Exam Roll - CSE218056

Class Roll - 001710501029

BCSE - IV - 2nd semester

Sub:- Optimization techniques & Operation Research, 1

2.

Algorithm of Big M :-

1. Multiply the inequality constraints to ensure that the right hand side is positive.

2. If the problem is of minimization, transform to maximization by multiplying the objective by -1.

3. For any greater than constraints, introduce surplus s_i and artificial variables a_i (as shown below).

4. Choose a large positive value M and introduce a term in the objective of the form -M multiplying the artificial variables.

5. For less than or equal constraints, introduce slack variables s_i so that all constraints are equalities.

6. Solve the problem using the usual simplex method.

2. Maximize $Z = 30x_1 + 50x_2$

s.t. $x_1 + RL * x_2 \leq 4 * RL$

$5RL\,x_1 + 3x_2 \geq 8 * 20$

$x_2, x_1 \geq 0$

$RL \rightarrow 29$ [Roll :- 00171050 10 $\underline{29}$]

$\therefore$ $5RL \Rightarrow 5 * 29 = 145$; $4 * RL \Rightarrow 4 \times 29 = 116$

Maximize $Z = 30x_1 + 50x_2$

S.t. $x_1 + 29x_2 \leq 116$

$145x_1 + 3x_2 \geq 160$

$x_1, x_2 \geq 0$

Converting into cannonical form by adding slack variable S_1 in the first constraint & surplus variable S_2 & artifical variable A_1 in the 2nd constraint.

Max $Z = 30x_1 + 50x_2 + 0.S_1 + 0.S_2 - MA_1$

Subject to

$x_1 + 29x_2 + S_1 = 116$

$145x_1 + 3x_2 - S_2 + A_1 = 160$

and $x_1, x_2, S_1, S_2, A_1 \geq 0$

Initial Tableau
(Iteration 1)

	Basic variable	x_1	x_2	s_1	s_2	A_1	RHS
$R_1 \rightarrow$	s_1	1	29	1	0	0	116
$R_2 \rightarrow$	A_1	145	3	0	-1	1	160
$R_3 \rightarrow$	Z	30	50	0	0	M	0

$R_3 \rightarrow R_3 - M R_2$

	B.V	x_1	x_2	s_1	s_2	A_1	RHS	Ratio
$R_1 \rightarrow$	s_1	1	29	1	0	0	116	$\frac{116}{1} = 116$
$R_2 \rightarrow$	A_1	145	3	0	-1	1	160	$\frac{160}{145} = 1.1$
$R_3 \rightarrow$	Z	$(30 - 145M)$	$(50 - 3M)$	0	$-(M)$	0	$-160M$	

$\therefore$ pivot element is 145.
Entering $= x_1$
Leaving $= A_1$

$\therefore R_2 \rightarrow R_2 / 145$.

$\Rightarrow R_2 = \left[1, \frac{3}{145}, 0, \frac{1}{145}, \frac{-1}{145}, \frac{160}{145} \right]$

$R_1 \rightarrow R_1 - 1.R_2$

$\Rightarrow R_1 = \left[1-1, 29 - \frac{3}{145}, 1-0, +\frac{1}{145}, \frac{-1}{145}, 116 - \frac{160}{145} \right]$

$= [0, 28.979, 1, 0.0069, -0.0069, 114.8966]$

$R_3 \rightarrow R_3 - (-30-145M) R_1$

$\therefore R_3 = \left[0, (-50-3M) - (-30-145M) \times \frac{3}{145}, 0, M - (-30-145M)\frac{1}{145}, 0 - (-30-145M) \times \frac{-1}{145}, \right.$

$\left. -160M - (-30-145M) \times \frac{160}{145} \right]$

Iteration - 2

Basic variables	x_1	x_2	s_1	s_2	A_1	RHS
S_1	0	29.979	1	0.0069	-0.0069	114.8966
x_1						

BV	x_1	x_2	S_1	S_2	A_1	RHS	ratio
S_1	0	28.979	1	0.0069	-0.0069	114.8966	$\frac{114.8966}{28.97} = 3.96$
x_1	1	0.0207	0	-0.0069	0.0069	1.1034	$\frac{1.1034}{0.0207} = 53.33$
Z	0	-49.379	0	-0.2069	M+0.2069	33.1634	

pivot element = 28.979

Entering variable x_2

Leaving variable S_1

$R_1 \to R_1 / 28.979$

$R_1 = \left[0, \ 1, \ \frac{1}{28.979}, \ \frac{0.0069}{28.979}, \ \frac{-0.0069}{28.979}, \ \frac{114.8966}{28.979} \right]$

$= [0, \ 1, \ 0.0345, \ (0.000238) \ -0.000238 \ 3.9648$

$R_2 \to R_2 - 0.0207 * R_1$

$R_2 = [1-0, \ 0.0207 - 0.0207(1), \ -0.0207 \times 0.0345,$

$-0.0069 - 0.0207 \times 0.00024,$

$0.0069 - 0.0207 \times (-0.000238)$

$1.1634 - 0.0207 \times (3.9648)]$

$= [1, \ 0, \ -0.0007, \ -0.0069, \ 0.0069, \ 1.0214$

$R_3 \to R_3 - (-49.347) \times R_1$

$R_3 = [0, \ 0, \ 0 - (-49.347) \times 0.0345, \ -0.2069 - (-49.347) \times 0.000238,$

$M + 0.2069 - (-49.347)(-0.000238)$

$33.1634 - (-49.347) \times 3.964]$

$\therefore R_3 = [0, 0, 1.704, -0.1951, M + 0.1951, 288.8818]$

Iteration 3 :

S.V	x_1	x_2	S_1	S_2	A_1	RHS	Ratio.
x_2	0	1	0.0345	0.000238	-0.000238	3.964	$\frac{3.9648}{0.000238}$
x_1	1	0	-0.0007	-0.0069	0.0069	1.0214	←
Z	0	0	1.704	-0.1951	(M+0.1951)	288.8815	

Pivot element = 0.0002

$\therefore$ Entering variable S_2,

leaving variable x_2

$\therefore R_1 \to R_1 \div 0.00024$

$R_1 \equiv [0, \frac{1}{0.000238} = 4200, \frac{0.0345}{0.000238} \approx 145, 1, -1, 16658.82]$

$R_2 \to R_2 + 0.0069(R_1)$

$R_2 = [1, 29, -0.0007 + 1, 1 - 1, 1.0214 + 0.0069 \times 16658.82 = 116]$

$R_3 \to R_3 + 0.1951(R_1)$

$R_3 = [0, 0.1951 \times 4166, 1.704 + 0.1951 \times 143.75 = 30$

$-0.1951 + 0.1951 \times 1 = 0$

$M + 0.1951 + 0.1951 \times -1 = M$

$288.88 + 0.1951 \times 16658.82 = 3288 \approx 3539.0672$

Iteration - 4

B.V	x_1	x_2	s_1	s_2	A_1	RHS
s_2	0	24800	145	1	-1	16688.82
x_1	1	29	1	0	0	116
Z	0	821.7866	30	0	M	~~[illegible]~~ 3454.0672

∴ optimal value is reached.

$x_1 = 116$, $x_2 = 0$

⇒ max Z = $30 \times x_1 = 3480$.

(according to table 3454.0672 due to decimal approximations)

4. Minimize $f(x) = 3.6x_1 - 0.4x_1^2 + 1.6x_2 - 0.2x_2^2$

$h(x) \Rightarrow$ s.t. $2x_1 + x_2 = 10$

$x_1, x_2 \geq 0$

$g_1(x) \Rightarrow -x_1 \leq 0$

$g_2(x) \Rightarrow -x_2 \leq 0$

The Lagrangian for this problem is :-

$$L(x, \lambda, \mu_1, \mu_2) = 3.6x_1 - 0.4x_1^2 + 1.6x_2 - 0.2x_2^2$$
$$+ \lambda(2x_1 + x_2 - 10)$$
$$+ \mu_1(-x_1) + \mu_2(-x_2).$$

First order necessary condition :-

$$\frac{\partial L}{\partial x_1} = 3.6 - 0.8x_1 + 2\lambda - \mu_1 = 0 \quad - (1)$$

$$\frac{\partial L}{\partial x_2} = 1.6 - 0.4x_2 + \lambda - \mu_2 = 0 \quad - (2)$$

$$\frac{\partial L}{\partial \lambda} = 2x_1 + x_2 - 10 = 0 \quad - (3).$$

$$\mu_1(-x) = 0 \quad - (4)$$

$$\mu_2(-x) = 0 \quad - (5)$$

solving the above equations to get candidate solution.

Case - 1 :- when $\mu_1 = 0$, $\mu_2 = 0$.

$\therefore$ (1) $- 2 \times$ (2)

$$\Rightarrow \quad 3.6 - 0.8x_1 + 2\lambda = 0$$
$$-(\ 3.2 - 0.8x_2 + 2\lambda = 0.$$

$$6.4 - 0.8x_1 + 0.8x_2 = 0.$$

$$\Rightarrow 1 - 2x_1 + 2x_2 = 0 \quad - (6)$$

Now, ⑥ − ③.

$$1 - 2x_1 + 2x_2 = 0$$
$$+(\ 2x_1 + x_2 - 10 = 0$$
$$-9 + 3x_2 = 0$$

$$x_2 = 3.$$
$$x_1 = 3.5$$

Putting the value of x_1 & x_2 in ① we get.

$$3.6 - 0.8 \times 3.5 + 2\lambda = 0$$
$$2\lambda = 2.8 - 3.6 = -0.8$$
$$\Rightarrow \lambda = -0.4$$

$$\left.\begin{array}{l} x_1 = 3.5 \\ x_2 = 3 \\ \lambda = -0.4 \\ \mu_1 = 0 \\ \mu_2 = 0 \end{array}\right\}$$ as $\lambda < 0$ it cannot be a minima point and hence this is not a candidate solution.

Case II when $\mu_1 = 0$, $\mu_2 \neq 0$.

from eq ③ $x_2 = 0$ — ⑦.

putting value of x_2 in ③

$$2x_1 - 10 = 0 \Rightarrow x_1 = 5 \quad - ⑧.$$

from ⑧ & ①

$$3.6 - 0.8 \times 5 + 2\lambda - 0 = 0$$
$$\Rightarrow 3.6 - 4 + 2\lambda = 0$$
$$\Rightarrow 2\lambda = 0.4$$
$$\lambda = 0.2 \quad - ⑨.$$

Putting ⑨ & ⑧ in ②.

$$1.6 - 0.4 \times 0 + 0.2 - \mu_2 = 0$$
$$\Rightarrow 1.6 + 0.2 = \mu_2$$
$$\Rightarrow \mu_2 = 1.8$$

(9) (8)

$\therefore$ let solution $x^{(1)}$ be

$x_1 = 5$

$x_2 = 0$

$\mu_1 = 0$

$\mu_2 = 1.8$

$\lambda = 0.2.$

Case = III when $\mu_1 \neq 0$ & $\mu_2 = 0$,

From eq (4), $x_1 = 0.$ — (10)

from eq (10) & (3).

$\therefore x_2 = 10$ — (11)

from (10), (11) in (2)

$1.6 - 0.4 \times 10 + \lambda - 0 = 0$

$\lambda = 4 - 1.6 = 2.4$ — (12)

from (10), (11), (12) in (1)

$3.6 - 0.8 \times 0 + 2 \times 2.4 - \mu_1 = 0$

$\Rightarrow \mu_1 = 3.6 + 4.8$
$= 8.4$

$\therefore$ let solution $x^{(2)}$ be,

$x_1 = 0$

$x_2 = 10$

$\mu_1 = 8.4$

$\mu_2 = 0$

& $\lambda = 2.4$

Now for solution $x^{(1)}$, $u_1 = 5$ and $u_2 = 0$

$h(x)$ and $g_2(x)$ are active.

$$J(x^{(1)}) = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}_{x^{(1)}} = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\therefore \quad J(x^{(1)}) \cdot y = 0$$

$$y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hessian of Lagrangian is

$$\nabla_x^2 L = \begin{bmatrix} -0.8 & 0 \\ 0 & -0.4 \end{bmatrix}$$

$$y^T \cdot \nabla_x^2 L \cdot y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \geq 0$$

$\therefore$ $x^{(1)}$ is still a candidate point by the second order necessary condition (SONC)

checking the ~~seco~~ second order sufficient condition (SOSC)

$y^T \cdot \nabla^2_x L \cdot y \ngtr 0$; $\therefore$ SOSC is not satisfied.

Now for solution $x^{(2)}$, $x_1 = 0$, $x_2 = 10$

$h(x)$ and $g_1(x)$ are active for the following candidate.

$$J(x^{(1)}) = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}\Bigg|_{x^{(2)}} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$\therefore$ $J(x^{(1)})\, y = 0$ gives

$$y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

~~hes~~ hessian of ~~lagrange~~ Lagrangian is

$$\nabla_x^2 L = \begin{bmatrix} -0.8 & 0 \\ 0 & -0.4 \end{bmatrix}$$

$$y^T . \nabla_x^2 L . y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -0.8 & 0 \\ 0 & -0.4 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ SONC is satisfied

But SOSC is not satisfied.

Therefore, both solution $x^{(1)}$ and $x^{(2)}$ may be local minimum, but we cannot be sure as SOSC is not satisfied.

Testing the solution on (z)

at $x^{(1)}$

$$z_1 = 3.6 \times 5 - 0.4 \times 5^2 + 1.6 \times 0 - 0.2 \times 0^2 = 8$$

at $x^{(2)}$

$$z_2 = 3.6 \times 0 - 0.4 \times 0^2 + 1.6 \times 10 + -0.2 \times 10^2 = -4$$

Hence on testing the candidate points,

z is min at $(x_1 = 0,\ x_2 = 10)$

5.

K → Kolkata

C → Cuttack

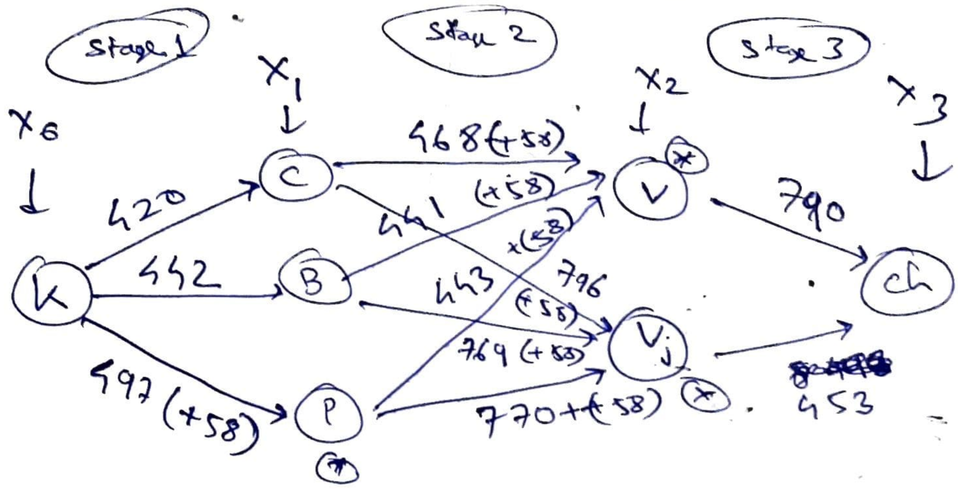
B → Bhubaneshwar

P → Puri

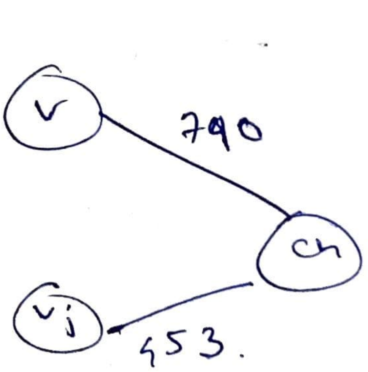
V → Vishakapatnam.

Vj → Vijayawada

Ch → Chennai

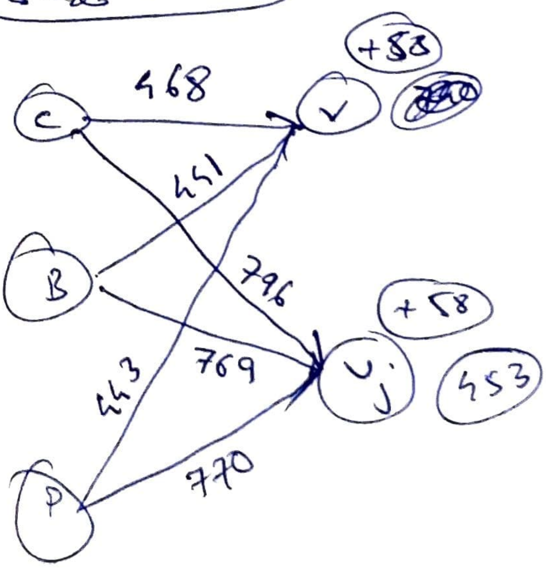


Stage 3

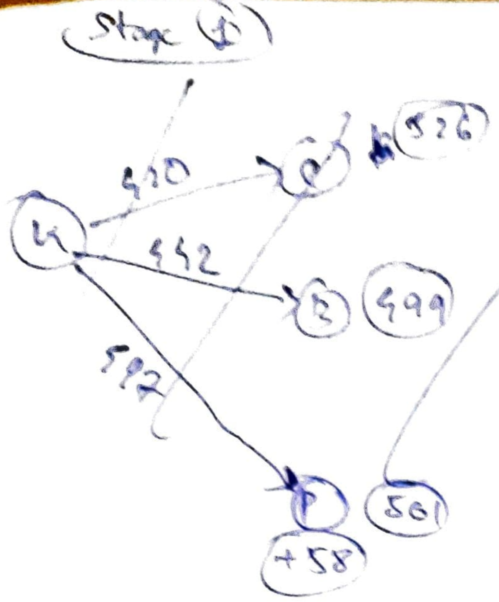


x_2	$F_3(x_2)$	x_3
V	790	Ch
Vj	453.	Ch

stage - 2



x_1	$F_2(x_1)$	x_2
C	468+58 = 526	V
	796+58 = 854	Vj (invalid)
B	441+58 = 499	V
	769+58 = 827	Vj (invalid)
P	443+58 = 501	V
	770+58 = 828	Vj (invalid)

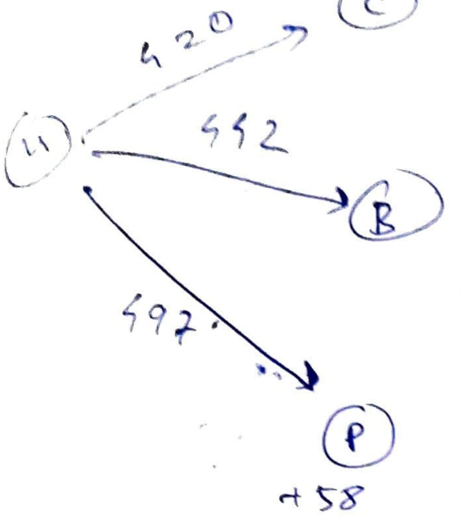


x_0	$F_1(x_0)$	x_1
K	420+526	C
K	442+499	B
K		P

x_1	$F_2(x_1)$	x_2
C	(468+58)+790 = 1316	✓
C	(796+58)+453 = 1307	v_i (invalid) ✗
B	(441+58)+790 = 1289	✓
B	(769+58)+453 = 1280	v_j (invalid) ✗
P	(443+58)+790 = 1291	✓
P	(770+58)+453 = 1281	v_j (invalid) ✗

All paths to v_j excep 800 km on one day ∴ those are marked invalid.

stage 1



x_0	$f_1(x)$	x_1
K	$420 + 1316 = 1736$	C
K	$442 + 1289 = 1731$	B ✓
K	$(497 + 58) + 1291 = 1846$	P

shortest path.

K → B → V → Ch

with distance 1731 km.

3. or

warehouse :

Factory	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
warehouse requirement	5	8	7	14	34

Rewriting above table :

To / From	A	B	C	D	Supply	Penalty cost.
1	19	30	50	10	7	9
2	70	30	40	60	9	10
3	40	8	70	20	18	12
Demand	5	8	7	14	34	
	21	22	10	10		

Largest penalty is 22

we allocate 8 to the cell

To / From	A	B	C	D	Supply	
1	$\boxed{19}$	$\boxed{30}$	$\boxed{50}$	$\boxed{10}$	7	9
2	$\boxed{70}$	$\boxed{30}$	$\boxed{40}$	$\boxed{60}$	9	10
3	$\boxed{40}$	$\boxed{8}$ 8	$\boxed{70}$	$\boxed{20}$	18	20
Demand	5	8	7	14	34	
	21	X	10	10		

largest ~~[illegible]~~ penalty is 21.

we allocate 5 to 1A. (minimum cost cell in col A).

To / From	A	B	C	D	Supply	
1	$\boxed{19}$ 5	$\boxed{30}$	$\boxed{50}$	$\boxed{10}$	7	20
2	$\boxed{70}$	$\boxed{30}$	$\boxed{40}$	$\boxed{60}$	9	10
3	$\boxed{40}$	$\boxed{8}$ 8	$\boxed{70}$	$\boxed{20}$	18	20
~~Remand~~ Demand	5	8	7	14	34	
	X	X	10	10		

largest cell is 20

lets take row 3 to fill for next iteration.

we assign 10 to 3D (min cost of unfilled cell in row 3)

To / From	A	B	C	D	Supply	
1	19 5	30	50	10	7	20
2	70	30	40	60	9	10
3	40	8 8	70	20 10	~~18~~	X
Demand	5	8	7	14	34	
	X	X	10	50		

Largest penalty cost is 50
next column to consider is D
we assign 2 to 1D (minimum cost cell in the column D)

To / From	A	B	C	D	Supply	
1	19 5	30	50	10 2	7	X
2	70	30	40	60	9	10
3	40	8 8	70	20	18	X
Demand	5	8	7	14	34	
	X	X	10	X		

largest penalty cost is 70
lets consider row 2
we assign 7 to 2C

To / From	A	B	C	D	Supply	
1	$\boxed{19}$ 5	$\boxed{30}$	$\boxed{50}$	$\boxed{10}$ 2	7	X
2	$\boxed{70}$	$\boxed{30}$	$\boxed{40}$ 7	$\boxed{60}$	9	X
3	$\boxed{40}$	$\boxed{8}$ 8	$\boxed{70}$	$\boxed{20}$ 10	18	X
Demand	5	8	7	14	34	
	X	X	X	X		

we now fill 2 in 2D to complete the process.

Initial basic feasible solution is :-

To / From	A	B	C	D	Supply
1	$\boxed{19}$ 5	$\boxed{30}$	$\boxed{50}$	$\boxed{10}$ 2	7.
2	$\boxed{70}$	$\boxed{30}$	$\boxed{40}$ 7	$\boxed{60}$ 2	3
3	$\boxed{40}$	$\boxed{8}$ 8	$\boxed{70}$	$\boxed{20}$ 10	18
Demand	5	8	7	14	34

We see the initial solution has 6 assignments which is equal to $m+n-1$ $(3+4-1)$.

Thus, the solution is not degenerate in nature.

Initial minim cost = 5×19 + 2×10 + 7×40 + 2×60 + 8×8 + 10×20

= 95 + 20 + 280 + 120 + 64 + 200

= 779

MODI method :-

u_i	v_j / To / From	$v_A = 19$ A	$v_B = 2$ B	$v_C = 10$ C	$v_D = 10$ D	Supply
$u_1 = 0$	1	[19] 5	[30]	[50]	[10] 2	7
$u_2 = 50$	2	[70]	[30]	[40] 7	[60] 2	9
$u_3 = 10$	3	[40]	[8] 8	[70]	[20] 10	18
	Demand	5	8	7	14	34

we know $u_i + v_j = c_{ij}$

where c_{ij} is the cost of cell (i, j)

we calculate u_i's and v_j's for allocated cells only

$u_1 + v_A = 19$ $u_2 + v_C = 40$ $u_3 + v_B = 8$

$u_1 + v_D = 10$ $u_2 + v_D = 60$ $u_3 + v_D = 20$

We have 7 variables and 6 equations.

Let $u_1 = 0$

20

$v_A = 19 - 0 = 19$ $\qquad u_2 = 60 - v_D = 60 - 10 = 50$

$v_D = 10 - 0 = 10$ $\qquad v_C = 50 - u_2 = 50 - 50 = -10$

$u_3 = 20 - v_D = 20 - 10 = 10$

$v_B = 8 - u_3 = 8 - 10 = -2$

We calculate $k_{ij} = c_{ij} - u_i - v_j$ for every empty cell.

↳ cost increase or decrease that would occur by allocating to the cell ij.

$\therefore$ $k_{1B} = 30 - 0 + 2 = 32$

$k_{1C} = 50 - 0 + 10 = 60$

$k_{2A} = 70 - 50 - 19 = 1$

$k_{2B} = 30 - 50 + 2 = -18$

$k_{3A} = 40 - 10 - 19 = 11$

$k_{3C} = 70 - 10 + 10 = 70$

We can allocate to cell 2B to reduce the cost.

u_i \ v_j	To / From	A ($V_A = 19$)	B ($V_B = -2$)	C ($V_C = 8$)	D ($V_D = 10$)	Supply
$u_1 = 0$	1	19 5	30	50	10 2	7
$u_2 = 32$	2	70	30 2	40 7	60	9
$u_3 = 10$	3	40	8 6	70	20 12	18
	Demand	5	8	7	14	34

$u_1 + V_A = 19$ $\quad$ $u_2 + V_B = 30$ $\quad$ $u_3 + V_B = 8$

$u_1 + V_D = 10$ $\quad$ $u_2 + V_C = 40$ $\quad$ $u_3 + V_D = 20$

We find u_is & V_js using above eqs.

Let $u_1 = 0$.

$V_A = 19$ $\quad$ $u_3 = 20 - 10 = 10$

$V_D = 10$, $\quad$ $V_B = 10 - u_3 = -2$

$u_2 = 30 - V_B = 32$

$V_C = 40 - u_2 = 8$.

Calculating k_{ij} for empty cells.

$k_{1B} = 30 - 0 + 2 = 32$

$k_{1C} = 50 - 0 - 8 = 32$

$k_{2A} = 70 - 32 - 19 = 19$

$k_{2D} = 60 - 32 - 10 = 18$

$k_{3A} = 40 - 10 - 19 = 11$

$k_{3C} = 70 - 10 - 8 = 52$

All the value of k_{ij} is +ve. Thus the solution is optimal

minimum $Z = 13 \times 5 + 16 \times 2 + 30 \times 2 + 50 \times 7 + 8 \times 6 + 20 \times 12$

$= 8 \cdot 2743$

with allocation shown in the last table.

---

6. given

$\lambda = 3$ units per hour (Roll - 001710501029)

service time $= \frac{1}{\mu} = 0.26 - 29 \times 0.0005$

$= 0.2455$ hour

$\mu = \frac{1}{0.2455} = 4.0733$ units per hour

$N = 2$

traffic intensity. $\rho = \frac{\lambda}{\mu} = \frac{3}{4.0733} = 0.7365$

The steady state probability distribution of n customers (calling units) in the system is

$$P_n = \frac{(1-\rho)\rho^n}{1-\rho^{N+1}} \qquad \rho \neq 1$$

$$= \frac{(1-0.7365) \times (0.7365)^n}{1-(0.7365)^{2+1}}$$

$$= \frac{0.2635 \times (0.7365)^n}{0.60049} = 0.43880(0.7365)^n$$

Expected number of calling units in the system

$$L_s = \sum_{n=1}^{N} nP_n = \sum_{n=1}^{2} n(0.53880)(0.7365)^n$$

$$= 0.53880\left[0.78365 + 2 \times 0.5525\right]$$

$$= 0.7991$$

1. or

<u>Relaxation</u> :- Feasible space increase or stays same (does not affect optimality)

<u>Restriction</u> :- Feasible space decreases or stays same (does not affect optimality).

Significance of relaxation and restriction for LPP :-

→ used for ~~sensitivity~~ sensitivity analysis.

→ observes how change in parameter affect optimal solution

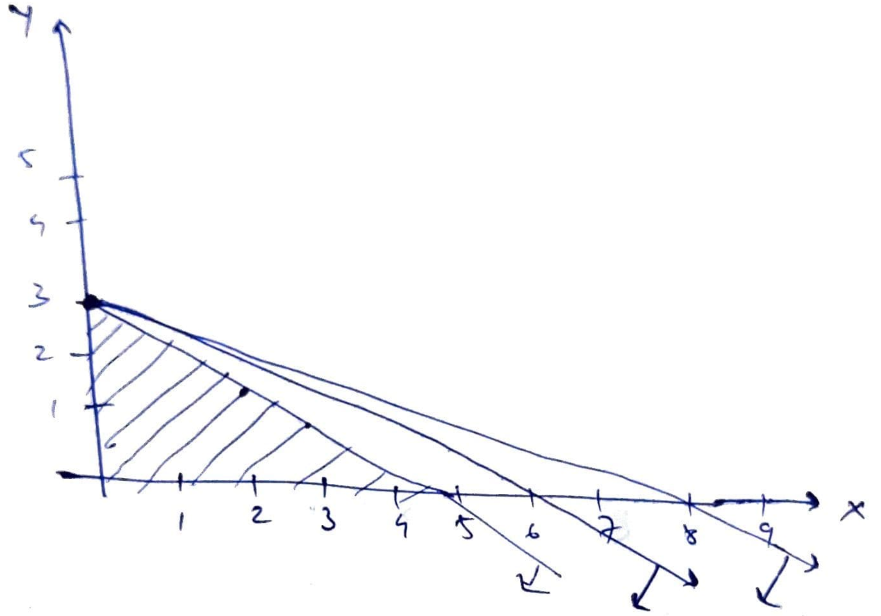
→ deals with uncertainty in economic problems.

Degeneracy:

It is a situation when the solution of the problem degenerates. A solution of the problem degenerates if value or values of basic variable(s) become zero.

It occurs due to redundant ~~consto~~ constraints

Eg:



This is redundant constraint.

In degenerate cases there will be a tie in basic variables. We can deal with this by slightly modifying the simplex method. In case of a tie, we go from left to right and divide the S_is and A_is (slack & artificial variables) with the denominator as the column value. We can choose the lowest amongst the calculated values and ~~per~~ prefer artificial values in case of equal values.

Dual Simplex vs Revised simplex :-

Similarities :-

→ Both are iterative methods to solve LPP problem

→ Both are a ~~derived form~~ ~~of~~ variants of the simplex method.

→ In both the methods we use the concepts of entering & leaving variables.

Differences :-

Dual simplex :-	Revised simplex :-
→ Starts with optimal but infeasible solution.	→ starts with suboptimal but feasible solution
→ Final sol^n may be super optimal	→ Final sol^n may be sub optimal.
→ It maintains optimality in successive iterations.	→ It maintains flexibility and tries to obtain optimality in successive iterations
→ It is inefficient compared to revised simplex	→ It is very efficient due to a modified representation of the tableau
→ may not be suitable for large problems (may take more time).	→ suitable for large LP problems.