Massachusetts Institute of Technology
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6.829 Fall 2017 **Problem Set 1 Solution** October 17, 2017

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1 Interference Cancellation

Consider 2 clients, Alice and Bob, who transmit to an access point (AP). Let P be the received signal power at the AP when Alice or Bob transmits alone. Let N be the noise power at the AP. Let W be the bandwidth of the wireless channel. Let R_{max} be the maximum rate at which either client alone (Alice or Bob) can deliver data successfully to the AP. Let $R_{Interference}$ be the maximum bit rate that the AP can correctly decode one client in the presence of interference from the other client.

1. Write the equation that gives R_{max} as a function of P, W and N.
   Answer: $R_{max} = W\log_2(1+\frac{P}{N})$

2. Write the equation that gives $R_{Interference}$ as a function of P, W and N.
   Answer: By treating the interference as noise, $R_{Interference} = W\log_2(1+\frac{P}{P+N})$

3. Consider low SNR scenarios (i.e., $\frac{P}{N} \to 0$) vs. high SNR scenarios (i.e., $\frac{N}{P} \to 0$). Define R_{opt} as the **maximum total bit rate** that can be successfully delivered to the AP (i.e., the maximum over one client transmitting alone and the two clients transmitting together). Which of these statements are true ? Give a one-line explanation of your answer.

   (a) In low SNRs, the AP can achieve R_{opt} if Alice and Bob, each transmits at R_{max} and ignores whether the other node is transmitting.
   Answer: True, because at low SNRs $R_{Interference}$ and R_{max} approach each other.

   (b) In high SNRs, the AP can achieve R_{opt} if Alice and Bob, each transmits at R_{max} and ignores whether the other node is transmitting.
   Answer: False, at high SNRs, if both nodes transmit at R_{max}, the total rate will be outside the capacity region and the AP cannot decode.

   (c) In both high and low SNRs, the AP can achieve R_{opt}, if Alice and Bob each transmits at $R_{Interference}$ and ignores whether the other node is transmitting.
   Answer: False If both Alice and Bob transmit at $R_{Interference}$, the capacity is under-utilized.

   (d) In both high and low SNRs, the AP can achieve R_{opt}, if Alice and Bob alternate between one client transmitting at R_{max} and the other transmitting at $R_{Interference}$.
   Answer: True If Alice and Bob transmit at R_{max} and $R_{Interference}$ alternatively, the capacity is fully utilized.

2 WiFi MAC

Consider an 802.11 network with 2 clients connected to an AP. One client has a good channel to the AP that can sustain a bit rate of 54 Mb/s, while the second client has a bad channel to the AP that sustains only a bit rate of 2 Mb/s. Assume the MAC is perfectly fair and efficient, and has no overhead. Also, assume that bitrate adaptation is perfect and has converged to the optimal rates mentioned above.

1. What is the throughput that each client achieves to the AP when operating individually?

Answer: In principle, they achieve throughputs of 54 Mb/s and 2 Mb/s respectively.

2. What is the throughput that each client achieves to the AP when the two clients operate jointly?

Answer: 802.11 MAC gives each node a fair chance to access the channel and then send their packets, i.e., it has packet-based fairness. Assuming packet sizes are the same, since we are told the MAC is fair, each node has a rate of $\frac{1}{\frac{1}{54}+\frac{1}{2}} = 1.93$ Mbps

3 OFDM

A 20 MHz channel between a single antenna transmitter and a single antenna receiver was observed to have the frequency response in Figure 1:

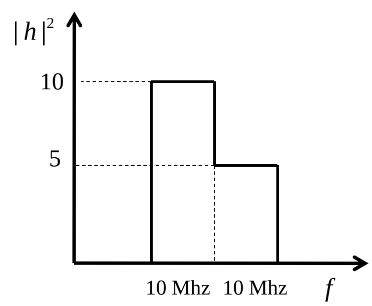

Figure 1: The figure shows the frequency response (i.e., the attenuation squared) of a 20 MHz wireless channel. The frequency response is flat for segments of 10 MHz each.

1. Assume that each of these subcarriers is a narrowband channel. Compute the capacity of each of these narrowband channels assuming that the transmit power in each of them is 1, and the noise power in each of them is also 1.
   Answer: We use the equation: $C = W\log_2(1 + SNR)$, where the $SNR = P_{received}/N$ and $P_{received} = P_{transmited} \times |h|^2$. Hence, the capacity of the first channel is $10M \times \log_2(1+10/1) = 34.59Mbps$, and the capacity of the second channel is $10M \times \log_2(1 + 5/1) = 25.85Mbps$.

2. Compute the capacity of the wideband channel which has both of these subcarriers, assuming that the entire transmit power is 1 across the whole wideband channel, and the entire noise power across the entire channel is also 1. You should also assume that the noise is divided

equally between the sub-carriers. **Hint:** Note that the capacity is the upper bound on the throughput, and that the two sub-carriers have different attenuations. Hence, the optimal solution may distribute the Tx power unequally between them.
Answer: The overall capacity is given by the equation:

$$C = W\log_2\left(1 + \frac{P_1|h_1|^2}{N_1}\right) + W\log_2\left(1 + \frac{P_2|h_2|^2}{N_2}\right) \tag{1}$$

where P_1 and P_2 are the power of the transmitted symbols on the first and second channel, and N_1 and N_2 are the noise variances on the first and second channels respectively.

Since the total power of the transmitter is 1, then:

$$P_1 + P_2 = 1 \tag{2}$$

And, since the noise is equally divided among both subcarriers, then:

$$N_1 = N_2 = 1/2 \tag{3}$$

Hence, we can rewrite the capacity equation as:

$$C = W\log_2\left(1 + \frac{10P_1}{0.5}\right) + W\log_2\left(1 + \frac{5(1-P_1)}{0.5}\right) \tag{4}$$

Since capacity is the maximum achievable rate, this becomes a maximization problem. Specifically,

$$\max_P \left\{W\log_2\left(1 + \frac{10P_1}{0.5}\right) + W\log_2\left(1 + \frac{5(1-P_1)}{0.5}\right)\right\} \tag{5}$$

To solve the maximization problem, we can simply differentiate with respect to P, i.e., $\frac{dC}{dP} = 0$, we obtain:

$$W\left(\frac{20}{(1+20P_1)} - \frac{10}{(1+10(1-P_1))}\right) = 0 \tag{6}$$

Solving this equation, we obtain $P = 21/40$. Now, by plugging it into the capacity equation, we obtain $C = 60.47Mbps$.

3. For the above channel, assume the transmitter and receiver use OFDM with two subcarriers, where the bandwidth of each OFDM subcarrier is 10 MHz. Alice remarks that the data in each OFDM subcarrier has to be transmitted at a different bit rate to achieve the capacity of the channel. Bob, however insists that a single bit rate should suffice to achieve the capacity of the channel. Who is right and why?
Answer: Alice is right. For any frequency band of interest, the maximum possible bit rate is the capacity of that frequency band. Since each of the 10Mhz bands have a different capacity, different bit rates are required to achieve the capacity.

4 Wireless

Grace Hacker has a new idea to improve wireless reliability.

Consider a wireless transmitter sending bit b_i, which could be 0 or 1 with equal probability. The transmitter uses BPSK modulation, i.e., it sends $x_i = -1$ if b_i is 0, and $x_i = +1$ if b_i is 1. Assume that the channel introduces additive Gaussian random noise with zero mean, and a standard deviation of σ, so that the received symbol $y_i = x_i + n_i$, where the n_i are i.i.d random Gaussian variables. The receiver knows σ for the channel, but does not know the noise n_i experienced by individual transmitted symbols.

1. What is the distribution of the received symbols y_i?

Answer: y_i is a mixture of two Gaussians. With probability 1/2, it is a Gaussian with mean 1 and standard deviation σ, and with probability 1/2, it is a Gaussian with mean -1 and standard deviation σ.

2. Given a y_i, what is the receiver's best estimate of the corresponding transmitted bit b_i?

Answer: Let y be the random variable corresponding to the received symbol. We predict $b_i = \beta, \beta = 0, 1$ such that $P(y = y_i | b_i = \beta)$ is maximized i.e. we predict 1 if $P(y = y_i | b_i = 1) > P(y = y_i | b_i = 0)$. This means that $e^{-\frac{(y_i-1)^2}{2\sigma^2}} > e^{-\frac{(y_i-(-1))^2}{2\sigma^2}}$. Simplifying, we get that we predict $b_i = 1$ if $y_i > 0$. Similarly, we predict $b_i = 0$ if $y_i < 0$. Note that $P(y_i = 0) = 0$, so you can assign it either way without affecting your success probability.

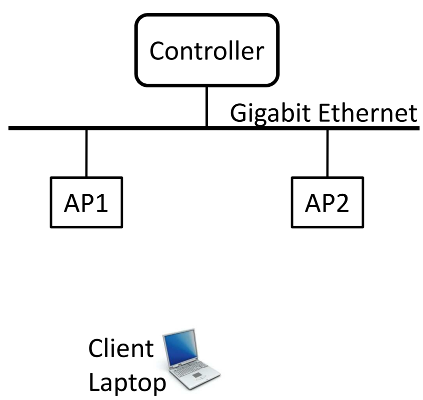

Figure 2: Wireless LAN Architecture

3. Consider the scenario in Fig. 2. When the laptop transmits a packet, it is received by both APs. The channels to the two APs both experience additive white Gaussian noise, but differ in the noise variance, i.e., AP1 receives $y_{i,1} = x_i + n_{i,1}$ and AP2 receives $y_{i,2} = x_i + n_{i,2}$, where $n_{i,1}$ is an i.i.d Gaussian with standard deviation σ_1 and $n_{i,2}$ is an i.i.d Gaussian with standard deviation σ_2, and these two Gaussians are independent of each other.

Grace leverages the Gb/s Ethernet to combine the received symbols across APs in order to increase reliability. For all bits in the packet, the two APs transmit their received symbols $y_{i,1}$ and $y_{i,2}$.

They also transmit their estimates of σ_1 and σ_2 to the controller. The controller, uses these values to produce an estimate of each transmitted bit b_i.

What is the controller's best estimate of b_i, given $y_{i,1}$, $y_{i,2}$, σ_1 and σ_2?

Answer: We predict $b_i = 1$ if $P(y_1 = y_{i,1}, y_2 = y_{i,2}|b = 1)$ is maximized. Since y_1 and y_2 are independent, this means that $P(y_1 = y_{i,1}|b = 1)P(y_2 = y_{i,2}|b = 1)$ is maximized. As above, this means that $e^{-\frac{(y_{i,1}-1)^2}{2\sigma_1^2}}.e^{-\frac{(y_{i,2}-1)^2}{2\sigma_2^2}} > e^{-\frac{(y_{i,1}+1)^2}{2\sigma_1^2}}.e^{-\frac{(y_{i,2}+1)^2}{2\sigma_2^2}}$. Simplifying, we get $\frac{(y_{i,1}+1)^2}{\sigma_1^2} + \frac{(y_{i,2}+1)^2}{\sigma_2^2} > \frac{(y_{i,1}-1)^2}{\sigma_1^2} + \frac{(y_{i,2}-1)^2}{\sigma_2^2}$. Hence, we predict $b_i = 1$ if $\frac{y_{i,1}}{\sigma_1^2} + \frac{y_{i,2}}{\sigma_2^2} > 0$, and $b_i = 0$ otherwise.

4. Now assume that you have a scenario with just one AP and one client and you want to improve reliability. In this question you are not allowed to change the client, you are only allowed to change the AP. Can you leverage the above combining technique to increase reliability? Explain how.

Answer: Retransmissions. If a packet has errors, the AP can combine the original packet and the retransmitted packet from the client using the technique above. This will not require any modifications to the client, but only needs the AP to implement the combining technique.

5 Angle of Arrival

Consider the case of a <u>two antenna receiver</u>. Recall that if the source is far away, then the wave is planer, and the difference between the phases of the signals on the two antennas is related to the spatial angle of the source by:

$$\frac{\Delta\phi}{2\pi} = \frac{D\cos\theta}{\lambda} \tag{7}$$

Where D is the separation between the two antennas, λ is the wavelength, $\Delta\phi$ is the difference between phase of signal received by the two antennas, and θ is the spatial direction of the source.

Is each of the following statements true or false? **Explain** your answer in no more than two lines.

- To identify the location of the source, we need to measure the the angle of arrival θ from at least two receiver locations.

  Answer: True. We intersect to angles of arrival from two receivers to identify the location.

- If D is multiple wavelengths, the system has zero noise, and the signal experiences no multipath, one can identify the direction of the source with very high accuracy from only two antennas using the equation above.

  Answer: False. Phase wraps around and creates ambiguity. One $\Delta\phi$ can be mapped to multiple possible θ. We need $D < \frac{\lambda}{2}$ to avoid such ambiguity.

- The above equation is not a suitable localization technique for scenarios with strong multipath

  Answer: True. Signals from multiple paths combine at the receivers making the estimation of θ incorrect with eq. 7.

- If there is a frequency offset between the transmitter and receiver, the receiver should first estimate the frequency offset and compensate for it before computing the direction using the above equation.

  Answer: False. The phase difference subtracts out the phase shift caused by the frequency offset.

6 WiTrack

For this problem, the following assumptions hold:

- There is no multipath.
- We operate with an ideal FMCW with no noise.
- The world is one-dimensional, so the localization problem is a ranging problem along 1 dimension (we care about finding a single distance to an object).
- The FMCW system has a single receive antenna, co-located with the transmit antenna.

1. Alissa wants to operate the system within the ISM band. This means that the total bandwidth allowed for the FMCW frequency sweep is 80 MHz. Can you tell Alissa the FMCW distance resolution that she would get based on this bandwidth, i.e., the minimum distance between 2 objects so that they may be located separably?

Answer: resolution $= \frac{c}{2B} = 1.875m$

2. After reading the WiTrack paper and learning about FMCW, Alissa is convinced that given that there is no multi-path, and for the scenario where there is a single object in the scene, she can localize that object with much higher accuracy than the minimum distance resolution computed in the previous question. Either provide a high-level description of how you could do that, or explain why this is not possible.

Answer: Correct. For the signle object case, one can estimate the phase of the signal to get a much accurate measurement. Let the phase $= e^{j2\pi\frac{d}{\lambda}} = e^{j2\pi f\tau}$, where d is the round trip distance from the antenna to the reflector, f is the frequency of the transmitted signal, τ is the time-of-flight. By using multiple f, we can estimate τ and thus d.

7 Distance-based Localization

Ben wants to build an RF-based localization system, where he attaches a transmitter to the object of interest and tries to localize it based on the received signal Assume for this question that localized object is far enough from the receiver such that the waves are planer. Ben is considering the following options for his system.

1. Assume that Ben is localizing a transmitter in an open field with no multipath. Further, Ben knows that the transmitter is between 100+epsilon meters in front of his receiver. He knows that epsilon is about +/-2 meters. Assume the noise is Gaussian and the transmitter does not move and the environment does not change. Ben claims that by making the transmitter transmits at low frequency like 27 MHz, he can find the value of epsilon to an accuracy of one millimeter using a single receiver with only two antennas. Is Ben right? Explain your answer.

Answer: Yes, Ben is right. The wavelength $\lambda = \frac{c}{f} = \frac{3*10^8}{27*x10^6} \simeq 11m$. Since the channel is $e^{-j\frac{2\pi d}{\lambda}}$, this implies ambiguity of 11 meters, i.e., the channel rotates by 2π every λ. However, $[-\epsilon, +\epsilon]$ is 4 meters wide which is smaller than 11 meters.

2. It turned out that it is very difficult to use low frequency due to FCC regulations. So Ben decided to operate his transmitter and receiver at 2.4 GHz, and use antenna arrays to obtain a good measure of direction. However, since it is very hard to build a very large antenna array, Ben decided to use SAR with a single moving antenna. Does Bens scheme work? Explain.

Answer: Antenna array provides only the direction. Hence, a single antenna array will not help in localization.

3. Ben clients changed their mind and became interested in locating people without requiring them to hold or carry any wireless device. Ben built a WiTrack device as described in the Witrack paper. However since he is operating in a planer domain (i.e., 2D localization) and he has no multipath effects in his setting, he removed the third RX antenna and operated using 2 RX antennas. Ben however failed to locate two people with his design. Can you explain to Ben why his device will not locate 2 people? Recall that the environment has no multipath.

Answer: At each receive antenna, we obtain two distance measurements. Two receive antennas give four possible locations.