

1) unrestricted variables
 $x_i \rightarrow$ unrestricted

$$\begin{cases} \geq 0 \\ \leq 0 \\ = 0 \end{cases}$$

$$x_r = x_r' - x_r''$$

$$x_r', x_r'' \geq 0$$

$$\begin{cases} x_r' \rightarrow +ve \\ x_r'' \rightarrow -ve \end{cases}$$

2) Yie in entering basic variables :- (key column)

i) Tie between two decision variables \rightarrow arbitrarily

ii) y_{ie} decision and slack variables \rightarrow decision variable

iii) y_{ie} " two slack variables \rightarrow arbitrarily
(surplus)

3) Tie for leaving variables: (key row) Degeneracy

Handwritten notes on grid paper showing a binary tree structure and calculations. The tree has a root node '1' with a left child '0' and a right child '0'. The left child '0' has a left child 'S₁' and a right child 'S₂'. The right child '0' has a left child 'S₃' and a right child 'b₁'. Below the tree, there are calculations for the nodes. For S₁, the calculation is 10 (1*5 + 0*5) and 14 (1*7 + 0*7). For S₂, the calculation is 0 (0*5 + 1*5) and 7 (0*7 + 1*7). For S₃, the calculation is 0 (0*5 + 1*5) and 7 (0*7 + 1*7). For b₁, the calculation is 25 (1*25 + 0*35) and 35 (1*35 + 0*25). The final result is 100.

Rules.

1. Divide the coefficients of slack variables in the simplex method's table where degeneracy is seen by the corresponding positive numbers of the key column, in the row, starting from left to right.
2. Compare the ratios in step (i) from left to right column wise, select the row that contains the smallest ratio.
if tie between slack and artificial variable
prefer

Ex

$$\text{Max } Z = 3x_1 + 9x_2$$

$$\text{s.t.} \quad \text{i) } x_1 + 4x_2 \leq 8$$

$$\text{ii) } x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Sensitivity analysis

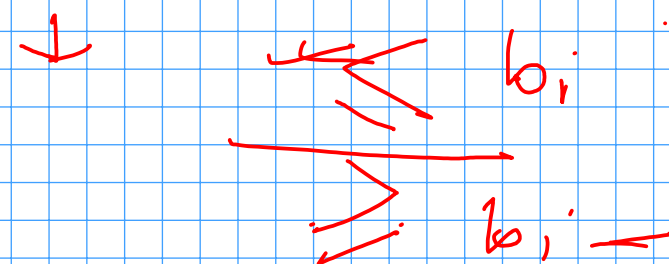
Relaxation: It is a modification of a problem instance PL that leads to a larger feasible set (increase the number of feasible solutions) or alternatively that the feasible set remain unchanged

Restriction: → a smaller feasible set (decrease the number of feasible solution)
 — alternatively feasible set remain unchanged

- $C_j \uparrow \downarrow$
- $b_i \uparrow \downarrow$
- $a_{ij} \uparrow \downarrow$
- \pm constraints
- \pm variable

$a_{ij} \uparrow$
 $a_{ij} \leq b_i$ → restriction

Max ↑ relaxation



Constraint type	$b_i \uparrow$	$b_i \downarrow$
\leq	RL	RS
\geq	RS	RL

$a_{ij} \downarrow \rightarrow$ Relaxation

$a_{ij} \geq b_i$

$a_{ij} \uparrow \rightarrow$ Relaxation

$a_{ij} \downarrow \rightarrow$ Restriction

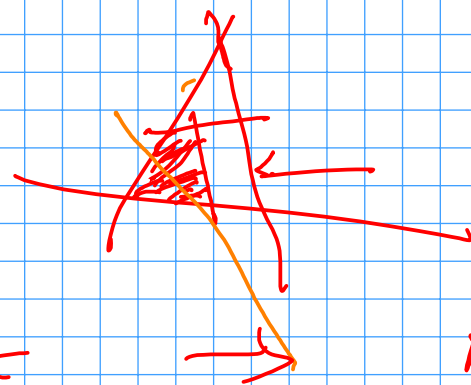
$c_{ij} \uparrow$

$z^* \rightarrow$ larger / unchanged

$c_{ij} \downarrow$

$z^* \Rightarrow$ unchanged or lower

+ new constraint \rightarrow Restriction



- constraint

Relaxation

+ new variable \rightarrow Relaxation

- a variable \rightarrow Restriction

Shadow price

* Economic interpretation of duality ?!

Objective:

To know how large the effect of a certain change becomes in the optimal objective function

Defⁿ:

The shadow price for a constraint is given by the change in objective function value when making a marginal increase of the right hand side

shadow price
marginal value
dual value

$$\text{Max } Z = 3x_1 + 4x_2$$

$$x_1 + x_2 \leq 7$$

$$x_1 + 5x_2 \leq 10$$

$$x_1, x_2 \geq 0$$

$$u_i = \frac{\partial Z}{\partial b_i}$$

$$\text{min } 7u_1 + 10u_2$$

$$u_1 + u_2 \geq 3$$

$$u_1 + 5u_2 \geq 4$$

$$u_1, u_2 \geq 0$$

Duality / Economic interpretation of dual variables

production manager

Determines quantities of each product to be produced

Optimizes resource utilization

To optimize the available resource

maximize profit

Cost of each unit of resource is equal to its marginal return

Shadow price indicates an additional price to be paid to obtain one additional unit of resource in order to maximize profit under resource constraint.

shadow price

=

change in optimal objective
function value
unit change in the avail-
ability of resource

Maximization problem

$$Z = \sum_{j=1}^n C_j x_j$$

Z = return

C_j = profit (or return)
per unit of variable (activity)
 x_j

x_j = number of unit variable j

b_i = maximum no of units for
a resource i available

a_{ij} =

units of resource i consumed (required) per unit
variable j

primal :

$$\text{Maximize (return). } Z \Rightarrow \sum_{j=1}^n C_j x_j = \sum_{j=1}^n (\text{profit per unit variable } x_j) (\text{unit of variable } x_j)$$

s.t.

$$\sum_{j=1}^n a_{ij} x_j \leq b_i$$

or
$$\sum_{j=1}^n \begin{matrix} \text{(unit of resource } i \text{ consumed per unit of variable } x_j) \\ \text{(unit of variable } x_j) \end{matrix} \leq \begin{matrix} \text{unit of resource } i \\ \text{available} \end{matrix}$$

$x_j \geq 0$ for all j

Dual LP problem.

$$\begin{aligned} \text{minimize (cost) } Z_y &= \sum_{i=1}^m b_i y_i \\ &= \sum_{i=1}^m \begin{matrix} \text{(unit of resource } i) \\ \text{(cost per unit of resource } i) \end{matrix} \end{aligned}$$

s.t.

$$\sum_{i=1}^m a_{ji} y_i \geq c_j$$

or
$$\sum_{i=1}^m \begin{matrix} \text{(unit of resource } i \text{ consumed per} \\ \text{unit of variable } y_i) \end{matrix} \text{(cost per unit of } y_i)$$

resource i $>$ profit per unit for each variable x_j

$$z_x, z_y$$

$$z_x \leq z_y$$

for feasible solution
for both primal &
optimal solution

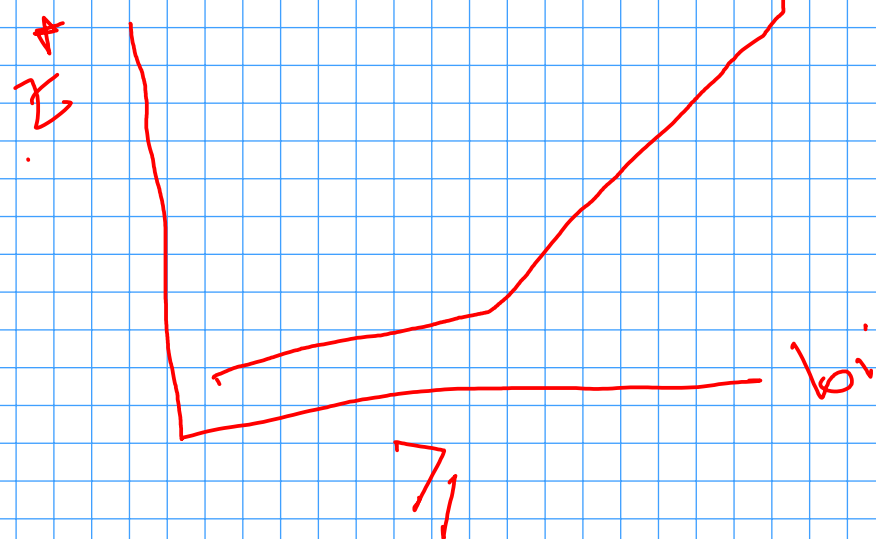
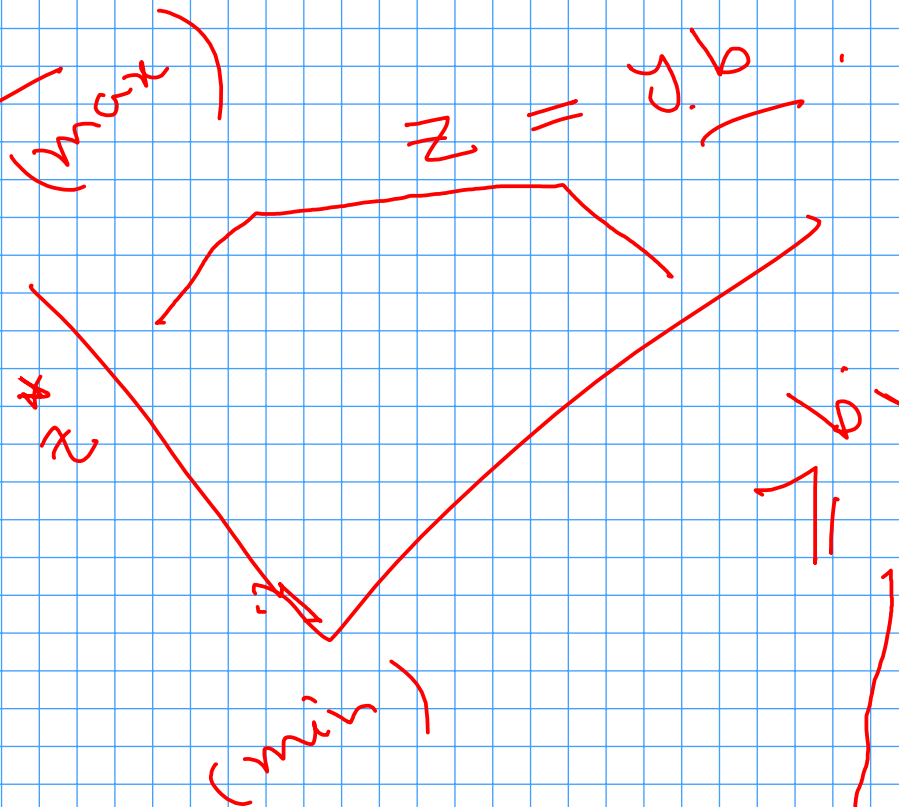
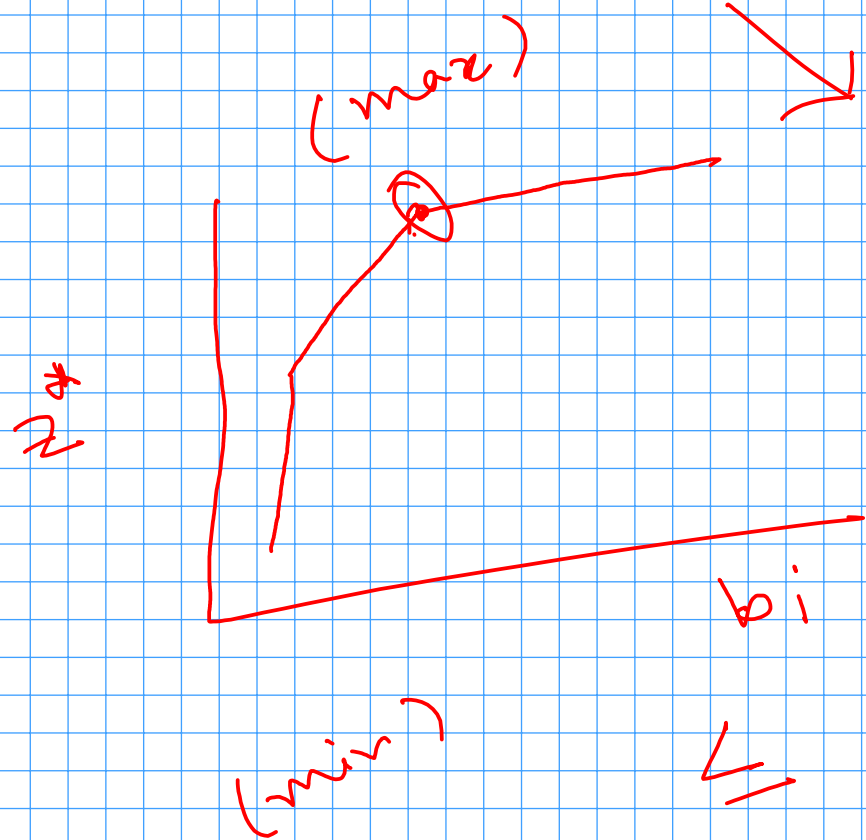
profit \leq worth of resources.

\rightarrow weak duality

profit $=$ worth of resources.

\rightarrow strong duality

$$\theta_i = \frac{\partial z}{\partial b_i}$$



A Change in right hand side with the maximize z

