Policy-Based Reinforcement Learning

Shusen Wang

Policy Function Approximation

Policy Function $\pi(a|s)$

- Policy function $\pi(a|s)$ is a probability density function (PDF).
- It takes state s as input.
- It output the probabilities for all the actions, e.g.,

$$\pi(\text{left}|s) = 0.2,$$
$$\pi(\text{right}|s) = 0.1,$$
$$\pi(\text{up}|s) = 0.7.$$

- The agent performs an action a random drawn from the distribution.

Can we directly learn a policy function $\pi(a|s)$?

- If there are only a few states and actions, then yes, we can.
- Draw a table (matrix) and learn the entries.

	Action a_1	Action a_2	Action a_3	Action a_4	$\cdots$
State s_1					
State s_2					
State s_3					
$\vdots$					

Can we directly learn a policy function $\pi(a|s)$?

- If there are only a few states and actions, then yes, we can.
- Draw a table (matrix) and learn the entries.
- What if there are too many (or infinite) states or actions?

	Action a_1	Action a_2	Action a_3	Action a_4	$\cdots$
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Policy Network $\pi(a|s; \boldsymbol{\theta})$

Policy network: Use a neural net to approximate $\pi(a|s)$.

- Use policy network $\pi(a|s; \boldsymbol{\theta})$ to approximate $\pi(a|s)$.
- $\boldsymbol{\theta}$: trainable parameters of the neural net.

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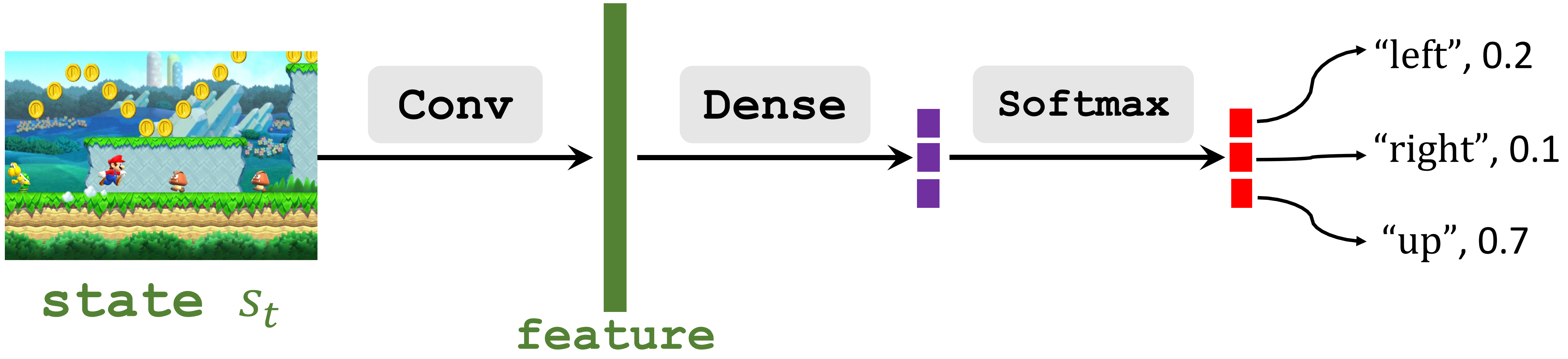
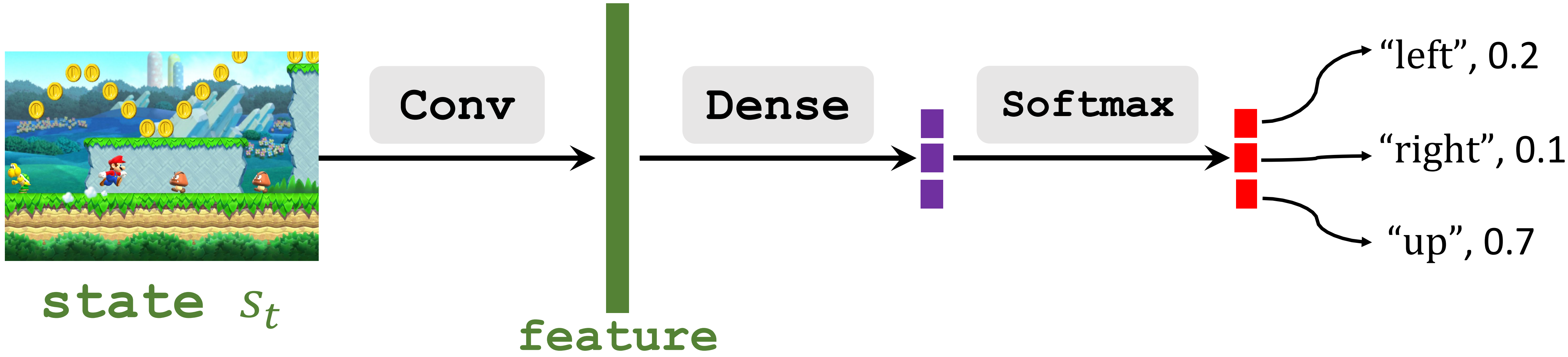


Policy Network $\pi(a|s;\boldsymbol{\theta})$

- $\sum_{a\in\mathcal{A}} \pi(a|s;\boldsymbol{\theta}) = 1.$
- Here, $\mathcal{A} = \{\text{“left”}, \text{“right”}, \text{“up”}\}$ is the set all actions.
- That is why we use softmax activation.



State-Value Function Approximation

Action-Value Function

Definition: Discounted return.

- $U_t = R_t + \gamma \cdot R_{t+1} + \gamma^2 \cdot R_{t+2} + \gamma^3 \cdot R_{t+3} + \cdots$

- The return depends on actions $A_t, A_{t+1}, A_{t+2}, \cdots$ and states $S_t, S_{t+1}, S_{t+2}, \cdots$
- Actions are random: $\mathbb{P}[A = a \mid S = s] = \pi(a|s)$. (Policy function.)
- States are random: $\mathbb{P}[S' = s'|S = s, A = a] = p(s'|s, a)$. (State transition.)

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- $Q_\pi(s_t, a_t) = \mathbb{E}\left[U_t | S_t = s_t, A_t = a_t\right].$

The expectation is taken w.r.t.
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Definition: State-value function.

- $V_\pi(s_t) = \mathbb{E}_A\left[Q_\pi(s_t, A)\right]$

Integrate out action $A \sim \pi(\cdot | s_t)$.

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- $V_\pi(s_t) = \mathbb{E}_A\left[Q_\pi(s_t, A)\right] = \sum_a \pi(a|s_t) \cdot Q_\pi(s_t, a).$

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- $V_\pi(s_t) = \mathbb{E}_A\left[Q_\pi(s_t, A)\right] = \sum_a \pi(a|s_t) \cdot Q_\pi(s_t, a)$.

Approximate state-value function.

- Approximate policy function $\pi(a|s_t)$ by policy network $\pi(a|s_t; \mathbf{\theta})$.

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Approximate state-value function.

- Approximate policy function $\pi(a|s_t)$ by policy network $\pi(a|s_t; \boldsymbol{\theta})$.
- Approximate value function $V_\pi(s_t)$ by:

$$V(s_t; \boldsymbol{\theta}) = \sum_a \pi(a|s_t; \boldsymbol{\theta}) \cdot Q_\pi(s_t, a).$$

Policy-Based Reinforcement Learning

Definition: Approximate state-value function.

- $V(s;\boldsymbol{\theta}) = \sum_a \pi(a|s;\boldsymbol{\theta}) \cdot Q_\pi(s,a)$.

Policy-based learning: Learn $\boldsymbol{\theta}$ that maximizes $J(\boldsymbol{\theta}) = \mathbb{E}_S[V(S;\boldsymbol{\theta})]$.

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How to improve $\boldsymbol{\theta}$? Policy gradient ascent!

- Observe state s.
- Update policy by: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \beta \cdot \boxed{\frac{\partial\, V(s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}}}$.

Policy gradient

Policy Gradient

Reference

- Sutton and others: Policy gradient methods for reinforcement learning with function approximation. In *NIPS*, 2000.

Policy Gradient

Definition: Approximate state-value function.

- $V(s; \mathbf{\theta}) = \sum_a \pi(a|s; \mathbf{\theta}) \cdot Q_\pi(s, a)$.

Policy gradient: Derivative of $V(s; \mathbf{\theta})$ w.r.t. $\mathbf{\theta}$.

- $\frac{\partial\, V(s;\mathbf{\theta})}{\partial\, \mathbf{\theta}}$

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Push derivative inside the summation

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Pretend Q_π is independent of $\boldsymbol{\theta}$.
(It may not be true.)

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Policy Gradient: Form 1

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Note: This derivation is over-simplified and not rigorous.

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- Chain rule: $\frac{\partial \log[\pi(\boldsymbol{\theta})]}{\partial \boldsymbol{\theta}} = \frac{1}{\pi(\boldsymbol{\theta})} \cdot \frac{\partial\, \pi(\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}}$.

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- Chain rule: $\frac{\partial \log[\pi(\boldsymbol{\theta})]}{\partial \boldsymbol{\theta}} = \frac{1}{\pi(\boldsymbol{\theta})} \cdot \frac{\partial\, \pi(\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}}$.
- $\Rightarrow$ $\pi(\boldsymbol{\theta}) \cdot \frac{\partial \log[\pi(\boldsymbol{\theta})]}{\partial \boldsymbol{\theta}} = \cancel{\pi(\boldsymbol{\theta})} \cdot \frac{1}{\cancel{\pi(\boldsymbol{\theta})}} \cdot \frac{\partial\, \pi(\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} = \frac{\partial\, \pi(\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}}$.

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$$= \mathbb{E}_A \left[\frac{\partial \log \pi(A|s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \cdot Q_\pi(s, A) \right].$$

The expectation is taken w.r.t. the random variable $A \sim \pi(\cdot\, |s; \boldsymbol{\theta})$.

Policy Gradient

Two forms of policy gradient:

- Form 1: $\frac{\partial\, V(s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} = \sum_{a} \frac{\partial \pi(a|s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \cdot Q_{\pi}(s, a)$.
- Form 2: $\frac{\partial\, V(s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} = \mathbb{E}_{A\sim\pi(\cdot|s;\boldsymbol{\theta})}\left[\frac{\partial \log \pi(A|s,\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \cdot Q_{\pi}(s, A)\right]$.

Calculate Policy Gradient for Discrete Actions

If the actions are discrete, e.g., action space $\mathcal{A} = \{\text{“left”}, \text{“right”}, \text{“up”}\}, \ldots$

Use Form 1: $\frac{\partial\, V(s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} = \sum_a \frac{\partial \pi(a|s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \cdot Q_\pi(s,a)$.

Calculate Policy Gradient for Discrete Actions

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1. Calculate $\mathbf{f}(a, \boldsymbol{\theta}) = \frac{\partial \pi(a|s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \cdot Q_\pi(s, a)$, for every action $a \in \mathcal{A}$.
2. Policy gradient: $\frac{\partial\, V(s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} = \mathbf{f}(\text{"left"}, \boldsymbol{\theta}) + \mathbf{f}(\text{"right"}, \boldsymbol{\theta}) + \mathbf{f}(\text{"up"}, \boldsymbol{\theta})$.

If $|\mathcal{A}|$ is big, this approach is costly.

Calculate Policy Gradient for Continuous Actions

If the actions are continuous, e.g., action space $\mathcal{A} = [0, 1]$, ...

Use Form 2: $$\frac{\partial\, V(s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} = \mathbb{E}_{A\sim\pi(\cdot|s;\boldsymbol{\theta})}\left[\frac{\partial \log \pi(A|s,\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \cdot Q_\pi(s, A)\right].$$

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1. Randomly sample an action $\hat{a}$ according to the PDF $\pi(\cdot\,|s;\boldsymbol{\theta})$.

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1. Randomly sample an action $\hat{a}$ according to the PDF $\pi(\cdot\,|s;\boldsymbol{\theta})$.
2. Calculate $\boxed{\mathbf{g}(\hat{a}, \boldsymbol{\theta}) = \frac{\partial \log \pi(\hat{a}|s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \cdot Q_\pi(s, \hat{a}).}$

- By the definition of $\mathbf{g}$, $\mathbb{E}_A[\mathbf{g}(A, \boldsymbol{\theta})] = \frac{\partial\, V(s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}}$.
- $\mathbf{g}(\hat{a}, \boldsymbol{\theta})$ is an unbiased estimate of $\frac{\partial\, V(s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}}$.

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2. Calculate $\mathbf{g}(\hat{a}, \boldsymbol{\theta}) = \frac{\partial \log \pi(\hat{a}|s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}} \cdot Q_\pi(s, \hat{a})$.
3. Use $\mathbf{g}(\hat{a}, \boldsymbol{\theta})$ as an approximation to the policy gradient $\frac{\partial\, V(s;\boldsymbol{\theta})}{\partial\, \boldsymbol{\theta}}$.

Update policy network using policy gradient

Algorithm

1. Observe the state s_t.
2. Randomly sample action a_t according to $\pi(\cdot\,|s_t;\boldsymbol{\theta}_t)$.

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2. Randomly sample action a_t according to $\pi(\cdot \mid s_t; \boldsymbol{\theta}_t)$.
3. Compute $q_t \approx Q_\pi(s_t, a_t)$ (some estimate).
4. Differentiate policy network: $\mathbf{d}_{\theta,t} = \frac{\partial \log \pi(a_t|s_t,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}_t}$.

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5. (Approximate) policy gradient: $\mathbf{g}(a_t, \boldsymbol{\theta}_t) = q_t \cdot \mathbf{d}_{\theta,t}$.
6. Update policy network: $\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \beta \cdot \mathbf{g}(a_t, \boldsymbol{\theta}_t)$.

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Algorithm

1. Observe the state s_t.
2. Randomly sample action a_t according to $\pi(\cdot \mid s_t; \theta_t)$.
3. $\boxed{\text{Compute } q_t \approx Q_\pi(s_t, a_t) \text{ (some estimate).}}$ How?

Option 1: REINFORCE.

- Play the game to the end and generate the trajectory:

$$s_1, a_1, r_1, \; s_2, a_2, r_2, \quad \cdots \quad , s_T, a_T, r_T.$$

- Compute the discounted return $u_t = \sum_{k=t}^{T} \gamma^{k-t} r_k$, for all t.
- Since $Q_\pi(s_t, a_t) = \mathbb{E}[U_t]$, we can use u_t to approximate $Q_\pi(s_t, a_t)$.
- ➔ Use $q_t = u_t$.

Algorithm

1. Observe the state s_t.
2. Randomly sample action a_t according to $\pi(\cdot \mid s_t; \boldsymbol{\theta}_t)$.
3. $\boxed{\text{Compute } q_t \approx Q_\pi(s_t, a_t) \text{ (some estimate).}}$ How?

Option 2: Approximate Q_π using a neural network.

- This leads to the actor-critic method.

Summary

Policy-Based Learning

- If a good policy function π is known, the agent can be controlled by the policy: randomly sample $a_t \sim \pi(\cdot\,|s_t)$.
- Approximate policy function $\pi(a|s)$ by policy network $\pi(a|s;\boldsymbol{\theta})$.
- Learn the policy network by policy gradient algorithm.
- Policy gradient algorithm learn $\boldsymbol{\theta}$ that maximizes $\mathbb{E}_S[V(S;\boldsymbol{\theta})]$.

Thank you!