production manager : optimize the available resources to maximize profit

$\downarrow$

Determine quantities of each product to be produced.

optimize the resource utilization so that marginal opportunity cost of each resource unit is equal to its marginal return (also called shadow prices)

$$\text{shadow price} = \frac{\text{change in optimal objective function value}}{\text{Unit change in availability of resource}}$$

+ve $\leq$

-ve $\geq$

$$\max \underbrace{Z}_{\text{return}} = \sum_{j=1}^{n} c_j x_j = \sum_{j=1}^{n} (\text{profit per unit of variable } x_j)(\text{unit of variable } x_j)$$

c_j: profit (or return) per unit of variable (activity) x_j

x_j: number of units of variable

s.t. constraints :

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$$

a_{ij}: unit of resource i consumed (required) per unit of variable

b_i: maximum unit of resource available

or $\sum_{j=1}^{n}$ (unit of resource i consumed (required) per unit of variable x_j) (unit of variable x_j) $\leq$ unit of resource i available

$x_j \geq 0$ for all j

Dual =

$$Z_y = \sum_{i=1}^{m} b_i y_i =$$

minimum (cost) $Z_y = \sum_{i=1}^{m}$ (unit of resource i) (cost per unit of resource i) y_i

s.t.

$$\sum_{i=1}^{m} a_{ji} y_i \geq c_j$$

or $\sum_{i=1}^{m}$ (units of resource i consumed per unit of variable y_i) (cost per unit of resource i) $\geq$ profit per unit for each variable x_j

$y_i \geq 0$ for all i

y_i in terms of return per unit of resource i

shadow price
or
simplex multiplier

$$Z_x \leq Z_y$$

$\Rightarrow$ profit $\leq$ worth of resources

$\geq$

$Z_x = Z_y \rightarrow$ for o

The optimality (maximum profit or return) is reached only when the resources are completely utilized.

$$\underbrace{\sum_{i=1}^{m} a_{ji} y_i}_{\mathrm{L}} - \underbrace{c_j}_{\mathrm{I}} \geq 0 \rightarrow c_j \geq \sum_{i=1}^{m} a_{ji} y_i$$

any non basic variable

Max $\quad z_j - c_j < 0$

c_j <

profit per unit of activity x_j $\geq$

Shadow price of resources used per unit of activity x_j.

Some standard results on duality

1. Dual of the dual LPP will be primal again
2. If either primal / dual has an unbounded objective function value, the ~~the~~ other problem has no feasible solution.
3. If either primal or dual has finite optimal solution, the other one also poses the same.
4.

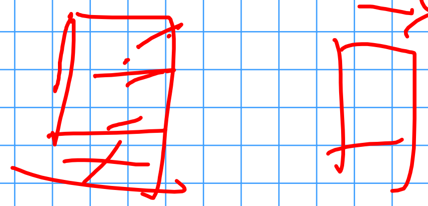
$n =$ $\qquad m > n$

$m =$

Simplex method :- → Sub optimal solution

→ Entering variable $\text{Min}(z_j - c_j < 0)$

→ leaving variable



$\frac{b}{\quad}$ $\frac{b_i}{\text{pivot column}}$

→ $x_B \geq 0$

Dual simplex :-

optimality criterion

$z_j - c_j \geq 0$

$x_B < 0$

1. Convert max / $-$min

2. $\geq$ → $\leq$

3. Slack variables

4. $z_j - c_j$

Case1: $z_j - c_j \geq 0$, $x_{B_i} \geq 0$

→ optimum

Case2: $z_j - c_j < 0$ → the method fails.

case 3 : $z_j - c_j \geq 0$ atleast one $\underline{x_{Bi}} < 0$

~~Step~~ 5. leaving variable

$\underline{x_{Bk}} = \underline{\underline{\min}} \left\{ x_{Bi}, \underline{x_{Bi}} < 0 \right\}$

$a_{ik} < 0$

pivot Row

6. Entering variable

$\text{Max} \left\{ \frac{z_j - c_j}{a_{ik}}, a_{ik} < 0 \right\}$

7. $\to$ +ve

$\underline{\quad}$ -

-ve

pivot : ve

element equal to 1

$x_{ij} = \frac{x_{ij}' - x_{ij}''}{\quad}$

+ve

-ve

0

x_{ij}''

x_{ij}''

x_{ij}'

x_{ij}'

> 0

> 0

x_{ij}''

$x_{ij}' =$