

# Diophantusi egyenlet

①

$$13x + 8y = 17$$

Hely  $x$  és  $y$  számokra teljesül?

Kiszámoljuk d-t.

$$(13, 8) = 1$$

$$13x + 8y = 17 \quad | :1$$

$$13x + 8y = 17 \quad \text{Euklidesz algoritmus}$$

$$13 = 2 \cdot 8 + 5$$

$$13 - 8 = 5$$

$$8 - (13 - 8) = 3$$

$$8 = 5 \cdot 1 + 3$$

$$8 - 5 = 3$$

$$8 - 13 + 8 = 3$$

$$5 = 3 \cdot 1 + 2$$

$$5 - 3 = 2$$

$$2 \cdot 8 - 13 = 3$$

$$3 = 2 \cdot 1 + 1$$

$$3 - 2 = 1$$

$$(13 - 8) - 3 = 2$$

$$2 = 2 \cdot 1 + 0$$

$$13 \cdot (-3) + 8 \cdot 5 = 1 \quad | \cdot 17$$

$$13 \cdot (-51) + 8 \cdot 85 = 17$$

$$x = -51 + 2 \cdot 8$$

$$y = 85 - 2 \cdot 13$$

Altalános alak:

$$ax + by = c$$

$$a, b, c \in \mathbb{Z}$$

$$x, y \in \mathbb{Z}$$

$$13 - 8 - (2 \cdot 8 - 13) = 2$$

$$13 - 8 - 2 \cdot 8 + 13 = 2$$

$$2 \cdot 13 - 3 \cdot 8 = 2$$

$$(2 \cdot 8 - 13) - 2 = 1$$

$$2 \cdot 8 - 13 - (2 \cdot 13 - 3 \cdot 8) = 1$$

$$2 \cdot 8 - 13 - 2 \cdot 13 + 3 \cdot 8 = 1$$

$$5 \cdot 8 - 3 \cdot 13 = 1$$

megkaptuk az eredeti alakot

$$\textcircled{2} \quad \boxed{12x + 8y = 10} \quad / :4 \quad (12, 8) = 4$$

$$3x + 2y = 4 \cdot 10 \rightarrow \text{neues meßda's} \quad \hookrightarrow$$

$$\textcircled{3} \quad \boxed{12x + 20y = 28} \quad / :4 \quad (12, 20) = 4$$

$$3x + 5y = 7 \quad / \text{ueb. alg.}$$

$$5 = 3 \cdot 1 + 2$$

$$5 - 3 \cdot 1 = 2$$

$$5 - 3 = 2 \quad \textcircled{2}$$

$$3 - 2 = 1 \quad \textcircled{1}$$

$$3 = 2 \cdot 1 + 1$$

$$3 - (5 - 3) = 1$$

$$2 = 1 \cdot 2 + 0$$

$$3 - 5 + 3 = 1$$

$$2 \cdot 3 - 5 = 1$$

↓

$$3 \cdot 2 + 5 \cdot (-1) = 1 \quad / \cdot 7$$

$$3 \cdot 14 + 5 \cdot (-7) = 7$$

$$\underline{\underline{x = 14 + 2 \cdot 5}}$$

$$\underline{\underline{y = -7 - 2 \cdot 12}}$$

$$\textcircled{4} \quad \boxed{10x + 4y = 12} \quad | :2 \quad (10, 4) = 2$$

$$5x + 2y = 6 \quad | \text{eucl. algo.}$$

$$5 = 2 \cdot 2 + 1$$

$$5 - 2 \cdot 2 = 1$$

$$2 = 1 \cdot 2 + 0$$

$$5 \cdot 1 + 2 \cdot (-2) = 1 \quad | \cdot 6$$

$$5 \cdot 6 + 2 \cdot (-12) = 6$$

$$\begin{array}{r} x = 6 \quad y = -12 \\ + \underline{2 \cdot 2} \quad - \underline{2 \cdot 5} \end{array}$$

$$\textcircled{5} \quad \boxed{26x + 10y = 12} \quad | :2 \quad (26, 10) = 2$$

$$13x + 5y = 6 \quad | \text{eucl. algo.}$$

$$13 = 5 \cdot 2 + 3$$

$$5 = 3 \cdot 1 + 2$$

$$3 = 1 \cdot 2 + 1$$

$$2 = 2 \cdot 1 + 0$$

$$\textcircled{3} \quad \begin{array}{l} 13 - 2 \cdot 5 = 3 \\ 5 - 3 \cdot 1 = 2 \\ 3 - 2 \cdot 1 = 1 \end{array}$$

$$5 - (13 - 2 \cdot 5) = 2$$

$$5 - 13 + 2 \cdot 5 = 2$$

$$3 \cdot 5 - 13 = 2$$

$$(13 - 2 \cdot 5) - 2 \cdot 1 = 1$$

$$13 - 2 \cdot 5 - 2 \cdot 1 = 1$$

$$13 - 2 \cdot 5 - 2 = 1$$

$$13 - 2 \cdot 5 - (3 \cdot 5 - 13) = 1$$

$$13 - 2 \cdot 5 - 3 \cdot 5 + 13 = 1$$

$$2 \cdot 13 - 5 \cdot 5 = 1 \quad | \cdot 6$$

$$12 \cdot 13 - 30 \cdot 5 = 1 \quad \begin{array}{r} x = 12 \quad y = -30 \\ + \underline{1 \cdot 5} \quad - \underline{2 \cdot 12} \end{array}$$

$$\textcircled{6} \quad \boxed{8x + 6y = 16} \quad | : 2 \quad (8, 6) = 2$$

$$4x + 3y = 8 \quad (\text{ext.-algo.})$$

$$4 = 3 \cdot 1 + 1$$

$$4 - 3 = 1$$

$$3 = 2 \cdot 1 + 1$$

$$4 \cdot 1 + 3 \cdot (-1) = 1 \quad | \cdot 8$$

$$2 = 2 \cdot 1 + 0$$

$$4 \cdot 8 + 3 \cdot (-8) = 8$$

$$x = 8 + 2 \cdot 3$$

$$x \rightarrow +2 \cdot B$$

$$y = -8 + 2 \cdot 4$$

$$y \rightarrow -2 \cdot A$$

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$$\textcircled{7} \quad \boxed{7x + 11y = 118} \quad (\text{Bezout-Lemma + ext.-algo.})$$

$$11 = 1 \cdot 7 + 4$$

$$7 = 4 \cdot 1 + 3$$

$$4 = 3 \cdot 1 + 1$$

$$3 = 3 \cdot 1 + 0$$

$$1 = 4 - 3 = 4 - (7 - 4) = 4 - 7 + 4 = 2 \cdot 4 - 7 =$$

$$= 2 \cdot (11 - 7) - 7 = 2 \cdot 11 - 2 \cdot 7 - 7 =$$

$$2 \cdot 11 + (-3) \cdot 7 = 1 \quad | \cdot 118$$

$$236 \cdot 11 + (-357) \cdot 7 = 118$$

$$\underline{\underline{x = -357}}$$

$$\underline{\underline{y = 236}}$$

$$\textcircled{8} \quad (4F) \quad \boxed{47x + 49y = 10.000}$$