Ex./IEBE/MATH/T/123/2018

BACHELOR OF (I.E.E.) ENGINEERING EXAMINATION, 2018

(1st Year, 2nd Semester)

Mathematics - III J

Time : Three hours | Full Marks : 100

Notations/Symbols have their usual meanings.

Answer any ***ten*** questions.

1. (a) What do you mean by the limit of a sequences ? Show that the limit of the sequence $\{x_n\}$, where $x_n = \left(a^n + b^n\right)^{1/n}$, $\forall n \in \mathbb{N}$, and 0 < a < b, is b.

   (b) Is the sequence $\{x_n\}$; $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots,$ $+ \frac{1}{2n}$, $\forall n \geq 1.$ monotonic ? Justify your answer.

   (c) Find $\lim_{n \to \infty} \left(\sqrt{n+1} - \sqrt{n}\right)$. 5+2+3

2. (a) Evaluate

$$\lim_{n \to \infty} \left[\frac{1}{n} \left\{ (a+1)(a+2)\ldots\ldots(a+n) \right\}^{1/n} \right] \quad a > 0$$

(Turn Over)

(b) State Raabe's test. Use this to find the convargence of the series

$$1+\frac{1}{2}\cdot\frac{1}{3}+\frac{1.3}{2.4}\cdot\frac{1}{5}+\frac{1.3.5}{2.4.6}\cdot\frac{1}{7}+.....$$ 4+6

3. (a) Discuss the convergence of the following series :

(i) $\sum_{n=1}^{\infty} a_n$, where $a_n=\sqrt{n^4+1}-\sqrt{n^4-1}$

(ii) $\sum_{n=2}^{\infty} \frac{1}{n(\log n)}p$, $p>0$

(b) Define sequence of partial sums of the infinite series

$\sum_{n=1}^{\infty} a_n$. 8+2

4. (a) Is the equation (cos y + y cos x) dx + (sin x – x sin y) dy = 0 exact ?

(b) Solve : $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2+y^2}$

(c) Use the method of variation of parameters to find the solution of $\frac{d^2y}{dx^2} + a^2y = tan\ ax$. 2+3+5

subject to the conditions

$u(0,y) = 0,\ u(a,y) = 0,\ 0 < y < b$

$u(x,0) = 0,\ u(x, b) = f(x),\ 0 < x < a.$ 10

12. Find the Fourier series expansion for the function $f(x) = x - x^2$ in the interval $-\pi \le x\ \pi$. 10

13. (a) Obtain the half range sine series for $f(x) = x$ in the interval $0 \le x \le \pi$.

(b) Find the half range cosine series for $f(x) = x^2$ in the interval $0 \le x \le \pi$. 5+5=10

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5. (a) Solve the ODE :

$$(D^2+4)y = \sin 2x, \text{ where } D \equiv \frac{d}{dx}.$$

(b) Solve the ODE :

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$ 4+6

6. (a) Define the regular singular point of the second order differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

(b) Find the regular singular point of

$(x^2-4)\,x^2y'' + 2x^3y' + 3y = 0$

(c) Use power series method to solve

$(1-x^2)y'' - 2xy' + 2y = 0$ 2+2+6

7. (a) Form a PDE by eliminating the arbitrary constants a and b from

$(x-a)^2 + (y-b)^2 + z^2 = 1$

(b) Form a PDE by eliminating the arbitrary function f from

$z = xy + f(x^2+y^2)$

(c) Form a PDE by eliminating the arbitrary functions f and g from

$z = yf(x) + x\,g(y)$ 3+3+4

8. (a) Solve the PDE $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

(b) Solve :

(i) $(x^2 - y^2 - z^2)p + 2xy\,q = 2xz$

(ii) $z(xp - yq) = y^2 - x^2$ 3+(4+3)

9. Obtain the various Possible solutions of the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, by the method of separation of variables. Identify the solution which is appropriate with the physical nature of the equation. Justify your answer. 10

10. Solve the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1, t > 0$, subject to the conditions : $u(0,t) = 0$ and $u(1,t) = 0$ for $t > 0$; and $u(x,0) = 3 \sin n\pi x$. 10

11. Solve the two-dimensional Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad 0 \le x \le a,\ 0 \le y \le b,$$