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M.E. MECHANICAL ENGINEERING FIRST YEAR SECOND SEMESTER – 2018

Subject: ROTOR DYNAMICS **Time: 3 Hours** **Full Marks: 100**

Answer any four questions.

Any missing information can be assumed suitably with appropriate justification.

1. (a) Derive (with neat sketches and stating relevant assumptions) the equations of motion for lateral vibration of a Jeffcott rotor mounted on rigid bearings, having the horizontal shaft axis (along X-direction), with constant angular acceleration $\alpha = \ddot{\Phi}$, under the action of unbalance excitation. Consider that m be the mass of the rotor, J_p be its polar mass moment of inertia, e be the unbalance eccentricity, k_S be the shaft stiffness, c be the constant damping coefficient against the vibration in both Y and Z directions, Φ be the angle of spin of the rotor mass centre about the X-axis. [10]

(b) In relation to the above system, establish also the equation for angular motion about X-axis under the influence of a resultant external torque T about X-axis. [5]

(c) Non-dimensionalize the two equations for lateral vibrations with introducing non-dimensional time $\tau = \omega_n t$ and non-dimensional angular acceleration $\overline{\alpha} = \dfrac{\alpha}{\omega_n^2}$ where ω_n be the undamped natural frequency of the rotor, such that the final form of the equations depends only on the parameters of $\overline{\alpha}$ and ς, the damping ratio. [5]

(d) With neat sketches, discuss briefly and qualitatively about the position and magnitude of the absolute maxima of the response in plot of the response against the parameter $\dfrac{\tau}{2\pi}$. Also mention the effects of $\overline{\alpha}$ and ς on it. [5]

2. (a) Consider a rotor disc is subject to gyroscopic action during vibration under the influence of unbalance excitation at a constant angular speed Ω about its axis, and consequently, in addition to the later vibration (characterized by v and w, the components of the translational motion of its geometric centre along Y- and Z-directions respectively), it undergoes small angular oscillation (characterized by B and Γ, the small angles of rotation about Y- and Z-directions respectively). Suitably **define** and **use the Euler's angles and the corresponding derivatives to derive the total angular velocity vector for this system.** During the derivation **clearly show different orientations of the body-fixed axes system (i.e., the axes fixed to the disc) and their transformation relationship. Obtain expressions of kinetic**

energy of the disc due to both translational and rotational vibration. Assume that m be the mass of the disc, J_p be its polar mass moment of inertia, e be the unbalance eccentricity. [25]

3. (a) Consider an undamped cantilever rotor with mass m, its polar and transverse mass moment of inertia be J_p and J_T respectively. Length of the shaft is l and the flexural rigidity is EI. **Determine the expressions of the stiffness components of the shaft for vibration in both *X-Y* plane and *X-Z* plane**, where X-axis is the direction of rotor spin. [16]

(b) Write (derivation is not required) the equations of free vibration of such a rotor with constant spin speed Ω in real Cartesian co-ordinates. Then transfer them into a set of equations with transverse displacement components and slopes expressed in appropriate complex coordinates. From these equations find the characteristics equation of the system. [9]

4. (a) With neat sketches and stating the appropriate assumptions deduce the Reynolds equations for a fluid film bearing in the form: $\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\eta U\frac{\partial h}{\partial x}+12\eta\frac{\partial h}{\partial t}$ where h be the local film thickness, p be the local fluid pressure, η be the fluid viscosity and U be the surface speed of the journal within the bearing. X- and Z-coordinates are corresponding to the circumferential and axial directions of the bearing respectively. [18]

(b) Simplifying the above equation find out an expression for pressure distribution around the journal for the case of a plane cylindrical **short** journal bearing at **steady state**. [7]

5. (a) Consider a Jeffcott rotor mounted on rigid bearings and running at a constant rotating speed with both non-rotating external damping and rotating internal damping of viscous type acting on it. **Deduce the equations of motion for such a rotor for its transverse vibration in inertial Cartesian coordinates**. Consider that m be the mass of the rotor, e be the unbalance eccentricity, k be the shaft stiffness, c_e and c_i be the constant external and internal damping coefficient against the vibration, Ω be the constant spin speed of the rotor. Deduce and expression of **stability limit of the spin speed** for such a rotor. [20]

(b) In case of above rotor, prove that the presence of internal damping has no role in the unbalance response amplitude of the rotor provided the stiffness of the rotor are same in both the planes of vibration. [5]

6. (a) With neat sketches describe the following types of mass unbalance present in the rotor:
(i) static unbalance, (ii) couple unbalance, (iii) quasi-static unbalance, (iv) dynamic unbalance. **[16]**
(b) With neat sketches describe the method of **single plane balancing of a rigid rotor using influence coefficient method**. **[9]**

7. Write a note on the finite element modelling of a laterally flexible rotor shaft for its transverse vibration mentioning the following: translational and rotational inertia matrices, gyroscopic matrix, bending stiffness matrix. Use the Rayleigh beam finite elements with two nodes and mention the corresponding shape functions. **[25]**