1) Unrestricted variables

$x_r \rightarrow$ unrestricted

$\left\{ \begin{array}{l} \geq 0 \\ \leq 0 \end{array} \right.$

$x_r = x_r' - x_r''$ $\rightarrow$ $x_r' > x_r'' \rightarrow +ve$

$x_r' < x_r'' \rightarrow -ve$

$x_r' = x_r''$

$x_r', x_r'' \geq 0$

2) Tie in entering basic variables :- (Key column)

i) Tie between two decision variables $\rightarrow$ arbitrarily

ii) Tie " decision and slack variables $\rightarrow$ decision variable

iii) Tie " two slack variables (surplus) $\rightarrow$ arbitrarily

3) Tie for leaving variables: (Key row) Degeneracy ??

left $\rightarrow$ right

	x_2	S_1	S_2	S_3	b_i
S_1	5	~~1~~ 5 10	0 5	0 5	25
S_2	7	~~0~~ 7 14	1 7	0	35
	9	0	0	1	

Rules:

1. Divide the coefficients of slack variables in the simplex method's table where degeneracy is seen by the corresponding positive numbers of the key column, in the row, starting from left to right.

2. Compare the ratios in step (i) from left to right column wise, select the row that contains the smallest ratio.

if tie between slack and artificial variable

$\underbrace{\text{artificial variable}}_{\text{prefer}}$

Ex

$$\text{Max } z = 3x_1 + 9x_2$$

s.t.: i) $x_1 + 4x_2 \leq 8$

ii) $x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

Sensitivity analysis

Relaxation: It is a modification of a problem instance that leads to a larger feasible set (increase the number of feasible solutions) or alternatively that the feasible set remain unchanged
RL

Restriction: → a smaller feasible set (decrease the number of feasible solution)
RS
– alternatively feasible set remain unchanged.

Max ↑ relaxation
↓

$\leq b_i$

$\geq b_i$

→ c_j ↑↓
→ b_i ↑↓ ⟶
→ a_{ij} ↑↓
→ ± constraints
→ ± variable

b_i ↑

Constraint type	b_i ↑	b_i ↓
$\leq$	RL	RS
$\geq$	RS	RL

a_{ij} ↑ ⟶ restriction

$a_{ij} \leq b_i$ ⟶

$a_{ij} \downarrow \rightarrow$ Relaxation

$a_{ij} \geq b_i$

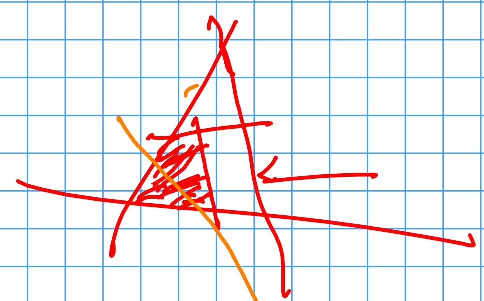
$a_{ij} \uparrow \rightarrow$ Relaxation

$a_{ij} \downarrow \rightarrow$ Restriction

$c_{ij} \uparrow$ $z^* \rightarrow$ larger/unchanged

$c_{ij} \downarrow$ $z^* \Rightarrow$ unchanged or lower

+ new constraint $\rightarrow$ Restriction



− constraint $\rightarrow$ Relaxation

+ new variable $\rightarrow$ Relaxation

− a variable $\rightarrow$ Restriction

Shadow price

* Economic interpretation of duality ?!

Objective: To know how large the effect of a certain change becomes in the optimal objective function

Def^n: The shadow price for a constraint is given by ~~the change~~ in objective function value when making a marginal increase of the right hand side

dual price

marginal price

Dual value

$$\text{Max } z = 3x_1 + 4x_2$$

$$x_1 + x_2 \le 2$$

$$x_1 + 5x_2 \le 10$$

$$x_1, x_2 \ge 0$$

$$u_i = \frac{\delta z}{\delta b_i}$$

$$\text{min } 2u_1 + 10u_2$$

$$u_1 + u_2 \ge 3$$

$$u_1 + 5u_2 \ge 4$$

$$u_1, u_2 \ge 0$$

Duality / Economic interpretation of Dual variables

production manager → Determines quantities of each product to be produced

⇓ optimizes resource utilization

⇓ marginal opportunity cost of each unit of resource is equal to its marginal return

⇓ To optimize the available resource

⇓ maximize profit

Shadow price indicates an additional price to be paid to obtain one additional unit of resource in order to maximize profit under resource constraint.

Shadow price

$$= \frac{\text{change in optimal objective function value}}{\text{unit change in the availability of resource}}$$

Maximization problem $\quad Z = \sum_{j=1}^{n} c_j x_j$

Z = return $\quad c_j$ = profit (or return) per unit of variable (activity) x_j

x_j = number of unit variable j

b_i = maximum no of units for a resource i available

a_{ij} = units of resource, i consumed (required) per unit variable j

Primal :

Maximize (return). $Z = \sum_{j=1}^{n} c_j x_j = \sum_{j=1}^{n}$ (profit per unit variable x_j) (unit of variable x_j)

s.t. $\quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i$

or $\sum_{j=1}^{n}$ (unit of resource i consumed per unit of variable x_j) (unit of variable x_j) $\leq$ unit of resource i available

$x_j \geq 0$ for al j

Dual LP problem:

minimize (cost) $z_y = \sum_{i=1}^{m} b_i y_i$

$= \sum_{i=1}^{m}$ (unit of resource i) (Cost per unit of resource i)

S.t.

$$\sum_{i=1}^{m} a_{ji} y_i \geq c_j$$

or $\sum_{i=1}^{m}$ (unit of a resource i consumed per unit of variable y_i) (Cost per unit of resource i) $\geq$ profit per unit for each variable x_j

$\hat{z}_x$, $\hat{z}_y$

$z_x \leq z_y$ for feasible solution for both primal & optimal solution

profit $\leq$ worth of resources.

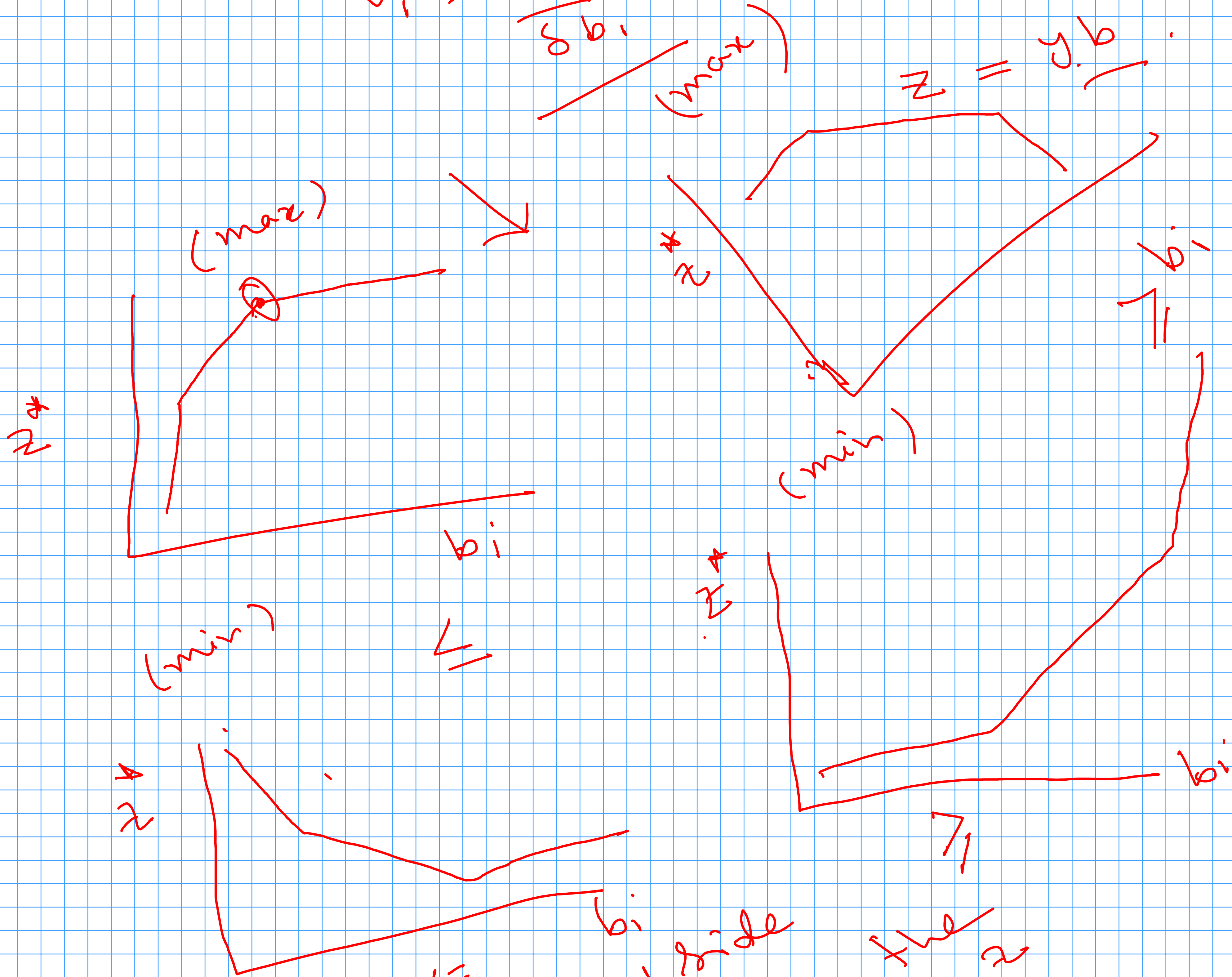
$\hookrightarrow$ weak duality

profit $=$ Worth of resources.

$\hookrightarrow$ strong duality

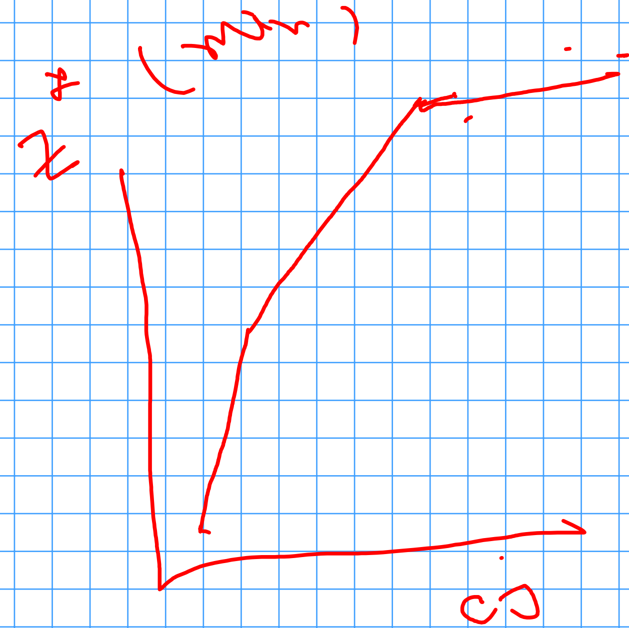
$$v_i = \frac{\delta z}{\delta b_i}$$

$$z = y_i b_i$$



A change in right hand side with the maximize z


max z
z*
cj


z* (min)
cj