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Frequent Itemset Mining & Association Rules

Mining of Massive Datasets

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Association Rule Discovery

Supermarket shelf management – Market-basket model:

- **Goal:** Identify items that are bought together by sufficiently many customers
- **Approach:** Process the sales data collected with barcode scanners to find dependencies among items
- **A classic rule:**
 - If someone buys diaper and milk, then he/she is likely to buy beer
 - Don't be surprised if you find six-packs next to diapers!

The Market-Basket Model

- A large set of **items**
 - e.g., things sold in a supermarket
- A large set of **baskets**
- Each basket is a **small subset of items**
 - e.g., the things one customer buys on one day
- Want to discover **association rules**
 - People who bought $\{x,y,z\}$ tend to buy $\{v,w\}$
 - Amazon!

Input:

<i>TID</i>	<i>Items</i>
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Output:

Rules Discovered:

$\{\text{Milk}\} \rightarrow \{\text{Coke}\}$

$\{\text{Diaper, Milk}\} \rightarrow \{\text{Beer}\}$

Applications – (1)

- **Items** = products; **Baskets** = sets of products someone bought in one trip to the store
- **Real market baskets:** Chain stores keep TBs of data about what customers buy together
 - Tells how typical customers navigate stores, lets them position tempting items
 - Suggests tie-in “tricks”, e.g., run sale on diapers and raise the price of beer
 - Need the rule to occur frequently, or no \$\$’s
- **Amazon’s people who bought X also bought Y**

Applications – (2)

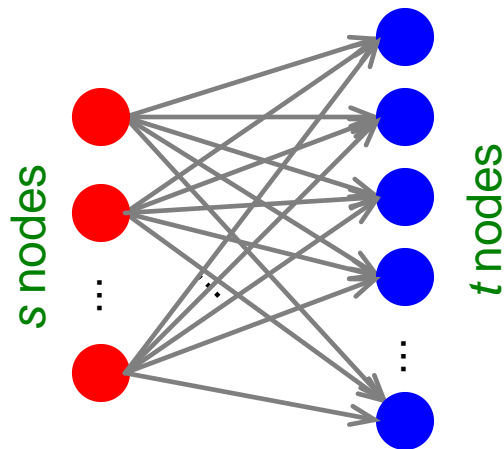
- **Baskets** = sentences; **Items** = documents containing those sentences
 - Items that appear together too often could represent plagiarism
 - Notice items do not have to be “in” baskets
- **Baskets** = patients; **Items** = drugs & side-effects
 - Has been used to detect combinations of drugs that result in particular side-effects
 - **But requires extension:** Absence of an item needs to be observed as well as presence

More generally

- **A general many-to-many mapping (association) between two kinds of things**
 - But we ask about connections among “items”, not “baskets”
- **For example:**
 - Finding communities in graphs (e.g., Twitter)

Example:

- Finding communities in graphs (e.g., Twitter)
- **Baskets** = nodes; **Items** = outgoing neighbors
 - Searching for complete bipartite subgraphs $K_{s,t}$ of a big graph



A dense 2-layer graph

■ How?

- View each node i as a basket B_i of nodes i it points to
- $K_{s,t}$ = a set Y of size t that occurs in s buckets B_i
- Looking for $K_{s,t} \rightarrow$ set of support s and look at layer t – all frequent sets of size t

Outline

First: Define

Frequent itemsets

Association rules:

Confidence, Support, Interestingness

Then: Algorithms for finding frequent itemsets

Finding frequent pairs

A-Priori algorithm

PCY algorithm + 2 refinements

Frequent Itemsets

- **Simplest question:** Find sets of items that appear together “frequently” in baskets
- **Support** for itemset I : Number of baskets containing all items in I
 - (Often expressed as a fraction of the total number of baskets)
- Given a **support threshold s** , then sets of items that appear in at least s baskets are called **frequent itemsets**

<i>TID</i>	<i>Items</i>
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Support of
 $\{\text{Beer, Bread}\} = 2$

Example: Frequent Itemsets

- **Items** = {milk, coke, pepsi, beer, juice}
- **Support threshold** = 3 baskets

$$B_1 = \{m, c, b\}$$

$$B_2 = \{m, p, j\}$$

$$B_3 = \{m, b\}$$

$$B_4 = \{c, j\}$$

$$B_5 = \{m, p, b\}$$

$$B_6 = \{m, c, b, j\}$$

$$B_7 = \{c, b, j\}$$

$$B_8 = \{b, c\}$$

- **Frequent itemsets:** {m}, {c}, {b}, {j}, {m,b} , {b,c} , {c,j}.

Association Rules

- **Association Rules:**

If-then rules about the contents of baskets

- $\{i_1, i_2, \dots, i_k\} \rightarrow j$ means: “if a basket contains all of i_1, \dots, i_k then it is *likely* to contain j ”

- **In practice there are many rules, want to find significant/interesting ones!**

- **Confidence** of this association rule is the probability of j given $I = \{i_1, \dots, i_k\}$

$$\text{conf}(I \rightarrow j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

Interesting Association Rules

- **Not all high-confidence rules are interesting**
 - The rule $X \rightarrow \textit{milk}$ may have high confidence for many itemsets X , because milk is just purchased very often (independent of X) and the confidence will be high
- **Interest** of an association rule $I \rightarrow j$:
difference between its confidence and the fraction of baskets that contain j
$$\text{Interest}(I \rightarrow j) = \text{conf}(I \rightarrow j) - \text{Pr}[j]$$
 - Interesting rules are those with high positive or negative interest values (usually above 0.5)

Example: Confidence and Interest

$$B_1 = \{m, c, b\}$$

$$B_2 = \{m, p, j\}$$

$$B_3 = \{m, b\}$$

$$B_4 = \{c, j\}$$

$$B_5 = \{m, p, b\}$$

$$B_6 = \{m, c, b, j\}$$

$$B_7 = \{c, b, j\}$$

$$B_8 = \{b, c\}$$

- Association rule: $\{m, b\} \rightarrow c$
 - **Confidence** = $2/4 = 0.5$
 - **Interest** = $|0.5 - 5/8| = 1/8$
 - Item c appears in $5/8$ of the baskets
 - Rule is not very interesting!

Finding Association Rules

- **Problem:** Find all association rules with support $\geq s$ and confidence $\geq c$
 - **Note:** Support of an association rule is the support of the set of items on the left side
- **Hard part:** Finding the frequent itemsets!
 - If $\{i_1, i_2, \dots, i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2, \dots, i_k\}$ and $\{i_1, i_2, \dots, i_k, j\}$ will be “frequent”

$$\text{conf}(I \rightarrow j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

Mining Association Rules

- **Step 1:** Find all frequent itemsets I
 - (we will explain this next)
- **Step 2: Rule generation**
 - For every subset A of I , generate a rule $A \rightarrow I \setminus A$
 - Since I is frequent, A is also frequent
 - **Variant 1:** Single pass to compute the rule confidence
 - $\text{confidence}(A, B \rightarrow C, D) = \text{support}(A, B, C, D) / \text{support}(A, B)$
 - **Variant 2:**
 - **Observation:** If $A, B, C \rightarrow D$ is below confidence, so is $A, B \rightarrow C, D$
 - Can generate “bigger” rules from smaller ones!
 - **Output the rules above the confidence threshold**

Example

$$B_1 = \{m, c, b\}$$

$$B_2 = \{m, p, j\}$$

$$B_3 = \{m, c, b, n\}$$

$$B_4 = \{c, j\}$$

$$B_5 = \{m, p, b\}$$

$$B_6 = \{m, c, b, j\}$$

$$B_7 = \{c, b, j\}$$

$$B_8 = \{b, c\}$$

- Support threshold $s = 3$, confidence $c = 0.75$

- 1) Frequent itemsets:

- $\{b, m\}$ $\{b, c\}$ $\{c, m\}$ $\{c, j\}$ $\{m, c, b\}$

- 2) Generate rules:

- ~~$b \rightarrow m: c=4/6$~~ $b \rightarrow c: c=5/6$ ~~$b, c \rightarrow m: c=3/5$~~
- $m \rightarrow b: c=4/5$... $b, m \rightarrow c: c=3/4$
- ~~$b \rightarrow c, m: c=3/6$~~

Compacting the Output

- To reduce the number of rules we can post-process them and only output:

- **Maximal frequent itemsets:**

No immediate superset is frequent

- Gives more pruning

or

- **Closed itemsets:**

No immediate superset has the same count (> 0)

- Stores not only frequent information, but exact counts

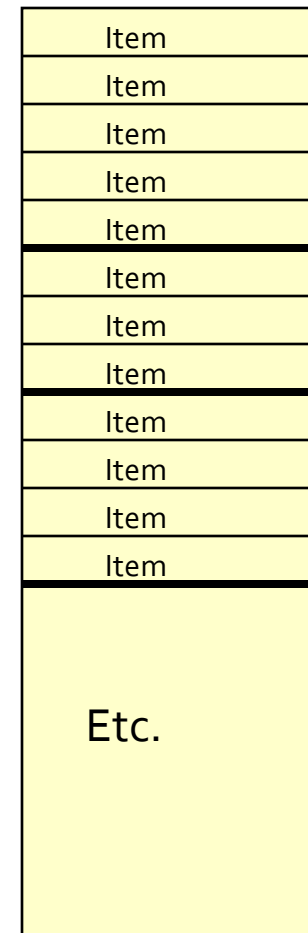
Example: Maximal/Closed

	Support	Maximal(s=3)	Closed	
A	4	No	No	Frequent, but superset BC also frequent.
B	5	No	Yes	Frequent, and its only superset, ABC, not freq.
C	3	No	No	Superset BC has same count.
AB	4	Yes	Yes	
AC	2	No	No	Its only super- set, ABC, has smaller count.
BC	3	Yes	Yes	
ABC	2	No	Yes	

Finding Frequent Itemsets

Itemsets: Computation Model

- **Back to finding frequent itemsets**
- Typically, data is kept in flat files rather than in a database system:
 - Stored on disk
 - Stored basket-by-basket
 - Baskets are **small** but we have many baskets and many items
 - Expand baskets into pairs, triples, etc. as you read baskets
 - Use **k** nested loops to generate all sets of size **k**



Items are positive integers, and boundaries between baskets are -1.

Note: We want to find frequent itemsets. To find them, we have to count them. To count them, we have to generate them.

Computation Model

- The true cost of mining disk-resident data is usually the **number of disk I/Os**
- In practice, association-rule algorithms read the data in *passes* – all baskets read in turn
- We measure the cost by the **number of passes** an algorithm makes over the data

Main-Memory Bottleneck

- For many frequent-itemset algorithms, **main-memory** is the critical resource
 - As we read baskets, we need to count something, e.g., occurrences of pairs of items
 - The number of different things we can count is limited by main memory
 - Swapping counts in/out is a disaster (**why?**)

Finding Frequent Pairs

- The hardest problem often turns out to be finding the frequent **pairs** of items $\{i_1, i_2\}$
 - **Why?** Freq. pairs are common, freq. triples are rare
 - **Why?** Probability of being frequent drops exponentially with size; number of sets grows more slowly with size
- **Let's first concentrate on pairs, then extend to larger sets**
- **The approach:**
 - We always need to generate all the itemsets
 - But we would only like to count (keep track) of those itemsets that in the end turn out to be frequent

Naïve Algorithm

- **Naïve approach to finding frequent pairs**
- Read file once, counting in main memory the occurrences of each pair:
 - From each basket of n items, generate its $n(n-1)/2$ pairs by two nested loops
- **Fails if $(\text{\#items})^2$ exceeds main memory**
 - **Remember:** \#items can be 100K (Wal-Mart) or 10B (Web pages)
 - Suppose 10^5 items, counts are 4-byte integers
 - Number of pairs of items: $10^5(10^5-1)/2 = 5 \cdot 10^9$
 - Therefore, $2 \cdot 10^{10}$ (20 gigabytes) of memory needed

Counting Pairs in Memory

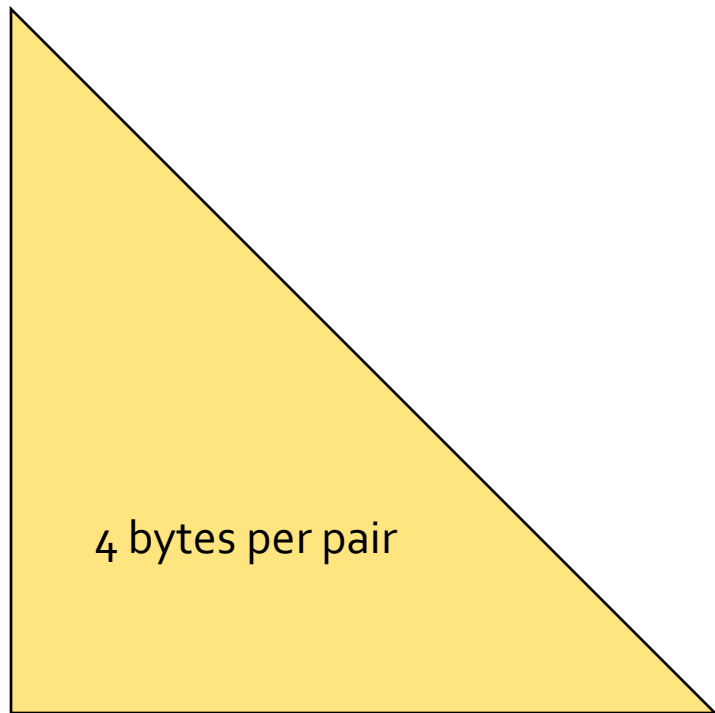
Two approaches:

- **Approach 1:** Count all pairs using a matrix
- **Approach 2:** Keep a table of triples $[i, j, c]$ = “the count of the pair of items $\{i, j\}$ is c .”
 - If integers and item ids are 4 bytes, we need approximately 12 bytes for pairs with count > 0
 - Plus some additional overhead for the hashtable

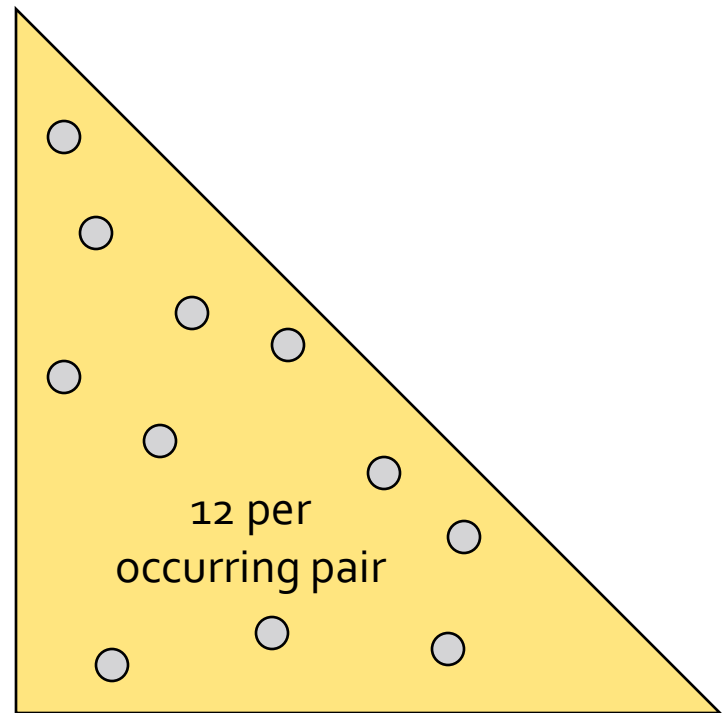
Note:

- **Approach 1** only requires 4 bytes per pair
- **Approach 2** uses 12 bytes per pair (but only for pairs with count > 0)

Comparing the 2 Approaches



Triangular Matrix



Triples

Comparing the two approaches

- **Approach 1: Triangular Matrix**
 - n = total number items
 - Count pair of items $\{i, j\}$ only if $i < j$
 - Keep pair counts in lexicographic order:
 - $\{1,2\}, \{1,3\}, \dots, \{1,n\}, \{2,3\}, \{2,4\}, \dots, \{2,n\}, \{3,4\}, \dots$
 - Pair $\{i, j\}$ is at position $(i-1)(n-i/2) + j-1$
 - Total number of pairs $n(n-1)/2$; total bytes = $2n^2$
 - **Triangular Matrix** requires 4 bytes per pair
- **Approach 2** uses **12 bytes** per occurring pair (*but only for pairs with count > 0*)
 - Beats Approach 1 if less than **1/3** of possible pairs actually occur

Comparing the two approaches

■ Approach 1: Triangular Matrix

- n = total number items

- Co

- k

- P

- T

- T

■ Approach 2

(but)

- Beats Approach 1 if less than $1/3$ of possible pairs actually occur

Problem is if we have too many items so the pairs do not fit into memory.

Can we do better?

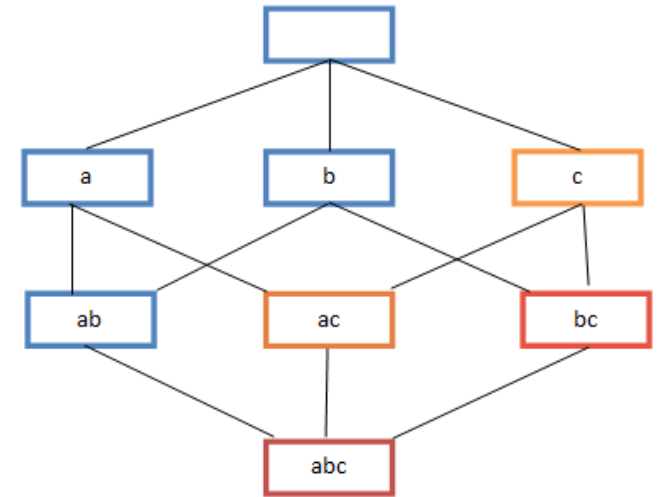
$2n^2$

A-Priori Algorithm

A-Priori Algorithm – (1)

- A **two-pass** approach called ***A-Priori*** limits the need for main memory
- **Key idea: *monotonicity***
 - If a set of items I appears at least s times, so does every **subset J** of I
- **Contrapositive for pairs:**

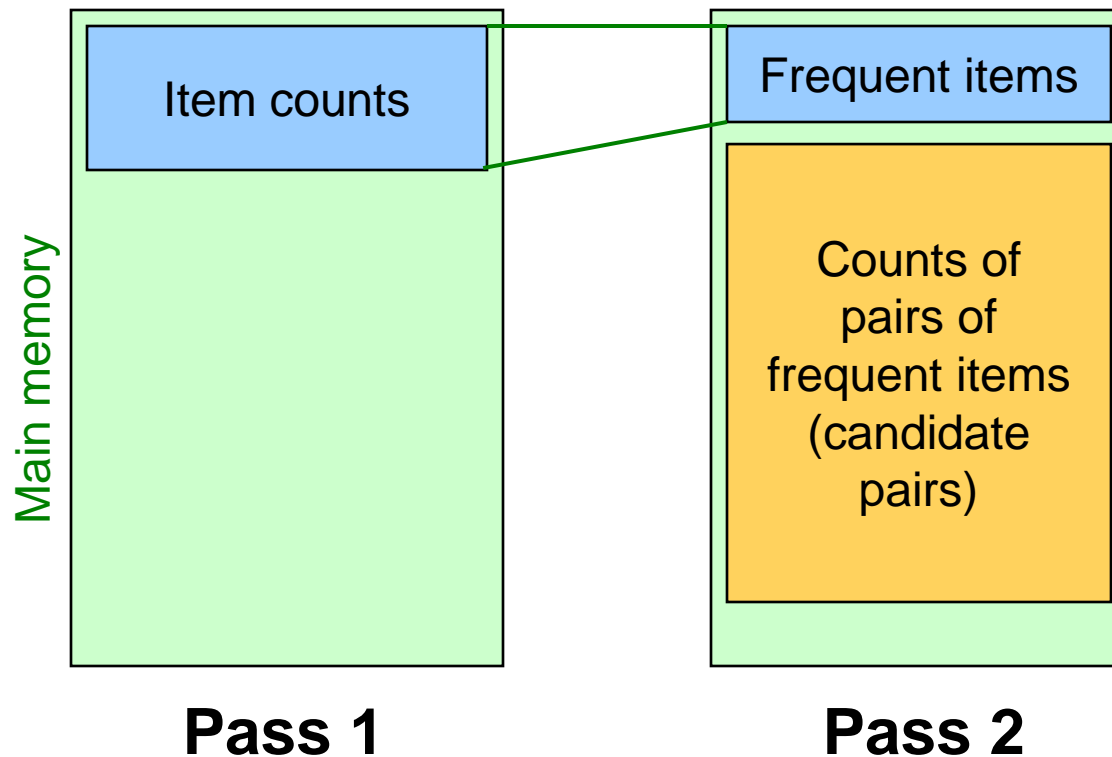
If item i does not appear in s baskets, then no pair including i can appear in s baskets
- **So, how does A-Priori find freq. pairs?**



A-Priori Algorithm – (2)

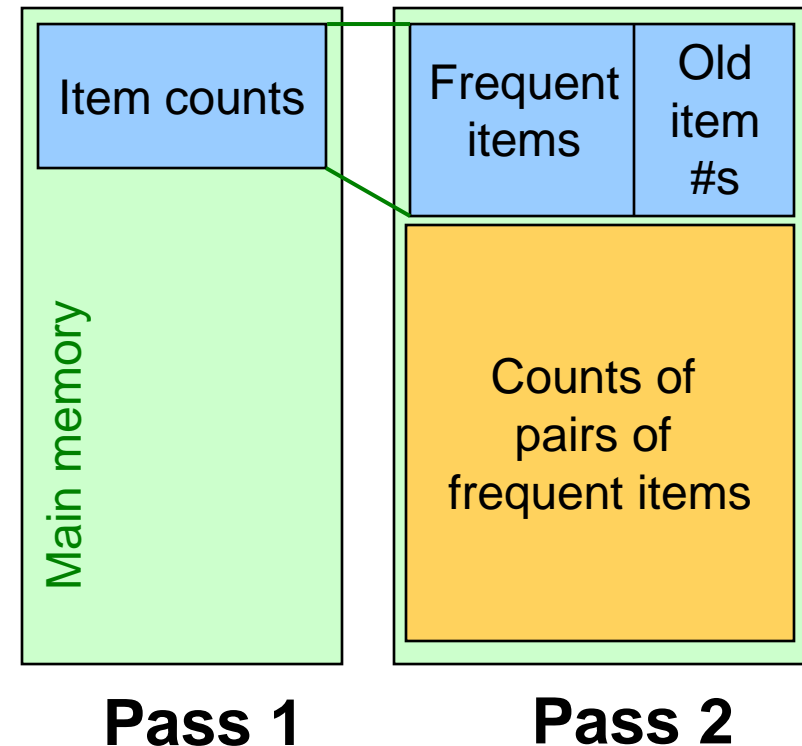
- **Pass 1:** Read baskets and count in main memory the occurrences of each **individual item**
 - Requires only memory proportional to #items
- **Items that appear $\geq s$ times are the frequent items**
- **Pass 2:** Read baskets again and count in main memory only those pairs where both elements are frequent (from Pass 1)
 - Requires memory proportional to square of **frequent** items only (for counts)
 - Plus a list of the frequent items (so you know what must be counted)

Main-Memory: Picture of A-Priori



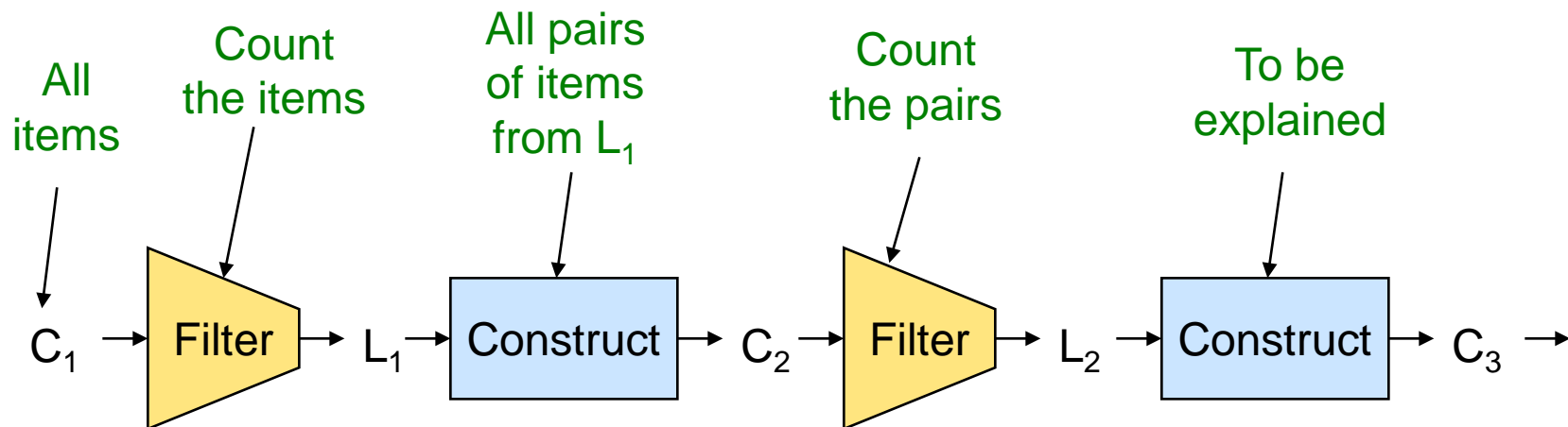
Detail for A-Priori

- You can use the triangular matrix method with n = number of frequent items
 - May save space compared with storing triples
- **Trick:** re-number frequent items 1,2,... and keep a table relating new numbers to original item numbers



Frequent Triples, Etc.

- For each k , we construct two sets of k -tuples (sets of size k):
 - C_k = *candidate k -tuples* = those that might be frequent sets (support $\geq s$) based on information from the pass for $k-1$
 - L_k = the set of truly frequent k -tuples



Example

** Note here we generate new candidates by generating C_k from L_{k-1} and L_1 .

But that one can be more careful with candidate generation. For example, in C_3 we know $\{b,m,j\}$ cannot be frequent since $\{m,j\}$ is not frequent

■ Hypothetical steps of the A-Priori algorithm

- $C_1 = \{ \{b\} \{c\} \{j\} \{m\} \{n\} \{p\} \}$
- Count the support of itemsets in C_1
- Prune non-frequent: $L_1 = \{ b, c, j, m \}$
- Generate $C_2 = \{ \{b,c\} \{b,j\} \{b,m\} \{c,j\} \{c,m\} \{j,m\} \}$
- Count the support of itemsets in C_2
- Prune non-frequent: $L_2 = \{ \{b,m\} \{b,c\} \{c,m\} \{c,j\} \}$
- Generate $C_3 = \{ \{b,c,m\} \{b,c,j\} \{b,m,j\} \{c,m,j\} \}$ **
- Count the support of itemsets in C_3
- Prune non-frequent: $L_3 = \{ \{b,c,m\} \}$

A-Priori for All Frequent Itemsets

- One pass for each k (itemset size)
- Needs room in main memory to count each candidate k -tuple
- For typical market-basket data and reasonable support (e.g., 1%), $k = 2$ requires the most memory
- **Many possible extensions:**
 - Association rules with intervals:
 - For example: Men over 65 have 2 cars
 - Association rules when items are in a taxonomy
 - Bread, Butter \rightarrow FruitJam
 - BakedGoods, MilkProduct \rightarrow PreservedGoods
 - Lower the support s as itemset gets bigger

PCY (Park-Chen-Yu) Algorithm

PCY (Park-Chen-Yu) Algorithm

- **Observation:**

In pass 1 of A-Priori, most memory is idle

- We store only individual item counts
- **Can we use the idle memory to reduce memory required in pass 2?**

- **Pass 1 of PCY:** In addition to item counts, maintain a hash table with as many buckets as fit in memory

- Keep a **count** for each bucket into which **pairs** of items are hashed
 - **For each bucket just keep the count, not the actual pairs that hash to the bucket!**

PCY Algorithm – First Pass

```
FOR (each basket) :  
    FOR (each item in the basket) :  
        add 1 to item's count;  
New in PCY { FOR (each pair of items) :  
                hash the pair to a bucket;  
                add 1 to the count for that bucket;
```

■ Few things to note:

- Pairs of items need to be generated from the input file; they are not present in the file
- We are not just interested in the presence of a pair, but we need to see whether it is present at least s (support) times

Observations about Buckets

- **Observation:** If a bucket contains a **frequent pair**, then the bucket is surely **frequent**
- However, even without any frequent pair, a bucket can still be frequent 😞
 - So, we cannot use the hash to eliminate any member (pair) of a “frequent” bucket
- **But, for a bucket with total count less than s , none of its pairs can be frequent 😊**
 - Pairs that hash to this bucket can be eliminated as candidates (even if the pair consists of 2 frequent items)
- **Pass 2:**
Only count pairs that hash to frequent buckets

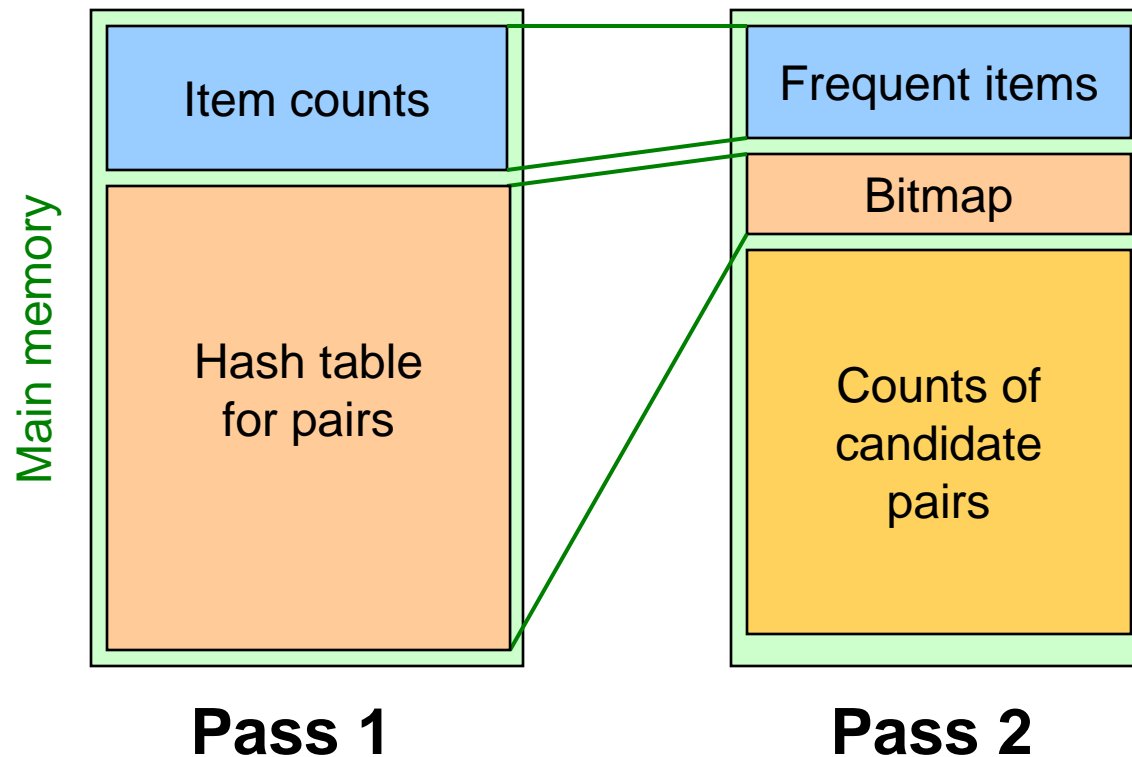
PCY Algorithm – Between Passes

- **Replace the buckets by a bit-vector:**
 - **1** means the bucket count exceeded the support s (call it a **frequent bucket**); **0** means it did not
- **4-byte integer counts are replaced by bits, so the bit-vector requires 1/32 of memory**
- **Also, decide which items are frequent and list them for the second pass**

PCY Algorithm – Pass 2

- Count all pairs $\{i, j\}$ that meet the conditions for being a **candidate pair**:
 1. Both i and j are frequent items
 2. The pair $\{i, j\}$ hashes to a bucket whose bit in the bit vector is **1** (i.e., a **frequent bucket**)
- **Both conditions are necessary for the pair to have a chance of being frequent**

Main-Memory: Picture of PCY



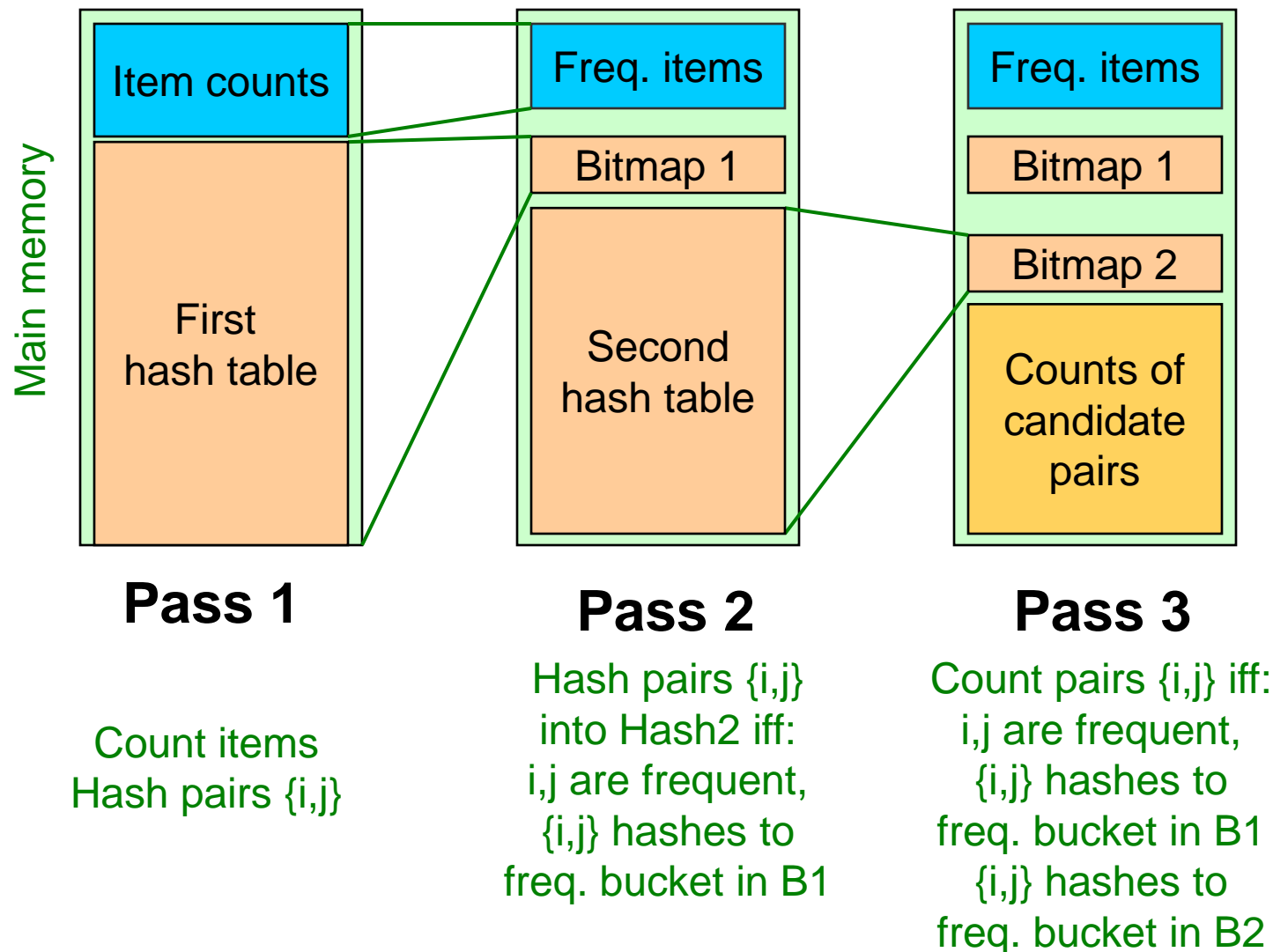
Main-Memory Details

- Buckets require a few bytes each:
 - **Note:** we do not have to count past s
 - #buckets is $O(\text{main-memory size})$
- On second pass, a table of (item, item, count) triples is essential (we cannot use triangular matrix approach, why?)
 - Thus, hash table must eliminate approx. 2/3 of the candidate pairs for PCY to beat A-Priori

Refinement: Multistage Algorithm

- **Limit the number of candidates to be counted**
 - **Remember:** Memory is the bottleneck
 - Still need to generate all the itemsets but we only want to count/keep track of the ones that are frequent
- **Key idea:** After Pass 1 of PCY, rehash only those pairs that **qualify** for Pass 2 of PCY
 - i and j are frequent, and
 - $\{i, j\}$ hashes to a frequent bucket from **Pass 1**
- On middle pass, fewer pairs contribute to buckets, so fewer **false positives**
- **Requires 3 passes over the data**

Main-Memory: Multistage



Multistage – Pass 3

- Count only those pairs $\{i, j\}$ that satisfy these **candidate pair conditions**:
 1. Both i and j are frequent items
 2. Using the first hash function, the pair hashes to a bucket whose bit in the first bit-vector is **1**
 3. Using the second hash function, the pair hashes to a bucket whose bit in the second bit-vector is **1**

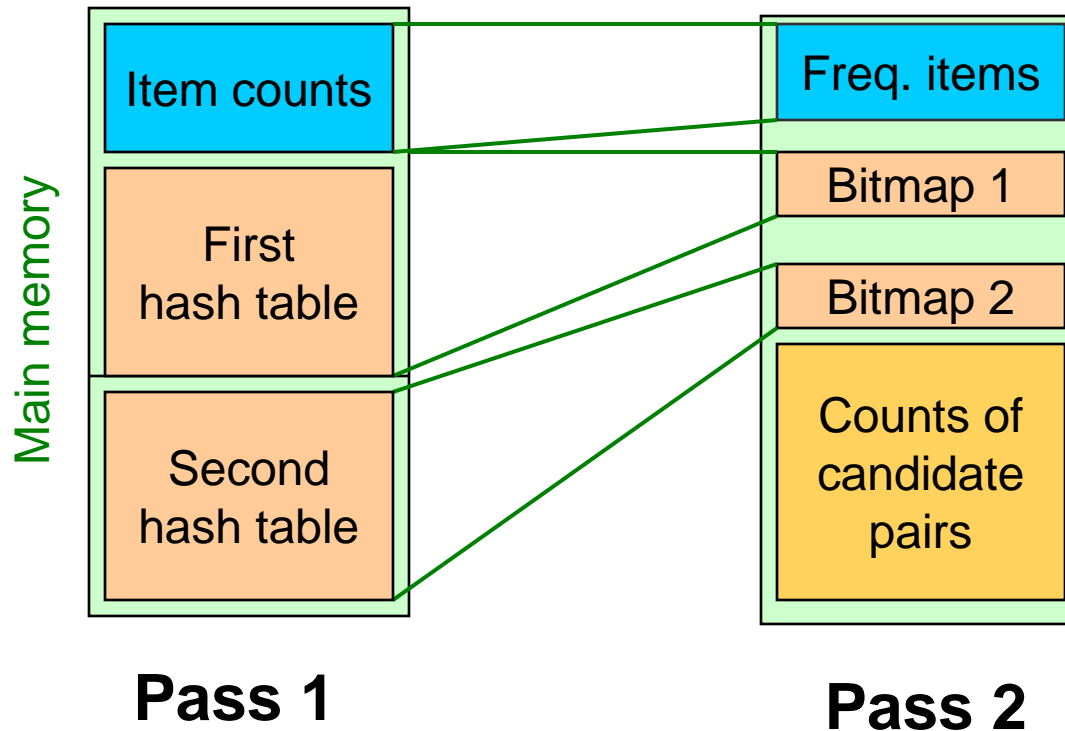
Important Points

1. The two hash functions have to be independent
2. We need to check both hashes on the third pass
 - If not, we would end up counting pairs of frequent items that hashed first to an infrequent bucket but happened to hash second to a frequent bucket

Refinement: Multihash

- **Key idea:** Use several independent hash tables on the first pass
- **Risk:** Halving the number of buckets doubles the average count
 - We have to be sure most buckets will still not reach count s
- If so, we can get a benefit like multistage, but in only 2 passes

Main-Memory: Multihash



PCY: Extensions

- Either **multistage** or **multihash** can use more than two hash functions
- In **multistage**, there is a point of diminishing returns, since the bit-vectors eventually consume all of main memory
- For **multihash**, the bit-vectors occupy exactly what one PCY bitmap does, but too many hash functions makes all counts $\geq s$

Frequent Itemsets in ≤ 2 Passes

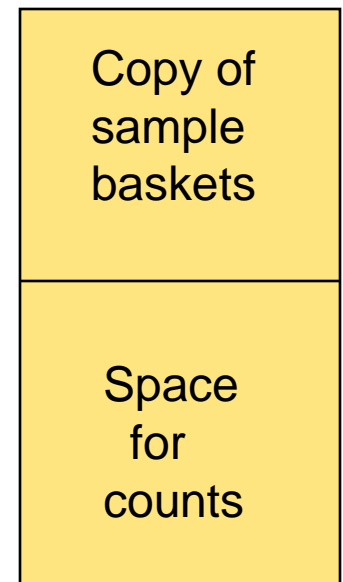
Frequent Itemsets in ≤ 2 Passes

- A-Priori, PCY, etc., take k passes to find frequent itemsets of size k
- Can we use fewer passes?
- Use 2 or fewer passes for all sizes, but may miss some frequent itemsets
 - Random sampling
 - SON (Savasere, Omiecinski, and Navathe)
 - Toivonen (see textbook)

Random Sampling (1)

- Take a random sample of the market baskets
- Run a-priori or one of its improvements in main memory
 - So we don't pay for disk I/O each time we increase the size of itemsets
 - Reduce support threshold proportionally to match the sample size

Main memory



Random Sampling (2)

- Optionally, verify that the candidate pairs are truly frequent in the entire data set by a second pass (avoid false positives)
- But you don't catch sets frequent in the whole but not in the sample
 - Smaller threshold, e.g., $s/125$, helps catch more truly frequent itemsets
 - But requires more space

SON Algorithm – (1)

- Repeatedly read small subsets of the baskets into main memory and run an in-memory algorithm to find all frequent itemsets
 - Note: we are not sampling, but processing the entire file in memory-sized chunks
- An itemset becomes a candidate if it is found to be frequent in *any* one or more subsets of the baskets.

SON Algorithm – (2)

- On a **second pass**, count all the candidate itemsets and determine which are frequent in the entire set
- **Key “monotonicity” idea:** an itemset cannot be frequent in the entire set of baskets unless it is frequent in at least one subset.

SON – Distributed Version

- SON lends itself to distributed data mining
- Baskets distributed among many nodes
 - Compute frequent itemsets at each node
 - Distribute candidates to all nodes
 - Accumulate the counts of all candidates

SON: Map/Reduce

- **Phase 1:** Find candidate itemsets
 - Map?
 - Reduce?
- **Phase 2:** Find true frequent itemsets
 - Map?
 - Reduce?