b) Use the frequency distribution of heights in the following table to find the mean height and median height of 100 students at the Department of Mechanical Engineering :

Height (inches)	Frequency (f)
60 - 62	5
63 - 65	18
66 - 68	42
69 - 71	27
72 - 79	8

4+6

Ex/ME/Math/T/221/2018

BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING EXAMINATION, 2018

(2nd Year, 2nd Semester)

MATHEMATICS - IV

Time : Three hours | Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer ***any five*** questions.

1. a) If $S = \{(x, yz) \in \mathbb{R}^3 / y = z = 0\}$, cheek whether S is a subspace of $\mathbb{R}^3$ or not.

   b) Let $W_1 = \{(x, y) \in \mathbb{R}^2 / y = 0\}$ and $W_2 = \{(x, y) \in \mathbb{R}^2 / x = 0\}$ be two subspaces $\mathbb{R}^2$.

   Show that $W_1 \cup W_2$ is not a subspace of $\mathbb{R}^2$, but $W_1 \cap W_2$ is a subspace of $\mathbb{R}^2$. 4+6

2. State and prove Replacement theorem. 2+8

3. a) Find a basis and the dimension of the subspace W of $\mathbb{R}^3$, Where $W = \{(x, y, z) \in \mathbb{R}^3 / x + y + z = 0\}$.

   b) State and prove parallelogram theorem for inner product space. 6+4

4. Let U and W be two subspaces of a finite dimensional vector space V over $\mathbb{R}$, then prove that

$$\dim(U+W) = \dim U + \dim W - \dim(U \cap W)$$ 10

5. a) If $\{\alpha_1, \alpha_2, \alpha_3\}$ is a basis of a real vector space V and $\beta_1 = \alpha_1 + \alpha_3$, $\beta_2 = 2\alpha_1 + 3\alpha_2 + 4\alpha_3$, $\beta_3 = \alpha_1 + 2\alpha_2 + 3\alpha_3$, then prove that $\{\beta_1, \beta_2, \beta_3\}$ is also a basis of V.

   b) Define orthogonal vectors in a Euclidean space V. If α, β are two orthogonal vectors in V, then prove that $\| \alpha + \beta \|^2 = \| \alpha \|^2 + \| \beta \|^2$. 5+5

6. Obtain an orthonormal basis of $\mathbb{R}^3$ using Gram-schmidt process for the vectors {(1, 0, 1), (1, 0, –1), (0, 3, 4)} explaining the process. 10

7. a) Prove that $\{(x, y, z) \in \mathbb{R}^3 / z^2 = x^2 + y^2\}$ is not a subspace of $\mathbb{R}^3$.

   b) Write a standard basis of $\mathbb{R}^3$ and express any vector in terms of standard basis.

   c) Check whether {(1, 2, 3), (4, 5, 6), (7, 8, 9)} is a basis of $\mathbb{R}^3$ or not.

13. a) The personal manager of the factory wants to find a measure which he can use to fix the monthly income of persons applying for a job in production department. As an experiment project he collected data on 7 persons from that department referring to years of service and their monthly income :

Years of service (x)	11	7	9	5	8	6	10
Income in Rs. 100 (y)	10	8	6	5	9	7	11

   i) Find $\bar{x}$ and $\bar{y}$.

   ii) Find the regression equation of y on x.

   b) Given a set of paired data (X, Y).

   i) If Y is independent of X, then what value of a correction coefficient would you expect ?

   ii) If Y is linearly dependent on X, then what value of a correlation coefficient would you expect ?

   iii) How could Y be closely dependent upon X yet correlation coefficient is 0 ? 7+3

14. a) Let X be an exponential random variable with pdf

$$f_X(x) = \begin{cases} e^{-x}, & x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

   Use Tehbyshev's inequality to bound $P(X \ge 3)$.

this output 5, 4 and 2 pieces are defective bolts. A bolt is drawn at random from the product and is found defective. What is the probability that it was manuactured by machine B ? 5+5

11. a) Define random variable. Use your definition to prove that the following function X is a random variable :

S = {HH, TT, HT, TH}, Δ = class of all subsets of S and X is defined as

X (w) : = number of H's in W, where $W \in S$.

b) The random variable X has the probability density function

$$f(x) = \begin{cases} \frac{1}{4}, & -2 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probabilities

(i) $P(X < 1)$, (ii) $P(|X| > 1)$. 5+5

12. a) Find the mode and variance of Binomial distribution having parameters n and p.

b) The probability of a man hitting a target is $\frac{1}{4}$. How many times he should fire so that the probability of his hitting the target at least once is greater than $\frac{2}{3}$? 5+5

PART - II

Symbols/Notations have their usual meanings.

Answer ***any five*** questions.

8. a) Give classical definition and axiomatic definition of probability.

b) Use axiomatic definition of probability to show that the probability space (Ω, S, P) with $H \in S$ and $P(H) \neq 0$,

$$P_H(A) = \{P(A/H) : A \in S\}$$

forms a probability space. 4+6

9. a) Given three events A, B, C with $P(A \cap B \cap C) = 0$, then show that

P(X/C) = P(A/C) + P(B/C),

where X = $A \cup B$.

b) Two players A and B alternately throw a pair of die ; A wins if A throws 6 before B throws 7, and B wins if B throws 7 before A throws 6. If A begins, then find the probability that A wins. 5+5

10. a) State and prove Baye's theorem.

b) In a bolt factory, machines A, B, C manufacture 25, 35 and 40 pieces of the total production, respectively. Of