Output Primitives

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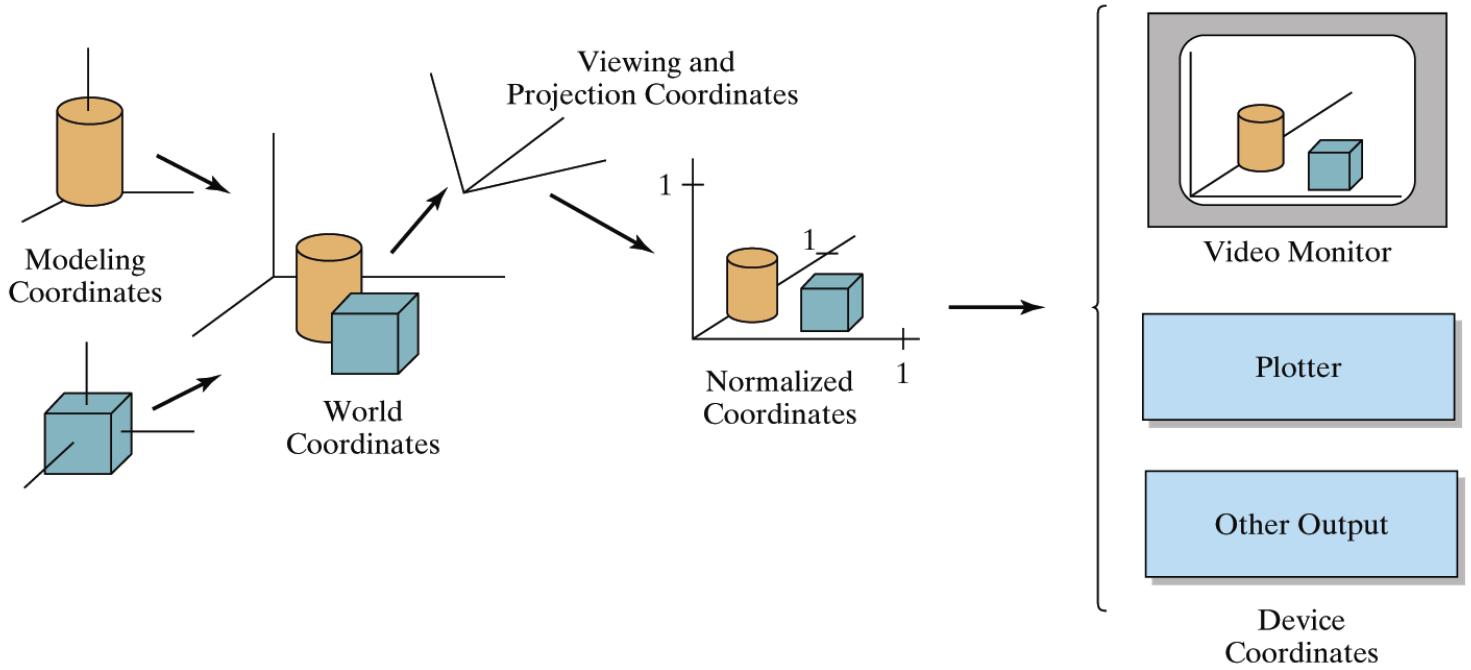
Contents (Part-I)

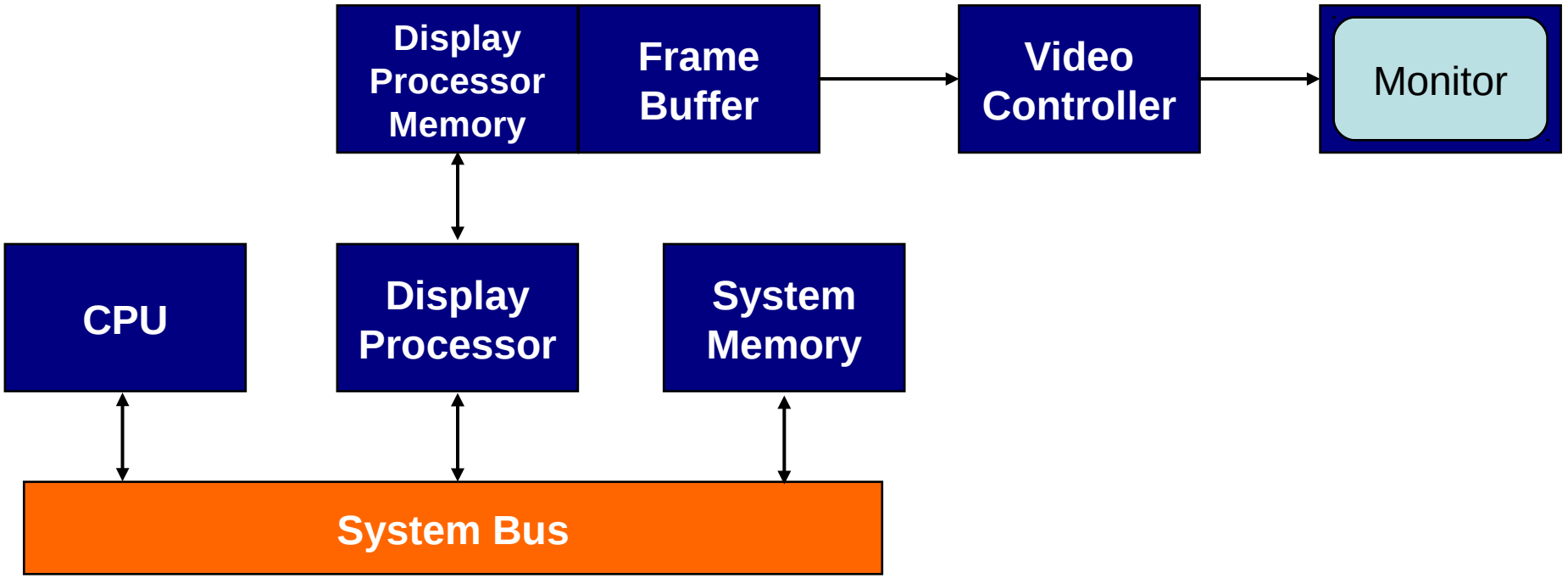
- Graphics hardware
- The problem of scan conversion
- Considerations
- Line equations
- Scan converting algorithms
  - A very simple solution
  - The DDA algorithm
- Conclusion

It's worth taking a little look at how graphics hardware works before we go any further

How do things end up on the screen?



Architecture Of A Graphics System



Output Devices

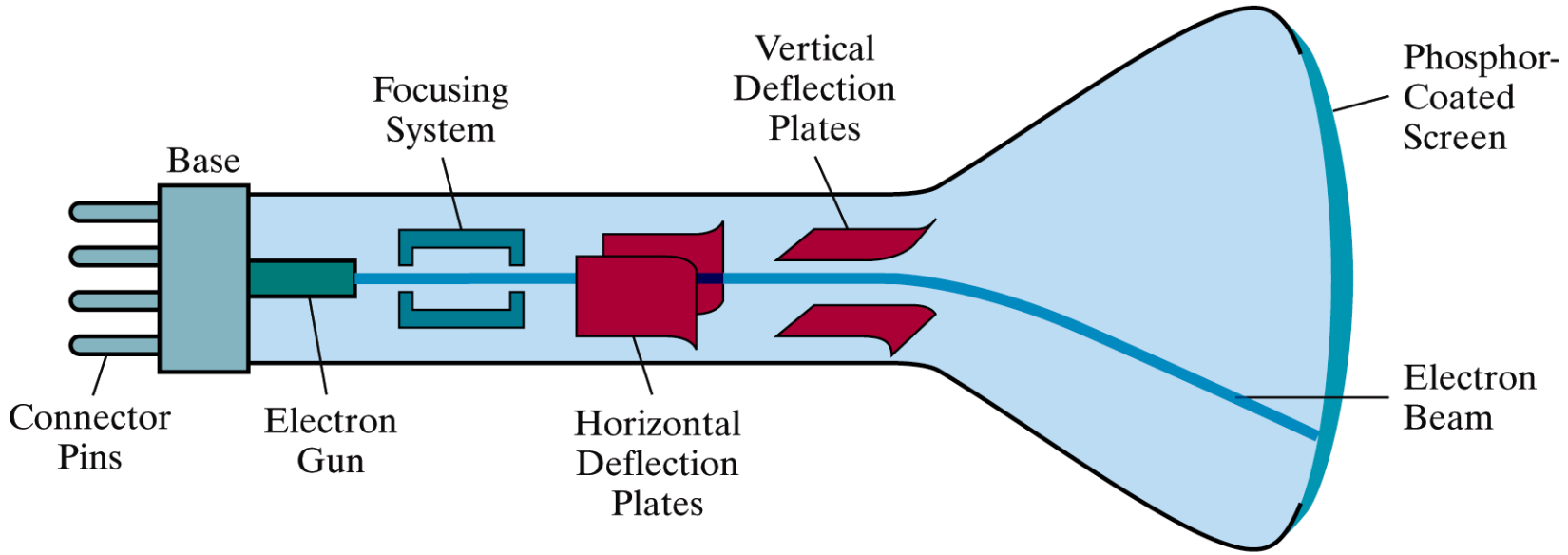
There are a range of output devices currently available:

- Printers/plotters
- Cathode ray tube displays
- Plasma displays
- LCD displays
- 3 dimensional viewers
- Virtual/augmented reality headsets

We will look briefly at some of the more common display devices

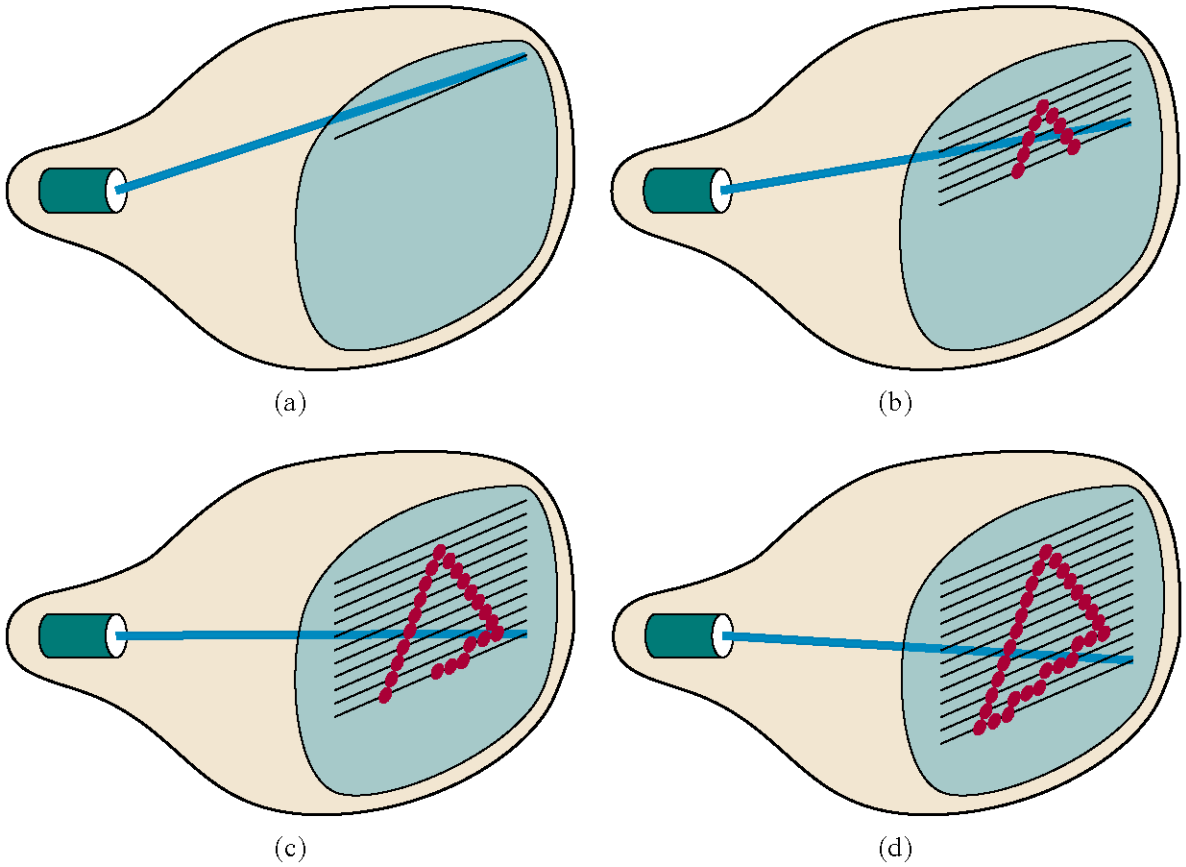
Basic Cathode Ray Tube (CRT)

Fire an electron beam at a phosphor coated screen



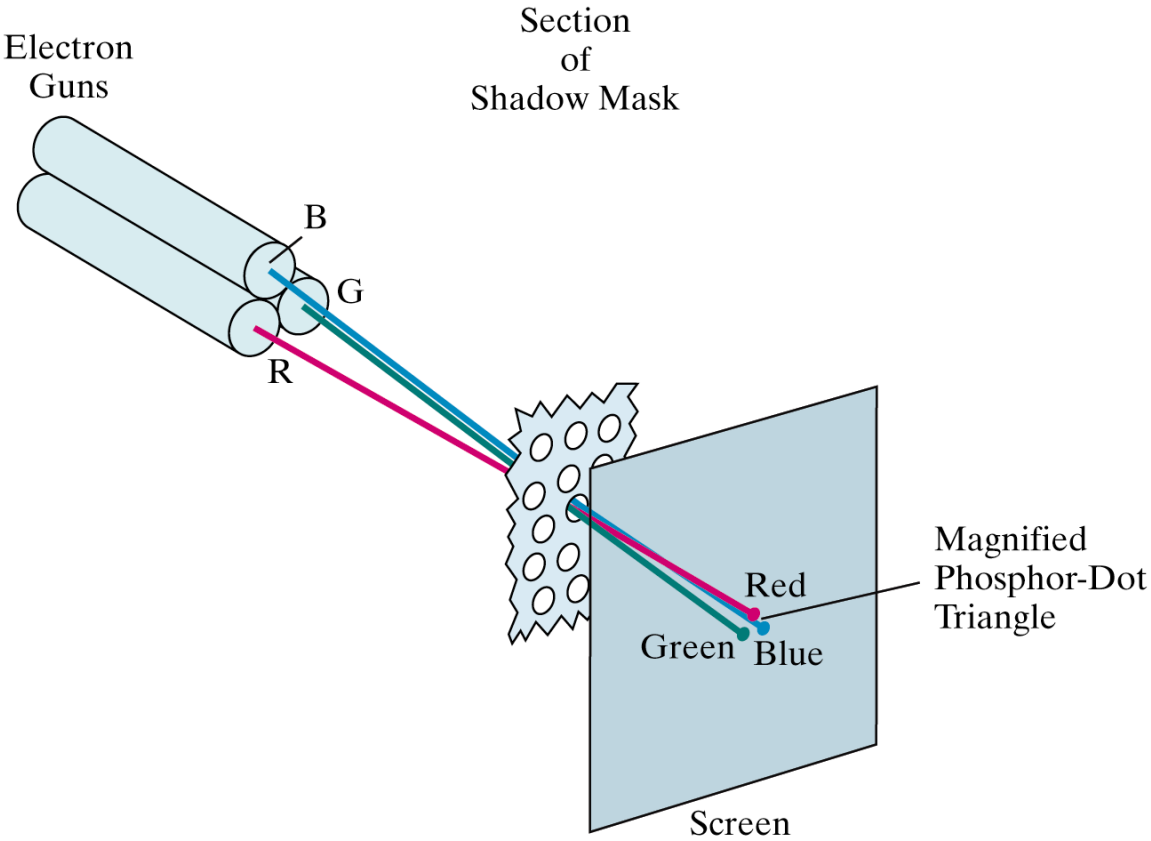
Images taken from Hearn & Baker, "Computer Graphics with OpenGL" (2004)

Draw one line at a time



Images taken from Hearn & Baker, "Computer Graphics with OpenGL" (2004)

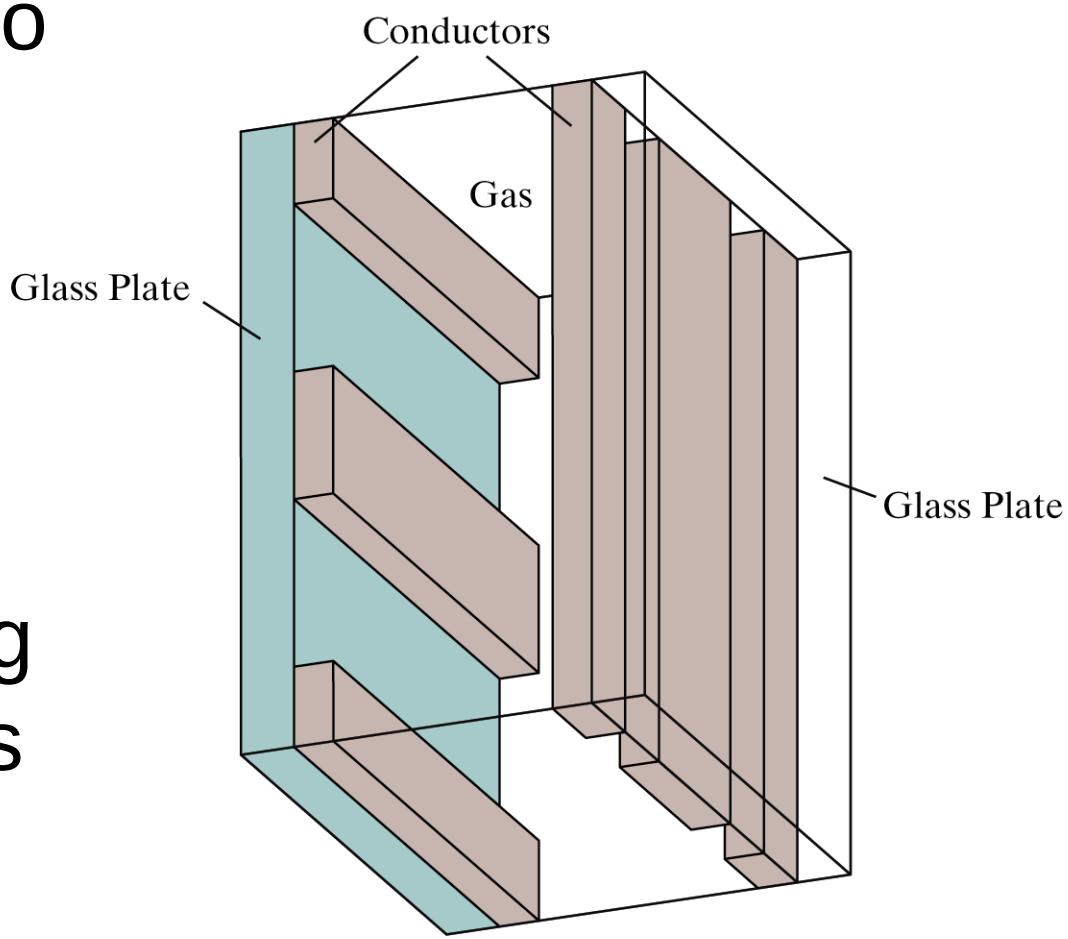
An electron gun for each colour – red, green and blue



Images taken from Hearn & Baker, "Computer Graphics with OpenGL" (2004)

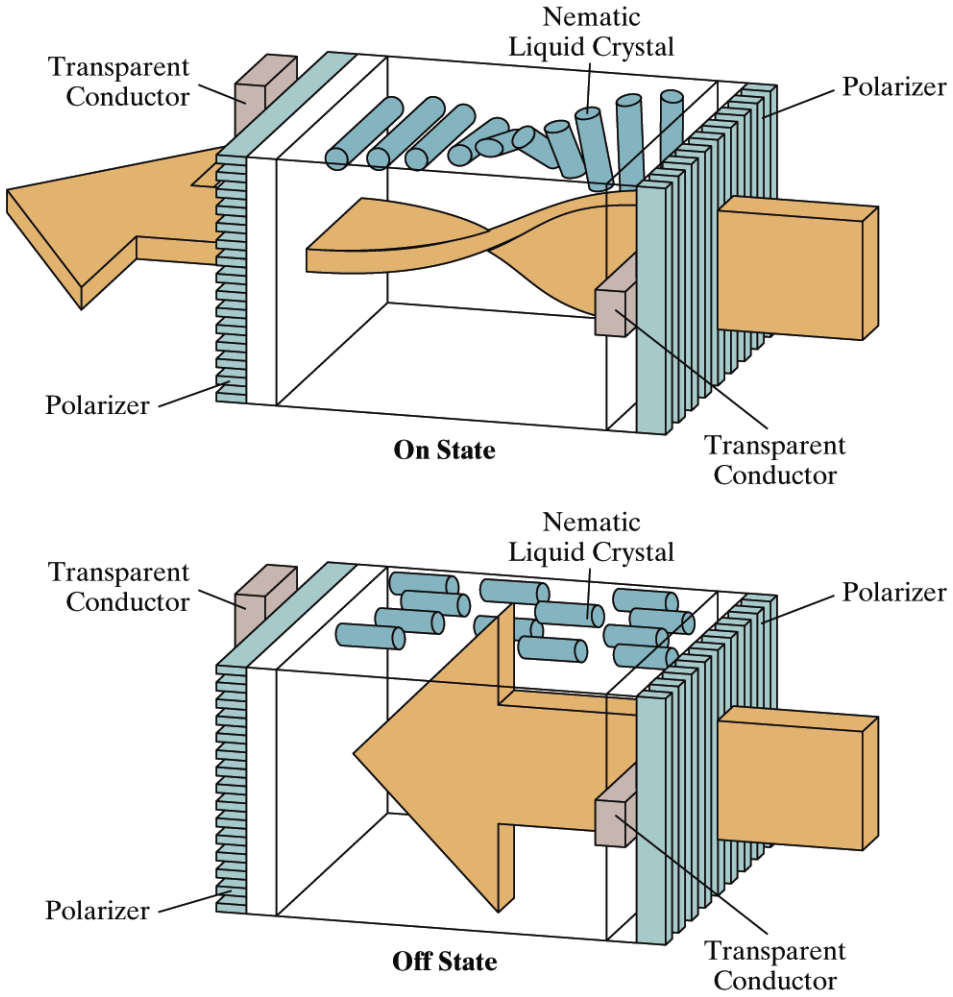
Plasma-Panel Displays

Applying voltages to crossing pairs of conductors causes the gas (usually a mixture including neon) to break down into a glowing plasma of electrons and ions



Images taken from Hearn & Baker, "Computer Graphics with OpenGL" (2004)

Liquid Crystal Displays



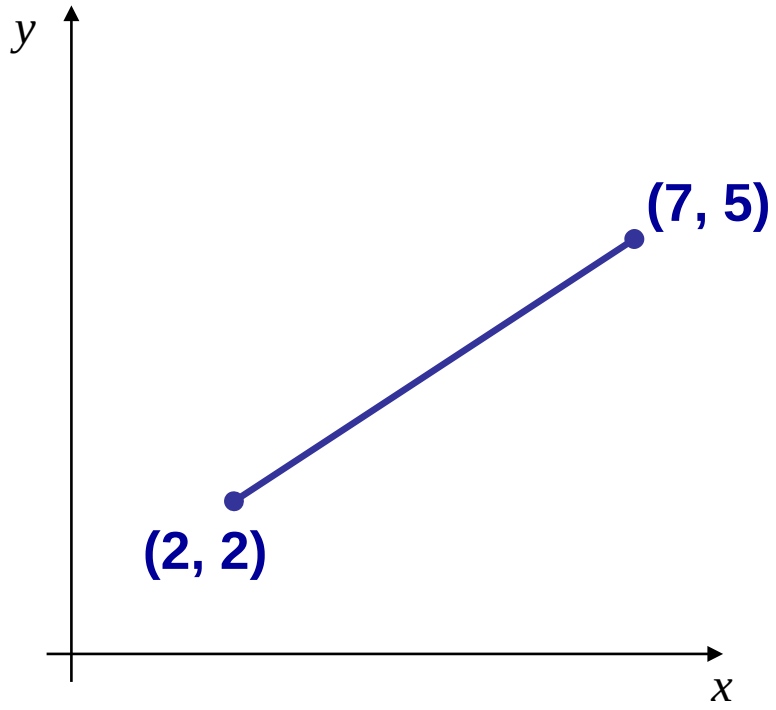
Light passing through the liquid crystal is twisted so it gets through the polarizer

A voltage is applied using the crisscrossing conductors to stop the twisting and turn pixels off

Images taken from Hearn & Baker, "Computer Graphics with OpenGL" (2004)

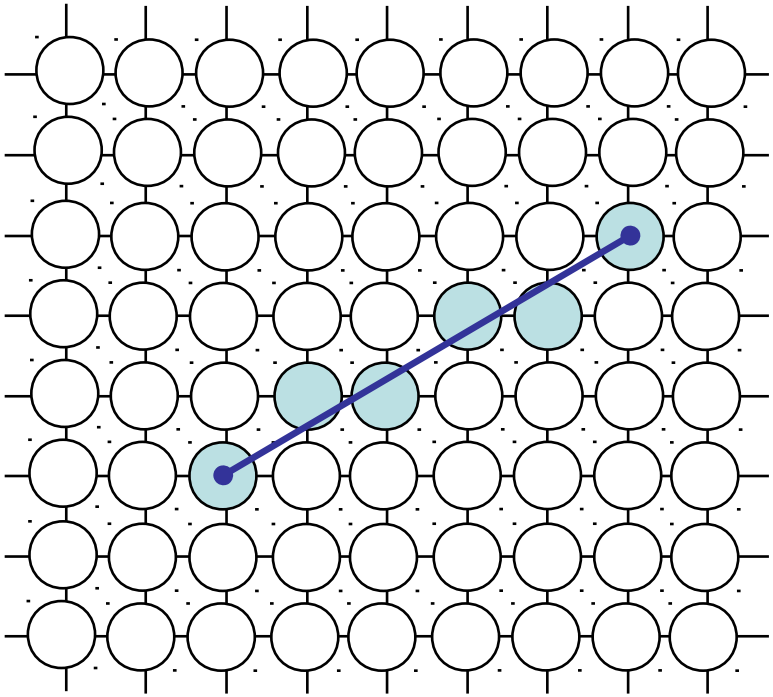
The Problem Of Scan Conversion

A line segment in a scene is defined by the coordinate positions of the line end-points



The Problem (cont…)

But what happens when we try to draw this on a pixel based display?



How do we choose which pixels to turn on?

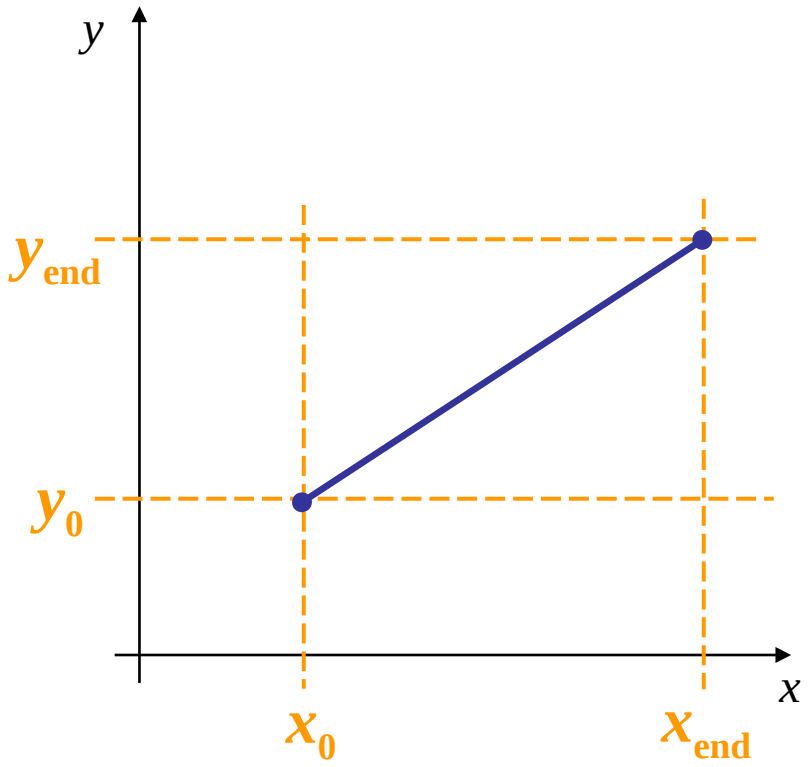
Considerations

Considerations to keep in mind:

- The line has to look good
  - Avoid *jaggies*
- It has to be lightening fast!
  - How many lines need to be drawn in a typical scene?
  - This is going to come back to bite us again and again

Line Equations

Let's quickly review the equations involved in drawing lines



Slope-intercept line equation:

$$y = m \cdot x + b$$

where:

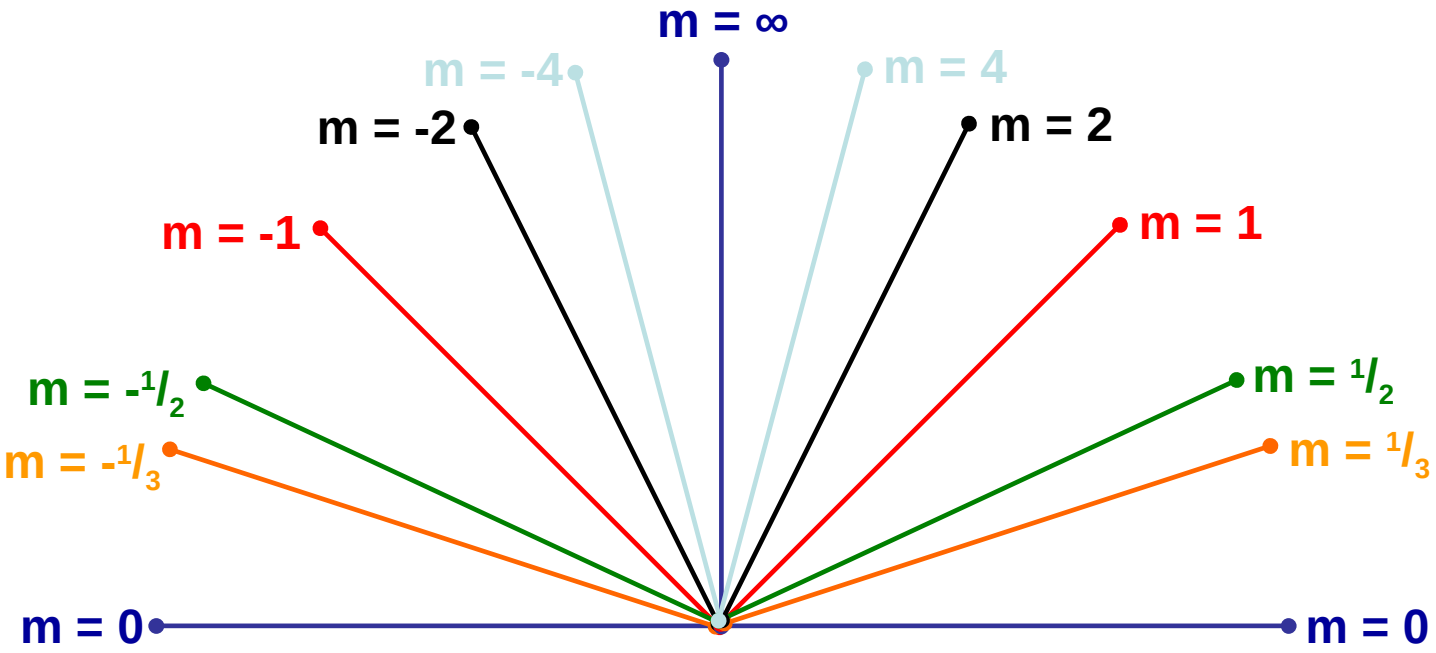
$$m = \frac{y_{end} - y_0}{x_{end} - x_0}$$

$$b = y_0 - m \cdot x_0$$

Lines & Slopes

The slope of a line (m) is defined by its start and end coordinates

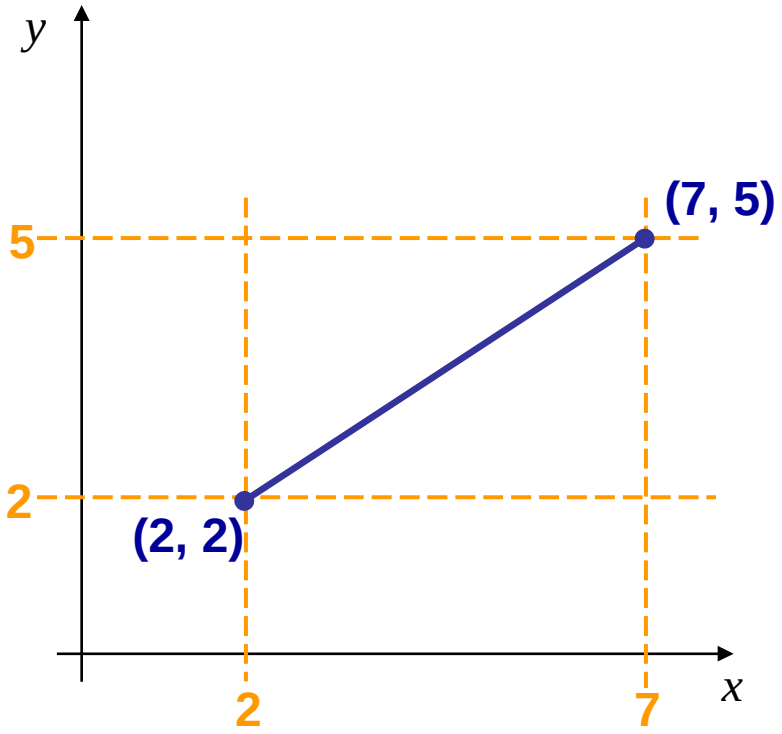
The diagram below shows some examples of lines and their slopes



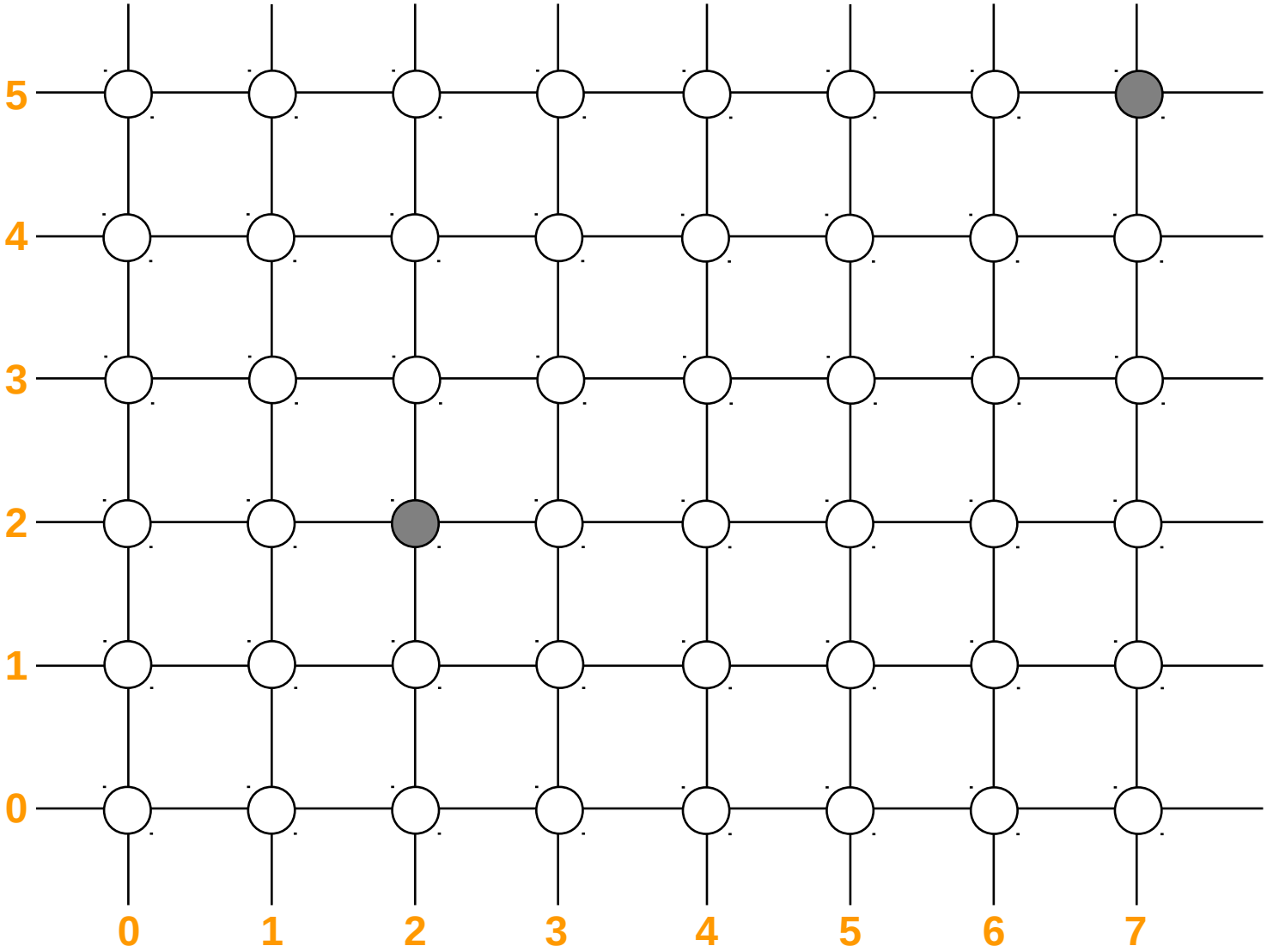
A Very Simple Solution

We could simply work out the corresponding y coordinate for each unit x coordinate

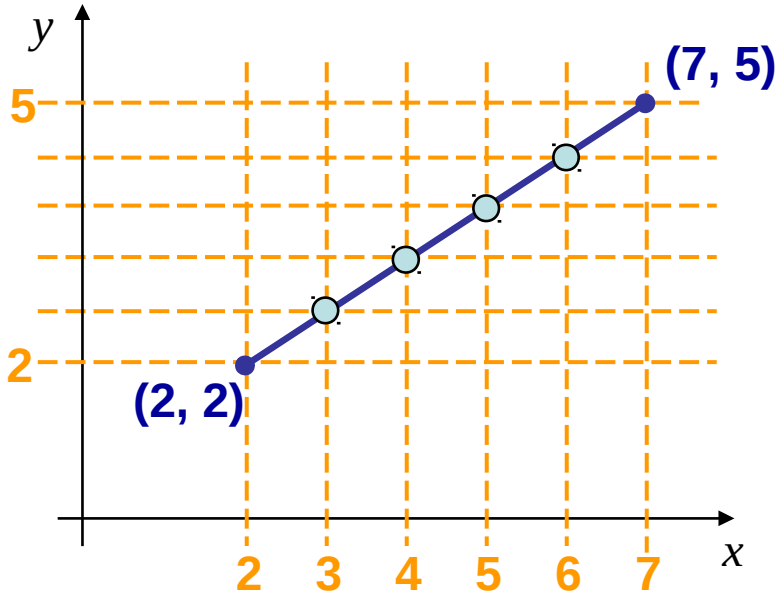
Let's consider the following example:



A Very Simple Solution (cont…)



A Very Simple Solution (cont…)



First work out m and b:

$$m = \frac{5 - 2}{7 - 2} = \frac{3}{5}$$

$$b = 2 - \frac{3}{5} \star 2 = \frac{4}{5}$$

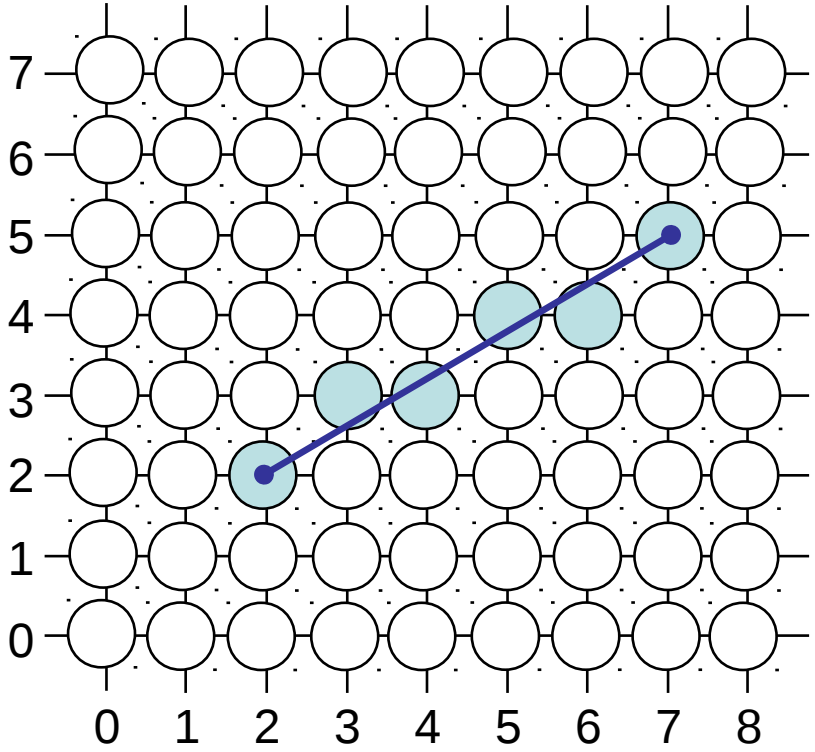
Now for each x value work out the y value:

$$y(3) = \frac{3}{5} \cdot 3 + \frac{4}{5} = 2\frac{3}{5} \qquad y(4) = \frac{3}{5} \cdot 4 + \frac{4}{5} = 3\frac{1}{5}$$

$$y(5) = \frac{3}{5} \cdot 5 + \frac{4}{5} = 3\frac{4}{5} \qquad y(6) = \frac{3}{5} \cdot 6 + \frac{4}{5} = 4\frac{2}{5}$$

A Very Simple Solution (cont…)

Now just round off the results and turn on these pixels to draw our line



$$y(3) = 2\frac{3}{5} \approx 3$$

$$y(4) = 3\frac{1}{5} \approx 3$$

$$y(5) = 3\frac{4}{5} \approx 4$$

$$y(6) = 4\frac{2}{5} \approx 4$$

Simple Algorithm

Start x at origin of line

Evaluate y value

Set nearest pixel (x,y)

Move x one pixel toward other endpoint

Repeat until done

$$y = m(x - x_0) + y_0$$

Implementation

```
// SimplePixelCalc - elementary pixel calculation using
// variation of y=mx+b
void SimplePixelCalc( int x0, int xn, int y0, int yn )
{
        for( int ix=x0; ix<=xn; ix++ )
        {
                float fy = y0 + float(yn-y0)/float(xn-x0)*ix;
                // round to nearest whole integer
                int iy = (int)(fy+0.5);
                SetPixel(ix, iy, 1); // write pixel value
        }
}
```

How well does this work?

Estimate the time taken to execute the task

A Very Simple Solution (cont…)

However, this approach is just way too slow

In particular look out for:

- The equation $y = mx + b$ requires the multiplication of m by x
- Rounding off the resulting y coordinates

We need a faster solution

A Quick Note About Slopes

In the previous example we chose to solve the parametric line equation to give us the y coordinate for each unit x coordinate

What if we had done it the other way around?

So this gives us: $x = \frac{y - b}{m}$

where: $m = \frac{y_{end} - y_0}{x_{end} - x_0}$ and $b = y_0 - m \cdot x_0$

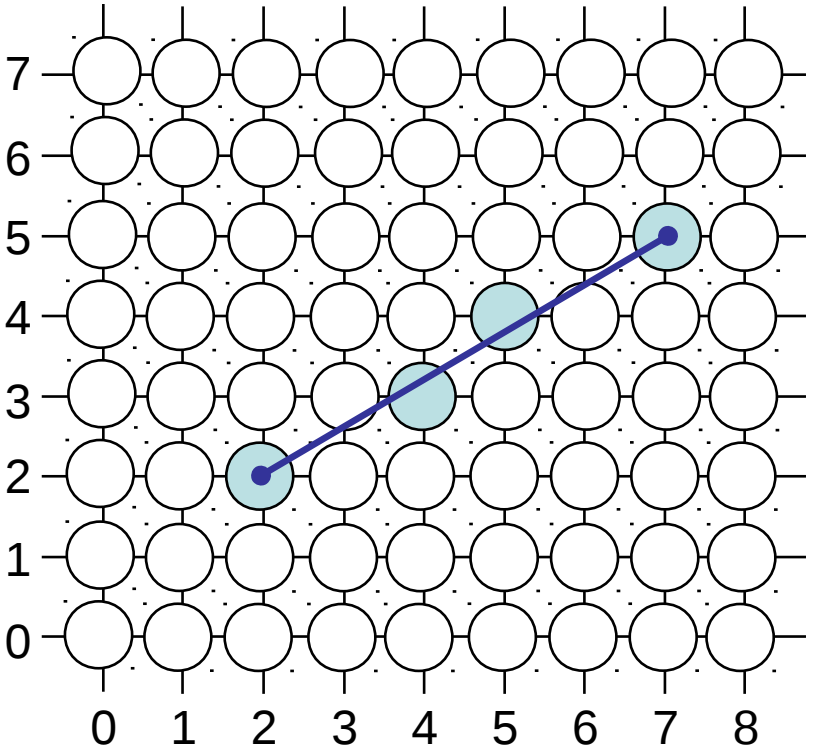
A Quick Note About Slopes (cont…)

Leaving out the details this gives us:

$$x(3) = 3\frac{2}{3} \approx 4 \qquad x(4) = 5\frac{1}{3} \approx 5$$

We can see easily that this line doesn't look very good!

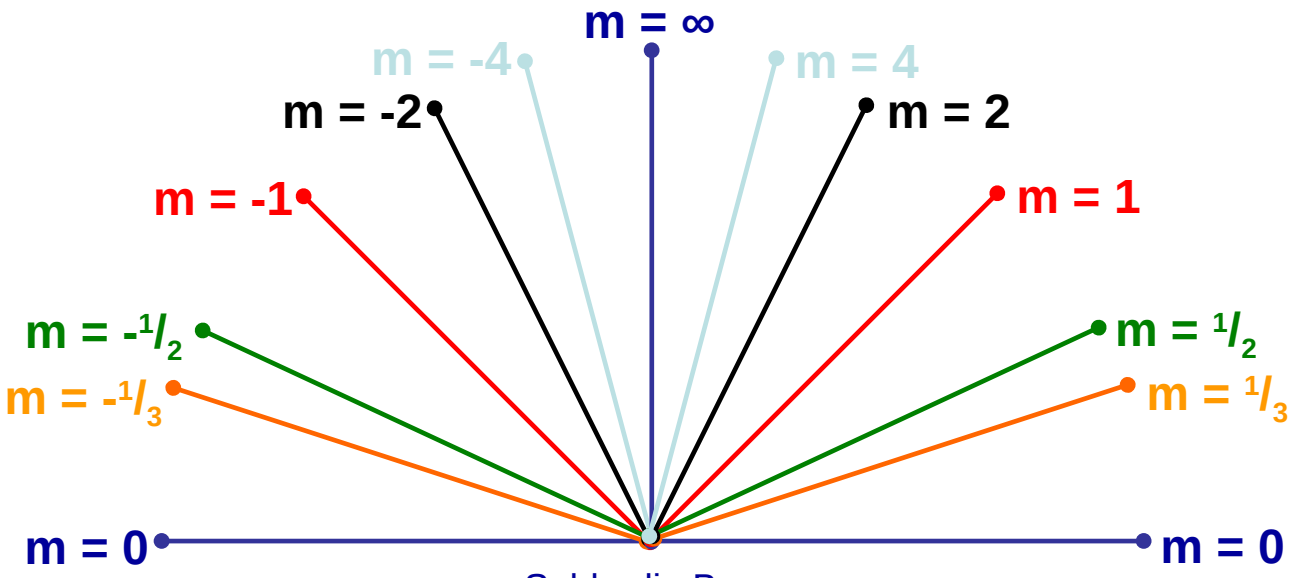
We choose which way to work out the line pixels based on the slope of the line



A Quick Note About Slopes (cont…)

If the slope of a line is between -1 and 1 then we work out the y coordinates for a line based on it's unit x coordinates

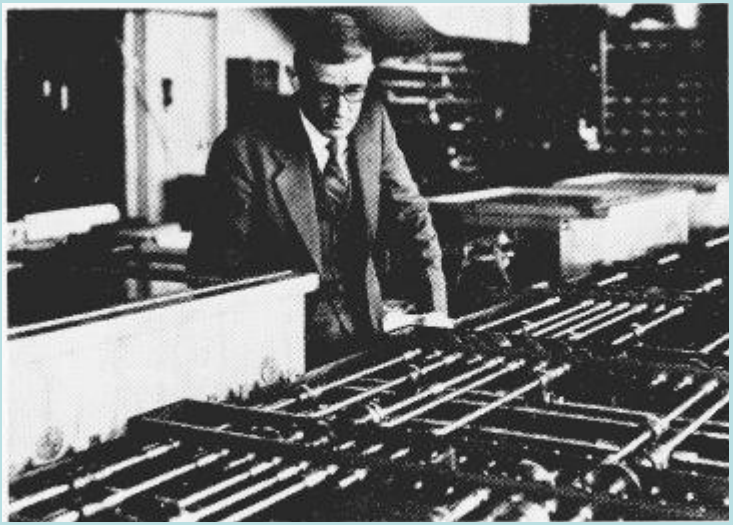
Otherwise we do the opposite – x coordinates are computed based on unit y coordinates



The DDA Algorithm

The *digital differential analyzer* (DDA) algorithm takes an incremental approach in order to speed up scan conversion

Simply calculate y_{k+1} based on y_k



The original differential analyzer was a physical machine developed by Vannevar Bush at MIT in the 1930's in order to solve ordinary differential equations.

More information here.

Digital Differential Analyzer (DDA) Algorithm

Step through either x or y based on slope

Build the line parametrically

- If |m|<1
  - $x_{k+1} = x_k + 1$
  - $y_{k+1} = y_k + m$
- If |m|>1
  - $x_{k+1} = x_k + 1/m$
  - $y_{k+1} = y_k + 1$
- If |m|=1
  - $x_{k+1} = x_k + 1$
  - $y_{k+1} = y_k + 1$

The DDA Algorithm (cont…)

Consider the list of points that we determined for the line in our previous example:

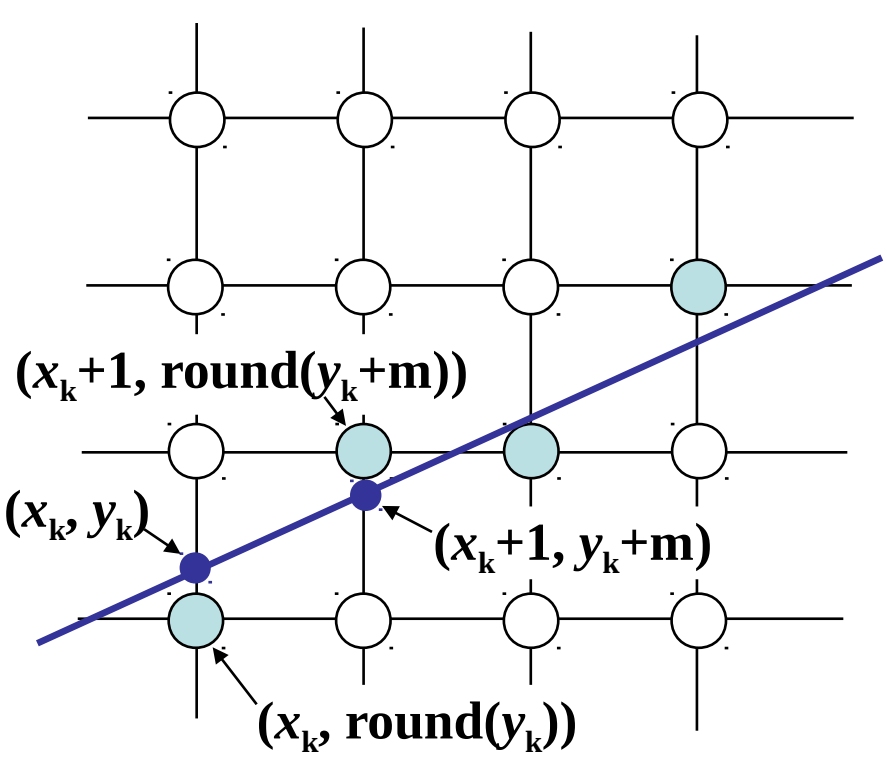
$(2, 2), (3, 2^3/_5), (4, 3^1/_5), (5, 3^4/_5), (6, 4^2/_5), (7, 5)$

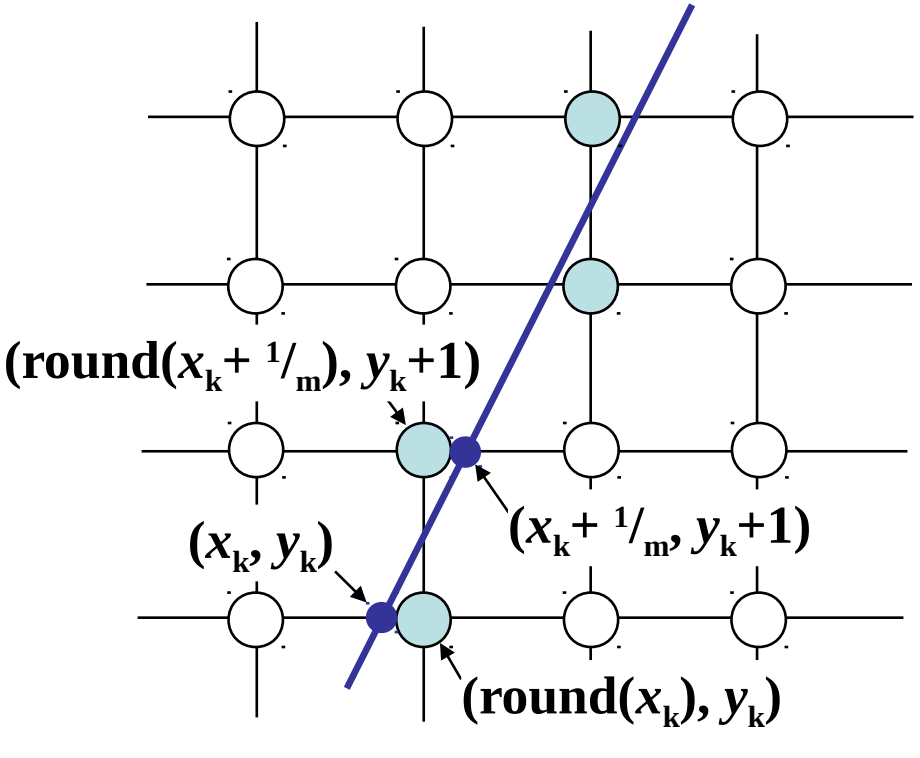
Notice that as the x coordinates go up by one, the y coordinates simply go up by the slope of the line

This is the key insight in the DDA algorithm

The DDA Algorithm (cont…)

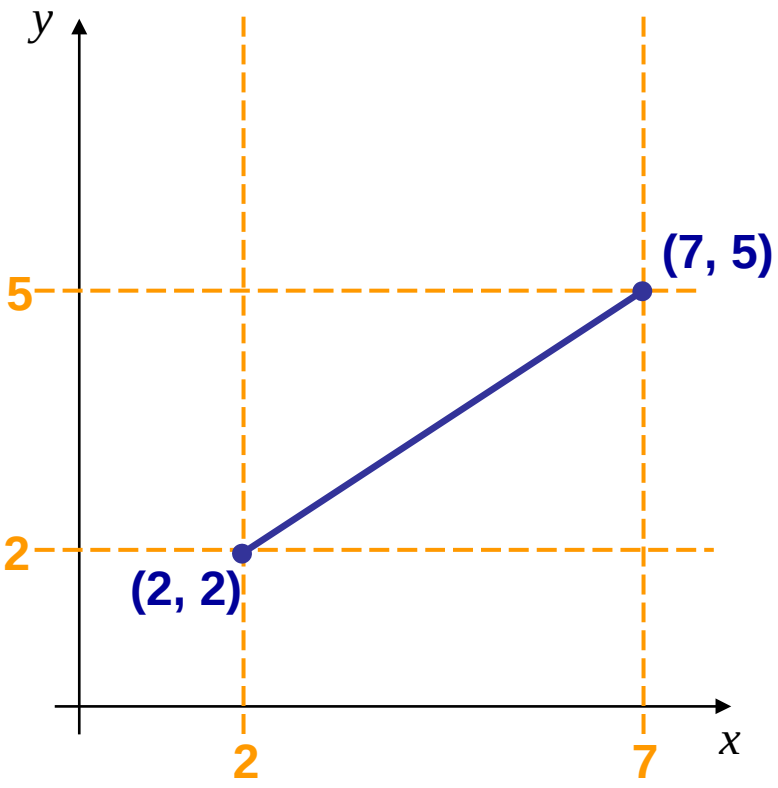
Again the values calculated by the equations used by the DDA algorithm must be rounded to match pixel values

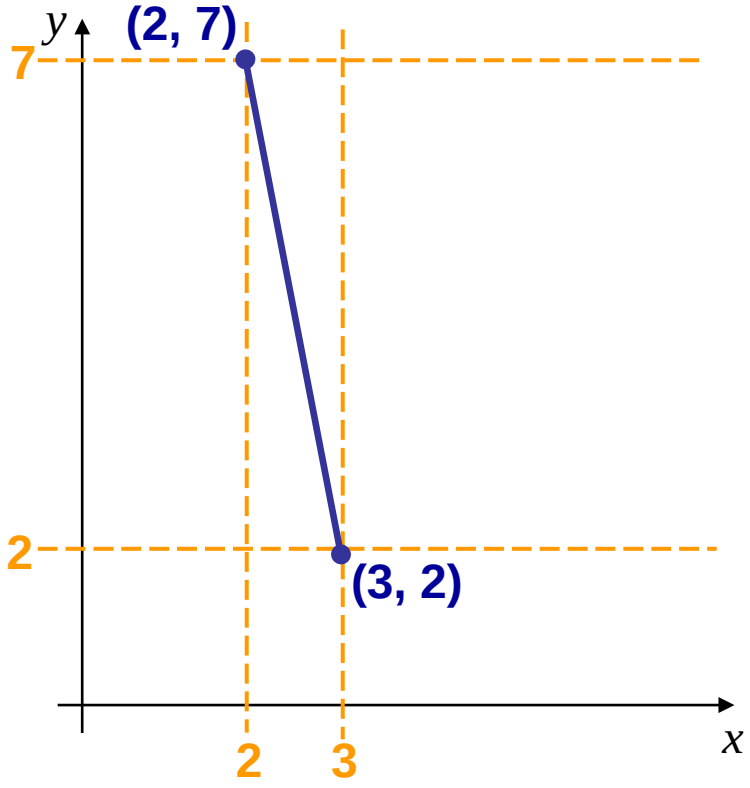




DDA Algorithm Example

Let's try out the following examples:





DDA Implementation

```
void dda( int x0, int xn, int y0, int yn )
{
          float dx = float(xn - x0); // total span in x
          float dy = float(yn - y0); // total span in y
          float y = float(y0);
          float x = float(x0);

          float Dx, Dy; // incremental steps in x & y
          // determine if slope m GT or LT 1
          if( dx > dy )
          {
             Dx = 1;
             Dy = dy/dx;// floating division, but only done once per line
          }
          else
          {
             Dx = dx/dy;
             Dy = 1;
          }
          int ix, iy; // pixel coords
          for( int k=0; k<=((dx>dy)? dx:dy); k++ )
          {
                    ix = int(x + 0.5); // round to nearest pixel coordinate
                    iy = int(y + 0.5);
                    x += Dx;  // floating point calculations
                    y += Dy;
          }
}
```

Estimate the time taken to execute the task

The DDA Algorithm Summary

The DDA algorithm is much faster than our previous attempt

- In particular, there are no longer any multiplications involved

However, there are still two big issues:

- Accumulation of round-off errors can make the pixelated line drift away from what was intended
- The rounding operations and floating point arithmetic involved are time consuming

Conclusion

- In this lecture we took a very brief look at how graphics hardware works
- Drawing lines to pixel based displays is time consuming so we need good ways to do it
- The DDA algorithm is pretty good – but we can do better
- Next time we'll like at the Bresenham line algorithm and how to draw circles, fill polygons and anti-aliasing

Line Drawing Algorithms

Part - 2

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Contents (Part-II)

In today's lecture we'll have a look at:

- Bresenham's line drawing algorithm
- Line drawing algorithm comparisons
- Circle drawing algorithms
  - A simple technique
  - The mid-point circle algorithm
- Polygon fill algorithms
- Summary of raster drawing algorithms

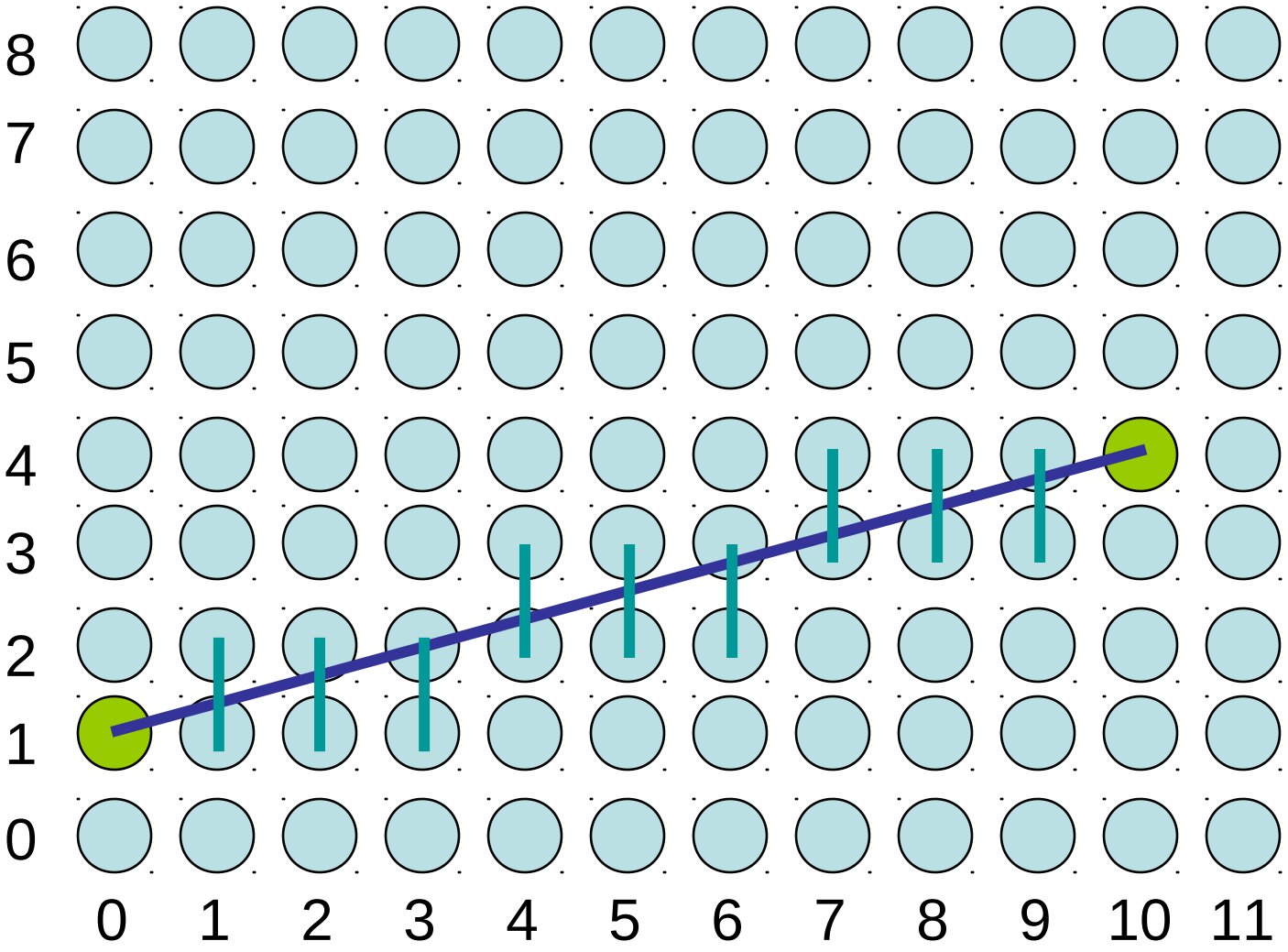
The Bresenham Line Algorithm

The Bresenham algorithm is another incremental scan conversion algorithm

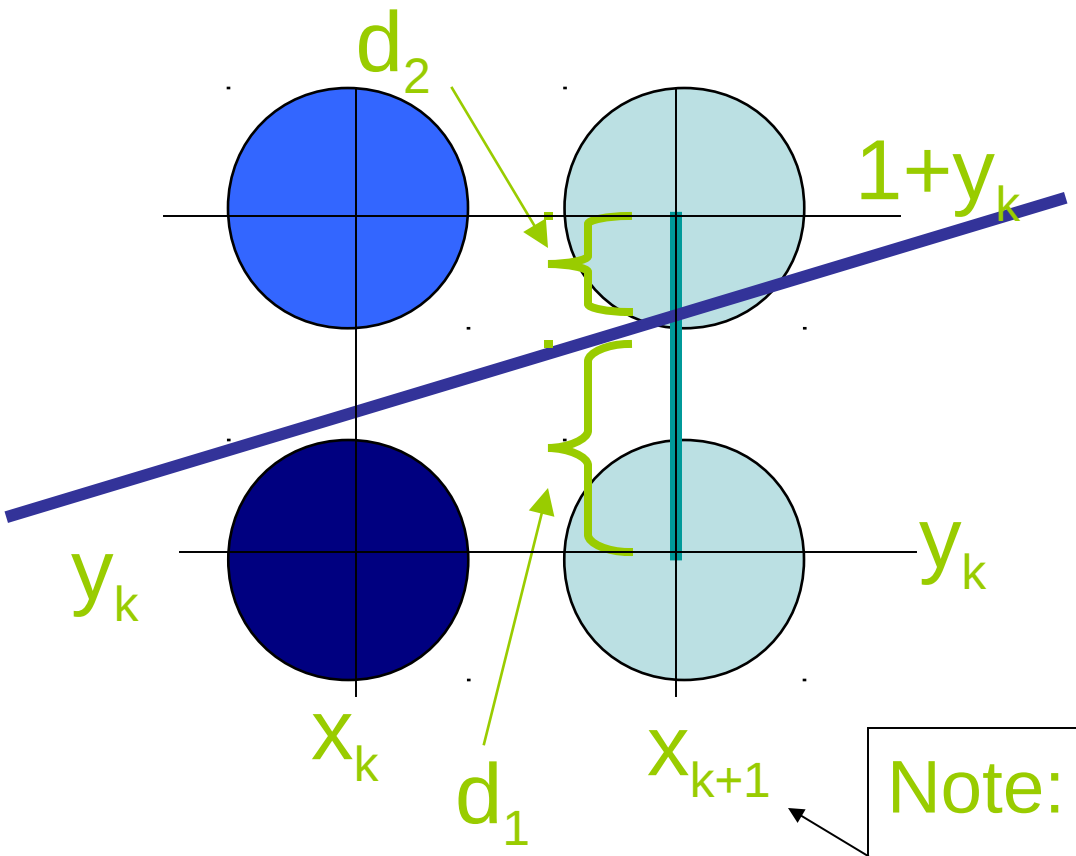
The big advantage of this algorithm is that it uses only integer calculations

Jack Bresenham worked for 27 years at IBM before entering academia. Bresenham developed his famous algorithms at IBM in the early 1960s

Analyzing errors at each point



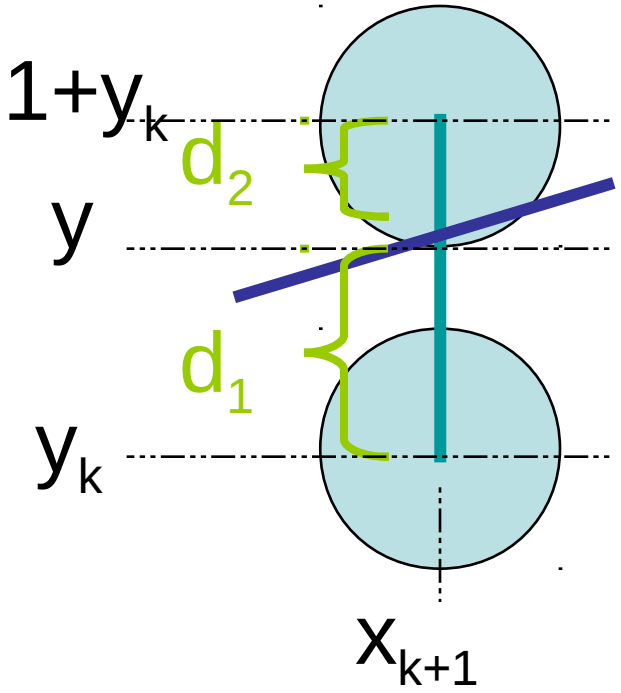
Error Analysis – for m<1



For |m|<1, there is only 1 pixel lit in each column

Note: when |m|<1, $x_{k+1} = 1+x_k$

Computation of error values



$$y = m(1 + x_k) + b$$

$$d_1 = y - y_k$$

Now substitute for y

$$= m(1 + x_k) + b - y_k$$

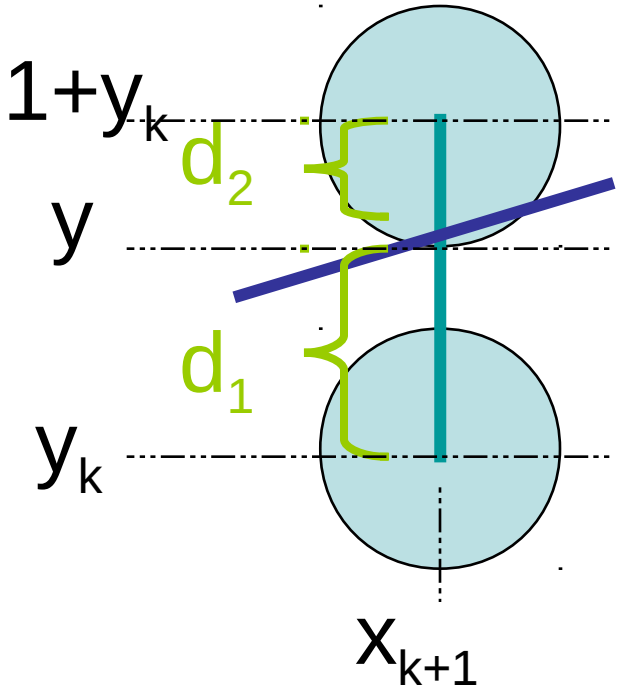
$$d_2 = (1 + y_k) - y$$

$$= y_k + 1 - m(1 + x_k) - b$$

Note: when $|m|<1$,

$x_{k+1} = 1+x_k$

Deciding on the Next Point



$$y = m(1 + x_k) + b$$

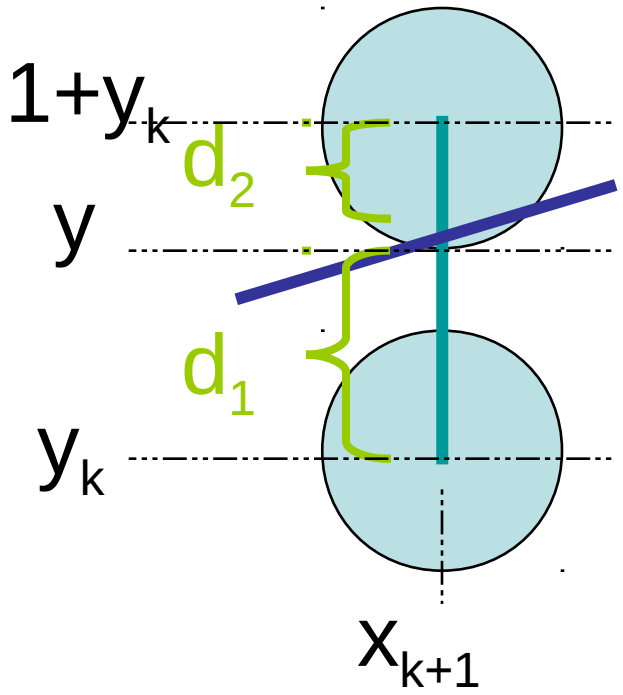
$$d_1 = m(1 + x_k) + b - y_k$$

$$d_2 = y_k + 1 - m(1 + x_k) - b$$

Decision value

$$d_1 - d_2 = 2m(1 + x_k) - 2y_k + 2b - 1$$

Decision point



If $d_2 > d_1$ (ie $d_1 - d_2 < 0$),

$y_{k+1} = y_k$

If $d_2 < d_1$ (ie $d_1 - d_2 > 0$),

$y_{k+1} = 1 + y_k$

Decision value

$$d_1 - d_2 = 2m(1 + x_k) - 2y_k + 2b - 1$$

Status Check

$$d_1 - d_2 = 2m(1 + x_k) - 2y_k + 2b - 1$$

What have we accomplished?

- One value to test (+/–)

But …

- Still have floating-point calculation
  - “m” and “b” are non-integer

Removing Non-Integral Values

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2b - 1$$

What is m?

$$d_1 - d_2 = 2\frac{\Delta y}{\Delta x}(x_k + 1) - 2y_k + 2b - 1$$

Now multiply throughout by Δx

$$\Delta x(d_1 - d_2) = 2\Delta y \cdot x_k + 2\Delta y - 2\Delta x \cdot y_k + \Delta x(2b - 1)$$

Which terms are constants?

Decision value → $p_k = 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$

c is constant, but non-integer

Recurrence Relation for p_k

$$p_k = 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$$

$$p_{k+1} = 2\Delta y \cdot x_{k+1} - 2\Delta x \cdot y_{k+1} + c$$

$$p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

$x_{k+1} - x_k = +1$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

$y_{k+1} - y_k = 0 \text{ or } +1$

Initializing the Algorithm

Initial value of decision parameter

$$p_0 = 2\Delta y \cdot x_0 - 2\Delta x \cdot y_0 + c$$

$$p_0 = 2\Delta y \cdot x_0 - 2\Delta x \cdot y_0 + 2\Delta y + \Delta x(2b - 1)$$

$$b = y_0 - (\Delta y / \Delta x) x_0$$

$$p_0 = 2\Delta y - \Delta x$$

Bresenham Algorithm Summary

Calculate once: $\Delta x, \Delta y, 2\Delta y, 2\Delta y - 2\Delta x, p_0$

At each point:

If $p_k < 0$:

$$\text{Plot}(x_k + 1, y_k)$$

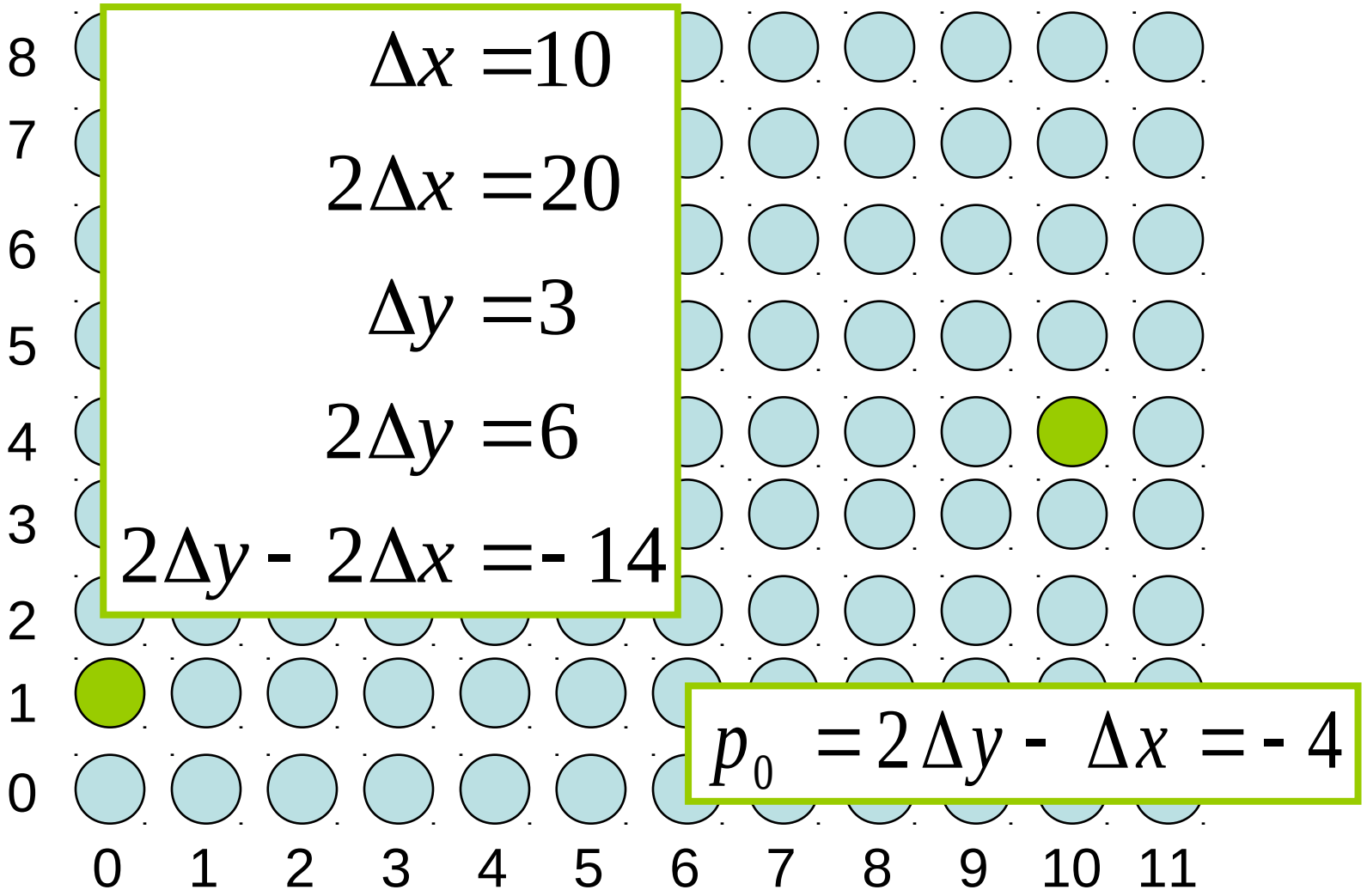
$$p_{k+1} = p_k + 2\Delta y$$

If $p_k >= 0$:

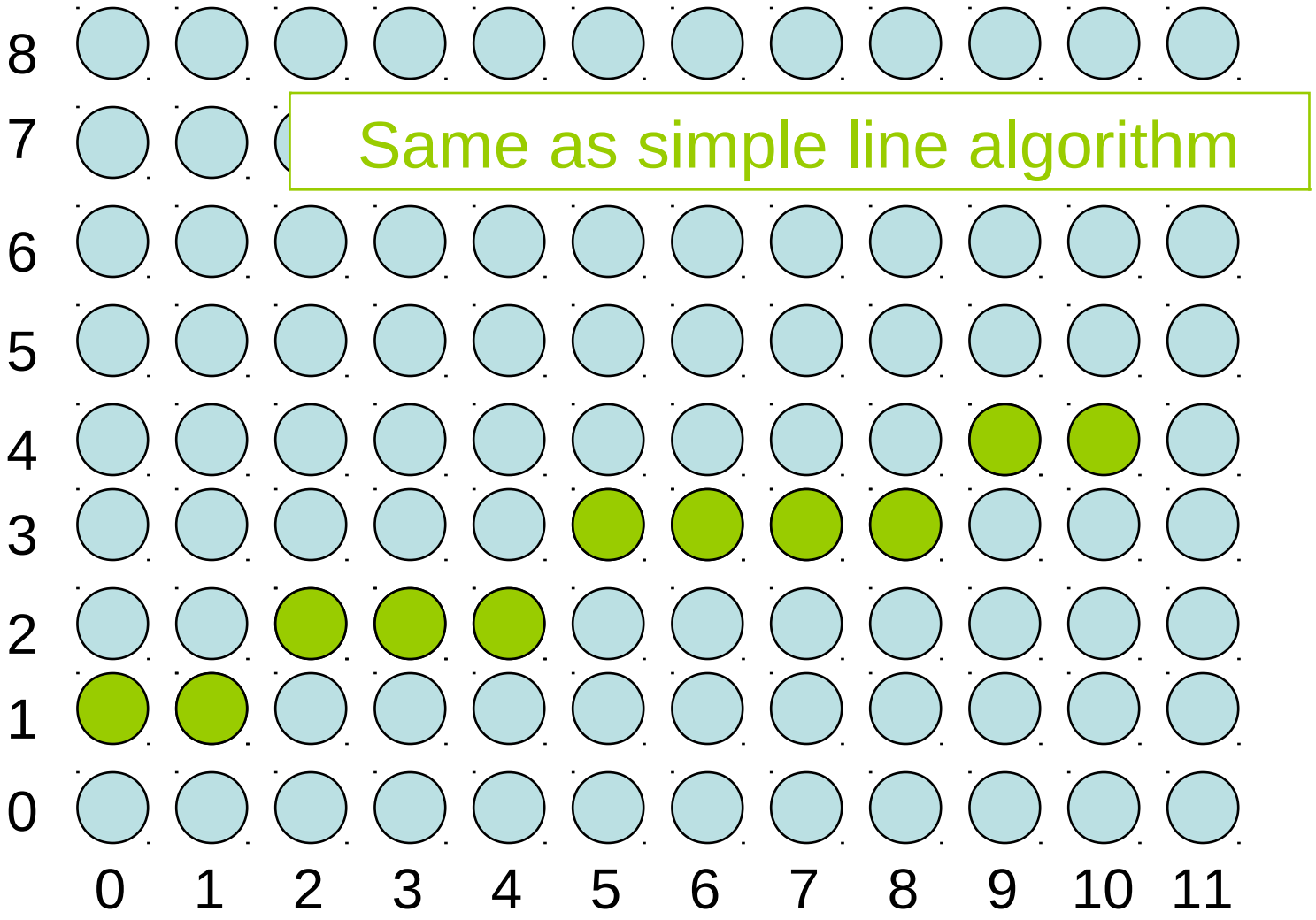
$$\text{Plot}(x_k + 1, y_k + 1)$$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

Example 1 Parameters



Example 1 Path



The Bresenham Line Algorithm

BRESENHAM'S LINE DRAWING ALGORITHM
(for $|m| < 1.0$)

1. Input the two line end-points, storing the left end-point in (x_0, y_0)
2. Plot the point (x_0, y_0)
3. Calculate the constants Δx, Δy, $2\Delta y$, and $(2\Delta y - 2\Delta x)$ and get the first value for the decision parameter as:

$$p_0 = 2\Delta y - \Delta x$$

4. At each x_k along the line, starting at $k = 0$, perform the following test. If $p_k < 0$, the next point to plot is (x_k+1, y_k) and:

$$p_{k+1} = p_k + 2\Delta y$$

The Bresenham Line Algorithm (cont…)

Otherwise, the next point to plot is (x_k+1, y_k+1) and:

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

5. Repeat step 4 (Δx – 1) times

NOTE! The algorithm and derivation above assumes slopes are less than 1. for other slopes we need to adjust the algorithm accordingly

Estimate the time taken to execute the task

Bresenham Modifications

So far, $0 < m < 1$, $\Delta x > 0$

How do we handle other cases?

- Horizontal
- Vertical
- Other octants
  - abs(Slope) $\in (1, \infty]$
    - Transpose x with y, Δx with Δy, etc.
    - Plot(y,x)
  - Slope < 0
    - Revise: $y_{k+1} - y_k = 0$ or -1

Bresenham Line Algorithm Summary

The Bresenham line algorithm has the following advantages:

- A fast incremental algorithm
- Uses only integer calculations

Comparing this to the DDA algorithm, DDA has the following problems:

- Accumulation of round-off errors can make the pixelated line drift away from what was intended
- The rounding operations and floating point arithmetic involved are time consuming

Drawing Circles and Arcs

Similar to line drawing

- In that you have to determine what pixels to activate

But non-linear

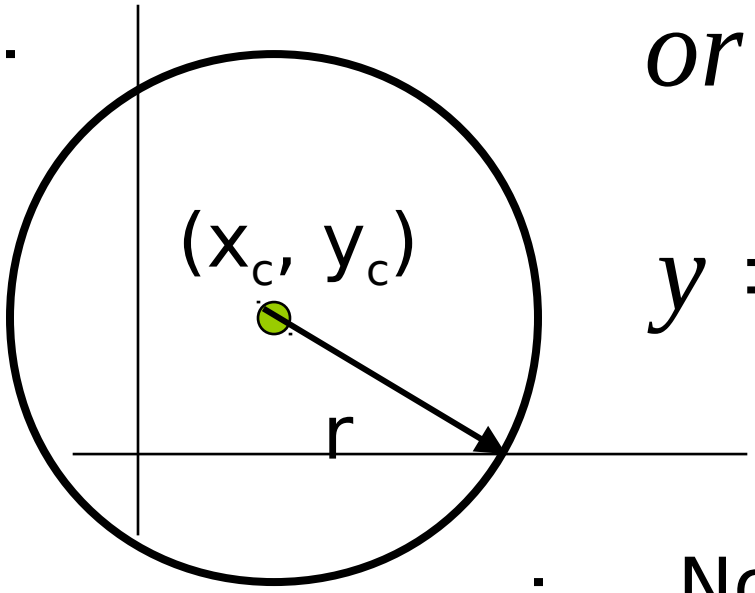
- Simple equation implementation
- Optimized

Cartesian form of Circle Equations

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

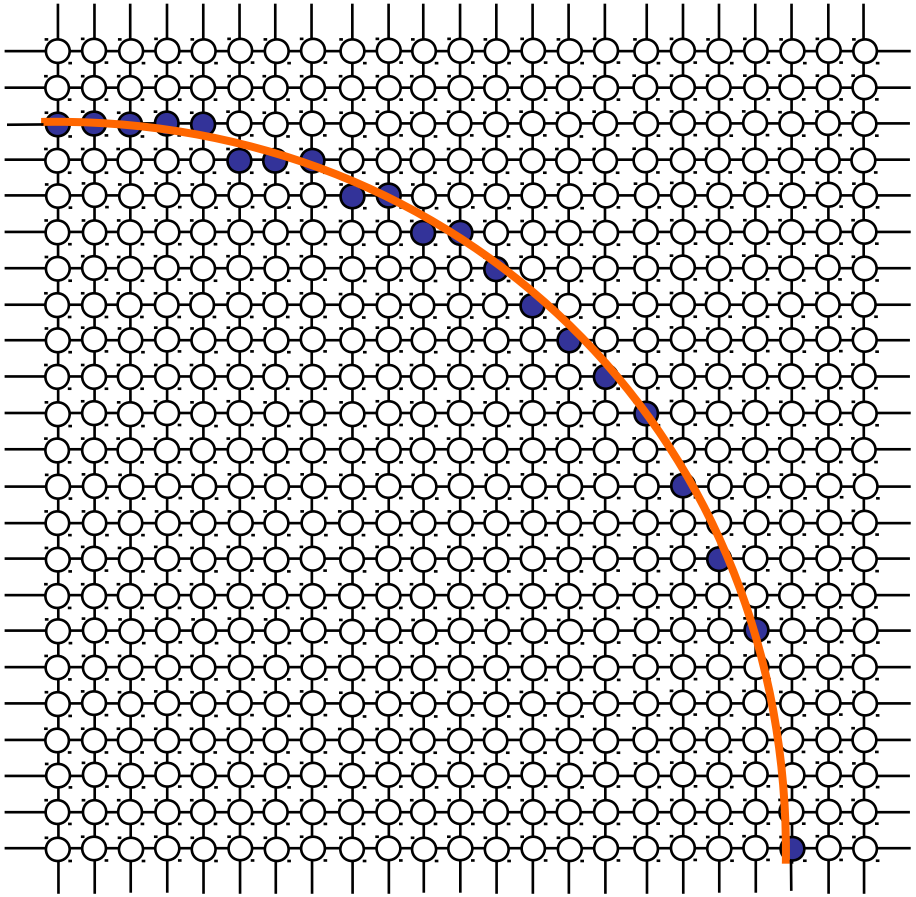
or

$$y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$$



- Not a very good method. Why?

A Simple Circle Drawing Algorithm (cont…)



$$y_0 = \sqrt{20^2 - 0^2} \approx 20$$

$$y_1 = \sqrt{20^2 - 1^2} \approx 20$$

$$y_2 = \sqrt{20^2 - 2^2} \approx 20$$

$$\vdots$$

$$y_{19} = \sqrt{20^2 - 19^2} \approx 6$$

$$y_{20} = \sqrt{20^2 - 20^2} \approx 0$$

A Simple Circle Drawing Algorithm (cont…)

However, unsurprisingly this is not a brilliant solution!

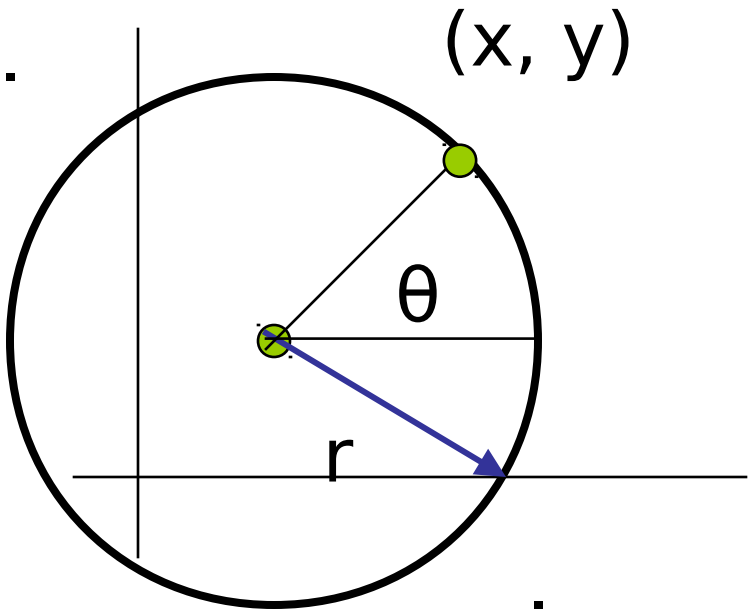
Firstly, the resulting circle has large gaps where the slope approaches the vertical

Secondly, the calculations are not very efficient

- The square (multiply) operations
- The square root operation – try really hard to avoid these!

We need a more efficient, more accurate solution

Polar Coordinate Form



$$x = x_c + r\cos\theta$$

$$y = y_c + r\sin\theta$$

$$\Delta s = r\theta$$

$$\Delta s \approx 1$$

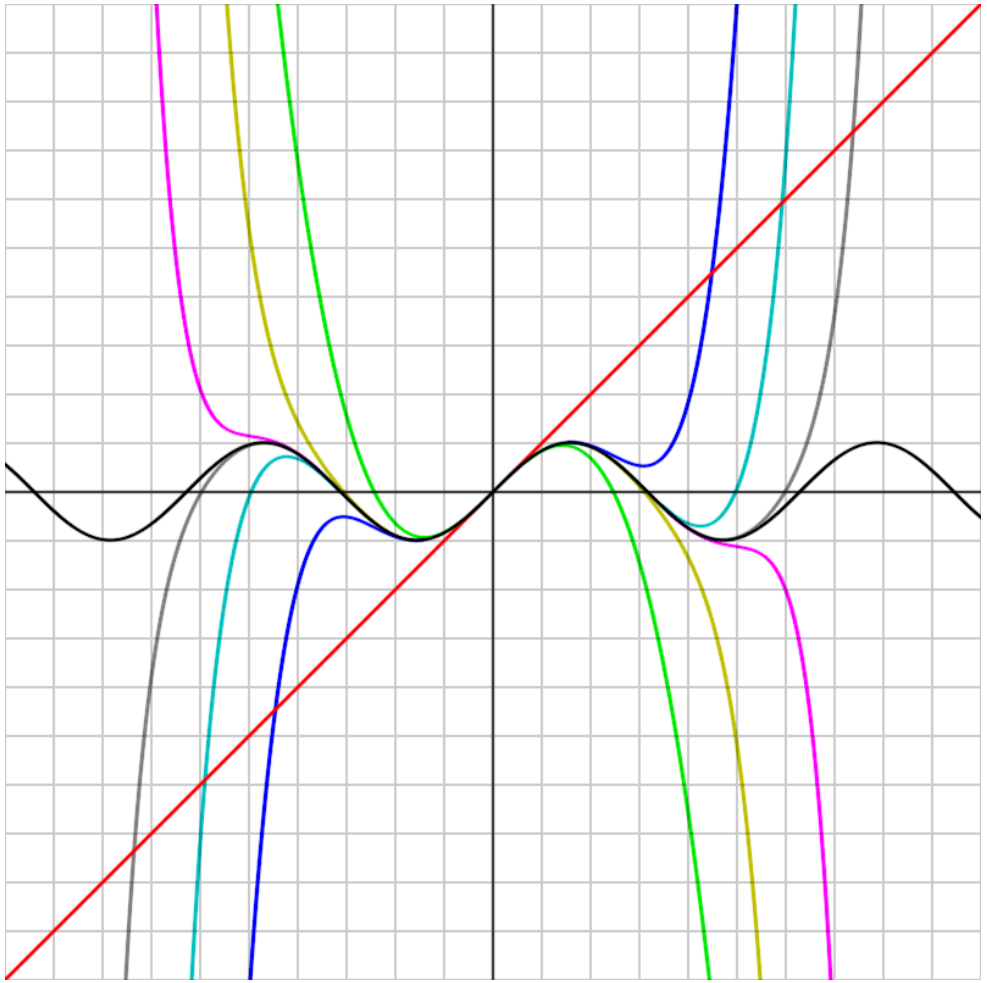
$$\Delta\theta \approx \frac{1}{r}$$

Simple method: plot directly from parametric equations

Trig functions are expensive

$$\sin\theta = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\theta^{2n+1}$$

$$\cos\theta = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\theta^{2n}$$

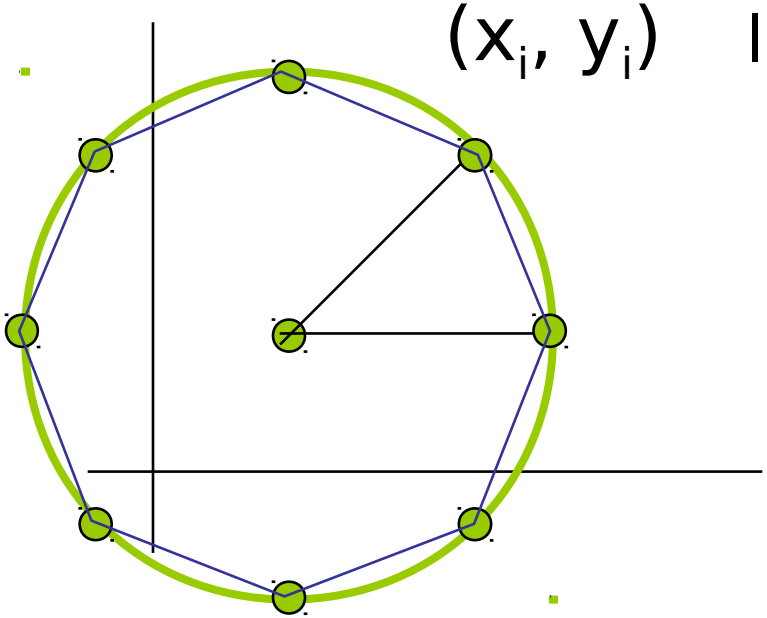


Plots of first 1, 3, 5, 7, 9, 11, and 13 terms in the Taylor's series for sin

Polygon Approximation

Calculate polygon vertices from polar equation; connect with line algorithm

(x_i, y_i)



Mid-Point Circle Algorithm

Similarly to the case with lines, there is an incremental algorithm for drawing circles – the *mid-point circle algorithm*

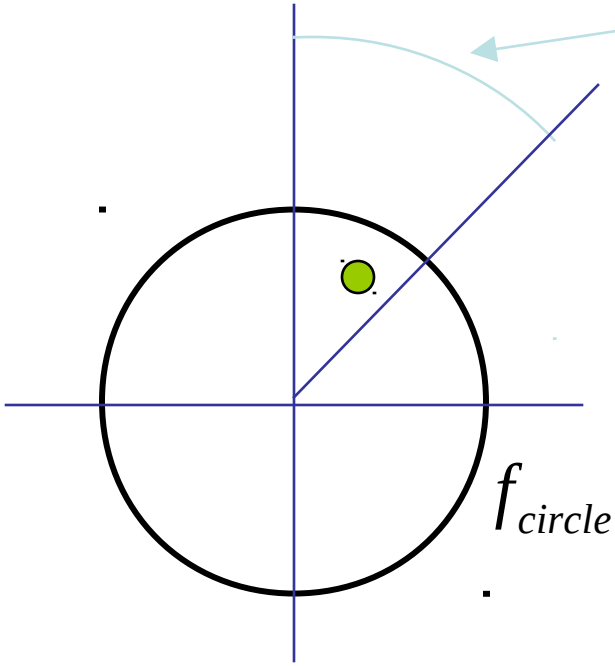
In the mid-point circle algorithm we use eight-way symmetry so only ever calculate the points for the top right eighth of a circle, and then use symmetry to get the rest of the points

The mid-point circle algorithm was developed by Jack Bresenham, who we heard about earlier. Bresenham's patent for the algorithm can be viewed here.

Bresenham's Midpoint Circle Algorithm

$$f_{circle}(x,y) = x^2 + y^2 - r^2$$

Consider this region



$$f_{circle}(x,y) = \begin{cases} < 0, & \text{if (x,y) inside circle} \\ = 0, & \text{if (x,y) on circle} \\ > 0, & \text{if (x,y) outside circle} \end{cases}$$

The Circle Decision Parameter

Calculate f_{circle} for point midway between candidate pixels

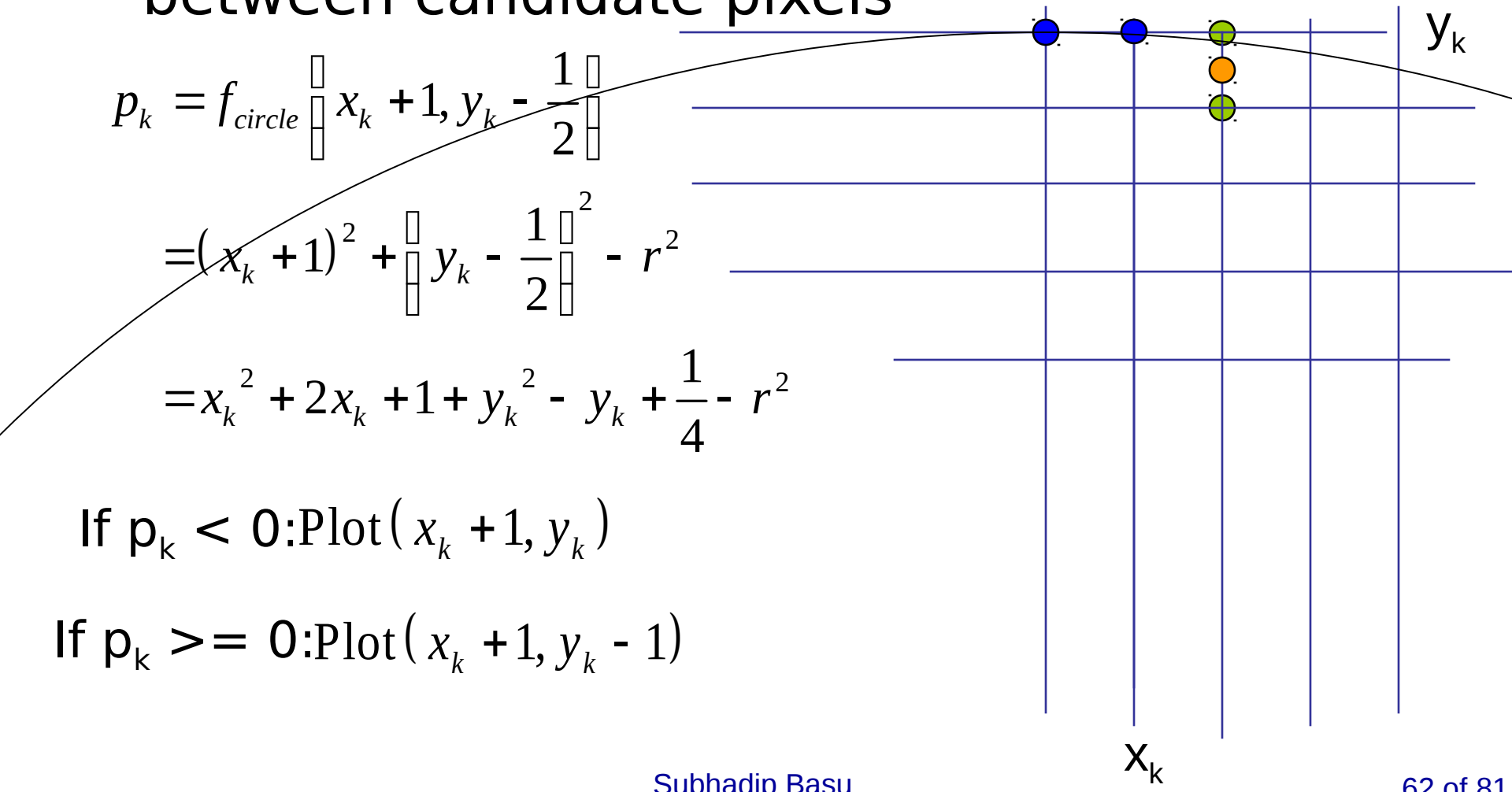
$$p_k = f_{circle}\left(x_k + 1, y_k - \frac{1}{2}\right)$$

$$= (x_k + 1)^2 + \left(y_k - \frac{1}{2}\right)^2 - r^2$$

$$= x_k^2 + 2x_k + 1 + y_k^2 - y_k + \frac{1}{4} - r^2$$

If $p_k < 0$: $\text{Plot}(x_k + 1, y_k)$

If $p_k >= 0$: $\text{Plot}(x_k + 1, y_k - 1)$



Calculating p_{k+1}

$$p_{k+1} = f_{circle}\left(x_{k+1}+1, y_{k+1}-\frac{1}{2}\right)$$

$$=(x_k+1+1)^2+\left(y_{k+1}-\frac{1}{2}\right)^2-r^2$$

$$=x_k^2+4x_k+4+y_{k+1}^2-y_{k+1}+\frac{1}{4}-r^2$$

Recurrence Relation for p_k

$$p_k = x_k^2 + 2x_k + 1 + y_k^2 - y_k + \frac{1}{4} - r^2$$

$$p_{k+1} = x_k^2 + 4x_k + 4 + y_{k+1}^2 - y_{k+1} + \frac{1}{4} - r^2$$

Compute $p_{k+1} - p_k$

$$p_{k+1} = p_k + (2x_k + 2) + 1 + \left(y_{k+1}^2 - y_k^2\right) - \left(y_{k+1} - y_k\right)$$

$$= p_k + 2x_{k+1} + 1 + \left(y_{k+1}^2 - y_k^2\right) - \left(y_{k+1} - y_k\right)$$

One Last Term …

$$\left(y_{k+1}^{\,2} - y_k^{\,2}\right) - \left(y_{k+1} - y_k\right) = ?$$

If $y_{k+1} = y_k$, then:

$$\left(y_k^{\,2} - y_k^{\,2}\right) - \left(y_k - y_k\right) = 0$$

If $y_{k+1} = y_k - 1$, then:

$$\left(\left(y_k - 1\right)^2 - y_k^{\,2}\right) - \left(\left(y_k - 1\right) - y_k\right) =$$

$$\left(y_k^{\,2} - 2y_k + 1 - y_k^{\,2}\right) - \left(-1\right) =$$

$$-2y_k + 2 = -2\left(y_k - 1\right)$$

Initial Values

$$(x_0, y_0) = (0, r)$$

$$p_0 = f_{circle}\left(0+1, r - \frac{1}{2}\right)$$

$$= 1 + \left(r - \frac{1}{2}\right)^2 - r^2$$

$$= 1 + r^2 - r + \frac{1}{4} - r^2$$

$$= \frac{5}{4} - r \approx 1 - r$$

Bresenham Midpoint Circle Algorithm Summary

At each point:

If $p_k < 0$:

$$\text{Plot}(x_k + 1, y_k)$$

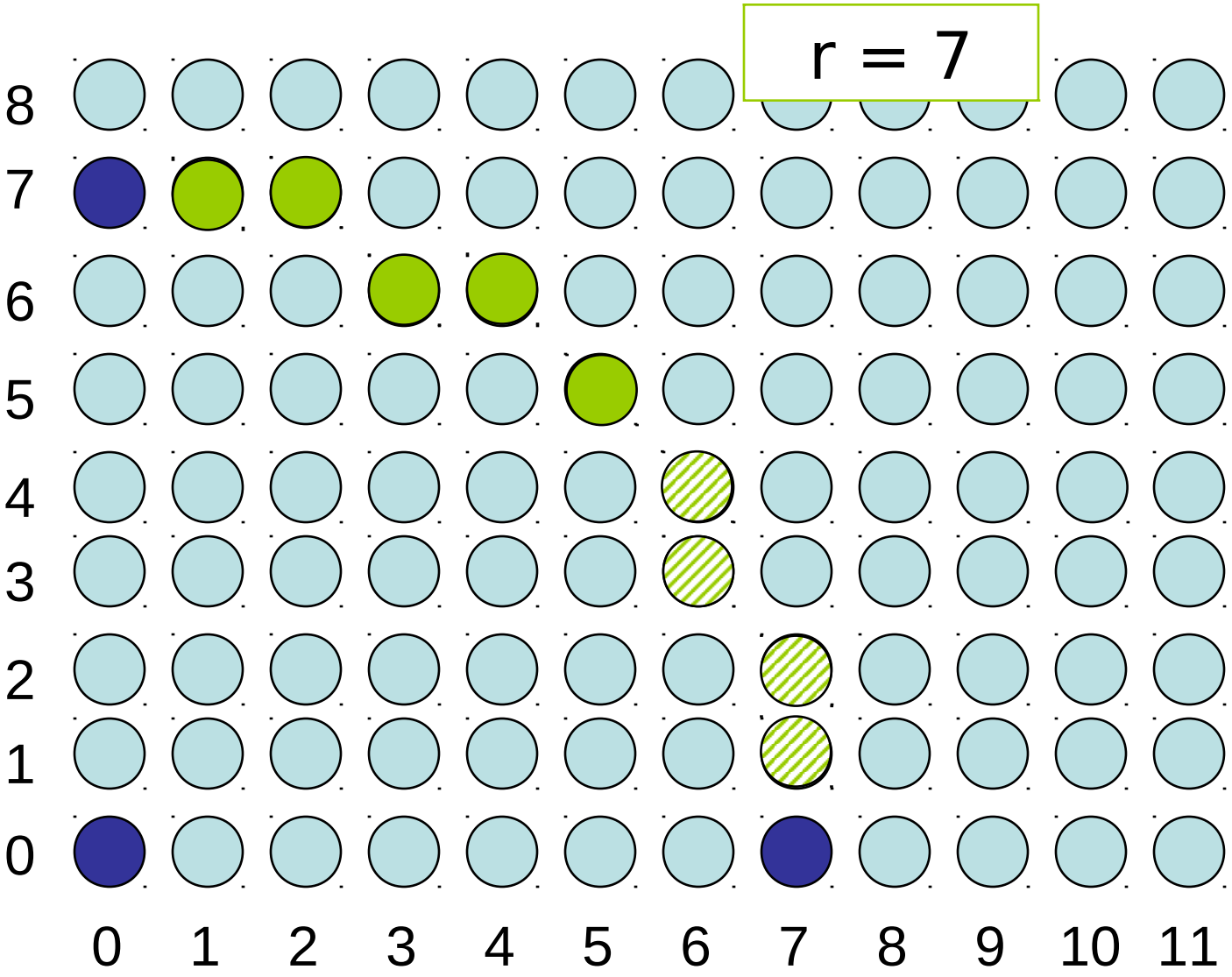
$$p_{k+1} = p_k + 2(x_k + 1) + 1$$

If $p_k >= 0$:

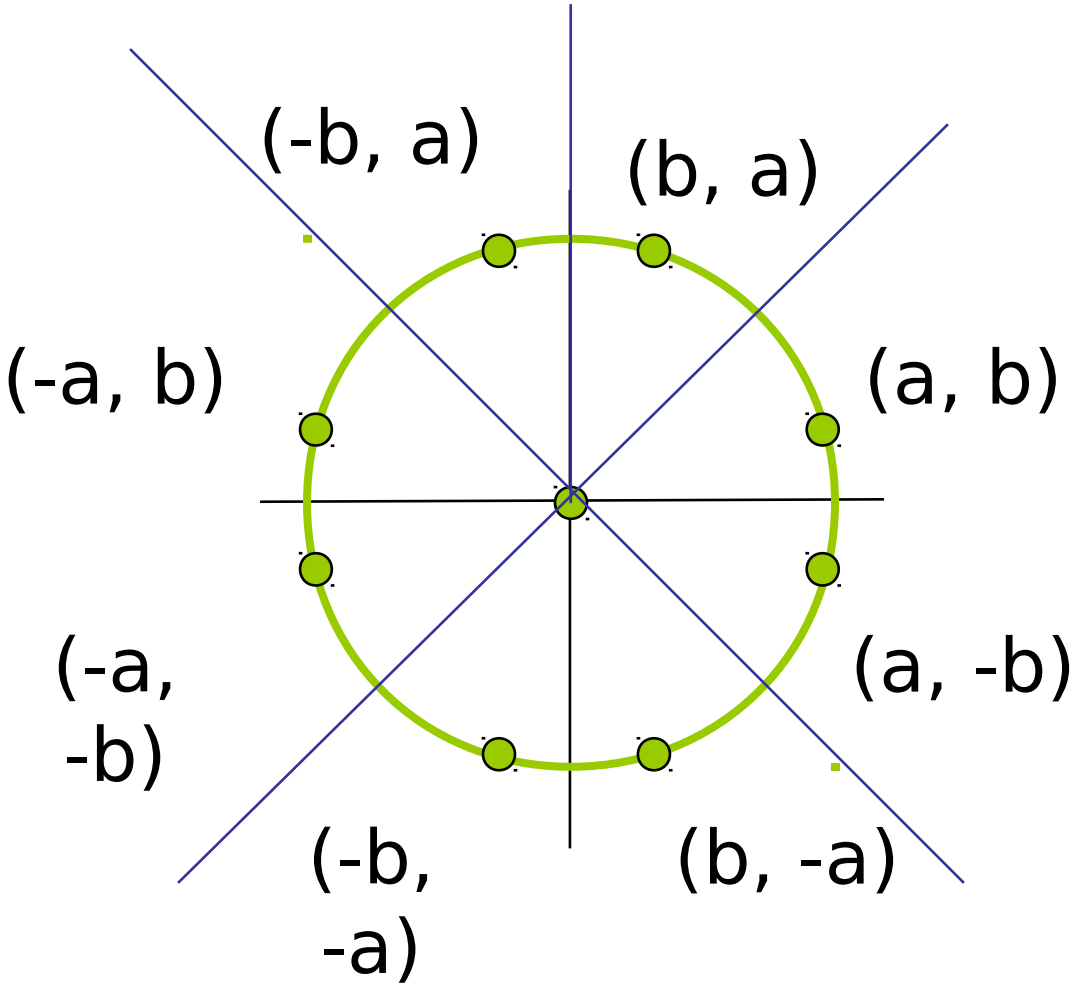
$$\text{Plot}(x_k + 1, y_k - 1)$$

$$p_{k+1} = p_k + 2(x_k + 1) + 1 - 2(y_k - 1)$$

Circle Example 1



Symmetry Optimization



Calculate points for one octant; replicate in other seven

The Mid-Point Circle Algorithm

MID-POINT CIRCLE ALGORITHM

- Input radius r and circle centre (x_c, y_c), then set the coordinates for the first point on the circumference of a circle centred on the origin as:

$$(x_0, y_0) = (0, r)$$

- Calculate the initial value of the decision parameter as:

$$p_0 = 1 - r$$

- Starting with $k = 0$ at each position x_k, perform the following test. If $p_k < 0$, the next point along the circle centred on $(0, 0)$ is $(x_k + 1, y_k)$ and:

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

The Mid-Point Circle Algorithm (cont…)

Otherwise the next point along the circle is (x_k+1, y_k-1) and:

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

4. Determine symmetry points in the other seven octants
5. Move each calculated pixel position (x, y) onto the circular path centred at (x_c, y_c) to plot the coordinate values:

$$x = x + x_c \qquad y = y + y_c$$

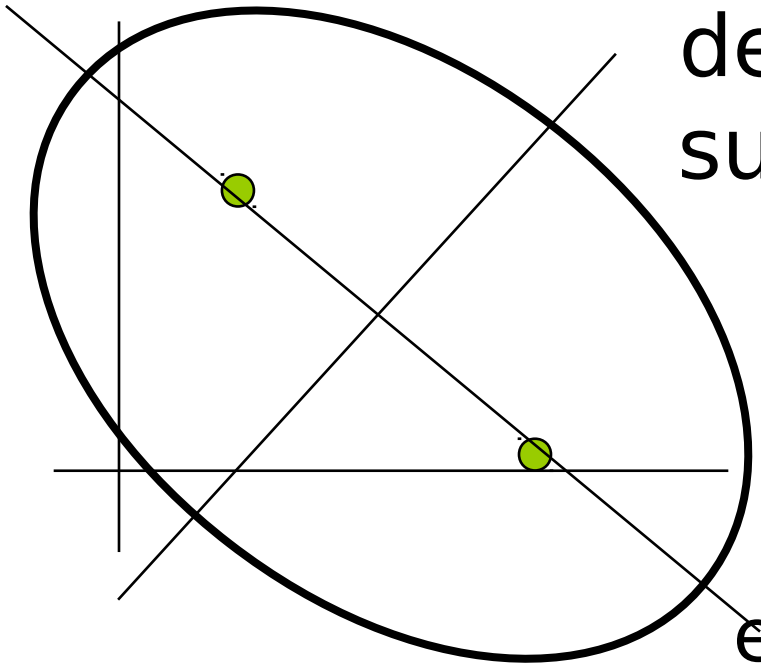
6. Repeat steps 3 to 5 until $x >= y$

Mid-Point Circle Algorithm Summary

The key insights in the mid-point circle algorithm are:

- Eight-way symmetry can hugely reduce the work in drawing a circle
- Moving in unit steps along the x axis at each point along the circle's edge we need to choose between two possible y coordinates

General Ellipse Equation



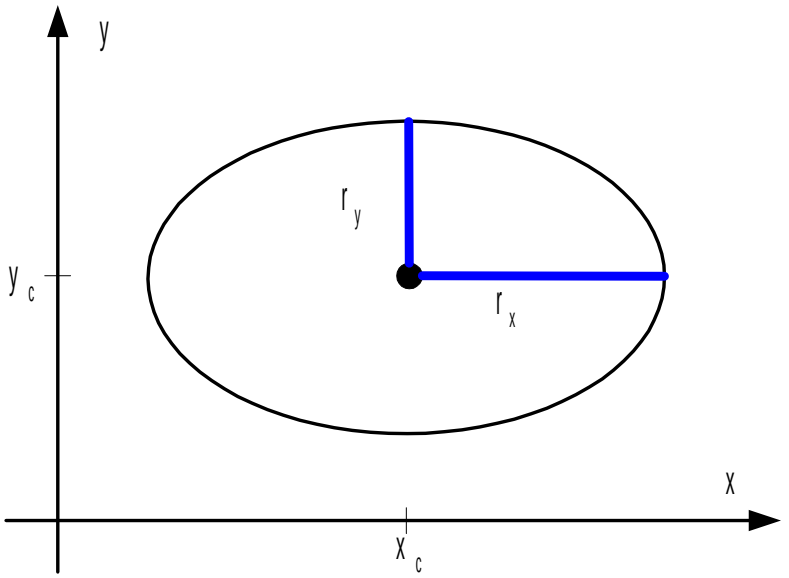
In general, ellipse described by constant sum of distances from foci

Difficult to solve equations and plot points (use parametric polar form?)

Properties of Ellipses

Equation simplified if ellipse axis parallel to coordinate axis

$$\left(\frac{x - x_c}{r_x}\right)^2 + \left(\frac{y - y_c}{r_y}\right)^2 = 1$$



Parametric form

$$x = x_c + r_x \cos\theta$$

$$y = y_c + r_y \sin\theta$$

Symmetry Considerations

4-way symmetry

Unit steps in x until reach region boundary

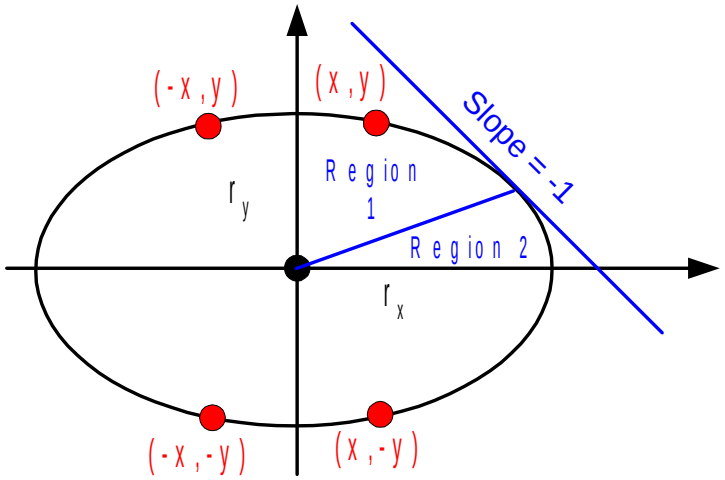
Switch to unit steps in y

$$f(x,y) = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2$$

$$\frac{dy}{dx} = -\frac{r_y^2 x}{r_x^2 y}$$

$$\frac{dy}{dx} = -1$$

$$r_y^2 x = r_x^2 y$$



- Step in x while

$$r_y^2 x < r_x^2 y$$

- Switch to steps in y when

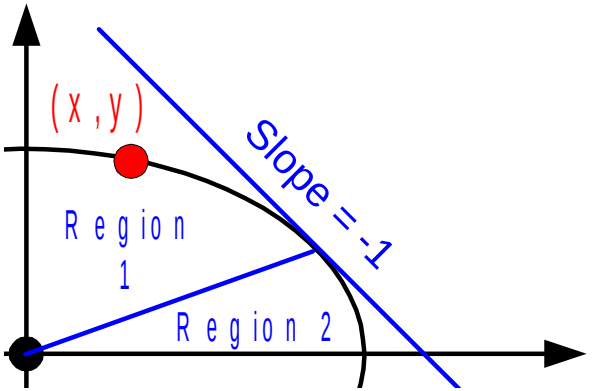
$$r_y^2 x \geq r_x^2 y$$

Midpoint Algorithm (initializing)

Similar to circles

The initial value for region 1

$$D_{init1} = f(1, r_y - \tfrac{1}{2})$$
$$= r_y^2 - r_x^2 r_y + \tfrac{1}{4} r_x^2$$

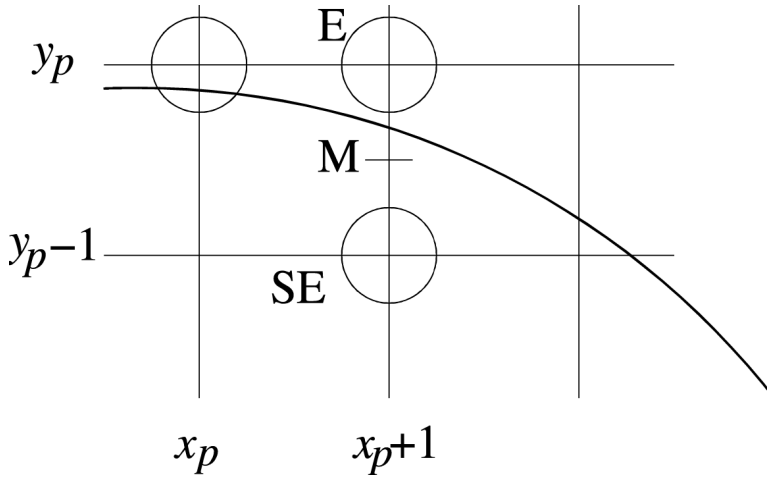


The initial value for region 2

$$D_{init2} = f(x_p + \tfrac{1}{2}, y_p - 1)$$
$$= r_y^2 (x_p + \tfrac{1}{2})^2 + r_x^2 (y_p - 1)^2 - r_x^2 r_y^2$$

We have initial values, now we need the increments

Making a Decision



Computing the decision variable

$$D = f(x_p + 1, y_p - \tfrac{1}{2})$$
$$= r_y^2(x_p + 1)^2 + r_x^2(y_p - \tfrac{1}{2})^2 - r_x^2 r_y^2$$

If $D < 0$ then *M* is *below* the arc,
hence the *E* pixel is closer to the line.

If $D \geq 0$ then *M* is *above* the arc,
hence the *SE* pixel is closer to the line.

Computing the Increment

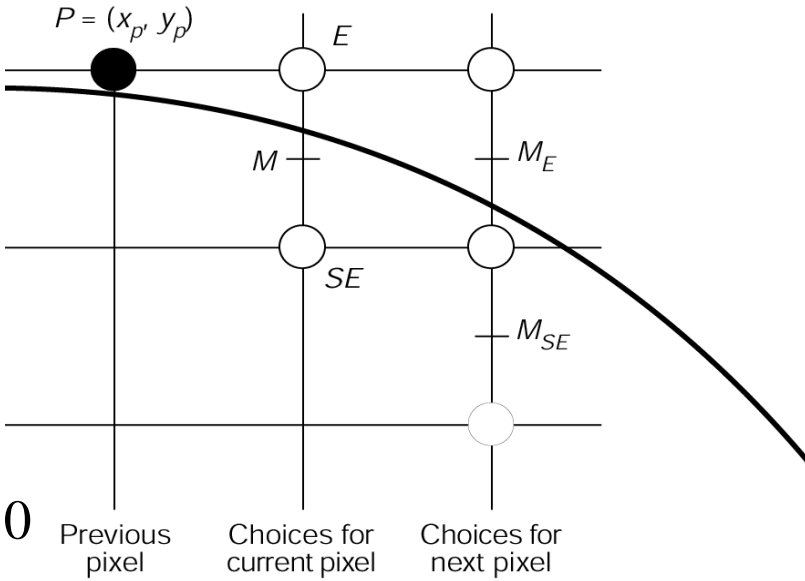
E case

$$D_{new} = f(x_p + 2, y_p - \tfrac{1}{2})$$
$$= r_y^2 (x_p + 2)^2 + r_x^2 (y_p - \tfrac{1}{2})^2 - r_x^2 r_y^2$$
$$= D_{old} + r_y^2 (2x_p + 3)$$

SE case

$$D_{new} = f(x_p + 2, y_p - \tfrac{3}{2})$$
$$= r_y^2 (x_p + 2)^2 + r_x^2 (y_p - \tfrac{3}{2})^2 - r_x^2 r_y^2$$
$$= D_{old} + r_y^2 (2x_p + 3) + r_x^2 (-2y_p + 2)$$

$$increment = \begin{cases} r_y^2 (2x_p + 3) & D_{old} < 0 \\ r_y^2 (2x_p + 3) + r_x^2 (-2y_p + 2) & D_{old} \geq 0 \end{cases}$$



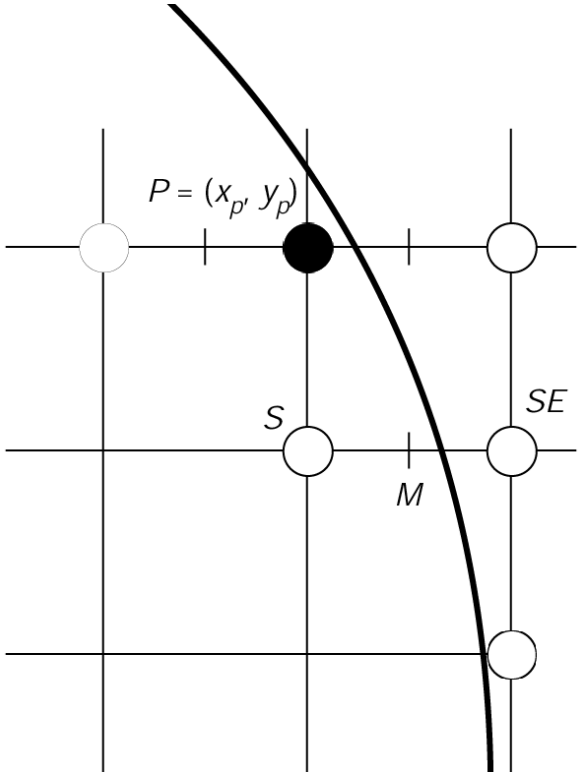
Computing the Increment in 2nd Region

Decision variable in 2nd region

$$D = f(x_p + \tfrac{1}{2}, y_p - 1)$$
$$= r_y^2 (x_p + \tfrac{1}{2})^2 + r_x^2 (y_p - 1)^2 - r_x^2 r_y^2$$

If $D < 0$ then *M* is *left of* the arc,
hence the *SE* pixel is closer to the line.

If $D \geq 0$ then *M* is *right of* the arc,
hence the *S* pixel is closer to the line.



Computing the Increment in 2nd Region

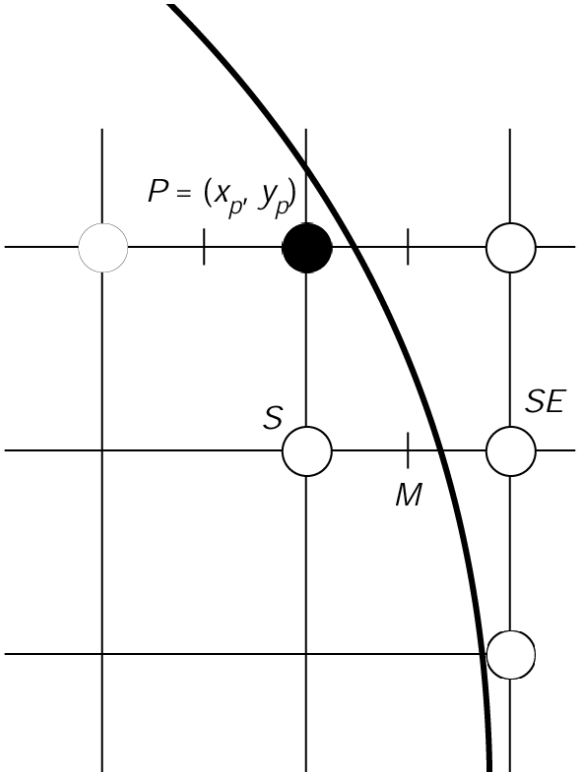
SE case

$$D_{new} = f(x_p + \tfrac{3}{2}, y_p - 2)$$
$$= r_y^2 (x_p + \tfrac{3}{2})^2 + r_x^2 (y_p - 2)^2 - r_x^2 r_y^2$$
$$= D_{old} + r_x^2 (-2y_p + 3) + r_y^2 (2x_p + 2)$$

S case

$$D_{new} = f(x_p + \tfrac{1}{2}, y_p - 2)$$
$$= r_y^2 (x_p + \tfrac{1}{2})^2 + r_x^2 (y_p - 2)^2 - r_x^2 r_y^2$$
$$= D_{old} + r_x^2 (-2y_p + 3)$$

$$increment = \begin{cases} r_x^2 (-2y_p + 3) + r_y^2 (2x_p + 2) & D_{old} < 0 \\ r_x^2 (-2y_p + 3) & D_{old} \geq 0 \end{cases}$$



Midpoint Ellipse Algorithm

```
void MidpointEllipse (int a, int b, int value)
/* Assumes center of ellipse is at the origin. Note that overflow may occur */
/* for 16-bit integers because of the squares. */
{
    double d2;

    int x = 0;
    int y = b;
    double d1 = b^2 - (a^2 b) + (0.25 a^2);
    EllipsePoints (x, y, value);                    /* The 4-way symmetrical WritePixel */

    /* Test gradient if still in region 1 */
    while ( a^2 (y - 0.5) > b^2 (x + 1) ) {         /* Region 1 */
        if (d1 < 0)                                 /* Select E */
            d1 += b^2 (2x + 3);
        else {                                      /* Select SE */
            d1 += b^2 (2x + 3) + a^2 (-2y + 2);
            y--;
        }
        x++;
        EllipsePoints (x, y, value);
    } /* Region 1 */
```

Midpoint Ellipse Algorithm (cont.)

```
d2 = b²(x + 0.5)² + a²(y − 1)² − a²b²;
while (y > 0) {                          /* Region 2 */
    if (d2 < 0) {                        /* Select SE */
        d2 += b²(2x + 2) + a²(−2y + 3);
        x++;
    } else
        d2 += a²(−2y + 3);               /* Select S */
    y−−;
    EllipsePoints (x, y, value);
}  /* Region 2 */
}  /* MidpointEllipse */
```