B E Mechanical Engineering 1st year 2nd Sem. Exam.2018(old).

MATHEMATICS – IVM

Time: Three hours Full Marks: 100

(Symbols /Notations have their meanings)

Answer any 10 questions

1. Find the Fourier Series for $f(x) = x - x^2$ in $-\pi < x < \pi$. Hence deduce that
$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots\ldots\ldots$$ 10

2. Obtain the Cosine series for $f(x) = x$ in $0 < x < \pi$ and deduce that
$$\sum_{n=0}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi}{96}$$ 10

3. A periodic function of period 2 is defined as :
$$f(x) = \begin{matrix} -1 < x \leq 0 \\ x + 2, 0 < x \leq 1 \end{matrix}$$
Find its Fourier series expansion. Hence, show that the sum of the series
$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \ldots\ldots = \frac{\pi}{4}$$ 10

4. Find the half range Cosine series for the function
$f(x) = (x-1)^2$ in the interval $0 < x < 1$ 10

5. A taut string of length l has its ends $x = 0$ and $x = l$ fixed. The midpoint is taken to a small height h and released from rest at time $t = 0$. Find the displacement function $y(x, t)$. 10

6 A homogeneous rod of conducting material of length l has its ends kept at zero temperature . The temperature at the center is T and falls uniform to zero at the two ends. Find the temperature function $u(x, t)$. 10

7 A rectangular metal plate is bounded by the lines $x = 0, x = a$, $y = 0$, and $y = b$. The three sides $x = 0, x = a$, and $y = b$. are insulated and the side $y = 0$ is kept at

temperatute $u_0 \cos(\frac{\pi x}{a})$. Show that the temperature in the steady state is
$$u(x, y) = u_0 \operatorname{sech}(\frac{(b-a)\pi}{a}) \cosh(\frac{(b-y)\pi}{a}) \cos(\frac{\pi x}{a})$$ 10

8a) Eliminate the constants a, b and c, from the relation

[Turn over

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ 5

b) Find the differential equations from
$\varphi(x+y+z, x^2+y^2-z^2)=0$ 5

9. Solve the following equations by Lagrange's method 2X5=10

a) $\frac{y^2z}{x}p+xzq=y^2$

b) $(x^2-yz)p+(y^2-zx)q=(z^2-xy)$

10. Find the complete solution of the following 2X5=10

a) $(p^2+q^2)x=pz$

b) $p^2x+qy=z$

11. Solve the following 2X5=10

a) $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\cos mx\cos ny$

b) $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=30(2x+y)$

12. Solve by Monge's method
$r+(a+b)s+abt=xy$ 10