Revised simplex

1) Row vector $X = [x_1, x_2, \cdots, x_n]$

2) column vector $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

3) Null vector $O = [0, 0, 0, \cdots, 0]$

$O = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

4) $m \times n$ matrix

→ rows → column

→ m row vector or n column vector

Inverse of a matrix

A

Inverse of matrix A $\Rightarrow A^{-1}$

$A \cdot A^{-1} = I$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Inverse

$AI \rightarrow I A^{-1}$

↳ non singular matrix

↳ Its determinant is non-zero

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

C_1, C_2, C_3

$A = [C_1 \; C_2 \; C_3]$

$A = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$

$$A = [A_1 \; A_2]$$

$$A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$$

$$A_1 = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}$$

AB A_1 A_2

$$A = \left[\begin{array}{c|cc} 2 & 1 & 0 \end{array}\right]$$

2x1 1

$$B = \left[\begin{array}{cc} 4 & 3 \\ \hline 0 & 1 \\ 1 & 0 \end{array}\right]$$

$$AB = [A_1 \; A_2] \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = A_1 B_1 + A_2 B_2$$

ok!

$$AB = \begin{bmatrix} 25 & 18 \\ 8 & 7 \end{bmatrix}$$

Max $Z = cx$

s.t. $Ax \le b$ & $x \ge 0$

$Ax \le b \rightarrow z$

$$\begin{bmatrix} A & I \end{bmatrix} \begin{bmatrix} x \\ x_s \end{bmatrix} = b \qquad \begin{bmatrix} x \\ x_s \end{bmatrix} \ge 0$$

Basic / Non Basic variable

I → identity matrix

$Z = c_B x_B + c_N x_N$

$$\begin{bmatrix} B, N \end{bmatrix} \begin{bmatrix} x_B \\ x_N \end{bmatrix} = b$$

$B x_B + N x_N = b$

$x_B = B^{-1} b$

$x_B = ?$

$I^{-1} = I$

original set of equation

$$\begin{bmatrix} 1 & -c & 0 \\ 0 & A & I \end{bmatrix} \begin{bmatrix} z \\ x \\ x_s \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$\begin{bmatrix} z - cx \\ Ax + Ix_s \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$c = c_B -$

$Z = c_B x_B$

$= c_B B^{-1} b$

$= c_B b$

$x_B = I^{-1} b$

$Z = c_B I^{-1} b$

$\begin{cases} z - cx = 0 \\ z = cx \end{cases}$

$c_B B^{-1} b$

→ original one

$B^{-1} b$

$$\begin{bmatrix} 1 & C_B B^{-1} \\ 0 & B^{-1} \end{bmatrix} \begin{bmatrix} 1 & C_B B^{-1} \\ 0 & B^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} C_B B^{-1} b \\ B^{-1} b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -C & 0 \\ 0 & A & I \end{bmatrix} * \begin{bmatrix} Z \\ X \\ X_S \end{bmatrix} = \begin{bmatrix} C_B B^{-1} b \\ B^{-1} b \end{bmatrix}$$

any iteration

$Z_j - C_j$

$$\Rightarrow \begin{bmatrix} Z \\ X_B \end{bmatrix} \begin{bmatrix} 1 & C_B B^{-1} A - C & -C_B B^{-1} \\ 0 & B^{-1} A & B^{-1} I \end{bmatrix} \begin{bmatrix} Z \\ X \\ X_S \end{bmatrix} = \begin{bmatrix} C_B B^{-1} b \\ B^{-1} b \end{bmatrix}$$

entering variable ≥ 0 — entering variable

Optimality Condition checking

$$C_B B^{-1} A - C$$

For leaving variable

$$B^{-1} A = \quad \frac{B^{-1} b}{} > 0$$

$$X_B = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$B = \rightarrow B^{-1}$$

$$B^{-1} = E \cdot B^{-1}_{old}$$

a_{12}, a_{22}, a_{32} — pivot element

pivot element col

$$\begin{bmatrix} 1 & -\frac{a_{12}}{a_{22}} & 0 \\ 0 & \frac{1}{a_{22}} & 0 \\ 0 & -\frac{a_{32}}{a_{22}} & 1 \end{bmatrix}$$

$$\text{Max} \quad Z = 3x_1 + 5x_2$$

$$\text{s.t.} \quad x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

S.E.

$$Z = 3x_1 + 5x_2$$

$$x_1 + x_3 = 4$$

$$2x_2 + x_4 = 12$$

$$3x_1 + 2x_2 + x_5 = 18$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 2 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad C = [3 \; 5]$$

$$b = \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad X_S = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$Z = C_B X_B + C_N X_N$$

Iteración 0

$$C = [3 \; 5] \qquad [A \; I] = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$X_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} \qquad B = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1} = I$$

$$C_B = [0 \; 0 \; 0] \qquad Z = [0 \; 0 \; 0] \, X_B$$

B^{-1} b

$$Z = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \end{bmatrix}$$

$$C_B B^{-1} A - C = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -5 \end{bmatrix}$$

→ The result is not optimum

Iteration 1: Identify the most negative cases from $C_B B^{-1} A - C$

-5 is the most negative then entering variable will be x_2

leaving variable:

$$B^{-1} A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

$$\frac{b_2}{a_{22}} = \frac{12}{2} = 6 \leftarrow \text{minimum}$$

$$\frac{b_3}{a_{32}} = \frac{18}{2} = 9$$

→ x_4

$x_4 \rightarrow$ leaving $x_2 =$ entering

$x_B = \begin{bmatrix} x_3 \\ x_2 \\ x_5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

$B^{-1}_{new} = E \, B^{-1}_{old}$

$\eta = \begin{bmatrix} -\frac{a_{12}}{a_{22}} \\ \frac{1}{a_{22}} \\ -\frac{a_{32}}{a_{22}} \end{bmatrix}$ $a_{22} \rightarrow$ pivot element

$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$E =$ unit matrix

$\rightarrow$ except for the pivot column

$= \begin{pmatrix} -\frac{a_{13}}{a_{33}} \\ -\frac{a_{23}}{a_{33}} \\ \frac{1}{a_{33}} \end{pmatrix}$ $\eta = \begin{matrix} 0 \\ 2 \\ 2 \end{matrix}$ $\begin{matrix} -\frac{0}{2} \\ \frac{1}{2} \\ -\frac{2}{2} \end{matrix} = \begin{matrix} 0 \\ \frac{1}{2} \\ -1 \end{matrix}$

$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$B^{-1}_{new} = E \cdot B^{-1}_{old}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$x_3\ x_4\ x_5$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_3 \\ x_2 \\ x_5 \end{bmatrix} = B^{-1}b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 6 \\ 6 \end{bmatrix}$$

optimality condition

$$C_B B^{-1} A - C$$

$$\begin{bmatrix} 0 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & - \\ 0 & - \\ 3 & - \end{bmatrix} - \begin{bmatrix} 3, & - \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{5}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & - \\ 0 & - \\ 3 & - \end{bmatrix} - \begin{bmatrix} 3, & - \end{bmatrix}$$

$$= 0 - \begin{bmatrix} 3, & - \end{bmatrix}$$

$$= \begin{bmatrix} -3 & - \end{bmatrix}$$

optimality condition is not satisfied

3. Iteration 2

1. Determine the entering variable

z_1

2. Identify the leaving variable

$$B^{-1}A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & \cdots & \\ 8 & & \\ 3 & & \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$$X_B = \begin{bmatrix} x_3 \\ x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ \leq \\ c \end{bmatrix} \rightarrow 4/1$$

$$c/3 = 2$$

leaving variable will be x_5

$$X_B = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} \quad \cdot X_B = B^{-1}b$$

$$n = \begin{bmatrix} -1/3 \\ 0 \\ 1/3 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$B^{-1}_{new} = E\, B^{-1}_{old}$$

$$= \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{bmatrix}$$

$$X_B = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$$

optimality test $\quad C_B B^{-1} A - C$

non basic variable x_4, x_5

$$\begin{bmatrix} 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{bmatrix}$$

$$Z = C_B X_B$$

$$= \begin{bmatrix} 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$$

$$= 36 \Rightarrow \text{optimum value}$$