

## Lecture 9: MIMO

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## 1 Overview

This lecture is on:

1. MIMO(802.11n)
2. Next Generation 802.11n or 802.11n<sup>+</sup>

MIMO is an acronym for Multiple Input Multiple Output. It is an implementation of the 802.11 protocol for use with access points with multiple antennae. In theory, having multiple inputs and outputs should be able to transmit more data, or transmit data more accurately, or both. In this lecture we will examine the math that allows for MIMO to work.

We will compare the previous SISO (Single Input Single Output) protocols we have been studying, to the potential throughput increase that MIMO allows for.

## 2 SISO

We first review the mechanics of SISO transmission. Before, when we considered access points with only one antenna, we said the capacity for the channel could be given as:

$$C = W \log 1 + SNR$$

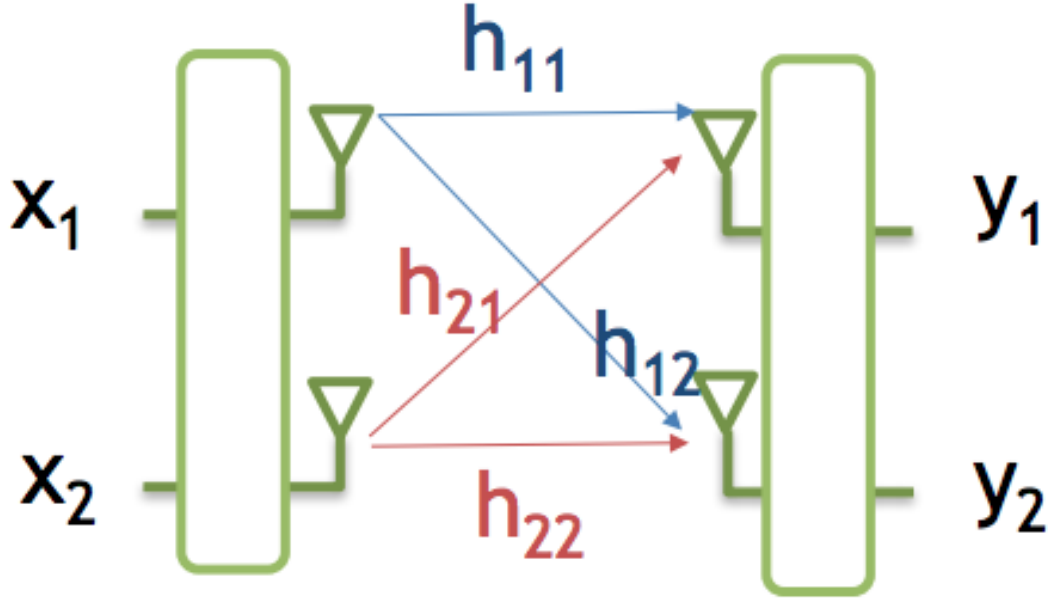
Where W is the bandwidth and SNR is the signal to noise ratio.

We were also able to say very simply, the received signal would be defined as:  $y = hx + n$  and we could estimate x as  $\hat{x} = \frac{y}{h}$ . Then, we can use previous lecture's methods to estimate h and transmit successfully.

## 3 MIMO

MIMO introduces the ability to multiply the capacity of a radio link through the use of multiple antennae to receive and transmit signals.

Very quickly, we run into problems when we realize that signals from the same access point will start to interfere with one another.



This means that at each antenna, the received signal will now be:

$$y_1 = h_{11}x_1 + h_{21}x_2 + n_1$$

$$y_2 = h_{12}x_1 + h_{22}x_2 + n_2$$

If we rewrite the equations as vectors and matrices, one of the outputs,  $\vec{y}$ , one of the inputs,  $\vec{x}$ , and a matrix of the different channels, we can rewrite this system of equations as:

$$\vec{y} = H\vec{x} + \vec{n}$$

Since we know the output vector and the channel matrix, we can estimate the value of  $\vec{x}$  as:

$$\hat{\vec{x}} = H^{-1}\vec{y}$$

If the channel matrix is unable to be invertible, we call the matrix unconditioned. To avoid scenarios where the channel matrix becomes unconditioned, one can separate the access point antennae by at least  $\frac{\lambda}{2}$ . Through this separation, we can prevent the channel matrix from being unconditioned.

## 4 Estimating the Channel

As before, we must use the preamble of a packet to estimate the channel it is being transmitted on. In this case however, since we have the interference of the two signals at once, we must send one preamble at a time. Once all preambles have gone through, we can simultaneously transmit all data.

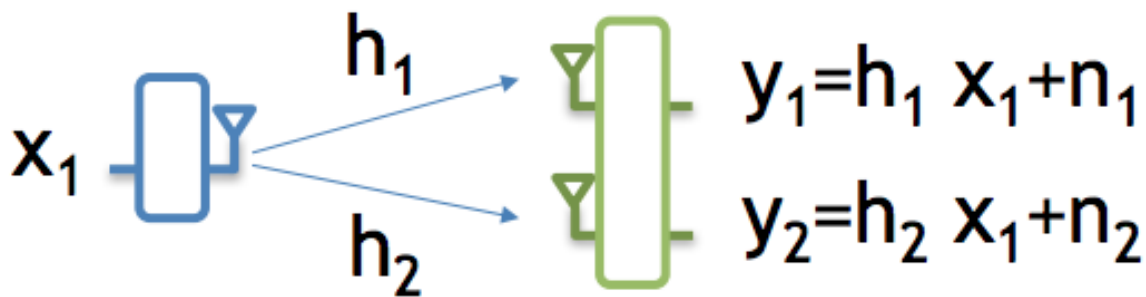
## 5 802.11n Implementations

### 5.1 Diversity Gain

In simple terms, Diversity Gain uses each available antenna and transmits the same packet across all of them.

We will consider two cases:

#### 5.1.1 Receive Diversity



In the case that we transmit the same packet to a receiver with two antennae, we must choose how to decode.

In class, we spoke of a variety of possible ways to decode and determined that the best decoder is a process called *Maximal Ratio Combining*

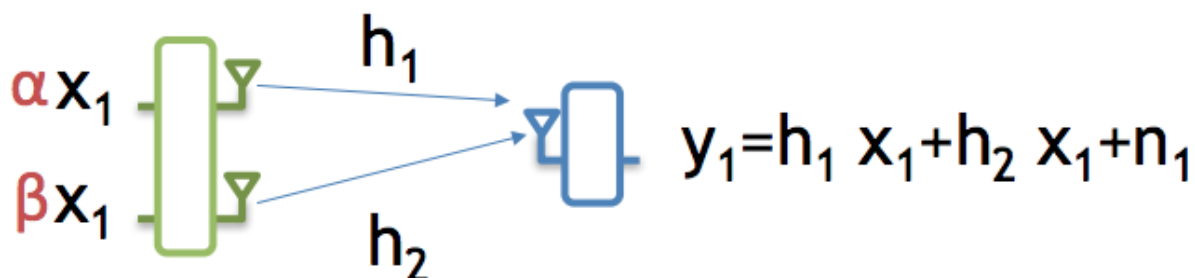
Given the channels,  $h_1$  and  $h_2$ :

1. Find the complex conjugates  $h_1^*, h_2^*$
2. Solve  $y_{est} = h_1^* y_1 + h_2^* y_2$

By using the complex conjugates, we are able to say that  $y_{est} = (|h_1|^2 + |h_2|^2)x_1 + h_1^* n_1 + h_2^* n_2$ . Using this fact, we are able to increase the SNR from  $|h_1|^2 \frac{P}{\sigma}$  to  $(|h_1|^2 + |h_2|^2) \frac{P}{\sigma}$ , essentially doubling the SNR.

While this improvement does seem huge, this only increases channel capacity by a logarithmic factor.

### 5.1.2 Transmit Diversity



This can cause problems however since we are transmitting the same packet to the same receiver. Potentially we could add the two signals destructively and be unable to read either signal.

We can solve this by pre-coding the transmissions of  $x_1$  with  $\alpha$  and  $\beta$  such that:

$$\alpha = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

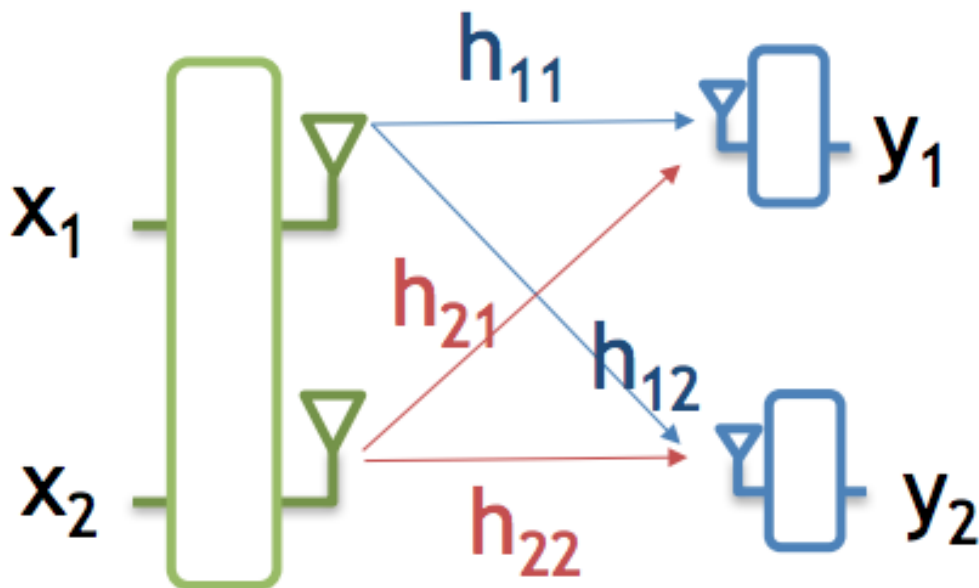
$$\beta = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

This is very similar to the work we do in Diversity Gain, but we realize the power from the transmission is limited, so we normalize the gain by a factor of  $\frac{1}{\sqrt{|h_1|^2 + |h_2|^2}}$ .

The SNR in this case is  $(|h_1|^2 + |h_2|^2) \frac{P}{\sigma}$  which is still only a logarithmic gain in the channel capacity.

## 5.2 Multiplexing Gain

In simple terms, Multiplexing Gain refers to sending more unique packets at once.



We still have to deal with the problem of interference. In this case, instead of the same packet interfering with itself in at a receiver, it will be interacting with the other packet being sent.

The solution to this is Interference Canceling.

## 5.3 Interference Canceling

The idea behind this is that as a signal traverses through the channel, the channel will modify the signal no matter what. If we use this to our advantage and pre-code the signal before we even send it, we can potentially have the channel decode for us.

We can use the idea to find how to pre-code the signal:

$$\begin{aligned} y_2 &= h_{12}(\alpha x_1) + h_{22}(\beta x_1) + n = 0 \\ h_{11}\alpha + h_{22}\beta &= 0 \\ \frac{\alpha}{\beta} &= \frac{-h_{22}}{h_{12}} \end{aligned}$$

This means that if we were to code each signal as follows:

$$\begin{aligned}x'_1 &= \alpha_2 x_2 + \alpha_1 x_1 \\x'_2 &= \beta_2 x_2 + \beta_1 x_1\end{aligned}$$

We will be able to accurately transmit the packets simultaneously and have them cancel at the receivers that shouldn't be interpreting them.

## 6 Next Generation 802.11n

We now discuss MIMO's relevance to 802.11. 802.11 was originally designed for 1 antenna nodes. This means that when a single antenna node transmits, all multi-antenna nodes refrain from transmitting.

### 6.1 802.11n<sup>+</sup>

We can avoid this problem of multi-antennae nodes waiting for lower antennae nodes to transmit by using interference canceling.

Often times however, it happens that this by itself is not enough and we must use Interference alignment to allow the signal to be interpreted. This is because it requires one full antennae to cancel out an interfering signal. Using only interference canceling alone quickly results in a loss of degrees of freedom and the system reverts to the normal 802.11.

Interference aligning uses only one antenna instead! By rotating the interfering signals to the same angle and canceling all signals at that angle, we are able to remove all interference in one go, and only lose one degree of freedom.

## References

- [1] Daniel Halperin, Wenjun Hu, Anmol Sheth, and David Wetherall. 802.11 with Multiple Antennas for Dummies.
- [2] Kate Ching-Ju Lin, Shyamnath Gollakota, and Dina Katabi. Random Access Heterogeneous MIMO Networks.