# Package 'SimCorrMix'

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Type Package

**Title** Simulation of Correlated Data of Multiple Variable Types including Continuous and Count Mixture Distributions

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Author Allison Cynthia Fialkowski

Maintainer Allison Cynthia Fialkowski <allijazz@uab.edu>

Description Generate continuous (normal, non-normal, or mixture distributions), binary, ordinal, and count (regular or zero-inflated, Poisson or Negative Binomial) variables with a specified correlation matrix, or one continuous variable with a mixture distribution. This package can be used to simulate data sets that mimic real-world clinical or genetic data sets (i.e., plasmodes, as in Vaughan et al., 2009 < DOI:10.1016/j.csda.2008.02.032>). The methods extend those found in the SimMultiCorrData R package. Standard normal variables with an imposed intermediate correlation matrix are transformed to generate the desired distributions. Continuous variables are simulated using either Fleishman (1978)'s third order <DOI:10.1007/BF02293811> or Headrick (2002)'s fifth order <DOI:10.1016/S0167-9473(02)00072-5> polynomial transformation method (the power method transformation, PMT). Non-mixture distributions require the user to specify mean, variance, skewness, standardized kurtosis, and standardized fifth and sixth cumulants. Mixture distributions require these inputs for the component distributions plus the mixing probabilities. Simulation occurs at the component level for continuous mixture distributions. The target correlation matrix is specified in terms of correlations with components of continuous mixture variables. These components are transformed into the desired mixture variables using random multinomial variables based on the mixing probabilities. However, the package provides functions to approximate expected correlations with continuous mixture variables given target correlations with the components. Binary and ordinal variables are simulated using a modification of package GenOrd's ordsample function. Count variables are simulated using the inverse CDF method. There are two simulation pathways which calculate intermediate correlations involving count variables differently. Correlation Method 1 adapts Yahav and Shmueli's 2012 method <DOI:10.1002/asmb.901>. Correlation Method 2 adapts Barbiero and Ferrari's 2015 modification of the GenOrd package <DOI:10.1002/asmb.2072>. The optional error loop may be used to improve the accuracy of the final correlation matrix. The package also contains functions to calculate the standardized cumulants of continuous mixture distributions, check parameter inputs, calculate feasible correlation boundaries, and summarize and plot simulated variables.

**Depends** R (>= 3.3.1),

SimMultiCorrData (>= 0.2.1)

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Imports BB, nleqslv, MASS, mvtnorm, psych, Matrix, VGAM, triangle, ggplot2, grid, stats, utils

Encoding UTF-8

LazyData true

Roxygen list(wrap = FALSE)

RoxygenNote 6.0.1

Suggests knitr,
rmarkdown, printr,
testthat

VignetteBuilder knitr

URL https://github.com/AFialkowski/SimCorrMix

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<del>-</del>	d Standardized Cumulants of a Continuous Mixture Distribution by thod of Moments
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## **Description**

This function uses the method of moments to calculate the expected mean, standard deviation, skewness, standardized kurtosis, and standardized fifth and sixth cumulants for a continuous mixture variable based on the distributions of its components. The result can be used as input to find\_constants or for comparison to a simulated mixture variable from contmixvar1, corrvar, or corrvar2. See the Expected Cumulants and Correlations for Continuous Mixture Variables vignette for equations of the cumulants.

## Usage

```
calc_mixmoments(mix_pis = NULL, mix_mus = NULL, mix_sigmas = NULL,
  mix_skews = NULL, mix_skurts = NULL, mix_fifths = NULL,
  mix_sixths = NULL)
```

#### **Arguments**

mix_pis	a vector of mixing probabilities that sum to 1 for the component distributions
mix_mus	a vector of means for the component distributions
mix_sigmas	a vector of standard deviations for the component distributions
mix_skews	a vector of skew values for the component distributions
mix_skurts	a vector of standardized kurtoses for the component distributions
mix_fifths	a vector of standardized fifth cumulants for the component distributions; keep NULL if using method = "Fleishman" to generate continuous variables
mix_sixths	a vector of standardized sixth cumulants for the component distributions; keep NULL if using method = "Fleishman" to generate continuous variables

# Value

A vector of the mean, standard deviation, skewness, standardized kurtosis, and standardized fifth and sixth cumulants

#### References

Davenport JW, Bezder JC, & Hathaway RJ (1988). Parameter Estimation for Finite Mixture Distributions. Computers & Mathematics with Applications, 15(10):819-28.

Headrick TC (2002). Fast Fifth-order Polynomial Transforms for Generating Univariate and Multivariate Non-normal Distributions. Computational Statistics & Data Analysis, 40(4):685-711. doi: 10.1016/S01679473(02)000725. (ScienceDirect)

Headrick TC, Kowalchuk RK (2007). The Power Method Transformation: Its Probability Density Function, Distribution Function, and Its Further Use for Fitting Data. Journal of Statistical Computation and Simulation, 77:229-249. doi: 10.1080/10629360600605065.

Headrick TC, Sheng Y, & Hodis FA (2007). Numerical Computing and Graphics for the Power Method Transformation Using Mathematica. Journal of Statistical Software, 19(3):1-17. doi: 10.18637/jss.v019.i03.

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Kendall M & Stuart A (1977). The Advanced Theory of Statistics, 4th Edition. Macmillan, New York.

Schork NJ, Allison DB, & Thiel B (1996). Mixture Distributions in Human Genetics Research. Statistical Methods in Medical Research, 5:155-178. doi: 10.1177/096228029600500204.

#### **Examples**

```
# Two mixture variables: 1st is mixture of Normal(-2, 1) and Normal(2, 1),
# 2nd is mixture of Logistic(0, 1), Chisq(4), and Beta(4, 1.5)
L <- calc_theory("Logistic", c(0, 1))
C <- calc_theory("Chisq", 4)</pre>
B <- calc_theory("Beta", c(4, 1.5))
mix_pis \leftarrow list(c(0.4, 0.6), c(0.3, 0.2, 0.5))
mix_mus \leftarrow list(c(-2, 2), c(L[1], C[1], B[1]))
mix_sigmas \leftarrow list(c(1, 1), c(L[2], C[2], B[2]))
mix_skews <- list(rep(0, 2), c(L[3], C[3], B[3]))
mix_skurts <- list(rep(0, 2), c(L[4], C[4], B[4]))
mix_fifths \leftarrow list(rep(0, 2), c(L[5], C[5], B[5]))
mix_sixths \leftarrow list(rep(0, 2), c(L[6], C[6], B[6]))
Nstcum <- calc_mixmoments(mix_pis[[1]], mix_mus[[1]], mix_sigmas[[1]],</pre>
  mix_skews[[1]], mix_skurts[[1]], mix_fifths[[1]], mix_sixths[[1]])
Nstcum
Mstcum <- calc_mixmoments(mix_pis[[2]], mix_mus[[2]], mix_sigmas[[2]],</pre>
  mix_skews[[2]], mix_skurts[[2]], mix_fifths[[2]], mix_sixths[[2]])
Mstcum
```

contmixvar1

Generation of One Continuous Variable with a Mixture Distribution Using the Power Method Transformation

## Description

This function simulates one continuous mixture variable. Mixture distributions describe random variables that are drawn from more than one component distribution. For a random variable  $Y_{mix}$  from a finite continuous mixture distribution with k components, the probability density function (PDF) can be described by:

$$h_Y(y) = \sum_{i=1}^k \pi_i f_{Yi}(y), \sum_{i=1}^k \pi_i = 1.$$

The  $\pi_i$  are mixing parameters which determine the weight of each component distribution  $f_{Yi}(y)$  in the overall probability distribution. As long as each component has a valid PDF, the overall distribution  $h_Y(y)$  has a valid PDF. The main assumption is statistical independence between the process of randomly selecting the component distribution and the distributions themselves. Each component  $Y_i$  is generated using either Fleishman's third-order (method = "Fleishman", doi: 10.1007/BF02293811) or Headrick's fifth-order (method = "Polynomial", doi: 10.1016/S01679473(02)00072-5) power method transformation (PMT). It works by matching standardized cumulants – the first four (mean, variance, skew, and standardized kurtosis) for Fleishman's method, or the first six (mean, variance, skew, standardized kurtosis, and standardized fifth and sixth cumulants) for Headrick's method. The transformation is expressed as follows:

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$$Y = c_0 + c_1 * Z + c_2 * Z^2 + c_3 * Z^3 + c_4 * Z^4 + c_5 * Z^5, Z \sim N(0, 1),$$

where  $c_4$  and  $c_5$  both equal 0 for Fleishman's method. The real constants are calculated by find\_constants. These components are then transformed to the desired mixture variable using a random multinomial variable generated based on the mixing probabilities. There are no parameter input checks in order to decrease simulation time. All inputs should be checked prior to simulation with validpar. Summaries for the simulation results can be obtained with summary\_var.

Mixture distributions provide a useful way for describing heterogeneity in a population, especially when an outcome is a composite response from multiple sources. The vignette **Variable Types** provides more information about simulation of mixture variables and the required parameters. The vignette **Expected Cumulants and Correlations for Continuous Mixture Variables** gives the equations for the expected cumulants of a mixture variable. In addition, Headrick & Kowalchuk (2007, doi: 10.1080/10629360600605065) outlined a general method for comparing a simulated distribution Y to a given theoretical distribution  $Y^*$ . These steps can be found in the **Continuous Mixture Distributions** vignette.

#### Usage

```
contmixvar1(n = 10000, method = c("Fleishman", "Polynomial"), means = 0,
  vars = 1, mix_pis = NULL, mix_mus = NULL, mix_sigmas = NULL,
  mix_skews = NULL, mix_skurts = NULL, mix_fifths = NULL,
  mix_sixths = NULL, mix_Six = list(), seed = 1234, cstart = list(),
  quiet = FALSE)
```

## Arguments

n

11	the sample size (i.e. the length of the simulated variable, default – 10000)
method	the method used to generate the component variables. "Fleishman" uses Fleishman's third-order polynomial transformation and "Polynomial" uses Headrick's fifth-order transformation.
means	mean for the mixture variable (default $= 0$ )
vars	variance for the mixture variable (default = 1)
mix_pis	a vector of mixing probabilities that sum to 1 for the component distributions
mix_mus	a vector of means for the component distributions
mix_sigmas	a vector of standard deviations for the component distributions
mix_skews	a vector of skew values for the component distributions
mix_skurts	a vector of standardized kurtoses for the component distributions
mix_fifths	a vector of standardized fifth cumulants for the component distributions; keep NULL if using method = "Fleishman" to generate continuous variables
mix_sixths	a vector of standardized sixth cumulants for the component distributions; keep NULL if using method = "Fleishman" to generate continuous variables
mix_Six	a list of vectors of sixth cumulant correction values for the component distributions of $Y_{mix}$ ; use NULL if no correction is desired for a given component; if no correction is desired for any component keep as $mix_Six = list()$ (not necessary for method = "Fleishman")
seed	the seed value for random number generation (default = 1234)

the sample size (i.e. the length of the simulated variable: default = 10000)

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cstart a list of length equal to the total number of mixture components containing

initial values for root-solving algorithm used in find\_constants. If user specified, each list element must be input as a matrix. For method = "Fleishman", each should have 3 columns for  $c_1, c_2, c_3$ ; for method = "Polynomial", each should have 5 columns for  $c_1, c_2, c_3, c_4, c_5$ . If no starting values are specified

for a given component, that list element should be NULL.

quiet if FALSE prints total simulation time

#### Value

A list with the following components:

constants a data.frame of the constants

Y\_comp a data.frame of the components of the mixture variable

Y\_mix a data.frame of the generated mixture variable

sixth\_correction the sixth cumulant correction values for Y\_comp

valid.pdf "TRUE" if constants generate a valid PDF, else "FALSE"

Time the total simulation time in minutes

#### Choice of Fleishman's third-order or Headrick's fifth-order method

Using the fifth-order approximation allows additional control over the fifth and sixth moments of the generated distribution, improving accuracy. In addition, the range of feasible standardized kurtosis values, given skew and standardized fifth  $(\gamma_3)$  and sixth  $(\gamma_4)$  cumulants, is larger than with Fleishman's method (see calc\_lower\_skurt). For example, the Fleishman method can not be used to generate a non-normal distribution with a ratio of  $\gamma_3^2/\gamma_4 > 9/14$  (see Headrick & Kowalchuk, 2007). This eliminates the Chi-squared family of distributions, which has a constant ratio of  $\gamma_3^2/\gamma_4 = 2/3$ . The fifth-order method also generates more distributions with valid PDF's. However, if the fifth and sixth cumulants are unknown or do not exist, the Fleishman approximation should be used.

#### **Overview of Simulation Process**

- 1) A check is performed to see if any distributions are repeated within the parameter inputs, i.e. if the mixture variable contains 2 components with the same standardized cumulants. These are noted so that the constants are only calculated once.
- 2) The constants are calculated for each component variable using find\_constants. If no solutions are found that generate a valid power method PDF, the function will return constants that produce an invalid PDF (or a stop error if no solutions can be found). Possible solutions include: 1) changing the seed, or 2) using a mix\_Six list with vectors of sixth cumulant correction values (if method = "Polynomial"). Errors regarding constant calculation are the most probable cause of function failure.
- 3) A matrix X\_cont of dim n x length(mix\_pis) of standard normal variables is generated and singular-value decomposition is done to remove any correlation. The constants are applied to X\_cont to create the component variables Y with the desired distributions.
- 4) A random multinomial variable M = rmultinom(n, size = 1, prob = mix\_pis) is generated using rmultinom. The continuous mixture variable Y\_mix is created from the component variables Y based on this multinomial variable. That is, if M[i, k\_i] = 1, then Y\_mix[i] = Y[i, k\_i]. A location-scale transformation is done on Y\_mix to give it mean means and variance vars.

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#### **Reasons for Function Errors**

1) The most likely cause for function errors is that no solutions to fleish or poly converged when using find\_constants. If this happens, the simulation will stop. It may help to first use find\_constants for each component variable to determine if a sixth cumulant correction value is needed. The solutions can be used as starting values (see cstart below). If the standardized cumulants are obtained from calc\_theory, the user may need to use rounded values as inputs (i.e. skews = round(skews, 8)). For example, in order to ensure that skew is exactly 0 for symmetric distributions.

2) The kurtosis may be outside the region of possible values. There is an associated lower boundary for kurtosis associated with a given skew (for Fleishman's method) or skew and fifth and sixth cumulants (for Headrick's method). Use calc\_lower\_skurt to determine the boundary for a given set of cumulants.

#### References

Davenport JW, Bezder JC, & Hathaway RJ (1988). Parameter Estimation for Finite Mixture Distributions. Computers & Mathematics with Applications, 15(10):819-28.

Everitt BS (1996). An Introduction to Finite Mixture Distributions. Statistical Methods in Medical Research, 5(2):107-127. doi: 10.1177/096228029600500202.

Fleishman AI (1978). A Method for Simulating Non-normal Distributions. Psychometrika, 43:521-532. doi: 10.1007/BF02293811.

Headrick TC (2002). Fast Fifth-order Polynomial Transforms for Generating Univariate and Multivariate Non-normal Distributions. Computational Statistics & Data Analysis, 40(4):685-711. doi: 10.1016/S01679473(02)000725. (ScienceDirect)

Headrick TC (2004). On Polynomial Transformations for Simulating Multivariate Nonnormal Distributions. Journal of Modern Applied Statistical Methods, 3(1):65-71. doi: 10.22237/jmasm/1083370080.

Headrick TC, Kowalchuk RK (2007). The Power Method Transformation: Its Probability Density Function, Distribution Function, and Its Further Use for Fitting Data. Journal of Statistical Computation and Simulation, 77:229-249. doi: 10.1080/10629360600605065.

Headrick TC, Sawilowsky SS (1999). Simulating Correlated Non-normal Distributions: Extending the Fleishman Power Method. Psychometrika, 64:25-35. doi: 10.1007/BF02294317.

Headrick TC, Sheng Y, & Hodis FA (2007). Numerical Computing and Graphics for the Power Method Transformation Using Mathematica. Journal of Statistical Software, 19(3):1 - 17. doi: 10.18637/jss.v019.i03.

Pearson, RK. 2011. "Exploring Data in Engineering, the Sciences, and Medicine." In. New York: Oxford University Press.

#### See Also

find\_constants, validpar, summary\_var

## **Examples**

```
## Not run:
# Mixture of Beta(6, 3), Beta(4, 1.5), and Beta(10, 20)
Stcum1 <- calc_theory("Beta", c(6, 3))
Stcum2 <- calc_theory("Beta", c(4, 1.5))
Stcum3 <- calc_theory("Beta", c(10, 20))
mix_pis <- c(0.5, 0.2, 0.3)</pre>
```

```
mix_mus <- c(Stcum1[1], Stcum2[1], Stcum3[1])</pre>
mix_sigmas <- c(Stcum1[2], Stcum2[2], Stcum3[2])</pre>
mix_skews <- c(Stcum1[3], Stcum2[3], Stcum3[3])</pre>
mix_skurts <- c(Stcum1[4], Stcum2[4], Stcum3[4])</pre>
mix_fifths <- c(Stcum1[5], Stcum2[5], Stcum3[5])</pre>
mix_sixths <- c(Stcum1[6], Stcum2[6], Stcum3[6])</pre>
mix_Six \leftarrow list(seq(0.01, 10, 0.01), c(0.01, 0.02, 0.03),
  seq(0.01, 10, 0.01))
Bstcum <- calc_mixmoments(mix_pis, mix_mus, mix_sigmas, mix_skews,</pre>
  mix_skurts, mix_fifths, mix_sixths)
Bmix <- contmixvar1(n = 10000, "Polynomial", Bstcum[1], Bstcum[2]^2,</pre>
  mix_pis, mix_mus, mix_sigmas, mix_skews, mix_skurts, mix_fifths,
  mix_sixths, mix_Six)
Bsum <- summary_var(Y_comp = Bmix$Y_comp, Y_mix = Bmix$Y_mix, means = means,</pre>
  vars = vars, mix_pis = mix_pis, mix_mus = mix_mus,
  mix_sigmas = mix_sigmas, mix_skews = mix_skews, mix_skurts = mix_skurts,
  mix_fifths = mix_fifths, mix_sixths = mix_sixths)
## End(Not run)
```

corrvar

Generation of Correlated Ordinal, Continuous (mixture and non-mixture), and/or Count (Poisson and Negative Binomial, regular and zero-inflated) Variables: Correlation Method 1

#### **Description**

This function simulates k\_cat ordinal ( $r \ge 2$  categories), k\_cont continuous non-mixture, k\_mix continuous mixture, k\_pois Poisson (regular and zero-inflated), and/or k\_nb Negative Binomial (regular and zero-inflated) variables with a specified correlation matrix rho. The variables are generated from multivariate normal variables with intermediate correlation matrix Sigma, calculated by intercorr, and then transformed. The intermediate correlations involving count variables are determined using **correlation method 1**. The *ordering* of the variables in rho must be 1st ordinal, 2nd continuous non-mixture, 3rd components of the continuous mixture, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, and 7th zero-inflated NB. Note that it is possible for k\_cat, k\_cont, k\_mix, k\_pois, and/or k\_nb to be 0. Simulation occurs at the component-level for continuous mixture distributions. The target correlation matrix is specified in terms of correlations with components of continuous mixture variables. There are no parameter input checks in order to decrease simulation time. All inputs should be checked prior to simulation with validpar and validcorr. Summaries for the simulation results can be obtained with summary\_var.

All continuous variables are simulated using either Fleishman's third-order (method = "Fleishman", doi: 10.1007/BF02293811) or Headrick's fifth-order (method = "Polynomial", doi: 10.1016/S0167-9473(02)000725) power method transformation. It works by matching standardized cumulants – the first four (mean, variance, skew, and standardized kurtosis) for Fleishman's method, or the first six (mean, variance, skew, standardized kurtosis, and standardized fifth and sixth cumulants) for Headrick's method. The transformation is expressed as follows:

$$Y = c_0 + c_1 * Z + c_2 * Z^2 + c_3 * Z^3 + c_4 * Z^4 + c_5 * Z^5, Z \sim N(0, 1),$$

where  $c_4$  and  $c_5$  both equal 0 for Fleishman's method. The real constants are calculated by find\_constants. Continuous mixture variables are generated componentwise and then transformed to the desired mixture variables based on random multinomial variables generated from

the mixing probabilities. Ordinal variables ( $r \geq 2$  categories) are generated by discretizing the standard normal variables at quantiles. These quantiles are determined by evaluating the inverse standard normal cdf at the cumulative probabilities defined by each variable's marginal distribution. Count variables are generated using the inverse cdf method. The CDF of a standard normal variable has a uniform distribution. The appropriate quantile function  $(F_Y)^{(-1)}$  is applied to this uniform variable with the designated parameters to generate the count variable:  $Y = (F_Y)^{(-1)}(Phi(Z))$ . The Negative Binomial variable represents the number of failures which occur in a sequence of Bernoulli trials before the target number of successes is achieved. Zero-inflated Poisson or NB variables are obtained by setting the probability of a structural zero to be greater than 0. The optional error loop attempts to correct the final pairwise correlations to be within a user-specified precision value (epsilon) of the target correlations.

The vignette **Variable Types** discusses how each of the different variables are generated and describes the required parameters.

The vignette **Overall Workflow for Generation of Correlated Data** provides a detailed example discussing the step-by-step simulation process and comparing correlation methods 1 and 2.

#### Usage

```
corrvar(n = 10000, k_cat = 0, k_cont = 0, k_mix = 0, k_pois = 0,
    k_nb = 0, method = c("Fleishman", "Polynomial"), means = NULL,
    vars = NULL, skews = NULL, skurts = NULL, fifths = NULL,
    sixths = NULL, Six = list(), mix_pis = list(), mix_mus = list(),
    mix_sigmas = list(), mix_skews = list(), mix_skurts = list(),
    mix_fifths = list(), mix_sixths = list(), mix_Six = list(),
    marginal = list(), support = list(), lam = NULL, p_zip = 0,
    size = NULL, prob = NULL, mu = NULL, p_zinb = 0, rho = NULL,
    seed = 1234, errorloop = FALSE, epsilon = 0.001, maxit = 1000,
    use.nearPD = TRUE, nrand = 1e+05, Sigma = NULL, cstart = list(),
    quiet = FALSE)
```

#### **Arguments**

n	the sample size (i.e. the length of each simulated variable; default = 10000)
k_cat	the number of ordinal ( $r \ge 2$ categories) variables (default = 0)
k_cont	the number of continuous non-mixture variables (default = 0)
k_mix	the number of continuous mixture variables (default = 0)
k_pois	the number of regular Poisson and zero-inflated Poisson variables (default = 0)
k_nb	the number of regular Negative Binomial and zero-inflated Negative Binomial variables (default = $0$ )
method	the method used to generate the $k\_cont$ non-mixture and $k\_mix$ mixture continuous variables. "Fleishman" uses Fleishman's third-order polynomial transformation and "Polynomial" uses Headrick's fifth-order transformation.
means	a vector of means for the k_cont non-mixture and k_mix mixture continuous variables (i.e. rep(0, (k_cont + k_mix)))
vars	a vector of variances for the $k\_cont$ non-mixture and $k\_mix$ mixture continuous variables (i.e. $rep(1, (k\_cont + k\_mix))$ )
skews	a vector of skewness values for the k_cont non-mixture continuous variables
skurts	a vector of standardized kurtoses (kurtosis - 3, so that normal variables have a value of 0) for the k_cont non-mixture continuous variables

the sample size (i.e. the length of each simulated variable; default - 10000)

fifths	a vector of standardized fifth cumulants for the $k\_cont$ non-mixture continuous variables (not necessary for method = "Fleishman")
sixths	a vector of standardized sixth cumulants for the k_cont non-mixture continuous variables (not necessary for method = "Fleishman")
Six	a list of vectors of sixth cumulant correction values for the k_cont non-mixture continuous variables if no valid PDF constants are found, ex: $Six = list(seq(0.01, 2, 0.01), seq(1, 10, 0.5))$ ; if no correction is desired for $Y_{cont_i}$ , set the i-th list component equal to NULL; if no correction is desired for any of the $Y_{cont}$ keep as $Six = list()$ (not necessary for method = "Fleishman")
mix_pis	a list of length k_mix with i-th component a vector of mixing probabilities that sum to 1 for component distributions of $Y_{mix_i}$
mix_mus	a list of length k_mix with i-th component a vector of means for component distributions of $Y_{mix_i}$
mix_sigmas	a list of length k_mix with i-th component a vector of standard deviations for component distributions of $Y_{mix_i}$
mix_skews	a list of length k_mix with i-th component a vector of skew values for component distributions of $Y_{mix_i}$
mix_skurts	a list of length k_mix with i-th component a vector of standardized kurtoses for component distributions of $Y_{mix_i}$
mix_fifths	a list of length k_mix with i-th component a vector of standardized fifth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")
mix_sixths	a list of length k_mix with i-th component a vector of standardized sixth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")
mix_Six	a list of length k_mix with i-th component a list of vectors of sixth cumulant correction values for component distributions of $Y_{mix_i}$ ; use NULL if no correction is desired for a given component or mixture variable; if no correction is desired for any of the $Y_{mix}$ keep as mix_Six = list() (not necessary for method = "Fleishman")
marginal	a list of length equal to k_cat; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain r - 1 probabilities (the r-th is assumed to be 1); for binary variables, these should be input the same as for ordinal variables with more than 2 categories (i.e. the user-specified probability is the probability of the 1st category, which has the smaller support value)
support	a list of length equal to $k_{cat}$ ; the i-th element is a vector containing the r ordered support values; if not provided (i.e. support = list()), the default is for the i-th element to be the vector 1,, r
lam	a vector of lambda (mean > 0) constants for the Poisson variables (see dpois); the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson variables
p_zip	a vector of probabilities of structural zeros (not including zeros from the Poisson distribution) for the zero-inflated Poisson variables (see <code>dzipois</code> ); if <code>p_zip = 0</code> , $Y_{pois}$ has a regular Poisson distribution; if <code>p_zip</code> is in (0, 1), $Y_{pois}$ has a zero-inflated Poisson distribution; if <code>p_zip</code> is in (-(exp(lam) - 1)^(-1), 0), $Y_{pois}$ has a zero-deflated Poisson distribution and <code>p_zip</code> is not a probability; if

p\_zip = -(exp(lam) - 1)^(-1),  $Y_{pois}$  has a positive-Poisson distribution (see dpospois); if length(p\_zip) < length(lam), the missing values are set to 0 (and ordered 1st) a vector of size parameters for the Negative Binomial variables (see dnbinom); size the order should be 1st regular NB variables, 2nd zero-inflated NB variables a vector of success probability parameters for the NB variables; order the same prob as in size a vector of mean parameters for the NB variables (\*Note: either prob or mu mu should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean of the NB distribution (see dzinegbin) p\_zinb a vector of probabilities of structural zeros (not including zeros from the NB distribution) for the zero-inflated NB variables (see dzinegbin); if p\_zinb = 0,  $Y_{nb}$ has a regular NB distribution; if p\_zinb is in (-prob^size/(1 - prob^size),  $\emptyset$ ),  $Y_{nb}$  has a zero-deflated NB distribution and p\_zinb is not a probability; if p\_zinb = -prob^size/(1 - prob^size),  $Y_{nb}$  has a positive-NB distribution (see dposnegbin); if length(p\_zinb) < length(size), the missing values are set to 0 (and ordered 1st) the target correlation matrix which must be ordered 1st ordinal, 2nd continurho ous non-mixture, 3rd components of continuous mixtures, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, 7th zero-inflated NB; note that rho is specified in terms of the components of  $Y_{mix}$ the seed value for random number generation (default = 1234) seed errorloop if TRUE, uses corr\_error to attempt to correct final pairwise correlations to be within epsilon of target pairwise correlations (default = FALSE) epsilon the maximum acceptable error between the final and target pairwise correlations (default = 0.001) in the calculation of ordinal intermediate correlations with ord\_norm or in the error loop maxit the maximum number of iterations to use (default = 1000) in the calculation of ordinal intermediate correlations with ord\_norm or in the error loop use.nearPD TRUE to convert the overall intermediate correlation matrix to the nearest positive definite matrix with Matrix::nearPD if necessary; if FALSE the negative eigenvalues are replaced with 0 if necessary nrand the number of random numbers to generate in calculating intermediate correlations with intercorr (default = 10000) Sigma an intermediate correlation matrix to use if the user wants to provide one, else it is calculated within by intercorr cstart a list of length equal to k\_cont + the total number of mixture components containing initial values for root-solving algorithm used in find\_constants. If user specified, each list element must be input as a matrix. For method = "Fleishman", each should have 3 columns for  $c_1, c_2, c_3$ ; for method = "Polynomial", each should have 5 columns for  $c_1, c_2, c_3, c_4, c_5$ . If no starting values are specified for a given component, that list element should be NULL. if FALSE prints simulation messages, if TRUE suppresses message printing quiet

#### Value

A list whose components vary based on the type of simulated variables.

If **ordinal variables** are produced: Y\_cat the ordinal variables,

If **continuous variables** are produced:

constants a data frame of the constants.

Y\_cont the continuous non-mixture variables,

Y\_comp the components of the continuous mixture variables,

Y\_mix the continuous mixture variables,

sixth\_correction a list of sixth cumulant correction values,

valid.pdf a vector where the i-th element is "TRUE" if the constants for the i-th continuous variable generate a valid PDF, else "FALSE"

If **Poisson variables** are produced: Y\_pois the regular and zero-inflated Poisson variables,

If **Negative Binomial variables** are produced: Y\_nb the regular and zero-inflated Negative Binomial variables,

Additionally, the following elements:

Sigma the intermediate correlation matrix (after the error loop),

Error\_Time the time in minutes required to use the error loop,

Time the total simulation time in minutes,

niter a matrix of the number of iterations used for each variable in the error loop,

#### Overview of Correlation Method 1

The intermediate correlations used in method 1 are more simulation based than those in method 2, which means that accuracy increases with sample size and the number of repetitions. In addition, specifying the seed allows for reproducibility. In addition, method 1 differs from method 2 in the following ways:

- 1) The intermediate correlation for **count variables** is based on the method of Yahav & Shmueli (2012, doi: 10.1002/asmb.901), which uses a simulation based, logarithmic transformation of the target correlation. This method becomes less accurate as the variable mean gets closer to zero.
- 2) The **ordinal count variable** correlations are based on an extension of the method of Amatya & Demirtas (2015, doi: 10.1080/00949655.2014.953534), in which the correlation correction factor is the product of the upper Frechet-Hoeffding bound on the correlation between the count variable and the normal variable used to generate it and a simulated upper bound on the correlation between an ordinal variable and the normal variable used to generate it (see Demirtas & Hedeker, 2011, doi: 10.1198/tast.2011.10090).
- 3) The **continuous count variable** correlations are based on an extension of the methods of Amatya & Demirtas (2015) and Demirtas et al. (2012, doi: 10.1002/sim.5362), in which the correlation correction factor is the product of the upper Frechet-Hoeffding bound on the correlation between the count variable and the normal variable used to generate it and the power method correlation between the continuous variable and the normal variable used to generate it (see Headrick & Kowalchuk, 2007, doi: 10.1080/10629360600605065). The intermediate correlations are the ratio of the target correlations to the correction factor.

Please see the **Comparison of Correlation Methods 1 and 2** vignette for more information and a step-by-step overview of the simulation process.

#### Choice of Fleishman's third-order or Headrick's fifth-order method

Using the fifth-order approximation allows additional control over the fifth and sixth moments of the generated distribution, improving accuracy. In addition, the range of feasible standardized kurtosis values, given skew and standardized fifth  $(\gamma_3)$  and sixth  $(\gamma_4)$  cumulants, is larger than with Fleishman's method (see calc\_lower\_skurt). For example, the Fleishman method can not be used to generate a non-normal distribution with a ratio of  $\gamma_3^2/\gamma_4 > 9/14$  (see Headrick & Kowalchuk, 2007). This eliminates the Chi-squared family of distributions, which has a constant ratio of  $\gamma_3^2/\gamma_4 = 2/3$ . The fifth-order method also generates more distributions with valid PDF's. However, if the fifth and sixth cumulants are unknown or do not exist, the Fleishman approximation should be used.

#### **Reasons for Function Errors**

- 1) The most likely cause for function errors is that no solutions to fleish or poly converged when using find\_constants. If this happens, the simulation will stop. It may help to first use find\_constants for each continuous variable to determine if a sixth cumulant correction value is needed. The solutions can be used as starting values (see cstart below). If the standardized cumulants are obtained from calc\_theory, the user may need to use rounded values as inputs (i.e. skews = round(skews, 8)). For example, in order to ensure that skew is exactly 0 for symmetric distributions.
- 2) The kurtosis may be outside the region of possible values. There is an associated lower boundary for kurtosis associated with a given skew (for Fleishman's method) or skew and fifth and sixth cumulants (for Headrick's method). Use calc\_lower\_skurt to determine the boundary for a given set of cumulants.
- 3) The feasibility of the final correlation matrix rho, given the distribution parameters, should be checked first using <code>validcorr</code>. This function either checks if a given rho is plausible or returns the lower and upper final correlation limits. It should be noted that even if a target correlation matrix is within the "plausible range," it still may not be possible to achieve the desired matrix. This happens most frequently when generating ordinal variables or using negative correlations. The error loop frequently fixes these problems.

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#### See Also

find\_constants, validpar, validcorr, intercorr, corr\_error, summary\_var

## **Examples**

```
## Not run:
# 2 continuous mixture, 1 binary, 1 zero-inflated Poisson, and
# 1 zero-inflated NB variable
n <- 10000
seed <- 1234
# Mixture variables: Normal mixture with 2 components;
# mixture of Logistic(0, 1), Chisq(4), Beta(4, 1.5)
# Find cumulants of components of 2nd mixture variable
L <- calc_theory("Logistic", c(0, 1))
C <- calc_theory("Chisq", 4)</pre>
B <- calc_theory("Beta", c(4, 1.5))</pre>
skews <- skurts <- fifths <- sixths <- NULL
Six <- list()
mix_pis \leftarrow list(c(0.4, 0.6), c(0.3, 0.2, 0.5))
mix_mus \leftarrow list(c(-2, 2), c(L[1], C[1], B[1]))
mix\_sigmas \leftarrow list(c(1, 1), c(L[2], C[2], B[2]))
mix_skews \leftarrow list(rep(0, 2), c(L[3], C[3], B[3]))
mix\_skurts \leftarrow list(rep(0, 2), c(L[4], C[4], B[4]))
mix_fifths \leftarrow list(rep(0, 2), c(L[5], C[5], B[5]))
mix_sixths \leftarrow list(rep(0, 2), c(L[6], C[6], B[6]))
mix_Six <- list(list(NULL, NULL), list(1.75, NULL, 0.03))</pre>
Nstcum <- calc_mixmoments(mix_pis[[1]], mix_mus[[1]], mix_sigmas[[1]],</pre>
  mix_skews[[1]], mix_skurts[[1]], mix_fifths[[1]], mix_sixths[[1]])
Mstcum <- calc_mixmoments(mix_pis[[2]], mix_mus[[2]], mix_sigmas[[2]],</pre>
  mix_skews[[2]], mix_skurts[[2]], mix_fifths[[2]], mix_sixths[[2]])
means <- c(Nstcum[1], Mstcum[1])</pre>
vars <- c(Nstcum[2]^2, Mstcum[2]^2)</pre>
marginal <- list(0.3)</pre>
support <- list(c(0, 1))
lam <- 0.5
p_zip <- 0.1
size <- 2
prob <- 0.75
p_zinb <- 0.2
k_cat <- k_pois <- k_nb <- 1
k_cont <- 0
k_mix <- 2
Rey <- matrix(0.39, 8, 8)
diag(Rey) <- 1
rownames(Rey) <- colnames(Rey) <- c("01", "M1_1", "M1_2", "M2_1", "M2_2",
  "M2_3", "P1", "NB1")
\mbox{\#} set correlation between components of the same mixture variable to 0
Rey["M1_1", "M1_2"] \leftarrow Rey["M1_2", "M1_1"] \leftarrow 0
\label{eq:rey_ma_1"} \text{Rey}["M2\_1", "M2\_2"] <- \ \text{Rey}["M2\_2", "M2\_1"] <- \ \text{Rey}["M2\_1", "M2\_3"] <- \ \emptyset
Rey["M2_3", "M2_1"] \leftarrow Rey["M2_2", "M2_3"] \leftarrow Rey["M2_3", "M2_2"] \leftarrow 0
# check parameter inputs
validpar(k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
```

```
mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal, support,
  lam, p_zip, size, prob, mu = NULL, p_zinb, rho = Rey)
# check to make sure Rey is within the feasible correlation boundaries
validcorr(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal,
  lam, p_zip, size, prob, mu = NULL, p_zinb, Rey, seed)
# simulate without the error loop
Sim1 <- corrvar(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal, support,
  lam, p_zip, size, prob, mu = NULL, p_zinb, Rey, seed, epsilon = 0.01)
names(Sim1)
# simulate with the error loop
Sim1_EL <- corrvar(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial",
  means, vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus,
  mix_sigmas, mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six,
  marginal, support, lam, p_zip, size, prob, mu = NULL, p_zinb, Rey,
  seed, errorloop = TRUE, epsilon = 0.01)
names(Sim1_EL)
## End(Not run)
```

corrvar2

Generation of Correlated Ordinal, Continuous (mixture and non-mixture), and/or Count (Poisson and Negative Binomial, regular and zero-inflated) Variables: Correlation Method 2

## Description

This function simulates k\_cat ordinal ( $r \ge 2$  categories), k\_cont continuous non-mixture, k\_mix continuous mixture, k\_pois Poisson (regular and zero-inflated), and/or k\_nb Negative Binomial (regular and zero-inflated) variables with a specified correlation matrix rho. The variables are generated from multivariate normal variables with intermediate correlation matrix Sigma, calculated by intercorr2, and then transformed. The intermediate correlations involving count variables are determined using **correlation method 2**. The *ordering* of the variables in rho must be 1st ordinal, 2nd continuous non-mixture, 3rd components of the continuous mixture, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, and 7th zero-inflated NB. Note that it is possible for k\_cat, k\_cont, k\_mix, k\_pois, and/or k\_nb to be 0. Simulation occurs at the component-level for continuous mixture distributions. The target correlation matrix is specified in terms of correlations with components of continuous mixture variables. There are no parameter input checks in order to decrease simulation time. All inputs should be checked prior to simulation with validpar and validcorr2. Summaries for the simulation results can be obtained with summary\_var.

All continuous variables are simulated using either Fleishman's third-order (method = "Fleishman", doi: 10.1007/BF02293811) or Headrick's fifth-order (method = "Polynomial", doi: 10.1016/S0167-9473(02)000725) power method transformation. It works by matching standardized cumulants –

the first four (mean, variance, skew, and standardized kurtosis) for Fleishman's method, or the first six (mean, variance, skew, standardized kurtosis, and standardized fifth and sixth cumulants) for Headrick's method. The transformation is expressed as follows:

$$Y = c_0 + c_1 * Z + c_2 * Z^2 + c_3 * Z^3 + c_4 * Z^4 + c_5 * Z^5, Z \sim N(0, 1),$$

where  $c_4$  and  $c_5$  both equal 0 for Fleishman's method. The real constants are calculated by find\_constants. Continuous mixture variables are generated componentwise and then transformed to the desired mixture variables based on random multinomial variables generated from the mixing probabilities. Ordinal variables ( $r \geq 2$  categories) are generated by discretizing the standard normal variables at quantiles. These quantiles are determined by evaluating the inverse standard normal cdf at the cumulative probabilities defined by each variable's marginal distribution. Count variables are generated using the inverse cdf method. The CDF of a standard normal variable has a uniform distribution. The appropriate quantile function  $(F_{-}Y)^{\wedge}(-1)$  is applied to this uniform variable with the designated parameters to generate the count variable:  $Y = (F_{-}Y)^{\wedge}(-1)(Phi(Z))$ . The Negative Binomial variable represents the number of failures which occur in a sequence of Bernoulli trials before the target number of successes is achieved. Zero-inflated Poisson or NB variables are obtained by setting the probability of a structural zero to be greater than 0. The optional error loop attempts to correct the final pairwise correlations to be within a user-specified precision value (epsilon) of the target correlations.

The vignette **Variable Types** discusses how each of the different variables are generated and describes the required parameters.

The vignette **Overall Workflow for Generation of Correlated Data** provides a detailed example discussing the step-by-step simulation process and comparing correlation methods 1 and 2.

#### Usage

```
corrvar2(n = 10000, k_cat = 0, k_cont = 0, k_mix = 0, k_pois = 0,
  k_nb = 0, method = c("Fleishman", "Polynomial"), means = NULL,
  vars = NULL, skews = NULL, skurts = NULL, fifths = NULL,
  sixths = NULL, Six = list(), mix_pis = list(), mix_mus = list(),
  mix_sigmas = list(), mix_skews = list(), mix_skurts = list(),
  mix_fifths = list(), mix_sixths = list(), mix_Six = list(),
  marginal = list(), support = list(), lam = NULL, p_zip = 0,
  size = NULL, prob = NULL, mu = NULL, p_zinb = 0, pois_eps = 1e-04,
  nb_eps = 1e-04, rho = NULL, seed = 1234, errorloop = FALSE,
  epsilon = 0.001, maxit = 1000, use.nearPD = TRUE, Sigma = NULL,
  cstart = list(), quiet = FALSE)
```

## **Arguments**

n	the sample size (i.e. the length of each simulated variable; default = 10000)
k_cat	the number of ordinal ( $r \ge 2$ categories) variables (default = 0)
k_cont	the number of continuous non-mixture variables (default = $0$ )
k_mix	the number of continuous mixture variables (default = 0)
k_pois	the number of regular Poisson and zero-inflated Poisson variables (default = 0)
k_nb	the number of regular Negative Binomial and zero-inflated Negative Binomial variables (default = $0$ )
method	the method used to generate the k_cont non-mixture and k_mix mixture continuous variables. "Fleishman" uses Fleishman's third-order polynomial transformation and "Polynomial" uses Headrick's fifth-order transformation.

means	a vector of means for the k_cont non-mixture and k_mix mixture continuous variables (i.e. $rep(0, (k\_cont + k\_mix))$ )
vars	a vector of variances for the k_cont non-mixture and k_mix mixture continuous variables (i.e. rep(1, (k_cont + k_mix)))
skews	a vector of skewness values for the k_cont non-mixture continuous variables
skurts	a vector of standardized kurtoses (kurtosis - 3, so that normal variables have a value of 0) for the $k\_cont$ non-mixture continuous variables
fifths	a vector of standardized fifth cumulants for the k_cont non-mixture continuous variables (not necessary for method = "Fleishman")
sixths	a vector of standardized sixth cumulants for the $k\_cont$ non-mixture continuous variables (not necessary for method = "Fleishman")
Six	a list of vectors of sixth cumulant correction values for the k_cont non-mixture continuous variables if no valid PDF constants are found, ex: $Six = list(seq(0.01, 2, 0.01), seq(1, 10, 0.5))$ ; if no correction is desired for $Y_{cont_i}$ , set the i-th list component equal to NULL; if no correction is desired for any of the $Y_{cont}$ keep as $Six = list()$ (not necessary for method = "Fleishman")
mix_pis	a list of length k_mix with i-th component a vector of mixing probabilities that sum to 1 for component distributions of $Y_{mix_i}$
mix_mus	a list of length k_mix with i-th component a vector of means for component distributions of $Y_{mix_i}$
mix_sigmas	a list of length k_mix with i-th component a vector of standard deviations for component distributions of $Y_{mix_i}$
mix_skews	a list of length k_mix with i-th component a vector of skew values for component distributions of $Y_{mix_i}$
mix_skurts	a list of length k_mix with i-th component a vector of standardized kurtoses for component distributions of $Y_{mix_i}$
mix_fifths	a list of length k_mix with i-th component a vector of standardized fifth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")
mix_sixths	a list of length k_mix with i-th component a vector of standardized sixth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")
mix_Six	a list of length k_mix with i-th component a list of vectors of sixth cumulant correction values for component distributions of $Y_{mix_i}$ ; use NULL if no correction is desired for a given component or mixture variable; if no correction is desired for any of the $Y_{mix}$ keep as mix_Six = list() (not necessary for method = "Fleishman")
marginal	a list of length equal to k_cat; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain $r$ - 1 probabilities (the r-th is assumed to be 1); for binary variables, these should be input the same as for ordinal variables with more than 2 categories (i.e. the user-specified probability is the probability of the 1st category, which has the smaller support value)
support	a list of length equal to $k_{cat}$ ; the i-th element is a vector containing the r ordered support values; if not provided (i.e. support = list()), the default is for the i-th element to be the vector 1,, r

lam a vector of lambda (mean > 0) constants for the Poisson variables (see dpois); the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson varia vector of probabilities of structural zeros (not including zeros from the Poisson p\_zip distribution) for the zero-inflated Poisson variables (see dzipois); if p\_zip = 0,  $Y_{pois}$  has a regular Poisson distribution; if p\_zip is in (0, 1),  $Y_{pois}$  has a zeroinflated Poisson distribution; if  $p_{zip}$  is in  $(-(exp(lam) - 1)^{-1}, 0)$ ,  $Y_{pois}$  has a zero-deflated Poisson distribution and p\_zip is not a probability; if p\_zip = -(exp(lam) - 1)^(-1),  $Y_{pois}$  has a positive-Poisson distribution (see dpospois); if length(p\_zip) < length(lam), the missing values are set to 0 (and ordered 1st) a vector of size parameters for the Negative Binomial variables (see dnbinom); size the order should be 1st regular NB variables, 2nd zero-inflated NB variables prob a vector of success probability parameters for the NB variables; order the same as in size a vector of mean parameters for the NB variables (\*Note: either prob or mu mu should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean of the NB distribution (see dzinegbin) a vector of probabilities of structural zeros (not including zeros from the NB disp\_zinb tribution) for the zero-inflated NB variables (see dzinegbin); if p\_zinb = 0,  $Y_{nh}$ has a regular NB distribution; if p\_zinb is in (-prob^size/(1 - prob^size), 0),  $Y_{nb}$  has a zero-deflated NB distribution and p\_zinb is not a probability; if p\_zinb = -prob^size/(1 - prob^size),  $Y_{nb}$  has a positive-NB distribution (see dposnegbin); if length(p\_zinb) < length(size), the missing values are set to 0 (and ordered 1st) a vector of length k\_pois containing total cumulative probability truncation valpois\_eps ues; if none are provided, the default is 0.0001 for each variable a vector of length k\_nb containing total cumulative probability truncation valnb\_eps ues; if none are provided, the default is 0.0001 for each variable rho the target correlation matrix which must be ordered 1st ordinal, 2nd continuous non-mixture, 3rd components of continuous mixtures, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, 7th zero-inflated NB; note that rho is specified in terms of the components of  $Y_{mix}$ seed the seed value for random number generation (default = 1234) errorloop if TRUE, uses corr\_error to attempt to correct final pairwise correlations to be within epsilon of target pairwise correlations (default = FALSE) the maximum acceptable error between the final and target pairwise correlations epsilon (default = 0.001) in the calculation of ordinal intermediate correlations with ord\_norm or in the error loop the maximum number of iterations to use (default = 1000) in the calculation of maxit ordinal intermediate correlations with ord\_norm or in the error loop use.nearPD TRUE to convert the overall intermediate correlation matrix to the nearest positive definite matrix with Matrix::nearPD if necessary; if FALSE the negative eigenvalues are replaced with 0 if necessary Sigma an intermediate correlation matrix to use if the user wants to provide one, else it is calculated within by intercorr2

cstart a list of length equal to k\_cont + the total number of mixture components con-

taining initial values for root-solving algorithm used in find\_constants. If user specified, each list element must be input as a matrix. For method = "Fleishman", each should have 3 columns for  $c_1, c_2, c_3$ ; for method = "Polynomial", each should have 5 columns for  $c_1, c_2, c_3, c_4, c_5$ . If no starting values are speci-

fied for a given component, that list element should be NULL.

quiet if FALSE prints simulation messages, if TRUE suppresses message printing

#### Value

A list whose components vary based on the type of simulated variables.

If **ordinal variables** are produced: Y\_cat the ordinal variables,

If **continuous variables** are produced:

constants a data.frame of the constants,

Y\_cont the continuous non-mixture variables,

Y\_comp the components of the continuous mixture variables,

Y\_mix the continuous mixture variables,

sixth\_correction a list of sixth cumulant correction values,

valid.pdf a vector where the i-th element is "TRUE" if the constants for the i-th continuous variable generate a valid PDF, else "FALSE"

If **Poisson variables** are produced: Y\_pois the regular and zero-inflated Poisson variables,

If **Negative Binomial variables** are produced: Y\_nb the regular and zero-inflated Negative Binomial variables,

Additionally, the following elements:

Sigma the intermediate correlation matrix (after the error loop),

Error\_Time the time in minutes required to use the error loop,

Time the total simulation time in minutes,

niter a matrix of the number of iterations used for each variable in the error loop,

#### Overview of Method 2

The intermediate correlations used in method 2 are less simulation based than those in method 1, and no seed is needed. Their calculations involve greater utilization of correction loops which make iterative adjustments until a maximum error has been reached (if possible). In addition, method 2 differs from method 1 in the following ways:

- 1) The intermediate correlations involving **count variables** are based on the methods of Barbiero & Ferrari (2012, doi: 10.1080/00273171.2012.692630, 2015, doi: 10.1002/asmb.2072). The Poisson or Negative Binomial support is made finite by removing a small user-specified value (i.e. 1e-06) from the total cumulative probability. This truncation factor may differ for each count variable. The count variables are subsequently treated as ordinal and intermediate correlations are calculated using the correction loop of ord\_norm.
- 2) The **continuous count variable** correlations are based on an extension of the method of Demirtas et al. (2012, doi: 10.1002/sim.5362), and the count variables are treated as ordinal. The correction factor is the product of the power method correlation between the continuous variable and the normal variable used to generate it (see Headrick & Kowalchuk, 2007, doi: 10.1080/10629360600605065) and the point-polyserial correlation between the ordinalized count variable

and the normal variable used to generate it (see Olsson et al., 1982, doi: 10.1007/BF02294164). The intermediate correlations are the ratio of the target correlations to the correction factor.

Please see the **Comparison of Correlation Methods 1 and 2** vignette for more information and a step-by-step overview of the simulation process.

#### Choice of Fleishman's third-order or Headrick's fifth-order method

Using the fifth-order approximation allows additional control over the fifth and sixth moments of the generated distribution, improving accuracy. In addition, the range of feasible standardized kurtosis values, given skew and standardized fifth  $(\gamma_3)$  and sixth  $(\gamma_4)$  cumulants, is larger than with Fleishman's method (see calc\_lower\_skurt). For example, the Fleishman method can not be used to generate a non-normal distribution with a ratio of  $\gamma_3^2/\gamma_4 > 9/14$  (see Headrick & Kowalchuk, 2007). This eliminates the Chi-squared family of distributions, which has a constant ratio of  $\gamma_3^2/\gamma_4 = 2/3$ . The fifth-order method also generates more distributions with valid PDF's. However, if the fifth and sixth cumulants are unknown or do not exist, the Fleishman approximation should be used.

#### **Reasons for Function Errors**

- 1) The most likely cause for function errors is that no solutions to fleish or poly converged when using find\_constants. If this happens, the simulation will stop. It may help to first use find\_constants for each continuous variable to determine if a sixth cumulant correction value is needed. The solutions can be used as starting values (see cstart below). If the standardized cumulants are obtained from calc\_theory, the user may need to use rounded values as inputs (i.e. skews = round(skews, 8)). For example, in order to ensure that skew is exactly 0 for symmetric distributions.
- 2) The kurtosis may be outside the region of possible values. There is an associated lower boundary for kurtosis associated with a given skew (for Fleishman's method) or skew and fifth and sixth cumulants (for Headrick's method). Use calc\_lower\_skurt to determine the boundary for a given set of cumulants.
- 3) The feasibility of the final correlation matrix rho, given the distribution parameters, should be checked first using validcorr2. This function either checks if a given rho is plausible or returns the lower and upper final correlation limits. It should be noted that even if a target correlation matrix is within the "plausible range," it still may not be possible to achieve the desired matrix. This happens most frequently when generating ordinal variables or using negative correlations. The error loop frequently fixes these problems.

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# See Also

find\_constants, validpar, validcorr2, intercorr2, corr\_error, summary\_var

#### **Examples**

```
## Not run:
# 2 continuous mixture, 1 binary, 1 zero-inflated Poisson, and
# 1 zero-inflated NB variable
n <- 10000
seed <- 1234
# Mixture variables: Normal mixture with 2 components;
# mixture of Logistic(0, 1), Chisq(4), Beta(4, 1.5)
# Find cumulants of components of 2nd mixture variable
L <- calc_theory("Logistic", c(0, 1))
C <- calc_theory("Chisq", 4)</pre>
B <- calc_theory("Beta", c(4, 1.5))</pre>
skews <- skurts <- fifths <- sixths <- NULL
Six <- list()
mix_pis \leftarrow list(c(0.4, 0.6), c(0.3, 0.2, 0.5))
mix_mus \leftarrow list(c(-2, 2), c(L[1], C[1], B[1]))
mix\_sigmas \leftarrow list(c(1, 1), c(L[2], C[2], B[2]))
mix_skews \leftarrow list(rep(0, 2), c(L[3], C[3], B[3]))
mix\_skurts \leftarrow list(rep(0, 2), c(L[4], C[4], B[4]))
mix_fifths <- list(rep(0, 2), c(L[5], C[5], B[5]))
mix_sixths \leftarrow list(rep(0, 2), c(L[6], C[6], B[6]))
mix_Six <- list(list(NULL, NULL), list(1.75, NULL, 0.03))</pre>
Nstcum <- calc_mixmoments(mix_pis[[1]], mix_mus[[1]], mix_sigmas[[1]],</pre>
  mix_skews[[1]], mix_skurts[[1]], mix_fifths[[1]], mix_sixths[[1]])
Mstcum <- calc_mixmoments(mix_pis[[2]], mix_mus[[2]], mix_sigmas[[2]],</pre>
  mix_skews[[2]], mix_skurts[[2]], mix_fifths[[2]], mix_sixths[[2]])
means <- c(Nstcum[1], Mstcum[1])</pre>
vars <- c(Nstcum[2]^2, Mstcum[2]^2)</pre>
marginal <- list(0.3)</pre>
support <- list(c(0, 1))
lam <- 0.5
p_zip <- 0.1
pois_eps <- 0.0001
size <- 2
prob <- 0.75
p_zinb <- 0.2
nb_eps <- 0.0001
k_cat <- k_pois <- k_nb <- 1
k_cont <- 0
k_mix <- 2
Rey <- matrix(0.39, 8, 8)
diag(Rey) <- 1
rownames(Rey) <- colnames(Rey) <- c("01", "M1_1", "M1_2", "M2_1", "M2_2",
  "M2_3", "P1", "NB1")
\mbox{\#} set correlation between components of the same mixture variable to 0
\label{eq:rey_model} $\operatorname{Rey}["M1\_1", "M1\_2"] <- \operatorname{Rey}["M1\_2", "M1\_1"] <- 0$
Rey["M2_1", "M2_2"] \leftarrow Rey["M2_2", "M2_1"] \leftarrow Rey["M2_1", "M2_3"] \leftarrow 0
Rey["M2_3", "M2_1"] <- Rey["M2_2", "M2_3"] <- Rey["M2_3", "M2_2"] <- 0</pre>
# check parameter inputs
```

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```
validpar(k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal, support,
  lam, p_zip, size, prob, mu = NULL, p_zinb, pois_eps, nb_eps, Rey)
# check to make sure Rey is within the feasible correlation boundaries
validcorr2(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal,
 lam, p_zip, size, prob, mu = NULL, p_zinb, pois_eps, nb_eps, Rey, seed)
# simulate without the error loop
Sim2 <- corrvar2(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal, support,
  lam, p_zip, size, prob, mu = NULL, p_zinb, pois_eps, nb_eps, Rey, seed,
  epsilon = 0.01)
names(Sim2)
# simulate with the error loop
Sim2_EL <- corrvar2(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial",
  means, vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus,
  mix_sigmas, mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six,
 marginal, support, lam, p_zip, size, prob, mu = NULL, p_zinb, pois_eps,
  nb_eps, Rey, seed, errorloop = TRUE, epsilon = 0.01)
names(Sim2_EL)
## End(Not run)
```

corr\_error

Error Loop to Correct Final Correlation of Simulated Variables

## **Description**

This function attempts to correct the final pairwise correlations of simulated variables to be within epsilon of the target correlations. It updates the intermediate normal correlation iteratively in a loop until either the maximum error is less than epsilon or the number of iterations exceeds maxit. This function would not ordinarily be called directly by the user. The function is a modification of Barbiero & Ferrari's ordcont function in GenOrd-package. The ordcont function has been modified in the following ways:

- 1) It works for continuous, ordinal ( $r \ge 2$  categories), and count (regular or zero-inflated, Poisson or Negative Binomial) variables.
- 2) The initial correlation check has been removed because this intermediate correlation Sigma from corrvar or corrvar2 has already been checked for positive-definiteness and used to generate variables.
- 3) Eigenvalue decomposition is done on Sigma to impose the correct interemdiate correlations on the normal variables. If Sigma is not positive-definite, the negative eigen values are replaced with 0
- 4) The final positive-definite check has been removed.
- 5) The intermediate correlation update function was changed to accommodate more situations.

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6) Allowing specifications for the sample size and the seed for reproducibility.

The vignette **Error Loop Algorithm** describes the algorithm used in the error loop.

## Usage

```
corr_error(n = 10000, k_cat = 0, k_cont = 0, k_pois = 0, k_nb = 0,
  method = c("Fleishman", "Polynomial"), means = NULL, vars = NULL,
  constants = NULL, marginal = list(), support = list(), lam = NULL,
  p_zip = 0, size = NULL, mu = NULL, p_zinb = 0, seed = 1234,
  epsilon = 0.001, maxit = 1000, rho0 = NULL, Sigma = NULL,
  rho_calc = NULL)
```

## Arguments

n	the sample size
k_cat	the number of ordinal ( $r \ge 2$ categories) variables
k_cont	the number of continuous variables (these may be regular continuous variables or components of continuous mixture variables)
k_pois	the number of Poisson (regular or zero-inflated) variables
k_nb	the number of Negative Binomial (regular or zero-inflated) variables
method	the method used to generate the continuous variables. "Fleishman" uses a third-order polynomial transformation and "Polynomial" uses Headrick's fifth-order transformation.
means	a vector of means for the continuous variables
vars	a vector of variances for the continuous variables
constants	a matrix with k_cont rows, each a vector of constants $c0$ , $c1$ , $c2$ , $c3$ (if method = "Fleishman") or $c0$ , $c1$ , $c2$ , $c3$ , $c4$ , $c5$ (if method = "Polynomial"), like that returned by find_constants
marginal	a list of length equal $k_{cat}$ ; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain r - 1 probabilities (the r-th is assumed to be 1)
support	a list of length equal $k\_cat$ ; the i-th element is a vector of containing the r ordered support values; if not provided, the default is for the i-th element to be the vector $1,, r$
lam	a vector of lambda (mean > 0) constants for the Poisson variables (see dpois); the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson variables
p_zip	a vector of probabilities of structural zeros (not including zeros from the Poisson distribution) for the zero-inflated Poisson variables (see dzipois)
size	a vector of size parameters for the Negative Binomial variables (see

epsilon the maximum acceptable error between the final and target pairwise correlation;

smaller epsilons take more time

maxit the maximum number of iterations to use to find the intermediate correlation; the

correction loop stops when either the iteration number passes maxit or epsilon

is reached

rho0 the target correlation matrix

Sigma the intermediate correlation matrix previously used in corrvar or corrvar2 rho\_calc the final correlation matrix calculated in corrvar or corrvar2 before execution

of corr\_error

#### Value

A list with the following components:

Sigma the intermediate MVN correlation matrix resulting from the error loop

rho\_calc the calculated final correlation matrix generated from Sigma

Y\_cat the ordinal variables

Y the continuous (mean 0, variance 1) variables

Y\_cont the continuous variables with desired mean and variance

Y\_pois the Poisson variables

Y\_nb the Negative Binomial variables

niter a matrix containing the number of iterations required for each variable pair

#### References

Please see references for SimCorrMix.

#### See Also

corrvar, corrvar2

intercorr	Calculate Intermediate MVN Correlation for Ordinal, Continuous,
	Poisson, or Negative Binomial Variables: Correlation Method 1

## **Description**

This function calculates a  $k \times k$  intermediate matrix of correlations, where  $k = k\_cat + k\_cont + k\_pois + k\_nb$ , to be used in simulating variables with corrvar. The k\\_cont includes regular continuous variables and components of continuous mixture variables. The ordering of the variables must be ordinal, continuous non-mixture, components of continuous mixture variables, regular Poisson, zero-inflated Poisson, regular Negative Binomial (NB), and zero-inflated NB (note that it is possible for k\\_cat, k\\_cont, k\_pois, and/or k\_nb to be 0). There are no parameter input checks in order to decrease simulation time. All inputs should be checked prior to simulation with validpar. There is a message given if the calculated intermediate correlation matrix Sigma is not positive-definite because it may not be possible to find a MVN correlation matrix that will produce the desired marginal distributions. This function is called by the simulation function corrvar, and would only be used separately if the user wants to first find the intermediate correlation matrix. This matrix Sigma can be used as an input to corrvar.

Please see the **Comparison of Correlation Methods 1 and 2** vignette for information about calculations by variable pair type and the differences between this function and intercorr2.

#### Usage

```
intercorr(k_cat = 0, k_cont = 0, k_pois = 0, k_nb = 0,
  method = c("Fleishman", "Polynomial"), constants = NULL,
  marginal = list(), support = list(), lam = NULL, p_zip = 0,
  size = NULL, prob = NULL, mu = NULL, p_zinb = 0, rho = NULL,
  seed = 1234, epsilon = 0.001, maxit = 1000, nrand = 1e+05)
```

## **Arguments**

k\_cat the number of ordinal ( $r \ge 2$  categories) variables (default = 0)

k\_cont the number of continuous non-mixture variables and components of continuous

mixture variables (default = 0)

k\_pois the number of regular and zero-inflated Poisson variables (default = 0)

k\_nb the number of regular and zero-inflated Negative Binomial variables (default =

0)

method the method used to generate the k\_cont continuous variables. "Fleishman" uses

a third-order polynomial transformation and "Polynomial" uses Headrick's fifth-

order transformation.

constants a matrix with k\_cont rows, each a vector of constants c0, c1, c2, c3 (if method

= "Fleishman") or c0, c1, c2, c3, c4, c5 (if method = "Polynomial") like that

returned by find\_constants

marginal a list of length equal to k\_cat; the i-th element is a vector of the cumulative

probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain r - 1 probabilities (the r-th is assumed

to be 1; default = list())

support a list of length equal to k\_cat; the i-th element is a vector of containing the r

ordered support values; if not provided (i.e. support = list()), the default is for

the i-th element to be the vector 1, ..., r

lam a vector of lambda (mean > 0) constants for the regular and zero-inflated Poisson

variables (see dpois); the order should be 1st regular Poisson variables, 2nd

zero-inflated Poisson variables

p\_zip a vector of probabilities of structural zeros (not including zeros from the Poisson

distribution) for the zero-inflated Poisson variables (see <code>dzipois</code>); if <code>p\_zip = 0</code>,  $Y_{pois}$  has a regular Poisson distribution; if <code>p\_zip</code> is in (0, 1),  $Y_{pois}$  has a zero-inflated Poisson distribution; if <code>p\_zip</code> is in (-(exp(lam) - 1)^(-1), 0),  $Y_{pois}$  has a zero-deflated Poisson distribution and <code>p\_zip</code> is not a probability; if <code>p\_zip = -(exp(lam) - 1)^(-1)</code>,  $Y_{pois}$  has a positive-Poisson distribution

(see dpospois); if length(p\_zip) < length(lam), the missing values are set

to 0 (and ordered 1st)

size a vector of size parameters for the Negative Binomial variables (see dnbinom);

the order should be 1st regular NB variables, 2nd zero-inflated NB variables

prob a vector of success probability parameters for the NB variables; order the same

 $as \ in \ {\tt size}$ 

mu a vector of mean parameters for the NB variables (\*Note: either prob or mu

should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean

of the NB distribution (see dzinegbin)

p_zinb	a vector of probabilities of structural zeros (not including zeros from the NB distribution) for the zero-inflated NB variables (see <code>dzinegbin</code> ); if <code>p_zinb = 0</code> , $Y_{nb}$ has a regular NB distribution; if <code>p_zinb</code> is in ( <code>-prob^size/(1 - prob^size)</code> , 0), $Y_{nb}$ has a zero-deflated NB distribution and <code>p_zinb</code> is not a probability; if <code>p_zinb = -prob^size/(1 - prob^size)</code> , $Y_{nb}$ has a positive-NB distribution (see <code>dposnegbin</code> ); if <code>length(p_zinb) &lt; length(size)</code> , the missing values are set to 0 (and ordered 1st)
rho	the target correlation matrix which must be ordered <i>1st ordinal, 2nd continuous non-mixture, 3rd components of continuous mixtures, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, 7th zero-inflated NB;</i> note that rho is specified in terms of the components of $Y_{mix}$
seed	the seed value for random number generation (default = 1234)
epsilon	the maximum acceptable error between the pairwise correlations (default = $0.001$ ) in the calculation of ordinal intermediate correlations with ord_norm
maxit	the maximum number of iterations to use (default = 1000) in the calculation of ordinal intermediate correlations with ord_norm
nrand	the number of random numbers to generate in calculating intermediate correlations (default = 10000)

#### Value

the intermediate MVN correlation matrix

## References

Please see corrvar for references.

## See Also

corrvar

# **Examples**

```
## Not run:
\# 1 continuous mixture, 1 binary, 1 zero-inflated Poisson, and
# 1 zero-inflated NB variable
seed <- 1234
# Mixture of N(-2, 1) and N(2, 1)
constants <- rbind(c(0, 1, 0, 0, 0, 0), c(0, 1, 0, 0, 0))
marginal <- list(0.3)</pre>
support <- list(c(0, 1))
lam <- 0.5
p_zip <- 0.1
size <- 2
prob <- 0.75
p_zinb <- 0.2
k_cat <- k_pois <- k_nb <- 1
k_cont <- 2
Rey <- matrix(0.35, 5, 5)
diag(Rey) <- 1</pre>
```

```
rownames(Rey) <- colnames(Rey) <- c("01", "M1_1", "M1_2", "P1", "NB1")

# set correlation between components of the same mixture variable to 0
Rey["M1_1", "M1_2"] <- Rey["M1_2", "M1_1"] <- 0

Sigma1 <- intercorr(k_cat, k_cont, k_pois, k_nb, "Polynomial", constants, marginal, support, lam, p_zip, size, prob, mu = NULL, p_zinb, Rey, seed)
Sigma1

## End(Not run)</pre>
```

intercorr2

Calculate Intermediate MVN Correlation for Ordinal, Continuous, Poisson, or Negative Binomial Variables: Correlation Method 2

## **Description**

This function calculates a  $k \times k$  intermediate matrix of correlations, where  $k = k\_cat + k\_cont + k\_pois + k\_nb$ , to be used in simulating variables with corrvar2. The k\\_cont includes regular continuous variables and components of continuous mixture variables. The ordering of the variables must be ordinal, continuous non-mixture, components of continuous mixture variables, regular Poisson, zero-inflated Poisson, regular Negative Binomial (NB), and zero-inflated NB (note that it is possible for k\\_cat, k\\_cont, k\_pois, and/or k\_nb to be 0). There are no parameter input checks in order to decrease simulation time. All inputs should be checked prior to simulation with validpar. There is a message given if the calculated intermediate correlation matrix Sigma is not positive-definite because it may not be possible to find a MVN correlation matrix that will produce the desired marginal distributions. This function is called by the simulation function corrvar2, and would only be used separately if the user wants to first find the intermediate correlation matrix. This matrix Sigma can be used as an input to corrvar2.

Please see the **Comparison of Correlation Methods 1 and 2** vignette for information about calculations by variable pair type and the differences between this function and intercorr.

## Usage

```
intercorr2(k_cat = 0, k_cont = 0, k_pois = 0, k_nb = 0,
  method = c("Fleishman", "Polynomial"), constants = NULL,
  marginal = list(), support = list(), lam = NULL, p_zip = 0,
  size = NULL, prob = NULL, mu = NULL, p_zinb = 0, pois_eps = 1e-04,
  nb_eps = 1e-04, rho = NULL, epsilon = 0.001, maxit = 1000)
```

## Arguments

k_cat	the number of ordinal ( $r \ge 2$ categories) variables (default = 0)
k_cont	the number of continuous non-mixture variables and components of continuous mixture variables (default $= 0$ )
k_pois	the number of regular and zero-inflated Poisson variables (default = 0)
k_nb	the number of regular and zero-inflated Negative Binomial variables (default = $0$ )
method	the method used to generate the k_cont continuous variables. "Fleishman" uses a third-order polynomial transformation and "Polynomial" uses Headrick's fifth-order transformation.

constants	a matrix with k_cont rows, each a vector of constants c0, c1, c2, c3 (if method = "Fleishman") or c0, c1, c2, c3, c4, c5 (if method = "Polynomial") like that returned by find_constants
marginal	a list of length equal to k_cat; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain r - 1 probabilities (the r-th is assumed to be 1; default = list())
support	a list of length equal to $k_{cat}$ ; the i-th element is a vector of containing the r ordered support values; if not provided (i.e. $support = list()$ ), the default is for the i-th element to be the vector 1,, r
lam	a vector of lambda (mean > 0) constants for the regular and zero-inflated Poisson variables (see dpois); the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson variables
p_zip	a vector of probabilities of structural zeros (not including zeros from the Poisson distribution) for the zero-inflated Poisson variables (see <code>dzipois</code> ); if <code>p_zip = 0</code> , $Y_{pois}$ has a regular Poisson distribution; if <code>p_zip</code> is in (0, 1), $Y_{pois}$ has a zero-inflated Poisson distribution; if <code>p_zip</code> is in (-(exp(lam) - 1)^(-1), 0), $Y_{pois}$ has a zero-deflated Poisson distribution and <code>p_zip</code> is not a probability; if <code>p_zip = -(exp(lam) - 1)^(-1)</code> , $Y_{pois}$ has a positive-Poisson distribution (see <code>dpospois</code> ); if <code>length(p_zip) &lt; length(lam)</code> , the missing values are set to 0 (and ordered 1st)
size	a vector of size parameters for the Negative Binomial variables (see

intercorr\_cat\_nb 31

#### Value

the intermediate MVN correlation matrix

#### References

Please see corryar2 for references.

#### See Also

corrvar2

#### **Examples**

```
## Not run:
# 1 continuous mixture, 1 binary, 1 zero-inflated Poisson, and
# 1 zero-inflated NB variable
# The defaults of pois_eps <- nb_eps <- 0.0001 are used.
# Mixture of N(-2, 1) and N(2, 1)
constants <- rbind(c(0, 1, 0, 0, 0, 0), c(0, 1, 0, 0, 0))
marginal <- list(0.3)</pre>
support <- list(c(0, 1))
lam <- 0.5
p_zip <- 0.1
size <- 2
prob <- 0.75
p_zinb <- 0.2
k_cat <- k_pois <- k_nb <- 1
k_cont <- 2
Rey <- matrix(0.35, 5, 5)
diag(Rey) <- 1</pre>
rownames(Rey) <- colnames(Rey) <- c("01", "M1_1", "M1_2", "P1", "NB1")</pre>
\mbox{\#} set correlation between components of the same mixture variable to 0
Rey["M1_1", "M1_2"] \leftarrow Rey["M1_2", "M1_1"] \leftarrow 0
Sigma2 <- intercorr2(k_cat, k_cont, k_pois, k_nb, "Polynomial", constants,</pre>
  marginal, support, lam, p_zip, size, prob, mu = NULL, p_zinb, rho = Rey)
Sigma2
## End(Not run)
```

Calculate Intermediate MVN Correlation for Ordinal - Negative Binomial Variables: Correlation Method 1

32 intercorr\_cat\_nb

## **Description**

This function calculates the k\_cat x k\_nb intermediate matrix of correlations for the k\_cat ordinal (r>=2 categories) and k\_nb Negative Binomial variables required to produce the target correlations in rho\_cat\_nb. It extends the method of Amatya & Demirtas (2015, doi: 10.1080/00949655.2014.953534) to ordinal - Negative Binomial pairs and allows for regular or zero-inflated NB variables. Here, the intermediate correlation between Z1 and Z2 (where Z1 is the standard normal variable discretized to produce an ordinal variable Y1, and Z2 is the standard normal variable used to generate a Negative Binomial variable via the inverse CDF method) is calculated by dividing the target correlation by a correction factor. The correction factor is the product of the upper Frechet-Hoeffding bound on the correlation between a Negative Binomial variable and the normal variable used to generate it and a simulated GSC upper bound on the correlation between an ordinal variable and the normal variable used to generate it (see Demirtas & Hedeker, 2011, doi: 10.1198/tast.2011.10090). The function is used in intercorr and corrvar. This function would not ordinarily be called by the user.

## Usage

```
intercorr_cat_nb(rho_cat_nb = NULL, marginal = list(), size = NULL,
  mu = NULL, p_zinb = 0, nrand = 1e+05, seed = 1234)
```

#### **Arguments**

rho_cat_nb	a k_cat $$ x k_nb matrix of target correlations among ordinal and Negative Binomial variables; the NB variables should be ordered 1st regular, 2nd zero-inflated
marginal	a list of length equal to $k\_cat$ ; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain $r$ - 1 probabilities (the r-th is assumed to be 1)
size	a vector of size parameters for the Negative Binomial variables (see <a href="mailto:dnbinom">dnbinom</a> ); the order should be 1st regular NB variables, 2nd zero-inflated NB variables
mu	a vector of mean parameters for the NB variables (*Note: either prob or mu should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean of the NB distribution (see dzinegbin)
p_zinb	a vector of probabilities of structural zeros (not including zeros from the NB distribution) for the zero-inflated NB variables (see dzinegbin); if p_zinb = 0, $Y_{nb}$ has a regular NB distribution; if p_zinb is in (-prob^size/(1 - prob^size), 0), $Y_{nb}$ has a zero-deflated NB distribution and p_zinb is not a probability; if p_zinb = -prob^size/(1 - prob^size), $Y_{nb}$ has a positive-NB distribution (see dposnegbin); if length(p_zinb) < length(size), the missing values are set to 0 (and ordered 1st)
nrand	the number of random numbers to generate in calculating the bound (default = $10000$ )
seed	the seed used in random number generation (default = 1234)

## Value

a  $k\_cat\ x\ k\_nb$  matrix whose rows represent the  $k\_cat$  ordinal variables and columns represent the  $k\_nb$  Negative Binomial variables

intercorr\_cat\_pois 33

#### References

Please see references for intercorr\_cat\_pois

#### See Also

intercorr, corrvar

intercorr\_cat\_pois Calculate Intermediate MVN Correlation for Ordinal - Poisson Variables: Correlation Method 1

## Description

This function calculates a k\_cat  $\, x \, k$ -pois intermediate matrix of correlations for the k\_cat ordinal (r >= 2 categories) and k\_pois Poisson variables required to produce the target correlations in rho\_cat\_pois. It extends the method of Amatya & Demirtas (2015, doi: 10.1080/00949655.2014.953534) to ordinal - Poisson pairs and allows for regular or zero-inflated Poisson variables. Here, the intermediate correlation between Z1 and Z2 (where Z1 is the standard normal variable discretized to produce an ordinal variable Y1, and Z2 is the standard normal variable used to generate a Poisson variable via the inverse CDF method) is calculated by dividing the target correlation by a correction factor. The correction factor is the product of the upper Frechet-Hoeffding bound on the correlation between a Poisson variable and the normal variable used to generate it and a simulated GSC upper bound on the correlation between an ordinal variable and the normal variable used to generate it (see Demirtas & Hedeker, 2011, doi: 10.1198/tast.2011.10090). The function is used in intercorr and corrvar. This function would not ordinarily be called by the user.

## Usage

```
intercorr_cat_pois(rho_cat_pois = NULL, marginal = list(), lam = NULL,
    p_zip = 0, nrand = 1e+05, seed = 1234)
```

#### Arguments

rho\_cat\_pois a k\_cat x k\_pois matrix of target correlations among ordinal and Poisson

variables; the Poisson variables should be ordered 1st regular, 2nd zero-inflated

marginal a list of length equal to k\_cat; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable

can take r values, the vector will contain r - 1 probabilities (the r-th is assumed

to be 1)

 $\label{eq:approx} \mbox{a vector of lambda (mean > 0) constants for the regular and zero-inflated Poisson}$ 

variables (see dpois); the order should be 1st regular Poisson variables, 2nd

zero-inflated Poisson variables

p\_zip a vector of probabilities of structural zeros (not including zeros from the Poisson

distribution) for the zero-inflated Poisson variables (see dzipois); if p\_zip = 0,  $Y_{pois}$  has a regular Poisson distribution; if p\_zip is in (0, 1),  $Y_{pois}$  has a zero-inflated Poisson distribution; if p\_zip is in (-(exp(lam) - 1)^(-1), 0),  $Y_{pois}$  has a zero-deflated Poisson distribution and p\_zip is not a probability; if p\_zip = -(exp(lam) - 1)^(-1),  $Y_{pois}$  has a positive-Poisson distribution (see dpospois); if length(p\_zip) < length(lam), the missing values are set

to 0 (and ordered 1st)

34 intercorr\_cont

nrand the number of random numbers to generate in calculating the bound (default =

10000)

seed the seed used in random number generation (default = 1234)

#### Value

a  $k\_cat \ x \ k\_pois \ matrix$  whose rows represent the  $k\_cat$  ordinal variables and columns represent the  $k\_pois \ Poisson \ variables$ 

#### References

Amatya A & Demirtas H (2015). Simultaneous generation of multivariate mixed data with Poisson and normal marginals. Journal of Statistical Computation and Simulation, 85(15):3129-39. doi: 10.1080/00949655.2014.953534.

Demirtas H & Hedeker D (2011). A practical way for computing approximate lower and upper correlation bounds. American Statistician, 65(2):104-109. doi: 10.1198/tast.2011.10090.

Frechet M (1951). Sur les tableaux de correlation dont les marges sont donnees. Ann. l'Univ. Lyon SectA, 14:53-77.

Hoeffding W. Scale-invariant correlation theory. In: Fisher NI, Sen PK, editors. The collected works of Wassily Hoeffding. New York: Springer-Verlag; 1994. p. 57-107.

Yahav I & Shmueli G (2012). On Generating Multivariate Poisson Data in Management Science Applications. Applied Stochastic Models in Business and Industry, 28(1):91-102. doi: 10.1002/asmb.901.

Yee TW (2017). VGAM: Vector Generalized Linear and Additive Models.

https://CRAN.R-project.org/package=VGAM.

## See Also

intercorr, corrvar

intercorr_cont	Calculate Intermediate MVN Correlation for Continuous Variables
	Generated by Polynomial Transformation Method

## **Description**

This function finds the intermediate correlation for standard normal random variables which are used in Fleishman's third-order (doi: 10.1007/BF02293811) or Headrick's fifth-order (doi: 10.1016/S01679473(02)000725) polynomial transformation method (PMT) using nleqslv. It is used in intercorr and intercorr2 and would not ordinarily be called by the user. The correlations are found pairwise so that eigen-value or principal components decomposition should be done on the resulting Sigma matrix. The Comparison of Correlation Methods 1 and 2 vignette contains the equations which were derived by Headrick and Sawilowsky (doi: 10.1007/BF02294317) or Headrick (doi: 10.1016/S01679473(02)000725).

### Usage

```
intercorr_cont(method = c("Fleishman", "Polynomial"), constants = NULL,
  rho_cont = NULL)
```

35 intercorr\_cont\_nb

## **Arguments**

method the method used to generate the continuous variables. "Fleishman" uses Fleish-

man's third-order polynomial transformation and "Polynomial" uses Headrick's

fifth-order transformation.

a matrix with each row a vector of constants c0, c1, c2, c3 (if method = "Fleishconstants

man") or c0, c1, c2, c3, c4, c5 (if method = "Polynomial"), like that returned by

a matrix of target correlations among continuous variables, does not have to be rho\_cont

symmetric

#### Value

the intermediate matrix of correlations with the same dimensions as rho\_cont

#### References

Berend H (2017). nleqslv: Solve Systems of Nonlinear Equations. R package version 3.2. https: //CRAN.R-project.org/package=nleqslv

Fleishman AI (1978). A Method for Simulating Non-normal Distributions. Psychometrika, 43, 521-532. doi: 10.1007/BF02293811.

Headrick TC (2002). Fast Fifth-order Polynomial Transforms for Generating Univariate and Multivariate Non-normal Distributions. Computational Statistics & Data Analysis, 40(4):685-711. doi: 10.1016/S01679473(02)000725. (ScienceDirect)

Headrick TC (2004). On Polynomial Transformations for Simulating Multivariate Nonnormal Distributions. Journal of Modern Applied Statistical Methods, 3(1), 65-71. doi: 10.22237/jmasm/ 1083370080.

Headrick TC, Kowalchuk RK (2007). The Power Method Transformation: Its Probability Density Function, Distribution Function, and Its Further Use for Fitting Data. Journal of Statistical Computation and Simulation, 77, 229-249. doi: 10.1080/10629360600605065.

Headrick TC, Sawilowsky SS (1999). Simulating Correlated Non-normal Distributions: Extending the Fleishman Power Method. Psychometrika, 64, 25-35. doi: 10.1007/BF02294317.

Headrick TC, Sheng Y, & Hodis FA (2007). Numerical Computing and Graphics for the Power Method Transformation Using Mathematica. Journal of Statistical Software, 19(3), 1 - 17. doi: 10.18637/ jss.v019.i03.

## See Also

intercorr, intercorr2, nleqslv

intercorr\_cont\_nb Calculate Intermediate MVN Correlation for Continuous - Negative

Binomial Variables: Correlation Method 1

36 intercorr\_cont\_nb

#### **Description**

This function calculates a k\_cont x k\_nb intermediate matrix of correlations for the k\_cont continuous and k\_nb Negative Binomial variables. It extends the method of Amatya & Demirtas (2015, doi: 10.1080/00949655.2014.953534) to continuous variables generated using Headrick's fifth-order polynomial transformation and regular or zero-inflated NB variables. Here, the intermediate correlation between Z1 and Z2 (where Z1 is the standard normal variable transformed using Headrick's fifth-order or Fleishman's third-order method to produce a continuous variable Y1, and Z2 is the standard normal variable used to generate a Negative Binomial variable via the inverse CDF method) is calculated by dividing the target correlation by a correction factor. The correction factor is the product of the upper Frechet-Hoeffding bound on the correlation between a Negative Binomial variable and the normal variable used to generate it and the power method correlation (described in Headrick & Kowalchuk, 2007, doi: 10.1080/10629360600605065) between Y1 and Z1. The function is used in intercorr and corrvar. This function would not ordinarily be called by the user.

# Usage

```
intercorr_cont_nb(method = c("Fleishman", "Polynomial"), constants = NULL,
  rho_cont_nb = NULL, size = NULL, mu = NULL, p_zinb = 0,
  nrand = 1e+05, seed = 1234)
```

## **Arguments**

method	the method used to generate the k_cont continuous variables. "Fleishman" us	es
--------	---	----

a third-order polynomial transformation and "Polynomial" uses Headrick's fifth-

order transformation.

constants a matrix with k\_cont rows, each a vector of constants c0, c1, c2, c3 (if method

= "Fleishman") or c0, c1, c2, c3, c4, c5 (if method = "Polynomial"), like that

returned by find\_constants

rho\_cont\_nb a k\_cont x k\_nb matrix of target correlations among continuous and Negative

Binomial variables; the NB variables should be ordered 1st regular, 2nd zero-

inflated

size a vector of size parameters for the Negative Binomial variables (see dnbinom);

the order should be 1st regular NB variables, 2nd zero-inflated NB variables

mu a vector of mean parameters for the NB variables (\*Note: either prob or mu

should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean

of the NB distribution (see dzinegbin)

p\_zinb a vector of probabilities of structural zeros (not including zeros from the NB dis-

tribution) for the zero-inflated NB variables (see dzinegbin); if p\_zinb = 0,  $Y_{nb}$  has a regular NB distribution; if p\_zinb is in (-prob^size/(1 - prob^size), 0),  $Y_{nb}$  has a zero-deflated NB distribution and p\_zinb is not a probability; if p\_zinb = -prob^size/(1 - prob^size),  $Y_{nb}$  has a positive-NB distribution (see dposnegbin); if length(p\_zinb) < length(size), the missing values

are set to 0 (and ordered 1st)

nrand the number of random numbers to generate in calculating the bound (default =

10000)

seed the seed used in random number generation (default = 1234)

intercorr\_cont\_nb2 37

#### Value

a k\_cont x k\_nb matrix whose rows represent the k\_cont continuous variables and columns represent the k\_nb Negative Binomial variables

#### References

Please see references for intercorr\_cont\_pois.

# See Also

find\_constants, intercorr, corrvar

intercorr\_cont\_nb2

Calculate Intermediate MVN Correlation for Continuous - Negative Binomial Variables: Correlation Method 2

# **Description**

This function calculates a k\_cont x k\_nb intermediate matrix of correlations for the k\_cont continuous and k\_nb Negative Binomial variables. It extends the methods of Demirtas et al. (2012, doi: 10.1002/sim.5362) and Barbiero & Ferrari (2015, doi: 10.1002/asmb.2072) by:

- 1) including non-normal continuous and regular or zero-inflated Negative Binomial variables
- 2) allowing the continuous variables to be generated via Fleishman's third-order or Headrick's fifth-order transformation, and
- 3) since the count variables are treated as ordinal, using the point-polyserial and polyserial correlations to calculate the intermediate correlations (similar to findintercorr\_cont\_cat in SimMultiCorrData).

Here, the intermediate correlation between Z1 and Z2 (where Z1 is the standard normal variable transformed using Headrick's fifth-order or Fleishman's third-order method to produce a continuous variable Y1, and Z2 is the standard normal variable used to generate a Negative Binomial variable via the inverse CDF method) is calculated by dividing the target correlation by a correction factor. The correction factor is the product of the point-polyserial correlation between Y2 and Z2 (described in Olsson et al., 1982, doi: 10.1007/BF02294164) and the power method correlation (described in Headrick & Kowalchuk, 2007, doi: 10.1080/10629360600605065) between Y1 and Z1. After the maximum support value has been found using maxcount\_support, the point-polyserial correlation is given by:

$$\rho_{Y2,Z2} = \frac{1}{\sigma_{Y2}} \sum_{j=1}^{r-1} \phi(\tau_j) (y2_{j+1} - y2_j)$$

where

$$\phi(\tau) = (2\pi)^{-1/2} * exp(-0.5\tau^2)$$

Here,  $y_j$  is the j-th support value and  $\tau_j$  is  $\Phi^{-1}(\sum_{i=1}^j Pr(Y=y_i))$ . The power method correlation is given by:

$$\rho_{Y1,Z1} = c_1 + 3c_3 + 15c_5,$$

where  $c_5 = 0$  if method = "Fleishman". The function is used in intercorr2 and corrvar2. This function would not ordinarily be called by the user.

38 intercorr\_cont\_pois

## **Usage**

```
intercorr_cont_nb2(method = c("Fleishman", "Polynomial"), constants = NULL,
  rho_cont_nb = NULL, nb_marg = list(), nb_support = list())
```

# **Arguments**

method the method used to generate the k\_cont continuous variables. "Fleishman"

uses Fleishman's third-order polynomial transformation and "Polynomial" uses

Headrick's fifth-order transformation.

constants a matrix with k\_cont rows, each a vector of constants c0, c1, c2, c3 (if method

= "Fleishman") or c0, c1, c2, c3, c4, c5 (if method = "Polynomial"), like that

returned by find\_constants

rho\_cont\_nb a k\_cont x k\_nb matrix of target correlations among continuous and Negative

Binomial variables; the NB variables should be ordered 1st regular, 2nd zero-

inflated

nb\_marg a list of length equal to k\_nb ordered 1st regular and 2nd zero-inflated; the i-th

element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain r - 1 probabilities (the r-th is assumed to be 1); this is created within

intercorr2 and corrvar2

 $nb\_support \qquad \quad a \ list \ of \ length \ equal \ to \ k\_nb \ ordered \ 1st \ regular \ and \ 2nd \ zero-inflated; \ the \ i-th$ 

element is a vector of containing the r ordered support values, with a minimum

of 0 and maximum determined via maxcount\_support

## Value

a  $k\_cont \times k\_nb$  matrix whose rows represent the  $k\_cont$  continuous variables and columns represent the  $k\_nb$  Negative Binomial variables

## References

Please see references in intercorr\_cont\_pois2.

# See Also

find\_constants, power\_norm\_corr, intercorr2, corrvar2

intercorr\_cont\_pois Calculate Intermediate MVN Correlation for Continuous - Poisson

Variables: Correlation Method 1

# **Description**

This function calculates a k\_cont x k\_pois intermediate matrix of correlations for the k\_cont continuous and k\_pois Poisson variables. It extends the method of Amatya & Demirtas (2015, doi: 10.1080/00949655.2014.953534) to continuous variables generated using Headrick's fifth-order polynomial transformation and zero-inflated Poisson variables. Here, the intermediate correlation between Z1 and Z2 (where Z1 is the standard normal variable transformed using Headrick's fifth-order or Fleishman's third-order method to produce a continuous variable Y1, and Z2 is the

intercorr\_cont\_pois 39

standard normal variable used to generate a Poisson variable via the inverse CDF method) is calculated by dividing the target correlation by a correction factor. The correction factor is the product of the upper Frechet-Hoeffding bound on the correlation between a Poisson variable and the normal variable used to generate it and the power method correlation (described in Headrick & Kowalchuk, 2007, doi: 10.1080/10629360600605065) between Y1 and Z1. The function is used in intercorr and corrvar. This function would not ordinarily be called by the user.

## **Usage**

```
intercorr_cont_pois(method = c("Fleishman", "Polynomial"), constants = NULL,
  rho_cont_pois = NULL, lam = NULL, p_zip = 0, nrand = 1e+05,
  seed = 1234)
```

# **Arguments**

method the	e method used to generate the k_	_cont continuous variables.	"Fleishman" uses
------------	----------------------------------	-----------------------------	------------------

a third-order polynomial transformation and "Polynomial" uses Headrick's fifth-

order transformation.

constants a matrix with k\_cont rows, each a vector of constants c0, c1, c2, c3 (if method

= "Fleishman") or c0, c1, c2, c3, c4, c5 (if method = "Polynomial"), like that

returned by find\_constants

rho\_cont\_pois a k\_cont x k\_pois matrix of target correlations among continuous and Poisson

variables; the Poisson variables should be ordered 1st regular, 2nd zero-inflated

lam a vector of lambda (mean > 0) constants for the regular and zero-inflated Poisson

variables (see dpois); the order should be 1st regular Poisson variables, 2nd

zero-inflated Poisson variables

p\_zip a vector of probabilities of structural zeros (not including zeros from the Poisson

distribution) for the zero-inflated Poisson variables (see dzipois); if p\_zip = 0,  $Y_{pois}$  has a regular Poisson distribution; if p\_zip is in (0, 1),  $Y_{pois}$  has a zero-inflated Poisson distribution; if p\_zip is in (-(exp(lam) - 1)^(-1), 0),  $Y_{pois}$  has a zero-deflated Poisson distribution and p\_zip is not a probability; if p\_zip = -(exp(lam) - 1)^(-1),  $Y_{pois}$  has a positive-Poisson distribution (see dpospois); if length(p\_zip) < length(lam), the missing values are set

to 0 (and ordered 1st)

nrand the number of random numbers to generate in calculating the bound (default =

10000)

seed the seed used in random number generation (default = 1234)

# Value

a  $k\_cont\ x\ k\_pois\ matrix$  whose rows represent the  $k\_cont\ continuous\ variables$  and columns represent the  $k\_pois\ Poisson\ variables$ 

# References

Amatya A & Demirtas H (2015). Simultaneous generation of multivariate mixed data with Poisson and normal marginals. Journal of Statistical Computation and Simulation, 85(15):3129-39. doi: 10.1080/00949655.2014.953534.

Demirtas H & Hedeker D (2011). A practical way for computing approximate lower and upper correlation bounds. American Statistician, 65(2):104-109. doi: 10.1198/tast.2011.10090.

Frechet M (1951). Sur les tableaux de correlation dont les marges sont donnees. Ann. l'Univ. Lyon SectA. 14:53-77.

Headrick TC, Kowalchuk RK (2007). The Power Method Transformation: Its Probability Density Function, Distribution Function, and Its Further Use for Fitting Data. Journal of Statistical Computation and Simulation, 77:229-249. doi: 10.1080/10629360600605065.

Hoeffding W. Scale-invariant correlation theory. In: Fisher NI, Sen PK, editors. The collected works of Wassily Hoeffding. New York: Springer-Verlag; 1994. p. 57-107.

Yahav I & Shmueli G (2012). On Generating Multivariate Poisson Data in Management Science Applications. Applied Stochastic Models in Business and Industry, 28(1):91-102. doi: 10.1002/asmb.901.

Yee TW (2017). VGAM: Vector Generalized Linear and Additive Models. https://CRAN.R-project.org/package=VGAM.

# See Also

power\_norm\_corr, find\_constants, intercorr, corrvar

intercorr\_cont\_pois2 Calculate Intermediate MVN Correlation for Continuous - Poisson Variables: Correlation Method 2

# **Description**

This function calculates a k\_cont x k\_pois intermediate matrix of correlations for the k\_cont continuous and k\_pois Poisson variables. It extends the methods of Demirtas et al. (2012, doi: 10.1002/sim.5362) and Barbiero & Ferrari (2015, doi: 10.1002/asmb.2072) by:

- 1) including non-normal continuous and regular or zero-inflated Poisson variables
- 2) allowing the continuous variables to be generated via Fleishman's third-order or Headrick's fifth-order transformation, and
- 3) since the count variables are treated as ordinal, using the point-polyserial and polyserial correlations to calculate the intermediate correlations (similar to findintercorr\_cont\_cat) in SimMultiCorrData).

Here, the intermediate correlation between Z1 and Z2 (where Z1 is the standard normal variable transformed using Headrick's fifth-order or Fleishman's third-order method to produce a continuous variable Y1, and Z2 is the standard normal variable used to generate a Poisson variable via the inverse CDF method) is calculated by dividing the target correlation by a correction factor. The correction factor is the product of the point-polyserial correlation between Y2 and Z2 (described in Olsson et al., 1982, doi: 10.1007/BF02294164) and the power method correlation (described in Headrick & Kowalchuk, 2007, doi: 10.1080/10629360600605065) between Y1 and Z1. After the maximum support value has been found using maxcount\_support, the point-polyserial correlation is given by:

$$\rho_{Y2,Z2} = \frac{1}{\sigma_{Y2}} \sum_{j=1}^{r-1} \phi(\tau_j) (y2_{j+1} - y2_j)$$

where

$$\phi(\tau) = (2\pi)^{-1/2} * exp(-0.5\tau^2)$$

Here,  $y_j$  is the j-th support value and  $\tau_j$  is  $\Phi^{-1}(\sum_{i=1}^j Pr(Y=y_i))$ . The power method correlation is given by:

$$\rho_{Y1,Z1} = c_1 + 3c_3 + 15c_5,$$

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where  $c_5 = 0$  if method = "Fleishman". The function is used in intercorr2 and corrvar2. This function would not ordinarily be called by the user.

# Usage

```
intercorr_cont_pois2(method = c("Fleishman", "Polynomial"),
  constants = NULL, rho_cont_pois = NULL, pois_marg = list(),
  pois_support = list())
```

# **Arguments**

method	the method used to generate the k_cont continuous variables. "Fleishman" uses Fleishman's third-order polynomial transformation and "Polynomial" uses Headrick's fifth-order transformation.
constants	a matrix with k_cont rows, each a vector of constants $c0$ , $c1$ , $c2$ , $c3$ (if method = "Fleishman") or $c0$ , $c1$ , $c2$ , $c3$ , $c4$ , $c5$ (if method = "Polynomial"), like that returned by find_constants
rho_cont_pois	a k_cont x k_pois matrix of target correlations among continuous and Poisson variables; the Poisson variables should be ordered 1st regular, 2nd zero-inflated
pois_marg	a list of length equal to k_pois ordered 1st regular and 2nd zero-inflated; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take $r$ values, the vector will contain $r$ - 1 probabilities (the r-th is assumed to be 1); this is created within intercorr2 and corrvar2
pois_support	a list of length equal to k_pois ordered 1st regular and 2nd zero-inflated; the i-th element is a vector of containing the r ordered support values, with a minimum

# Value

a  $k\_cont\ x\ k\_pois\ matrix$  whose rows represent the  $k\_cont\ continuous\ variables$  and columns represent the  $k\_pois\ Poisson\ variables$ 

of 0 and maximum determined via maxcount\_support

# References

Please see additional references in intercorr\_cont\_pois.

Barbiero A & Ferrari PA (2015). Simulation of correlated Poisson variables. Applied Stochastic Models in Business and Industry, 31:669-80. doi: 10.1002/asmb.2072.

# See Also

```
find_constants, power_norm_corr, intercorr2, corrvar2
```

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intercorr_nb	Calculate Intermediate MVN Correlation for Negative Binomial Vari-
	ables: Correlation Method 1

# **Description**

This function calculates a k\_nb x k\_nb intermediate matrix of correlations for the Negative Binomial variables by extending the method of Yahav & Shmueli (2012, doi: 10.1002/asmb.901). The intermediate correlation between Z1 and Z2 (the standard normal variables used to generate the Negative Binomial variables Y1 and Y2 via the inverse CDF method) is calculated using a logarithmic transformation of the target correlation. First, the upper and lower Frechet-Hoeffding bounds (mincor, maxcor) on  $\rho_{Y1,Y2}$  are simulated. Then the intermediate correlation is found as follows:

$$\rho_{Z1,Z2} = \frac{1}{b} * log(\frac{\rho_{Y1,Y2} - c}{a}),$$

where a = -(maxcor \* mincor)/(maxcor + mincor), b = log((maxcor + a)/a), and c = -a. The function adapts code from Amatya & Demirtas' (2016) package PoisNor-package by:

- 1) allowing specifications for the number of random variates and the seed for reproducibility
- 2) providing the following checks: if  $Sigma_(Z1, Z2) > 1$ ,  $Sigma_(Z1, Z2)$  is set to 1; if  $Sigma_(Z1, Z2) < -1$ ,  $Sigma_(Z1, Z2)$  is set to -1
- 3) simulating regular and zero-inflated Negative Binomial variables.

The function is used in intercorr and corrvar and would not ordinarily be called by the user.

# Usage

```
intercorr_nb(rho_nb = NULL, size = NULL, mu = NULL, p_zinb = 0,
    nrand = 1e+05, seed = 1234)
```

# **Arguments**

rho_nb	a k_nb $$ x $$ k_nb matrix of target correlations ordered 1st regular and 2nd zero-inflated
size	a vector of size parameters for the Negative Binomial variables (see <a href="mailto:dnbinom">dnbinom</a> ); the order should be 1st regular NB variables, 2nd zero-inflated NB variables
mu	a vector of mean parameters for the NB variables (*Note: either prob or mu should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean of the NB distribution (see dzinegbin)
p_zinb	a vector of probabilities of structural zeros (not including zeros from the NB distribution) for the zero-inflated NB variables (see dzinegbin); if p_zinb = 0, $Y_{nb}$ has a regular NB distribution; if p_zinb is in (-prob^size/(1 - prob^size), 0), $Y_{nb}$ has a zero-deflated NB distribution and p_zinb is not a probability; if p_zinb = -prob^size/(1 - prob^size), $Y_{nb}$ has a positive-NB distribution (see dposnegbin); if length(p_zinb) < length(size), the missing values are set to 0 (and ordered 1st)
nrand	the number of random numbers to generate in calculating the bound (default = 10000)
seed	the seed used in random number generation (default = 1234)

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#### Value

the k\_nb x k\_nb intermediate correlation matrix for the Negative Binomial variables

#### References

Please see references for intercorr\_pois.

## See Also

intercorr\_pois, intercorr\_pois\_nb, intercorr, corrvar

intercorr\_pois

Calculate Intermediate MVN Correlation for Poisson Variables: Correlation Method 1

# **Description**

This function calculates a k\_pois x k\_pois intermediate matrix of correlations for the Poisson variables using the method of Yahav & Shmueli (2012, doi: 10.1002/asmb.901). The intermediate correlation between Z1 and Z2 (the standard normal variables used to generate the Poisson variables Y1 and Y2 via the inverse CDF method) is calculated using a logarithmic transformation of the target correlation. First, the upper and lower Frechet-Hoeffding bounds (mincor, maxcor) on  $\rho_{Y1,Y2}$  are simulated. Then the intermediate correlation is found as follows:

$$\rho_{Z1,Z2} = \frac{1}{b} * log(\frac{\rho_{Y1,Y2} - c}{a}),$$

where a = -(maxcor \* mincor)/(maxcor + mincor), b = log((maxcor + a)/a), and c = -a. The function adapts code from Amatya & Demirtas' (2016) package PoisNor-package by:

- 1) allowing specifications for the number of random variates and the seed for reproducibility
- 2) providing the following checks: if  $Sigma_(Z1, Z2) > 1$ ,  $Sigma_(Z1, Z2)$  is set to 1; if  $Sigma_(Z1, Z2) < -1$ ,  $Sigma_(Z1, Z2)$  is set to -1
- 3) simulating regular and zero-inflated Poisson variables.

The function is used in intercorr and corrvar and would not ordinarily be called by the user.

# Usage

```
intercorr_pois(rho_pois = NULL, lam = NULL, p_zip = 0, nrand = 1e+05,
  seed = 1234)
```

# **Arguments**

rho\_pois a k\_pois x k\_pois matrix of target correlations ordered 1st regular and 2nd

zero-inflated

 $\label{eq:constants} \mbox{lam} \quad \mbox{a vector of lambda (mean > 0) constants for the regular and zero-inflated Poisson}$ 

variables (see dpois); the order should be 1st regular Poisson variables, 2nd

zero-inflated Poisson variables

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p_zip	a vector of probabilities of structural zeros (not including zeros from the Poisson distribution) for the zero-inflated Poisson variables (see $dzipois$ ); if $p_zip = 0$ ,
	$Y_{pois}$ has a regular Poisson distribution; if p_zip is in (0, 1), $Y_{pois}$ has a zero-
	inflated Poisson distribution; if $p_zip$ is in $(-(exp(lam) - 1)^{(-1)}, 0)$ , $Y_{pois}$ has a zero-deflated Poisson distribution and $p_zip$ is not a probability; if
	$p_{pois}$ has a zero-denated Poisson distribution and $p_{2}$ 1p is not a probability, if $p_{zip} = -(\exp(\text{lam}) - 1)^{-1}$ , $Y_{pois}$ has a positive-Poisson distribution (see dpospois); if length( $p_{zip}$ ) < length(lam), the missing values are set to 0 (and ordered 1st)
nrand	the number of random numbers to generate in calculating the bound (default = 10000)
seed	the seed used in random number generation (default = $1234$ )

## Value

the k\_pois x k\_pois intermediate correlation matrix for the Poisson variables

#### References

Amatya A & Demirtas H (2015). Simultaneous generation of multivariate mixed data with Poisson and normal marginals. Journal of Statistical Computation and Simulation, 85(15):3129-39. doi: 10.1080/00949655.2014.953534.

Demirtas H & Hedeker D (2011). A practical way for computing approximate lower and upper correlation bounds. American Statistician, 65(2):104-109.

Frechet M (1951). Sur les tableaux de correlation dont les marges sont donnees. Ann. l'Univ. Lyon SectA, 14:53-77.

Hoeffding W. Scale-invariant correlation theory. In: Fisher NI, Sen PK, editors. The collected works of Wassily Hoeffding. New York: Springer-Verlag; 1994. p. 57-107.

Yahav I & Shmueli G (2012). On Generating Multivariate Poisson Data in Management Science Applications. Applied Stochastic Models in Business and Industry, 28(1):91-102. doi: 10.1002/asmb.901.

Yee TW (2017). VGAM: Vector Generalized Linear and Additive Models. https://CRAN.R-project.org/package=VGAM.

# See Also

 $intercorr\_nb, intercorr\_pois\_nb, intercorr, corrvar$ 

# Description

This function calculates a k\_pois x k\_nb intermediate matrix of correlations for the Poisson and Negative Binomial variables by extending the method of Yahav & Shmueli (2012, doi: 10.1002/asmb.901). The intermediate correlation between Z1 and Z2 (the standard normal variables used to generate the Poisson and Negative Binomial variables Y1 and Y2 via the inverse CDF method) is calculated using a logarithmic transformation of the target correlation. First, the upper and lower

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Frechet-Hoeffding bounds (mincor, maxcor) on  $\rho_{Y1,Y2}$  are simulated. Then the intermediate correlation is found as follows:

$$\rho_{Z1,Z2} = \frac{1}{b} * log(\frac{\rho_{Y1,Y2} - c}{a}),$$

where a = -(maxcor \* mincor)/(maxcor + mincor), b = log((maxcor + a)/a), and c = -a. The function adapts code from Amatya & Demirtas' (2016) package PoisNor-package by:

- 1) allowing specifications for the number of random variates and the seed for reproducibility
- 2) providing the following checks: if Sigma\_(Z1, Z2) > 1, Sigma\_(Z1, Z2) is set to 1; if  $Sigma_(Z1, Z2) < -1$ ,  $Sigma_(Z1, Z2)$  is set to -1
- 3) simulating regular and zero-inflated Poisson and Negative Binomial variables.

The function is used in intercorr and corrvar and would not ordinarily be called by the user.

## Usage

```
intercorr_pois_nb(rho_pois_nb = NULL, lam = NULL, p_zip = 0,
  size = NULL, mu = NULL, p_zinb = 0, nrand = 1e+05, seed = 1234)
```

## **Arguments**

a k\_pois x k\_nb matrix of target correlations; order of each type should be 1st rho\_pois\_nb regular, 2nd zero-inflated

a vector of lambda (mean > 0) constants for the regular and zero-inflated Poisson variables (see dpois); the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson variables

> a vector of probabilities of structural zeros (not including zeros from the Poisson distribution) for the zero-inflated Poisson variables (see dzipois); if p\_zip = 0,  $Y_{pois}$  has a regular Poisson distribution; if p\_zip is in (0, 1),  $Y_{pois}$  has a zeroinflated Poisson distribution; if  $p_{zip}$  is in  $(-(exp(lam) - 1)^{(-1)}, 0)$ ,  $Y_{pois}$  has a zero-deflated Poisson distribution and p\_zip is not a probability; if p\_zip = -(exp(lam) - 1)^(-1),  $Y_{pois}$  has a positive-Poisson distribution (see dpospois); if length(p\_zip) < length(lam), the missing values are set to 0 (and ordered 1st)

> a vector of size parameters for the Negative Binomial variables (see dnbinom); the order should be 1st regular NB variables, 2nd zero-inflated NB variables

> a vector of mean parameters for the NB variables (\*Note: either prob or mu should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean of the NB distribution (see dzinegbin)

> a vector of probabilities of structural zeros (not including zeros from the NB distribution) for the zero-inflated NB variables (see dzinegbin); if p\_zinb = 0,  $Y_{nh}$ has a regular NB distribution; if p\_zinb is in (-prob^size/(1 - prob^size), 0),  $Y_{nb}$  has a zero-deflated NB distribution and p\_zinb is not a probability; if p\_zinb = -prob^size/(1 - prob^size),  $Y_{nb}$  has a positive-NB distribution (see dposnegbin); if length(p\_zinb) < length(size), the missing values are set to 0 (and ordered 1st)

> the number of random numbers to generate in calculating the bound (default =

the seed used in random number generation (default = 1234)

lam

p\_zip

size

mu

p\_zinb

nrand

seed

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#### Value

the k\_pois x k\_nb intermediate correlation matrix whose rows represent the k\_pois Poisson variables and columns represent the k\_nb Negative Binomial variables

## References

Please see references for intercorr\_pois.

## See Also

intercorr\_pois, intercorr\_nb, intercorr, corrvar

# **Description**

This function calculates the maximum support value for count variables by extending the method of Barbiero & Ferrari (2015, doi: 10.1002/asmb.2072) to include regular and zero-inflated Poisson and Negative Binomial variables. In order for count variables to be treated as ordinal in the calculation of the intermediate MVN correlation matrix, their infinite support must be truncated (made finite). This is done by setting the total cumulative probability equal to 1 - a small user-specified value (pois\_eps or nb\_eps). The maximum support value equals the inverse CDF applied to this result. The truncation values may differ for each variable. The function is used in intercorr2 and corrvar2 and would not ordinarily be called by the user.

# Usage

```
maxcount_support(k_pois = 0, k_nb = 0, lam = NULL, p_zip = 0,
    size = NULL, prob = NULL, mu = NULL, p_zinb = 0, pois_eps = NULL,
    nb_eps = NULL)
```

# **Arguments**

k_pois	the number of Poisson variables
k_nb	the number of Negative Binomial variables
lam	a vector of lambda (mean > 0) constants for the regular and zero-inflated Poisson variables (see dpois); the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson variables
p_zip	a vector of probabilities of structural zeros (not including zeros from the Poisson distribution) for the zero-inflated Poisson variables (see dzipois); if p_zip = 0, $Y_{pois}$ has a regular Poisson distribution; if p_zip is in (0, 1), $Y_{pois}$ has a zero-inflated Poisson distribution; if p_zip is in (-(exp(lam) - 1)^(-1), 0), $Y_{pois}$ has a zero-deflated Poisson distribution and p_zip is not a probability; if p_zip = -(exp(lam) - 1)^(-1), $Y_{pois}$ has a positive-Poisson distribution (see dpospois); if length(p_zip) < length(lam), the missing values are set to 0 (and ordered 1st)
size	a vector of size parameters for the Negative Binomial variables (see dnbinom); the order should be 1st regular NB variables, 2nd zero-inflated NB variables

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a vector of success probability parameters for the NB variables; order the same prob as in size a vector of mean parameters for the NB variables (\*Note: either prob or mu mu should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean of the NB distribution (see dzinegbin) a vector of probabilities of structural zeros (not including zeros from the NB disp\_zinb tribution) for the zero-inflated NB variables (see dzinegbin); if p\_zinb = 0,  $Y_{nb}$ has a regular NB distribution; if p\_zinb is in (-prob^size/(1 - prob^size), 0),  $Y_{nb}$  has a zero-deflated NB distribution and p\_zinb is not a probability; if p\_zinb = -prob^size/(1 - prob^size),  $Y_{nb}$  has a positive-NB distribution (see dposnegbin); if length(p\_zinb) < length(size), the missing values are set to 0 (and ordered 1st) a vector of length k\_pois containing total cumulative probability truncation valpois\_eps ues; if none are provided, the default is 0.0001 for each variable a vector of length k\_nb containing total cumulative probability truncation valnb\_eps

ues; if none are provided, the default is 0.0001 for each variable

## Value

a data.frame with k\_pois + k\_nb rows; the column names are:
Distribution Poisson or Negative Binomial
Number the variable index
Max the maximum support value

## References

Barbiero A & Ferrari PA (2015). Simulation of correlated Poisson variables. Applied Stochastic Models in Business and Industry, 31:669-80. doi: 10.1002/asmb.2072.

Ferrari PA, Barbiero A (2012). Simulating ordinal data, Multivariate Behavioral Research, 47(4):566-589. doi: 10.1080/00273171.2012.692630.

Yee TW (2017). VGAM: Vector Generalized Linear and Additive Models.

https://CRAN.R-project.org/package=VGAM.

# See Also

intercorr2, corrvar2

norm_ord Calculate Correlations of Ordinal Variables Obtained from Discretiz- ing Normal Variables	norm_ord	Calculate Correlations of Ordinal Variables Obtained from Discretizing Normal Variables
---	----------	---

# **Description**

This function calculates the correlation of ordinal variables (or variables treated as "ordinal"), with given marginal distributions, obtained from discretizing standard normal variables with a specified correlation matrix. The function modifies Barbiero & Ferrari's contord function in GenOrd-package. It uses pmvnorm function from the mvtnorm package to calculate multivariate normal cumulative probabilities defined by the normal quantiles obtained at marginal and the supplied correlation matrix Sigma. This function is used within ord\_norm and would not ordinarily be called by the user.

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## **Usage**

```
norm_ord(marginal = list(), Sigma = NULL, support = list(),
   Spearman = FALSE)
```

# **Arguments**

marginal a list of length equal to the number of variables; the i-th element is a vector of the

cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain r - 1 probabilities (the r-th

is assumed to be 1)

Sigma the correlation matrix of the multivariate standard normal variable

support a list of length equal to the number of variables; the i-th element is a vector of

containing the r ordered support values; if not provided (i.e. support = list()),

the default is for the i-th element to be the vector 1, ..., r

Spearman if TRUE, Spearman's correlations are used (and support is not required); if

FALSE (default) Pearson's correlations are used

# Value

the correlation matrix of the ordinal variables

#### References

Please see references in ord\_norm.

Genz A, Bretz F, Miwa T, Mi X, Leisch F, Scheipl F, Hothorn T (2017). mvtnorm: Multivariate Normal and t Distributions. R package version 1.0-6. http://CRAN.R-project.org/package=mvtnorm

Genz A, Bretz F (2009), Computation of Multivariate Normal and t Probabilities. Lecture Notes in Statistics, Vol. 195., Springer-Verlag, Heidelberg. ISBN 978-3-642-01688-2

# See Also

ord\_norm

ord_norm	Calculate	Intermediate	MVN	Correlation	to	Generate	Variables	
	Treated as	Ordinal						

## **Description**

This function calculates the intermediate MVN correlation needed to generate a variable described by a discrete marginal distribution and associated finite support. This includes ordinal ( $r \geq 2$  categories) variables or variables that are treated as ordinal (i.e. count variables in the Barbiero & Ferrari, 2015 method used in corrvar2, doi: 10.1002/asmb.2072). The function is a modification of Barbiero & Ferrari's ordcont function in GenOrd-package. It works by setting the intermediate MVN correlation equal to the target correlation and updating each intermediate pairwise correlation until the final pairwise correlation is within epsilon of the target correlation or the maximum number of iterations has been reached. This function uses norm\_ord to calculate the ordinal correlation obtained from discretizing the normal variables generated from the intermediate correlation matrix. The ordcont has been modified in the following ways:

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1) the initial correlation check has been removed because this is done within the simulation functions

- 2) the final positive-definite check has been removed
- 3) the intermediate correlation update function was changed to accommodate more situations

This function would not ordinarily be called by the user. Note that this will return a matrix that is NOT positive-definite because this is corrected for in the simulation functions corrvar and corrvar2 using the method of Higham (2002) and the nearPD function.

## Usage

```
ord_norm(marginal = list(), rho = NULL, support = list(),
  epsilon = 0.001, maxit = 1000, Spearman = FALSE)
```

# **Arguments**

_	
marginal	a list of length equal to the number of variables; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take $r$ values, the vector will contain $r$ - 1 probabilities (the $r$ -th is assumed to be 1)
rho	the target correlation matrix
support	a list of length equal to the number of variables; the i-th element is a vector of containing the r ordered support values; if not provided (i.e. support = list()), the default is for the i-th element to be the vector $1,, r$
epsilon	the maximum acceptable error between the final and target pairwise correlations (default = $0.001$ ); smaller values take more time
maxit	the maximum number of iterations to use (default = 1000) to find the intermediate correlation; the correction loop stops when either the iteration number passes maxit or epsilon is reached
Spearman	if TRUE, Spearman's correlations are used (and support is not required); if FALSE (default) Pearson's correlations are used

# Value

A list with the following components:

SigmaC the intermediate MVN correlation matrix

rho0 the calculated final correlation matrix generated from SigmaC

rho the target final correlation matrix

niter a matrix containing the number of iterations required for each variable pair maxerr the maximum final error between the final and target correlation matrices

# References

Barbiero A, Ferrari PA (2015). Simulation of correlated Poisson variables. Applied Stochastic Models in Business and Industry, 31:669-80. doi: 10.1002/asmb.2072.

Barbiero A, Ferrari PA (2015). GenOrd: Simulation of Discrete Random Variables with Given Correlation Matrix and Marginal Distributions. R package version 1.4.0.

https://CRAN.R-project.org/package=GenOrd

Ferrari PA, Barbiero A (2012). Simulating ordinal data, Multivariate Behavioral Research, 47(4):566-589. doi: 10.1080/00273171.2012.692630.

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## See Also

corrvar, corrvar2, norm\_ord, intercorr, intercorr2

# **Description**

This plots the PDF of simulated continuous or count (regular or zero-inflated, Poisson or Negative Binomial) data and overlays the target PDF (if overlay = TRUE), which is specified by distribution name (plus up to 4 parameters) or PDF function fx (plus support bounds). If a continuous target distribution is provided (cont\_var = TRUE), the simulated data y is scaled and then transformed (i.e. y = sigma \* scale(y) + mu) so that it has the same mean (mu) and variance  $(sigma^2)$  as the target distribution. The PDF's of continuous variables are shown as lines (using geom\_density and geom\_line). It works for valid or invalid power method PDF's. The PMF's of count variables are shown as vertical bar graphs (using geom\_col). The function returns a ggplot2-package object so the user can save it or modify it as necessary. The graph parameters (i.e. title, power\_color, target\_color, target\_lty, legend.position, legend.justification, legend.text.size, title.text.size, axis.text.size, and axis.title.size) are inputs to the ggplot2-package functions so information about valid inputs can be obtained from that package's documentation.

# Usage

```
plot_simpdf_theory(sim_y, title = "Simulated Probability Density Function",
   ylower = NULL, yupper = NULL, power_color = "dark blue",
   overlay = TRUE, cont_var = TRUE, target_color = "dark green",
   target_lty = 2, Dist = c("Benini", "Beta", "Beta-Normal",
   "Birnbaum-Saunders", "Chisq", "Dagum", "Exponential", "Exp-Geometric",
   "Exp-Logarithmic", "Exp-Poisson", "F", "Fisk", "Frechet", "Gamma", "Gaussian",
   "Gompertz", "Gumbel", "Kumaraswamy", "Laplace", "Lindley", "Logistic",
   "Loggamma", "Lognormal", "Lomax", "Makeham", "Maxwell", "Nakagami",
   "Paralogistic", "Pareto", "Perks", "Rayleigh", "Rice", "Singh-Maddala",
   "Skewnormal", "t", "Topp-Leone", "Triangular", "Uniform", "Weibull",
   "Poisson", "Negative_Binomial"), params = NULL, fx = NULL, lower = NULL,
   upper = NULL, legend.position = c(0.975, 0.9),
   legend.justification = c(1, 1), legend.text.size = 10,
   title.text.size = 15, axis.text.size = 10, axis.title.size = 13)
```

# **Arguments**

sim_y	a vector of simulated data
title	the title for the graph (default = "Simulated Probability Density Function")
ylower	the lower y value to use in the plot (default = $NULL$ , uses minimum simulated y value) on the x-axis
yupper	the upper y value (default = $NULL$ , uses maximum simulated y value) on the x-axis
power_color	the line color for the simulated variable (or column fill color in the case of Dist = "Poisson" or "Negative Binomial")

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overlay if TRUE (default), the target distribution is also plotted given either a distribution

name (and parameters) or PDF function fx (with bounds = ylower, yupper)

cont\_var TRUE (default) for continuous variables, FALSE for count variables

target\_color the line color for the target PDF (or column fill color in the case of Dist =

"Poisson" or "Negative\_Binomial")

target\_lty the line type for the target PDF (default = 2, dashed line)

Dist name of the distribution. The possible values are: "Benini", "Beta", "Beta-

Normal", "Birnbaum-Saunders", "Chisq", "Exponential", "Exp-Geometric", "Exp-Logarithmic", "Exp-Poisson", "F", "Fisk", "Frechet", "Gamma", "Gaussian", "Gompertz", "Gumbel", "Kumaraswamy", "Laplace", "Lindley", "Logistic", "Loggamma", "Lognormal", "Lomax", "Makeham", "Maxwagami", "Nakagami", "Data in the "Bara", "Bara", "Bara", "Grandelle, "Recommendation of the "Bara", "Maxwagami", "Recommendation of the "Bara", "Recommendation of the "Bara", "Bara", "Recommendation of the "Bara", "Bara", "Bara", "Recommendation of the "Bara", "B

"Paralogistic", "Pareto", "Perks", "Rayleigh", "Rice", "Singh-Maddala", "Skewnormal", "t", "Topp-Leone", "Triangular", "Uniform", "Weibull", "Pois-

son", and "Negative\_Binomial". Please refer to the documentation for each package (either stats-package, VGAM-package, or triangle) for information

on appropriate parameter inputs.

params a vector of parameters (up to 4) for the desired distribution (keep NULL if fx

supplied instead); for Poisson variables, must be lambda (mean) and the probability of a structural zero (use 0 for regular Poisson variables); for Negative Binomial variables, must be size, mean and the probability of a structural zero

(use 0 for regular NB variables)

fx a PDF input as a function of x only, i.e.  $fx = function(x) 0.5 * (x - 1)^2$ ;

must return a scalar (keep NULL if Dist supplied instead)

lower the lower support bound for fx upper the upper support bound for fx

legend.position

the position of the legend

legend.justification

the justification of the legend

legend.text.size

the size of the legend labels

title.text.size

the size of the plot title

axis.text.size the size of the axes text (tick labels)

axis.title.size

the size of the axes titles

## Value

A ggplot2-package object.

## References

Please see the references for plot\_simtheory.

Wickham H. ggplot2: Elegant Graphics for Data Analysis. Springer-Verlag New York, 2009.

# See Also

```
calc_theory, ggplot, geom_line, geom_density, geom_col
```

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## **Examples**

```
## Not run:
# Mixture of Beta(6, 3), Beta(4, 1.5), and Beta(10, 20)
Stcum1 <- calc_theory("Beta", c(6, 3))</pre>
Stcum2 <- calc_theory("Beta", c(4, 1.5))</pre>
Stcum3 <- calc_theory("Beta", c(10, 20))</pre>
mix_pis <- c(0.5, 0.2, 0.3)
mix_mus <- c(Stcum1[1], Stcum2[1], Stcum3[1])</pre>
mix_sigmas <- c(Stcum1[2], Stcum2[2], Stcum3[2])</pre>
mix_skews <- c(Stcum1[3], Stcum2[3], Stcum3[3])</pre>
mix_skurts <- c(Stcum1[4], Stcum2[4], Stcum3[4])</pre>
mix_fifths <- c(Stcum1[5], Stcum2[5], Stcum3[5])</pre>
mix_sixths <- c(Stcum1[6], Stcum2[6], Stcum3[6])</pre>
mix_Six \leftarrow list(seq(0.01, 10, 0.01), c(0.01, 0.02, 0.03),
  seq(0.01, 10, 0.01))
Bstcum <- calc_mixmoments(mix_pis, mix_mus, mix_sigmas, mix_skews,</pre>
 mix_skurts, mix_fifths, mix_sixths)
Bmix <- contmixvar1(n = 10000, "Polynomial", Bstcum[1], Bstcum[2]^2,</pre>
  mix_pis, mix_mus, mix_sigmas, mix_skews, mix_skurts, mix_fifths,
  mix_sixths, mix_Six)
plot_simpdf_theory(Bmix$Y_mix[, 1], title = "Mixture of Beta Distributions",
  fx = function(x) mix_pis[1] * dbeta(x, 6, 3) + mix_pis[2] *
    dbeta(x, 4, 1.5) + mix_pis[3] * dbeta(x, 10, 20), lower = 0, upper = 1)
## End(Not run)
```

plot\_simtheory

Plot Simulated Data and Target Distribution Data by Name or Function for Continuous or Count Variables

# Description

This plots simulated continuous or count (regular or zero-inflated, Poisson or Negative Binomial) data and overlays data (if overlay = TRUE) generated from the target distribution. The target is specified by name (plus up to 4 parameters) or PDF function fx (plus support bounds). Due to the integration involved in finding the CDF from the PDF supplied by fx, only continuous fx may be supplied. Both are plotted as histograms (using geom\_histogram). If a continuous target distribution is specified (cont\_var = TRUE), the simulated data y is scaled and then transformed (i.e. y = sigma\*scale(y) + mu) so that it has the same mean (mu) and variance  $(sigma^2)$  as the target distribution. It works for valid or invalid power method PDF's. It returns a ggplot2-package object so the user can save it or modify it as necessary. The graph parameters (i.e. title, power\_color, target\_color, target\_lty, legend.position, legend.justification, legend.text.size, title.text.size, axis.text.size, and axis.title.size) are inputs to the ggplot2-package functions so information about valid inputs can be obtained from that package's documentation.

# Usage

```
plot_simtheory(sim_y, title = "Simulated Data Values", ylower = NULL,
  yupper = NULL, power_color = "dark blue", overlay = TRUE,
  cont_var = TRUE, target_color = "dark green", nbins = 100,
  Dist = c("Benini", "Beta", "Beta-Normal", "Birnbaum-Saunders", "Chisq",
```

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```
"Dagum", "Exponential", "Exp-Geometric", "Exp-Logarithmic", "Exp-Poisson", "F", "Fisk", "Frechet", "Gamma", "Gaussian", "Gompertz", "Gumbel", "Kumaraswamy", "Laplace", "Lindley", "Logistic", "Loggamma", "Lognormal", "Lomax", "Makeham", "Maxwell", "Nakagami", "Paralogistic", "Pareto", "Perks", "Rayleigh", "Rice", "Singh-Maddala", "Skewnormal", "t", "Topp-Leone", "Triangular", "Uniform", "Weibull", "Poisson", "Negative_Binomial"), params = NULL, fx = NULL, lower = NULL, upper = NULL, seed = 1234, sub = 1000, legend.position = c(0.975, 0.9), legend.justification = c(1, 1), legend.text.size = 10, title.text.size = 15, axis.text.size = 10, axis.title.size = 13)
```

# Arguments

Dist

sim\_y a vector of simulated data

title the title for the graph (default = "Simulated Data Values")

ylower the lower y value to use in the plot (default = NULL, uses minimum simulated

y value) on the y-axis

yupper the upper y value (default = NULL, uses maximum simulated y value) on the

y-axis

power\_color the histogram fill color for the simulated variable (default = "dark blue")

overlay if TRUE (default), the target distribution is also plotted given either a distribution

name (and parameters) or PDF function fx (with support bounds = lower, upper)

cont\_var TRUE (default) for continuous variables, FALSE for count variables

target\_color the histogram fill color for the target distribution (default = "dark green")

nbins the number of bins to use when creating the histograms (default = 100)

name of the distribution. The possible values are: "Benini", "Beta", "Beta-Normal", "Birnbaum-Saunders", "Chisq", "Exponential", "Exp-Geometric", "Exp-Logarithmic", "Exp-Poisson", "F", "Fisk", "Frechet", "Gamma", "Gaussian", "Gompertz", "Gumbel", "Kumaraswamy", "Laplace", "Lindley", "Logistic", "Loggamma", "Lognormal", "Lomax", "Makeham", "Maxwell", "Nakagami", "Paralogistic", "Pareto", "Perks", "Rayleigh", "Rice", "Singh-Maddala",

"Skewnormal", "t", "Topp-Leone", "Triangular", "Uniform", "Weibull", "Poisson", and "Negative\_Binomial". Please refer to the documentation for each package (either stats-package, VGAM-package, or triangle) for information

on appropriate parameter inputs.

params a vector of parameters (up to 4) for the desired distribution (keep NULL if fx

supplied instead); for Poisson variables, must be lambda (mean) and the probability of a structural zero (use 0 for regular Poisson variables); for Negative Binomial variables, must be size, mean and the probability of a structural zero

(use 0 for regular NB variables)

fx a PDF input as a function of x only, i.e.  $fx = function(x) \ 0.5 * (x - 1)^2$ ;

must return a scalar (keep NULL if Dist supplied instead)

lower the lower support bound for a supplied fx, else keep NULL (note: if an error is

thrown from uniroot, try a slightly higher lower bound; i.e., 0.0001 instead of

0)

upper the upper support bound for a supplied fx, else keep NULL (note: if an error is

thrown from uniroot, try a lower upper bound; i.e., 100000 instead of Inf)

seed the seed value for random number generation (default = 1234)

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```
the number of subdivisions to use in the integration to calculate the CDF from fx; if no result, try increasing sub (requires longer computation time; default = 1000)

legend.position
the position of the legend

legend.justification
the justification of the legend

legend.text.size
the size of the legend labels

title.text.size
the size of the plot title

axis.text.size the size of the axes text (tick labels)

axis.title.size
the size of the axes titles
```

#### Value

A ggplot2-package object.

## References

Carnell R (2017). triangle: Provides the Standard Distribution Functions for the Triangle Distribution. R package version 0.11. https://CRAN.R-project.org/package=triangle.

Headrick TC, Sheng Y, & Hodis FA (2007). Numerical Computing and Graphics for the Power Method Transformation Using Mathematica. Journal of Statistical Software, 19(3):1-17. doi: 10.18637/jss.v019.i03.

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Yee TW (2017). VGAM: Vector Generalized Linear and Additive Models.

https://CRAN.R-project.org/package=VGAM.

# See Also

```
calc_theory, ggplot, geom_histogram
```

# **Examples**

```
## Not run:
# Mixture of Beta(6, 3), Beta(4, 1.5), and Beta(10, 20)
Stcum1 <- calc_theory("Beta", c(6, 3))</pre>
Stcum2 <- calc_theory("Beta", c(4, 1.5))</pre>
Stcum3 <- calc_theory("Beta", c(10, 20))</pre>
mix_pis <- c(0.5, 0.2, 0.3)
mix_mus <- c(Stcum1[1], Stcum2[1], Stcum3[1])</pre>
mix_sigmas <- c(Stcum1[2], Stcum2[2], Stcum3[2])</pre>
mix_skews <- c(Stcum1[3], Stcum2[3], Stcum3[3])</pre>
mix_skurts <- c(Stcum1[4], Stcum2[4], Stcum3[4])</pre>
mix_fifths <- c(Stcum1[5], Stcum2[5], Stcum3[5])</pre>
mix_sixths <- c(Stcum1[6], Stcum2[6], Stcum3[6])</pre>
mix_Six \leftarrow list(seq(0.01, 10, 0.01), c(0.01, 0.02, 0.03),
  seq(0.01, 10, 0.01))
Bstcum <- calc_mixmoments(mix_pis, mix_mus, mix_sigmas, mix_skews,</pre>
  mix_skurts, mix_fifths, mix_sixths)
```

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```
Bmix <- contmixvar1(n = 10000, "Polynomial", Bstcum[1], Bstcum[2]^2,
    mix_pis, mix_mus, mix_sigmas, mix_skews, mix_skurts, mix_fifths,
    mix_sixths, mix_Six)
plot_simtheory(Bmix$Y_mix[, 1], title = "Mixture of Beta Distributions",
    fx = function(x) mix_pis[1] * dbeta(x, 6, 3) + mix_pis[2] *
        dbeta(x, 4, 1.5) + mix_pis[3] * dbeta(x, 10, 20), lower = 0, upper = 1)
## End(Not run)</pre>
```

rho\_M1M2

Approximate Correlation between Two Continuous Mixture Variables M1 and M2

# **Description**

This function approximates the expected correlation between two continous mixture variables M1 and M2 based on their mixing proportions, component means, component standard deviations, and correlations between components across variables. The equations can be found in the **Expected Cumulants and Correlations for Continuous Mixture Variables** vignette. This function can be used to see what combination of component correlations gives a desired correlation between M1 and M2.

# Usage

```
rho_M1M2(mix_pis = list(), mix_mus = list(), mix_sigmas = list(),
p_M1M2 = NULL)
```

# Arguments

mix_pis	a list of length 2 with 1st component a vector of mixing probabilities that sum to 1 for component distributions of $M1$ and likewise for 2nd component and $M2$
mix_mus	a list of length 2 with 1st component a vector of means for component distributions of $M1$ and likewise for 2nd component and $M2$
mix_sigmas	a list of length 2 with 1st component a vector of standard deviations for component distributions of $M1$ and likewise for 2nd component and $M2$
p_M1M2	a matrix of correlations with rows corresponding to $M1$ and columns corresponding to $M2$ ; i.e., p_M1M2[1, 2] is the correlation between the 1st component of $M1$ and the 2nd component of $M2$

# Value

the expected correlation between M1 and M2

# References

Davenport JW, Bezder JC, & Hathaway RJ (1988). Parameter Estimation for Finite Mixture Distributions. Computers & Mathematics with Applications, 15(10):819-28.

Pearson RK (2011). Exploring Data in Engineering, the Sciences, and Medicine. In. New York: Oxford University Press.

Schork NJ, Allison DB, & Thiel B (1996). Mixture Distributions in Human Genetics Research. Statistical Methods in Medical Research, 5:155-178. doi: 10.1177/096228029600500204.

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## See Also

rho\_M1Y

# **Examples**

```
# M1 is mixture of N(-2, 1) and N(2, 1);
# M2 is mixture of Logistic(0, 1), Chisq(4), and Beta(4, 1.5)
# pairwise correlation between components across M1 and M2 set to 0.35
L <- calc_theory("Logistic", c(0, 1))
C <- calc_theory("Chisq", 4)
B <- calc_theory("Beta", c(4, 1.5))
mix_pis <- list(c(0.4, 0.6), c(0.3, 0.2, 0.5))
mix_mus <- list(c(-2, 2), c(L[1], C[1], B[1]))
mix_sigmas <- list(c(1, 1), c(L[2], C[2], B[2]))
p_M11M21 <- p_M11M22 <- p_M11M23 <- 0.35
p_M12M21 <- p_M12M22 <- p_M12M23 <- 0.35
p_M1M2 <- matrix(c(p_M11M21, p_M11M22, p_M11M23, p_M12M21, p_M12M22, p_M12M23), 2, 3, byrow = TRUE)
rhoM1M2 <- rho_M1M2(mix_pis, mix_mus, mix_sigmas, p_M1M2)
rhoM1M2</pre>
```

rho\_M1Y

Approximate Correlation between Continuous Mixture Variable M1 and Random Variable Y

# Description

This function approximates the expected correlation between a continous mixture variables M1 and another random variable Y based on the mixing proportions, component means, and component standard deviations of M1 and correlations between components of M1 and Y. The equations can be found in the **Expected Cumulants and Correlations for Continuous Mixture Variables** vignette. This function can be used to see what combination of correlations between components of M1 and Y gives a desired correlation between M1 and Y.

## **Usage**

```
rho_M1Y(mix_pis = NULL, mix_mus = NULL, mix_sigmas = NULL, p_M1Y = NULL)
```

# **Arguments**

mix_pis	a vector of mixing probabilities that sum to 1 for component distributions of $M1$
mix_mus	a vector of means for component distributions of $M1$
mix_sigmas	a vector of standard deviations for component distributions of $M1$
p_M1Y	a vector of correlations between the components of $M1$ and $Y$ ; i.e., p_M1Y[1] is the correlation between the 1st component of $M1$ and $Y$

# Value

the expected correlation between M1 and Y

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#### References

Please see references for rho\_M1M2.

## See Also

rho\_M1Y

# **Examples**

```
# M1 is mixture of N(-2, 1) and N(2, 1); C1 is a continuous non-mixture
# variable (but could also be an ordinal or count variable)
# pairwise correlation between components of M1 and C1 set to 0.35
L <- calc_theory("Logistic", c(0, 1))
C <- calc_theory("Chisq", 4)
B <- calc_theory("Beta", c(4, 1.5))
mix_pis <- list(c(0.4, 0.6), c(0.3, 0.2, 0.5))
mix_mus <- list(c(-2, 2), c(L[1], C[1], B[1]))
mix_sigmas <- list(c(1, 1), c(L[2], C[2], B[2]))
p_M11C1 <- p_M12C1 <- 0.35
p_M1C1 <- c(p_M11C1, p_M12C1)
rho_M1C1 <- rho_M1Y(mix_pis[[1]], mix_mus[[1]], mix_sigmas[[1]], p_M1C1)
rho_M1C1</pre>
```

SimCorrMix

Simulation of Correlated Data of Multiple Variable Types including Continuous and Count Mixture Distributions

# Description

SimCorrMix generates continuous (normal, non-normal, or mixture distributions), binary, ordinal, and count (Poisson or Negative Binomial, regular or zero-inflated) variables with a specified correlation matrix, or one continuous variable with a mixture distribution. This package can be used to simulate data sets that mimic real-world clinical or genetic data sets (i.e. plasmodes, as in Vaughan et al., 2009, doi: 10.1016/j.csda.2008.02.032). The methods extend those found in the SimMultiCorrData package. Standard normal variables with an imposed intermediate correlation matrix are transformed to generate the desired distributions. Continuous variables are simulated using either Fleishman's third-order (doi: 10.1007/BF02293811) or Headrick's fifth-order (doi: 10.1016/S01679473(02)000725) power method transformation (PMT). Non-mixture distributions require the user to specify mean, variance, skewness, standardized kurtosis, and standardized fifth and sixth cumulants. Mixture distributions require these inputs for the component distributions plus the mixing probabilities. Simulation occurs at the component-level for continuous mixture distributions. The target correlation matrix is specified in terms of correlations with components of continuous mixture variables. These components are transformed into the desired mixture variables using random multinomial variables based on the mixing probabilities. However, the package provides functions to approximate expected correlations with continuous mixture variables given target correlations with the components. Binary and ordinal variables are simulated using a modification of GenOrd-package's ordsample function. Count variables are simulated using the inverse CDF method. There are two simulation pathways which calculate intermediate correlations involving count variables differently. Correlation Method 1 adapts Yahav and Shmueli's 2012 method (doi: 10.1002/asmb.901). Correlation Method 2 adapts Barbiero and Ferrari's 2015 modification of GenOrd-package (doi: 10.1002/asmb.2072). The optional error loop may be used to improve the 58 SimCorrMix

accuracy of the final correlation matrix. The package also provides functions to calculate the standardized cumulants of continuous mixture distributions, check parameter inputs, calculate feasible correlation boundaries, and summarize and plot simulated variables.

# **Vignettes**

There are several vignettes which accompany this package to help the user understand the simulation and analysis methods.

- 1) **Calculation of Correlation Boundaries** explains how the feasible correlation boundaries are calculated for each of the two simulation pathways.
- 2) **Comparison of Correlation Methods 1 and 2** describes the two simulation pathways that can be followed for generation of correlated data.
- 3) **Continuous Mixture Distributions** demonstrates how to simulate one continuous mixture variable using contmixvar1 and gives a step-by-step guideline for comparing a simulated distribution to the target distribution.
- 4) **Error Loop Algorithm** details the algorithm involved in the optional error loop that helps to minimize correlation errors.
- 5) **Expected Cumulants and Correlations for Continuous Mixture Variables** derives the equations used by the function calc\_mixmoments to find the mean, standard deviation, skew, standardized kurtosis, and standardized fifth and sixth cumulants for a continuous mixture variable. The vignette also explains how the functions rho\_M1M2 and rho\_M1Y approximate the expected correlations with continuous mixture variables based on the target correlations with the components.
- 6) **Overall Workflow for Generation of Correlated Data** gives a step-by-step guideline to follow with an example containing continuous non-mixture and mixture, ordinal, zero-inflated Poisson, and zero-inflated Negative Binomial variables. It executes both correlated data simulation functions with and without the error loop.
- 7) Variable Types describes the different types of variables that can be simulated in SimCorrMix.

# **Functions**

ord\_norm, norm\_ord

```
This package contains 3 simulation functions:

contmixvar1, corrvar, and corrvar2

4 data description (summary) function:

calc_mixmoments, summary_var, rho_M1M2, rho_M1Y

2 graphing functions:

plot_simpdf_theory, plot_simtheory

3 support functions:

validpar, validcorr, validcorr2

and 16 auxiliary functions (should not normally be called by the user, but are called by other functions):

corr_error, intercorr, intercorr2, intercorr_cat_nb, intercorr_cat_pois, intercorr_cont_nb, intercorr_cont_pois, intercorr_cont_pois, intercorr_cont_pois, intercorr_cont_pois, intercorr_cont_pois, intercorr_cont_nb, maxcount_support,
```

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Zhang X, Mallick H, & Yi N (2016). Zero-Inflated Negative Binomial Regression for Differential Abundance Testing in Microbiome Studies. Journal of Bioinformatics and Genomics 2(2):1-9. doi: 10.18454/jbg.2016.2.2.1.

# See Also

Useful link: https://github.com/AFialkowski/SimMultiCorrData, https://github.com/AFialkowski/ SimCorrMix

summary_var	Summary of Simulated Variables	
-------------	--------------------------------	--

# **Description**

This function summarizes the results of contmixvar1, corrvar, or corrvar2. The inputs are either the simulated variables or inputs for those functions. See their documentation for more information. If summarizing result from contmixvar1, mixture parameters may be entered as vectors instead of lists.

# Usage

```
summary_var(Y_cat = NULL, Y_cont = NULL, Y_comp = NULL, Y_mix = NULL,
Y_pois = NULL, Y_nb = NULL, means = NULL, vars = NULL, skews = NULL,
skurts = NULL, fifths = NULL, sixths = NULL, mix_pis = list(),
mix_mus = list(), mix_sigmas = list(), mix_skews = list(),
mix_skurts = list(), mix_fifths = list(), mix_sixths = list(),
marginal = list(), lam = NULL, p_zip = 0, size = NULL, prob = NULL,
mu = NULL, p_zinb = 0, rho = NULL)
```

# **Arguments**

Y_cat	a matrix of ordinal variables
Y_cont	a matrix of continuous non-mixture variables
Y_comp	a matrix of components of continuous mixture variables
Y_mix	a matrix of continuous mixture variables
Y_pois	a matrix of Poisson variables
Y_nb	a matrix of Negative Binomial variables
means	a vector of means for the k_cont non-mixture and k_mix mixture continuous variables (i.e. $rep(0, (k_cont + k_mix))$ )
vars	a vector of variances for the $k\_cont$ non-mixture and $k\_mix$ mixture continuous variables (i.e. $rep(1, (k\_cont + k\_mix))$ )
skews	a vector of skewness values for the k_cont non-mixture continuous variables
skurts	a vector of standardized kurtoses (kurtosis - 3, so that normal variables have a value of 0) for the $k\_cont$ non-mixture continuous variables
fifths	a vector of standardized fifth cumulants for the k_cont non-mixture continuous variables (not necessary for method = "Fleishman")
sixths	a vector of standardized sixth cumulants for the k_cont non-mixture continuous variables (not necessary for method = "Fleishman")
mix_pis	a list of length k_mix with i-th component a vector of mixing probabilities that sum to 1 for component distributions of $Y_{mix_i}$
mix_mus	a list of length k_mix with i-th component a vector of means for component distributions of $Y_{mix_i}$
mix_sigmas	a list of length k_mix with i-th component a vector of standard deviations for component distributions of $Y_{mix_i}$
mix_skews	a list of length k_mix with i-th component a vector of skew values for component distributions of $Y_{mix_i}$

mix\_skurts a list of length k\_mix with i-th component a vector of standardized kurtoses for component distributions of  $Y_{mix_i}$ mix\_fifths a list of length k\_mix with i-th component a vector of standardized fifth cumulants for component distributions of  $Y_{mix_i}$  (not necessary for method = "Fleishman") mix\_sixths a list of length k\_mix with i-th component a vector of standardized sixth cumulants for component distributions of  $Y_{mix_i}$  (not necessary for method = "Fleisha list of length equal to k\_cat; the i-th element is a vector of the cumulative marginal probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain r - 1 probabilities (the r-th is assumed to be 1); for binary variables, these should be input the same as for ordinal variables with more than 2 categories (i.e. the user-specified probability is the probability of the 1st category, which has the smaller support value) a vector of lambda (mean > 0) constants for the Poisson variables (see dpois); lam the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson variables a vector of probabilities of structural zeros (not including zeros from the Poisson p\_zip distribution) for the zero-inflated Poisson variables (see dzipois); if p\_zip = 0,  $Y_{pois}$  has a regular Poisson distribution; if p\_zip is in (0, 1),  $Y_{pois}$  has a zeroinflated Poisson distribution; if  $p_{zip}$  is in  $(-(exp(lam) - 1)^{-1}, 0)$ ,  $Y_{pois}$  has a zero-deflated Poisson distribution and p\_zip is not a probability; if p\_zip = -(exp(lam) - 1)^(-1),  $Y_{pois}$  has a positive-Poisson distribution (see dpospois); if length(p\_zip) < length(lam), the missing values are set to 0 (and ordered 1st) a vector of size parameters for the Negative Binomial variables (see dnbinom); size the order should be 1st regular NB variables, 2nd zero-inflated NB variables a vector of success probability parameters for the NB variables; order the same prob as in size a vector of mean parameters for the NB variables (\*Note: either prob or mu mu should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean of the NB distribution (see dzinegbin) p\_zinb a vector of probabilities of structural zeros (not including zeros from the NB distribution) for the zero-inflated NB variables (see dzinegbin); if p\_zinb = 0,  $Y_{nb}$ has a regular NB distribution; if p\_zinb is in (-prob^size/(1 - prob^size), 0),  $Y_{nb}$  has a zero-deflated NB distribution and p\_zinb is not a probability; if  $p_zinb = -prob^size/(1 - prob^size), Y_{nb}$  has a positive-NB distribution (see dposnegbin); if length(p\_zinb) < length(size), the missing values are set to 0 (and ordered 1st) the target correlation matrix which must be ordered 1st ordinal, 2nd continurho ous non-mixture, 3rd components of continuous mixtures, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, 7th zero-inflated NB; note that rho is specified in terms of the components of  $Y_{mix}$ 

# Value

A list whose components vary based on the type of simulated variables.

If **ordinal variables** are produced:

ord\_sum a list, where the i-th element contains a data.frame with target and simulated cumulative probabilities for ordinal variable Y\_i

# If **continuous variables** are produced:

```
cont_sum a data.frame summarizing Y_cont and Y_comp,
```

target\_sum a data.frame with the target distributions for Y\_cont and Y\_comp,

mix\_sum a data.frame summarizing Y\_mix,

target\_mix a data.frame with the target distributions for Y\_mix,

# If **Poisson variables** are produced:

pois\_sum a data.frame summarizing Y\_pois

# If Negative Binomial variables are produced:

nb\_sum a data.frame summarizing Y\_nb

Additionally, the following elements:

rho\_calc the final correlation matrix for Y\_cat, Y\_cont, Y\_comp, Y\_pois, and Y\_nb rho\_mix the final correlation matrix for Y\_cat, Y\_cont, Y\_mix, Y\_pois, and Y\_nb maxerr the maximum final correlation error of rho\_calc from the target rho.

#### References

See references for SimCorrMix.

## See Also

```
contmixvar1, corrvar, corrvar2
```

## **Examples**

```
## Not run:
# 2 continuous mixture, 1 binary, 1 zero-inflated Poisson, and
# 1 zero-inflated NB variable
n <- 10000
seed <- 1234
# Mixture variables: Normal mixture with 2 components;
# mixture of Logistic(0, 1), Chisq(4), Beta(4, 1.5)
# Find cumulants of components of 2nd mixture variable
L <- calc_theory("Logistic", c(0, 1))
C <- calc_theory("Chisq", 4)</pre>
B <- calc_theory("Beta", c(4, 1.5))</pre>
skews <- skurts <- fifths <- sixths <- NULL
Six <- list()
mix_pis \leftarrow list(c(0.4, 0.6), c(0.3, 0.2, 0.5))
mix_mus \leftarrow list(c(-2, 2), c(L[1], C[1], B[1]))
mix_sigmas \leftarrow list(c(1, 1), c(L[2], C[2], B[2]))
mix\_skews \leftarrow list(rep(0, 2), c(L[3], C[3], B[3]))
mix\_skurts \leftarrow list(rep(0, 2), c(L[4], C[4], B[4]))
mix_fifths \leftarrow list(rep(0, 2), c(L[5], C[5], B[5]))
mix_sixths <- list(rep(0, 2), c(L[6], C[6], B[6]))
mix_Six <- list(list(NULL, NULL), list(1.75, NULL, 0.03))</pre>
Nstcum <- calc_mixmoments(mix_pis[[1]], mix_mus[[1]], mix_sigmas[[1]],</pre>
```

```
mix_skews[[1]], mix_skurts[[1]], mix_fifths[[1]], mix_sixths[[1]])
Mstcum <- calc_mixmoments(mix_pis[[2]], mix_mus[[2]], mix_sigmas[[2]],</pre>
  mix_skews[[2]], mix_skurts[[2]], mix_fifths[[2]], mix_sixths[[2]])
means <- c(Nstcum[1], Mstcum[1])</pre>
vars <- c(Nstcum[2]^2, Mstcum[2]^2)</pre>
marginal <- list(0.3)</pre>
support <- list(c(0, 1))
lam <- 0.5
p_zip < -0.1
size <- 2
prob <- 0.75
p_zinb <- 0.2
k_cat <- k_pois <- k_nb <- 1
k_cont <- 0
k_mix <- 2
Rey <- matrix(0.39, 8, 8)
diag(Rey) < -1
rownames(Rey) \leftarrow colnames(Rey) \leftarrow c("01", "M1_1", "M1_2", "M2_1", "M2_2",
  "M2_3", "P1", "NB1")
\# set correlation between components of the same mixture variable to 0
Rey["M1_1", "M1_2"] \leftarrow Rey["M1_2", "M1_1"] \leftarrow 0
Rey["M2_1", "M2_2"] <- Rey["M2_2", "M2_1"] <- Rey["M2_1", "M2_3"] <- 0
Rey["M2_3", "M2_1"] <- Rey["M2_2", "M2_3"] <- Rey["M2_3", "M2_2"] <- 0
# check parameter inputs
validpar(k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal, support,
  lam, p_zip, size, prob, mu = NULL, p_zinb, rho = Rey)
# check to make sure Rey is within the feasible correlation boundaries
validcorr(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal,
  lam, p_zip, size, prob, mu = NULL, p_zinb, Rey, seed)
# simulate without the error loop
Sim1 <- corrvar(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal, support,
  lam, p_zip, size, prob, mu = NULL, p_zinb, Rey, seed, epsilon = 0.01)
Summ1 <- summary_var(Sim1$Y_cat, Y_cont = NULL, Sim1$Y_comp, Sim1$Y_mix,</pre>
  Sim1$Y_pois, Sim1$Y_nb, means, vars, skews, skurts, fifths, sixths,
  mix_pis, mix_mus, mix_sigmas, mix_skews, mix_skurts, mix_fifths,
  mix_sixths, marginal, lam, p_zip, size, prob, mu = NULL, p_zinb, Rey)
Sim1_error <- abs(Rey - Summ1$rho_calc)</pre>
summary(as.numeric(Sim1_error))
## End(Not run)
```

validcorr	Determine Correlation Bounds for Ordinal, Continuous, Poisson,	
	and/or Negative Binomial Variables: Correlation Method 1	

# **Description**

This function calculates the lower and upper correlation bounds for the given distributions and checks if a given target correlation matrix rho is within the bounds. It should be used before simulation with corrvar. However, even if all pairwise correlations fall within the bounds, it is still possible that the desired correlation matrix is not feasible. This is particularly true when ordinal variables ( $r \geq 2$  categories) are generated or negative correlations are desired. Therefore, this function should be used as a general check to eliminate pairwise correlations that are obviously not reproducible. It will help prevent errors when executing the simulation. The *ordering* of the variables in rho must be 1st ordinal, 2nd continuous non-mixture, 3rd components of continuous mixture, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, and 7th zero-inflated NB. Note that it is possible for k\_cat, k\_cont, k\_mix, k\_pois, and/or k\_nb to be 0. The target correlations are specified with respect to the components of the continuous mixture variables. There are no parameter input checks in order to decrease simulation time. All inputs should be checked prior to simulation with validpar.

Please see the **Comparison of Correlation Methods 1 and 2** vignette for the differences between the two correlation methods, and the **Calculation of Correlation Boundaries** vignette for a detailed explanation of how the correlation boundaries are calculated.

# Usage

```
validcorr(n = 10000, k_cat = 0, k_cont = 0, k_mix = 0, k_pois = 0,
    k_nb = 0, method = c("Fleishman", "Polynomial"), means = NULL,
    vars = NULL, skews = NULL, skurts = NULL, fifths = NULL,
    sixths = NULL, Six = list(), mix_pis = list(), mix_mus = list(),
    mix_sigmas = list(), mix_skews = list(), mix_skurts = list(),
    mix_fifths = list(), mix_sixths = list(), mix_Six = list(),
    marginal = list(), lam = NULL, p_zip = 0, size = NULL, prob = NULL,
    mu = NULL, p_zinb = 0, rho = NULL, seed = 1234, use.nearPD = TRUE,
    quiet = FALSE)
```

# **Arguments**

n	the sample size (i.e. the length of each simulated variable; default = 10000)
k_cat	the number of ordinal $(r \ge 2 \text{ categories})$ variables (default = 0)
k_cont	the number of continuous non-mixture variables (default = 0)
k_mix	the number of continuous mixture variables (default = 0)
k_pois	the number of regular Poisson and zero-inflated Poisson variables (default = 0)
k_nb	the number of regular Negative Binomial and zero-inflated Negative Binomial variables (default $= 0$ )
method	the method used to generate the k_cont non-mixture and k_mix mixture continuous variables. "Fleishman" uses Fleishman's third-order polynomial transformation and "Polynomial" uses Headrick's fifth-order transformation.
means	a vector of means for the k_cont non-mixture and k_mix mixture continuous variables (i.e. $rep(0, (k_cont + k_mix))$ )

the sample size (i.e. the length of each simulated variables default = 10000)

vars	a vector of variances for the k_cont non-mixture and k_mix mixture continuous variables (i.e. rep(1, (k_cont + k_mix)))
skews	a vector of skewness values for the k_cont non-mixture continuous variables
skurts	a vector of standardized kurtoses (kurtosis - 3, so that normal variables have a value of 0) for the k_cont non-mixture continuous variables
fifths	a vector of standardized fifth cumulants for the $k\_cont$ non-mixture continuous variables (not necessary for method = "Fleishman")
sixths	a vector of standardized sixth cumulants for the $k\_cont$ non-mixture continuous variables (not necessary for method = "Fleishman")
Six	a list of vectors of sixth cumulant correction values for the k_cont non-mixture continuous variables if no valid PDF constants are found, ex: $Six = list(seq(0.01, 2, 0.01), seq(1, 10, 0.5))$ ; if no correction is desired for variable $Y_{cont_i}$ , set set the i-th list component equal to NULL; if no correction is desired for any of the $Y_{cont}$ keep as $Six = list()$ (not necessary for method = "Fleishman")
mix_pis	a list of length k_mix with i-th component a vector of mixing probabilities that sum to 1 for component distributions of $Y_{mix_i}$
mix_mus	a list of length k_mix with i-th component a vector of means for component distributions of $Y_{mix_i}$
mix_sigmas	a list of length k_mix with i-th component a vector of standard deviations for component distributions of $Y_{mix_i}$
mix_skews	a list of length k_mix with i-th component a vector of skew values for component distributions of $Y_{mix_i}$
mix_skurts	a list of length k_mix with i-th component a vector of standardized kurtoses for component distributions of $Y_{mix_i}$
mix_fifths	a list of length k_mix with i-th component a vector of standardized fifth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")
mix_sixths	a list of length k_mix with i-th component a vector of standardized sixth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")
mix_Six	a list of length k_mix with i-th component a list of vectors of sixth cumulant correction values for component distributions of $Y_{mix_i}$ ; use NULL if no correction is desired for a given component or mixture variable; if no correction is desired for any of the $Y_{mix}$ keep as mix_Six = list() (not necessary for method = "Fleishman")
marginal	a list of length equal to k_cat; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain r - 1 probabilities (the r-th is assumed to be 1); for binary variables, these should be input the same as for ordinal variables with more than 2 categories (i.e. the user-specified probability is the probability of the 1st category, which has the smaller support value)
lam	a vector of lambda (> 0) constants for the Poisson variables (see dpois); the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson variables
p_zip	a vector of probabilities of structural zeros (not including zeros from the Poisson distribution) for the zero-inflated Poisson variables (see <code>dzipois</code> ); if <code>p_zip = 0</code> , $Y_{pois}$ has a regular Poisson distribution; if <code>p_zip</code> is in (0, 1), $Y_{pois}$ has a zero-inflated Poisson distribution; if <code>p_zip</code> is in (-(exp(lam) - 1)^(-1), 0),

 $Y_{pois}$  has a zero-deflated Poisson distribution and p\_zip is not a probability; if p\_zip = -(exp(lam) - 1)^(-1),  $Y_{pois}$  has a positive-Poisson distribution (see dpospois); if length(p\_zip) < length(lam), the missing values are set to 0 (and ordered 1st)

size a vector of size parameters for the Negative Binomial variables (see dnbinom);

the order should be 1st regular NB variables, 2nd zero-inflated NB variables

prob a vector of success probability parameters for the NB variables; order the same

as in size

mu a vector of mean parameters for the NB variables (\*Note: either prob or mu

should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean

of the NB distribution (see dzinegbin)

p\_zinb a vector of probabilities of structural zeros (not including zeros from the NB dis-

tribution) for the zero-inflated NB variables (see dzinegbin); if p\_zinb = 0,  $Y_{nb}$  has a regular NB distribution; if p\_zinb is in (-prob^size/(1 - prob^size), 0),  $Y_{nb}$  has a zero-deflated NB distribution and p\_zinb is not a probability; if p\_zinb = -prob^size/(1 - prob^size),  $Y_{nb}$  has a positive-NB distribution (see dposnegbin); if length(p\_zinb) < length(size), the missing values

are set to 0 (and ordered 1st)

rho the target correlation matrix which must be ordered *1st ordinal*, *2nd continu-*

ous non-mixture, 3rd components of continuous mixtures, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, 7th zero-inflated NB; note that rho is

specified in terms of the components of  $Y_{mix}$ 

seed the seed value for random number generation (default = 1234)

use.nearPD TRUE to convert rho to the nearest positive definite matrix with Matrix::nearPD

if necessary

quiet if FALSE prints messages, if TRUE suppresses message printing

# Value

A list with components:

rho the target correlation matrix, which will differ from the supplied matrix (if provided) if it was converted to the nearest positive-definite matrix

L\_rho the lower correlation bound

U\_rho the upper correlation bound

If continuous variables are desired, additional components are:

constants the calculated constants

sixth\_correction a vector of the sixth cumulant correction values

 $\verb|valid.pdf| a vector with i-th component equal to "TRUE" if variable Y\_i has a valid power method PDF, else "FALSE" \\$ 

If a target correlation matrix rho is provided, each pairwise correlation is checked to see if it is within the lower and upper bounds. If the correlation is outside the bounds, the indices of the variable pair are given.

valid. rho TRUE if all entries of rho are within the bounds, else FALSE

#### **Reasons for Function Errors**

1) The most likely cause for function errors is that no solutions to fleish or poly converged when using find\_constants. If this happens, the function will stop. It may help to first use find\_constants for each continuous variable to determine if a sixth cumulant correction value is needed. If the standardized cumulants are obtained from calc\_theory, the user may need to use rounded values as inputs (i.e. skews = round(skews, 8)). For example, in order to ensure that skew is exactly 0 for symmetric distributions.

2) The kurtosis may be outside the region of possible values. There is an associated lower boundary for kurtosis associated with a given skew (for Fleishman's method) or skew and fifth and sixth cumulants (for Headrick's method). Use calc\_lower\_skurt to determine the boundary for a given set of cumulants.

## References

Please see corrvar for additional references.

Demirtas H & Hedeker D (2011). A practical way for computing approximate lower and upper correlation bounds. American Statistician, 65(2):104-109. doi: 10.1198/tast.2011.10090.

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Frechet M (1951). Sur les tableaux de correlation dont les marges sont donnees. Ann. l'Univ. Lyon SectA, 14:53-77.

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# See Also

find\_constants, corrvar, validpar

# **Examples**

```
## Not run:
# 2 continuous mixture, 1 binary, 1 zero-inflated Poisson, and
# 1 zero-inflated NB variable
n <- 10000
seed <- 1234

# Mixture variables: Normal mixture with 2 components;
# mixture of Logistic(0, 1), Chisq(4), Beta(4, 1.5)
# Find cumulants of components of 2nd mixture variable
L <- calc_theory("Logistic", c(0, 1))
C <- calc_theory("Chisq", 4)
B <- calc_theory("Beta", c(4, 1.5))

skews <- skurts <- fifths <- sixths <- NULL
Six <- list()
mix_pis <- list(c(0.4, 0.6), c(0.3, 0.2, 0.5))
mix_mus <- list(c(-2, 2), c(L[1], C[1], B[1]))</pre>
```

```
mix_sigmas <- list(c(1, 1), c(L[2], C[2], B[2]))
mix_skews \leftarrow list(rep(0, 2), c(L[3], C[3], B[3]))
mix_skurts <- list(rep(0, 2), c(L[4], C[4], B[4]))
mix_fifths <- list(rep(0, 2), c(L[5], C[5], B[5]))
mix_sixths \leftarrow list(rep(0, 2), c(L[6], C[6], B[6]))
mix_Six <- list(list(NULL, NULL), list(1.75, NULL, 0.03))</pre>
Nstcum <- calc_mixmoments(mix_pis[[1]], mix_mus[[1]], mix_sigmas[[1]],</pre>
  mix_skews[[1]], mix_skurts[[1]], mix_fifths[[1]], mix_sixths[[1]])
Mstcum <- calc_mixmoments(mix_pis[[2]], mix_mus[[2]], mix_sigmas[[2]],</pre>
  mix_skews[[2]], mix_skurts[[2]], mix_fifths[[2]], mix_sixths[[2]])
means <- c(Nstcum[1], Mstcum[1])</pre>
vars <- c(Nstcum[2]^2, Mstcum[2]^2)</pre>
marginal <- list(0.3)</pre>
support <- list(c(0, 1))
lam <- 0.5
p_zip < -0.1
size <- 2
prob <- 0.75
p_zinb <- 0.2
k_cat <- k_pois <- k_nb <- 1
k_cont <- 0
k_mix <- 2
Rey <- matrix(0.39, 8, 8)
diag(Rey) <- 1
rownames(Rey) <- colnames(Rey) <- c("01", "M1_1", "M1_2", "M2_1", "M2_2",
  "M2_3", "P1", "NB1")
\# set correlation between components of the same mixture variable to 0
Rey["M1_1", "M1_2"] \leftarrow Rey["M1_2", "M1_1"] \leftarrow 0
Rey["M2_1", "M2_2"] <- Rey["M2_2", "M2_1"] <- Rey["M2_1", "M2_3"] <- 0
Rey["M2_3", "M2_1"] <- Rey["M2_2", "M2_3"] <- Rey["M2_3", "M2_2"] <- 0
# check parameter inputs
validpar(k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal, support,
  lam, p_zip, size, prob, mu = NULL, rho = Rey)
# check to make sure Rey is within the feasible correlation boundaries
validcorr(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal,
  lam, p_zip, size, prob, mu = NULL, p_zinb, Rey, seed)
## End(Not run)
```

validcorr2

Determine Correlation Bounds for Ordinal, Continuous, Poisson, and/or Negative Binomial Variables: Correlation Method 2

# **Description**

This function calculates the lower and upper correlation bounds for the given distributions and checks if a given target correlation matrix rho is within the bounds. It should be used before sim-

ulation with corrvar2. However, even if all pairwise correlations fall within the bounds, it is still possible that the desired correlation matrix is not feasible. This is particularly true when ordinal variables ( $r \geq 2$  categories) are generated or negative correlations are desired. Therefore, this function should be used as a general check to eliminate pairwise correlations that are obviously not reproducible. It will help prevent errors when executing the simulation. The *ordering* of the variables in rho must be 1st ordinal, 2nd continuous non-mixture, 3rd components of continuous mixture, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, and 7th zero-inflated NB. Note that it is possible for k\_cat, k\_cont, k\_mix, k\_pois, and/or k\_nb to be 0. The target correlations are specified with respect to the components of the continuous mixture variables. There are no parameter input checks in order to decrease simulation time. All inputs should be checked prior to simulation with validpar.

Please see the **Comparison of Correlation Methods 1 and 2** vignette for the differences between the two correlation methods, and the **Calculation of Correlation Boundaries** vignette for a detailed explanation of how the correlation boundaries are calculated.

# Usage

```
validcorr2(n = 10000, k_cat = 0, k_cont = 0, k_mix = 0, k_pois = 0,
    k_nb = 0, method = c("Fleishman", "Polynomial"), means = NULL,
    vars = NULL, skews = NULL, skurts = NULL, fifths = NULL,
    sixths = NULL, Six = list(), mix_pis = list(), mix_mus = list(),
    mix_sigmas = list(), mix_skews = list(), mix_skurts = list(),
    mix_fifths = list(), mix_sixths = list(), mix_Six = list(),
    marginal = list(), lam = NULL, p_zip = 0, size = NULL, prob = NULL,
    mu = NULL, p_zinb = 0, pois_eps = 1e-04, nb_eps = 1e-04, rho = NULL,
    seed = 1234, use.nearPD = TRUE, quiet = FALSE)
```

# **Arguments** n

••	the sample size (not the length of each simulated variable, default
k_cat	the number of ordinal ( $r \ge 2$ categories) variables (default = 0)
k_cont	the number of continuous non-mixture variables (default = $0$ )
k_mix	the number of continuous mixture variables (default = 0)
k_pois	the number of regular Poisson and zero-inflated Poisson variables (default = 0)
k_nb	the number of regular Negative Binomial and zero-inflated Negative Binomial variables (default = $0$ )
method	the method used to generate the k_cont non-mixture and k_mix mixture continuous variables. "Fleishman" uses Fleishman's third-order polynomial transformation and "Polynomial" uses Headrick's fifth-order transformation.
means	a vector of means for the k_cont non-mixture and k_mix mixture continuous variables (i.e. rep(0, (k_cont + k_mix)))
vars	a vector of variances for the k_cont non-mixture and k_mix mixture continuous variables (i.e. rep(1, $(k_cont + k_mix))$ )
skews	a vector of skewness values for the k_cont non-mixture continuous variables
skurts	a vector of standardized kurtoses (kurtosis - 3, so that normal variables have a value of $0$ ) for the k_cont non-mixture continuous variables
fifths	a vector of standardized fifth cumulants for the $k\_cont$ non-mixture continuous variables (not necessary for method = "Fleishman")

the sample size (i.e. the length of each simulated variable; default = 10000)

sixths	a vector of standardized sixth cumulants for the k_cont non-mixture continuous variables (not necessary for method = "Fleishman")
Six	a list of vectors of sixth cumulant correction values for the k_cont non-mixture continuous variables if no valid PDF constants are found, ex: Six = list(seq(0.01, 2, 0.01), seq(1, 10, 0.5)); if no correction is desired for variable $Y_{cont_i}$ , set set the i-th list component equal to NULL; if no correction is desired for any of the $Y_{cont}$ keep as Six = list() (not necessary for method = "Fleishman")
mix_pis	a list of length k_mix with i-th component a vector of mixing probabilities that sum to 1 for component distributions of $Y_{mix_i}$
mix_mus	a list of length k_mix with i-th component a vector of means for component distributions of $Y_{mix_i}$
mix_sigmas	a list of length k_mix with i-th component a vector of standard deviations for component distributions of $Y_{mix_i}$
mix_skews	a list of length k_mix with i-th component a vector of skew values for component distributions of $Y_{mix_i}$
mix_skurts	a list of length k_mix with i-th component a vector of standardized kurtoses for component distributions of $Y_{mix_i}$
mix_fifths	a list of length k_mix with i-th component a vector of standardized fifth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")
mix_sixths	a list of length k_mix with i-th component a vector of standardized sixth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")
mix_Six	a list of length k_mix with i-th component a list of vectors of sixth cumulant correction values for component distributions of $Y_{mix_i}$ ; use NULL if no correction is desired for a given component or mixture variable; if no correction is desired for any of the $Y_{mix}$ keep as mix_Six = list() (not necessary for method = "Fleishman")
marginal	a list of length equal to k_cat; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable can take r values, the vector will contain r - 1 probabilities (the r-th is assumed to be 1); for binary variables, these should be input the same as for ordinal variables with more than 2 categories (i.e. the user-specified probability is the probability of the 1st category, which has the smaller support value)
lam	a vector of lambda (> 0) constants for the Poisson variables (see dpois); the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson variables
p_zip	a vector of probabilities of structural zeros (not including zeros from the Poisson distribution) for the zero-inflated Poisson variables (see dzipois); if p_zip = 0, $Y_{pois}$ has a regular Poisson distribution; if p_zip is in (0, 1), $Y_{pois}$ has a zero-inflated Poisson distribution; if p_zip is in (-(exp(lam) - 1)^(-1), 0), $Y_{pois}$ has a zero-deflated Poisson distribution and p_zip is not a probability; if p_zip = -(exp(lam) - 1)^(-1), $Y_{pois}$ has a positive-Poisson distribution (see dpospois); if length(p_zip) < length(lam), the missing values are set to 0 (and ordered 1st)
size	a vector of size parameters for the Negative Binomial variables (see

mu a vector of mean parameters for the NB variables (\*Note: either prob or mu should be supplied for all Negative Binomial variables, not a mixture; default =

NULL); order the same as in size; for zero-inflated NB this refers to the mean

of the NB distribution (see dzinegbin)

p\_zinb a vector of probabilities of structural zeros (not including zeros from the NB dis-

tribution) for the zero-inflated NB variables (see dzinegbin); if p\_zinb = 0,  $Y_{nb}$  has a regular NB distribution; if p\_zinb is in (-prob^size/(1 - prob^size), 0),  $Y_{nb}$  has a zero-deflated NB distribution and p\_zinb is not a probability; if p\_zinb = -prob^size/(1 - prob^size),  $Y_{nb}$  has a positive-NB distribution (see dposnegbin); if length(p\_zinb) < length(size), the missing values

are set to 0 (and ordered 1st)

pois\_eps a vector of length k\_pois containing total cumulative probability truncation val-

ues; if none are provided, the default is 0.0001 for each variable

nb\_eps a vector of length k\_nb containing total cumulative probability truncation val-

ues; if none are provided, the default is 0.0001 for each variable

rho the target correlation matrix which must be ordered *1st ordinal*, *2nd continu*-

ous non-mixture, 3rd components of continuous mixtures, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, 7th zero-inflated NB; note that rho is

specified in terms of the components of  $Y_{mix}$ 

seed the seed value for random number generation (default = 1234)

use.nearPD TRUE to convert rho to the nearest positive definite matrix with Matrix::nearPD

if necessary

quiet if FALSE prints messages, if TRUE suppresses message printing

## Value

A list with components:

rho the target correlation matrix, which will differ from the supplied matrix (if provided) if it was converted to the nearest positive-definite matrix

L\_rho the lower correlation bound

U\_rho the upper correlation bound

If continuous variables are desired, additional components are:

constants the calculated constants

sixth\_correction a vector of the sixth cumulant correction values

 $\verb|valid.pdf| a vector with i-th component equal to "TRUE" if variable Y\_i has a valid power method PDF, else "FALSE" |$ 

If a target correlation matrix rho is provided, each pairwise correlation is checked to see if it is within the lower and upper bounds. If the correlation is outside the bounds, the indices of the variable pair are given.

valid, rho TRUE if all entries of rho are within the bounds, else FALSE

# **Reasons for Function Errors**

1) The most likely cause for function errors is that no solutions to fleish or poly converged when using find\_constants. If this happens, the function will stop. It may help to first use find\_constants for each continuous variable to determine if a sixth cumulant correction value is needed. If the standardized cumulants are obtained from calc\_theory, the user may need to use

rounded values as inputs (i.e. skews = round(skews, 8)). For example, in order to ensure that skew is exactly 0 for symmetric distributions.

2) The kurtosis may be outside the region of possible values. There is an associated lower boundary for kurtosis associated with a given skew (for Fleishman's method) or skew and fifth and sixth cumulants (for Headrick's method). Use calc\_lower\_skurt to determine the boundary for a given set of cumulants.

## References

Please see corryar2 and validcorr for references.

## See Also

```
find_constants, corrvar2, validpar
```

## **Examples**

```
## Not run:
# 2 continuous mixture, 1 binary, 1 zero-inflated Poisson, and
# 1 zero-inflated NB variable
n <- 10000
seed <- 1234
# Mixture variables: Normal mixture with 2 components;
# mixture of Logistic(0, 1), Chisq(4), Beta(4, 1.5)
# Find cumulants of components of 2nd mixture variable
L <- calc_theory("Logistic", c(0, 1))
C <- calc_theory("Chisq", 4)</pre>
B <- calc_theory("Beta", c(4, 1.5))
skews <- skurts <- fifths <- sixths <- NULL
Six <- list()
mix_pis \leftarrow list(c(0.4, 0.6), c(0.3, 0.2, 0.5))
mix_mus \leftarrow list(c(-2, 2), c(L[1], C[1], B[1]))
mix_sigmas \leftarrow list(c(1, 1), c(L[2], C[2], B[2]))
mix_skews \leftarrow list(rep(0, 2), c(L[3], C[3], B[3]))
mix_skurts \leftarrow list(rep(0, 2), c(L[4], C[4], B[4]))
mix_fifths \leftarrow list(rep(0, 2), c(L[5], C[5], B[5]))
mix_sixths \leftarrow list(rep(0, 2), c(L[6], C[6], B[6]))
mix_Six <- list(list(NULL, NULL), list(1.75, NULL, 0.03))</pre>
Nstcum <- calc_mixmoments(mix_pis[[1]], mix_mus[[1]], mix_sigmas[[1]],</pre>
  \label{limit_skews} \begin{subarray}{ll} mix\_skews[[1]], mix\_skews[[1]], mix\_sixths[[1]]) \\ \end{subarray}
Mstcum <- calc_mixmoments(mix_pis[[2]], mix_mus[[2]], mix_sigmas[[2]],</pre>
  mix_skews[[2]], mix_skurts[[2]], mix_fifths[[2]], mix_sixths[[2]])
means <- c(Nstcum[1], Mstcum[1])</pre>
vars <- c(Nstcum[2]^2, Mstcum[2]^2)</pre>
marginal <- list(0.3)</pre>
support <- list(c(0, 1))
lam <- 0.5
p_zip < -0.1
pois_eps <- 0.0001
size <- 2
prob <- 0.75
p_zinb <- 0.2
```

```
nb_eps <- 0.0001
k_cat <- k_pois <- k_nb <- 1
k_cont <- 0
k_mix <- 2
Rey <- matrix(0.39, 8, 8)
diag(Rey) <- 1
rownames(Rey) <- colnames(Rey) <- c("01", "M1_1", "M1_2", "M2_1", "M2_2",
  "M2_3", "P1", "NB1")
\# set correlation between components of the same mixture variable to 0
Rey["M1_1", "M1_2"] <- Rey["M1_2", "M1_1"] <- 0</pre>
Rey["M2_1", "M2_2"] <- Rey["M2_2", "M2_1"] <- Rey["M2_1", "M2_3"] <- 0</pre>
Rey["M2_3", "M2_1"] <- Rey["M2_2", "M2_3"] <- Rey["M2_3", "M2_2"] <- 0
# check parameter inputs
validpar(k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal, support,
  lam, p_zip, size, prob, mu = NULL, p_zinb, pois_eps, nb_eps, Rey)
# check to make sure Rey is within the feasible correlation boundaries
validcorr2(n, k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal,
  lam, p_zip, size, prob, mu = NULL, p_zinb, pois_eps, nb_eps, Rey, seed)
## End(Not run)
```

validpar

Parameter Check for Simulation or Correlation Validation Functions

## **Description**

This function checks the parameter inputs to the simulation functions <code>contmixvar1</code>, <code>corrvar</code>, and <code>corrvar2</code> and to the correlation validation functions <code>validcorr</code> and <code>validcorr2</code>. It should be used prior to execution of these functions to ensure all inputs are of the correct format. Those functions do not contain parameter checks in order to decrease simulation time. This would be important if the user is running several simulation repetitions so that the inputs only have to be checked once. Note that the inputs do not include all of the inputs to the simulation functions. See the appropriate function documentation for more details about parameter inputs.

# **Usage**

```
validpar(k_cat = 0, k_cont = 0, k_mix = 0, k_pois = 0, k_nb = 0,
  method = c("Fleishman", "Polynomial"), means = NULL, vars = NULL,
  skews = NULL, skurts = NULL, fifths = NULL, sixths = NULL,
  Six = list(), mix_pis = list(), mix_mus = list(), mix_sigmas = list(),
  mix_skews = list(), mix_skurts = list(), mix_fifths = list(),
  mix_sixths = list(), mix_Six = list(), marginal = list(),
  support = list(), lam = NULL, p_zip = 0, size = NULL, prob = NULL,
  mu = NULL, p_zinb = 0, pois_eps = 1e-04, nb_eps = 1e-04, rho = NULL,
  Sigma = NULL, cstart = list(), quiet = FALSE)
```

# Arguments

_	,	
	k_cat	the number of ordinal ( $r \ge 2$ categories) variables (default = 0)
	k_cont	the number of continuous non-mixture variables (default = 0)
	k_mix	the number of continuous mixture variables (default = $0$ )
	k_pois	the number of regular Poisson and zero-inflated Poisson variables (default = 0)
	k_nb	the number of regular Negative Binomial and zero-inflated Negative Binomial variables (default = $0$ )
	method	the method used to generate the k_cont non-mixture and k_mix mixture continuous variables. "Fleishman" uses Fleishman's third-order polynomial transformation and "Polynomial" uses Headrick's fifth-order transformation.
	means	a vector of means for the k_cont non-mixture and k_mix mixture continuous variables (i.e. rep(0, (k_cont + k_mix)))
	vars	a vector of variances for the k_cont non-mixture and k_mix mixture continuous variables (i.e. rep(1, (k_cont + k_mix)))
	skews	a vector of skewness values for the $k\_cont$ non-mixture continuous variables
	skurts	a vector of standardized kurtoses (kurtosis - 3, so that normal variables have a value of 0) for the $k\_cont$ non-mixture continuous variables
	fifths	a vector of standardized fifth cumulants for the k_cont non-mixture continuous variables (not necessary for method = "Fleishman")
	sixths	a vector of standardized sixth cumulants for the k_cont non-mixture continuous variables (not necessary for method = "Fleishman")
	Six	a list of vectors of sixth cumulant correction values for the k_cont non-mixture continuous variables if no valid PDF constants are found, ex: $\text{Six} = \text{list}(\text{seq}(0.01, 2, 0.01), \text{seq}(1, 10, 0.5));$ if no correction is desired for variable $Y_{cont_i}$ , set set the i-th list component equal to NULL; if no correction is desired for any of the $Y_{cont}$ keep as $\text{Six} = \text{list}()$ (not necessary for method = "Fleishman")
	mix_pis	a vector if using contmixvar1 or a list of length k_mix with i-th component a vector of mixing probabilities that sum to 1 for component distributions of $Y_{mix_i}$
	mix_mus	a vector if using <code>contmixvar1</code> or a list of length k_mix with i-th component a vector of means for component distributions of $Y_{mix_i}$
	mix_sigmas	a vector if using <code>contmixvar1</code> or a list of length k_mix with i-th component a vector of standard deviations for component distributions of $Y_{mix_i}$
	mix_skews	a vector if using <code>contmixvar1</code> or a list of length k_mix with i-th component a vector of skew values for component distributions of $Y_{mix_i}$
	mix_skurts	a vector if using <code>contmixvar1</code> or a list of length k_mix with i-th component a vector of standardized kurtoses for component distributions of $Y_{mix_i}$
	mix_fifths	a vector if using <code>contmixvar1</code> or a list of length k_mix with i-th component a vector of standardized fifth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")
	mix_sixths	a vector if using <code>contmixvar1</code> or a list of length k_mix with i-th component a vector of standardized sixth cumulants for component distributions of $Y_{mix_i}$ (not necessary for method = "Fleishman")

mix\_Six if using contmixvar1, a list of vectors of sixth cumulant corrections for the components of the continuous mixture variable; else a list of length k\_mix with i-th component a list of vectors of sixth cumulant correction values for component distributions of  $Y_{mix_i}$ ; use NULL if no correction is desired for a given

> component or mixture variable; if no correction is desired for any of the  $Y_{mix}$ keep as mix\_Six = list() (not necessary for method = "Fleishman")

marginal a list of length equal to k\_cat; the i-th element is a vector of the cumulative probabilities defining the marginal distribution of the i-th variable; if the variable

can take r values, the vector will contain r - 1 probabilities (the r-th is assumed to be 1; default = list()); for binary variables, these should be input the same as for ordinal variables with more than 2 categories (i.e. the user-specified probability

is the probability of the 1st category, which has the smaller support value)

a list of length equal to k\_cat; the i-th element is a vector containing the r ordered support values; if not provided (i.e. support = list()), the default is

for the i-th element to be the vector 1, ..., r

a vector of lambda (mean > 0) constants for the Poisson variables (see dpois); the order should be 1st regular Poisson variables, 2nd zero-inflated Poisson vari-

ables

a vector of probabilities of structural zeros (not including zeros from the Poisson distribution) for the zero-inflated Poisson variables (see dzipois); if  $p_zip = 0$ ,  $Y_{pois}$  has a regular Poisson distribution; if p\_zip is in (0, 1),  $Y_{pois}$  has a zeroinflated Poisson distribution; if  $p_{zip}$  is in  $(-(exp(lam) - 1)^{-1}, 0)$ ,  $Y_{pois}$  has a zero-deflated Poisson distribution and p\_zip is not a probability; if

p\_zip = -(exp(lam) - 1)^(-1),  $Y_{pois}$  has a positive-Poisson distribution (see dpospois); if length(p\_zip) < length(lam), the missing values are set

to 0 (and ordered 1st)

a vector of size parameters for the Negative Binomial variables (see dnbinom); the order should be 1st regular NB variables, 2nd zero-inflated NB variables

a vector of success probability parameters for the NB variables; order the same

as in size

a vector of mean parameters for the NB variables (\*Note: either prob or mu should be supplied for all Negative Binomial variables, not a mixture; default = NULL); order the same as in size; for zero-inflated NB this refers to the mean

of the NB distribution (see dzinegbin)

a vector of probabilities of structural zeros (not including zeros from the NB distribution) for the zero-inflated NB variables (see dzinegbin); if p\_zinb = 0,  $Y_{nb}$ has a regular NB distribution; if p\_zinb is in (-prob^size/(1 - prob^size),

0),  $Y_{nb}$  has a zero-deflated NB distribution and p\_zinb is not a probability; if p\_zinb = -prob^size/(1 - prob^size),  $Y_{nb}$  has a positive-NB distribution (see dposnegbin); if length(p\_zinb) < length(size), the missing values

are set to 0 (and ordered 1st)

a vector of length k\_pois containing total cumulative probability truncation values; if none are provided, the default is 0.0001 for each variable

> a vector of length k\_nb containing total cumulative probability truncation values; if none are provided, the default is 0.0001 for each variable

the target correlation matrix which must be ordered 1st ordinal, 2nd continuous non-mixture, 3rd components of continuous mixtures, 4th regular Poisson, 5th zero-inflated Poisson, 6th regular NB, 7th zero-inflated NB; note that rho is

specified in terms of the components of  $Y_{mix}$ 

support

lam

p\_zip

size

prob

mu

p\_zinb

pois\_eps

nb\_eps

rho

Sigma an intermediate correlation matrix to use if the user wants to provide one, else it is calculated within by <code>intercorr</code> a list of length equal to <code>k\_cont+</code> the total number of mixture components containing initial values for root-solving algorithm used in <code>find\_constants</code>. If user specified, each list element must be input as a matrix. For <code>method="Fleishman"</code>, each should have 3 columns for c1, c2, c3; for <code>method="Polynomial"</code>, each should have 5 columns for c1, c2, c3, c4, c5. If no starting values are specified for a given component, that list element should be <code>NULL</code>.

quiet if <code>FALSE</code> prints messages, if <code>TRUE</code> suppresses message printing

# Value

TRUE if all inputs are correct, else it will stop with a correction message

# See Also

contmixvar1, corrvar, corrvar2, validcorr, validcorr2

## **Examples**

```
## Not run:
# 2 continuous mixture, 1 binary, 1 zero-inflated Poisson, and
# 1 zero-inflated NB variable
# Mixture variables: Normal mixture with 2 components;
# mixture of Logistic(0, 1), Chisq(4), Beta(4, 1.5)
# Find cumulants of components of 2nd mixture variable
L <- calc_theory("Logistic", c(0, 1))
C <- calc_theory("Chisq", 4)</pre>
B <- calc_theory("Beta", c(4, 1.5))
skews <- skurts <- fifths <- sixths <- NULL
Six <- list()
mix_pis \leftarrow list(c(0.4, 0.6), c(0.3, 0.2, 0.5))
mix_mus \leftarrow list(c(-2, 2), c(L[1], C[1], B[1]))
mix_sigmas \leftarrow list(c(1, 1), c(L[2], C[2], B[2]))
mix_skews \leftarrow list(rep(0, 2), c(L[3], C[3], B[3]))
mix_skurts \leftarrow list(rep(0, 2), c(L[4], C[4], B[4]))
mix_fifths <- list(rep(0, 2), c(L[5], C[5], B[5]))
mix_sixths \leftarrow list(rep(0, 2), c(L[6], C[6], B[6]))
mix_Six \leftarrow list(list(NULL, NULL), list(1.75, NULL, 0.03))
Nstcum <- calc_mixmoments(mix_pis[[1]], mix_mus[[1]], mix_sigmas[[1]],</pre>
  mix_skews[[1]], mix_skurts[[1]], mix_fifths[[1]], mix_sixths[[1]])
Mstcum <- calc_mixmoments(mix_pis[[2]], mix_mus[[2]], mix_sigmas[[2]],</pre>
  mix_skews[[2]], mix_skurts[[2]], mix_fifths[[2]], mix_sixths[[2]])
means <- c(Nstcum[1], Mstcum[1])</pre>
vars <- c(Nstcum[2]^2, Mstcum[2]^2)</pre>
marginal <- list(0.3)</pre>
support <- list(c(0, 1))
lam <- 0.5
p_{zip} < -0.1
size <- 2
prob <- 0.75
p_zinb <- 0.2
```

```
k_cat <- k_pois <- k_nb <- 1
k_cont <- 0
k_mix <- 2
Rey <- matrix(0.39, 8, 8)
diag(Rey) <- 1
rownames(Rey) <- colnames(Rey) <- c("01", "M1_1", "M1_2", "M2_1", "M2_2",
  "M2_3", "P1", "NB1")
\# set correlation between components of the same mixture variable to 0
\label{eq:rey_model} $$ \text{Rey["M1\_1", "M1\_2"]} < - \text{Rey["M1\_2", "M1\_1"]} < - \emptyset $$
Rey["M2_1", "M2_2"] <- Rey["M2_2", "M2_1"] <- Rey["M2_1", "M2_3"] <- 0
Rey["M2_3", "M2_1"] <- Rey["M2_2", "M2_3"] <- Rey["M2_3", "M2_2"] <- 0
# use before contmixvar1 with 1st mixture variable:
# change mix_pis to not sum to 1
check1 <- validpar(k_mix = 1, method = "Polynomial", means = Nstcum[1],</pre>
  vars = Nstcum[2]^2, mix_pis = C(0.4, 0.5), mix_mus = mix_mus[[1]],
  mix_sigmas = mix_sigmas[[1]], mix_skews = mix_skews[[1]],
 mix_skurts = mix_skurts[[1]], mix_fifths = mix_fifths[[1]],
  mix_sixths = mix_sixths[[1]])
# use before validcorr: should return TRUE
check2 <- validpar(k_cat, k_cont, k_mix, k_pois, k_nb, "Polynomial", means,</pre>
  vars, skews, skurts, fifths, sixths, Six, mix_pis, mix_mus, mix_sigmas,
  mix_skews, mix_skurts, mix_fifths, mix_sixths, mix_Six, marginal, support,
 lam, p_zip, size, prob, mu = NULL, p_zinb, rho = Rey)
## End(Not run)
```

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