BACHELOR OF (I.E.E.) ENGINEERING EXAMINATION, 2018

(1st Year, 2nd Semester)

Mathematics - III J

Time: Three hours Full Marks: 100

Notations/Symbols have their usual meanings.

Answer any *ten* questions.

- 1. (a) What do you mean by the limit of a sequences? Show that the limit of the sequence $\{x_n\}$, where $x_n = (a^n + b^n)^{1/n}$, $\forall n \in \mathbb{N}$, and 0 < a < b, is b.
 - (b) Is the sequence $\{x_n\}$; $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots$, $+\frac{1}{2n}$, $\forall n \ge 1$. monotonic? Justify your answer.

(c) Find
$$\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n}\right)$$
. 5+2+3

2. (a) Evaluate

$$\lim_{n \to \infty} \left[\frac{1}{n} \{ (a+1)(a+2).....(a+n) \}^{1/n} \right] \quad a > 0$$

(b) State Raabe's test. Use this to find the convargence of the series

$$1 + \frac{1}{2} \cdot \frac{1}{3} + \frac{1.3}{2.4} \cdot \frac{1}{5} + \frac{1.3.5}{2.4.6} \cdot \frac{1}{7} + \dots$$
 4+6

3. (a) Discuss the convergence of the following series :

(i)
$$\sum_{n=1}^{\infty} a_n$$
, where $a_n = \sqrt{n^4 + 1} - \sqrt{n^4 - 1}$

(ii)
$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)} p$$
, $p > 0$

(b) Define sequence of partial sums of the infinite series

$$\sum_{n=1}^{\infty} a_n.$$
 8+2

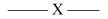
- 4. (a) Is the equation $(\cos y + y \cos x) dx + (\sin x x \sin y) dy = 0$ exact?
 - (b) Solve: $x dx + y dy + \frac{x dy y dx}{x^2 + y^2}$
 - (c) Use the method of variation of parameters to find the

solution of
$$\frac{d^2y}{dx^2} + a^2y = \tan ax$$
. $2+3+5$

subject to the conditions

$$u(0,y) = 0$$
, $u(a,y) = 0$, $0 < y < b$
 $u(x,0) = 0$, $u(x,b) = f(x)$, $0 < x < a$.

- 12. Find the Fourier series expansion for the function $f(x) = x x^2$ in the interval $-\pi \le x \pi$.
- 13. (a) Obtain the half range sine series for f(x) = x in the interval $0 \le x \le \pi$.
 - (b) Find the half range cosine series for $f(x) = x^2$ in the interval $0 \le x \le \pi$. 5+5=10



- 8. (a) Solve the PDE $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when x = 0, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$.
 - (b) Solve:

(i)
$$(x^2 - y^2 - z^2)p + 2xy q = 2xz$$

(ii) $z(xp - yq) = y^2 - x^2$ 3+(4+3)

- 9. Obtain the various Possible solutions of the onedimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, by the method of separation of variables. Identify the solution which is appropriate with the physical nature of the equation. Justify your answer.
- 10. Solve the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$, t > 0, subject to the conditions : u(0,t) = 0 and u(1,t) = 0 for t > 0; and $u(x,0) = 3 \sin n \pi x$.
- 11. Solve the two-dimensional Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad 0 \le x \le a, \ 0 \le y \le b,$$

5. (a) Solve the ODE:

$$(D^2+4)y = \sin 2x$$
, where $D = \frac{d}{dx}$.

(b) Solve the ODE:

$$x^{3} \frac{d^{3}y}{dx^{3}} + 2x^{2} \frac{d^{2}y}{dx^{2}} + 2y = 10\left(x + \frac{1}{x}\right)$$
 4+6

6. (a) Define the regular singular point of the second order differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

(b) Find the regular singular point of

$$(x^2-4) x^2y'' + 2x^3y' + 3y = 0$$

(c) Use power series method to solve

$$(1-x^2)y'' - 2xy' + 2y = 0$$
 2+2+6

7. (a) Form a PDE by eliminating the arbitrary constants a and b from

$$(x-a)^2 + (y-b)^2 + z^2 = 1$$

(b) Form a PDE by eliminating the arbitrary function f from

$$z = xy + f(x^2 + y^2)$$

(c) Form a PDE by eliminating the arbitrary functions f and g from

$$z = yf(x) + x g(y)$$
 3+3+4

(Turn Over)