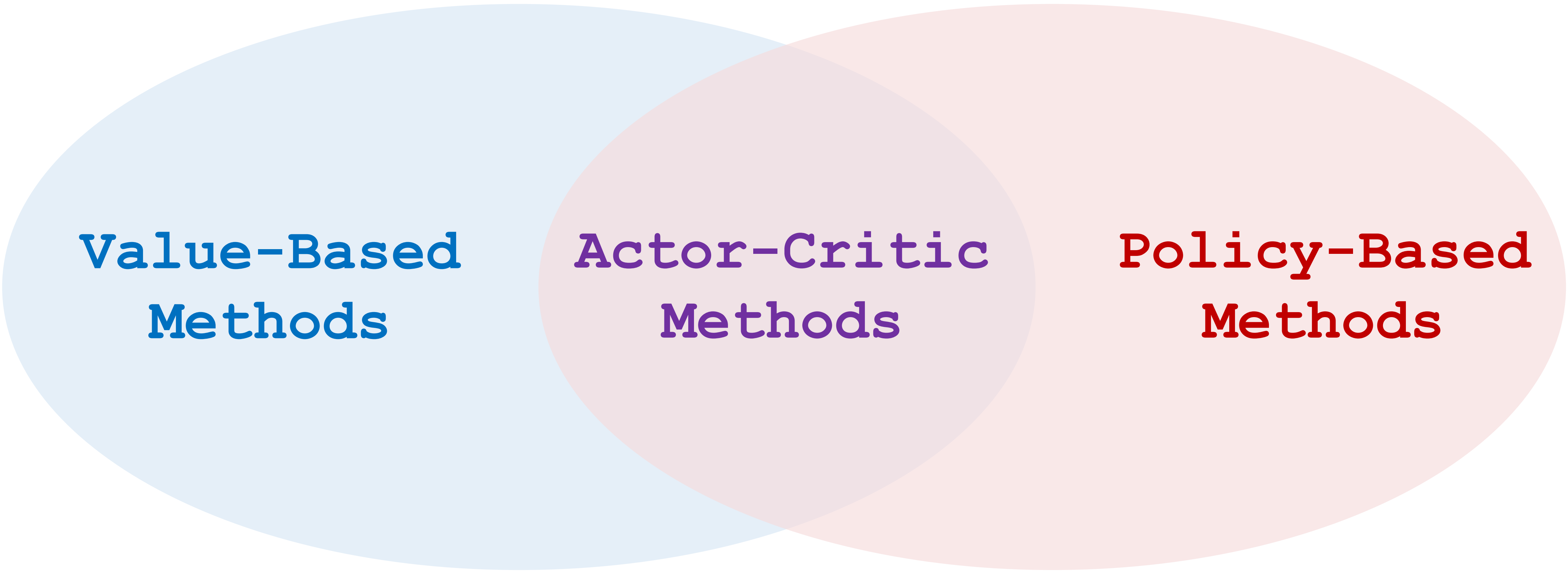
Actor-Critic Methods

Shusen Wang


Value-Based Methods
Actor-Critic Methods
Policy-Based Methods

Value Network and Policy Network

State-Value Function Approximation

Definition: State-value function.

- $V_\pi(s) = \sum_a \pi(a|s) \cdot Q_\pi(s, a)$.

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Policy network (actor):

- Use neural net $\pi(a|s; \boldsymbol{\theta})$ to approximate $\pi(a|s)$.
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Value network (critic):

- Use neural net $q(s, a; \mathbf{w})$ to approximate $Q_\pi(s, a)$.
- $\mathbf{w}$: trainable parameters of the neural net.

State-Value Function Approximation

Definition: State-value function.

- $V_\pi(s) = \sum_a \pi(a|s) \cdot Q_\pi(s,a)$ $\approx \sum_a \pi(a|s;\boldsymbol{\theta}) \cdot q(s,a;\mathbf{w})$.

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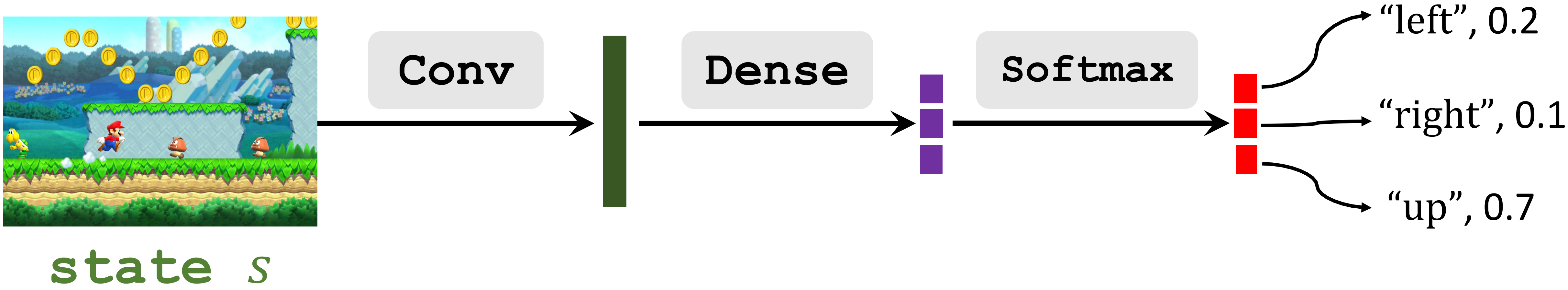
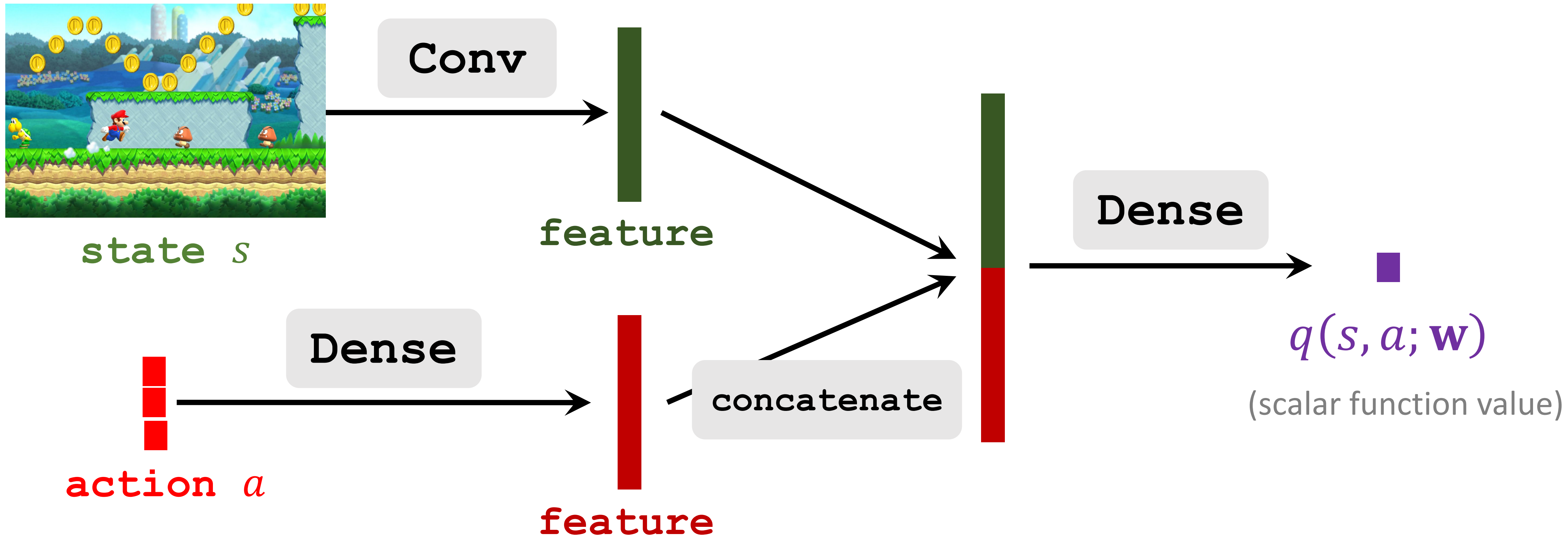
Policy Network (Actor): $\pi(a|s, \boldsymbol{\theta})$

- Input: state s, e.g., a screenshot of Super Mario.
- Output: probability distribution over the actions.
- Let $\mathcal{A}$ be the set all actions, e.g., $\mathcal{A} = \{\text{“left”}, \text{“right”}, \text{“up”}\}$.
- $\sum_{a \in \mathcal{A}} \pi(a|s, \boldsymbol{\theta}) = 1$. (That is why we use softmax activation.)



Value Network (Critic): $q(s, a; \mathbf{w})$

- Inputs: state s and action a.
- Output: approximate action-value (scalar).



Actor-Critic Method

policy network (actor)

value network (critic)

Train the Neural Networks

Train the networks

Definition: State-value function approximated using neural networks.

- $V(s;\boldsymbol{\theta},\mathbf{w}) = \sum_a \pi(a|s;\boldsymbol{\theta}) \cdot q(s,a;\mathbf{w})$.

Training: Update the parameters $\boldsymbol{\theta}$ and $\mathbf{w}$.

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- Update policy network $\pi(a|s;\boldsymbol{\theta})$ to increase the state-value $V(s;\boldsymbol{\theta},\mathbf{w})$.
  - Actor gradually performs better.
  - Supervision is purely from the value network (critic).

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  - Actor gradually performs better.
  - Supervision is purely from the value network (critic).
- Update value network $q(s, a; \mathbf{w})$ to better estimate the return.
  - Critic's judgement becomes more accurate.
  - Supervision is purely from the rewards.

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Training: Update the parameters $\mathbf{\theta}$ and $\mathbf{w}$.

1. Observe the state s_t.
2. Randomly sample action a_t according to $\pi(\cdot\,|s_t; \mathbf{\theta}_t)$.
3. Perform a_t and observe new state s_{t+1} and reward r_t.
4. Update $\mathbf{w}$ (in value network) using temporal difference (TD).
5. Update $\mathbf{\theta}$ (in policy network) using policy gradient.

Update value network q using TD

- Compute $q(s_t, a_t; \mathbf{w}_t)$ and $q(s_{t+1}, a_{t+1}; \mathbf{w}_t)$.
- TD target: $y_t = r_t + \gamma \cdot q(s_{t+1}, a_{t+1}; \mathbf{w}_t)$.

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- Loss: $L(\mathbf{w}) = \frac{1}{2}[q(s_t, a_t; \mathbf{w}) - y_t]^2$.
- Gradient descent: $\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \cdot \frac{\partial\, L(\mathbf{w})}{\partial\, \mathbf{w}} \Big|_{\mathbf{w}=\mathbf{w}_t}$.

Update policy network π using policy gradient

Definition: State-value function approximated using neural networks.

- $V(s;\boldsymbol{\theta},\mathbf{w}) = \sum_a \pi(a|s;\boldsymbol{\theta}) \cdot q(s,a;\mathbf{w})$.

Policy gradient: Derivative of $V(s_t;\boldsymbol{\theta},\mathbf{w})$ w.r.t. $\boldsymbol{\theta}$.

- Let $\mathbf{g}(a,\boldsymbol{\theta}) = \frac{\partial \log \pi(a|s,\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \cdot q(s_t,a;\mathbf{w})$.
- $\frac{\partial\, V(s;\boldsymbol{\theta},\mathbf{w}_t)}{\partial\, \boldsymbol{\theta}} = \mathbb{E}_A[\mathbf{g}(A,\boldsymbol{\theta})]$.

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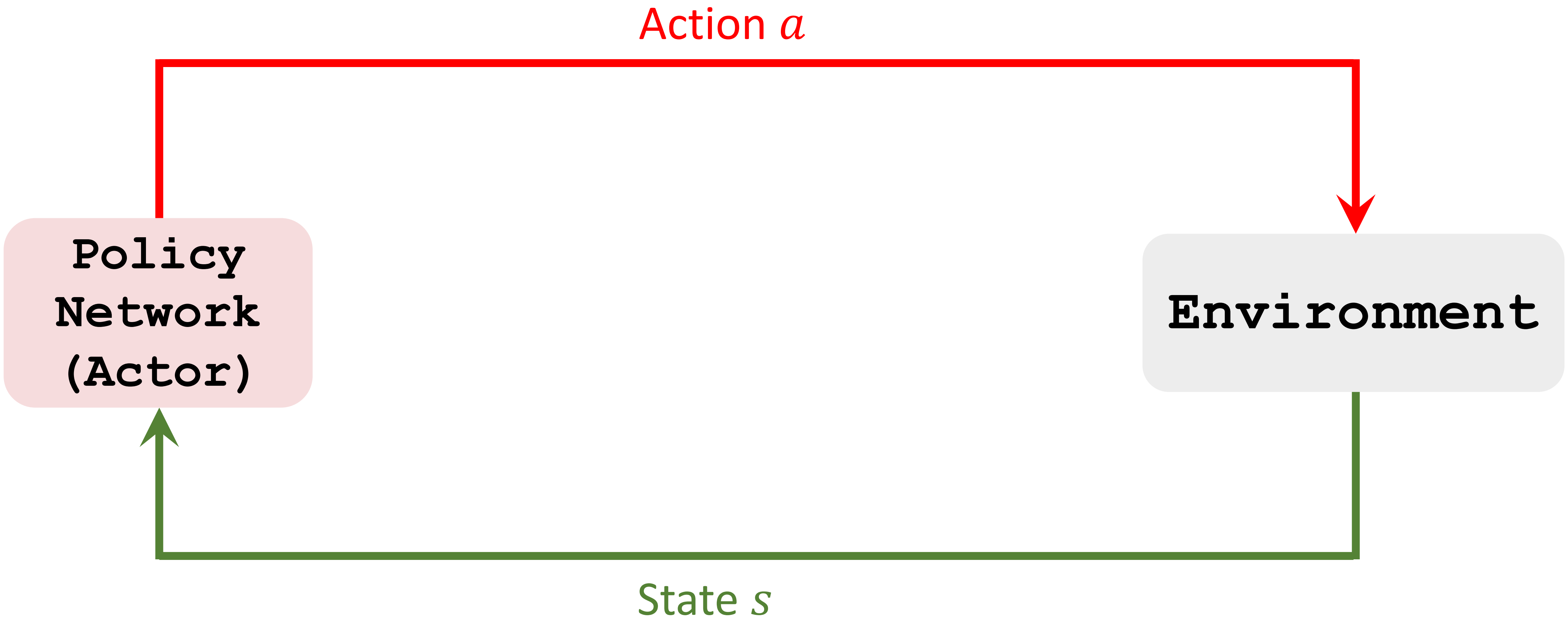
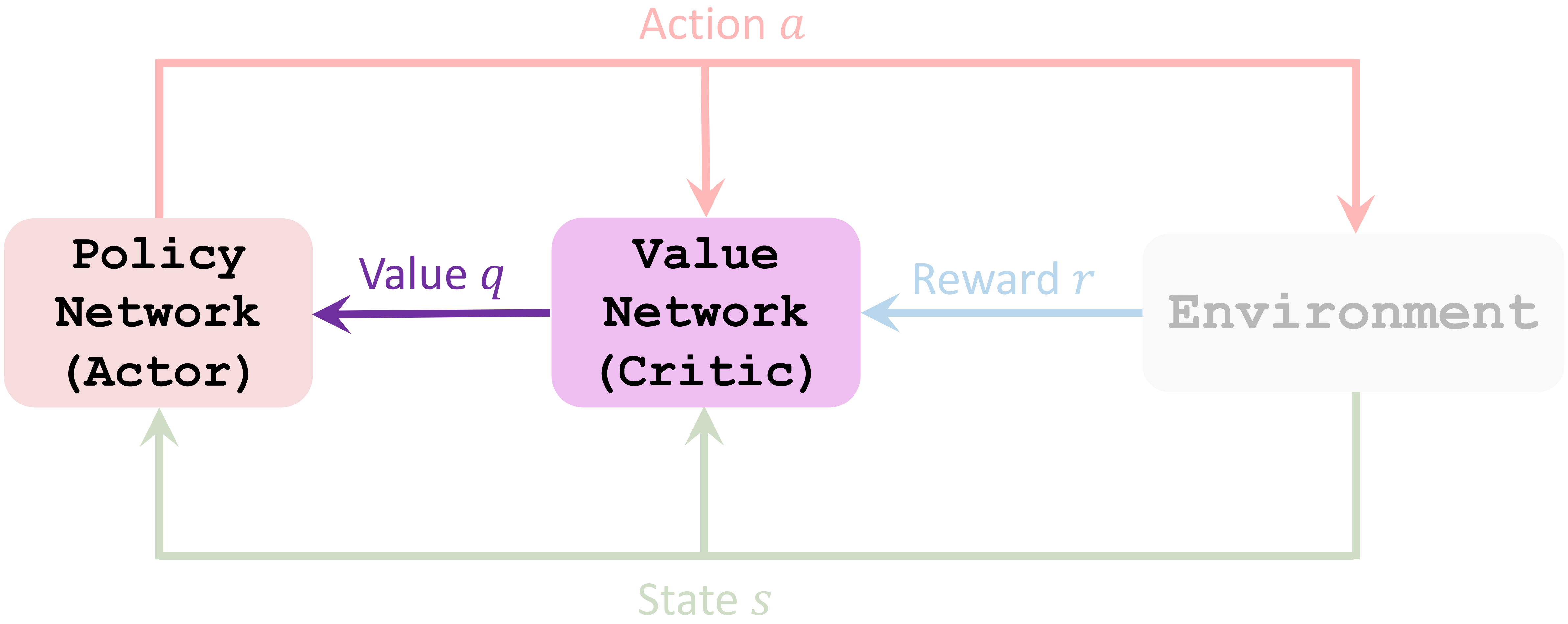
Algorithm: Update policy network using stochastic policy gradient.

- Random sampling: $a \sim \pi(\cdot\,|s_t;\boldsymbol{\theta}_t)$. (Thus $\mathbf{g}(a,\boldsymbol{\theta})$ is unbiased.)
- Stochastic gradient ascent: $\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \beta \cdot \mathbf{g}(a,\boldsymbol{\theta}_t)$.

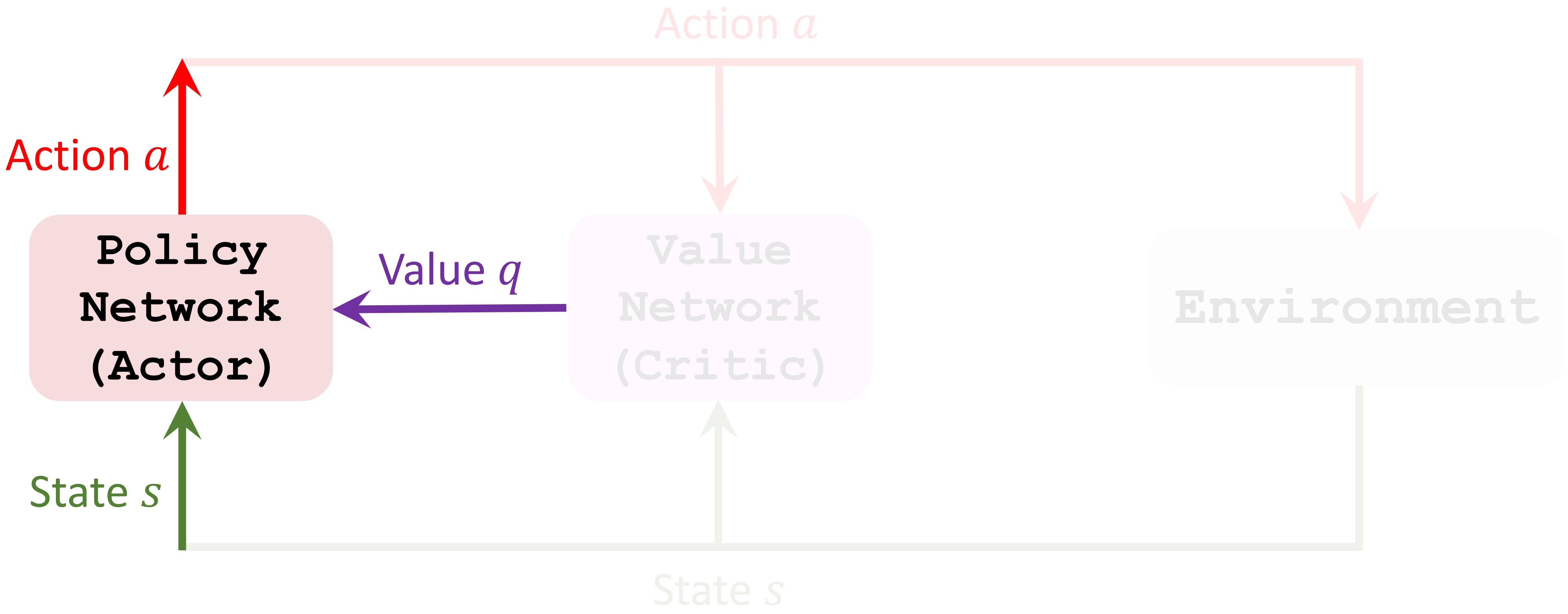
Actor-Critic Method



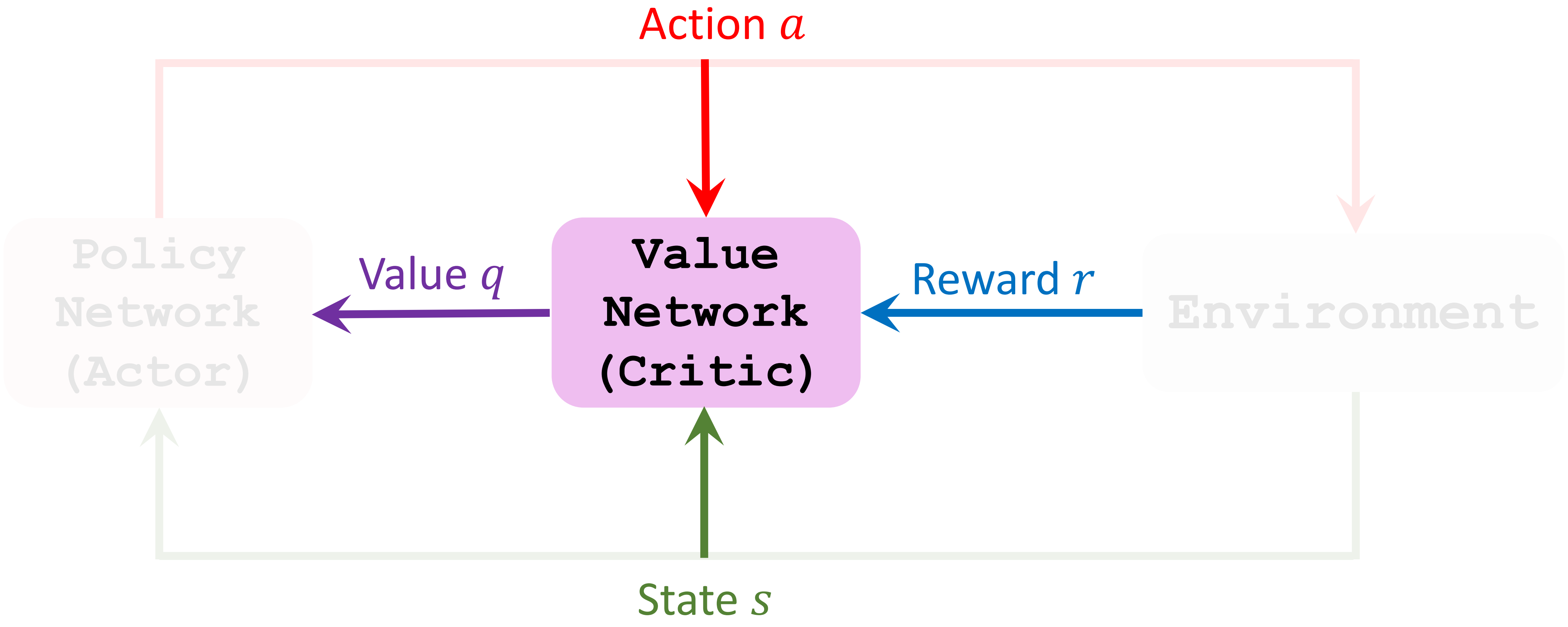
Actor-Critic Method



Actor-Critic Method: Update Actor



Actor-Critic Method: Update Critic



Summary of Algorithm

1. Observe state s_t and randomly sample $a_t \sim \pi(\cdot\,|s_t;\boldsymbol{\theta}_t)$.
2. Perform a_t; then environment gives new state s_{t+1} and reward r_t.
3. Randomly sample $\tilde{a}_{t+1} \sim \pi(\cdot\,|s_{t+1};\boldsymbol{\theta}_t)$. (Do not perform $\tilde{a}_{t+1}$!)

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4. Evaluate value network: $q_t = q(s_t, a_t; \mathbf{w}_t)$ and $q_{t+1} = q(s_{t+1}, \tilde{a}_{t+1}; \mathbf{w}_t)$.
5. Compute TD error: $\delta_t = q_t - \underbrace{(r_t + \gamma \cdot q_{t+1})}_{\text{TD Target}}$.

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6. Differentiate value network: $\mathbf{d}_{w,t} = \frac{\partial q(s_t, a_t; \mathbf{w})}{\partial\, \mathbf{w}} \Big|_{\mathbf{w}=\mathbf{w}_t}$.
7. Update value network: $\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \cdot \delta_t \cdot \mathbf{d}_{w,t}$.

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8. Differentiate policy network: $\mathbf{d}_{\theta,t} = \frac{\partial \log \pi(a_t|s_t, \boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}_t}$.
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Policy Gradient with Baseline

1. Observe state s_t and randomly sample $a_t \sim \pi(\cdot\,|s_t;\boldsymbol{\theta}_t)$.
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3. Randomly sample $\tilde{a}_{t+1} \sim \pi(\cdot\,|s_{t+1};\boldsymbol{\theta}_t)$. (Do not perform $\tilde{a}_{t+1}$!)
4. Evaluate value network: $q_t = q(s_t, a_t; \mathbf{w}_t)$ and $q_{t+1} = q(s_{t+1}, \tilde{a}_{t+1}; \mathbf{w}_t)$.
5. Compute TD error: $\delta_t = q_t - \underbrace{(r_t + \gamma \cdot q_{t+1})}_{\text{Baseline}}$.
6. Differentiate value network: $\mathbf{d}_{w,t} = \frac{\partial q(s_t, a_t, \mathbf{w})}{\partial\, \mathbf{w}}\Big|_{\mathbf{w}=\mathbf{w}_t}$.
7. Update value network: $\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \cdot \delta_t \cdot \mathbf{d}_{w,t}$.
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Summary

Policy Network and Value Network

Definition: State-value function.

- $V_\pi(s) = \sum_a \pi(a|s) \cdot Q_\pi(s, a)$.

Definition: function approximation using neural networks.

- Approximate policy function $\pi(a|s)$ by $\pi(a|s; \boldsymbol{\theta})$ (actor).
- Approximate value function $Q_\pi(s, a)$ by $q(s, a; \mathbf{w})$ (critic).

Roles of Actor and Critic

During training

- Agent is controlled by policy network (actor): $a_t \sim \pi(\cdot \mid s_t; \boldsymbol{\theta})$.
- Value network q (critic) provides the actor with supervision.

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- Agent is controlled by policy network (actor): $a_t \sim \pi(\cdot\,|s_t;\boldsymbol{\theta})$.
- Value network q (critic) provides the actor with supervision.

After training

- Agent is controlled by policy network (actor): $a_t \sim \pi(\cdot\,|s_t;\boldsymbol{\theta})$.
- Value network q (critic) will not be used.

Training

Learning: Update the policy network (actor) by policy gradient.

- Seek to increase state-value: $V(s; \mathbf{\theta}, \mathbf{w}) = \sum_a \pi(a|s; \mathbf{\theta}) \cdot q(s, a; \mathbf{w})$.
- Compute policy gradient: $\frac{\partial\, V(s;\mathbf{\theta})}{\partial\, \mathbf{\theta}} = \mathbb{E}_A \left[\frac{\partial \log \pi(A|s,\mathbf{\theta})}{\partial\, \mathbf{\theta}} \cdot q(s, A; \mathbf{w}) \right]$.
- Perform gradient ascent.

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- Perform gradient ascent.

Learning: Update the value network (critic) by TD learning.

- Predicted action-value: $q_t = q(s_t, a_t; \mathbf{w})$.
- TD target: $y_t = r_t + \gamma \cdot q(s_{t+1}, a_{t+1}; \mathbf{w})$
- Gradient: $\frac{\partial (q_t - y_t)^2/2}{\partial\, \mathbf{w}} = (q_t - y_t) \cdot \frac{\partial\, q(s_t,a_t;\mathbf{w})}{\partial\, \mathbf{w}}$.
- Perform gradient descent.

Thank you!