M.C.A. SECOND YEAR SECOND SEMESTER EXAMINATION-2018
THEORY OF COMPUTING

Time: Three hours | Full Marks: 100

Answer any **FIVE** questions.

1. a)Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.
(i) Can a Turing machine ever write the blank symbol on its tape?
(ii) Can the tape alphabet Γ be the same as the input alphabet Σ?
(iii) Can a Turing machine contain just a single state?
b) Construct a Turing machine for the function f(n), where

$$f(n) = \begin{cases} x - y & if\ x \geq y \\ 0 & otherwise \end{cases}$$

(Validate the construction taking at least one valid and another invalid nontrivial string)

(2 + 2 + 2) + 14

2. a) Why do we design Nondeterministic Finite Automata although Deterministic Finite Automata is used?
b) Let L and M be regular languages. Then show that the following languages are also regular:
i) Union: L ∪ M (ii) Intersection: L ∩ M (iii) Complement: N (iv) Reversal: $L^R = \{w^R : w \in L\}$ (v) Closure: L* (vi) Concatenation: L.M
c) Convert the Mealy machine given in Fig. A to corresponding Moore machine

Present State	Next State			
	a = 0		a = 1	
	State	Output	State	Output
->q0	q1	0	q3	0
q1	q3	1	q2	0
q2	q4	1	q0	0
q3	q0	0	q4	1
q4	q2	0	q1	1

Fig. A

12 + 8

3. a) Let A_{DFA} = {(B, w) | B accepts the input string w}. Show that A_{DFA} is undecidable.
b) Let A_{TM} = {(M, w) |The TM M accepts w}. Show that A_{TM} is undecidable.
c) "If A is reducible to B and B is decidable then A is decidable. If A is reducible to B and A is undecidable, then B is undecidable."--- Justify

8 + 8 + 4

4. a) Find a regular expression for the language consisting of alternating zeroes and ones.
b) Construct regular expressions representing languages, over the alphabet {a, b, c}, in which for every string w it holds that |w| = 3i. (i ≥ 0)
c) Find the equivalent Regular Expression for the DFA given in Fig. B.

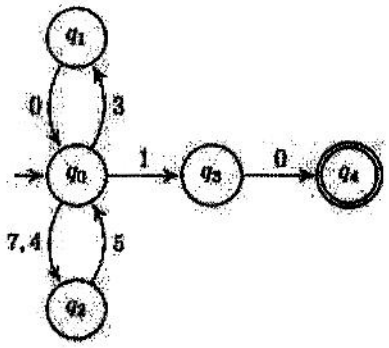

Fig. B

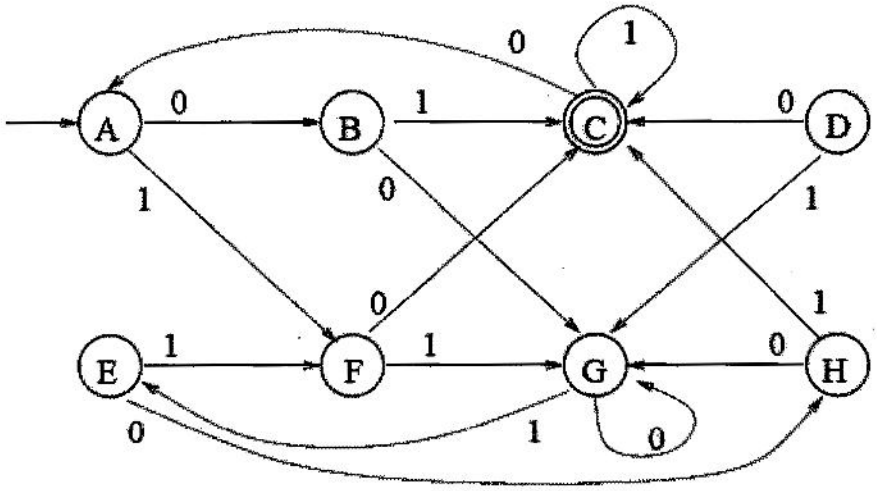

Fig. C

d) Minimize the DFA given in Fig. C. **4 + 4 + 4 + 8**

5. Construct one context-free grammar for each of the following languages:

a) $L = \{a^n b^n c^m d^m \mid m, n \geq 1\}$

b) The regular language corresponding to the Regular Expression 00^*11^*

c) All binary strings with both an even number of zeroes and an even number of ones.

d) All strings of the form $0^a1^b0^c$ where $a + c = b$.

4 × 5

6. a) Describe a TM that decides the language $L = \{w \in \{0, 1\}^* \mid |w|_0 = |w|_1\}$ where $|w|_0$ and $|w|_1$ are respectively the number of 0's and 1's in w.
(Validate the construction taking at least one valid and another invalid nontrivial string)
(b) Show that the set of decidable languages is closed under the union and complementation operation.

14 + 6

7. Construct a PDA for the following languages

a) $L = \{a^i b^{2i} c^j \mid i, j \geq 0\}$

b) $L = \{1^k 0^i 1^i 0^j 1^j 0^k \mid i, j, k > 0\}$

(Validate the construction taking at least one valid and another invalid nontrivial string)

10 +10

8. Using pumping lemma show that the language $L = \{ a^{i^2} \mid i \geq 1\}$ is neither regular nor context free. **10 +10**