Ref. NO.: EX/PG/ME/T/129A/2018

M.E. Mechanical Engineering Examination, 2018

(1st Year, 2nd Semester)

FINITE ELEMENT ANALYSIS IN ENGINEERING

Time: Three hours Full Marks: 100

Answer any *five* questions. All questions carry equal marks.

Q1. Consider a beam element with two degrees of freedom per node comprising of deflection and rotation. Assume the following shape functions as

$$N_1(\xi) = 1 - 3\xi^2 + 2\xi^3$$

$$N_2(\xi) = \xi - 2\xi^2 + \xi^3$$

$$N_3(\xi) = 3\xi^2 - 2\xi^3$$

$$N_4(\xi) = -\xi^2 + \xi^3$$

where ξ is the natural coordinate. Determine the element stiffness matrix and the deflection at the mid-span of a fixed-fixed beam subjected to uniformly distributed load.

Q2. Consider a constant strain triangle (CST) finite element. Express its shape functions in terms of natural coordinates. Show that all the elements of strain-displacement matrix of CST with isoparametric formulation are constants and are expressed in terms of nodal coordinates.

Q3. The shape functions of a 4-node isoparametric quadrilateral element having two degrees of freedom per node are expressed as $N_i = (1 + \xi\, \xi_i)\,(1 + \eta\, \eta_i)/4$, where ξ and η are the natural coordinates. Derive the strain- displacement matrix for this element. Also show that the element stiffness matrix with usual meanings of the symbols can be expressed as

$$[K]_e = t_e \int\int [B]^T[D][B]\det[J]\,d\xi\, d\eta$$

Q4. (a) Determine the weight factor and the location of the sampling point for 2-point Gauss quadrature.

(b) Derive the element body force vector for an isoparametric quadrilateral element.

(c) For an axially symmetric finite element of rectangular cross-section the shape functions with ususal notations are given by $N_i = (a \pm x)(b \pm z)/4ab$, where $x = r - r_m$ and $r_m = (r_1 + r_2 + r_3 + r_4)/4$, where r_m is the mean radius. Deduce the strain-displacement matrix for this element.

Q5. Explain the basic differences in the deformations based on Kirchhoff Theory and Mindlin Theory with suitable diagrams. Also express the strain-displacement relations in both the cases. For isotropic material what are the moment-curvature relations according to Kirchhoff Theory. Discuss the procedure for deriving the element stiffness matrix of a Kirchhoff plate element.

Q6. (a) Consider a 8-node isoparametric degenerated solid shell element. What are the degrees of freedom for the element? Express the displacements and cartesian coordinates of an arbitrary point in the element. Illustrate the above parameters with suitable diagrams. Also determine the Jacobian matrix of this element.

(b) Express briefly about generalized eigenvalue problem and standard eigenvalue problem. Discuss about mesh convergence and skyline storage scheme.