B.E. MECHANICAL ENGINEERING 3RD YEAR 2ND SEMESTER , 2018
SUBJECT: Introduction to Finite Element Method for Engineers

Time: Three Hours Full Marks 100

Answer questions 1a OR 1b (CO1), 2a OR 2b (CO2), 3a OR 3b (CO3) and 4 (CO4)
All questions carry equal marks

Question 1a

State the principle of virtual work

Derive the stiffness matrix and force vector for a bar element (of cross-sectional area, modulus of elasticity and length A, E & l respectively) with an axial distributed load using the principle of virtual work and Galerkin's method. Refer Figure 1.

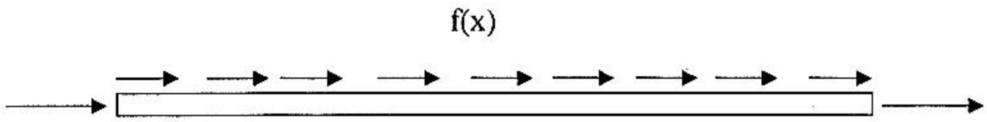

Figure 1

OR

Question 1b

State the principle of minimization of potential energy

Using this principle derive the expression for stiffness matrix and equivalent nodal forces for a two-dimensional beam element

The shape functions are given below. Only indicate the steps for deriving them.

$$N_1 = \left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right) \quad N_2 = \left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right)$$

$$N_3 = \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3}\right) \quad N_4 = \left(\frac{x^3}{L^2} - \frac{x^2}{L}\right)$$

Show the degrees of freedom of a three-dimensional beam element in a sketch

What is a 3rd point for a three-dimensional beam element?

Question 2a

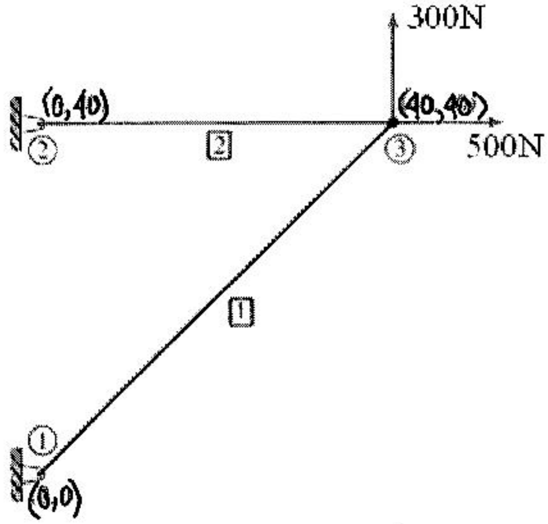

Figure 2a

Consider the truss structure shown in Figure 2a. The coordinates are given in centimeters. The modulus of elasticity and cross-sectional area are $2 \times 10^7\ N/cm^2$ and $2cm^2$ respectively.

The joint is pinned joint and the supports are hinges

a) Find out the displacements at node 3

b) Find out the stresses in the elements.

You may use the following relation:-

$$[K^e] = \frac{AE}{l}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ & s^2 & -cs & -s^2 \\ & & c^2 & cs \\ & sym. & & s^2 \end{bmatrix}$$

OR

Question 2b

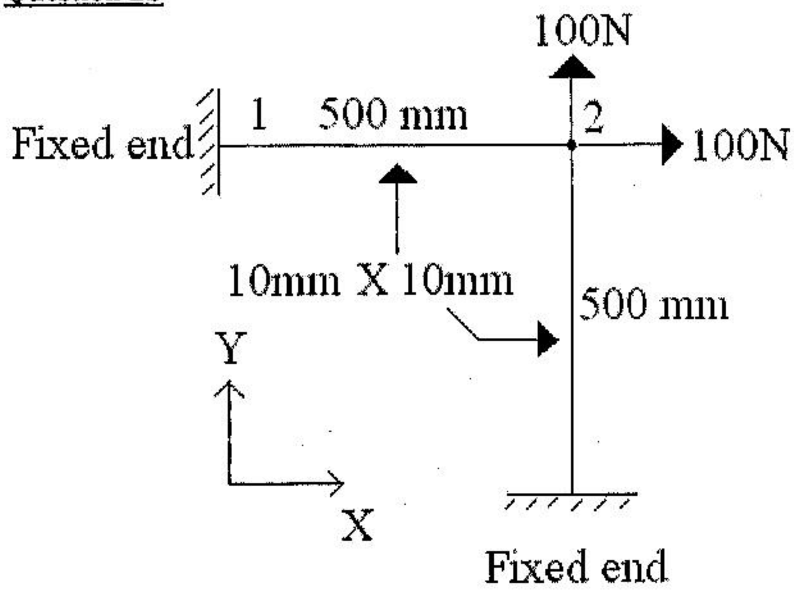

Figure 2b

A two element steel **FRAME** in Figure 2b is subjected to external loading as shown. *The joint is a rigid one.*

a. Compute the element stiffness matrix
b. Assemble the element stiffness matrix and form the system stiffness matrix
c. Incorporate the boundary conditions using the method of row column deletion
d. Indicate how to obtain bending moment within an element

Solution of equations is not required. You may use the following relation

$$\underline{k} = \frac{E}{L} \times$$

$$\begin{bmatrix} AC^2 + \frac{12I}{L^2}S^2 & \left(A - \frac{12I}{L^2}\right)CS & -\frac{6I}{L}S & -\left(AC^2 + \frac{12I}{L^2}S^2\right) & -\left(A - \frac{12I}{L^2}\right)CS & -\frac{6I}{L}S \\ & AS^2 + \frac{12I}{L^2}C^2 & \frac{6I}{L}C & -\left(A - \frac{12I}{L^2}\right)CS & -\left(AS^2 + \frac{12I}{L^2}C^2\right) & \frac{6I}{L}C \\ & & 4I & \frac{6I}{L}S & -\frac{6I}{L}C & 2I \\ & & & AC^2 + \frac{12I}{L^2}S^2 & \left(A - \frac{12I}{L^2}\right)CS & \frac{6I}{L}S \\ & & & & AS^2 + \frac{12I}{L^2}C^2 & -\frac{6I}{L}C \\ \text{Symmetry} & & & & & 4I \end{bmatrix}$$

Question 3a

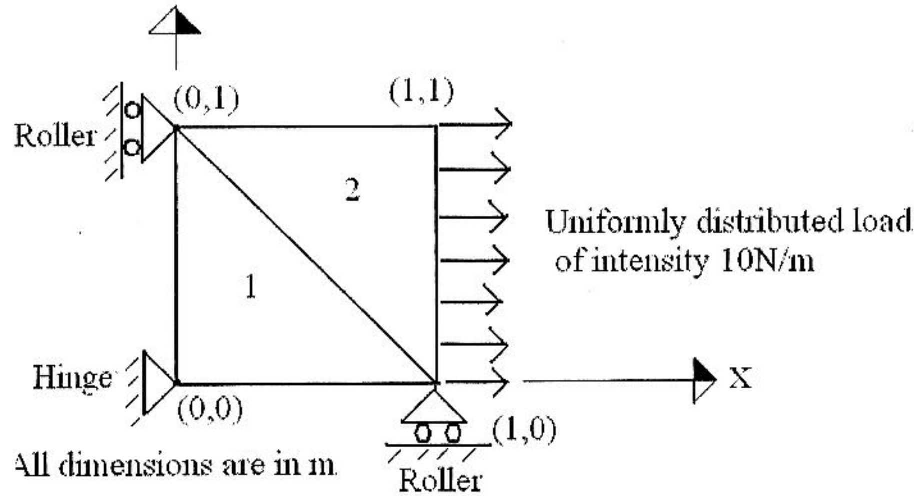

Figure 3

An assembly of two constant-strain triangles is shown in Figure 3. Assume **plane stress** conditions. Take thickness as $t = 0.001m$. **All dimensions are in meters**.

For the sake of calculation take $\dfrac{E}{1-\mu^2} = 200GPa$ and $\mu = 0.25$

Use the relation $N_i = \dfrac{1}{2\Delta}(a_i + b_i x + c_i y)$

Where, $a_1 = x_2 y_3 - x_3 y_2 \quad b_1 = y_2 - y_3 \quad c_1 = x_3 - x_2$

a) How many degrees of freedom does this system have after elimination of the boundary conditions?
b) Assemble the element stiffness and the force vector only for the effective (free) degrees of freedom. *Solution of equations is not required.*
c) Indicate how to compute stress in the elements

OR

Question 3b

a. Show that a straight edge in parametric space is mapped into a straight edge in object space for a four-node isoparametric quadrilateral element.

b. If the coordinates of a quadrilateral four-node isoparametric element is (0,0), (a,0), (a,b) and (0,b) respectively compute its Jacobian matrix.

c. Indicate how do you compute the numerical value of $\frac{\partial N_i}{\partial x}$?

d. Evaluate the integral $\int_{-1}^{1}\int_{-1}^{1} r^3 s^3 drds$. Use 2 point and 3 point Gauss quadrature rule. Use the data given in Table 1. Are the results same? Explain your answer.

Table 1. Data for 2 point and 3 point Gauss quadrature rule

Number of points	Locations	Weights
2	$\pm$0.57735 02691 89626	1.00000 00000 00000
3	$\pm$0.77459 66692 41483 0.00000 00000 00000	0.55555 55555 55556 0.88888 88888 88889

Question 4

What are the assumptions in Kirchoff plate bending theory?

What are the moment curvature relations for a Kirchoff plate bending element?

Show the degrees of freedom of a rectangular thin plate bending element in a neat sketch

Describe in details the process of obtaining the finite element equations for a rectangular thin plate bending element

Is this element a compatible one? Explain.