

CS350 – Fall 2018

Homework 1

Due Thursday, October 11th, hard copy in class or alternatively you can submit it as a pdf on D2L. Please make sure you write/print your name in the document, otherwise the homework will not be graded.

1. For each of the following pairs of functions $f(n)$ and $g(n)$, state whether $f(n) \in O(g(n))$, $f(n) \in \Omega(g(n))$, $f(n) \in \Theta(g(n))$, or none of the above. Briefly explain your reasoning.

(a) $f(n) = 2n^3 + 3n + 4$,	$g(n) = 57n + 75$
(b) $f(n) = \lg(2n + 1)$,	$g(n) = 3\sqrt{n}$
(c) $f(n) = \frac{n^2 + 2}{7 + 3^{-n}}$,	$g(n) = n(2n + 1)$
(d) $f(n) = \frac{2^n - n^2}{100}$,	$g(n) = 5n^4 + 3n^2 + 7$

2. What is the worst-case running time of the following function? Use big- O notation and show your work.

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pesky(n):  
  r ← 0  
  for i ← 1 to n do:  
    for j ← 1 to i do:  
      for k ← j to (i + j) do:  
        r ← r + 1
```

3. Consider the following eighteen functions:

\sqrt{n}	n	2^n
$n \log n$	$n - n^3 + 7n^5$	$n^2 + \log n$
n^2	n^3	$\log n$
$n^{1/3} + \log n$	$(\log n)^2$	$n!$
$\ln n$	$\frac{n}{\log n}$	$\log \log n$
$\left(\frac{1}{3}\right)^n$	$\left(\frac{3}{2}\right)^n$	6

Group these functions so that any two functions f and g are in the same group if and only if f and g have the same rate of growth, that is, iff $f(n) \in O(g(n))$ and $g(n) \in O(f(n))$. List the groups in increasing order.

4. Solve the following recurrences exactly.

(a) $T(1) = 8$, and for all $n \geq 2$, $T(n) = 3T(n-1) - 15$
(b) $T(1) = 3$, and for all $n \geq 2$, $T(n) = T(n-1) + 2n - 3$

- (c) $T(1) = 1$, and for all $n \geq 2$ such that n is a power of 2, $T(n) = 2T\left(\frac{n}{2}\right) + 6n - 1$
- (d) $T(1) = 1$, and for all $n \geq 2$ such that n is a power of 3, $T(n) = 4T\left(\frac{n}{3}\right) + n^2 - 7n + 5$