



# A Comparison of SLAM Algorithms Based on a Graph of Relations

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Wolfram Burgard, Cyrill Stachniss, Giorgio Grisetti,  
Bastian Steder, Rainer Kümmmerle, Christian Dornhege,  
Michael Ruhnke, Alexander Kleiner, Juan D. Tardos

University of Freiburg, Germany and University of  
Zaragoza, Spain

# RAWSEEDS



- Robotics Advancement through Web-publishing of Sensorial and Elaborated Extensive Data Sets
  - University of Freiburg
  - University of Zaragoza
  - Politecnico di Milano
  - Università degli Studi di Milano-Bicocca
- Funded by the European Commission
- <http://www.rawseeds.org/>



# Motivation

- No gold standard to evaluate and compare the results of SLAM algorithms
- Most approaches depend on the used sensor setup, landmark detector/grid map, and estimation algorithm
- Often, ground truth is not available
- Even if there is ground truth information: how can we compare the results of SLAM algorithms?

# Related Work

- **SLAM approaches**  
(Smith&Cheeseman, Lu&Milios, Frese, Duckett et al., Dissanayake et al., Howard et al., Eustice et al., Grisetti et al., Stachniss et al., Olson, ...)
- **Competitions**  
(DGC, RoboCup, NIST, ...)
- **Data repositories**  
(Radish, RAWSEEDS, OpenSLAM, ...)

# Measures

- **Uncertainty**  
(Entropy, KL-Divergence, ...)
- **Variance of estimators**  
(Cramer-Rao-bound, Fisher information, ...)
- **Quality of estimates**  
(observation likelihood, normalized estimation error squared, the measure described in this paper, ...)

# Sum of Squared Distances between Estimated and Ground Truth Poses

- Let  $x_{1:T}$  be the estimated poses from time 1 to  $T$ .
- Let  $x_{1:T}^*$  be the reference poses.
- Let  $\oplus$  be the standard motion composition operator and  $\ominus$  its inverse.
- Resulting error function

$$\varepsilon(x_{1:T}) = \sum_{t=1}^T ||x_t \ominus x_t^*||^2$$

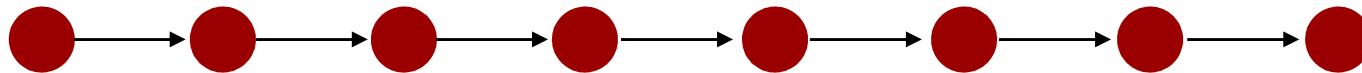
## Normalized Estimation Error Squared: Taking into Account Covariance

$$\varepsilon(k) = [x(k) - x^*(k)]^T P^{-1}(k | k) [x(k) - x^*(k)]$$

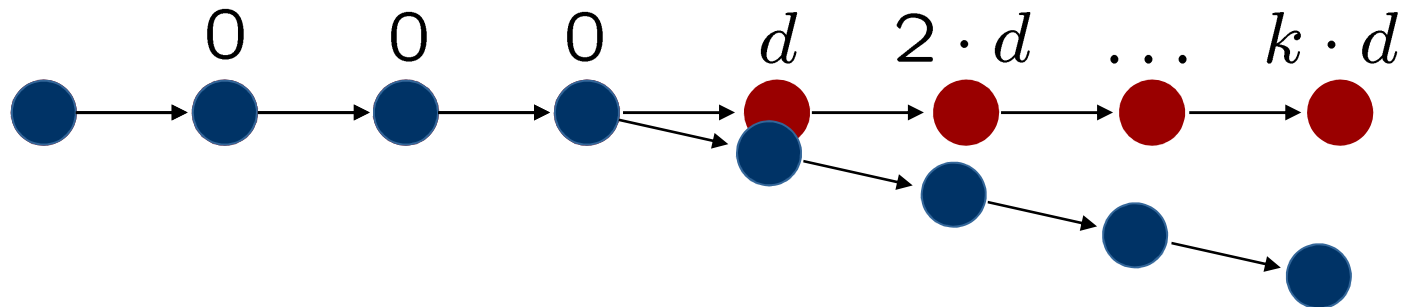
In SLAM, this measures the absolute error of the landmark estimates (weighted by inverse covariance).

# Problems with this Measure

$x_{1:T}^*$ :



$x_{1:T}$ :

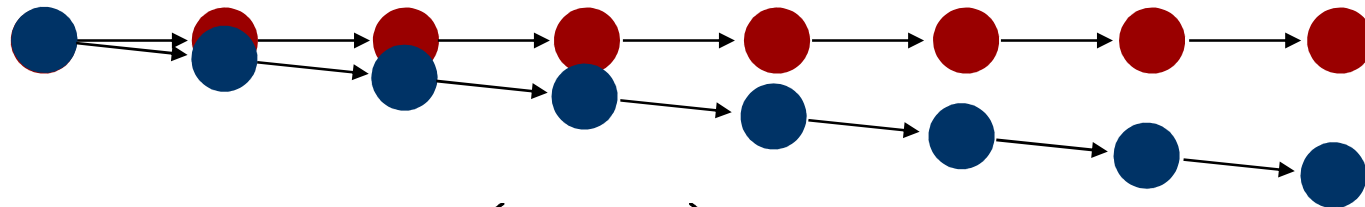


$$\varepsilon(x_{1:T}) = \frac{k \cdot (k + 1)}{2} d$$



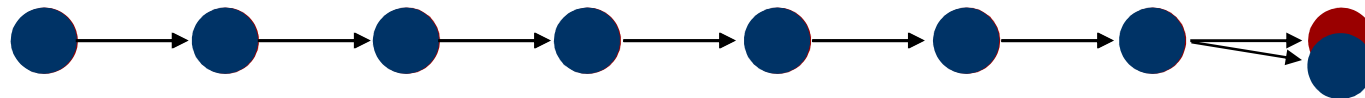
# Problems with this Measure

- Error in the beginning:



$$\varepsilon(x_{1:T}) = \frac{(n-1) \cdot n}{2} d$$

- Error in the very end:



$$\varepsilon(x_{1:T}) = d$$

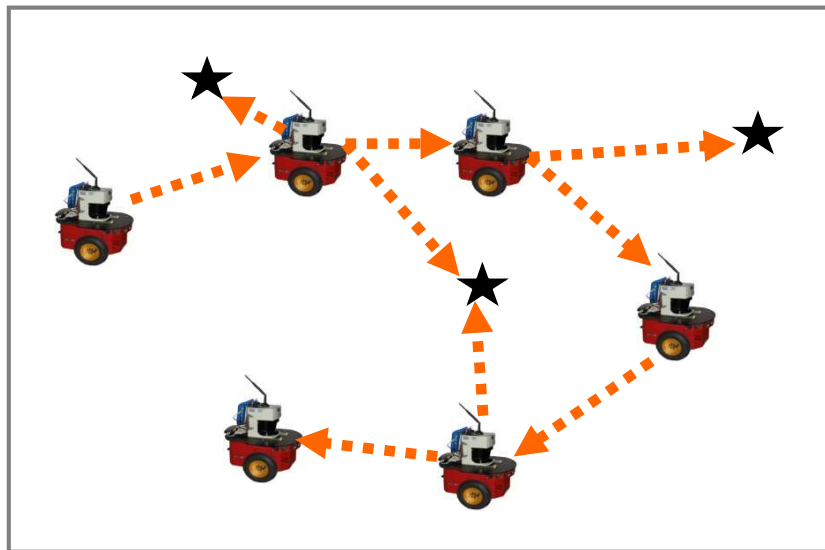
# Desired Measure

- that does not automatically count errors multiple times,
- whose results does not depend on the order data is processed, and
- that is compatible with what is actually optimized in SLAM approaches.

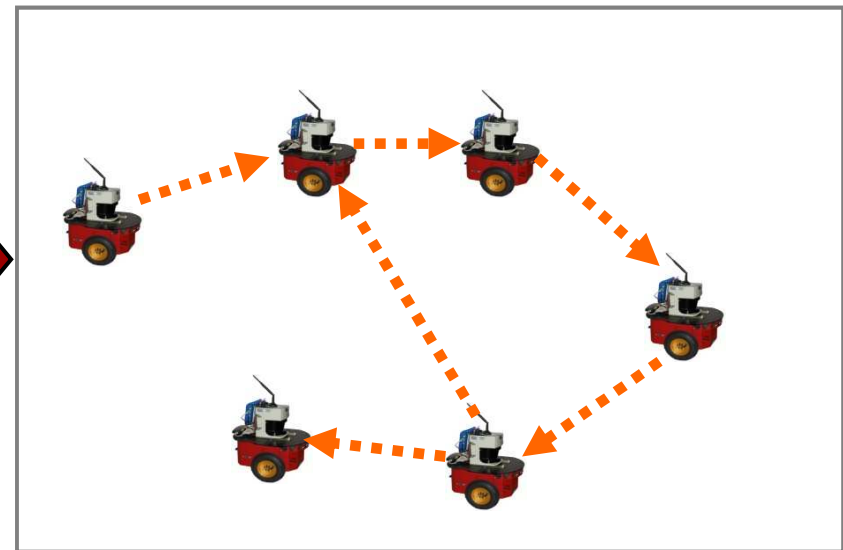
**Inspired by graph-based SLAM:  
consider the energy needed to  
transform the **pose graph** into ground  
truth**

# Pose Graph

- A pose-graph encodes the poses of the robot during mapping as well as constraints resulting from observations
- Independent of the kind of observations
- and the type of the map (landmarks, grids, ...)



Robot poses and observations



Pose graph

# Our Approach

- measures the relative errors between (consecutive) poses

$$\delta_{i,j} \ominus \delta_{i,j}^*$$

- which is exactly what approaches to graph-based SLAM seek to minimize.

# Metric Based on Relative Poses

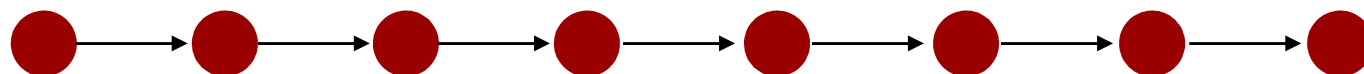
estimated relative movement

ground truth

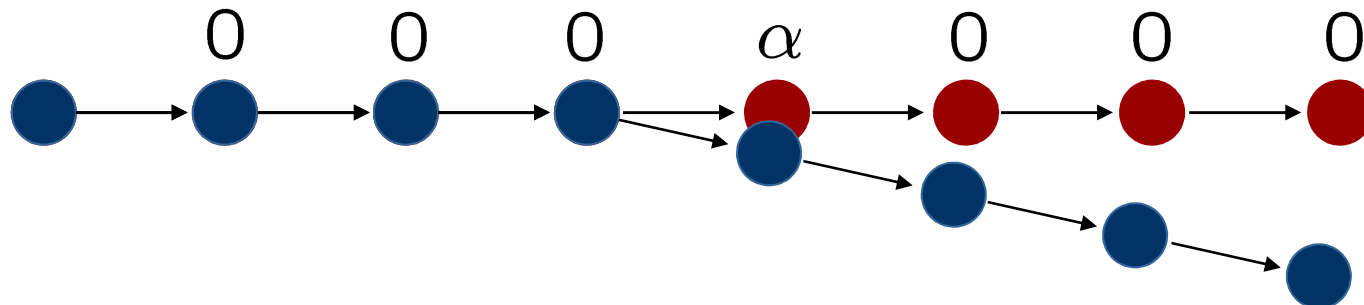
$$\begin{aligned}\varepsilon(\delta) &= \frac{1}{N} \sum_{i,j} \|\delta_{i,j} \ominus \delta_{i,j}^*\|^2 \\ &= \frac{1}{N} \sum_{i,j} \text{trans}(\delta_{i,j} \ominus \delta_{i,j}^*)^2 \\ &\quad + \text{rot}(\delta_{i,j} \ominus \delta_{i,j}^*)^2\end{aligned}$$

# Application to the Example

$x_{1:T}^*$ :



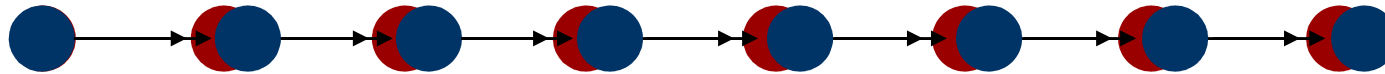
$x_{1:T}$ :



$$\varepsilon(x_{1:T}) = \alpha$$

# Translational Error

- Error in the beginning:



$$\varepsilon(x_{1:T}) = d \quad \text{vs.} \quad \varepsilon(x_{1:T}) = (n - 1) \cdot d$$

- Error in the very end:



$$\varepsilon(x_{1:T}) = d$$

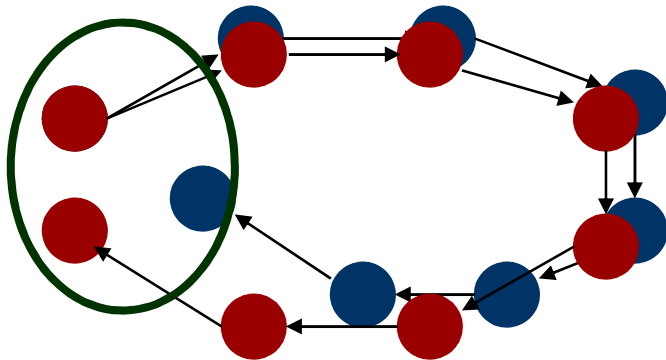
# Properties

- Good for incremental errors
- Thus far, we only considered incremental links
- This can be sufficient for navigation,
- but does not help to close loops and achieve global consistency.



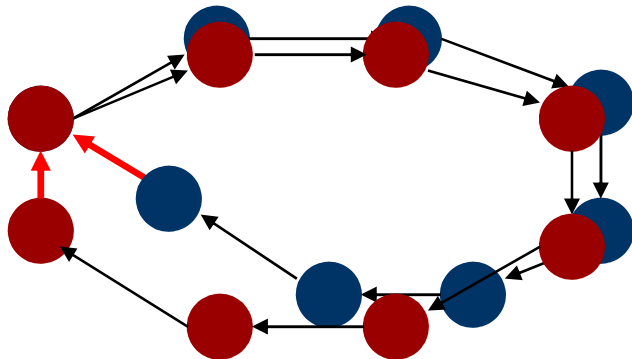
# Addition of Loop Closure Links

In the loop closure area, the map will be locally inconsistent, which this is not reflected by



$$\varepsilon(x_{1:T}) = \sum_{t=1}^T \varepsilon_t$$

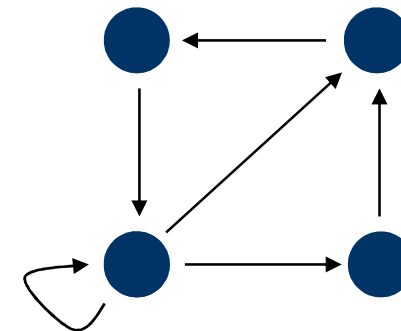
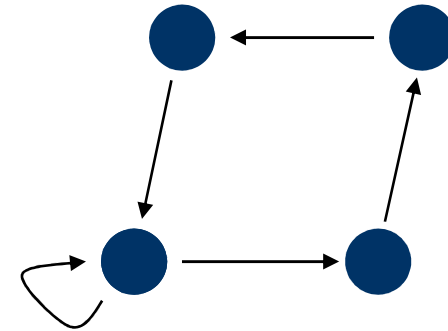
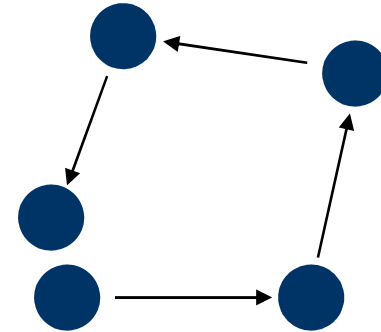
If we introduce an appropriate link, we increase the error, representing the faulty part of the map



$$\varepsilon(x_{1:T}) = \varepsilon_{\text{loop}} + \sum_{t=1}^T \varepsilon_t$$

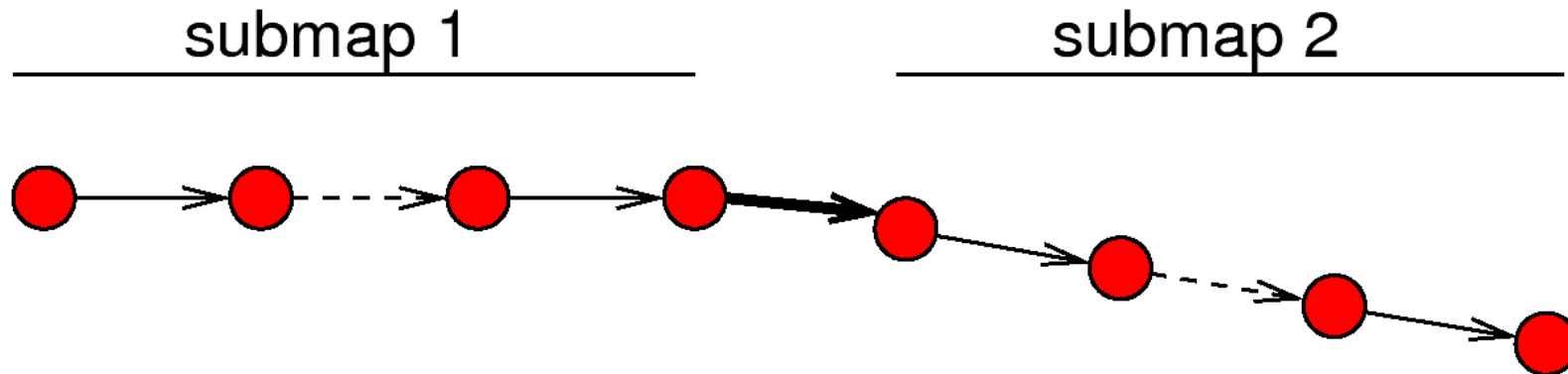
# Selection of Relations

- No loop closure links:  
local consistency only  
(wrt. time)
- Loop closure links:  
local consistency  
(wrt. space)
- Links between far away nodes:  
global consistency



# Negative example for naive metric

- Minor angular error somewhere in the trajectory leads to a high error in the naive metric

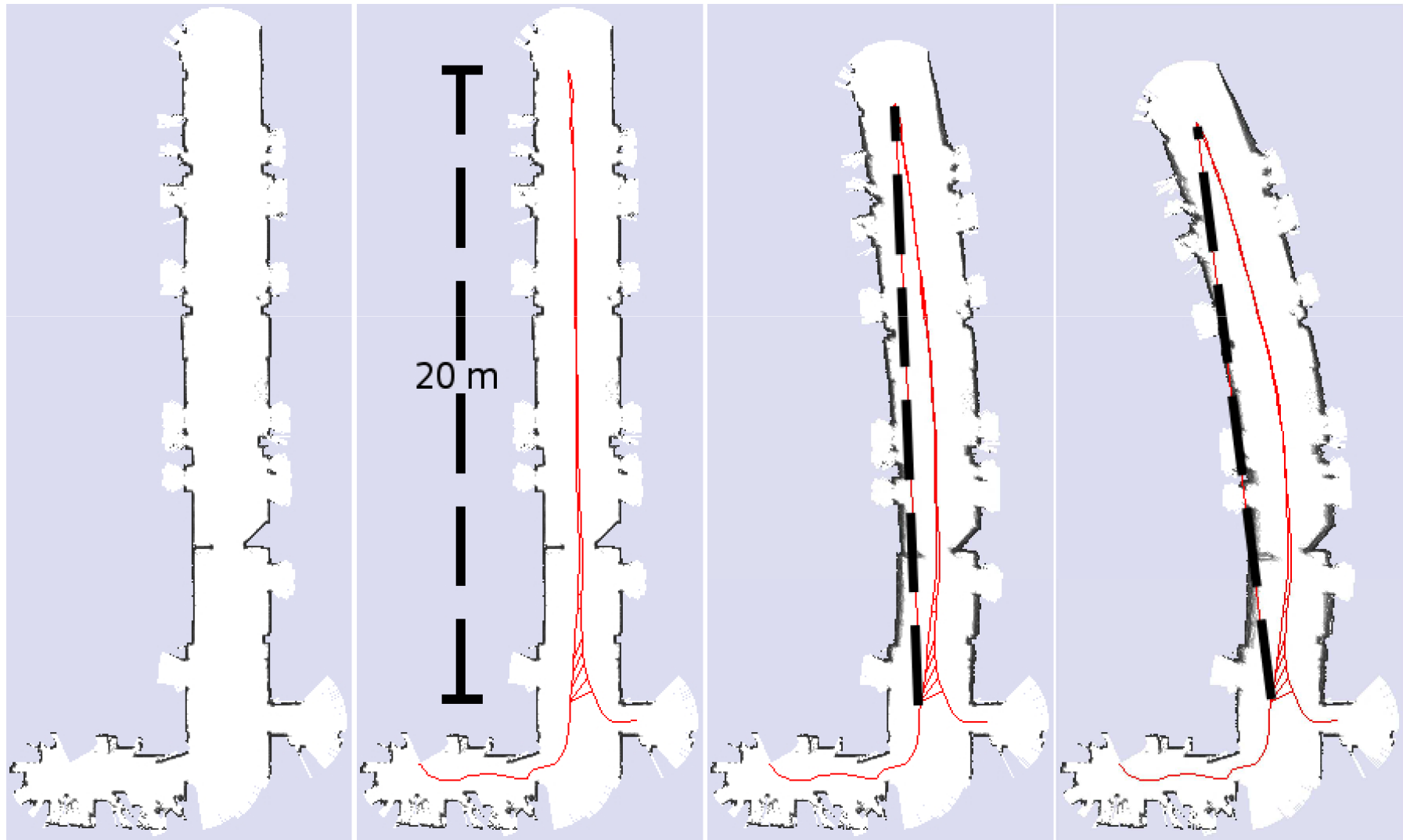


# Our Approach

- Consider the energy needed to transform the result of a SLAM algorithm into the ground truth
- Inspiration from the ideas of graph-based SLAM

**[ADD EXAMPLE FROM THE PAPER]**

# Benefit of (Manually Added) Relations for Global Consistency



# Relative Relations from a Pose Graph

- Generate the true relative relations given background knowledge
- Compute the error of these relations given the robot's poses estimated by the SLAM approach

**[ADD FORMULAR FROM THE PAPER]**

# Obtaining Ground Truth Relations

- Besides in simulation, true relations are hard to obtain
- Highly accurate measurement device (e.g., Simeo positioning system or any other sources)
- How can we use existing datasets for evaluations (Intel Research, MIT infinite corridor, ...)?
- For laser-based SLAM, perform scan alignment paired with manual inspection

# Determine Relations Manually

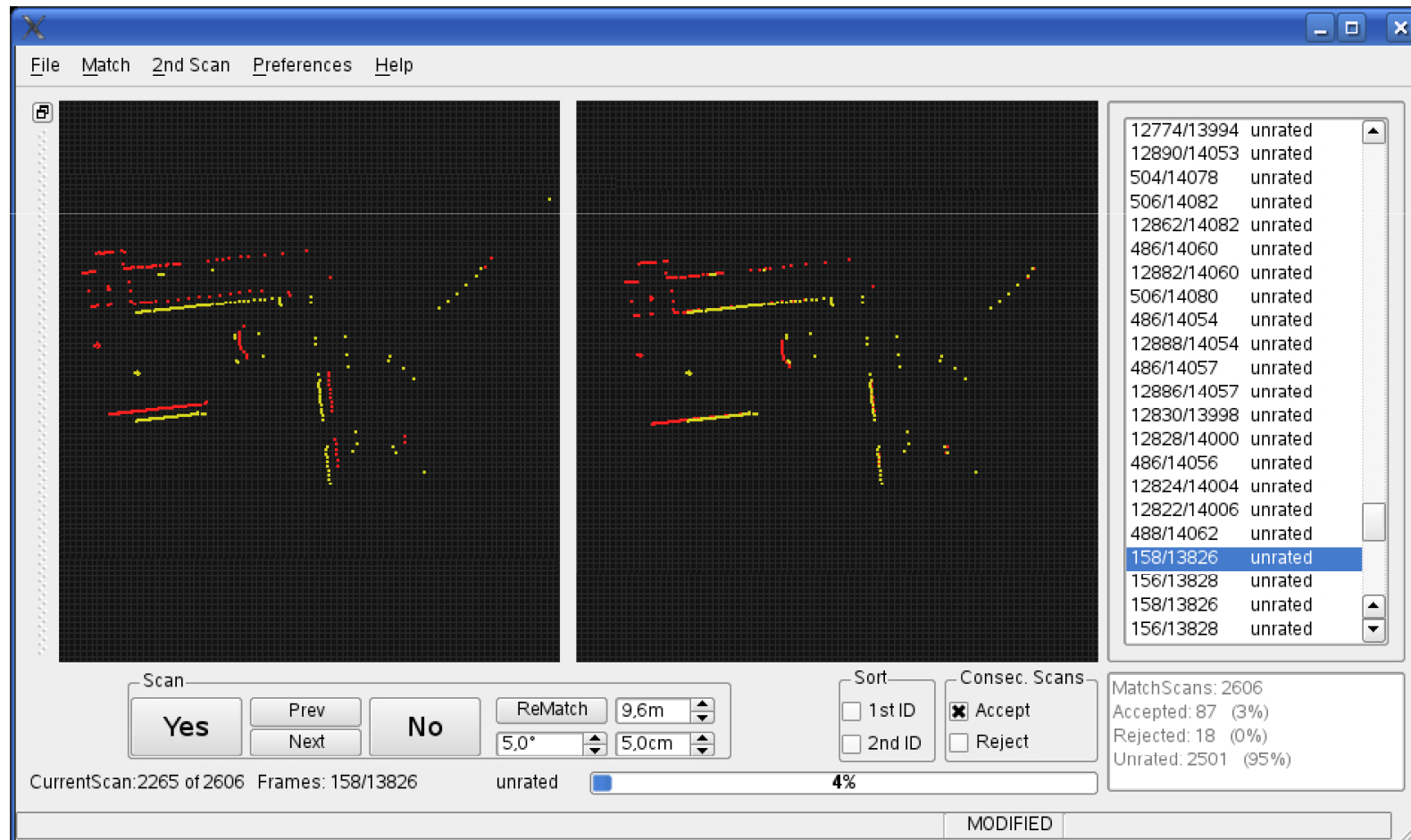
- Scan alignment paired with manual inspection can generate close to ground truth relations
- Significant manual efforts but need to be done only once per dataset

**[ADD GUI IMAGE FROM THE PAPER]**



# Graphical Interface to Confirm & Correct Constraints

- Scan alignment paired with manual inspection to generate close to ground truth relations



# Experiments

- Application of measure to evaluate measure for different algorithms and environments
- Demonstrate that the scores correlate with the quality of the map
- Demonstrate that the plots allows us to localize errors in the alignments

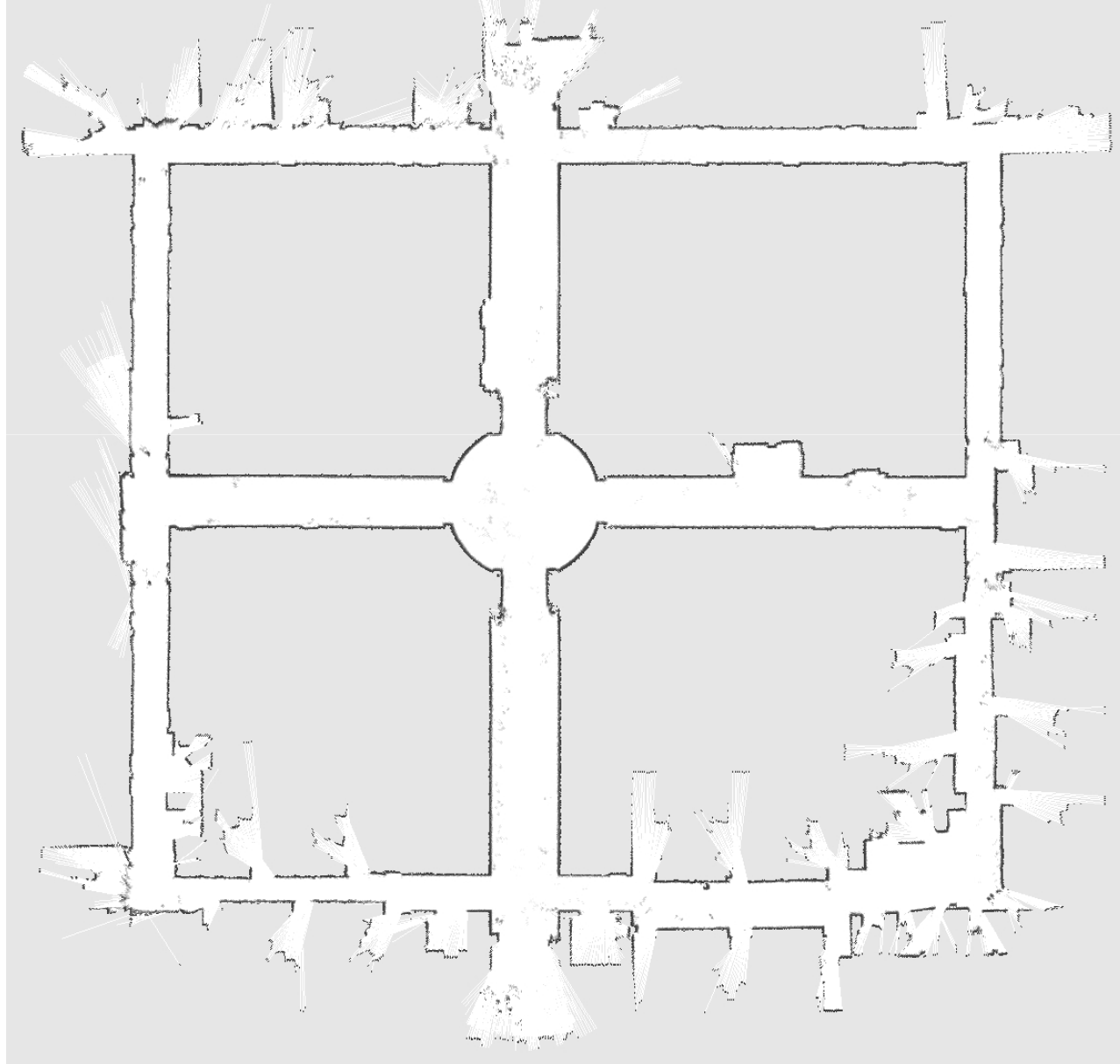
# Relations Available Online

- We generated relations in this way for
  - MIT Killian Court
  - ACES Building at the University of Texas
  - Intel Research Lab Seattle
  - MIT CSAIL Building
  - Building 079 University of Freiburg

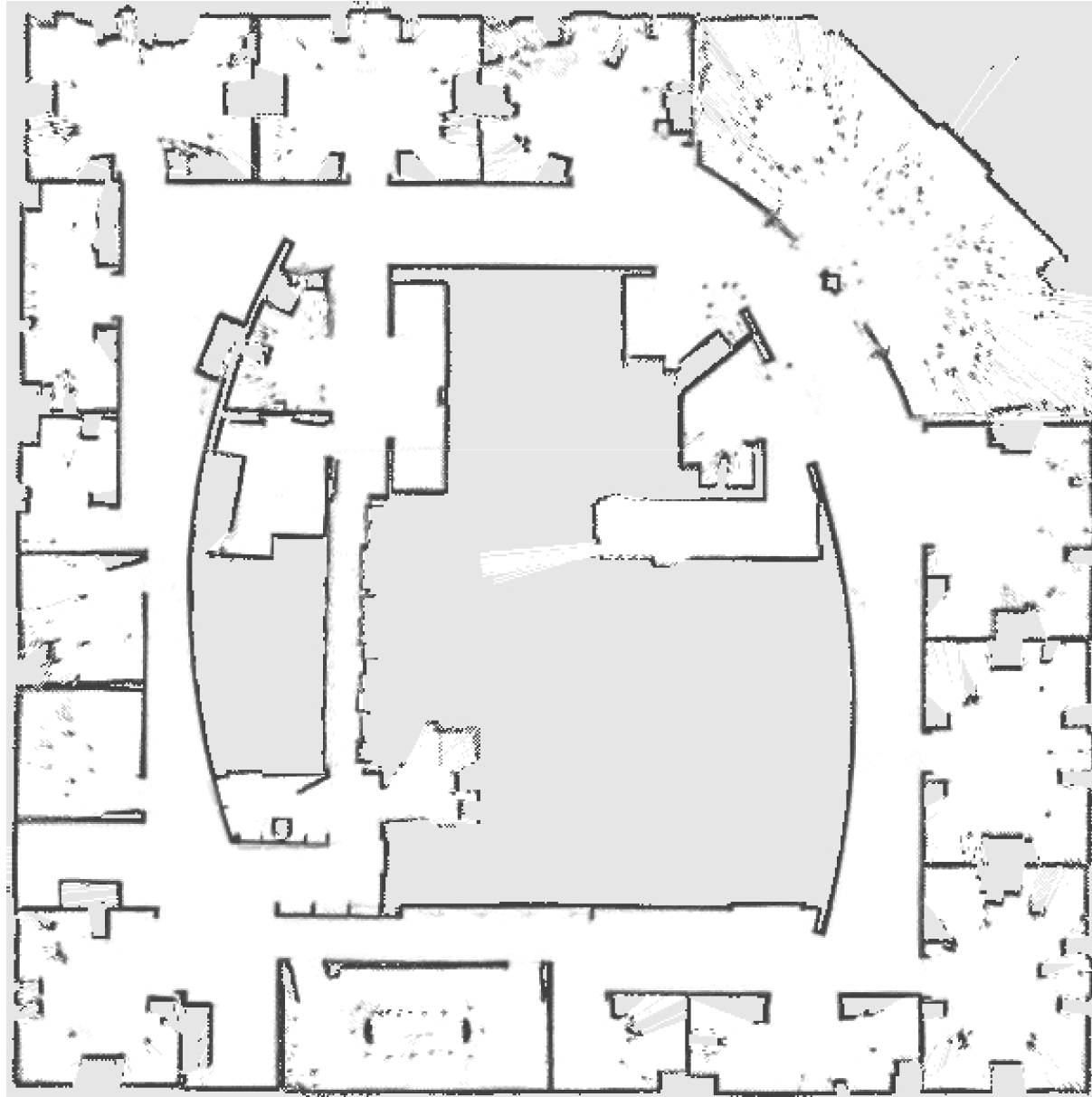
These are online available at

<http://ais.informatik.uni-freiburg.de/slamevaluation/>

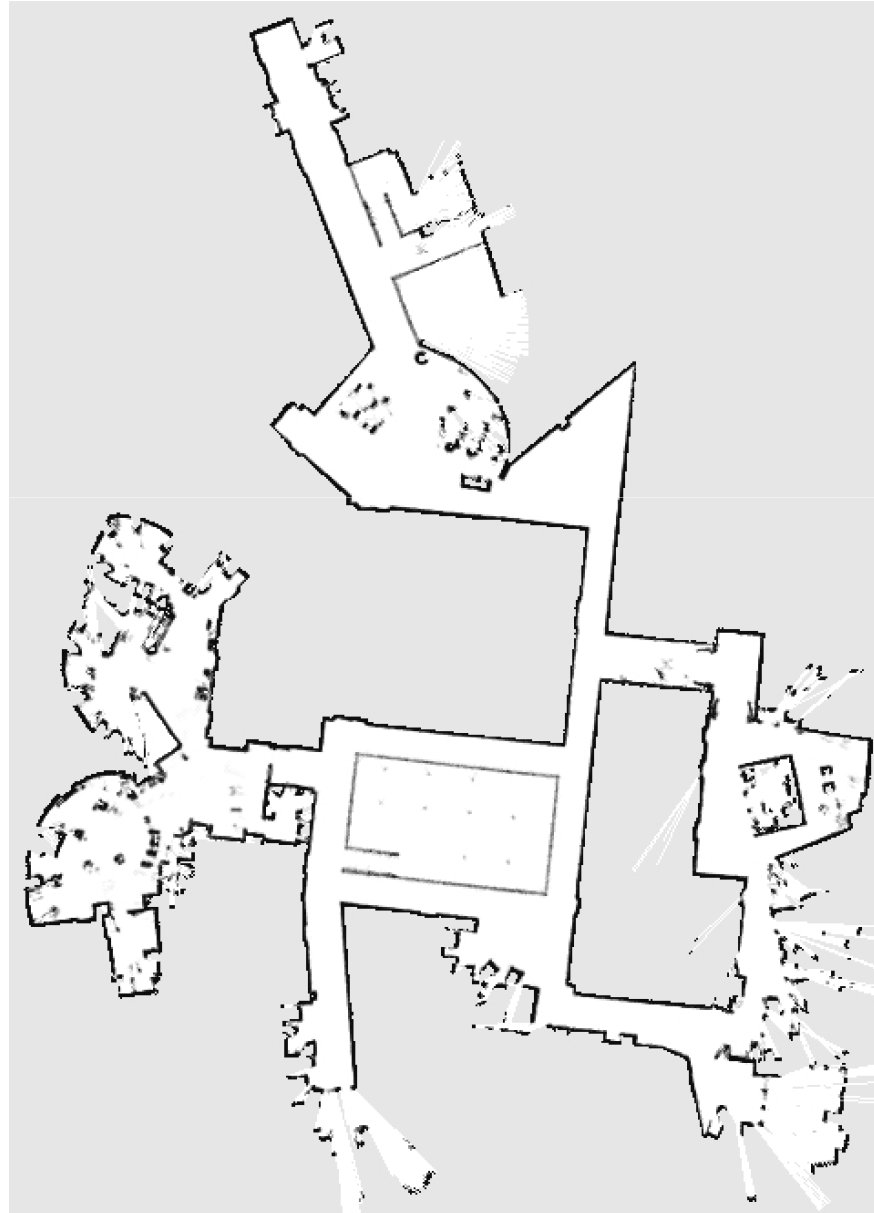
# ACES Building



# Intel Lab, Seattle



# MIT CSAIL



# Building 79 Univ. of Freiburg

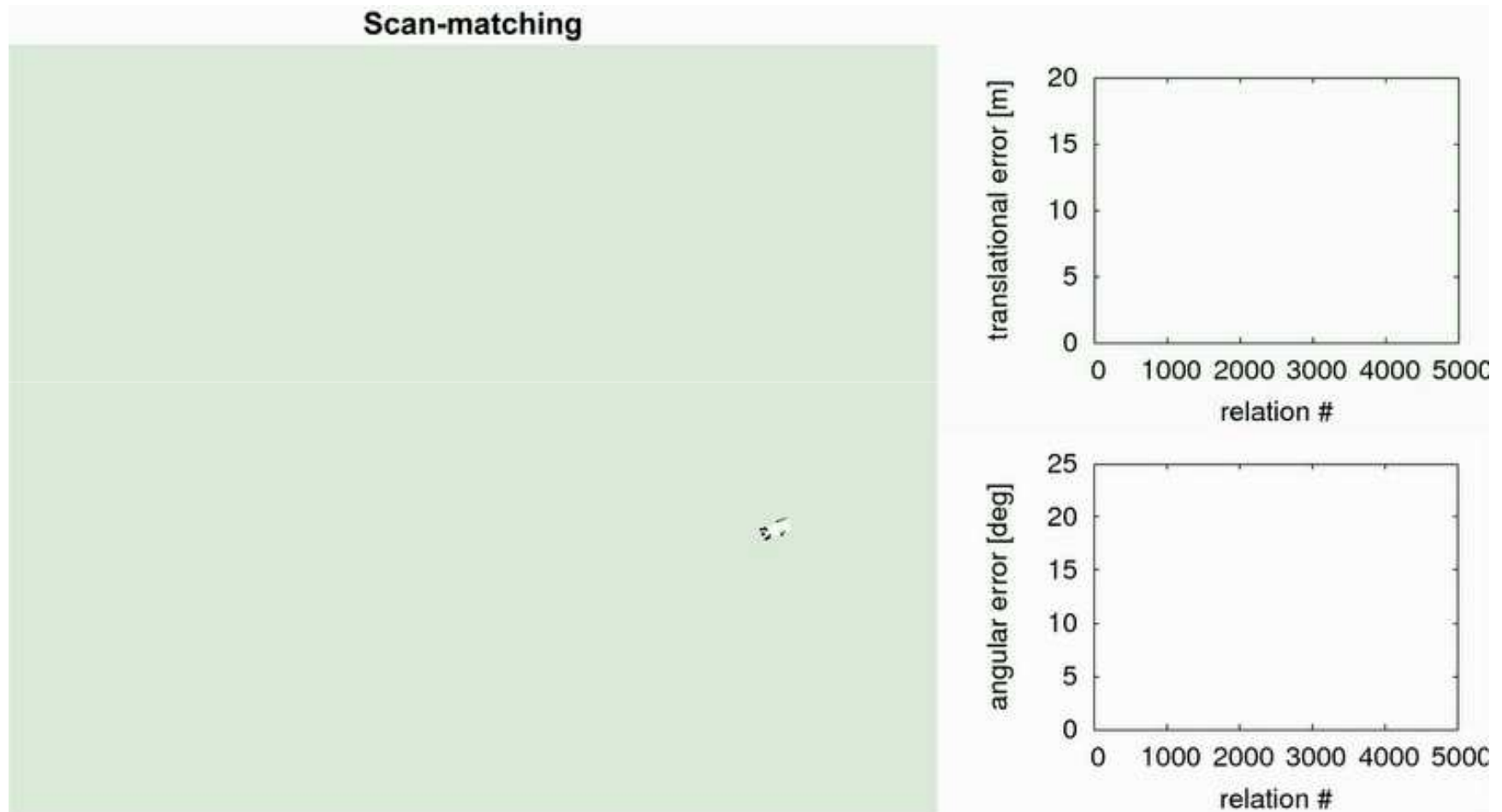


# Experimental Evaluation

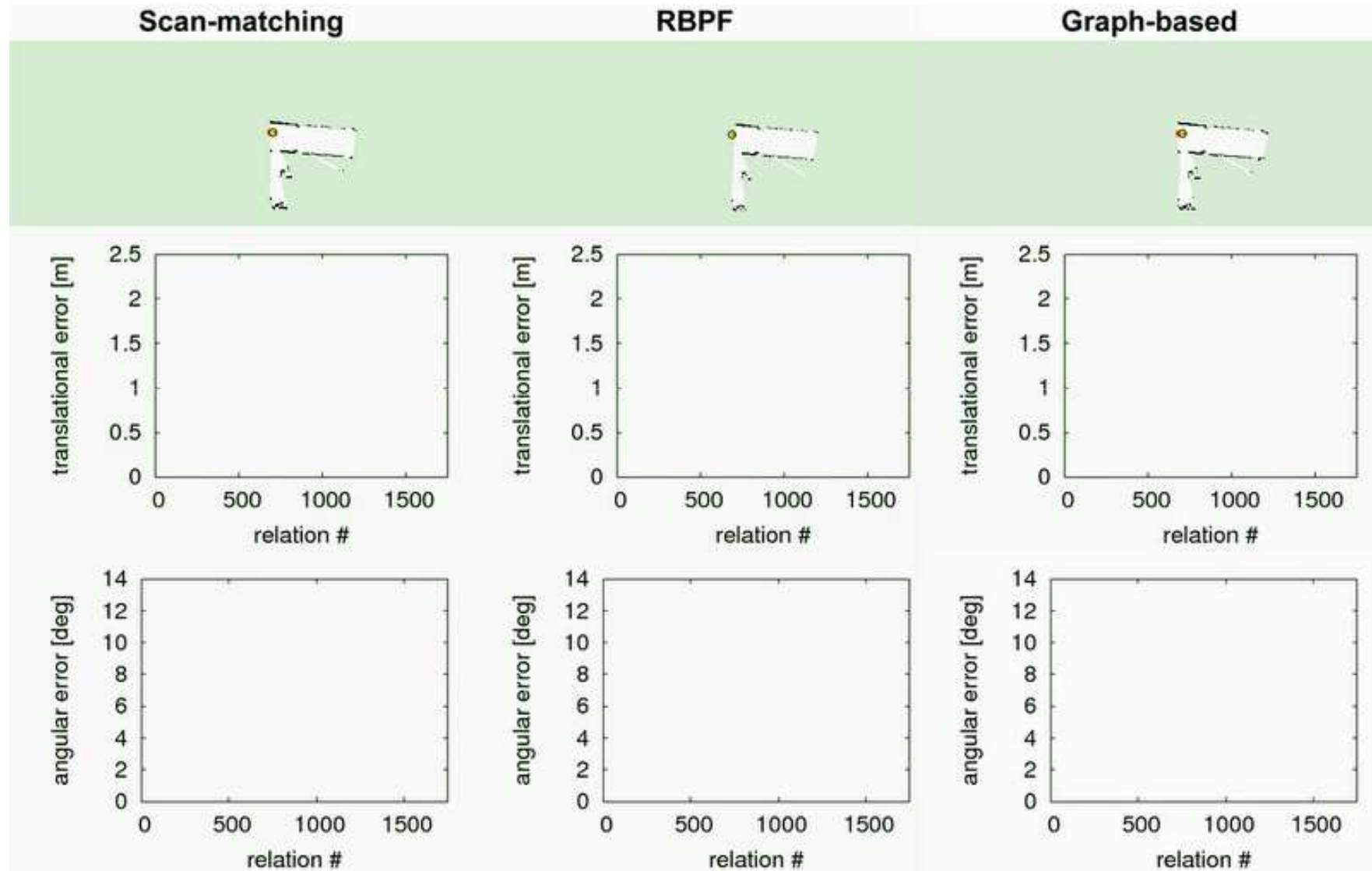
- Compared three approach using all datasets
  - Incremental scan-matching
  - GMapping (see <http://www.openslam.org>)
  - Graph-based SLAM system based on TORO (see <http://www.openslam.org>)
- We provide resulting scores for all algorithms and datasets



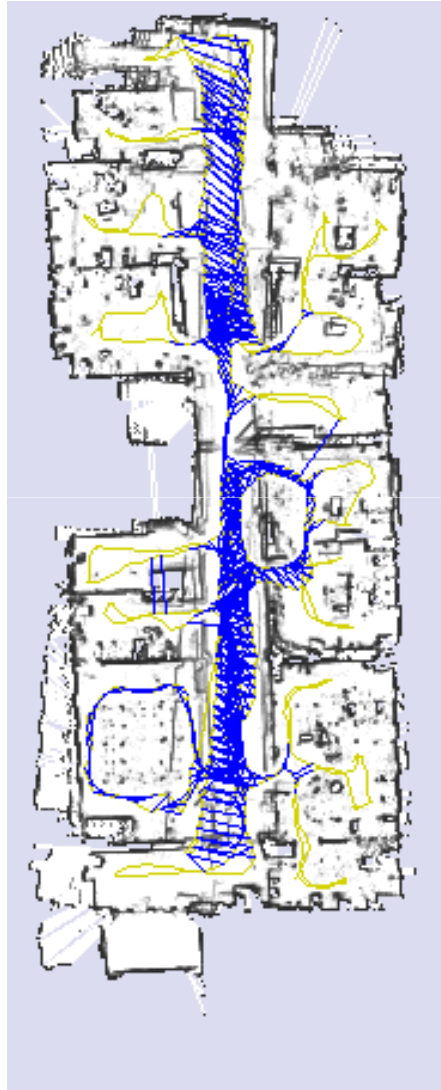
# Example



# Example



# Maps Obtained



Scan Matching

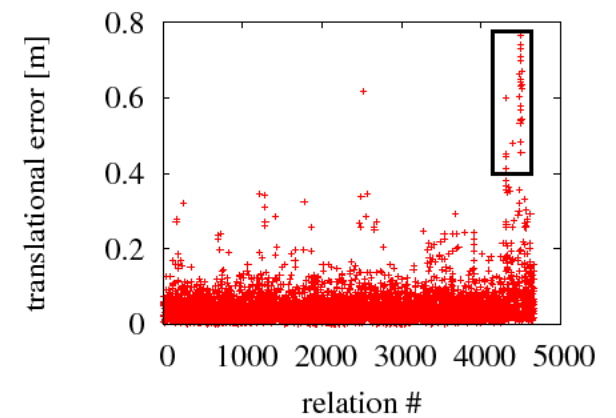
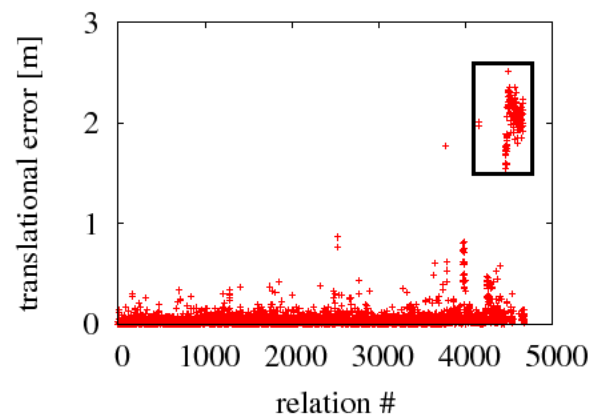
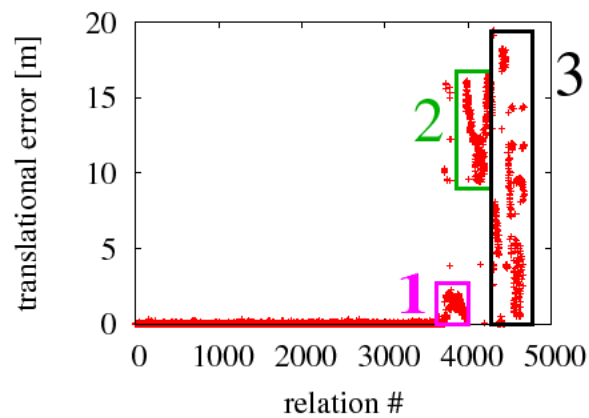
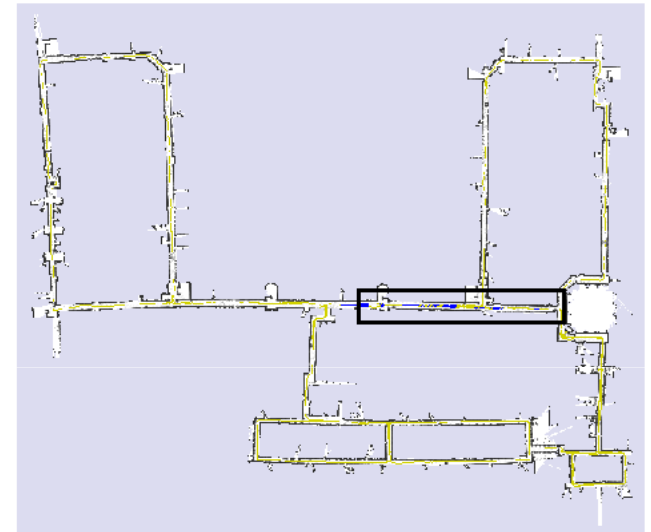
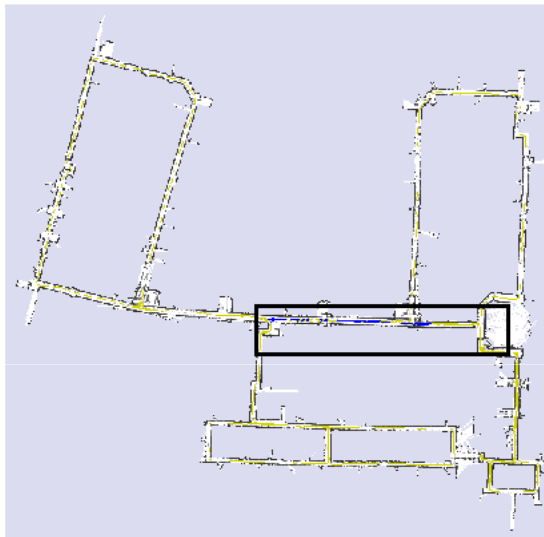
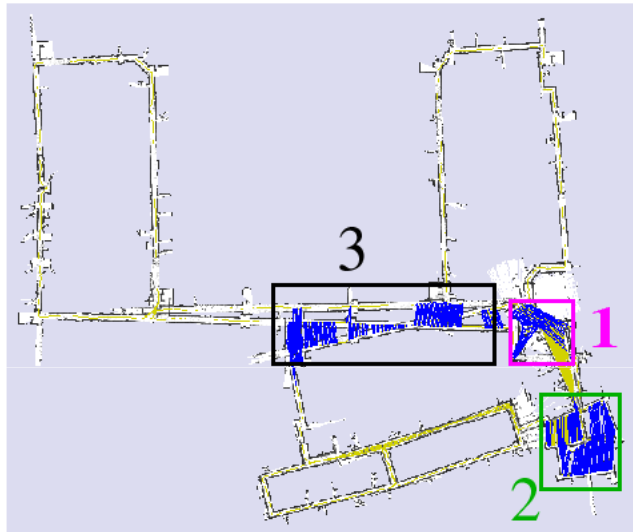


RBPF

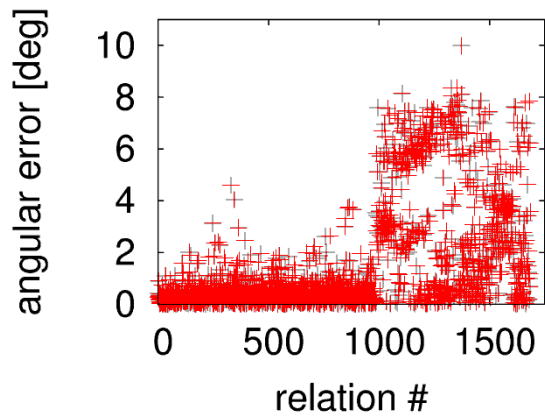
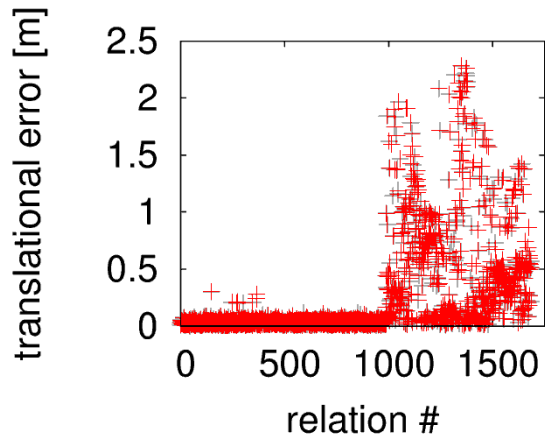


Graph mapping

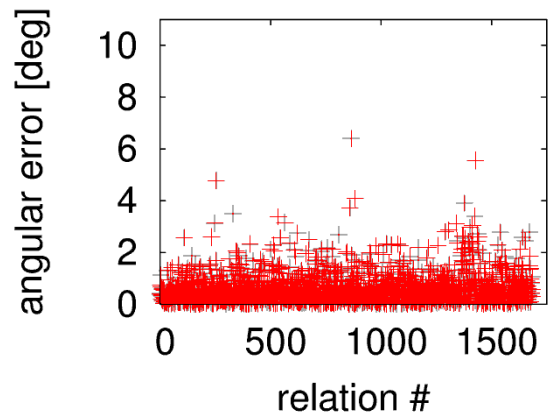
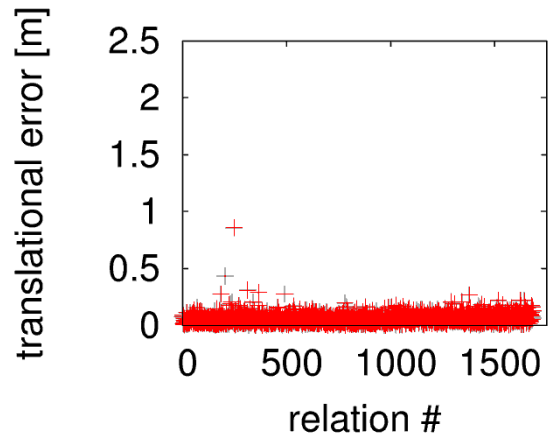
# Analyzing Error Plots



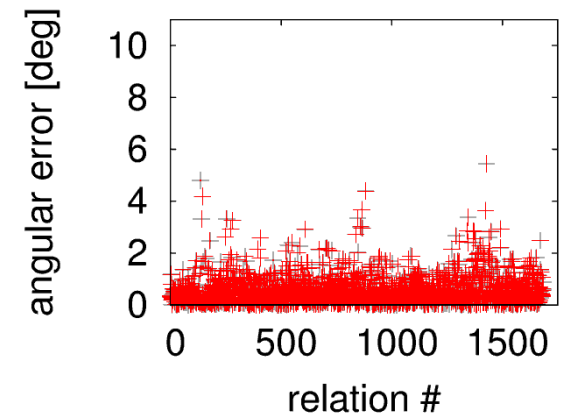
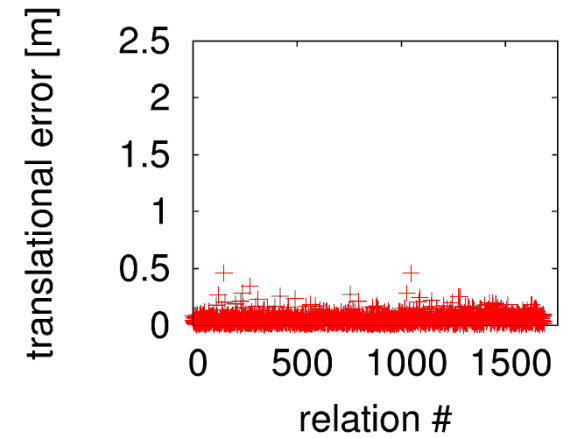
# Error Plots



Scan Matching



RBPF



Graph mapping

# Numerical Evaluation

Trans. error $m / m^2$	Scan Matching	RBPF (50 part.)	Graph Mapping
Aces (abs)	$0.173 \pm 0.614$	$0.060 \pm 0.049$	$0.044 \pm 0.044$
Aces (sqr)	$0.407 \pm 2.726$	$0.006 \pm 0.011$	$0.004 \pm 0.009$
Intel (abs)	$0.220 \pm 0.296$	$0.070 \pm 0.083$	$0.031 \pm 0.026$
Intel (sqr)	$0.136 \pm 0.277$	$0.011 \pm 0.034$	$0.002 \pm 0.004$
MIT (abs)	$1.651 \pm 4.138$	$0.122 \pm 0.386^1$	$0.050 \pm 0.056$
MIT (sqr)	$19.85 \pm 59.84$	$0.164 \pm 0.814^1$	$0.006 \pm 0.029$
CSAIL (abs)	$0.106 \pm 0.325$	$0.049 \pm 0.049^1$	$0.004 \pm 0.009$
CSAIL (sqr)	$0.117 \pm 0.728$	$0.005 \pm 0.013^1$	$0.0001 \pm 0.0005$
FR 79 (abs)	$0.258 \pm 0.427$	$0.061 \pm 0.044^1$	$0.056 \pm 0.042$
FR 79 (sqr)	$0.249 \pm 0.687$	$0.006 \pm 0.020^1$	$0.005 \pm 0.011$

# Numerical Evaluation

Rot. error <i>deg / deg<sup>2</sup></i>	Scan Matching	RBPF (50 part.)	Graph Mapping
Aces (abs)	$1.2 \pm 1.5$	$1.2 \pm 1.3$	$0.4 \pm 0.4$
Aces (swr)	$3.7 \pm 10.7$	$3.1 \pm 7.0$	$0.3 \pm 0.8$
Intel (abs)	$1.7 \pm 4.8$	$3.0 \pm 5.3$	$1.3 \pm 4.7$
Intel (sqr)	$25.8 \pm 170.9$	$36.7 \pm 187.7$	$24.0 \pm 166.1$
MIT (abs)	$2.3 \pm 4.5$	$0.8 \pm 0.8^1$	$0.5 \pm 0.5$
MIT (sqr)	$25.4 \pm 65.0$	$0.9 \pm 1.7^1$	$0.9 \pm 0.9$
CSAIL (abs)	$1.4 \pm 4.5$	$0.6 \pm 1.2^1$	$0.05 \pm 0.08$
CSAIL (sqr)	$22.3 \pm 111.3$	$1.9 \pm 17.3^1$	$0.01 \pm 0.04$
FR 79 (abs)	$1.7 \pm 2.1$	$0.6 \pm 0.6^1$	$0.6 \pm 0.6$
FR 79 (sqr)	$7.3 \pm 14.5$	$0.7 \pm 2.0^1$	$0.7 \pm 1.7$

# What if no Trajectory is Available?

- Algorithms such as EKF-based SLAM often do not store the whole trajectory of the robot
- No relations can be evaluated
- Two solutions:
  - Recover the trajectory by localization
  - Transfer the idea of relations to landmarks (which yields a landmark-pose-graph)



# Offline Trajectory Recovery by Localization

- EKF-based approaches provide landmark estimates
- Data associations are made during SLAM
- To recover the poses of the robot during mapping, run EKF-based localization given the landmark locations, data associations, and motion/sensor model used during SLAM
- Approach using relation can be applied directly

# Relations based on Landmark Locations

- Alternatively, one can retrieve from known landmark locations relative relations via a triangle mesh
- Take care: data associations between landmarks might need to be done manually

# Towards the Rescue of the Sum of Squared Distances

- Let  $x_{1:T}$  be the estimated poses from time 1 to  $T$ .
- Let  $x_{1:T}^*$  be the reference poses.
- Error function

$$\varepsilon(x_{1:T}) = \min_A \sum_{t=1}^T \|x_t \ominus Ax_t^*\|^2$$

where  $A$  is the set of affine transformations

# Conclusions

- Approach to comparing SLAM algorithms
- Based on relative relations between poses
- Independent of the used algorithm
- Independent of the sensor setup since it operates on the trajectory estimate
- Manually corrected datasets available online
- Results for comparisons available online

# Limitations

- The measure mostly captures localization
- Assumption is that good localization implies accurate map
- It does not directly capture, whether the map contains all obstacles in the real world.