative k_{ij})
5. Repeat steps 2 through 4 until all k_{ij} values are positive or zero.

THE UNBALANCED TRANSPORTATION MODEL (1 OF 2)

- When demand exceeds supply a dummy row is added to the tableau.

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Dummy	0	0	0	50
Demand	200	100	350	650

An Unbalanced Model (Demand . Supply)

THE UNBALANCED TRANSPORTATION MODEL (2 OF 2)

- When supply exceeds demand, a dummy column is added to the tableau.

- The dummy column (or dummy row) has no effect on the initial solution methods or the optimal solution methods.

From \ To	A	B	C	Dummy	Supply
1	6	8	10	0	150
2	7	11	11	0	175
3	4	5	12	0	375
Demand	200	100	300	100	700

An Unbalanced Model (Supply . Demand)

DEGENERACY
(1 OF 3)

- In a transportation tableau with *m* rows and *n* columns, there must be *m* + *n* - 1 cells with allocations; if not, it is *degenerate*.

- The tableau in the figure does not meet the condition since 3 + 3 -1 = 5 cells and there are only 4 cells with allocations.

From \ To	A	B	C	Supply
1	(6)	(8) 100	(10) 50	150
2	(7)	(11)	(11) 250	250
3	(4) 200	(5)	(12)	200
Demand	200	100	300	600

The Minimum Cell Cost Initial Solution

DEGENERACY (2 OF 3)

- In a degenerate tableau, all the stepping-stone paths or MODI equations cannot be developed.

-To rectify a degenerate tableau, an empty cell must artificially be treated as an occupied cell.

The Initial Solution

From \ To	A	B	C	Supply
1	6 0	8 100	10 50	150
2	7	11	11 250	250
3	4 200	5	12	200
Demand	200	100	300	600

DEGENERACY (3 OF 3)

- The stepping-stone path s and cost changes for this tableau:

2A 2C 1C 1A

x_{2A}: 7 - 11 + 10 - 6 = 0

2B 2C 1C 1B

x_{2B}: 11 - 11 + 10 - 8 = + 2

3B 1B 1A 3A

x_{3B}: 5 - 8 + 6 - 4 = - 1

3C 1C 1A 3A

x_{3C}: 12 - 10 + 6 - 4 = + 4

From \ To	A	B	C	Supply
1	[6] 100	[8]	[10] 50	150
2	[7]	[11]	[11] 250	250
3	[4] 100	[5] 100	[12]	200
Demand	200	100	300	600

The Second Stepping-Stone Iteration

PROHIBITED ROUTES

- A prohibited route is assigned a large cost such as *M*.
- When the prohibited cell is evaluated, it will always contain the cost M, which will keep it from being selected as an entering variable.

THE ASSIGNMENT MODEL

CHARACTERISTICS

- Special form of linear programming model similar to the transportation model.
- Supply at each source and demand at each destination limited to one unit.
- In a balanced model supply equals demand.
- In an unbalanced model supply does not equal demand.

THE ASSIGNMENT MODEL
EXAMPLE PROBLEM DEFINITION AND DATA

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

	Game Sites			
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

THE ASSIGNMENT MODEL
EXAMPLE PROBLEM MODEL FORMULATION

Minimize $Z = 210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} + 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} + 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC}$

subject to

$$x_{AR} + x_{AA} + x_{AD} + x_{AC} = 1$$
$$x_{BR} + x_{BA} + x_{BD} + x_{BC} = 1$$
$$x_{CR} + x_{CA} + x_{CD} + x_{CC} = 1$$
$$x_{DR} + x_{DA} + x_{DD} + x_{DC} = 1$$
$$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$$
$$x_{AA} + x_{BA} + x_{CA} + x_{DA} = 1$$
$$x_{AD} + x_{BD} + x_{CD} + x_{DD} = 1$$
$$x_{AC} + x_{BC} + x_{CC} + x_{DC} = 1$$
$$x_{ij} \geq 0$$

SOLUTION OF THE ASSIGNMENT MODEL (1 OF 7)

- An *assignment problem* is a special form of the transportation problem where all supply and demand values equal one.

- Example: assigning four teams of officials to four games in a way that will minimize distance traveled by the officials.

	Game Sites			
Officials	Raleigh	Atlanta	Durham	Clemson
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

The Travel Distances to Each Game for Each Team of Officials

Solution of the Assignment Model (2 of 7)

- An *opportunity cost table* is developed by first subtracting the minimum value in each row from all other row values *(row reductions*) and then repeating this process for each column.

	Game Sites			
Officials	Raleigh	Atlanta	Durham	Clemson
A	120	0	90	70
B	30	0	60	130
C	70	0	35	65
D	15	0	40	55

The Assignment Tableau with Row Reductions

SOLUTION OF THE ASSIGNMENT MODEL (3 OF 7)

- The minimum value in each column is subtracted from all column values (*column reductions*).

- Assignments can be made in the table wherever a zero is present.

- An *optimal solution* results when each of the four teams can be assigned to a different game.

- Table 36 does not contain an optimal solution

	Game Sites			
Officials	Raleigh	Atlanta	Durham	Clemson
A	105	0	55	15
B	15	0	25	75
C	55	0	0	10
D	0	0	5	0

The Tableau with Column Reductions

SOLUTION OF THE ASSIGNMENT MODEL (4 OF 7)

- An optimal solution occurs when the number of independent unique assignments equals the number of rows and columns.

- If the number of unique assignments is less than the number of rows (or columns) a line test must be used.

	Game Sites			
Officials	Raleigh	Atlanta	Durham	Clemson
A	105	0	55	15
B	15	0	25	75
C	35	0	0	10
D	0	0	5	0

The Opportunity Cost Table with the Line Test

SOLUTION OF THE ASSIGNMENT MODEL (5 OF 7)

- In a line test all zeros are crossed out by horizontal and vertical lines; the minimum uncrossed value is subtracted from all other uncrossed values and added to values where two lines cross.

	Game Sites			
Officials	Raleigh	Atlanta	Durham	Clemson
A	90	0	40	0
B	0	0	10	60
C	55	15	0	10
D	0	15	5	0

The Second Iteration

SOLUTION OF THE ASSIGNMENT MODEL (6 OF 7)

- At least four lines are required to cross out all zeros in table 38.

- This indicates an optimal solution has been reached.

- Assignments and distances:

Assignment	Distance	Assignment	Distance
Team A $\rightarrow$ Atlanta	90	Team A $\rightarrow$ Clemson	160
Team B $\rightarrow$ Raleigh	100	Team B $\rightarrow$ Atlanta	70
Team C $\rightarrow$ Durham	140	Team C $\rightarrow$ Durham	140
Team D $\rightarrow$ Clemson	120	Team D $\rightarrow$ Raleigh	80
Total	450 miles	Total	450 miles

- If in initial assignment team A went to Clemson, result is the same; resulting assignments represent multiple optimal solutions.

SOLUTION OF THE ASSIGNMENT MODEL (7 OF 7)

- When supply exceeds demand, a dummy column is added to the tableau.
- When demand exceeds supply, a dummy row is added to the tableau.
- The addition of a dummy row or column does not affect the solution method.
- A prohibited assignment is given a large relative cost of *M* so that it will never be selected.

	Game Sites				
Officials	Raleigh	Atlanta	Durham	Clemson	Dummy
A	210	90	180	160	0
B	100	70	130	200	0
C	175	105	140	170	0
D	80	65	105	120	0
E	95	115	120	100	0

An Unbalanced Assignment Tableau with a Dummy Column

SOLUTION OF THE ASSIGNMENT MODEL
SUMMARY OF SOLUTION STEPS

1. Perform row reductions.

2. Perform column reductions.

3 In the completed opportunity cost table, cross out all zeros using the minimum number of horizontal and/or vertical lines.

4. If fewer than *m* lines are required, subtract the minimum uncrossed value from all other uncrossed values, and add the same value to all cells where two lines intersect.

5. Leave all other values unchanged and repeat step 3.

6. If *m* lines are required, the tableau contains the optimal solution. If fewer than *m* lines are required, repeat step 4.

THE ASSIGNMENT PROBLEM WITH MORE EXAMPLES

THE ASSIGNMENT PROBLEM

- In many business situations, management needs to assign - personnel to jobs, - jobs to machines, - machines to job locations, or - salespersons to territories.
- Consider the situation of assigning n jobs to n machines.
- When a job i (=1,2,....,n) is assigned to machine j (=1,2, ...n) that incurs a cost Cij.
- The objective is to assign the jobs to machines at the least possible total cost.

THE ASSIGNMENT PROBLEM

- This situation is a special case of the Transportation Model And it is known as the *assignment problem*.
- Here, jobs represent "sources" and machines represent "destinations."
- The supply available at each source is 1 unit And demand at each destination is 1 unit.

THE ASSIGNMENT PROBLEM

		Machine				
		1	2	....	n	Source
	1	C11	C12	....	C1n	1
	2	C21	C22	....	C2n	1
Job	.	.	.		.	.
	.	.	.		.	.
	.	.	.		.	.
	n	Cn1	Cn2	....	Cnn	1
Destination		1	1	......	1	

The assignment model can be expressed mathematically as follows:

X_{ij}= 0, if the job j is not assigned to machine i

1, if the job j is assigned to machine i

THE ASSIGNMENT PROBLEM

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} Cij\ Xij$$

(Sum of assignments from a source should be exactly equal to 1):

$$\sum_{j=1}^{n} Xij = 1 \qquad \text{For } i=1,2,\ldots,n$$

(Sum of assignments to a destination should be equal to the demanded quantity by that destination):

$$\sum_{i=1}^{n} Xij = 1 \qquad \text{For } j=1,2,\ldots,n$$

(Quantities to be assigned can be either 0 or 1):

$$Xij = 0 \quad \text{or} \quad 1 \qquad \text{For all } i \text{ and } j.$$

THE ASSIGNMENT PROBLEM EXAMPLE

- Ballston Electronics manufactures small electrical devices.
- Products are manufactured on five different assembly lines (1,2,3,4,5).
- When manufacturing is finished, products are transported from the assembly lines to one of the five different inspection areas (A,B,C,D,E).
- Transporting products from five assembly lines to five inspection areas requires different times (in minutes)

THE ASSIGNMENT PROBLEM EXAMPLE

	Inspection Area				
Assembly Line	A	B	C	D	E
1	10	4	6	10	12
2	11	7	7	9	14
3	13	8	12	14	15
4	14	16	13	17	17
5	19	11	17	20	19

Under current arrangement, assignment of inspection areas to the assembly lines are 1 to A, 2 to B, 3 to C, 4 to D, and 5 to E.

This arrangement requires 10+7+12+17+19 = 65 man minutes.

HUNGARIAN METHOD EXAMPLE

		Machine			
		1	2	3	
	1	5	7	9	1
Job	2	14	10	12	1
	3	15	13	16	1
		1	1	1	

Step 1: Select the smallest value in each row. Subtract this value from each value in that row

Step 2: Do the same for the columns that do not have any zero value.

HUNGARIAN METHOD EXAMPLE

		Machine		
		1	2	3
Job	1	**5**	7	9
	2	14	**10**	12
	3	15	**13**	16

		Machine		
		1	2	3
Job	1	0	2	4
	2	4	0	**2**
	3	2	0	3

	Machine		
	1	2	3
1	**0**	2	2
2	4	0	**0**
3	2	**0**	1

If not finished, continue with other columns.

HUNGARIAN METHOD EXAMPLE

Step 3: Assignments are made at zero values.

- Therefore, we assign job 1 to machine 1; job 2 to machine 3, and job 3 to machine 2.
- Total cost is 5+12+13 = 30.
- It is not always possible to obtain a feasible assignment as in here.

HUNGARIAN METHOD EXAMPLE 2

	1	2	3	4
1	***1***	4	6	3
2	9	***7***	10	9
3	***4***	5	11	7
4	8	7	8	***5***

	1	2	**3**	4
1	0	3	**5**	2
2	2	0	**3**	2
3	0	1	**7**	3
4	3	2	**3**	0

	1	2	3	4
1	***0***	3	2	2
2	2	***0***	0	2
3	0	1	4	3
4	3	2	***0***	0

HUNGARIAN METHOD EXAMPLE 2

- A feasible assignment is not possible at this moment.
- In such a case, The procedure is to draw a ***minimum*** number of ***lines*** through some of the rows and columns, ***Such that all zero values are crossed out***.

HUNGARIAN METHOD EXAMPLE 2

	1	2	3	4
1	0	3	2	2
2	2	0	0	2
3	0	1	4	3
4	3	2	0	0

The next step is to select the smallest uncrossed out element. This element is ***subtracted from every uncrossed out element*** and ***added to every element at the intersection*** of two lines.

	1	2	3	4
1	***0***	2	1	1
2	3	0	***0***	2
3	0	***0***	3	2
4	4	2	0	***0***

HUNGARIAN METHOD EXAMPLE 2

- We can now easily assign to the zero values. Solution is to assign (1 to 1), (2 to 3), (3 to 2) and (4 to 4).
- If drawing lines do not provide an easy solution, then we should perform the task of drwaing lines one more time.
- Actually, we should continue drawing lines until a feasible assignment is possible.