

B.TECH INSTRUMENTATION AND ELECTRONICS ENGINEERING
SECOND YEAR SECOND SEMESTER – 2018

Subject : DIGITAL SIGNAL PROCESSING

Time : Three hours Full Marks :100

Answer any ten. All questions carry equal marks.

1. a) Determine the impulse response of a causal linear time invariant system which produces the output signal

$$y(n) = 6\delta(n) + 10\delta(n-1) - 9\delta(n-2) - 14\delta(n-3) + 5\delta(n-4) - \delta(n-6)$$

when excited by the input signal $x(n) = 3\delta(n) + 5\delta(n-1) - \delta(n-3)$

- b) Consider a sinusoidal signal sampled at 8000 Hz. This signal is applied to a system which produces the following output signal

$$y(n) = x(n) \text{ when } n \text{ is even integer} \\ = 0 \text{ otherwise.}$$

What is the frequency of the output signal when the input signal is 400Hz?

2. a) Consider a causal LTI system whose output is

$$y(n) - (3/4)y(n-1) + (1/8)y(n-2) = 2x(n)$$

determine its impulse response.

If the input to this system is $x(n) = (1/4)^n u(n)$. What will be its output?

- b) What is the stability criterion of a discrete time system?

3. a) Consider the finite sequence

$$x(n) = 1 \text{ for } -1 \leq n \leq 1 \\ = 0 \text{ otherwise}$$

Determine its DTFT and DFT. Plot them.

- b) Find the time domain signal $x(n)$ given its Fourier series coefficients

$$X(k) = j\delta(k-1) - j\delta(k+1) + \delta(k-3) + \delta(k+3) \text{ and } \omega_0 = \pi.$$

Repeat the problem for $\omega_0 = 3\pi$.

4. Find out whether the system $y(n) = x^2(n)u(n)$ is

a) linear, b) causal, c) stable, d) time invariant, d) memory less.

5. a) Find the convolution of $x_1(n) = \{1 \ 2 \ 3\}$ with $x_2(n) = \{2 \ 2 \ 5\}$ graphically. Origin is at 2nd instant.

- b) Express the following signal as weighted sum of time shifted impulses

$$x(n) = 1 \text{ for } n = \pm 1, \pm 3 \\ = 2 \text{ for } n = \pm 2 \\ = 0 \text{ otherwise.}$$

6. a) Prove that $x(n) * \delta(n-m) = x(n-m)$.

- b) Convolute $x(n) = n+1$ for $n = 0, 1, 2$
 $= 0$ otherwise

with $h(n) = 2^n u(n)$.

7. Use the z-transform to perform the convolution of the following two sequences

$$h(n) = \begin{cases} (1/2)^n & 0 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{And } x(n) = \delta(n) + \delta(n-1) + 4\delta(n-2).$$

8. An FIR filter is given by the system function

$$H(z) = 1 + 16.0625z^{-4} + z^{-8}.$$

Draw the filter structure.

9. Design a digital Butterworth filter to meet the constraint
- $$\begin{aligned} 0.9 \leq |H(e^{j\omega})| &\leq 1, 0 \leq \omega \leq \pi/2 \\ |H(e^{j\omega})| &\leq 0.2, 3\pi/4 \leq \omega \leq \pi \end{aligned}$$
10. Design a single pole low pass digital filter with a 3-dB bandwidth of 0.2π , using the bilinear transformation applied to the analog filter
- $$H(s) = \Omega_c / (s + \Omega_c)$$
- where Ω_c is the 3dB bandwidth of the analog filter.
11. A low pass filter is to be designed with the following desired frequency response
- $$\begin{aligned} H_d(e^{j\omega}) &= e^{-2j\omega}, -\pi/4 \leq \omega \leq \pi/4 \\ &= 0, \pi/4 < \omega \leq \pi \end{aligned}$$
- Determine the filter coefficient $h_d(n)$ if the window function is defined as
- $$\begin{aligned} w(n) &= 1, 0 \leq n \leq 4 \\ &= 0, \text{ otherwise.} \end{aligned}$$
12. Write short notes on: (any two)
- Merits and demerits of DFT method.
 - Process of Sampling an analog signal and its reconstruction
 - Comparison between linear convolution and circular convolution.
 - Phase delay and group delay.
- Compare Fourier transform, Laplace Transform and Z-transform.