9. a) Define linear span L(S) of a subset S of a vector space. Let $S=(\alpha,\beta)$ and $T=(\alpha,\beta,\alpha+\beta)$, then show that $L(S)=L(T)$.

   b) Find a basis containing the vectors (1, 1, 0) and (1, 1, 1).

10. What do you mean by inner product space ? Define norm of a vector. Prove that

$$|(\alpha,\beta)| \le \|\alpha\|\|\beta\|.$$

11. a) Define linear mapping. Show that $T: R^3 \to R^3$ defined by $T(x,y,z)=(x+1,\ y+1,\ z+1)$

   is a not linear mapping.

   b) Show that $T: R^2 \to R^3$ defined by

$$T(x,y,z)=(2x+y+z,\ x+2y+z),$$

   is a linear mapping. Find Ker T and dim Ker T.

Ex/ME/Math/5/T/121/2018

Bachelor of Engineering in Mechanical (Evening) Engineering Examination, 2018

(1st Year, 2nd Semester)

Mathematics - IV

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer ***any four*** questions. 12.5×4=50

1. Define Mean. Show that the sum of the difference of each item from its mean is zero. Let a and b be two positive numbers, then show that A.M. > G.M. > H.M..

2. a) Define Mode. State it advantages and disadvantages. Also describe it uses.

   b) From the following distribution of scores calculate the mean.

Scores :	50-59	60-69	70-79	80-89	90-99	100-109
Frequency :	6	20	40	50	30	6

3. What is advantages of sandard deviation of a set of observations ? Find the mean and standard deviation of the uniform distribution

$$f(x)=\frac{1}{x};\ (x=1,2,3,\cdots n).$$

4. a) State the axioms of probability.

   b) If A and B are two events which are mutually exclusive, then prove that

   $$P(A \cup B) = P(A) + P(B)$$

   c) If A be an event and its complementary event is A^C, show that $P(A^C) = 1 - P(A)$

5. a) One card is drawn from a standard pack of 52. What is the chance that either a king or a queen ?

   b) Two die are tossed. What is the probability that the sum is divisible by 3 or 4 ?

PART - II

Answer ***any four*** questions. 12.5×4=50

6. a) Show that any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix. Express

   $$\begin{bmatrix} -7 & 3 & 9 \\ -4 & 5 & -6 \\ 4 & 8 & -2 \end{bmatrix}$$

   as the sum of a symmetric and a skew symmetric matrix.

   b) Show that inverse of an orthogonal matrix orthogonal.

7. State Cayley-Hamilton theorem for matrix. If

   $$A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix},$$

   then show that A satisfies Cayley Hamilton Theorem.

   Hence find A^{-1}.

8. a) Define vector space. What do you mean by subspace of a vector space ? Show that intersecion of two subspaces is also a subspace but the union of two subspaces may not be a subspace.

   b) Express (3, 4, 5) as a linear combination of (1, 2, 3), (2, 3, 4) and (4, 3, 2).