1) A and B toss a coin alternately and the first to obtain a head wins the toss. If A starts the game, find the probability of his winning.

$\boxed{\text{ans:- } 2/3}$

2) An urn contains n tickets numbered 1 to n, from which a ticket is drawn and replaced r times. What is the prob that the greatest number drawn is i? ans $\left[i^r - (i-1)^r\right]/n^r$

3) Find the prob that a natural number chosen at random from the set $\{1, 2 - N\}$ is divisible by a fixed natural number K.

$\boxed{\text{ans } 1/K}$

4) Two numbers x and y are chosen without replacement from the set $\{1, 2, - N\}$. Find the probability that $|x-y| \geq m$, a fixed natural number.

$\boxed{\text{ans } \dfrac{(N-m)(N-m+1)}{N(N-1)}}$

5) The prob of detecting tuberculosis in X-ray examination of a person suffering from the disease is $1-b$. The prob. of diagnosing a healthy person as tubercular is a. If the ratio of tubercular patients to the whole population is c. Then show that the probability (prob) that a person is healthy if after examination he is diagnosed as tubercular is $\dfrac{a(1-c)}{(1-b)c + a(1-c)}$

6) One worker can manufacture 120 articles during a shift, another worker 140 articles, the probabilities of the articles being of a high quality are 0·94 and 0·8 respectively. Determine the most probable number of high quality articles manufactured by each worker.

~~ans 111 ≤ i ≤ 1~~

ans : = 112

7) In Banach's match-box problem, find the probability that when the first box is just emptied (ie the last match is drawn from the first box) the second box contains exactly i matches.

$\boxed{\underline{\text{ans}} \quad \binom{2n-i-1}{n-1}\left(\frac{1}{2}\right)^{2n-i}}$

8) A player repeatedly throws a coin and scores one point for a head and two points for a tail. ~~P~~ ~~Pn~~ If p_n denotes the probability of scoring n points, then show that $2p_n = p_{n-1} + p_{n-2}$

Hence deduce ~~as~~ an expression for p_n, and find ~~ans~~ its limiting value as n tends to infinity.

$\boxed{\underline{\text{ans}} \quad p_n = \frac{1}{3}\left\{2 + (-1)^n/2^n\right\} \to 2/3 \text{ as } n \to \infty}$

9) If n objects are distributed at random among a men and $b\,(<a)$ women, then show that the probability that the women will get an odd number of objects is

$$\left\{(a+b)^n - (a-b)^n\right\}/2(a+b)^n$$

10) A and B alternately throw a pair of dice. A starting the game. A wins if ~~the throws~~ he throws six before B throws seven, and B wins if he throws seven before A throws six. What is the prob. of A's winning.

$\boxed{\underline{\text{ans}} \quad 30/61}$

11) If a die is thrown n times, show that the prob. of an even number of sixes is $\left\{1+\left(\frac{2}{3}\right)^n\right\}/2$.

12) Let $F(x)$ be a distribution func. Prove that for any fixed $h \neq 0$, the func

$$G(x) = \frac{1}{2h}\int_{x-h}^{x+h} F(t)\,dt$$ is also a distribution func.

[Hint. To show $G(x)$ a distribution func, try to prove that $G(x)$ satisfies the properties of distribution func. ie $G(x)$ is monotonic non decreasing, $G(\infty)=1$, $G(-\infty)=0$]

13) The prob. mass func of a random variable X is given by $p(i) = c\lambda^i/i!$, $i = 0, 1, 2 \ldots$, where λ is some positive value

1. Find $P\{X=0\}$.
2. Find $P\{X=1\}$.

14) Three balls are randomly chosen from an urn containing 3 white, 3 red, 5 black balls. Suppose that we win \$1 for each white ball selected and loss \$1 for each red selected. If we let X denote our total winnings from the experiment, then ~~X is a random~~ What is the probability that we win money? $\boxed{\text{ans:- } \frac{1}{3}}$

15) Three balls are to be randomly selected without replacement from an urn containing 20 balls numbered 1 through 20. If we bet that at least one of the drawn balls has a number as large or larger than ~~17~~ 17, what is the prob. that we win the bet.

ans: 0.508

ans: 0.508

16) Suppose that an airplane engine will fail, when in flight, with probability $1-p$ independently from engine to engine; suppose that the airplane will make a successful flight if atleast 50% of its engines remain operative. For what values of p is a 4-engine plane preferable to a 2-engine plane?

ans: 4-engine plane is safer when $p \geq 2/3$
2-engine plane is safer when $p < 2/3$

17) In a certain daycare class, 30% of the children have grey eyes 50% of them have blue and the other 20% eyes are in other colours. One day they play a game together. In the 1st run, 65% of the grey eyed ones, 82% of the blue eyed ones and 50% of the children with other eye colour were selected. Now, if a child is selected randomly from the class, and we know that he/she was not in the 1st game, what is the prob. that the child has blue eyes?

ans. .305

18> Suppose that the prob. that an item produced by a certain machine will be defective is ·1. Find the prob. that a sample of 10 items will contain at most 1 defective items.

$\boxed{\text{ans } \cdot 7358}$

19> Consider an experiment that consists of counting the number of α-particles given off in a 1 second interval by 1 gram of radioactive material. If we know from past experience that, on the average, 3·2 such α-particles are given off, what is a good approximation to the probability that no more ~~thd~~ than 2 α-particles will appear?

$\boxed{\text{ans. } \cdot 382}$

20> An urn contains N white and M black balls. Balls are randomly selected, one at a time, ~~ttr~~ until a black one is obtained. If we assume that each selected ball is replaced before the next one is drawn, what is the prob. that

1) exactly ~~on~~ n draws are needed, and

2) at least K draws are needed?

21> A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot and to accept the lot only if all 3 are nondefective. If 30% of the lots have 4 ~~effective~~ defective components and 70% have only 1, what proportion of lots does the purchaser reject?

$\boxed{\text{ans. } 46\%}$

22) From the numbers $\{1, 2, \dots (2n+1)\}$ three numbers are chosen at random. Prove that the probability that, they are in ~~arithmet~~ arithmetic progression is $\frac{3n}{4n^2-1}$.

23) The outcome of an experiment is equally likely to be if one of the four points in 3-D space with ~~coo~~ rectangular coordinates $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and $(1,1,1)$. If A, B, C denotes the events x coordinate is unity, y coordinate is unity and z coordinate is unity respectively, then check A, B, C are mutually independent.

24) If a day is dry, the conditional prob. that the next day will also be dry is p; if a day is wet, the conditional probability that the next day will be dry is p'. If u_n is the probability that the nth day will be dry, prove that

$$u_n - (p-p')u_{n-1} - p' = 0$$

If the 1st day is sure to be dry and $p = 3/4$, $p' = 1/4$. find u_n.

25) The spectrum of the random variable X consists of the points $1, 2, \ldots, n$ and $P(X = i)$ is proportional to $\frac{1}{i(i+1)}$. Determine the distribution function of X. Compute $P(3 < X \leq n)$ and $P(X > 5)$.

~~2c) Thousand tickets are showed in a lottery, in which there is one prize of rupees 500, 4 prizes of rupees one hundred each & 5 prizes of Rs. 10/- each. A ticket costs Rs. 1/-. If X is your net gain, when you buy one ticket.~~

26) Among n coins $(n-1)$ coins are of usual type & the remaining are head of both side. A coin is chosen at random & toss K times. If the coin falls head each time, what is the probability that it is an unusual coin? [unusual coin means head in both side].

27) If 2 cells are chosen at random on a chess board What is the probability that they will have a common side?

$\boxed{\text{ans: } 1/18}$

28) The King comes from a family of 2 children. What is the probability that the other child is his sister?

29) Urn I contains 2 white and 4 red balls, whereas urn II contains 1 white and 1 red balls. A ball is randomly chosen from urn I and put into urn II, and a ball is then randomly selected from urn II.

1. What is the probability that the ball selected from urn II is white?

2. What is the conditional probability that the transformed ball was white, given that a white ball is selected from urn II?

$\boxed{\text{ans. } 4/9,\ \ 1/2}$

3.) It is known that screws produced by a certain company will be defective with prob. ·01 independently of each other. The company sells the screws in packages of 10 and offers a money-back guarantee that at most 1 of the 10 screws is defective. What proportion of packages sold must the company replace?

$\boxed{\text{ans:- } .7\%}$