

Revised simplex

1) Row vector

$$X = [x_1, x_2, \dots, x_n]$$

2) column vector

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

3) null vector

$$0 = [0, 0, 0, \dots, 0]$$

$$0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

4) $m \times n$ matrix

↓
rows ↓
 column



m row vector

or

n column vector

Inverse of a matrix

A Inverse of matrix $A \Rightarrow A^{-1}$

Inverse $\left\{ \begin{array}{l} A \cdot A^{-1} = I \\ A I \rightarrow I A \end{array} \right.$

$$A \cdot A^{-1} = I \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

\rightarrow non singular matrix

\rightarrow Its determinant is non-zero

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Columns are labeled C_1, C_2, C_3 above and below the matrix.

$$A = [C_1 \ C_2 \ C_3]$$

$$A = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

$$A = [A_1 \ A_2]$$

$$A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 3 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$

$$AB = [A_1 \ A_2] \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = A_1 B_1 + A_2 B_2$$

$$AB = \begin{bmatrix} 25 & 18 \\ 8 & 7 \end{bmatrix}$$

$$\text{Max } z = c^T x$$

$$\text{s.t. } Ax \leq b \text{ \& } x \geq 0$$

Basic
Non Basic
variable

$$\begin{bmatrix} A \\ I \end{bmatrix} \begin{bmatrix} x \\ x_s \end{bmatrix} = b \quad \begin{bmatrix} z \\ x_s \end{bmatrix} \geq 0$$

identity matrix

$$c = c_B -$$

$$z = c_B^T x_B + c_N^T x_N$$

$$z = c_B^T x_B$$

$$\begin{bmatrix} B & N \end{bmatrix} \begin{bmatrix} x_B \\ x_N \end{bmatrix} = b$$

$$= c_B^T B^{-1} b$$

$$\begin{bmatrix} B \end{bmatrix} x_B + \begin{bmatrix} N \end{bmatrix} x_N = b$$

$$= c_B^T b$$

$$x_B = ?$$

$$x_B = B^{-1} b$$

$$x_B = I^{-1} b$$

$$z = c_B^T I^{-1} b$$

$$I^{-1} = I$$

original set of equation

$$\begin{cases} z - c^T x = 0 \\ z = c^T x \end{cases}$$

$$\begin{bmatrix} 1 & -c & 0 \\ 0 & A & I \end{bmatrix} \begin{bmatrix} z \\ x \\ x_s \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ b \end{bmatrix} \xrightarrow{c_B^T B^{-1} b}$$

$$\begin{bmatrix} z - c^T x \\ Ax + I x_s \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ b \end{bmatrix}$$

original one

$$B^{-1} b$$

$$\begin{bmatrix} 1 & c_B B^{-1} \\ 0 & B^{-1} \end{bmatrix} \begin{bmatrix} 1 & c_B B^{-1} \\ 0 & B^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} c_B B^{-1} b \\ B^{-1} b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -c & 0 \\ 0 & A & I \end{bmatrix} \begin{bmatrix} z \\ x \\ x_s \end{bmatrix} = \begin{bmatrix} c_B B^{-1} b \\ B^{-1} b \end{bmatrix}$$

$z_j = c_j$ any iteration

$$\Rightarrow \begin{bmatrix} 1 & c_B B^{-1} A - c & -c_B B^{-1} \\ 0 & B^{-1} A & B^{-1} I \end{bmatrix} \begin{bmatrix} z \\ x \\ x_s \end{bmatrix} = \begin{bmatrix} c_B B^{-1} b \\ B^{-1} b \end{bmatrix}$$

entering variable ≥ 0 entering variable \rightarrow

Optimality Condition checking

$$\frac{c_B B^{-1} A - c}{\text{For leaving variable}}$$

For leaving variable

$$B^{-1} A = | \quad \quad \quad \frac{B^{-1} b}{\quad} > 0$$

$x_B = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$ $B = \begin{bmatrix} \vdots & \vdots \\ \vdots & \vdots \end{bmatrix} \rightarrow B^{-1}$

$B^{-1} = B \cdot c \cdot B_{old}^{-1}$

a_{12}
 a_{22}
 a_{32} → pivot element

pivot element comp

$$\begin{bmatrix} 1 & -\frac{a_{12}}{a_{22}} & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{a_{32}}{a_{22}} & 1 \end{bmatrix}$$

$$\text{Max } z = 3x_1 + 5x_2$$

$$\text{s.t. } x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

S.F.

$$z = 3x_1 + 5x_2$$

$$x_1 + x_3 = 4$$

$$2x_2 + x_4 = 12$$

$$3x_1 + 2x_2 + x_5 = 18$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 2 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c = [3 \ 5]$$

$$b = \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_s = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$z = c_B x_B + c_N x_N$$

Iteration 0

$$c = [3 \ 5]$$

$$[A \ I] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$B = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1} = I$$

$$c_B = [0 \ 0 \ 0]$$

$$z = [0 \ 0 \ 0] x_B$$

$$Z = [0 \ 0 \ 0] \overset{B^{-1}b}{\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}}}_{\text{min}} \\ = 0$$

$$C_B B^{-1} A - C = [0 \ 0 \ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 2 \end{bmatrix} - 0 \\ = \begin{bmatrix} -3 & -5 \end{bmatrix} \quad \begin{bmatrix} 3 & 5 \end{bmatrix}$$

The result is not optimum

Iteration 1: Identify the most negative cases from $C_B B^{-1} A - C$

-5 is the most negative then entering variable will be x_2

leaving variable:

$$B^{-1}A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

$$\frac{b_2}{a_{22}} = \frac{12}{2} = 6 \leftarrow \text{minimum}$$

$$\frac{b_3}{a_{32}} = \frac{18}{2} = 9 \rightarrow x_4$$

$x_1 \rightarrow$ leaving

$x_2 =$ entering

$$x_B = \begin{bmatrix} x_3 \\ x_2 \\ x_5 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

$$B_{new}^{-1} = E B_{old}^{-1}$$

$$E = \begin{bmatrix} -\frac{a_{12}}{a_{22}} & \dots \\ 1 & \frac{1}{a_{22}} \\ -\frac{a_{32}}{a_{22}} & \dots \end{bmatrix}$$

$$a_{22}$$

\rightarrow pivot element

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$E =$ unit matrix

except for the pivot column


$$E = \begin{bmatrix} -\frac{a_{13}}{a_{33}} & \dots \\ 0 & \frac{1}{a_{33}} \\ -\frac{a_{23}}{a_{33}} & \dots \end{bmatrix}$$

$$E =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$B_{new}^{-1} = E \cdot B_{old}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x_3 \ x_4 \ x_5$$


$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = B^{-1}b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 6 \end{bmatrix}$$

optimality condition

$$C_B B^{-1} A - C$$

$$\begin{bmatrix} 0 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & - \\ 0 & - \\ 3 & - \end{bmatrix} = [3, -]$$

$$= \begin{bmatrix} 0 & 5\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & - \\ 0 & - \\ 3 & - \end{bmatrix} = [3, -]$$

$$= 0 - [3, -]$$

$$= [-3, -]$$

optimality condition is not satisfied

3. Iteration 2

1. determine the entering variable x_1

2. Identify the leaving variable

$$B^{-1}A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 8 \\ 3 \end{bmatrix} =$$

$$x_B = \begin{bmatrix} x_3 \\ x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \rightarrow \begin{matrix} 4/1 = 4 \\ 5/1 = 5 \\ 6/3 = 2 \end{matrix}$$

leaving variable will be x_5

$$x_B = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} \quad x_B = B^{-1}b$$

$$r = \begin{bmatrix} -1/3 \\ 0 \\ 1/3 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$B_{\text{new}}^{-1} = E B_{\text{old}}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$$

optimality test

$$c_B B^{-1} A - c$$

non basic variable
 x_4, x_5

$$\begin{bmatrix} 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{bmatrix}$$

✓

✓

✓

$$Z = c_B x_B$$

$$= \begin{bmatrix} 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$$

$$= 36 \Rightarrow \text{optimum value}$$