# M.E. (Water Resources & Hydraulic Engineering) Examination, 2018

(2nd Semester)

## COMPUTATIONAL HYDRO DYNAMICS

**(Paper- VII)**

Time : Three Hours Full Marks : 100

Answer any ***four*** questions.

1. a) What do you mean by Finite Difference Method? Why it is named so?

   b) Explain what do you mean by "well posed" and "ill posed" problem?

   c) What do you mean by explicit and implicit approaches?

   d) Consider the function $f\left(x, y\right) = e^{2x} + e^{-3y}$ Consider the point (x,y) = 2,1.

   Assume $\Delta x = \Delta y = 0.01$.

   (i) Calculate the exact values of $f'\left(x\right)$ and $f'\left(y\right)$ at this point.

   (ii) Calculate the same using 1st order accurate forward and backward differences. Also calculate the percentage difference compared with the exact values.

   (iii) Calculate the same using 2nd order accurate central differences. Also calculate the percentage difference compared with the exact values.

   3+4+6+12 = 25

2. a) By Taylor series expansion show how can you obtain 2nd order accurate finite difference expressions for the terms $\frac{\partial u}{\partial z}$ and $\frac{\partial^2 w}{\partial y^2}$ at the forward and backward boundaries of the flow domain.

   b) By Taylor series expansion show how can you obtain 2nd order accurate finite difference expressions for the term $\frac{\partial^2 v}{\partial x \partial y}$ at the forward, backward and central boundaries of the flow domain.

   12+13 = 25

3. a) Determine the stability requirement for the solution of a finite difference equation $\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial y^2}$ for von Neumann stability method.

   b) Find out the numerical solution for the unsteady two-dimensional inviscid flow by MacCormack's technique. Use Euler equations in non-conservation form.

   13+12 = 25

4. a) Give examples of elliptic, hyperbolic and parabolic partial differential equations as found in fluid dynamics.

   b) By Polynomial approach show how can you obtain 1st order, 2nd order and 3rd order accurate finite difference expressions for the terms $\frac{\partial u}{\partial x}$ at the forward boundaries of the flow domain.

   c) Consider the viscous flow of water over a river bed. At a given station in the flow direction, the variation of the flow velocity (u) in the direction perpendicular to the bed (the z direction) is given by the expression u = $100(2p^3 - p^2 + 0.5p)$ where p = z/L and L = total depth of water = 1.0 m, velocity along the flow (u) is in meter per second. The dynamic coefficient of viscosity of water at $20^0$C (μ) = $1.002\times10^{-3}$ kg/(m·s). Values of u were measured at discrete grid points with $\Delta z$ = 1 cm. Calculate the shear stress ($\tau_o$) at the wall using the following ways: (a) Using 1st order accurate forward difference, (b) Using the 2nd order accurate forward difference and using the 3rd order accurate forward difference using Polynomial approach. Finally, compare these calculated finite-difference results with the exact value of $\tau_o$.

3+12+10 = 25

5. a) What do you mean by a staggered grid? Explain with examples.

   b) Find out the Tri-diagonal Matrix and the solution of the system of equations using Crank-Nicolson form of the 1-D linear heat conduction equation. Assume T is known at all grid points at the time level n. Choose to distribute seven grid points along x axis.

5+20= 25

6. a) Deduce the pressure correction formula using the x- and y-momentum equations (non-conservation form) for an incompressible viscous flow.

   b) Briefly discuss about the SIMPLE algorithm

20+5 = 25