Shusen Wang

Value-Based Methods Actor-Critic Methods

Policy-Based Methods

Value Network and Policy Network

Definition: State-value function.

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Policy network (actor):

- Use neural net $\pi(a|s; \theta)$ to approximate $\pi(a|s)$.
- θ : trainable parameters of the neural net.

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Value network (critic):

- Use neural net $q(s, \mathbf{a}; \mathbf{w})$ to approximate $Q_{\pi}(s, \mathbf{a})$.
- w : trainable parameters of the neural net.

Definition: State-value function.

• $V_{\pi}(s) = \sum_{a} \pi(a|s) \cdot Q_{\pi}(s,a) \approx \sum_{a} \pi(a|s;\theta) \cdot q(s,a;\mathbf{w}).$

Policy network (actor):

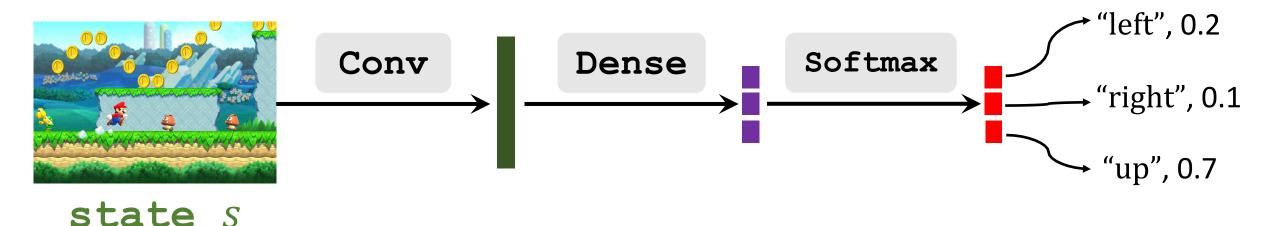
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Value network (critic):

- Use neural net $q(s, \mathbf{a}; \mathbf{w})$ to approximate $Q_{\pi}(s, \mathbf{a})$.
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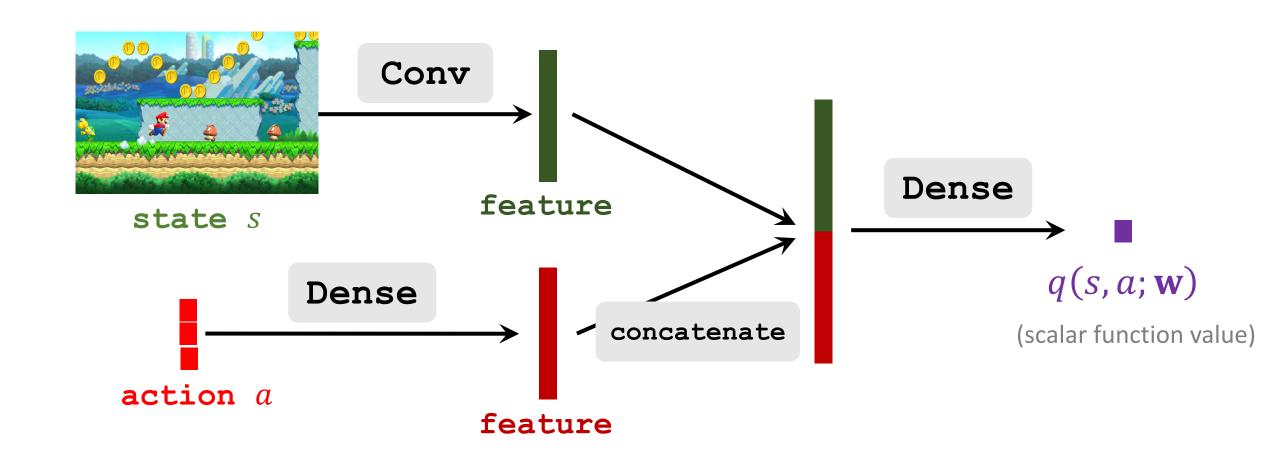
Policy Network (Actor): $\pi(a|s,\theta)$

- Input: state s, e.g., a screenshot of Super Mario.
- Output: probability distribution over the actions.
- Let \mathcal{A} be the set all actions, e.g., $\mathcal{A} = \{\text{"left", "right", "up"}\}$.
- $\sum_{a \in \mathcal{A}} \pi(a|s, \theta) = 1$. (That is why we use softmax activation.)



Value Network (Critic): q(s, a; w)

- Inputs: state s and action a.
- Output: approximate action-value (scalar).



policy network (actor)



value network (critic)



Train the Neural Networks

Definition: State-value function approximated using neural networks.

• $V(s; \theta, \mathbf{w}) = \sum_{\mathbf{a}} \pi(\mathbf{a}|s; \theta) \cdot q(s, \mathbf{a}; \mathbf{w}).$

Definition: State-value function approximated using neural networks.

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- Update policy network $\pi(a|s; \theta)$ to increase the state-value $V(s; \theta, \mathbf{w})$.
 - Actor gradually performs better.
 - Supervision is purely from the value network (critic).

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- Update policy network $\pi(a|s; \theta)$ to increase the state-value $V(s; \theta, \mathbf{w})$.
 - Actor gradually performs better.
 - Supervision is purely from the value network (critic).
- Update value network $q(s, \mathbf{a}; \mathbf{w})$ to better estimate the return.
 - Critic's judgement becomes more accurate.
 - Supervision is purely from the rewards.

Definition: State-value function approximated using neural networks.

• $V(s; \boldsymbol{\theta}, \mathbf{w}) = \sum_{\boldsymbol{a}} \pi(\boldsymbol{a}|s; \boldsymbol{\theta}) \cdot q(s, \boldsymbol{a}; \mathbf{w}).$

- 1. Observe the state s_t .
- 2. Randomly sample action a_t according to $\pi(\cdot | s_t; \theta_t)$.
- 3. Perform a_t and observe new state s_{t+1} and reward r_t .
- 4. Update w (in value network) using temporal difference (TD).
- 5. Update θ (in policy network) using policy gradient.

Update value network q using TD

- Compute $q(s_t, a_t; \mathbf{w}_t)$ and $q(s_{t+1}, a_{t+1}; \mathbf{w}_t)$.
- TD target: $y_t = r_t + \gamma \cdot q(s_{t+1}, a_{t+1}; \mathbf{w}_t)$.

Update value network q using TD

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- TD target: $y_t = r_t + \gamma \cdot q(s_{t+1}, a_{t+1}; \mathbf{w}_t)$.
- Loss: $L(\mathbf{w}) = \frac{1}{2} [q(s_t, a_t; \mathbf{w}) y_t]^2$.
- Gradient descent: $\mathbf{w}_{t+1} = \mathbf{w}_t \alpha \cdot \frac{\partial L(\mathbf{w})}{\partial \mathbf{w}} \mid_{\mathbf{w} = \mathbf{w}_t}$.

Update policy network π using policy gradient

Definition: State-value function approximated using neural networks.

• $V(s; \boldsymbol{\theta}, \mathbf{w}) = \sum_{\boldsymbol{a}} \pi(\boldsymbol{a}|s; \boldsymbol{\theta}) \cdot q(s, \boldsymbol{a}; \mathbf{w}).$

Policy gradient: Derivative of $V(s_t; \theta, \mathbf{w})$ w.r.t. θ .

- Let $\mathbf{g}(\mathbf{a}, \mathbf{\theta}) = \frac{\partial \log \pi(\mathbf{a}|s, \mathbf{\theta})}{\partial \mathbf{\theta}} \cdot q(s_t, \mathbf{a}; \mathbf{w}).$
- $\frac{\partial V(s; \theta, \mathbf{w}_t)}{\partial \theta} = \mathbb{E}_{\mathbf{A}}[\mathbf{g}(\mathbf{A}, \theta)].$

Update policy network π using policy gradient

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Policy gradient: Derivative of $V(s_t; \theta, \mathbf{w})$ w.r.t. θ .

- Let $\mathbf{g}(\mathbf{a}, \mathbf{\theta}) = \frac{\partial \log \pi(\mathbf{a}|s, \mathbf{\theta})}{\partial \mathbf{\theta}} \cdot q(s_t, \mathbf{a}; \mathbf{w}).$
- $\frac{\partial V(s;\theta,\mathbf{w}_t)}{\partial \theta} = \mathbb{E}_{\mathbf{A}}[\mathbf{g}(\mathbf{A},\mathbf{\theta})].$

Algorithm: Update policy network using stochastic policy gradient.

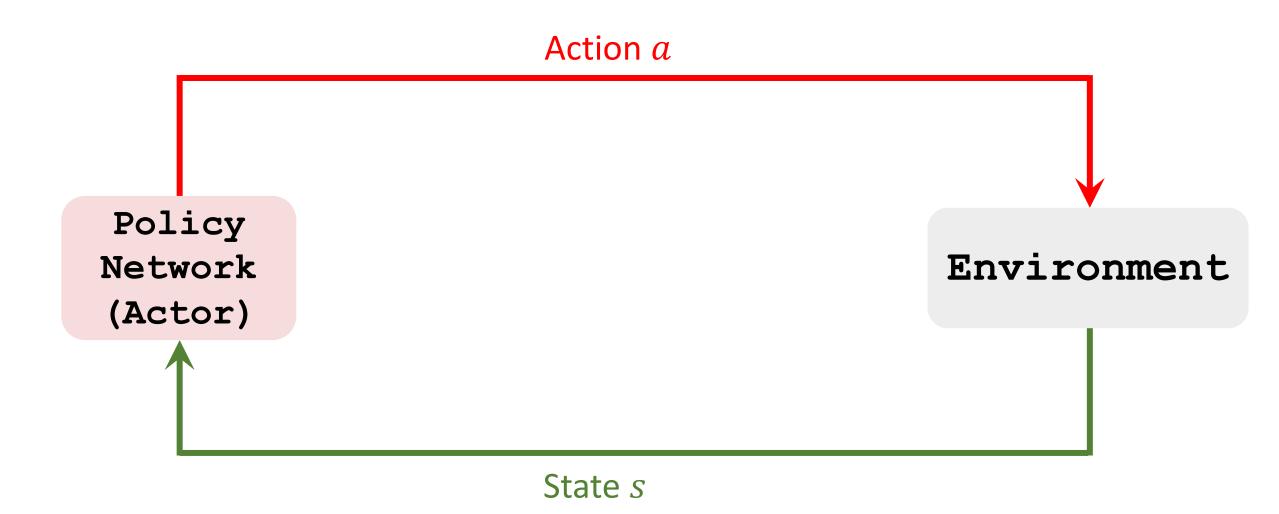
- Random sampling: $a \sim \pi(\cdot | s_t; \theta_t)$. (Thus $g(a, \theta)$ is unbiased.)
- Stochastic gradient ascent: $\theta_{t+1} = \theta_t + \beta \cdot \mathbf{g}(\mathbf{a}, \theta_t)$.

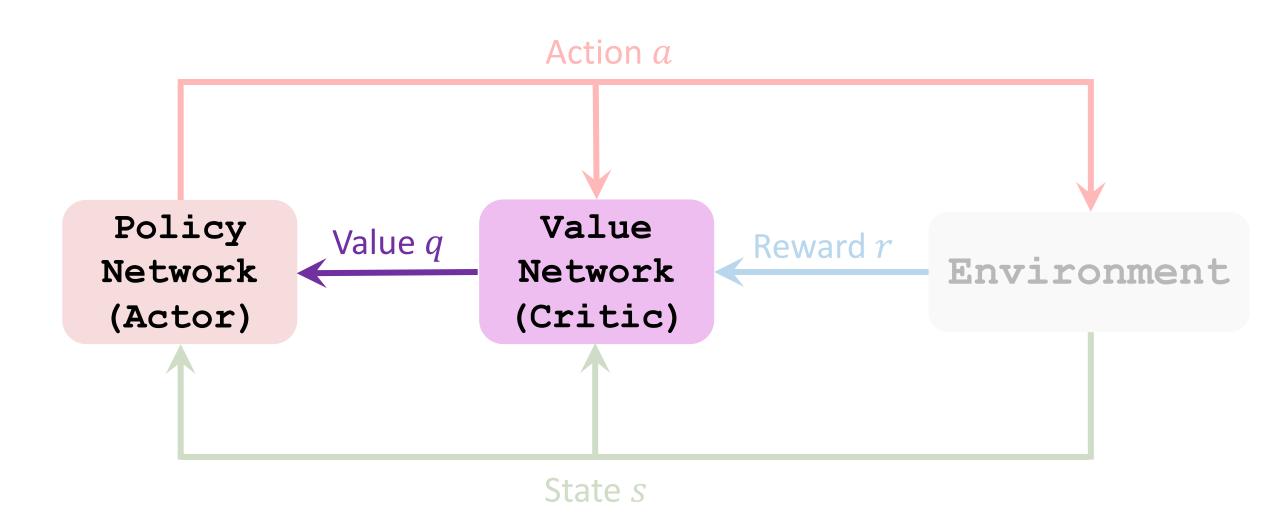
policy network (actor)



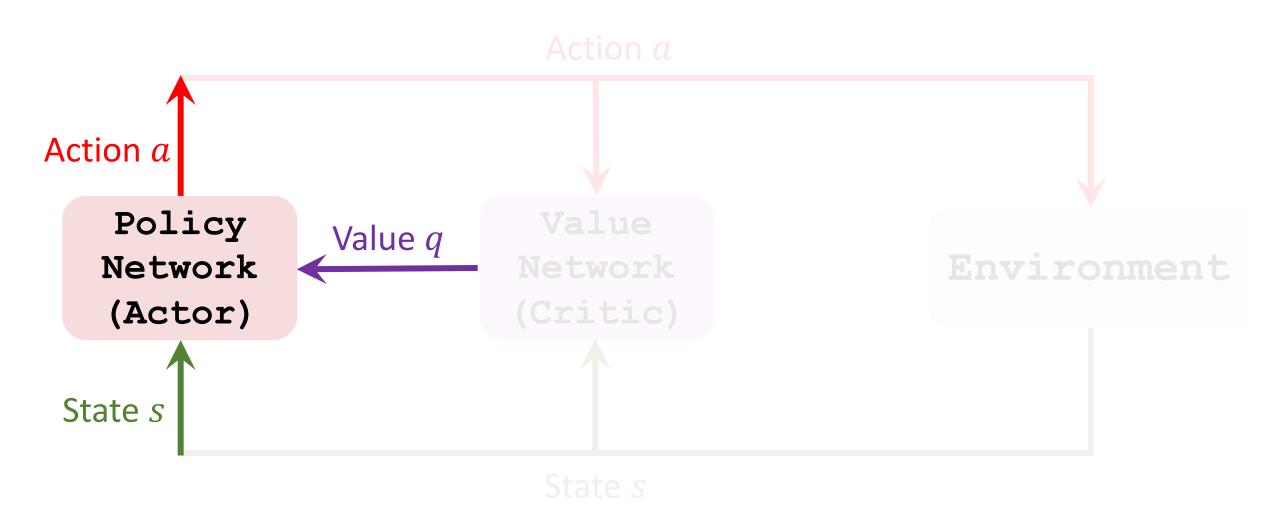
value network (critic)



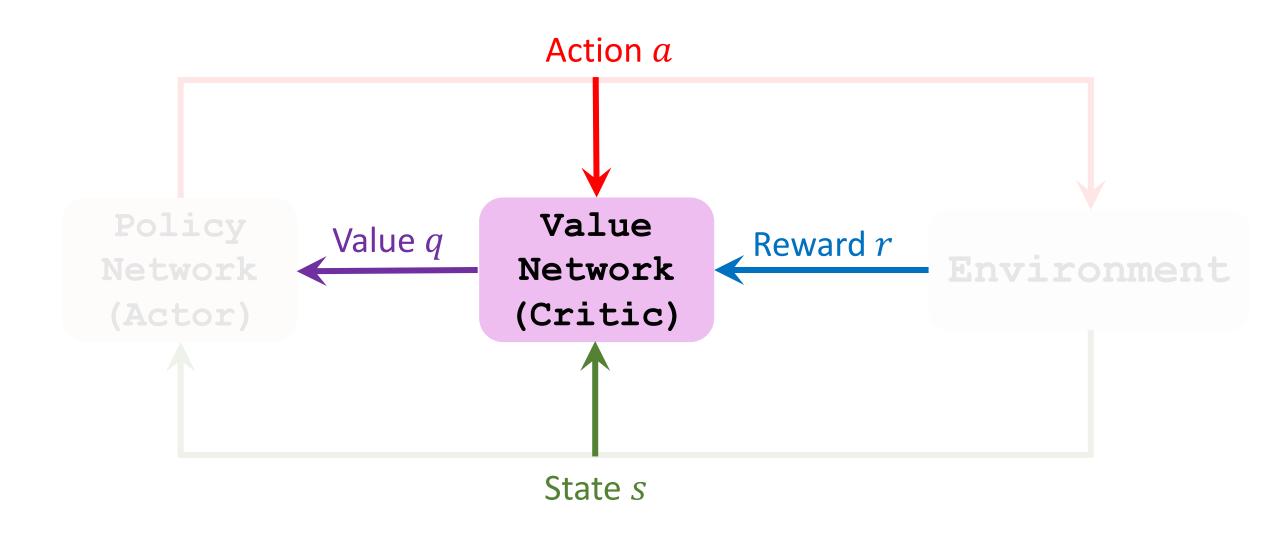




Actor-Critic Method: Update Actor



Actor-Critic Method: Update Critic



- 1. Observe state s_t and randomly sample $a_t \sim \pi(\cdot | s_t; \theta_t)$.
- 2. Perform a_t ; then environment gives new state s_{t+1} and reward r_t .
- 3. Randomly sample $\tilde{a}_{t+1} \sim \pi(\cdot | s_{t+1}; \theta_t)$. (Do not perform $\tilde{a}_{t+1}!$)

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- 4. Evaluate value network: $q_t = q(s_t, a_t; \mathbf{w}_t)$ and $q_{t+1} = q(s_{t+1}, \tilde{a}_{t+1}; \mathbf{w}_t)$.
- 5. Compute TD error: $\delta_t = q_t (r_t + \gamma \cdot q_{t+1})$.

 TD Target

- 1. Observe state s_t and randomly sample $a_t \sim \pi(\cdot | s_t; \theta_t)$.
- 2. Perform a_t ; then environment gives new state s_{t+1} and reward r_t .
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- 5. Compute TD error: $\delta_t = q_t (r_t + \gamma \cdot q_{t+1})$.
- 6. Differentiate value network: $\mathbf{d}_{w,t} = \frac{\partial q(s_t, \mathbf{a_t}; \mathbf{w})}{\partial \mathbf{w}} \big|_{\mathbf{w} = \mathbf{w}_t}$.
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Policy Gradient with Baseline

- 1. Observe state s_t and randomly sample $a_t \sim \pi(\cdot | s_t; \theta_t)$.
- 2. Perform a_t ; then environment gives new state s_{t+1} and reward r_t .
- 3. Randomly sample $\tilde{a}_{t+1} \sim \pi(\cdot | s_{t+1}; \theta_t)$. (Do not perform $\tilde{a}_{t+1}!$)
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- 5. Compute TD error: $\delta_t = q_t (r_t + \gamma \cdot q_{t+1})$.
- 6. Differentiate value network: $\mathbf{d}_{w,t} = \frac{\partial q(s_t, \alpha_t, \mathbf{w})}{\mathsf{Baseline}^{\mathbf{w}}} |_{\mathbf{w} = \mathbf{w}_t}$.
- 7. Update value network: $\mathbf{w}_{t+1} = \mathbf{w}_t \alpha \cdot \delta_t \cdot \mathbf{d}_{w,t}$.
- 8. Differentiate policy network: $\mathbf{d}_{\theta,t} = \frac{\partial \log \pi(a_t|s_t,\theta)}{\partial \theta} \big|_{\theta=\theta_t}$.
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Summary

Policy Network and Value Network

Definition: State-value function.

•
$$V_{\pi}(s) = \sum_{a} \pi(a|s) \cdot Q_{\pi}(s,a)$$
.

Definition: function approximation using neural networks.

- Approximate policy function $\pi(a|s)$ by $\pi(a|s;\theta)$ (actor).
- Approximate value function $Q_{\pi}(s, \mathbf{a})$ by $q(s, \mathbf{a}; \mathbf{w})$ (critic).

Roles of Actor and Critic

During training

- Agent is controlled by policy network (actor): $a_t \sim \pi(\cdot | s_t; \theta)$.
- Value network q (critic) provides the actor with supervision.

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- Value network q (critic) provides the actor with supervision.

After training

- Agent is controlled by policy network (actor): $a_t \sim \pi(\cdot | s_t; \theta)$.
- Value network q (critic) will not be used.

Training

Learning: Update the policy network (actor) by policy gradient.

- Seek to increase state-value: $V(s; \theta, \mathbf{w}) = \sum_{a} \pi(a|s; \theta) \cdot q(s, a; \mathbf{w})$.
- Compute policy gradient: $\frac{\partial V(s;\theta)}{\partial \theta} = \mathbb{E}_{A} \left[\frac{\partial \log \pi(A|s,\theta)}{\partial \theta} \cdot q(s,A;\mathbf{w}) \right].$
- Perform gradient ascent.

Training

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- Compute policy gradient: $\frac{\partial V(s;\theta)}{\partial \theta} = \mathbb{E}_{A} \left[\frac{\partial \log \pi(A|s,\theta)}{\partial \theta} \cdot q(s,A;\mathbf{w}) \right].$
- Perform gradient ascent.

Learning: Update the value network (critic) by TD learning.

- Predicted action-value: $q_t = q(s_t, \mathbf{a_t}; \mathbf{w})$.
- TD target: $y_t = r_t + \gamma \cdot q(s_{t+1}, a_{t+1}; \mathbf{w})$
- Gradient: $\frac{\partial (q_t y_t)^2/2}{\partial \mathbf{w}} = (q_t y_t) \cdot \frac{\partial q(s_t, a_t; \mathbf{w})}{\partial \mathbf{w}}$.
- Perform gradient descent.

Thank you!

Policy Gradient with Baseline

Policy Gradient with Baseline

Definition: Approximated state-value function.

- $V(s; \boldsymbol{\theta}) b = \sum_{\boldsymbol{a}} \pi(\boldsymbol{a}|s; \boldsymbol{\theta}) \cdot [Q_{\pi}(s, \boldsymbol{a}) b].$
- Here, the baseline b must be independent of θ and a.

Policy Gradient with Baseline

Definition: Approximated state-value function.

- $V(s; \boldsymbol{\theta}) b = \sum_{\boldsymbol{a}} \pi(\boldsymbol{a}|s; \boldsymbol{\theta}) \cdot [Q_{\pi}(s, \boldsymbol{a}) b].$
- Here, the baseline b must be independent of θ and a.

Policy gradient: Derivative of $V(s; \theta)$ w.r.t. θ .

$$\bullet \frac{\partial V(s;\theta)}{\partial \theta} = \frac{\partial [V(s;\theta)-b]}{\partial \theta} = \mathbb{E}_{\boldsymbol{a} \sim \pi(\cdot|s;\theta)} \left[\frac{\partial \log \pi(\boldsymbol{a}|s,\theta)}{\partial \theta} \cdot [Q_{\pi}(s,\boldsymbol{a})-b] \right].$$

- The baseline b does not affect correctness.
- A good baseline b can reduce variance.
- We can use $b = r_t + \gamma \cdot q_{t+1}$ (TD target) as the baseline.

Actor Critic Update (without baseline)

- 1. Observe the state s_t ; randomly sample action a_t according to $\pi(\cdot | s_t; \theta_t)$.
- 2. Perform a_t ; observe new state s_{t+1} and reward r_t .
- 3. Randomly sample a_{t+1} according to $\pi(\cdot | s_{t+1}; \theta_t)$. (Do not perform a_{t+1} .)
- 4. Evaluate value network: $q_t = q(s_t, a_t; \mathbf{w}_t)$ and $q_{t+1} = q(s_{t+1}, a_{t+1}; \mathbf{w}_t)$.
- 5. Compute the TD error: $\delta_t = q_t (r_t + \gamma \cdot q_{t+1})$.
- 6. Differentiate value network: $\mathbf{d}_{w,t} = \frac{\partial q(s_t, a_t; \mathbf{w})}{\partial \mathbf{w}} |_{\mathbf{w} = \mathbf{w}_t}$.
- 7. Update value network: $\mathbf{w}_{t+1} = \mathbf{w}_t \alpha \cdot \delta_t \cdot \mathbf{d}_{w,t}$.
- 8. Differentiate policy network: $\mathbf{d}_{\theta,t} = \frac{\partial \log \pi(a_t|s_t,\theta)}{\partial \theta} \big|_{\theta=\theta_t}$.
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- 1. Observe the state s_t ; randomly sample action a_t according to $\pi(\cdot | s_t; \theta_t)$.
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- 4. Evaluate value network: $q_t = q(s_t, a_t; \mathbf{w}_t)$ and $q_{t+1} = q(s_{t+1}, a_{t+1}; \mathbf{w}_t)$.
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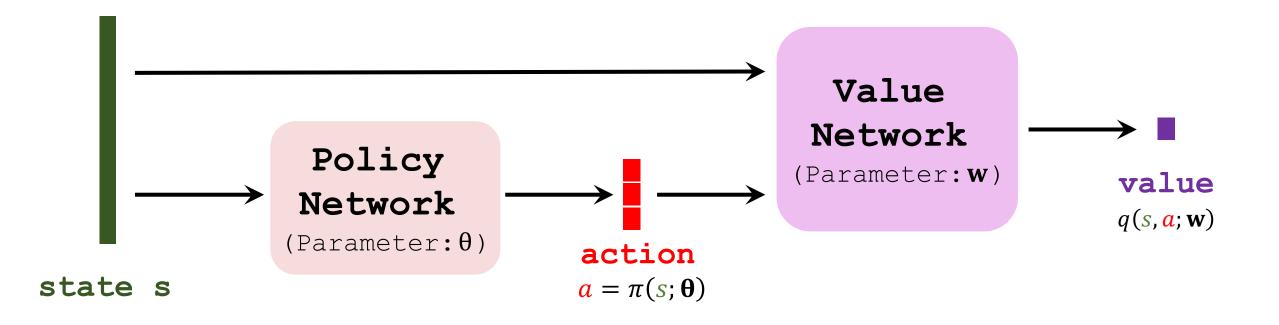
Deterministic Policy Gradient (DPG)

Reference

- Silver and others: Deterministic Policy Gradient Algorithms. In ICML, 2014.
- Lillicrap and others: Continuous control with deep reinforcement learning. arXiv:1509.02971. 2015.

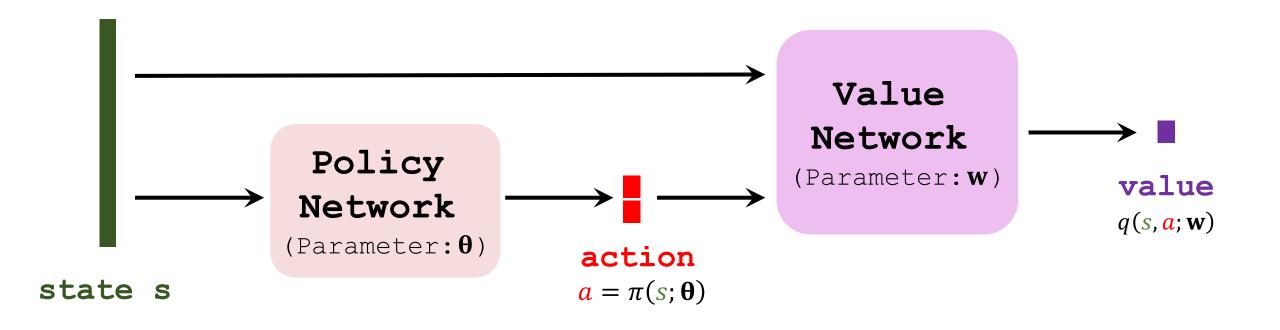
Deterministic Policy Gradient (DPG)

- DPG is a actor-critic method.
- The policy network is deterministic: $a = \pi(s; \theta)$.



Deterministic Policy Gradient (DPG)

- DPG is a actor-critic method.
- The policy network is deterministic: $a = \pi(s; \theta)$.
- Trained value network by TD learning.
- Train policy network to maximize the value $q(s, \mathbf{a}; \mathbf{w})$.



Train Policy Network

- Train policy network to maximize the value q(s, a; w).
- Gradient: $\frac{\partial q(s,a;w)}{\partial \theta} = \frac{\partial a}{\partial \theta} \cdot \frac{\partial q(s,a;w)}{\partial a}$.
- Update θ using gradient ascent.

