BCSE Examination, 2016
(2nd Year, 2nd Semester)
MATHEMATICS - V

Attempt any five questions. Each question carries 20 marks

Time: Three Hours Full Marks 100

1. a) The sum of two non negative quantities is equal to $2n$. Find the chance that their product is not less than $\frac{3}{4}$ times their greatest product.

   b) In random arrangements of the letters of the word "ENGINEERING", what is the probability that vowels always occurs together.

   c) What is the chance that two numbers, chosen at random, will be prime to each other?

2. a) If A, B, C are mutually independent events then $A \cup B$ and C are also independent.

   b) The contents of urns I, II and III are as follows:
   1 white, 2 black and 3 red balls,
   2 white, 1 black and 1 red balls, and
   4 white, 5 black and 3 red balls.
   One urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from urns I, II or III ?

   c) Find the mean of Normal distribution($N(m, \sigma)$).

3. a) Prove that in n Bernoulli trials with probability of failure q, the probability of at most k successes is

$$\frac{\int_0^q x^{(n-k-1)}(1-x)^k dx}{\int_0^1 x^{(n-k-1)}(1-x)^k dx}.$$

   b) If X is normal in $(0, 1)$. Find the distribution of e^X.

4. a) State and prove Chebyshev's inequality.

   b) Show that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least 19/20.

5. a) Joint distribution of X and Y is given by

$$f(x, y) = 4xye^{-x^2-y^2}; \quad x \geq 0, y \geq O.$$

Test whether X and Y are independent. For the above joint distribution, find the conditional density of X given $Y = y$.

b) Show that the expectation of the number of failures preceding the first success in an infinite sequence of Bernoulli trials with probability of success p is $\frac{q}{p}$.

6. a) State and prove Central limit theorem[De Moivre's and Laplace form].

b) The diameter of an electric cable; say X, is assumed to be a continuous random variable with p.df. $f(x) = 6x(1-x), 0 \le x \le 1$.
(i) Check that above is p.d.f.
(ii) Determine a number b such that $P(X < b) = P(X > b)$

7. a) Write down the comparison between central limit theorem and weak law of large number.

b) Prove that two random variables X and Y with joint p.d.f $f(x, y)$ are stochastically independent iff $f_{X,Y}(x, y)$ can be expressed as the product of a non-negative function of x alone and a non-negative function of y alone, i.e.
$f_{X,Y}(x, y) = h_X(x)k_Y(y)$.

8. a) A telephone booth with Poisson arrivals spaced 10 minutes apart on the average and exponential call lengths averaging 3 minutes is being considered. What is the probability that an arrival will have to wait more than 10 minutes before the phone is free.

b) Derive and solve the steady state difference equations governing the queueing model $(M/M/I) : (\infty/FIFO)$.