Department of Electrical Engineering and Computer Science

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

6.829 Fall 2015

Quiz Solutions

There are 5 problems and 13 pages in this quiz booklet. You have **80 minutes** to answer the questions.

If you find a question ambiguous, be sure to write down any assumptions you make. **Be neat and legible.**

Please write your name in the space below.

This is an open-papers, open-notes quiz. Internet access however is not allowed.

Do not write in the boxes below

1-2 (x/8)	3-8 (x/24)	9-13 (x/22)	14-17 (x/8)	18-19 (x/14)	Total (x/76)

Name:

I MIMO

Consider a scenario where a 2-antenna sender wants to transmit a packet p_1 to its 2-antenna receiver. Concurrently, there is a 2-antenna interferer transmitting a packet p_2 to its own receiver. In principle, a 2-antenna receiver should be able to decode two independent signals, and hence, our receiver should be able to receive from its transmitter in the presence of the interfering signal. However, in this case, the receiver has a software bug that prevents it from getting any signal from its second antenna.

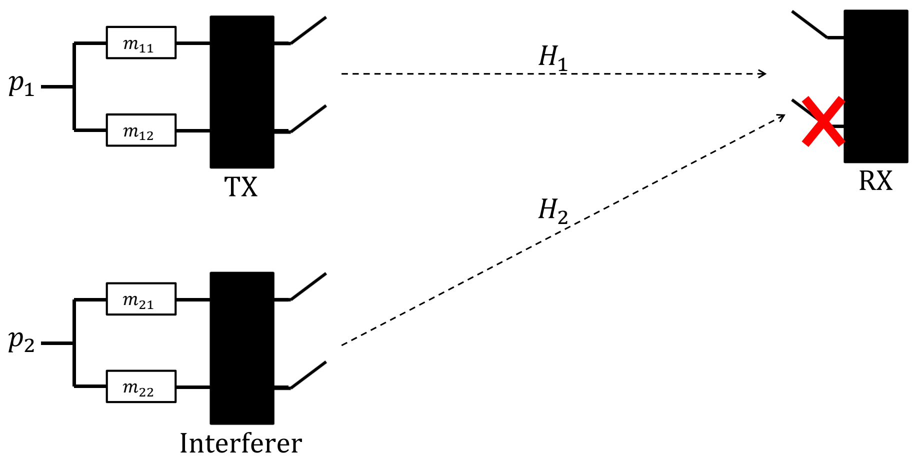

Figure 1: Two antenna transmitter (TX) transmitting to a two antenna receiver (RX). A two antenna interferer is transmitting concurrently with the transmitter TX.

The channel matrix from the transmitter to its receiver is H_1 and the channel matrix from the interferer to the receiver is H_2.

$$H_1 = \begin{pmatrix} \frac{\sqrt{3}+3i}{6} & \frac{1-i}{\sqrt{2}} \\ 2 & \frac{1+i}{2} \end{pmatrix} \qquad H_2 = \begin{pmatrix} \frac{\sqrt{3}-i}{2} & i\sqrt{3} \\ 1+i & \frac{1+i\sqrt{3}}{4} \end{pmatrix}$$

1. [4 points]: Compute a pre-coding vector $(m_{21}, m_{22})^T$ that when used by the interferer to pre-code its packet p_2, the receiver will be able to decode its desired transmitter despite the software bug.

Answer: Let y_1 and y_2 be the signal received at the first and second antenna at the receiver.

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = H_1 \begin{pmatrix} m_{11} \\ m_{12} \end{pmatrix} p_1 + H_2 \begin{pmatrix} m_{21} \\ m_{22} \end{pmatrix} p_2$$

Here, for each channel matrix H, H_{ij} is the channel from transmit antenna j to receive antenna i. Since the receiver only has one receive antenna working (i.e., only y_1), the interferer needs to null at RX's first antenna so that the receiver can still decode the transmitter's packet. Thus, we want

$$H_{2_{11}} m_{21} + H_{2_{12}} m_{22} = 0$$
$$\frac{\sqrt{3} - i}{2} m_{21} + i\sqrt{3} m_{22} = 0$$

If we assume the total amount of power transmitted is constant, i.e.,

$$\sqrt{m_{21}^2 + m_{22}^2} = 1$$

We can solve m_{21} and m_{22}.

2. [4 points]: Assume the interferer uses the above pre-coding vector, compute a second pre-coding vector $(m_{11}, m_{12})^T$ that when used by the transmitter to pre-code p_1 maximizes the SNR at the receiver.

Answer: Since the interferer nulls at the receiver's first antenna,

$$y_1 = (H_{1_{11}} m_{11} + H_{1_{12}} m_{12}) p_1$$

The goal is to

$$\begin{aligned} \underset{v_1, v_2}{\text{maximize}} \quad & |H_{1_{11}} m_{11} + H_{1_{12}} m_{12}|^2 \\ \text{subject to} \quad & \sqrt{m_{11}^2 + m_{11}^2} = 1 \end{aligned} \tag{1}$$

$H_{1_{11}} m_{11} + H_{1_{12}} m_{12}$ is the dot product of vector $\vec{h} = (H_{1_{11}}, H_{1_{12}})$ and $\vec{v} = (m_{11}, m_{12})$ and $\vec{v}$ lies on a unit circle. The best way to maximize the dot product is to align $\vec{v}$ with $\vec{h}$. Thus,

$$m_{11} = \frac{H_{1_{11}}^*}{\sqrt{|H_{1_{11}}|^2 + |H_{1_{12}}|^2}}$$
$$m_{12} = \frac{H_{1_{12}}^*}{\sqrt{|H_{1_{11}}|^2 + |H_{1_{12}}|^2}}$$

II Congestion Control

In this problem, you will design a congestion control protocol where the router measures the amount of excess traffic (or spare bandwidth) and explicitly tells each sender how much to decrease or increase their congestion window ($cwnd$). In this protocol, every packet has a header with two fields: $current\_cwnd$, and $current\_feedback$. When sending a packet, the sender sets $current\_cwnd$ to its $cwnd$. The router reads the $current\_cwnd$ and sets $current\_feedback$ to ask the sender to decrease or increase its $cwnd$. The receiver, conveys the $current\_feedback$ to the sender in the ack of the packet.

Assume N flows share the same bottleneck link, which has a capacity of C packets/second. Assume the buffer size is several times larger than the delay bandwidth product. Assume all flows have the same RTT of d seconds. Note that the system should work even if the flows started with different initial $cwnd$.
The protocol operates in rounds, which involve the router and senders as follows:

The Router

- In every round, n, the router computes the total number of packets it receives during that round which we call $load$. It also measures the queue size at the end of the round which we call $queue$. The router then computes a function called the $spare$. This function captures the amount of increase or decrease in the traffic rate that the router would like to achieve over the next round. It is computed as:

  $$spare[n] = \alpha(C \times d - load[n-1]) - \beta\, queue[n-1],$$

  where α and β are positive constants smaller than 1, which capture the fact that the router does not want to allocate all the spare in just one round.

- Now that the router knows the spare, it needs to decide how much each sender should increase or decrease by. We call that The **Control Law**:
  - If $spare[n] > 0$, each flow should increase to $cwnd[n+1] = cwnd[n] + spare[n]/N$
  - If $spare[n] < 0$, each flow should decrease to $cwnd[n+1] = cwnd[n] + (spare[n] \times cwnd[n]/load[n])$

The Sender

The sender collects the feedback it receives from the router for each packet it sent. At the beginning of the next round, each sender sets its congestion window to

$$cwnd[n+1] = cwnd[n] + \Sigma_i feedback[n, i],$$

where the sum is taken over all packets in the last round and $feedback[n, i]$ is the feedback for i^{th} packet. (If after update $cwnd$ is negative, it is set to 1 packet).

Answer the following questions:

3. [4 points]: How does the router set $current\_feedback$ in each packet so that the behavior of the senders match the Control Law above? Consider the case of $spare[n] < 0$. Compute $current\_feedback$ as a function of $current\_cwnd$ and $spare[n]$.

Answer: We want the total amount of feedbacks in a round matches the Control Law, i.e.,

$$\Sigma_i feedback[n, i] = \frac{spare[n] \times cwnd[n]}{load[n]},$$

The number of packets transmitted in a round from a single sender is cwnd[n]. Thus, we can let

$$feedback[n, i] = \frac{spare[n]}{load[n]}$$

4. [4 points]: Consider the case of $spare[n] > 0$, compute $current\_feedback$ as a function of $current\_cwnd$ and $spare[n]$ so that the behavior of the senders matches the Control Law. Note that the router does not know a priori the number of flows in the network N, yet still wants to divide the increase equally between the N flows. Thus to solve this part, the router first needs to compute N.

Answer: When $spare[n] > 0$, we want

$$\Sigma_i feedback[n, i] = \frac{spare[n]}{N}$$

We need to estimate the number of flows N first. Since each sender puts its $cwnd$ in the packet header, the router can use this information to estimate the number of flows. For a given sender with $cwnd$, the router will receive $cwnd$ number of packets with $cwnd$ in the packet header. Thus,

$$N = \sum_{packet_k} \frac{1}{cwnd_k}$$

where $cwnd_k$ is the $cwnd$ set in the packet header for $packet_k$. Here, we ignore the round index n for notation simplicity.

Note: The rest of the questions may be answered without computing the $current\_feedback$ by referring to the control law itself.

Given the right choice of α and β (e.g., $\alpha = 0.5$ and $\beta = 0.35$) the above protocol converges to a stable setting of $cwnd$ of all senders, independent of the initial values of their cwnds.

5. [4 points]: Knowing that the protocol converges, does it always converge to optimal efficiency (i.e., full link utilization and minimal queue size)?

Answer: Yes. When the protocol converges, $cwnd[n+1] = cwnd[n]$ and therefore $spare[n] = 0$, which gives us the constraint

$$\alpha(C \times d - load[n-1]) = \beta \, queue[n-1]$$

If $queue[n-1] > 0$, then $load[n-1]/d < C$. This means that the rate of packets arriving at the router is smaller than the rate the router sends packets to the link. Thus, the $queue[n]$ will become smaller, and $spare[n+1]$ will be positive again, making the senders send more packets (increases $load[n+1]$). At the end, this converges to

$$queue[n-1] = 0$$

$$load[n-1] = C \times d,$$

and achieves full link utilization and minimal queue size.

6. [4 points]: Knowing that it converges, does it always converge to fairness (i.e., all flows have the same $cwnd$)?

Answer: No. $load[n]$ is from all the senders and they don't need to have equal shares of the $load[n]$ to make the protocol converges. For example, with two senders, we can make

$$cwnd_{sender1} = load[n] = C \times d - \frac{\beta}{\alpha} queue[n-1]$$

and

$$cwnd_{sender2} = 0$$

7. [4 points]: Say that a flow lies to the router about its $cwnd$ value and put a value in the header that is half the actual current value, how does that affect fairness? i.e., how larger or smaller its $cwnd$ would be, in comparison to telling the truth?

Answer: The sender can emulate two flows by putting a value that is half the actual current value. Thus, it can have two times the $cwnd$ compared to telling the truth.

8. [4 points]: Say the above N flows share the bottleneck with a TCP sender, which ignores the routers feedback and applies the TCP congestion control law. Which of the following best describes the throughput of the flows (CIRCLE one).

(a) TCP will have a comparable throughput to the non-TCP flows

(b) TCP will have a significantly higher throughput than the non-TCP flows

(c) TCP will have a significantly smaller throughput than the non-TCP flows

Answer: b . The design of the protocol decreases the queue. If the queue is long, it will have smaller $spare[n]$ and ask senders to send fewer packets. However, TCP will not drop packets until the queue is full. Thus, a TCP sender will just continue sending packets and have much higher throughput than the others.

III RF-based Localization

9. [8 points]: Indicate whether each of the following statements are true or false by circling **True** or **False**. Please justify your answer in one/two lines.

A. True/False PinIt shows that the multipath profile of nearby RFID tags is more correlated than the multipath profiles of further away tags.
Answer: Out of scope.

B. True/False The accuracy of Arraytrack increases in multipath environments.
Answer: False. The mechanism for dealing with multipath may not work as well if there are too many multipaths.

C. True/False Both Arraytrack and PinIt localize an item with respect to some references in the environment. **Answer:** Out of scope.

D. True/False If WiFi nodes were as cheap and small as RFIDs, PinIt can be used to localize WiFi nodes in the same way as it is used to localize RFIDs **Answer:** Out of scope.

Consider a single antenna transmitter and a single antenna receiver. The receiver wants to measure its distance from the transmitter. Consider a multi-sub-carrier system like OFDM. However, unlike OFDM where the sub-carriers are continuous in frequency, in this system there are only two subcarriers at non-contiguous frequencies, f_1 and f_2. Assume that there is no noise, no multipath, and no CFO. Also, assume that the initial phase of the transmitted signal is zero and hence the phase of the signal at the receiver is due purely to the channel. Assume, the speed of light is 3×10^8. Further, the receiver knows that the transmitter must be within $200m$ from it.

10. [2 points]: First, assume that the receiver has access to the phase of the signal for f_1 but not f_2 and that $f_1 = 1$ MHz, can the receiver compute its distance from the transmitter? If yes, explain.

Answer: Yes.

$$\lambda_1 = \frac{c}{f_1} = \frac{3e^8}{1e^6} = 300m$$

The wave rotates less than a full cycle in $200m$. We can find a one-to-one mapping from the value of the phase to the receiver's distance to the transmitter.

For what follows, the receiver has access to the phase of the signal for both sub-carrier f_1 and f_2.

11. [4 points]: Let $f_1 = 3$ MHz, and $f_2 = 6$ MHz. Can the receiver compute its distance from the transmitter? If yes, explain.

Answer: No.

$$\lambda_1 = \frac{c}{f_1} = \frac{3e^8}{3e^6} = 100m$$

$$\lambda_2 = \frac{c}{f_2} = \frac{3e^8}{6e^6} = 50m$$

If we can find a one-to-one mappi6ng from the two phases (ϕ_1, ϕ_2) to the distance to the transmitter, we can compute the distnace to the transmitter without ambiguity. However, in this case, for any distance between $0m$ to $100m$, we can find another distance between $100m$ to $200m$ that gives us the same phase tuple (ϕ_1, ϕ_2).

12. [4 points]: Let $f_1 = 3$ MHz, and $f_2 = 5$ MHz. Can the receiver compute its distance from the transmitter? If yes, explain.

Answer: Yes.

$$\lambda_1 = \frac{c}{f_1} = \frac{3e^8}{3e^6} = 100m$$

$$\lambda_2 = \frac{c}{f_2} = \frac{3e^8}{5e^6} = 60m$$

The least common multiple for λ_1 and λ_2 is $300m$. Thus, we can find a one-to-one mapping from (ϕ_1, ϕ_2) to the distance to the transmitter within $200m$.

13. [4 points]: Let $f_1 = 3$ MHz, and $f_2 = 7$ MHz, what is the maximum distance between transmitter and receiver such that the receiver can still estimate its distance from the transmitter?

Answer: around 4285m

$$\lambda_1 = \frac{c}{f_1} = \frac{3e^8}{3e^6} = 100m$$

$$\lambda_2 = \frac{c}{f_2} = \frac{3e^8}{7e^6} = 42.85m$$

One can think of λ_2 is close to a prime number. Thus, the least common multiple for λ_1 and λ_2 is

$$\lambda_1 \times \lambda_2 = 4285m.$$

IV WiTrack/FMCW

WiTrack is based on a radar system called FMCW. Consider a scenario where you have two metallic balls in an empty field. Assume the field also has an FMCW system as descried in the WiTrack. The FMCW system has a single Tx antenna and a single Rx antenna, and is situated x_1 away from the first ball and x_2 away from the second ball. Assume that x_1 and x_2 are significantly larger than the distance between the Tx and Rx antennas, and that x_2 is significantly larger than x_1. Answer YES or NO. Explain your answers with no more than two sentences.

14. [2 points]: Can the system compute the distance of the two balls from itself (x_1 and x_2)? Explain in one sentence.

Answer: Yes. Assume no multipath and the separation between the two balls is larger than FMCW resolution ($\frac{c}{2BW}$). We get two distance measurements and the closer one is x_1 and the further one is x_2.

15. [2 points]: Assume there is a WiFi transmitter in the field at a third location x_3, can the system estimate its distance (assume no CFO)? Explain in one sentence.

Answer: No. Assume the WiFi transmitter does NOT reflect, the WiFi signal will only interfere with the FMCW signal. (If you assume the WiFi transmitter reflects in the same way as the metallic balls, you can still estimate it's distance.)

16. [2 points]: Replace the two metallic balls with two static humans. Can the system measure the distance of the static humans? Explain in one sentence.

Answer: Yes. The scenario is in an empty field, so we don't need to subtract consecutive measurements to remove environmental reflections. If we do need to separate human reflections from environmental reflections, we need to make sure our subtraction window captures breathing motion.

17. [2 points]: Assume the two humans start walking around along a random trajectory, can the system trace their distance from itself over time? Explain in one sentence.

Answer: Yes. We can trace their distances except when two people went to similar distances ($x_1 \approx x_2$). In that case, we will confuse which person is at x_1 and which is at x_2.

V MegaMIMO, Interference Alignment, Interference Nulling

For this problem, assume that we have two 3-antenna APs, connected via Ethernet and two 2-antenna clients.

18. [8 points]: What is the maximum number of concurrent packets/streams from the APs to the clients in each of the following systems:

Key ideas:

- At the transmitter, the number of antennas gives the number of degrees of freedom (DOF) the transmitter has.
- At the receiver, the number of antenna gives the number of equations to solve the unknown packets.
- Interference nulling at one receive antenna requires 1 DOF. Interference alignment requires 1 DOF.
- If all multipliers at the antennas of a transmitter are zero, the transmitter cannot transmit any packet.

A. 802.11n MIMO

Answer: 2 streams. 802.11n is single-user MIMO, so two APs cannot transmit at the same time. When a single AP is transmitting to one clinent, the client can decode two packets at the same time because it has two antennas.

B. JMB/MegaMIMO

Answer: Out of scope.

C. Interference Nulling

Answer: 2 streams. Let AP_1 transmit one stream to client 1 (C_1) and AP_2 tranmit a second stream two client 2 (C_2). AP_1 nulls at the two receive antennas at C_2 (using 2 DOF) and use the remaining DOF to transmit one stream of data to C_1. So deos AP_2.

D. Interference Alignment

Answer: 3 streams.

Let AP_1 transmit two streams to C_1 (using 2 DOF) and use the remaining DOF to align the two packets along the same direction in C_2's antenna space. AP_2 nulls at the two antennas at C_1 (using 2 DOF) and uses the remaining DOF to transmit one stream to C_2. Since AP_1 aligns its two packets as a single vector in C_2's antenna space, C_2 can decode AP_2's packet (two antennas can solve two unknown packets).

19. [6 points]: Which of the following requires the *senders* to compensate for any CFO they have before transmitting their packets. Circle the correct choices.

Answer: Out of scope.

(a) Interference alignment and nulling
(b) JMB/MegaMIMO
(c) Interference alignment
(d) Interference nulling
(e) ZigZag Decoding
(f) FICA

End of Quiz