Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.829 Fall 2017 **Problem Set 2 Solutions** Nov 11, 2017

This problem set has 6 questions, each with several parts. Answer them as clearly and concisely as possible. You may discuss ideas with others in the class, but your solutions and presentation must be your own. Do not look at anyone else's solutions or copy them from anywhere.

Turn in your solutions on **Friday, November 7, 2017 before 11:59pm** by uploading it online.

1 Pot Pourri

Answer true of false and provide a short justification. Each question is independent from the others.

(a) If the speakers did not have any non-linearities, the Backdoor technique described in the readings could use amplitude modulation (AM).

Answer: True. If there are no speaker non-linearities, mutiplying the carrier signal with another sinusoid (AM) will not generate audible signals.

(b) Despite using frequency modulation (FM), the Backdoor paper still needs to use multiple speakers to transmit the signal.

Answer: True. We need the second speaker to transmit so that its signal sums up with the first speaker's signal at the receiver and generate audible sound.

(c) In the Backscatter Gen 2 Protocol, if there is one tag only but the reader does not know the number a priori, the protocol achieves 100% efficiency since there will be no collisions.

Answer: False. The reader still needs to estimate the number of slots to use.

(d) In the Buzz paper, the code that is generated is rateless, which means that it works regardless of how many bits each transmitted symbol has.

Answer: False. In Buzz, each transmitter still sends one bit per symbol. The rateless property comes from its ability to deal with multiple tags colliding. In such case, Buzz keep the tags colliding until it is able to decode.

2 MIMO

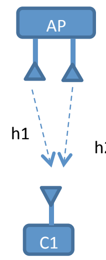
Recall that the **precoding vector** of a packet is a vector whose elements are the multipliers applied to the complex values transmitted by each antenna for the duration of that packet

1. Consider the system in the figure below. Let us refer to the pre-coding vector by (v1,v2). Assume the AP knows all channels in the system. What set of values for (v1,v2) ensure that the client in the figure does not hear any signal from the AP?

Answer: $v_1 \times h_1 + v_2 \times h_2 = 0$ Let the signal receive at c1 be y and the packet AP sends be x. Then

$$y = h_1(v_1 x) + h_2(v_2 x) = (h_1 v_1 + h_2 v_2)x$$

To ensure y is always zero, we choose $v_1 \times h_1 + v_2 \times h_2 = 0$.



2. Consider the same system in the above figure. Again assume the AP knows the channels. Pick values for (v1,v2) to maximize the SNR from the AP to the client, subject to the constraint that the power at the transmitter is 1 (i.e., subject to the constraint that the L2 norm of the pre-coding vector is 1).

Answer: The goal is to

$$\begin{aligned} \underset{v_1, v_2}{\text{maximize}} \quad & |h_1 v_1 + h_2 v_2|^2 \\ \text{subject to} \quad & \sqrt{v_1^2 + v_2^2} = 1 \end{aligned} \tag{1}$$

$h_1 v_1 + h_2 v_2$ is the dot product of vector $\vec{h} = (h_1, h_2)$ and $\vec{v} = (v_1, v_2)$ and $\vec{v}$ lies on a unit circle. The best way to maximize the dot product is to align $\vec{v}$ with $\vec{h}$. Thus,

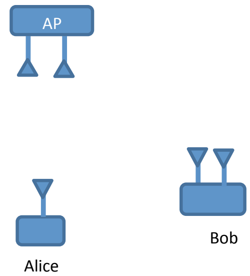
$$v_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}, v_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

(h_1^* and h_2^* can also be h_1 and h_2)

3. Consider the system in the figure below. Assume that the AP broadcasts the channels it measures from Alice and Bob in its beacons. Let the channels from Alice to the AP be (a1,a2) and from Bob to the AP (b1,b2;b1',b2'). Also assume the channels do not change. Also assume the AP decodes using **zero-forcing** for decoding and you cannot change the AP.

Say that Alice senses the medium as idle and starts transmitting her packet. Bob would like to transmit a packet concurrently with Alice. Alice transmits her packet at the maximum bit rate that is supported by her own channels to the AP, without knowing that Bob may concurrently transmit. Is there a pre-coding vector that Bob can use for his transmission that would allow the AP to decode Alice's packet and Bob's packet concurrently using zero-forcing? If the answer is yes, what is the precoding vector? If no, explain why. (Hint: Draw Alice's signal as a vector in the 2-dimensional space created by the AP's two antennas. Try to think how the vector representing Bob's signal should look like in this 2-dimensional space.)

Answer: Since Alice is already transmitting at the maximum bit rate, any additional interference will make the AP unable to decode Alice's packet. Thus, we need to precode Bob's packet such that it's orthogonal to Alice's packet in the 2-dimensional antenna space, i.e., let $\frac{b_1+b_1'}{b_2+b_2'}$ be proportional to $-\frac{a_2}{a_1}$



3 Full Duplex

1. After reading the MobiCom'10 full duplex paper, Ben Bitdiddle felt that it is impractical to require the receive antenna to be at a particular exact distance from the transmit antennas. Ben argues that it is easy to replace antenna cancellation with a different design that does not require a particular positioning of the receive antenna. As in the MobiCom paper, Ben's solution uses two transmit antennas and one receive antenna. It also keeps the hardware cancellation and the digital cancellation components with no modification. Describe a modification to antenna cancellation that matches the characteristics of Ben's solution.

Answer: We can choose a precoding multiplier for one of the antennas so that it nulls the other transmitted signal (combined destructively) at the receiver.

2. Describe an extension to Ben's solution so that it would stay accurate even for wideband 802.11 (i.e. , a large bandwidth of 40MHz or 80 MHz).

Answer: Estimate the channel at each frequency using OFDM, and then null at each frequency independently. That is, we can choose frequency specific precoding multipliers.

4 Multi-Hop Routing

1. Consider the wireless network shown below in Figure 1. The delivery probabilities shown are the combined delivery probabilities (i.e., the product of the forward and reverse link delivery probabilities). If no delivery probability is specified, the nodes are out of range. Node S is sending packets to D using a shortest path routing protocol based on the ETX metric. What is the path S will choose? What is the ETX metric of this path?

Answer: Node S will choose the path S-B-D. The ETX metric of the path is $1 + (1/0.7) = 2.43$

2. Consider the chain of 5 equally spaced nodes shown in Figure 2. The radio range is slightly larger than the inter-node distance, i.e., each node can sense his left and right neighbors only. Each link is perfect, i.e., the delivery probability is always 1.0 for any link. Node S is sending packets to D.

(a) What is the ETX metric of the 4-hop path in Figure 2?

Answer: 4

(b) Assume that the capacity of the wireless medium is 1 packet/second and there is a MAC which schedules transmissions optimally. What is the maximum throughput of the flow from S to D in Figure 2?

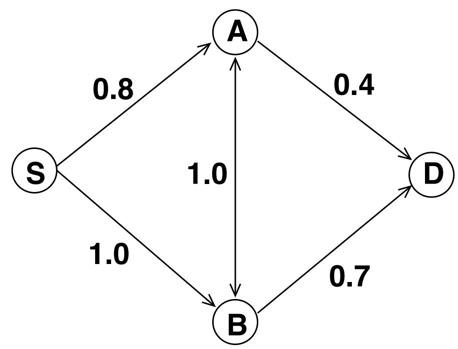


Figure 1: Simple wireless topology. The labels on the links are the delivery probabilities.

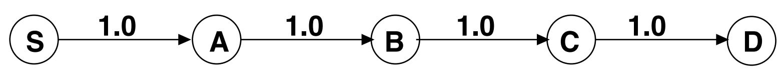


Figure 2: Simple wireless topology. The labels on the links are the delivery probabilities.

Answer: The maximum throughput of the path is 1/3 packets/second. This is because nodes S and C can send at the same time, hence the destination will receive 1 packet every 3 seconds.

(c) Why ETX cannot accurately predict the throughput of the path in Figure 2 whereas it correctly predicts the throughput of the path in Figure 1?

Answer: ETX does not take into account the spatial reuse possible in a wireless network. In the example above, ETX cannot account for the fact that S and C can send at the same time.

5 Congestion Control

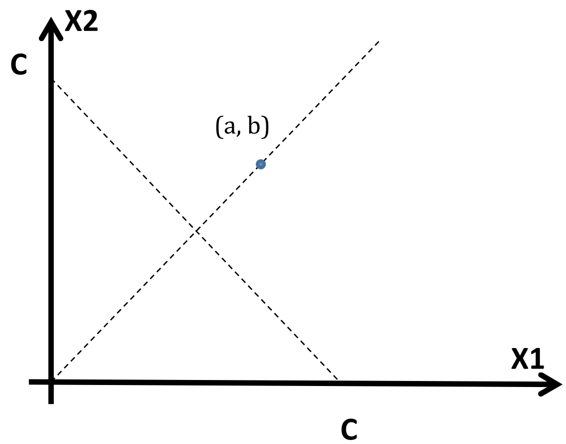
Alissa Hacker was hired by a client to improve congestion control in the client's network. Alissa's client complained that the TCP congestion control algorithm reacts too aggressively to a packet drop, which causes video streaming applications to stall. To address the problem, Alissa decided to redesign the congestion control algorithm. Alissa's new algorithm replaces TCP's additive increase and multiplicative decrease by the following two rules:

$$Increase\ Rule : cwnd = cwnd + \frac{1}{\sqrt{cwnd}} \tag{2}$$

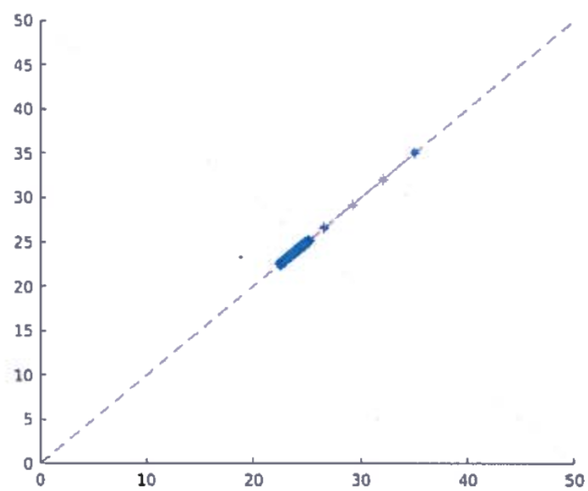
$$Decrease\ Rule : cwnd = cwnd - 0.5\sqrt{cwnd} \tag{3}$$

Where the "Increase Rule" refers to the increase in the congestion window as a result of receipt of one window of acknowledgements in an RTT, and the "Decrease Rule" refers to the decrease in the congestion window on detection of a loss by the sender, and where the congestion window is measured in number of packets.

1. Alissa decides to use phase-plots to check her intuition for the two-user case. The figure below shows a simple phase plot. In the graph, the state of the system is represented by a point (x1, x2), where x1 is flow 1's current rate, and x2 is flow 2's current rate. Assume the two flows have the exact same RTT. The total capacity of the bottleneck link is C bits/s. Let the initial state of the system be the tuple (a,b) shown in the figure below. Draw on the figure below how the system will evolve starting from (a,b) until it converges to its final state or a final set of states that it oscillates between.



Answer: The *cwnd* changes less aggresively than TCP's AIMD. A simulated example is shown in below:

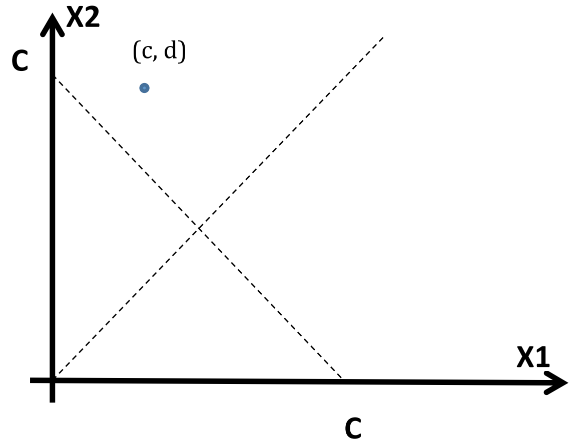


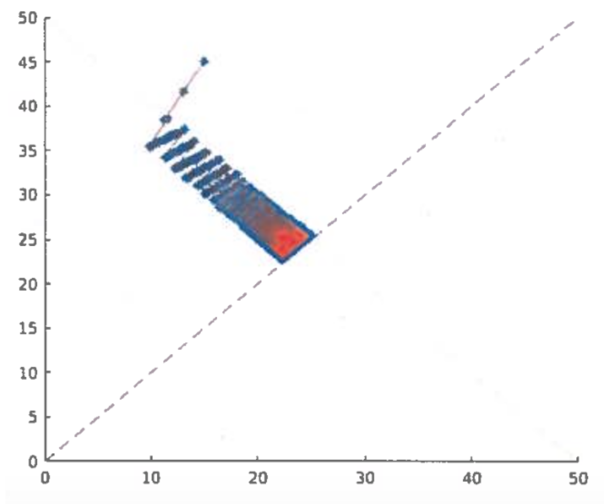
2. Now, let the initial state of the system be the tuple (c,d) shown in the figure below. Draw on the figure below how the system will evolve starting from (c,d) until it converges to its final state or a final set of states that it oscillates between.

Answer: The *cwnd* changes less aggresively than TCP's AIMD. A simulated example is shown below:

3. Does Alissa's algorithm provide fairness between flows? Does it provide efficient utilization of the bottleneck capacity? Justify your answer with a brief explanation (just two lines).

Answer: Yes for both. It converges to the intersection of the fairness line ($x1 = x2$) and the efficiency line ($x1 + x2 = C$).





6 BGP Madness

A network has four autonomous systems (ASs): W, X, Y, and Z. AS W has a direct route to some prefix p. These 4 ASs systems exchange routes to p **only** in the following manner (i.e., no other advertisements exist):

- AS W advertises p to X, Y, and Z.
- AS X advertises p to Y and Z.
- AS Y advertises p to X and Z.

1. Which of the following transit/peering relationships between the four ASs is consistent with the set of routing exchanges shown above (i.e., it would have generated the above advertisements and no more advertisements)? Provide a short explanation for each.

(A) W, X, and Y are providers for Z, and X and Y peer with each other.

Answer: True. Note that W can be a customer of X and Y.

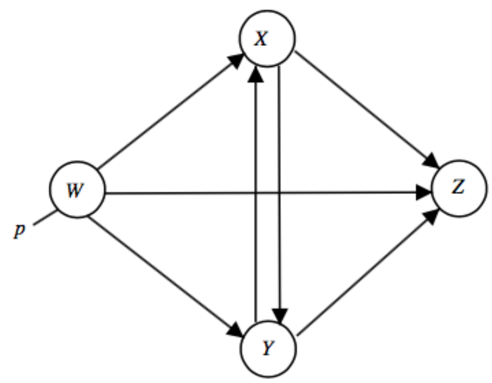


Figure 3: AS Topology. Arrows show direction of routing advertisements **(not business relationships)**.

(B) X, Y, and Z are providers for W, and X and Y peer with each other.

Answer: False. If Z is a provider for W, it will advertise p to X and Y.

(C) W is a provider for X, Y, and Z, and X and Y peer with each other.

Answer: False. If W is a provider for X, to generate arrows $X \to Z$ and $X \to Y$, X must be a provider for either Y or Z. In such case, however, it would generate $X \to W$.

(D) The set of exchanges is not consistent with any set of relationships.

Answer: False. Because A is true.

2. Eager B. Eaver notices that Y has a one-AS-hop route to p via W, and a two-AS-hop route to p via XW. He says If W issues a withdrawal to Y for prefix p, then Y knows that path XW is invalid as well, since they both go through W. Therefore, after W issues a withdrawal to Y, Y can remove all routes to p from its routing tables. However, Eager is incorrect. Explain why.

Answer: This is because W's intra-AS route to Y can be different from W's intra-AS route to X. For example, there might be a partition that can cause the iBGP sessions of the routers peering with Y and X to see different views. This can cause W to issue a withdrawal to Y, while still having a route to X.

3. In the steady state, X and Y each have two routes to p, and Z has three routes to p. Now W issues a withdrawal for p to X, Y, and Z. As these messages propagate, each AS will update its routes to p based on advertisements it has heard up to that time, select a new best route to p, and readvertise its new best route to its neighbors. We have shown the initial state of the routing tables and the first set of withdrawal messages in the first row of the table below. Fill in the rest of the table, proceeding until the state converges (i.e., until every AS has no route to p).

Notation: To indicate a routing announcement such as Y advertises path YW to Z and to X, please use the notation $Y \to \{Z, X\}:\ YW$.

Note: If you need more time-steps, feel free to add rows.

Answer:

Time step	X's routes	Y's routes	Z's routes	*Routing messages*
1	W, YW	W, XW	W, YW, XW	$W \rightarrow X, Y, Z$: withdraw p
2	YW	XW	YW, XW	$X \rightarrow Y : YW, X \rightarrow Z : YW$ $Y \rightarrow X : XW, Y \rightarrow Z : XW$
3	-	-	YW, XW	$X \rightarrow Y : -, X \rightarrow Z : -$ $Y \rightarrow X : -, Y \rightarrow Z : -$
4				