

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.829 Fall 2017

Problem Set 1

September 19, 2017

This problem set has 7 questions, each with several parts. Answer them as clearly and concisely as possible. You may discuss ideas with others in the class, but your solutions and presentation must be your own. Do not look at anyone else's solutions or copy them from anywhere.

Turn in your solutions on **Friday, September 29, 2017 before 11:59pm** by uploading it online.

1 Interference Cancellation

Consider 2 clients, Alice and Bob, who transmit to an access point (AP). Let P be the received signal power at the AP when Alice or Bob transmits alone. Let N be the noise power at the AP. Let W be the bandwidth of the wireless channel. Let R_{max} be the maximum rate at which either client alone (Alice or Bob) can deliver data successfully to the AP. Let $R_{Interference}$ be the maximum bit rate that the AP can correctly decode one client in the presence of interference from the other client.

1. Write the equation that gives R_{max} as a function of P , W and N .
2. Write the equation that gives $R_{Interference}$ as a function of P , W and N .
3. Consider low SNR scenarios (i.e., $\frac{P}{N} \rightarrow 0$) vs. high SNR scenarios (i.e., $\frac{N}{P} \rightarrow 0$). Define R_{opt} as the **maximum total bit rate** that can be successfully delivered to the AP (i.e., the maximum over one client transmitting alone and the two clients transmitting together). Which of these statements are true? Give a one-line explanation of your answer.
 - (a) In low SNRs, the AP can achieve R_{opt} if Alice and Bob, each transmits at R_{max} and ignores whether the other node is transmitting.
 - (b) In high SNRs, the AP can achieve R_{opt} if Alice and Bob, each transmits at R_{max} and ignores whether the other node is transmitting.
 - (c) In both high and low SNRs, the AP can achieve R_{opt} , if Alice and Bob each transmits at $R_{Interference}$ and ignores whether the other node is transmitting.
 - (d) In both high and low SNRs, the AP can achieve R_{opt} , if Alice and Bob alternate between one client transmitting at R_{max} and the other transmitting at $R_{Interference}$.

2 WiFi MAC

Consider an 802.11 network with 2 clients connected to an AP. One client has a good channel to the AP that can sustain a bit rate of 54 Mb/s, while the second client has a bad channel to the AP that sustains only a bit rate of 2 Mb/s. Assume the MAC is perfectly fair and efficient, and has no overhead. Also, assume that bitrate adaptation is perfect and has converged to the optimal rates mentioned above.

1. What is the throughput that each client achieves to the AP when operating individually?
2. What is the throughput that each client achieves to the AP when the two clients operate jointly?

3 OFDM

A 20 MHz channel between a single antenna transmitter and a single antenna receiver was observed to have the frequency response in Figure 1:

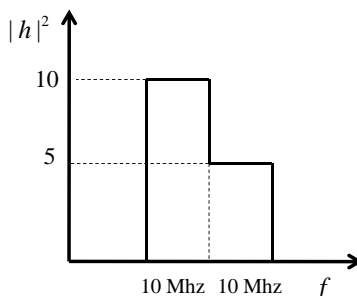


Figure 1: The figure shows the frequency response (i.e., the attenuation squared) of a 20 MHz wireless channel. The frequency response is flat for segments of 10 MHz each.

1. Assume that each of these subcarriers is a narrowband channel. Compute the capacity of each of these narrowband channels assuming that the transmit power in each of them is 1, and the noise power in each of them is also 1.
2. Compute the capacity of the wideband channel which has both of these subcarriers, assuming that the total transmit power is 1 across the whole wideband channel, and the total noise power across the entire channel is also 1. You should also assume that the noise is divided equally between the sub-carriers. **Hint:** Note that the capacity is the upper bound on the throughput, and that the two sub-carriers have different attenuations. Hence, the optimal solution may distribute the Tx power unequally between them.
3. For the above channel, assume the transmitter and receiver use OFDM with two subcarriers, where the bandwidth of each OFDM subcarrier is 10 MHz. Alice remarks that the data in each OFDM subcarrier has to be transmitted at a different bit rate to achieve the capacity of the channel. Bob, however insists that a single bit rate should suffice to achieve the capacity of the channel. Who is right and why?

4 Wireless

Grace Hacker has a new idea to improve wireless reliability.

Consider a wireless transmitter sending bit b_i , which could be 0 or 1 with equal probability. The transmitter uses BPSK modulation, i.e., it sends $x_i = -1$ if b_i is 0, and $x_i = +1$ if b_i is 1. Assume that the channel introduces additive Gaussian random noise with zero mean, and a standard deviation of σ , so that the received symbol $y_i = x_i + n_i$, where the n_i are i.i.d random Gaussian variables.

The receiver knows σ for the channel, but does not know the noise n_i experienced by individual transmitted symbols.

1. What is the distribution of the received symbols y_i ?
2. Given a y_i , what is the receiver's best estimate of the corresponding transmitted bit b_i ?

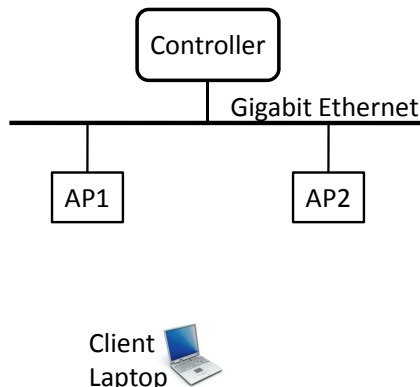


Figure 2: Wireless LAN Architecture

3. Consider the scenario in Fig. 2. When the laptop transmits a packet, it is received by both APs. The channels to the two APs both experience additive white Gaussian noise, but differ in the noise variance, i.e., AP1 receives $y_{i,1} = x_i + n_{i,1}$ and AP2 receives $y_{i,2} = x_i + n_{i,2}$, where $n_{i,1}$ is an i.i.d Gaussian with standard deviation σ_1 and $n_{i,2}$ is an i.i.d Gaussian with standard deviation σ_2 , and these two Gaussians are independent of each other.

Grace leverages the Gb/s Ethernet to combine the received symbols across APs in order to increase reliability. For all bits in the packet, the two APs transmit their received symbols $y_{i,1}$ and $y_{i,2}$. They also transmit their estimates of σ_1 and σ_2 to the controller. The controller, uses these values to produce an estimate of each transmitted bit b_i .

What is the controller's best estimate of b_i , given $y_{i,1}$, $y_{i,2}$, σ_1 and σ_2 ?

4. Now assume that you have a scenario with just one AP and one client and you want to improve reliability. In this question you are not allowed to change the client, you are only allowed to change the AP. Can you leverage the above combining technique to increase reliability? Explain how.

5 Angle of Arrival

Consider the case of a two antenna receiver. Recall that if the source is far away, then the wave is planar, and the difference between the phases of the signals on the two antennas is related to the spatial angle of the source by:

$$\frac{\Delta\phi}{2\pi} = \frac{D \cos \theta}{\lambda} \quad (1)$$

Where D is the separation between the two antennas, λ is the wavelength, $\Delta\phi$ is the difference between phase of signal received by the two antennas, and θ is the spatial direction of the source.

Is each of the following statements true or false? **Explain** your answer in no more than two lines.

- To identify the location of the source, we need to measure the the angle of arrival θ from at least two receiver locations.
- If D is multiple wavelengths, the system has zero noise, and the signal experiences no multipath, one can identify the direction of the source with very high accuracy from only two antennas using the equation above.
- The above equation is not a suitable localization technique for scenarios with strong multipath
- If there is a frequency offset between the transmitter and receiver, the receiver should first estimate the frequency offset and compensate for it before computing the direction using the above equation.

6 WiTrack

For this problem, the following assumptions hold:

- There is no multipath.
 - We operate with an ideal FMCW with no noise.
 - The world is one-dimensional, so the localization problem is a ranging problem along 1 dimension (we care about finding a single distance to an object).
 - The FMCW system has a single receive antenna, co-located with the transmit antenna.
1. Alissa wants to operate the system within the ISM band. This means that the total bandwidth allowed for the FMCW frequency sweep is 80 MHz. Can you tell Alissa the FMCW distance resolution that she would get based on this bandwidth, i.e., the minimum distance between 2 objects so that they may be located separably?
 2. After reading the WiTrack paper and learning about FMCW, Alissa is convinced that given that there is no multi-path, and for the scenario where there is a single object in the scene, she can localize that object with much higher accuracy than the minimum distance resolution computed in the previous question. Either provide a high-level description of how you could do that, or explain why this is not possible.

7 Distance-based Localization

Ben wants to build an RF-based localization system, where he attaches a transmitter to the object of interest and tries to localize it based on the received signal. Assume for this question that localized object is far enough from the receiver such that the waves are planar. Ben is considering the following options for his system.

1. Assume that Ben is localizing a transmitter in an open field with no multipath. Further, Ben knows that the transmitter is between $100 \pm \epsilon$ meters in front of his receiver. He knows that ϵ is about ± 2 meters. Assume the noise is Gaussian and the transmitter does not move and the environment does not change. Ben claims that by making the transmitter transmit at low frequency like 27 MHz, he can find the value of ϵ to an accuracy of one millimeter using a single receiver with only two antennas. Is Ben right? Explain your answer.

2. It turned out that it is very difficult to use low frequency due to FCC regulations. So Ben decided to operate his transmitter and receiver at 2.4 GHz, and use antenna arrays to obtain a good measure of direction. However, since it is very hard to build a very large antenna array, Ben decided to use SAR with a single moving antenna. Does Bens scheme work? Explain.

3. Ben clients changed their mind and became interested in locating people without requiring them to hold or carry any wireless device. Ben built a WiTrack device as described in the Witrack paper. However since he is operating in a planer domain (i.e., 2D localization) and he has no multipath effects in his setting, he removed the third RX antenna and operated using 2 RX antennas. Ben however failed to locate two people with his design. Can you explain to Ben why his device will not locate 2 people? Recall that the environment has no multipath.