B.E. Production Engineering 1st year 2nd 2018

MATHEMATICS – IVS

Time: Three hours | Full Marks: 100

Answer any 10 questions.

(Symbols/Notations have their usual meanings)

1. Find the Fourier Series for $f(x) = x - x^2$ in $-\pi < x < \pi$. Hence deduce that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots\ldots\ldots$$ 10

2. Obtain the Cosine series for $f(x) = x$ in $0 < x < \pi$ and deduce that

$$\sum_{n=0}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi}{96}$$ 10

3. A periodic function of period 2 is defined as :

$$f(x) = \begin{array}{l} -1 < x \leq 0 \\ x + 2\,, 0 < x \leq 1 \end{array}$$

Find its Fourier series expansion. Hence, show that the sum of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \ldots\ldots = \frac{\pi}{4}$$ 10

4a. If $F(s)$ is Fourier transform of $f(x)$, then show that $F[f(x-a)] = e^{ias}F(s)$. 5.

b. Show that $F[x^n f(x)] = (-i)^n \frac{d^n F(s)}{ds^n}$ 5.

5. Find Fourier transform of

$$f(x) = \begin{array}{ll} 1 & if\ |x| < a \\ 0 & if\ |x| \geq a \end{array}$$

Where "a" is a positive real number. Hence deduce that $\int_Q^{\infty} \frac{Sin\, t}{t}\, dt = \frac{\pi}{2}$ 10.

6a. What is the directional derivative of $\varphi(x,y,z) = 4xz^3 - 3\,x^2y^2z$ at the point (2,-1, 2) in the direction $2\bar{\imath} - 3\bar{j} + 6\bar{k}$. 5

b. If $\bar{a}$ is a constant vector, prove that

$$\bar{\nabla} \times \left(\frac{\bar{a} \times \bar{r}}{r}\right) = \frac{\bar{a}}{r} + \frac{\bar{a}\cdot\bar{r}}{r^2}\ \bar{a}$$ 5

7 a. Evaluate $\nabla^2(log r)$ 5

b. Find the equation of the tangent plane and normal line to the surface $x^2 - y^2 = 4z$ at the point (3,1,2). 5

[Turn over

8. Verify Green's theorem in the xy –plane

$$\oint_{c} [(xy^2 - 2xy)dx + (x^2y + 3)dy],$$

where c is the boundary of the region enclosed by $y^2 = 8x$, and $x = 2$. 10

9. Using divergence theorem, evaluate $\iint_{S} \bar{\bar{F}} . \hat{n}\, ds$, where $\bar{F} = 4xyz\bar{\imath}\ xyz^2\bar{\jmath} + 3z\,\bar{k}$ above xy – plane bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$. 10

10. A taut string of length l has its ends $x = 0$ and $x = l$ fixed. The midpoint is taken to a small height h and released from rest at time $t = 0$. Find the displacement function $y(x,t)$. 10

11. A homogeneous rod of conducting material of length l has its ends kept at zero temperature. The temperature at the centre is T and falls uniform to zero at the two ends. Find the temperature function $u(x,t)$. 10

12 A rectangular metal plate is bounded by the lines $x = 0, x = a,\ y = 0$, and $y = b$. The three sides $x = 0, x = a$, and $y = b$. are insulated and the side $y = 0$ is kept at temperatute $u_0 \cos(\frac{\pi x}{a})$. Show that the temperature in the steady state is

$$u(x,y) = u_0 \operatorname{sech}(\frac{(b-a)\pi}{a}) \cosh(\frac{(b-y)\pi}{u}) \cos(\frac{\pi x}{a})$$ 10

13.a Using convolution theorem evaluate $Z^{-1}[\frac{z^2}{(z-1)(z-3)}]$ 5

b. Solve $U_{k+1} + U_k = 1,\ if\ u_0 = 0$ 5