Parallel Computing for Machine Learning
(Part 1)

Shusen Wang

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- Parallel computing: using multiple processors to make the computation faster (in terms of wall-clock time.)

Toy Example: Least Squares Regression

Linear Predictor

- Inputs: $\mathbf{x} \in \mathbb{R}^d$ (e.g., features of a house).
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  - $f(\mathbf{x}) = w_1 x_1 + w_2 x_2 + \cdots + w_d x_d$
  - $w_1, w_2, \cdots, w_d$: weights
  - x_1: # of bedrooms
  - x_2: # of bathroom
  - x_3: square feet
  - x_4: age of house
  - …

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$\cdots$


totally n houses

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- Least squares regression: $\mathbf{w}^\star = \min_{\mathbf{w}} L(\mathbf{w})$.

Parallel Gradient Descent for Least Squares

Parallel Gradient Descent

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Gradient: $g(\mathbf{w}) = \frac{\partial\, L(\mathbf{w})}{\partial\, \mathbf{w}} = \sum_{i=1}^{n} \frac{\partial\, \frac{1}{2}\left(\mathbf{x}_i^T \mathbf{w} - y_i\right)^2}{\partial\, \mathbf{w}} = \sum_{i=1}^{n} \left(\mathbf{x}_i^T \mathbf{w} - y_i \right) \mathbf{x}_i$

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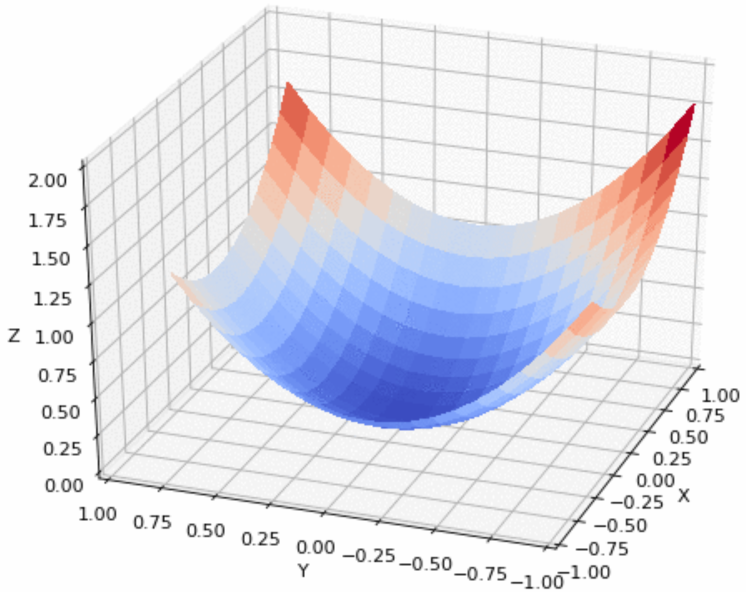
- Gradient descent: $\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \cdot g(\mathbf{w}_t).$



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- The bottleneck of GD is at computing the gradient.
- It is expensive if #samples and #parameters are both big.

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Parallel GD using 2 processors

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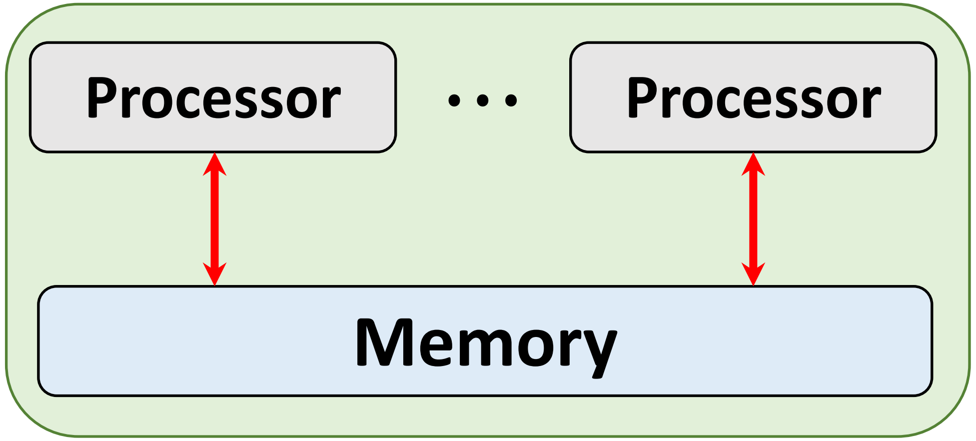
Aggregate: $g(\mathbf{w}) = \tilde{\mathbf{g}}_1 + \tilde{\mathbf{g}}_2$.

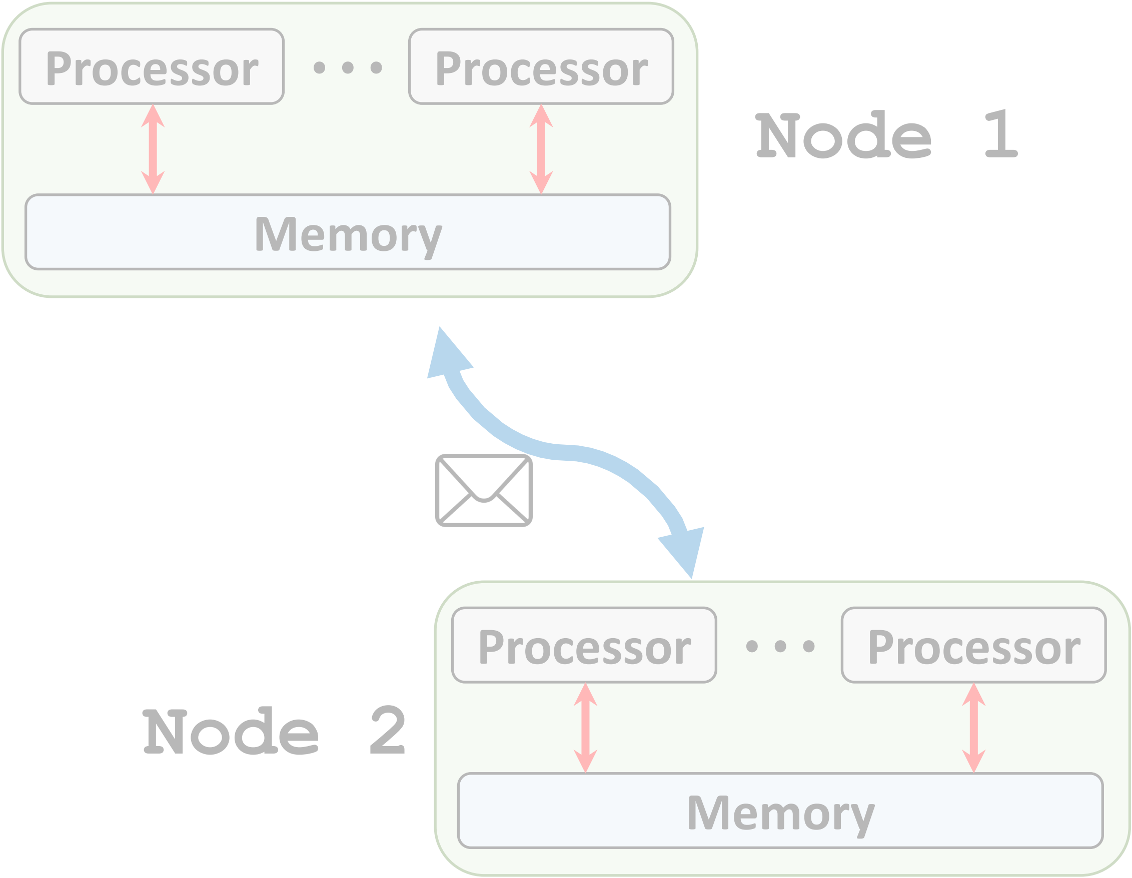
Communication

Two Ways of Communication

Share memory:



Message passing:



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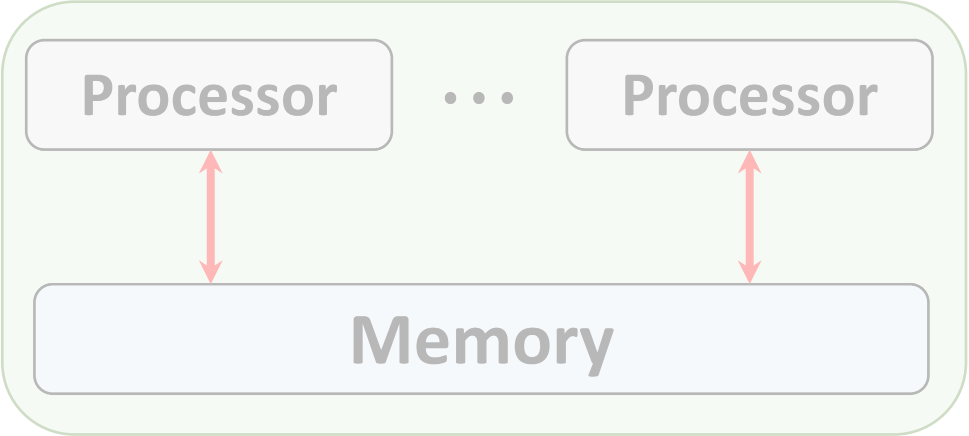

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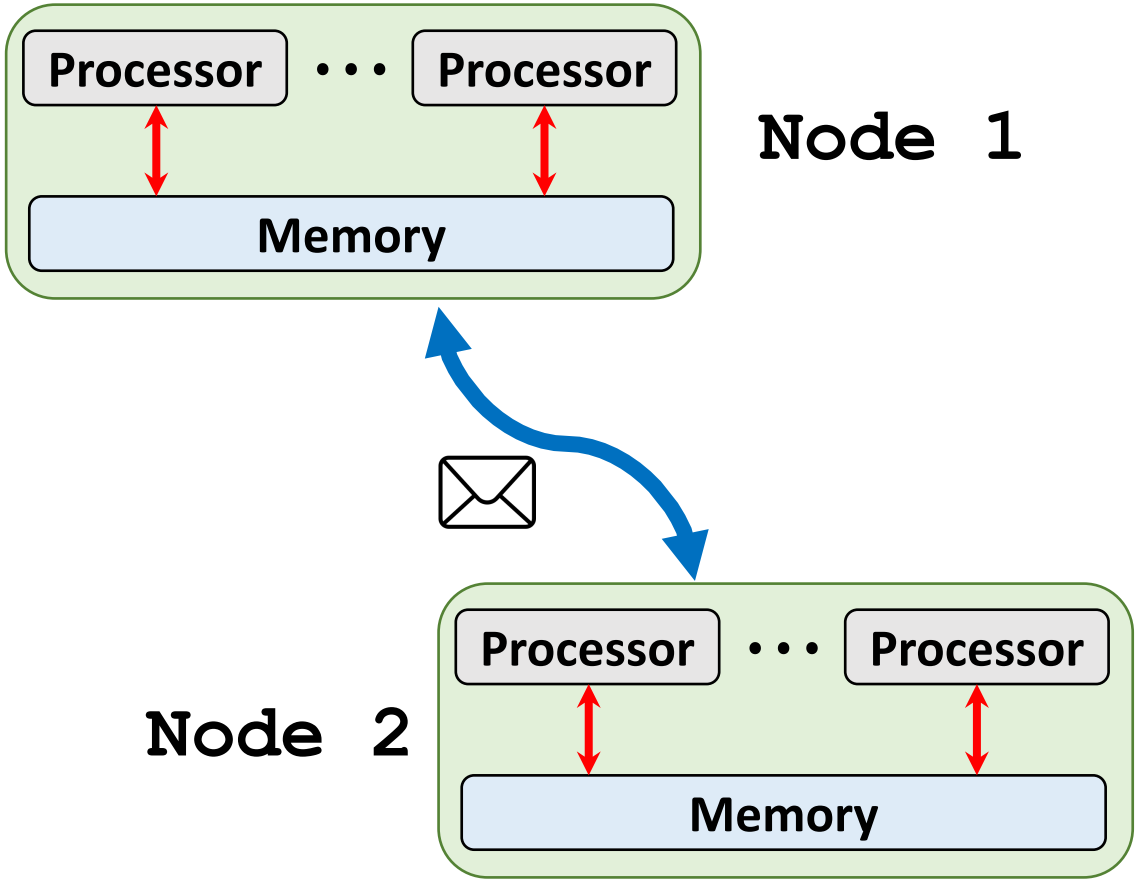
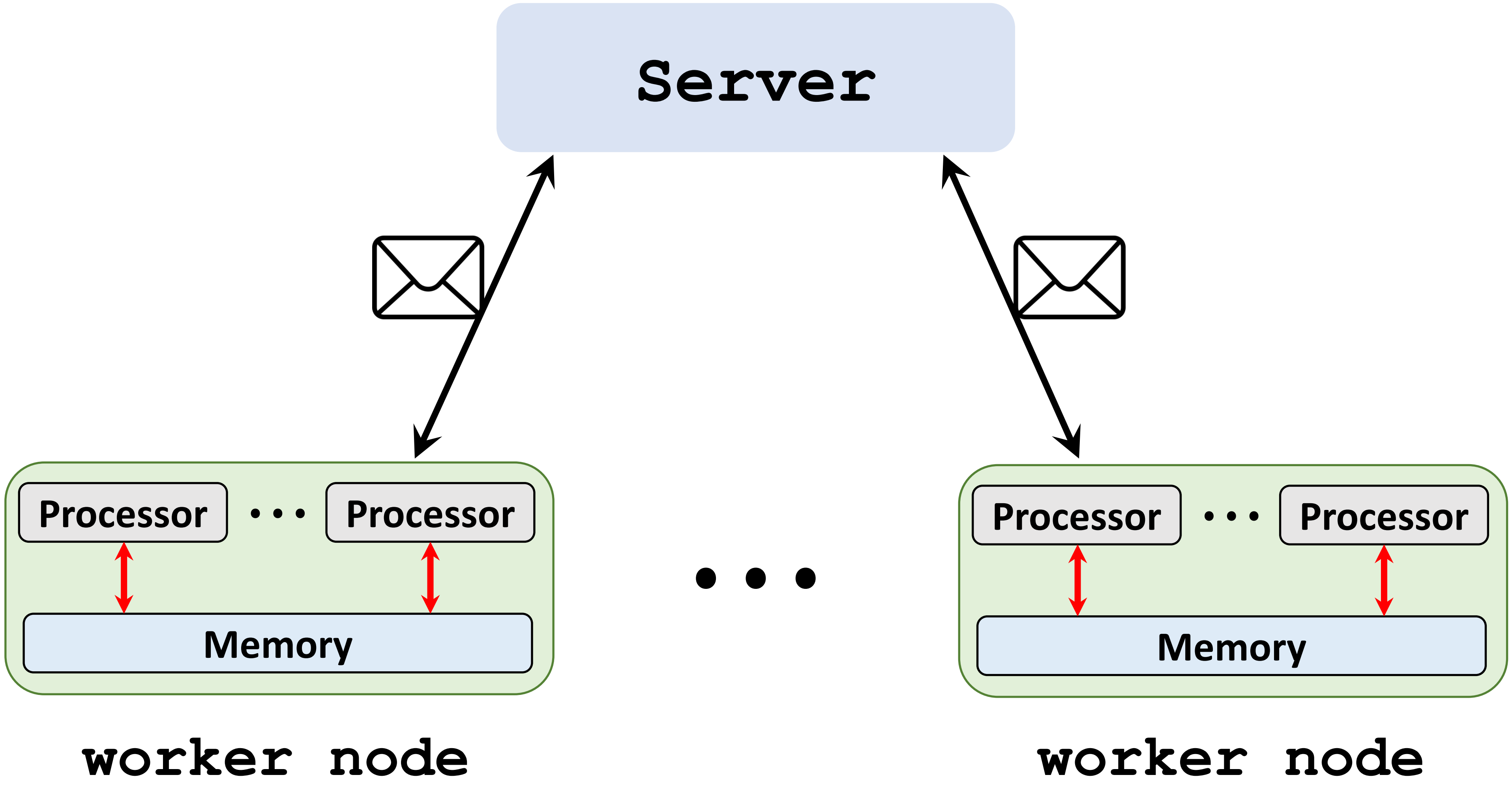
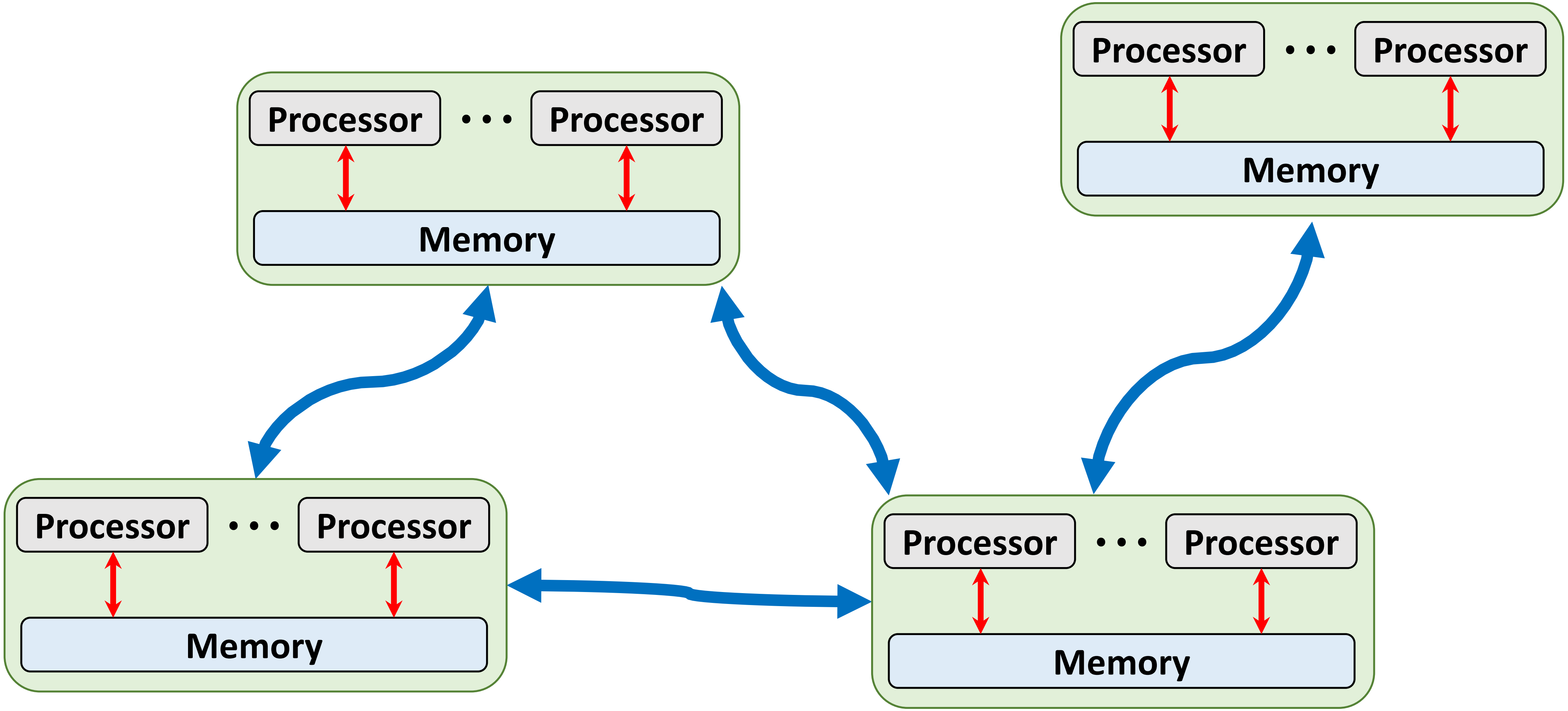


Client-Server Architecture



Peer-to-Peer Architecture



Synchronous Parallel Gradient Descent
Using MapReduce

MapReduce

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- **Characters:** client-server architecture, message-passing communication, and bulk synchronous parallel.

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- **Apache Spark** [3] is an improved open-source MapReduce.

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3. Zaharia and others: Apache Spark: a unified engine for big data processing. *Communications of the ACM*, 2016.

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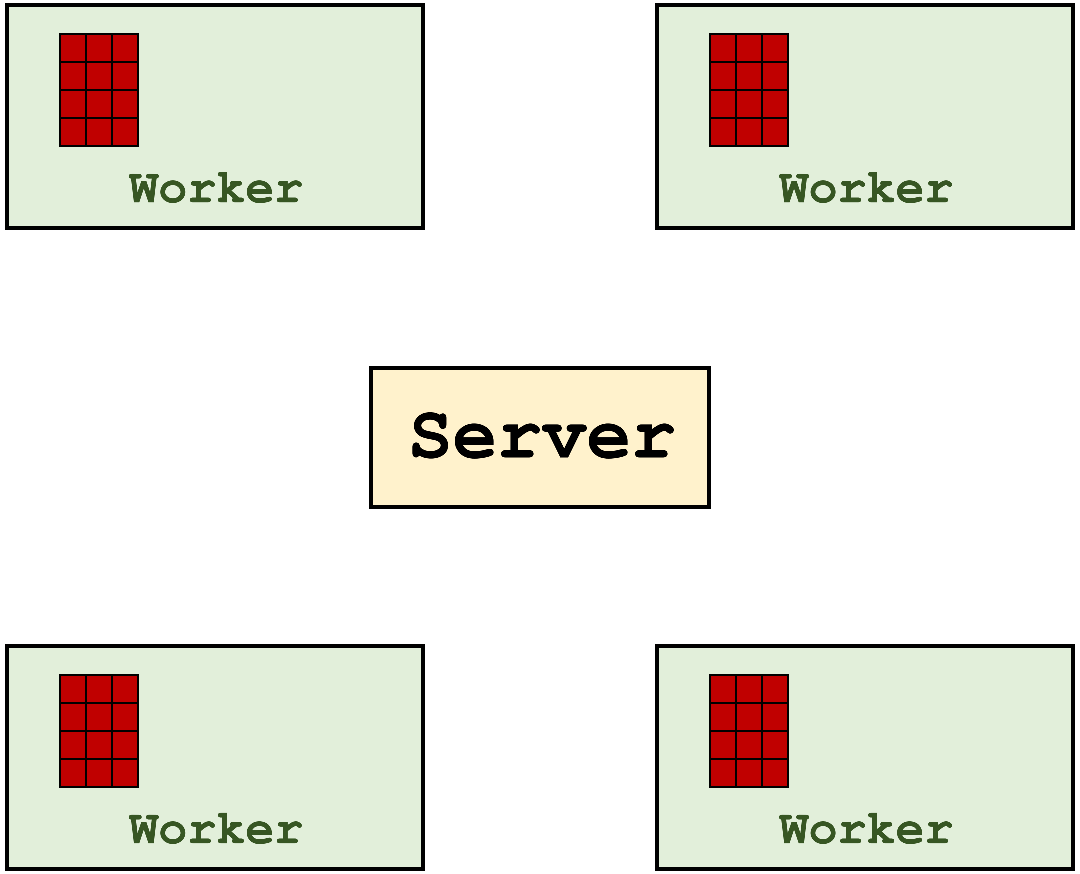

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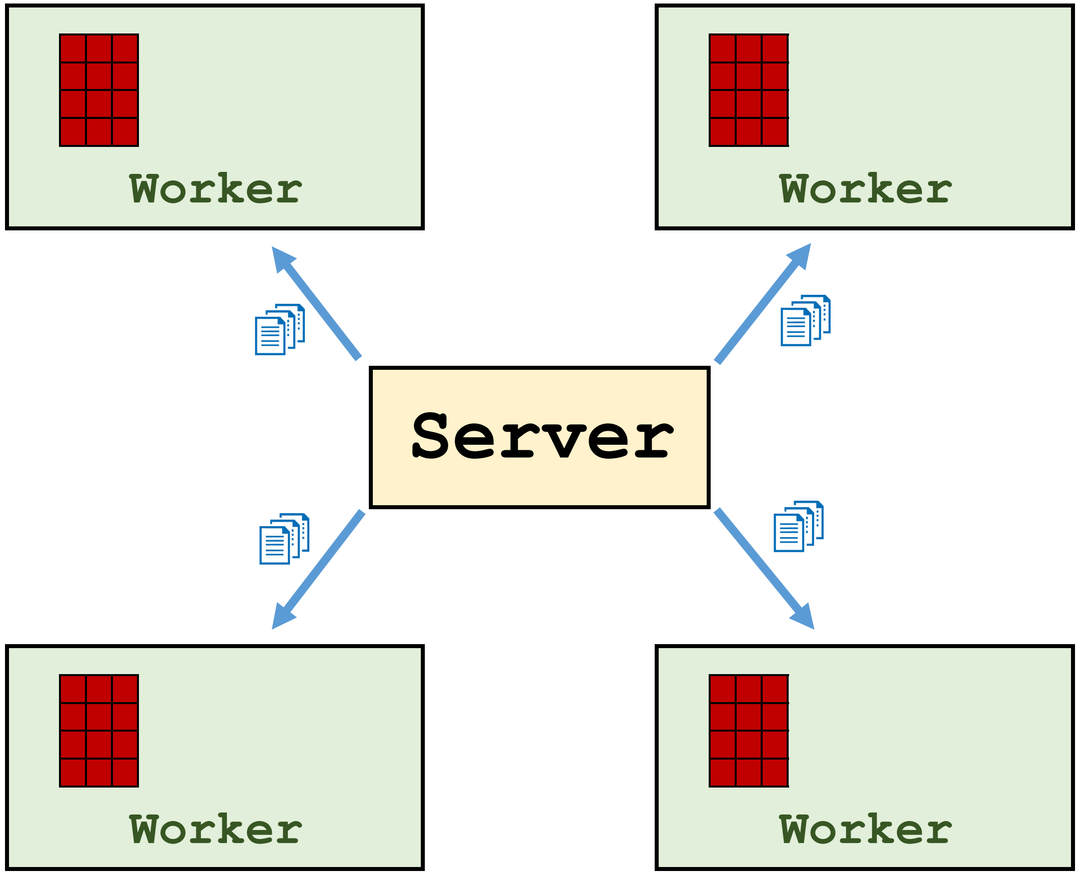
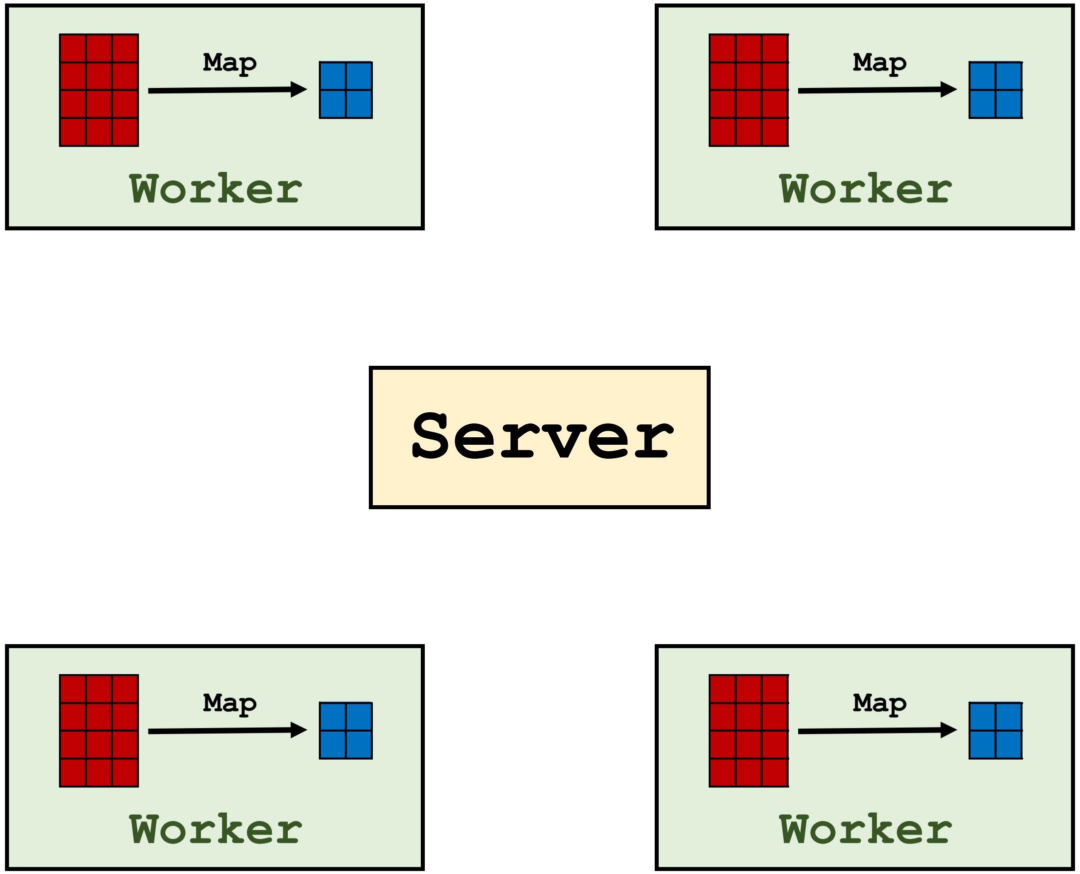


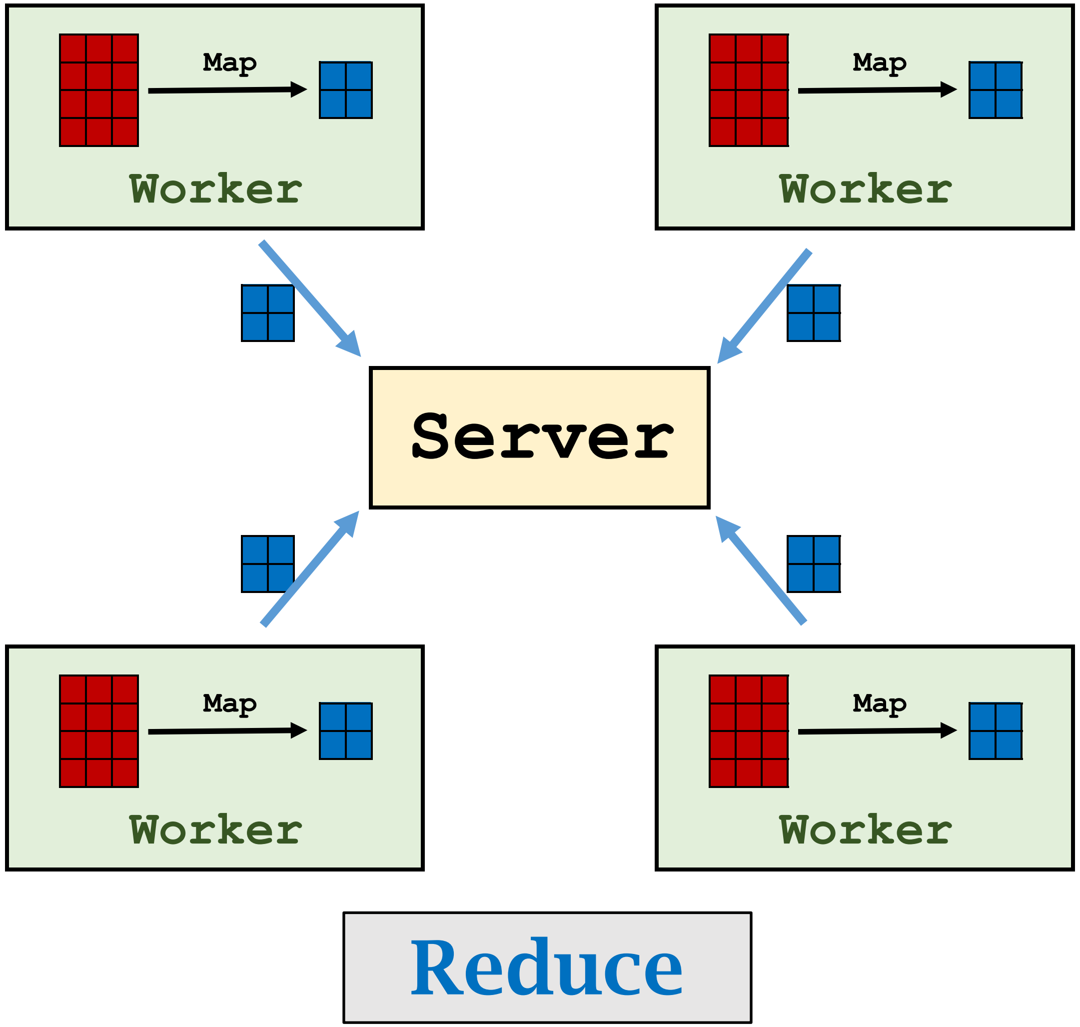
Broadcast

MapReduce

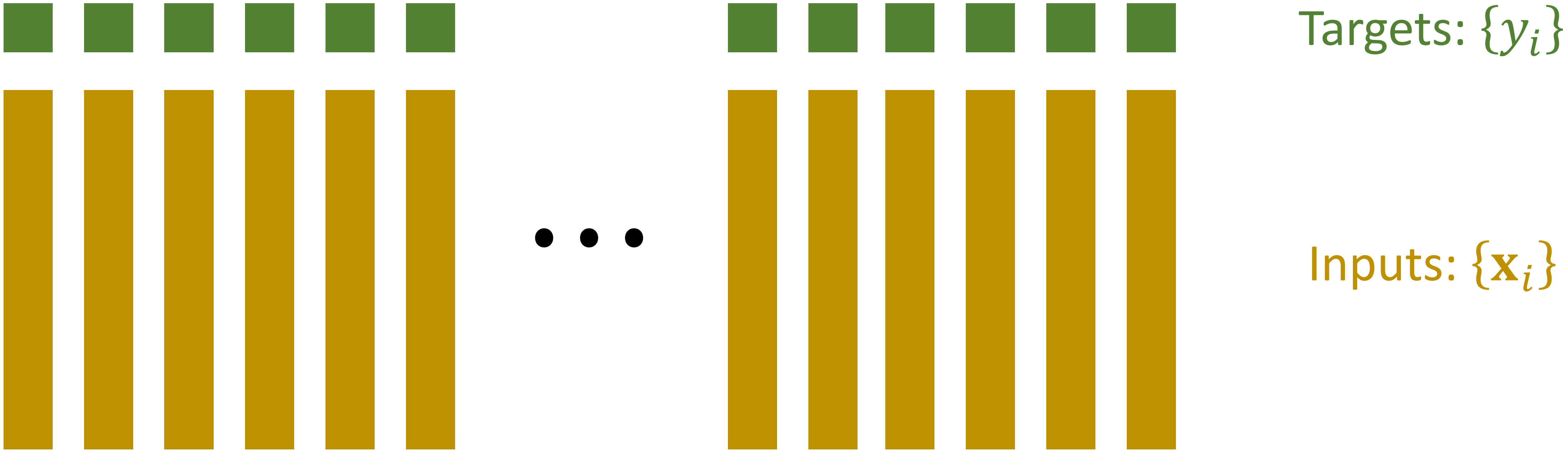


Map

MapReduce

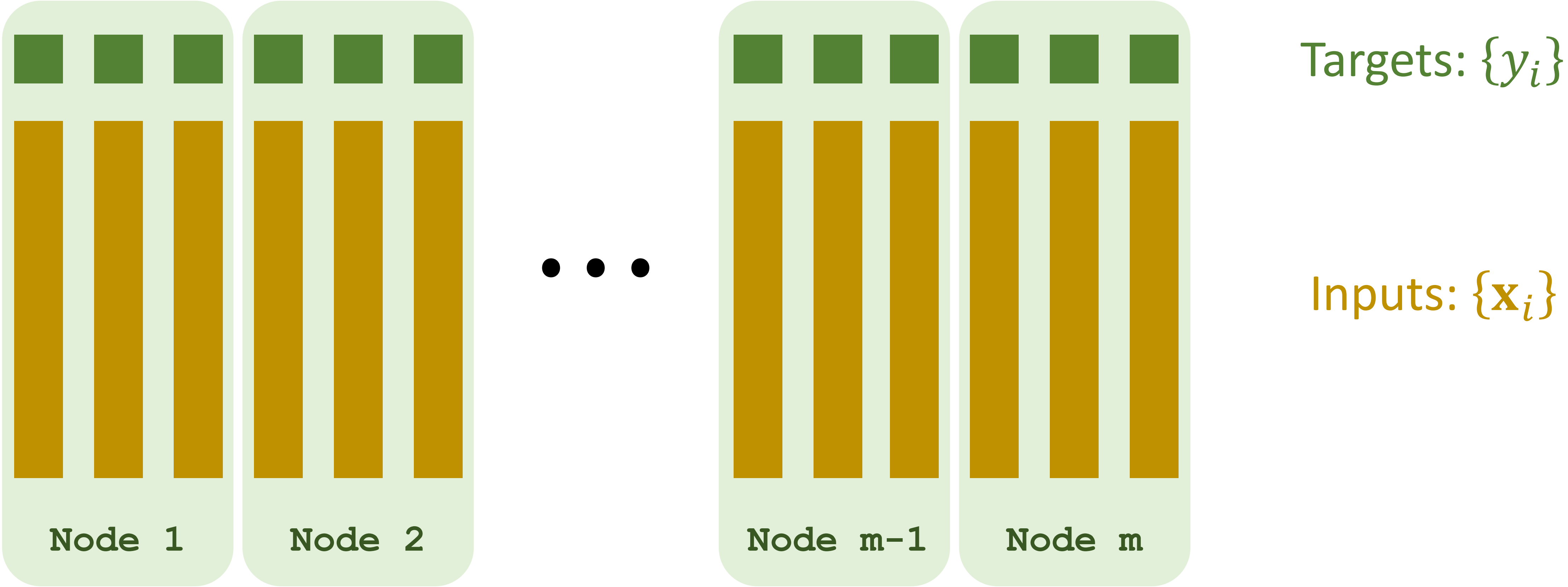


Data Parallelism



Data Parallelism

- Partition the data among worker nodes. (A node has a subset of data.)



Parallel Gradient Descent Using MapReduce

- Broadcast: Server broadcast the up-to-date parameters $\mathbf{w}_t$ to workers.
- Map: Workers do computation locally.
  - Map $(\mathbf{x}_i, y_i, \mathbf{w}_t)$ to $\mathbf{g}_i = \left(\mathbf{x}_i^T \mathbf{w} - y_i\right) \mathbf{x}_i$.
  - Obtain n vectors: $\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3, \cdots, \mathbf{g}_n$.

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  - Obtain n vectors: $\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3, \cdots, \mathbf{g}_n$.
- Reduce: Compute the sum: $\mathbf{g} = \sum_{i=1}^{n} \mathbf{g}_i$.
  - Every worker sums all the $\{\mathbf{g}_i\}$ stored in its local memory to get a vector.
  - Then, the server sums the resulting m vectors. (There are m workers.)
- Server updates the parameters: $\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \cdot \mathbf{g}$.

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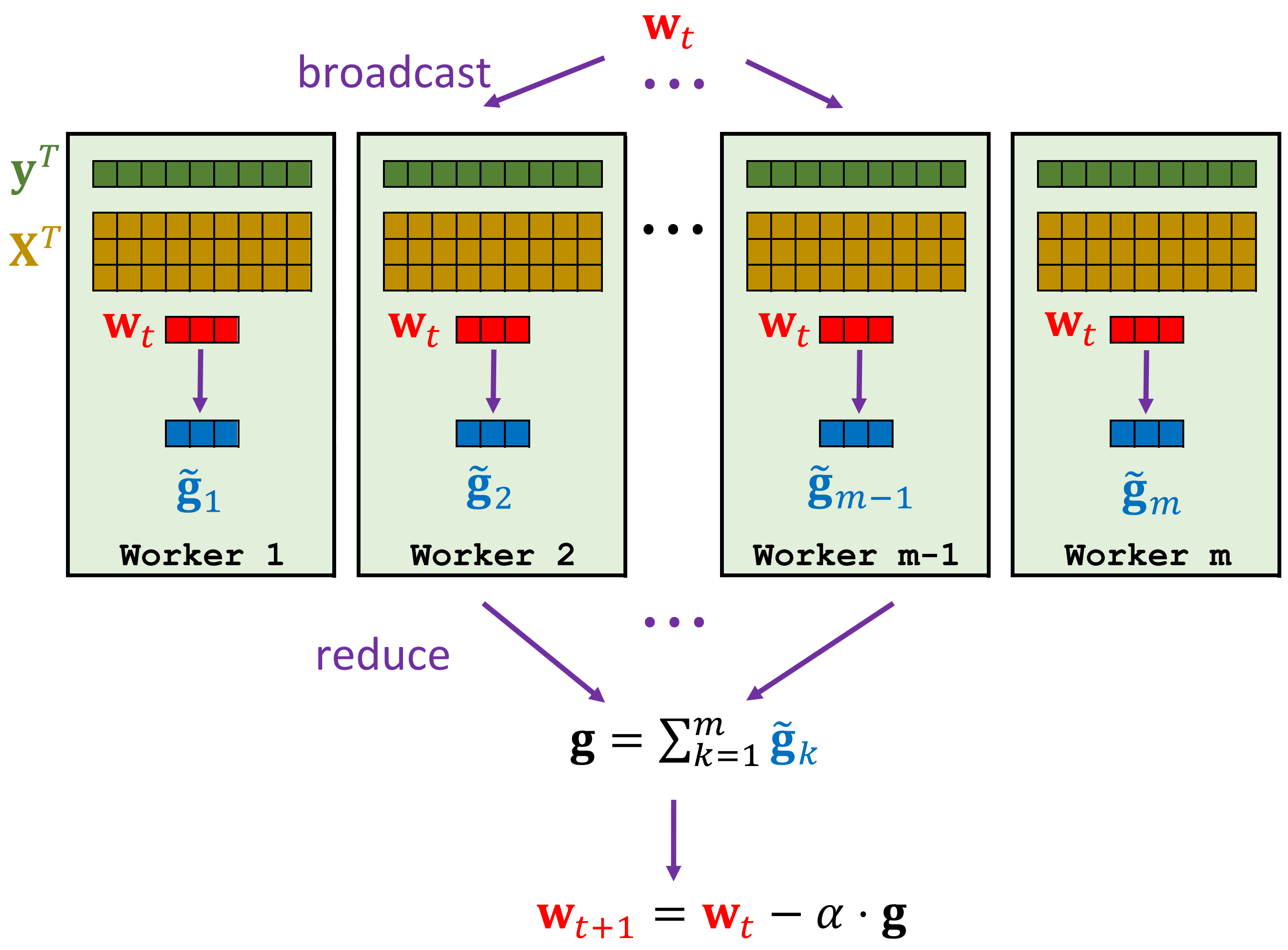

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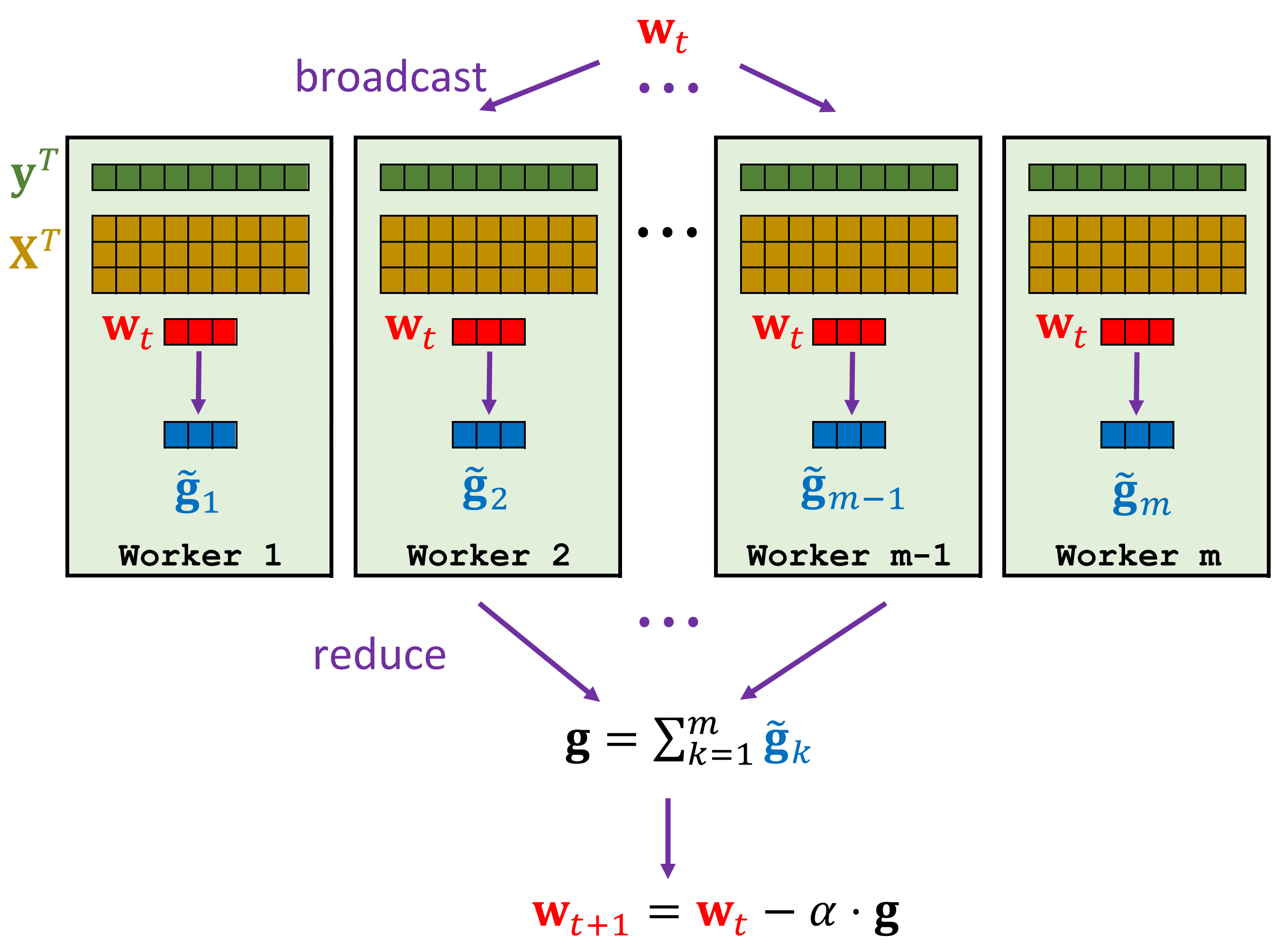
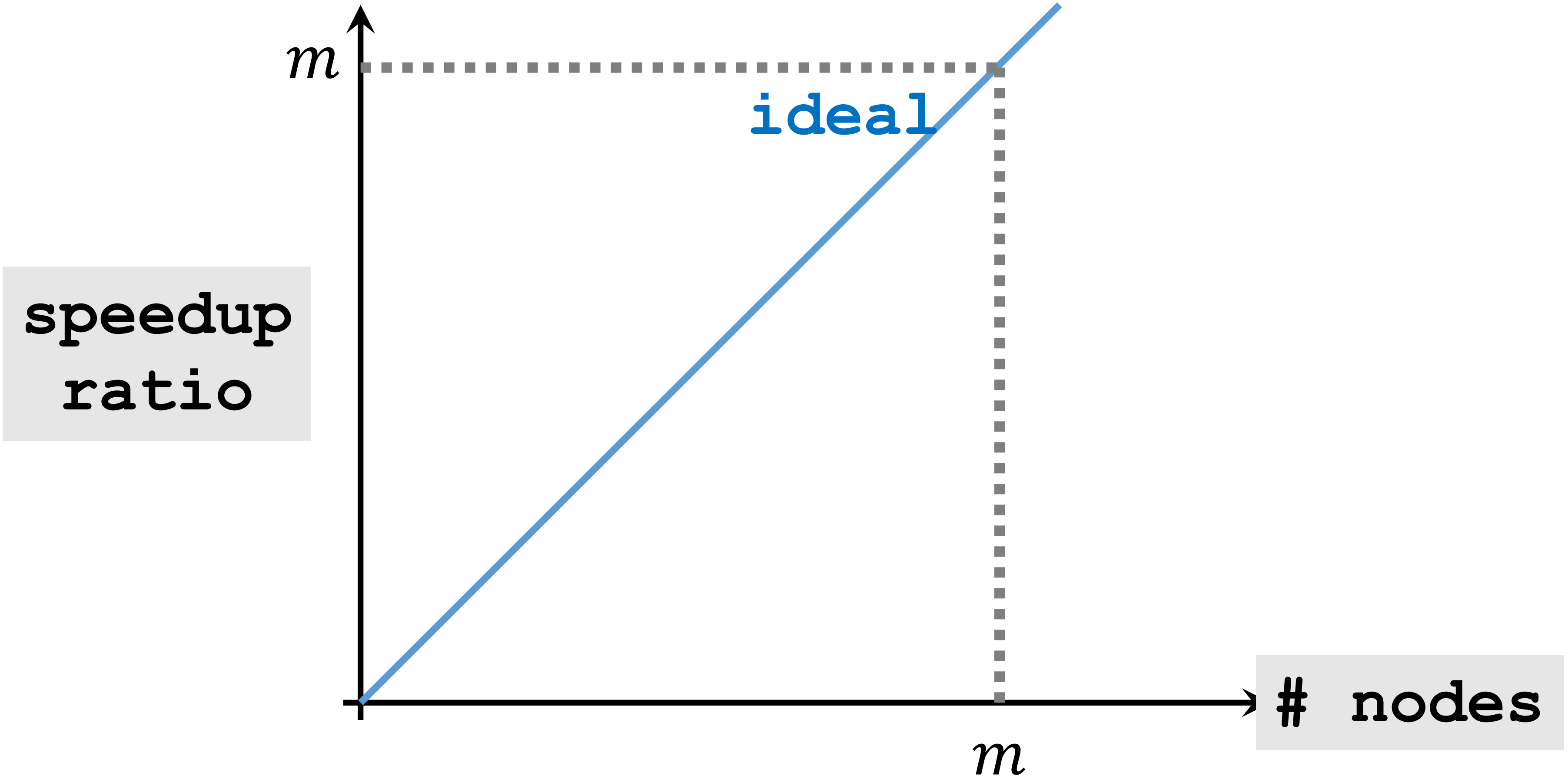


- Every worker stores $\frac{1}{m}$ of the data.
- Every worker does $\frac{1}{m}$ of the computation.
- Is the runtime reduced to $\frac{1}{m}$?
- No. Because communication and synchronization must be considered.

Speedup Ratio

$$\text{speedup ratio} = \frac{\text{wall clock time using one node}}{\text{wall clock time using } m \text{ nodes}}$$



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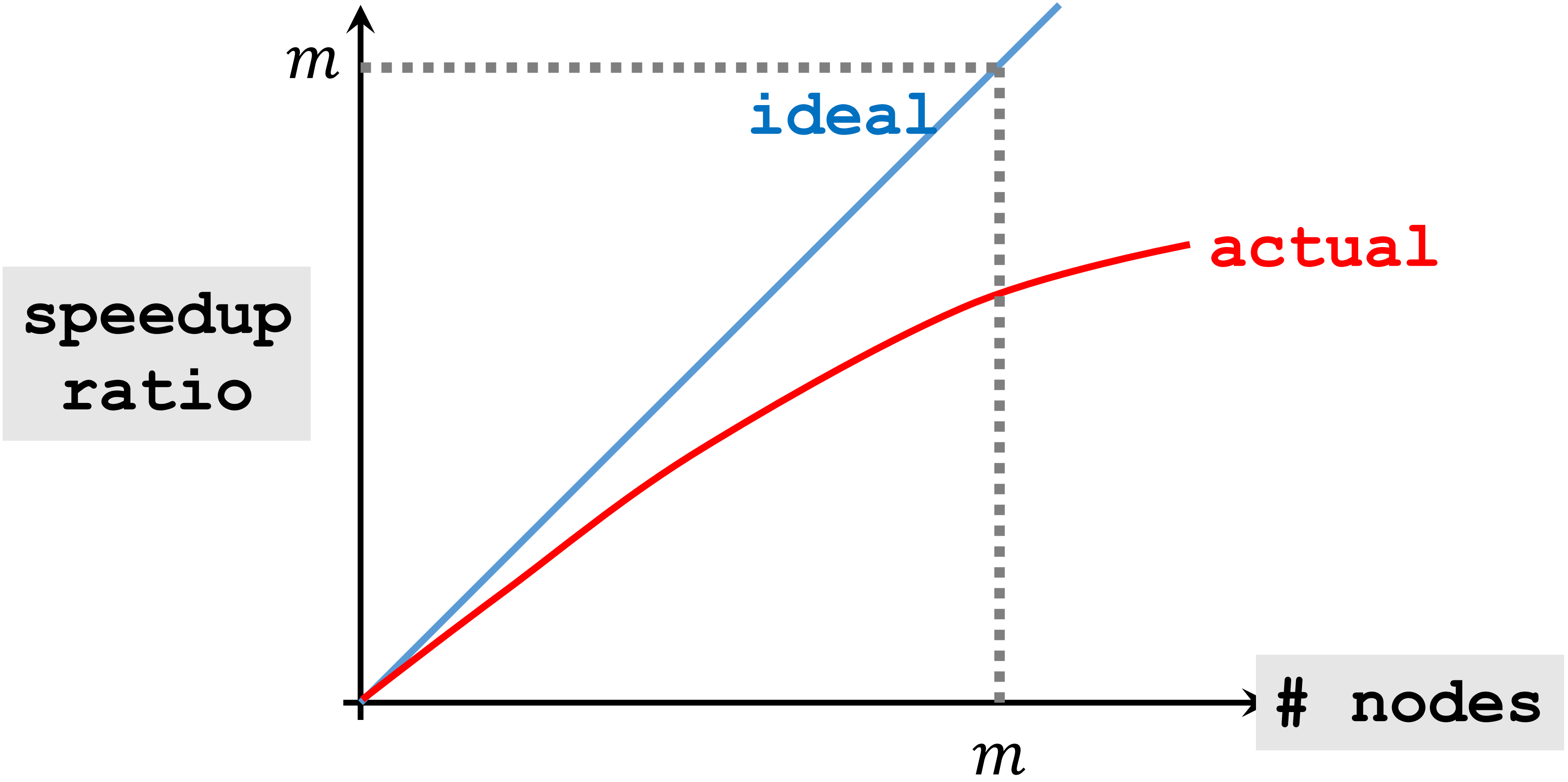

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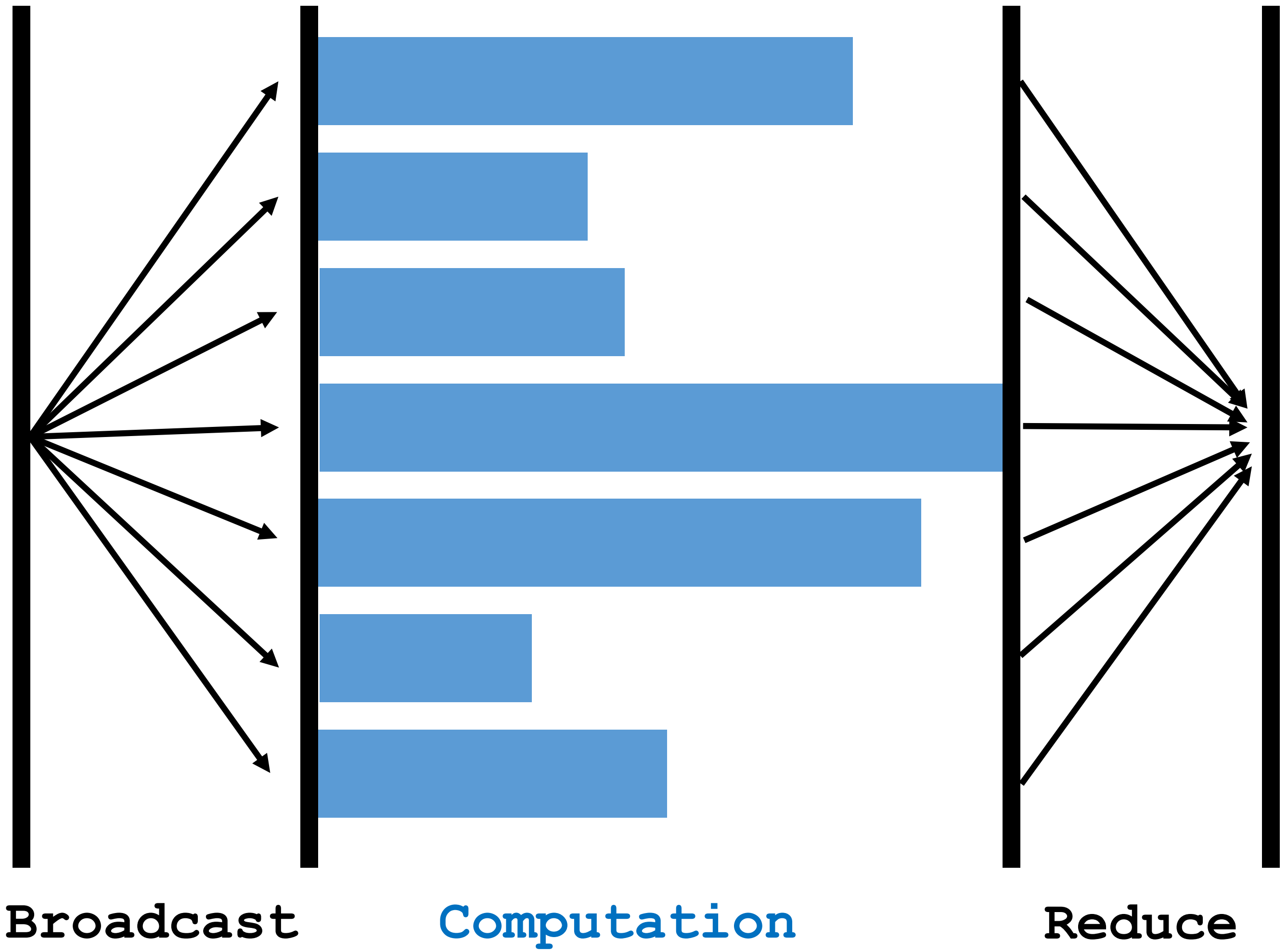
- Communication time: $\frac{\text{complexity}}{\text{bandwith}} + \text{latency}$.

Bulk Synchronous



Synchronization Cost

Question: What if a node fails and then restart?

- This node will be much slower than all the others.
- It is called straggler.
- Straggler effect:
  - The wall-clock time is determined by the slowest node.
  - It is a consequence of synchronization.

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- Data parallelism: Data are partitioned among the workers.
- One gradient descent step requires a broadcast, a map, and a reduce.
- Cost: computation, communication, and synchronization.
- Using m workers, the speedup ratio is lower than m.