Reinforcement Learning

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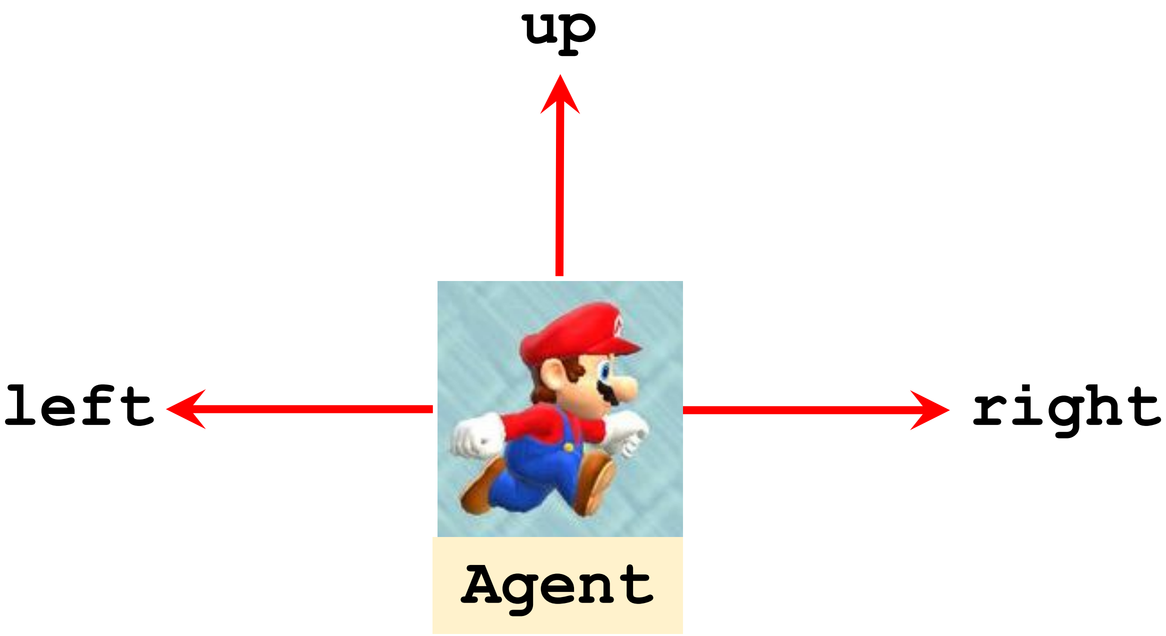
Terminologies

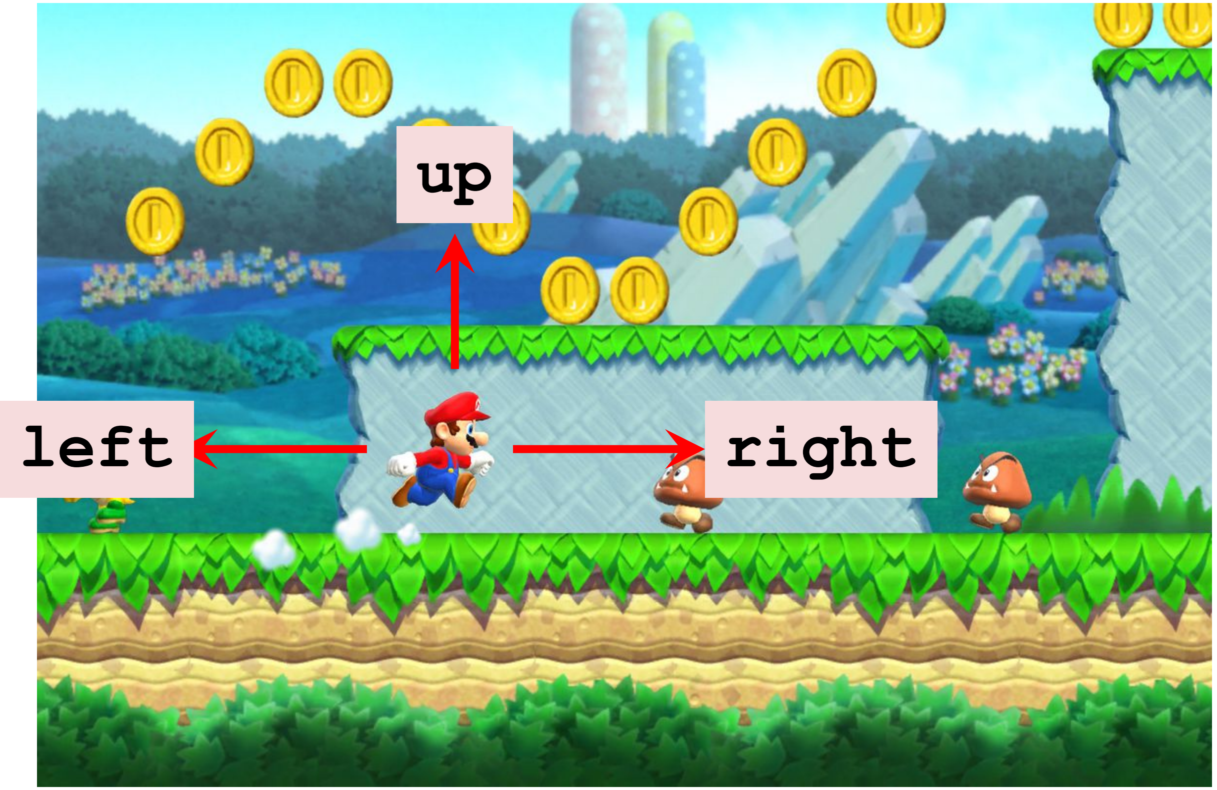
Terminology: state and action

state s (this frame)

Action $a \in \{\text{left}, \text{right}, \text{up}\}$



Terminology: policy



policy π

- Policy function $\pi: (s, a) \mapsto [0,1]$:

$$\pi(a \mid s) = \mathbb{P}(A = a \mid S = s).$$

- It is the probability of taking action a given state s and policy π.
- Upon observing state s, the agent's action a can be random, e.g.,
  - $\pi(\text{left} \mid s) = 0.2$,
  - $\pi(\text{right} \mid s) = 0.1$,
  - $\pi(\text{up} \mid s) = 0.7$.

Terminology: reward

reward r

- Collect a coin: $r = +1$

Terminology: reward

reward r

- Collect a coin: $r = +1$
- Win the game: $r = +10000$

Terminology: reward

reward r

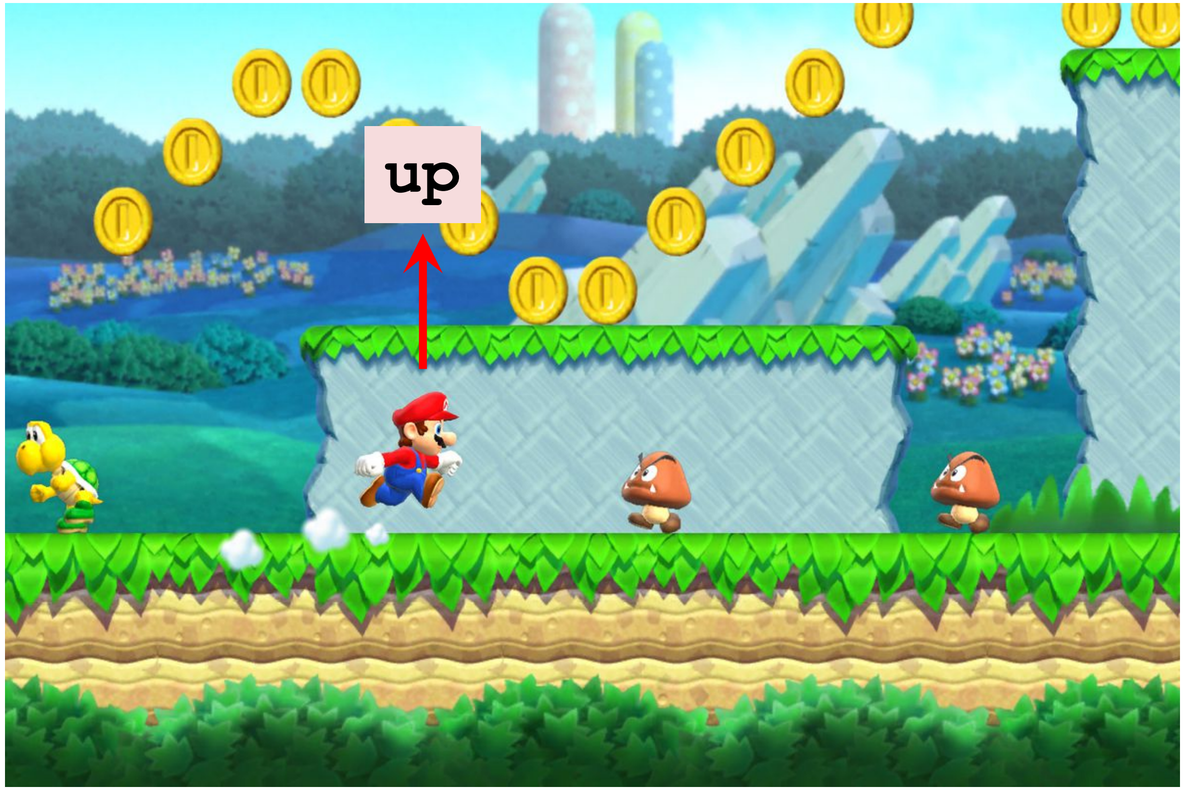
- Collect a coin: $r = +1$
- Win the game: $r = +10000$
- Touch a Goomba: $r = -10000$ (game over).

Terminology: reward

reward r

- Collect a coin: $r = +1$
- Win the game: $r = +10000$
- Touch a Goomba: $r = -10000$ (game over).
- Nothing happens: $r = 0$

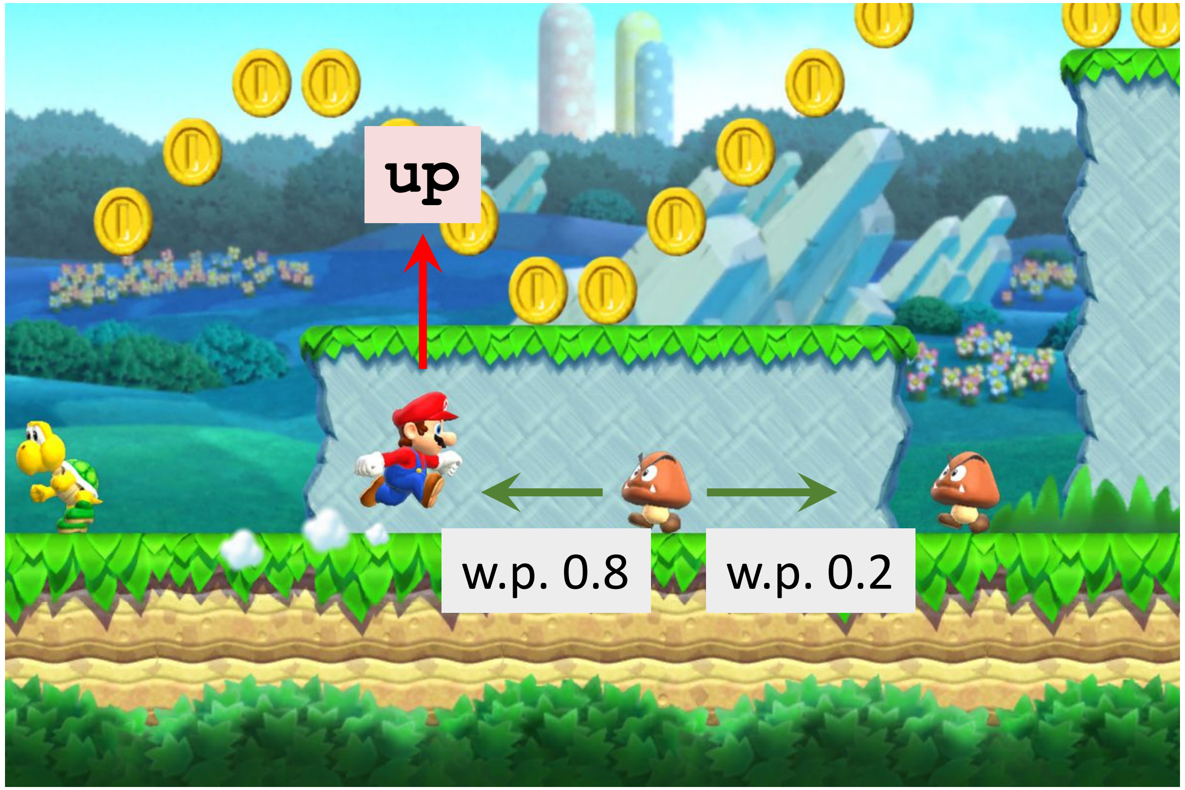
Terminology: state transition



state transition

- $(s, a) \mapsto s'$.
- E.g., "up" action leads to a new state.

Terminology: state transition

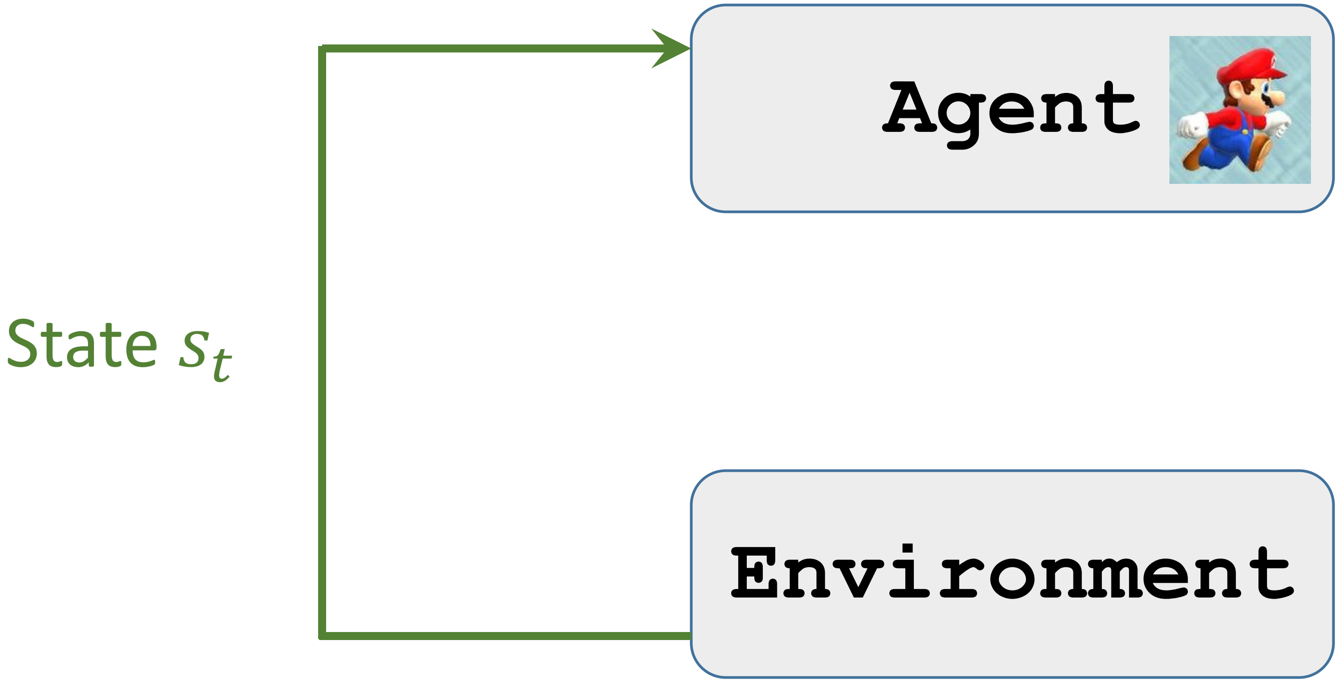


state transition

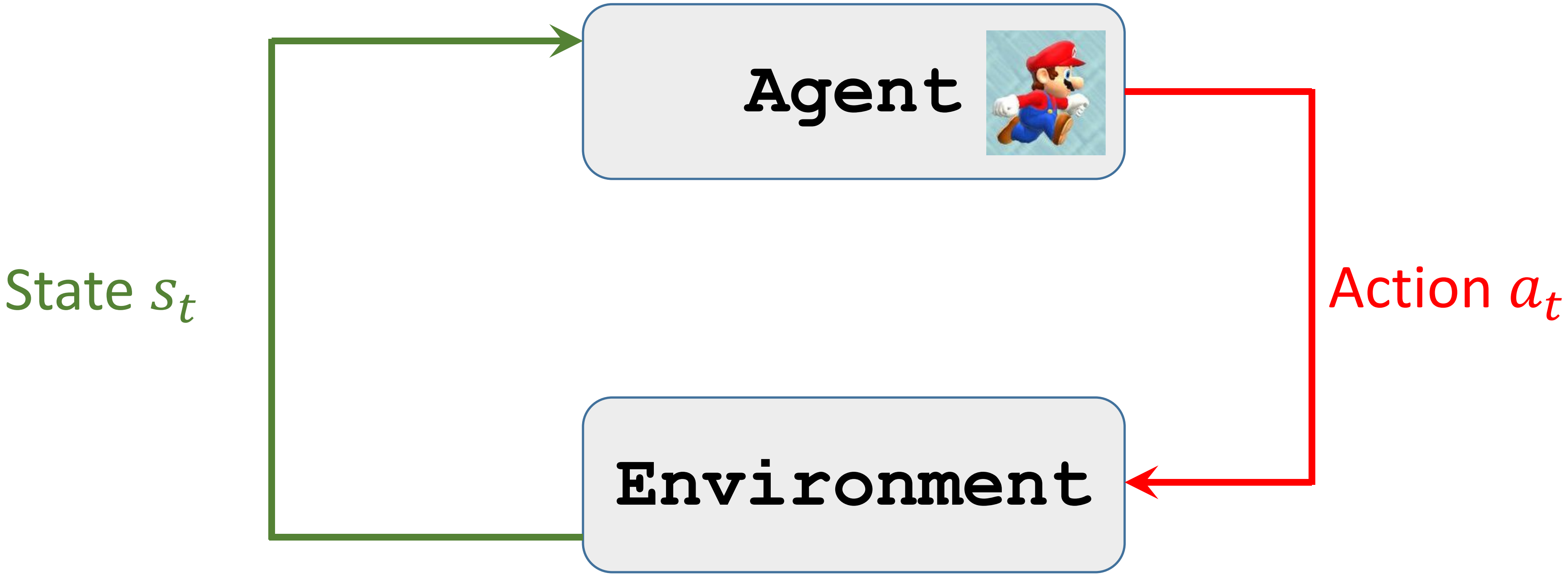
- $(s, a) \mapsto s'$.
- E.g., "up" action leads to a new state.

- State transition can be random.
- Randomness is from the environment.
- $p(s'|s, a) = \mathbb{P}(S' = s'|S = s, A = a)$.

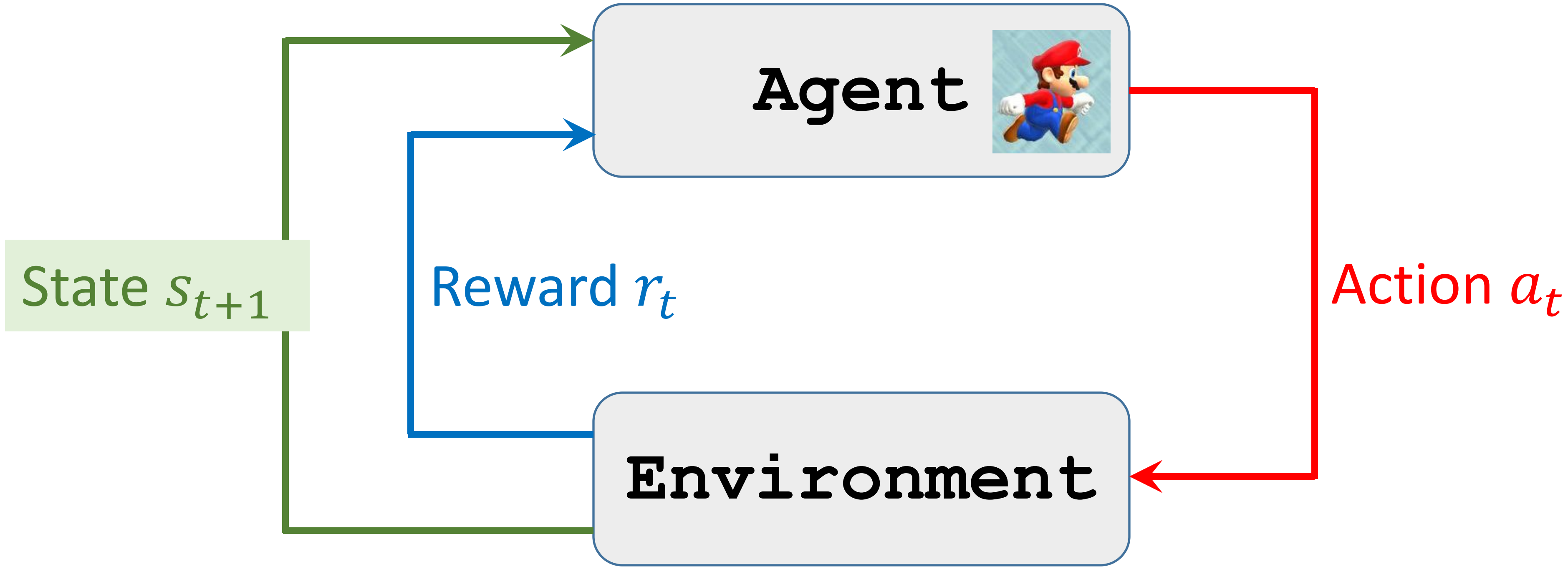
Terminology: agent environment interaction



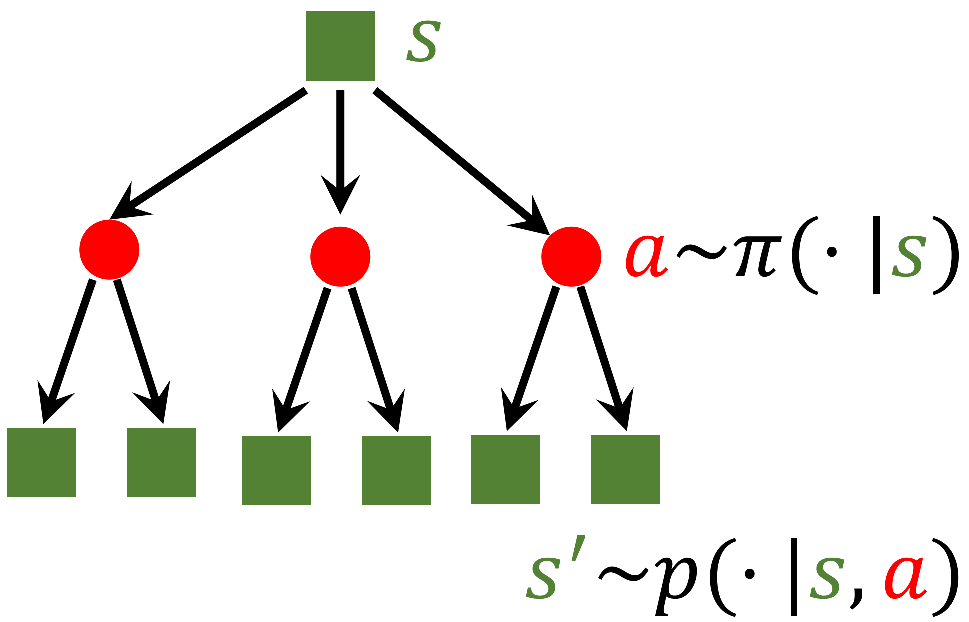
Terminology: agent environment interaction



Terminology: agent environment interaction



Randomness in Reinforcement Learning



Policy $\pi(a|s)$ can be random.

- Given state s and policy π, the action a can be random, e.g., .
  - $\pi(\text{“left”}|s) = 0.2$,
  - $\pi(\text{“right”}|s) = 0.1$,
  - $\pi(\text{“up”}|s) = 0.7$.

State transition $p(s'|s, a)$ can be random.

- Given state s and action a, the environment randomly generates a new state s'.

Play the game using AI

- Observe a frame (state s_0)
- ➔ Make action a_0 (left, right, or up)
- ➔ Observe a new frame (state s_1) and reward r_0
- ➔ Make action a_1
- ➔ …

- (state, action, reward) trajectory:

$$s_0, a_0, r_0, s_1, a_1, r_1, \quad \cdots \quad , s_T, a_T, r_T.$$

Rewards and Returns

Return

Definition: Return (aka cumulative future reward).

- $R_t = r_t + r_{t+1} + r_{t+2} + r_{t+3} + \cdots$ (to infinity.)

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Question: Are r_t and r_{t+1} equally important?

- Which of the followings do you prefer?
  - I give you \$100 right now.
  - I will give you \$100 one year later.

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- $R_t = r_t + r_{t+1} + r_{t+2} + r_{t+3} + \cdots$ (to infinity.)

Question: Are r_t and r_{t+1} equally important?

- Which of the followings do you prefer?
  - I give you \$100 right now.
  - I will give you \$100 one year later.

- Future value is less valuable than present value.
- r_{t+1} should be given less weight than r_t.

Discounted Return

Definition: Discounted return (aka cumulative discounted future reward).

- γ: discount rate (tuning hyper-parameter).
- $R_t = r_t + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \gamma^3\, r_{t+3} + \cdots$ (to infinity.)

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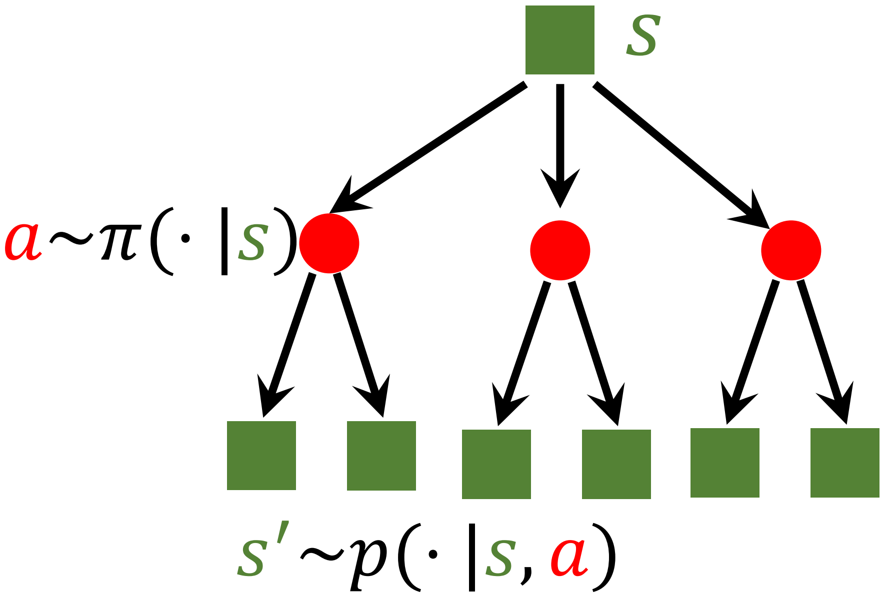
R_t can be random. (Two sources of randomness.)

1. Action can be random: $a \sim \pi(\cdot\,|s)$, where
$$\pi(a|s) = \mathbb{P}(A = a \,|\, S = s).$$
2. State transition can be random: $s' \sim p(\cdot\,|s, a)$, where
$$p(s'|s, a) = \mathbb{P}(S' = s' \,|\, S = s, A = a).$$



Value Functions

Action-Value Function $Q(s, a)$

Definition: Discounted return (aka cumulative discounted future reward).

- $R_t = r_t + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \gamma^3\, r_{t+3} + \cdots$ (to infinity.)

Definition: Action-value function for policy π.

- $Q_\pi(s_t, a_t) = \mathbb{E}\,[R_t | s_t, a_t, \pi]$.

  - Taken w.r.t. actions $a_{t+1}, a_{t+2}, a_{t+3}, \cdots$ and states $s_{t+1}, s_{t+2}, s_{t+3}, \cdots$
  - Actions are randomly sampled: $a_t \sim \pi(\cdot\, | s_t)$. (Policy function.)
  - States are randomly sampled: $s_{t+1} \sim p(\cdot\, | s_t, a_t)$. (State transition.)

Action-Value Function $Q(s, a)$

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- $R_t = r_t + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \gamma^3\, r_{t+3} + \cdots$ (to infinity.)

Definition: Action-value function for policy π.

- $Q_\pi(s_t, a_t) = \mathbb{E}\left[R_t | s_t, a_t, \pi\right]$.

Definition: Optimal action-value function.

- $Q^\star(s_t, a_t) = \max_\pi Q_\pi(s_t, a_t)$.

State-Value Function $V(s, \pi)$

Definition: Discounted return (aka cumulative discounted future reward).

- $R_t = r_t + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \gamma^3\, r_{t+3} + \cdots$ (to infinity.)

Definition: Action-value function for policy π.

- $Q_\pi(s_t, a_t) = \mathbb{E}\,[R_t | s_t, a_t, \pi]$.

Definition: State-value function.

- $V(s_t, \pi) = \mathbb{E}_{a_t}\,[Q_\pi(s_t, a_t)]$.
    - Taken w.r.t. the action $a_t \sim \pi(\cdot\,|s_t)$.

State-Value Function $V(s, \pi)$

Definition: Discounted return (aka cumulative discounted future reward).

- $R_t = r_t + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \gamma^3\, r_{t+3} + \cdots$ (to infinity.)

Definition: Action-value function for policy π.

- $Q_\pi(s_t, a_t) = \mathbb{E}\,[R_t | s_t, a_t, \pi]$.

Definition: State-value function.

- $V(s_t, \pi) = \mathbb{E}_{a_t}\,[Q_\pi(s_t, a_t)] = \sum_a \pi(a|s_t) \cdot Q_\pi(s_t, a)$.

Explaining Value Functions

- Optimal action-value function: $Q^{\star}(s_t, a_t) = \max_{\pi} Q_{\pi}(s_t, a_t)$.
- How good it is for an agent to pick action a_t while being in state s_t.

- State-value function: $V(s_t, \pi) = \mathbb{E}\left[Q_{\pi}(s_t, a_t)\right]$
- For policy π, how good the situation is in state s_t.

Goals

Goal of agent: Make actions $a_1, a_2, a_3, \cdots$ to maximize the return R_t.

Goal of reinforcement learning: Guide the agent to make good actions.

Suppose we have a good policy $\pi(a|s)$.

- Upon observe the state s_t,
- random sampling: $a_t \sim \pi(\cdot\,|s_t)$.

Goals

Goal of agent: Make actions $a_1, a_2, a_3, \cdots$ to maximize the return R_t.

Goal of reinforcement learning: Guide the agent to make good actions.

Suppose we have a good policy $\pi(a|s)$.

Alternatively, suppose we know the optimal value function $Q^{\star}(s, a)$.

- Upon observe the state s_t,
- choose the action that maximizes the value: $a_t = \text{argmax}_a \, Q^{\star}(s_t, a)$.

Goals

Goal of agent: Make actions $a_1, a_2, a_3, \cdots$ to maximize the return R_t.

Goal of reinforcement learning: Guide the agent to make good actions.

Suppose we have a good policy $\pi(a|s)$.

Alternatively, suppose we know the optimal value function $Q^\star(s, a)$.

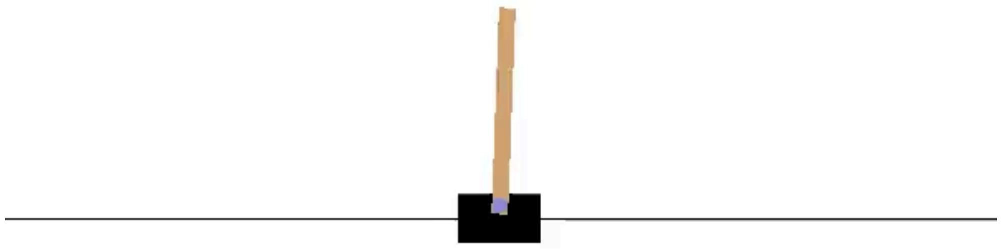
Learn either $\pi(a|s)$ or $Q^\star(s, a)$ from the rewards: $r_1, r_2, r_3, \cdots$

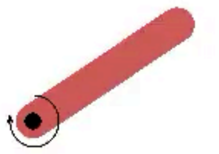
Play games using reinforcement learning

OpenAI Gym

- Gym is a toolkit for developing and comparing reinforcement learning algorithms.
- https://gym.openai.com/

Classical control problems





Cart Pole	Pendulum

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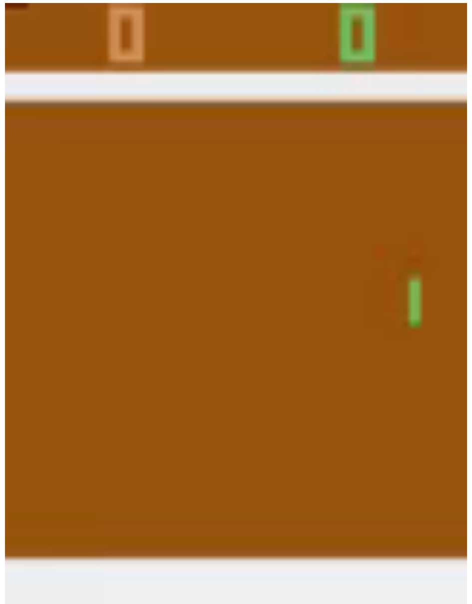
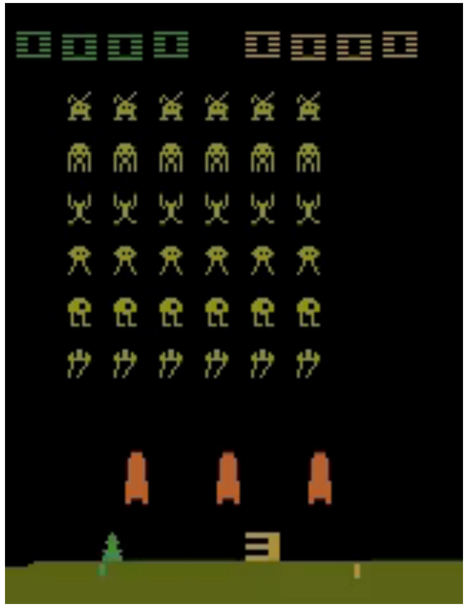
Atari Games		
		
Pong	Space Invader	Breakout

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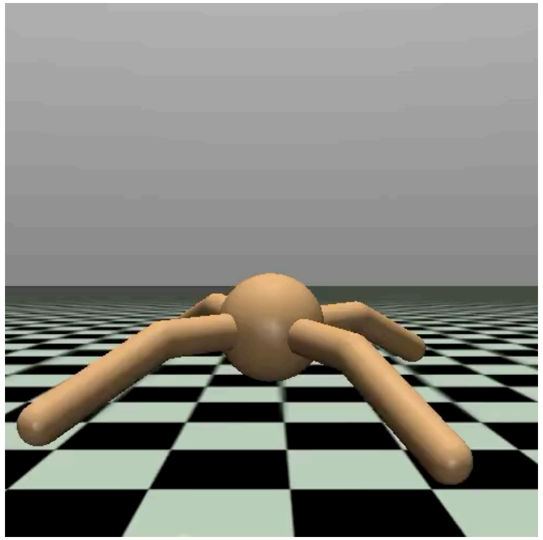
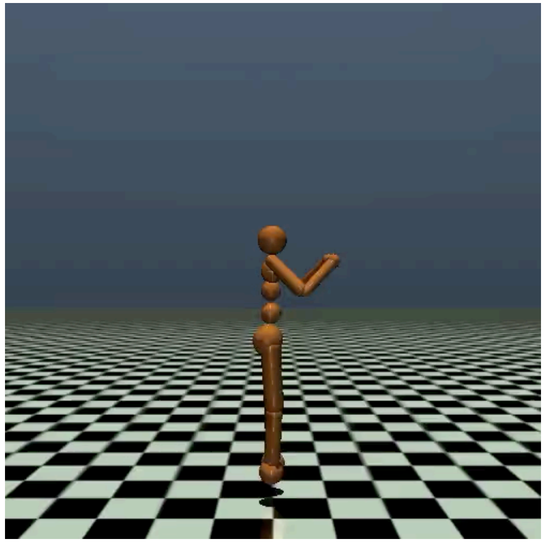
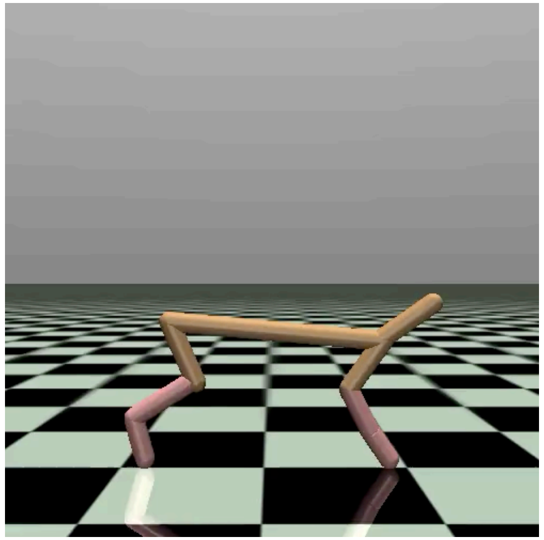
MuJoCo (Continuous control tasks.)



Ant



Humanoid

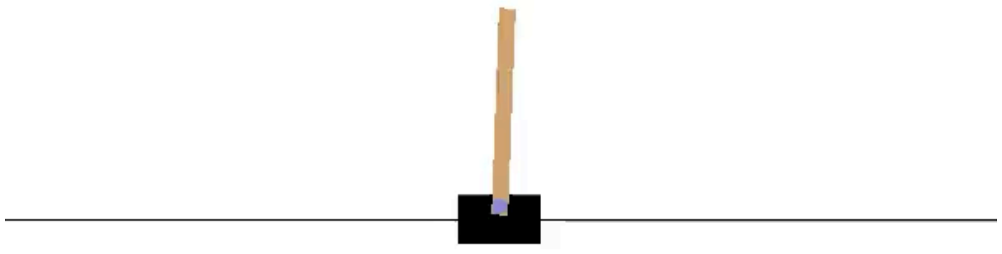


Half Cheetah

Play CartPole Game

```python
import gym
env = gym.make('CartPole-v0')
```

- Get the environment of CartPole from Gym.
- “env” provides states and reward.



OpenAI Gym

```
state = env.reset()

for t in range(100):
    env.render()
    print(state)

    action = env.action_space.sample()
    state, reward, done, info = env.step(action)

    if done:
        print('Finished')
        break

env.close()
```

A window pops up rendering CartPole.

A random action.

“done=0” means finished (win or lose the game)

Summary

Summary

Terminologies

- Agent 
- Environment
- State s.
- Action a.
- Reward r.
- Policy $\pi(a|s)$
- State transition $p(s'|s,a)$.

Return and Value

- Return:

$$R_t = r_t + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \cdots$$

- Action-value function:

$$Q_\pi(s,a) = \mathbb{E}\,[R_t|s,a,\pi].$$

- Optimal action-value function:

$$Q^\star(s,a) = \max_\pi Q_\pi(s,a).$$

- State-value function:

$$V(s,\pi) = \mathbb{E}\,[R_t|s,\pi].$$

Play game using reinforcement learning

- Observe state s_t, make action a_t, environment gives s_{t+1} and reward r_t.

$$
\begin{tikzcd}
s_t \arrow[r] & a_t \arrow[r] \arrow[dr] & s_{t+1} \arrow[r] & a_{t+1} \arrow[r] \arrow[dr] & s_{t+2} \arrow[r] & a_{t+2} \arrow[r] \arrow[dr] & s_{t+3} & \cdots \\
& & r_t & & r_{t+1} & & r_{t+2} &
\end{tikzcd}
$$

- Suppose we know either policy function $\pi(a|s)$ or the optimal action-value function $Q^\star(s,a)$.
- Then action a can be make according to $\pi(a|s)$ or $Q^\star(s,a)$.

We are going to study…

- Deep Q network (DQN) for approximating $Q^{\star}(s, a)$.
- Learn the network parameters using temporal different (TD).

- Policy network for approximating $\pi(a|s)$.
- Learn the network parameters using policy gradient.
- Actor-critic method. (Policy network + value network.)