b) Investigte for what values of a, b the euqations

$$\begin{aligned} x+2y+3z &= 4 \\ x+3y+4z &= 5 \\ x+3y+az &= b \end{aligned}$$

have (i) no solution, (ii) unique solution and (iii) an infinite number of solutions. 5+5

13. a) Find the eigen values of the matrix

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}.$$

b) Find the characteristic equation of the matrix

$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{pmatrix}$ and hence complete A^3. 4+6

14. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$. Hence find A^{-1} and also evaluate

$A^5 - 27A^3 + 65A^2$. 10

Ex/Met/Math//T/122/2018

Bachelor of Metallurgical Engineering Examination, 2018

(1st Year, 2ndSemester)

Mathematics - II N

Time : Three hours — Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

(Notation / Symbols have their usual meanings)

PART - I

Answer ***any five*** questions.

1. a) If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, find the angle between

i) $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$

ii) $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$.

b) Show, by vector method, that the diagonals of a rohmbus intersect at right angles. (3+3)+4

2. a) Show that the straight line joining the mid points of two non-parallel sides of a trapezium is parallel to the parallel sides and half of their sum.

b) Determine the unit vector, which is perpendicular to the vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - 3\hat{k}$.

c) A particle acted on by a constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. Find the total work done by the forces.

4+3+3

3. a) Show that the necessary and sufficient condition for two proper vectors to be parallel is that their cross product must vanish.

b) Show that $(\vec{a}\times\vec{b})^2 = |\vec{a}|^2 \ |b|^2 - (\vec{a}\cdot\vec{b})^2$.

c) If $\vec{u}(t)$ and $\vec{v}(t)$ are two vector functions of the scalar variable t, then show that

$$\frac{d}{dt}(\vec{u}\times\vec{v}) = \vec{u}\times\frac{d\vec{v}}{dt}+\frac{d\vec{u}}{dt}\times\vec{v}.$$

2+3+5

4. a) If $\vec{a}=5t^2\hat{i}+t^3\hat{j}-t\hat{k}$ and $\vec{b}=2\hat{i}\sin t-\hat{j}\cos t+5t\hat{k}$

find i) $\frac{d}{dt}(\vec{a}\cdot\vec{b})$

ii) $\frac{d}{dt}(\vec{a}\times\vec{b})$.

b) Find the directional derivative of $\varphi = x^2yz+4xz^2$ at the point (1, –2, 1) in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$.

(3+3)+4

10. a) Prove that A is involuntary matrix if and only if

(I+A) (I–A) = 0.

b) Solve by matrix method the system of equation

$$x-3z=1$$
$$2x-y-4z=2$$
$$y+2z=4.$$

4+6

11. a) If $A=\begin{bmatrix}1 & 0 & 0\\ -1 & -2 & -1\\ 2 & 3 & 2\end{bmatrix}$, show that $A^3-A=A^2-I$ and hence find A^{-1}.

b) Determine the values of α, β, γ when the matrix

$$\begin{bmatrix}0 & 2\beta & \gamma\\ \alpha & \beta & -\gamma\\ \alpha & -\beta & \gamma\end{bmatrix}$$

is orthogonal.

5+5

12. a) Reduce the following matrix to normal form and find its rank

$$\begin{pmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 2 & -2\\ 6 & 3 & 0 & -7\end{pmatrix}$$

[Turn over

PART - II

(Notation / Symbols have their usual meanings)

Answer ***any five*** :

8. a) Prove that

$$\begin{vmatrix} a^2 & a^2 & (b+c)^2 \\ (a+c)^2 & b^2 & b^2 \\ c^2 & (a+b)^2 & c^2 \end{vmatrix} = 2abc(a+b+c)^3$$

b) Solve by Cramer's rule, the following system of equation

$$x+y+z=3$$
$$2x+3y+4z=9$$
$$x+2y-4z=-1$$

5+5

9. a) If x, y, z are distinct real numbers and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0,$$

then prove that $1+xyz=0$

b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ -2 & 3 & -2 \end{bmatrix}$. Express the matrix A as sum of symmetric and skew symmetric matrix.

5+5

5. a) Find div $\vec{F}$ and Curl $\vec{F}$, where

$$\vec{F} = \text{grad } (x^3+y^3+z^3-3xyz).$$

b) Show that div Curl $\vec{F} = 0$

5+5

6. a) What do you mean by a solenoidal vector ? Find the value of 'a' such that the vector

$$(ax^2y+yz)\hat{i}+(xy^2-xz^2)\hat{j}+(2xyz-2x^2y^2)\hat{k}$$

is solenoidal.

b) If $\vec{F} = (x+y+az)\hat{i}(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$, find a, b, c such that curl $\vec{F} = \vec{0}$. What do you mean by an irrotational vector ?

5+5

7. a) Using Green's theorem, evaluate

$$\int_C [(y-\sin x)dx + \cos x dy],$$

where C is the triangle enclosed by the lines $y=0$, $x=\frac{\pi}{2}$ and $y=\frac{2}{\pi}x$.

b) A vector field is given by

$$\vec{F} = (x^2-y^2+x)\hat{i}-(2xy+y)\hat{j}.$$

Show that $\vec{F}$ is irrotational and find φ such that $\vec{F} = \text{grad } \varphi$.

5+5

[Turn over

PART - II

(Notation / Symbols have their usual meanings)

Answer ***any five*** :

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