Ref. No. : Ex/IEBE/T/323/2018

B.E. INSTRUMENTATION AND ELECTRONICS ENGINEERING THIRD YEAR SECOND SEMESTER – 2018

SUBJECT: Digital Signal Processing **Time: Three hours** **Full Marks 100**

Each question carries 10 marks.

Module:1

1. a) Express the sequence

$$x(n) = \begin{cases} 1 & n = 0 \\ 2 & n = 1 \\ 3 & n = 2 \\ 0 & else \end{cases}$$

as a sum of scaled and shifted unit steps.

b) Consider the discrete time sequence

$$x(n) = \cos\left(\frac{n\pi}{8}\right)$$

Find two different continuous time signals that would produce this sequence when sampled at fs=10kHz.

Or

a) Consider a sinusoidal signal $\sin(\omega_0 t)$ of period $T_0 = 2\pi/\omega_0$. It is sampled with period T_s. Find the condition to be satisfied from the sampled signal to be periodic.

b) Find whether i) $\sin(0.1\pi k)$ and ii) $\sin(0.3\pi k)$ are periodic or not. Also compare the properties of these two signals in analog domain and digital domain.

Module:2

2. a) The input to a linear shift-invariant system is $x(n)=\cos(n\omega_0)$. Find the output if the real valued unit sample response of the system is h(n).

b) Determine the response of the system with impulse response

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

to the input signal

$$x(n) = 5 - 5\sin\left(\frac{\pi}{2}n\right) + 10\cos(\pi n), -\infty < n < \infty$$

Or

a) Determine whether the system is shift-invariant: $y(n)=x(n)+x(n-1)+x(n-2)$.
b) Determine whether the system is linear: $y(n) = \log(x(n))$.
c) Determine whether the system is causal: $y(n)=x(n)+x(n-1)+x(n-2)$.
d) Determine whether the system is stable: $y(n)=\cos(x(n))$.

Module:3: Answer any three

3. a) Find the 10-point IDFT of

$$X(k) = \begin{cases} 3, & k = 0 \\ 1, & 1 \le k \le 9 \end{cases}$$

b) Consider the finite length sequence

$x(n)= \delta(n) + 2\,\delta(n-5)$

Find the 10 point discrete Fourier transform.

4. a) Determine the z-transform of

$$x(n) = -\left(\frac{1}{2}\right)^n u(-n-1) + 2^n u(-n-1)$$

and depict the ROC and the locations of poles and zeros in z-plane.

b) Determine the z-transform of

$$x(n) = \left(\frac{1}{2}\right)^n u(n) + 2^n u(-n-1)$$

and depict the ROC and the locations of poles and zeros in z-plane.

c) Determine the z-transform and the pole-zero plot for the signal

$$x(n) = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & elsewhere \end{cases}$$

5. Write short notes on (any two)
   a) Gibb's phenomenon
   b) Phase delay and group delay
   c) Limitations of DFT.

6. Find the transfer function and then pole zero pattern of the following sequence:

	h(-2)	h(-1)	h(0)	h(1)	h(2)	h(3)	h(4)	h(n) otherwise
a)	1	-2.5	5.25	-2.5	1	0	0	0
b)	0	1	-.5	1	0	0	0	0
c)	0	0	1	-2.5	5.25	-2.5	1	0

Comment on the locations of the singularities and type of sequences, its specialties.

Module:4: Answer any five

7. A second order continuous time filter has a system function

$$H_a(s) = \frac{1}{s-a} + \frac{1}{s-b}$$

where a<0 and b<0 are real.

a) Determine the locations of the poles and zeros of H(z) if the filter is designed using the bilinear transformation with T_s=2.

b) Repeat part (a) for the impulse invariance technique.

c) Comment on the locations of singularities.

8. Design a BPF of unity gain to pass frequencies in the range 1-4 rad/sample using Hanning window M=7. The causal Hanning window is given as

$$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{M-1}\right) & 0 \le n \le M-1 \\ 0 & otherwise \end{cases}$$

9. Design an IIR low-pass filter for the following specification:
   Pass-band gain required : -1dB
   Frequency up to which pass-band gain must remain more or less steady : 25Hz
   Amount of attenuation required : -30dB
   Frequency from which the attenuation must start : 75 Hz
   Assume the sampling frequency to be 300Hz.

10. Determine cut off frequency/frequencies for a FIR filter whose transfer function is
   a) $H(z) = \frac{1}{2}(1 - z^{-2})$
   b) $H(z) = \frac{1}{2}(1 - z^{-1})$
   c) $H(z) = \frac{1}{2}(1 + z^{-2})$
   d) $H(z) = \frac{1}{2}(1 + z^{-1})$

11. a) Show that the bilinear transformation maps the jΩ-axis in the s-plane *onto* the unit circle, $|z|=1$, and maps the left-half s-plane, Re(s) < 0 *inside* the unit circle, $|z|<1$.
   b) What order Butterworth filter is necessary to design a digital low pass filter that has a passband cut off frequency
   $\omega_p = 0.375\pi \; with \; \delta_p = 0.01$
   and stopband cut off frequency
   $\omega_s = 0.5\pi \; with \; \delta_s = 0.01.$

12. a) An FIR linear phase filter has a unit sample response that is real with h(n)=0 for n<0 and n>7. If h(0) = 1 and the system function has a zero at $z=0.4\, e^{j\pi/3}$ and a zero at z=3, what is H(z)?
   b) The relationship between input and output of an FIR system is

$$y(n) = \sum_{k=0}^{N} b(k)x(n-k)$$

   Find the coefficients b(k) of the smallest order filter that satisfies following condition:
   i) The filter has (generalized) linear phase
   ii) It completely rejects a sinusoid of frequency $\pi/3$
   iii) The magnitude of the frequency response is equal to unity at $\omega=0$ and $\omega=\pi$.