MIT 6.829: Computer Networks Fall 2017

Lecture 5: Localization: Angle of Arrival

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1 Overview

Indoor localization has great potential in areas such as smart home, indoor navigation, easy-loss item marking, warehouse management and accurate advertisement. It is a hot area both in academia and industry. Especially a lot of start-ups pumps out.

This lecture is mainly on indoor localization using wireless signal. Three main topics have been discussed:

1. Why wireless localization for indoor situation
2. Localization use receive signal strength indicator (RSSI)
3. Localization use angle of arrival (AoA)[1]

2 Localization

2.1 Why wireless?

Nowadays the most widely used way of localization is GPS. However, it is not suitable for indoor situation because of great attenuation of concrete walls.

Two typical alternative are video localization and wireless location. Wireless has its great advantages mainly in 3 ways: ubiquity, portability, and its ability to localize through walls.

2.2 RSSI

FCC regulations require radiation power of certain devices at some certain level, and the power attenuation of electromagnetic wave though air is $P(d) \propto \frac{1}{d^2} P_{source}$ at far field, thus we can use received power information to know the the distance from a client to the access point (AP). If the distance of the client to at least 3 APs are know, we can then use a method called trilateration[2] to know where the client is.

An example application of RSSI is that when you are dialing 911, the base stations will measure their received power and tell the police what is your location.

The pros of this method is its easiness in adoption. However, multiple path result in constructively addition and destructively subtraction will make the power distribution much more complex than the equation described above. Thus for the usual indoor application, a lot of start-ups are tying

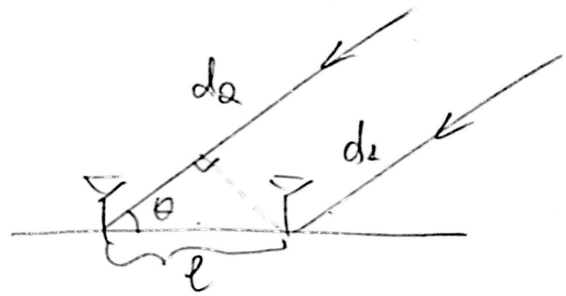

Figure 1: Time delay for signal arriving two antennas.

a method called fingerprinting to overcome such problem. They train the system by measurement RSSI at different positions in the room. However, this training procedure is intense and also the final result can not survive if the environment changes, which is a usual case in indoor places such as homes and offices where frequent furniture relocation will happen and people are walking around. Also RSSI is not robust to noise at faraway places because the power drops with distance while noise usually not. Low SNR brings in more uncertainty in localization.

2.3 AoA

Angle of Arrival methods are used to indicate the direction of the signal sources. In the following part, we will assume the signal source which is also our target for localization is far away from our receiver. It is a quite fair approximation in many real scenarios since the distance between our antennas are several centimeters while our targets are several meters away usually. Now, we will start by introducing the basic principle of AoA and then demonstrate how an antenna array works.

Basic idea of AoA is that the delay of the signal arrives two antennas varies corresponding to the direction of the transmitter. Thus by measuring the phase difference between received signals of two antennas we can recover the direction of the coming signal. Equations in (3) gives the specific procedure to calculate the direction θ from the channels h_i measured by two antennas. Fig. 1 demonstrates all the variables. d_i is the distance from the signal source to i'th antenna while h_i, Φ_i are the channel and its phase.

$$
\begin{aligned}
h_i &= \frac{1}{d_i} e^{-j\frac{2\pi d_i}{\lambda}} = \frac{1}{d_i} e^{-j\Phi_i} && (1)\\
\Delta d &= d_2 - d_1 = l\cos(\theta) && (2)\\
\Delta \Phi &= \Phi_2 - \Phi_1 = 2\pi \frac{l\cos(\theta)}{\lambda} && (3)
\end{aligned}
$$

We need to be careful when choosing the distance l between two antennas. In order to make sure we can recover θ from $\Delta\Phi$ uniquely, l should not be larger than $l_{max} = \frac{\lambda}{2}$. We can prove by simple math that this upper bound of l prevents the unwanted case where different source direction θ_1, θ_2 leads the same phase difference Δ of channels.

To detect the direction of signal by this AoA method is effective but also suffers from some problems:

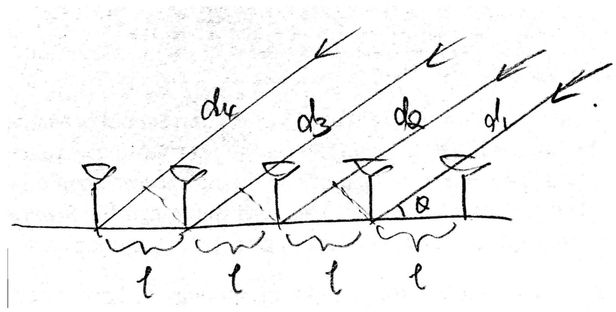

Figure 2: Antenna Array.

- *Multi-path:* When there exist many reflection paths, the channel will be much complicated than the idea case showed in Equ.(1). Thus the estimated angle will be very unrobust.
- *Nonuniform resolution:* Since $\cos(\theta)$ is not linear to θ, the resolution will be much lower when the angle of arrival is close to 0 or π, i.e., $\cos(0+\Delta\theta) - \cos(0) \ll \cos(\frac{\pi}{2}+\Delta\theta) - \cos(\frac{\pi}{2})$.
- *Half circle vision:* Due to $\cos(\theta) = \cos(-\theta)$, the range of degree that AoA can detect is from 0 to π, which means we can not distinguish whether the signal is from one side or the other if the antennas are isotropic.

Antenna Array is a natural extension from two antennas to n antennas. We are going to use the simplest antenna array system, *Uniformly spaced linear antenna array* as an example. As showed in Fig. 2, all antennas in an uniformly spaced linear antenna array was spaced uniformly by distance l. Similar to the two antenna case, we have formulas (4) to (7) for the channel phase of k'th antenna.

$$\Phi_1 = -2\pi\frac{d_1}{\lambda} \tag{4}$$

$$\Phi_2 = -2\pi\frac{d_1}{\lambda} = -2\pi\frac{d_1 + l\cos\theta}{\lambda} \tag{5}$$

$$\cdots \tag{6}$$

$$\Phi_k = -2\pi\frac{d_1}{\lambda} = -2\pi\frac{d_1 + (k-1)l\cos\theta}{\lambda} \tag{7}$$

Then we define a potential function $p(\theta)$ by eqn.(8) to indicate the power of the signal come from the direction angle θ. This potential function has a very important property: *Assuming the signal direction is θ^*, $p(\theta = \theta^*)$ will be extremely high while $p(\theta \neq \theta^*) \approx 0$.* Eqn. (9) to (13) gives a math explanation of that property. When $\theta = \theta^*$, the summation in eqn.(13) becomes n; While for $\theta \neq \theta^*$, the result of the summation approximates to zero. (Drawing the adding terms on a circle is a good way to visualize it.)

$$p(\theta) = \|\sum_{k=1}^{n} h_k e^{j2\pi k\frac{l\cos(\theta)}{\lambda}}\|^2 \tag{8}$$

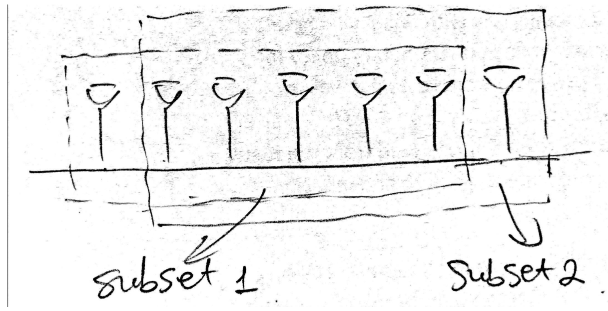

Figure 3: Choose two subsets of antennas.

$$
\begin{aligned}
p(\theta) &= \|\sum_{k=1}^{n} h_k e^{j2\pi k \frac{l\cos(\theta)}{\lambda}}\|^2 && (9)\\
&= \|\sum_{k=1}^{n} \frac{1}{d_k} e^{\Phi_k} e^{j2\pi k \frac{l\cos(\theta)}{\lambda}}\|^2 && (10)\\
&= \|\sum_{k=1}^{n} \frac{1}{d_k} e^{-2\pi \frac{d_1+(k-1)l\cos(\theta^*)}{\lambda}} e^{j2\pi k \frac{l\cos(\theta)}{\lambda}}\|^2 && (11)\\
&= \|\sum_{k=1}^{n} \frac{1}{d_k} e^{-2\pi \frac{d_1-l\cos(\theta^*)}{\lambda}} e^{j2\pi k \frac{l}{\lambda}(\cos(\theta)-\cos(\theta^*))}\|^2 && (12)\\
&\approx \|\frac{1}{d_1} e^{-2\pi \frac{d_1-l\cos(\theta^*)}{\lambda}} \sum_{k=1}^{n} e^{j2\pi k \frac{l}{\lambda}(\cos(\theta)-\cos(\theta^*))}\|^2 \text{ (approximating } \tfrac{1}{d_k} \text{ by } \tfrac{1}{d_1}) && (13)
\end{aligned}
$$

So far, we have demonstrated the principle for an antenna array to indicate the source direction. One main challenge remains unsolved is the Multi-path problem. In the ArrayTrack paper[1], a multi-path suppression algorithm is proposed based on the observation that the angle of the direct path is much more stable than the angle of the reflected path when a small relative movement happens between the transmitter and receiver pair. In order to simulate this small movement, the authors of [1] come up with a smart trick: Switching between different subsets of antennas to simulate the small relative movement. Fig. 3 illustrates the procedure where we switch between two subsets of an antenna array to simulate a tiny shift of the receiver.

References

[1] J. Xiong and K. Jamieson. ArrayTrack: A Fine-Grained Indoor Location System. In NSDI, 2013.

[2] Wikipedia contributors. "Trilateration." Wikipedia, The Free Encyclopedia. Wikipedia, The Free Encyclopedia, 8 Sep. 2017. Web.24 Sep. 2017