Ref. No.: Ex/PG/IEE/T/128B/2018

M. Tech. Instrumentation and Electronics Engg. 1st Year 2nd Semester Examination 2018

Digital Filtering and Control

Time: 3 hours **Full Marks: 100**

Attempt any five questions from the following

1.

(a) Design a first-order high-pass IIR digital filter with a 3-dB cut-off frequency of 0.5π.

(b) The frequency response $H(e^{j\omega})$ of a length-4 FIR filter with real impulse response has the following specific values: $H(e^{j0}) = 2, H(e^{j\pi/2}) = 7 - j3, and\ H(e^{j\pi}) = 0$. Determine $H(z)$.

10+10

2.

(a) An FIR LTI discrete-time system is described by the difference equation:

$y[n] = a_1x[n+k] + a_2x[n+k-1] + a_3x[n+k-2] + a_2x[n+k-3] + a_1x[n+k-4]$

Where $y[n]\ and\ x[n]$ denote, respectively, the output and the input sequences. Determine the expression for its frequency response $H(e^{j\omega})$. For what values of the constant k will the system have a frequency response $H(e^{j\omega})$ that is a real function of ω?

(b) A linear shift-invariant system is having a system function

$$H(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{3}z^{-1}}$$

The system is excited by zero mean exponentially correlated noise $x(n)$ with an autocorrelation sequence $r_x(k) = (\frac{1}{2})^{|k|}$. Let $y(n)$ be the output process with $y(n) = x(n) * h(n)$. Find the autocorrelation sequence $r_y(k)$ and power spectrum $P_y(z)$ of $y(n)$.

8+12

3. A random process $x(n)$ is generated as follows:

$x(n) = \alpha x(n-1) + v(n) + \beta v(n-1)$; where $v(n)$ is white noise with mean m_v and variance σ_v^2.

(a) Design a first-order linear predictor $\hat{x}(n+1) = w(0)x(n) + w(1)x(n-1)$ that minimizes the mean-square error in the prediction of $x(n+1)$, and find the minimum mean-square error.

(b) Consider a predictor of the form $\hat{x}(n+1) = c + w(0)x(n) + w(1)x(n-1)$

Find the values for $c, w(0)$ and $w(1)$ those minimize the mean-square error, and compare the mean-square error of this predictor with that found in part (a).

Derive all the expressions which you may require for your calculation.

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4. A signal $x(n)$ is observed in a noisy and reverberant environment: $y(n) = x(n) + 0.8x(n-1) + v(n)$ where $v(n)$ is white noise with variance $\sigma_v^2 = 1$ that is uncorrelated with $x(n)$. $x(n)$ is a wide-sense stationary AR(1) random process with autocorrelation values $\boldsymbol{r_x} = [4,\ 2,\ 1,\ 0.5]^T$. Design a noncausal IIR Wiener filter that produces the minimum mean-square estimate of $x(n)$. Derive all the necessary equations for your calculations.

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5. Find out the expression of system function for a causal IIR Wiener filter that produces the minimum mean-square estimate of a desired signal $d(n)$ from $x(n)$. For the unit variance white noise $\varepsilon(n)$, if $r_{d\varepsilon}(k) = \delta(k)$ *and* $P_x(z) = \frac{4}{(1-0.5z^{-1})(1-0.5z)}$, calculate the unit sample response of the causal Wiener filter.

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6. Consider the problem of estimating the value of an unknown constant x given measurements that are corrupted by uncorrelated, zero mean white noise $v(n)$ having a variance σ_v^2. With the help of discrete Kalman filter, show that the estimate at time n is given by:

$$\hat{x}(n) = \hat{x}(n-1) + \frac{P(0)}{n\,P(0) + \sigma_v^2}[y(n) - \hat{x}(n-1)]$$

Where $P(n)$ is the error covariance at time n. Derive the necessary equations for your calculation.

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7. Write short notes on
   (i) Symmetric and antisymmetric FIR filters
   (ii) Wiener deconvolution

2 X10

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