ECE 514 MINI PROJECT

BIT ERROR ESTIMATOR

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BACKGROUND:

In digital transmission, the number of bit errors is the number of received bits of a data stream over a communication channel that have been altered due to noise, interference, distortion or bit synchronization errors.

The bit error rate (BER) is the number of bit errors per sample size i.e. if total number of bits received are N and the total numbers of bits which has error are ne than the,

$$BER = \frac{n_e}{N} \ldots. (1)$$

With sufficiently large value of N, we can use the strong law of large numbers to conclude that,

$$\lim_{N \to \infty} BER = p_e \ldots (2)$$

Where p_e is the probability of the received bit being inaccurate.
If for a given received signal,

$$R = \sqrt{E_b} B + \sqrt{\frac{N0}{2}} N \ldots (3)$$

Where, B = bit value transferred,
N = Noise in the channel
E_b = Power density of signal
N_0 = Power density of the Noise

The p_e can be estimated as follows,

$$P_e = P(R < 0 | B = +1) + P(R > 0 | B = -1)$$

Which after solving comes out to be,

$$p_e = 0.5 \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \ldots (4)$$

In this project I have tried to simulate the received signal R and then compute the BER with the help of Mean estimators with known variance and unknown variance both to see the working of mentioned estimators and analyze their performance.

Experimental Setup:

To perform the mentioned simulation, I am using MATLAB® (ML) as my platform to work. To estimate the Bit error rate, I am performing 100 trials and doing the simulation for three different values of receive signal(n) which are n = 10, n = 100, n = 1000 and I am following the below mentioned steps,

1. Simulating the received signal R.
2. Calculating the X matrix where X is a random variable which is equal to 1 if there is an error and equal to 0 if there is no error.
3. Estimating the BER (Mean estimate).
4. Estimating the confidence interval using the known variance.
5. Estimating the confidence interval using the estimated variance.
6. Plotting the graph of Trial vs Mean estimate
7. Plotting the graph of Trial vs Confidence interval (calculated based on known and estimated variance method)
8. Plotting the graph of true mean.
9. Calculating the fraction of times true mean lies in confidence interval (calculated based on known and estimated variance method)

Simulation of Received Signal:

The receive signal is given by,

$$R = \sqrt{E_b}B + \sqrt{\frac{N0}{2}}N$$

Where, B = bit value transferred,
N = Noise in the channel
Eb = Power density of signal
N0 = Power density of the Noise

Where,
$E_b = 1$
$N_0 = 2$
B = -1
And N is calculated using a randn() function in MATLAB® (ML) with seed value = 592.
R is a 2-D matrix for each trial as column and signal as row.

Calculation of X:

Bit values are equi-likely with a detection threshold of 0 so that the BER is the same for +1- and -1-bit values. I am sending a bit B =-1 and modelling received bit or sample by above mentioned model for R. I have defined a BER as the event R>0|B=-1. I will define X if no error will occur as 0 and if error occur as 1. X is a 2-D matrix for each trial as column.

Here total number of inaccurate bits in a given trial can be calculated by summing all the values in one column of X.

$$n_e = \sum_{i=1}^{N} x_i \ldots (5)$$

Estimation BER:

As we have already seen that,

$$BER = \frac{n_e}{N}$$

From equation 5 we can also write,

$$BER = \frac{\sum x_i}{N} \dots (6)$$

Which is nothing else but the value of Mean estimator for any given data set.

So, the estimate for BER can be calculated using the mean estimator.

Estimating Confidence interval using known variance:

It is clear that the distribution of X is Bernoulli with a probability equal to BER which means the variance of X will be given by,

$$\sigma^2 = BER(1 - BER) \dots (7)$$

The confidence interval can be calculated using the below process,

We now summarize the procedure. Fix a confidence level $1-\alpha$. Find the corresponding $y_{\alpha/2}$ from Table 6.2. Then write

$$m = M_n \pm \frac{\sigma y_{\alpha/2}}{\sqrt{n}} \quad \text{with } 100(1-\alpha)\% \text{ probability,} \tag{6.11}$$

and the corresponding confidence interval is

$$\left[M_n - \frac{\sigma y_{\alpha/2}}{\sqrt{n}}, M_n + \frac{\sigma y_{\alpha/2}}{\sqrt{n}}\right]. \tag{6.12}$$

Here we are using 68.3% confidence interval so the value of y from z table comes out to be 1.

Estimating Confidence interval using estimated variance:

While calculating the confidence interval using the estimated variance, we can use the same method as above and just replace actual variance with estimated variance given below,

$$S_n^2 = \frac{1}{n-1}\Sigma(x_i - M_N)^2$$

RESULTS:

Table:

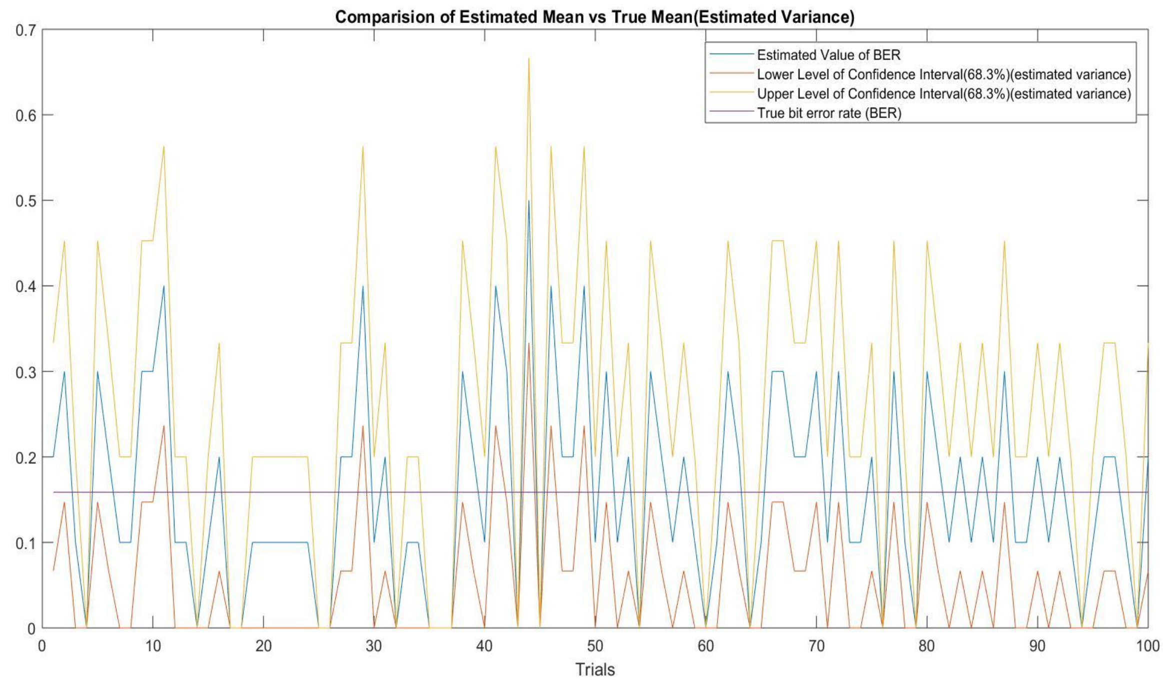
Number of Bits	Fraction within confidence interval (Known Variance)	Fraction within confidence interval (Estimated Variance)
10	0.59	0.75
100	0.60	0.60
1000	0.68	0.68

As we can see that from the table that with the increase in value of n the fraction is growing closer to our desired value of confidence interval that is 68.3%. This implies that with the increasing value of n we can be certain that our method of calculating confidence interval gives us pretty certain idea about how confident can we be about our estimate. The other interesting thing to note is that the value of both the fraction (estimated vs known mean) becomes equal with increasing value of n (at n = 100 and n = 1000) which again help us confirm that even if we don't know the variance of the data(which we generally don't) we can be fairly certain about the confidence interval if our sample is big enough.

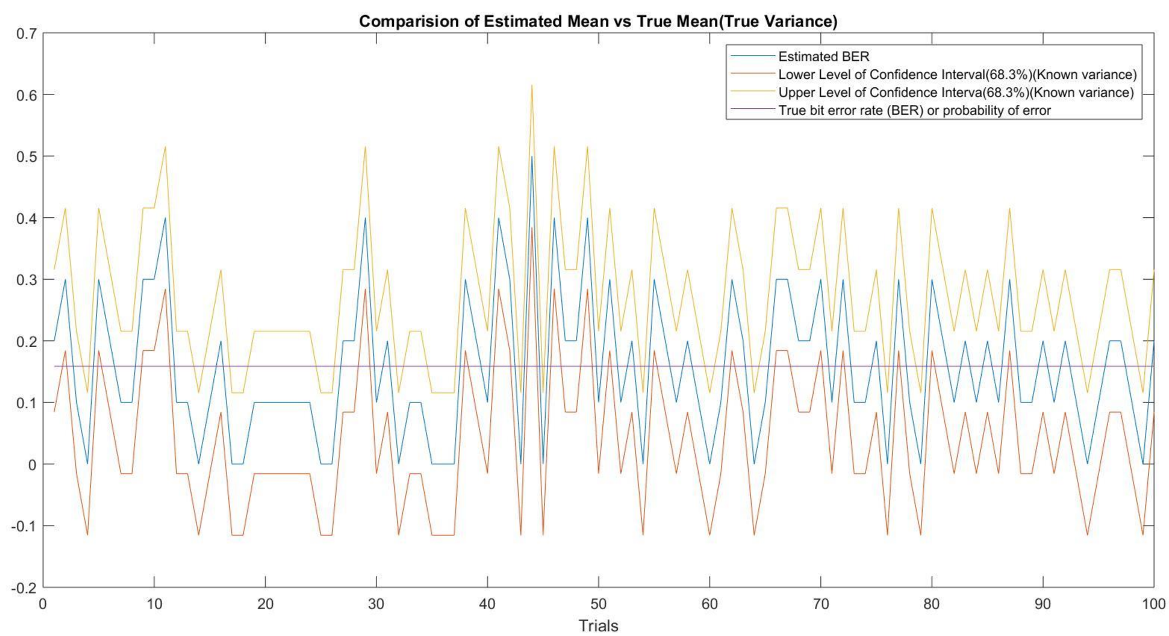
The only thing which problematic is that at smaller values of n we have very different values of fraction for estimated variance and known variance, also I was expecting for fraction to be either similar or better for known variance as in the known variance case we at least have some information about the sample.

PLOTS:

a) N = 10

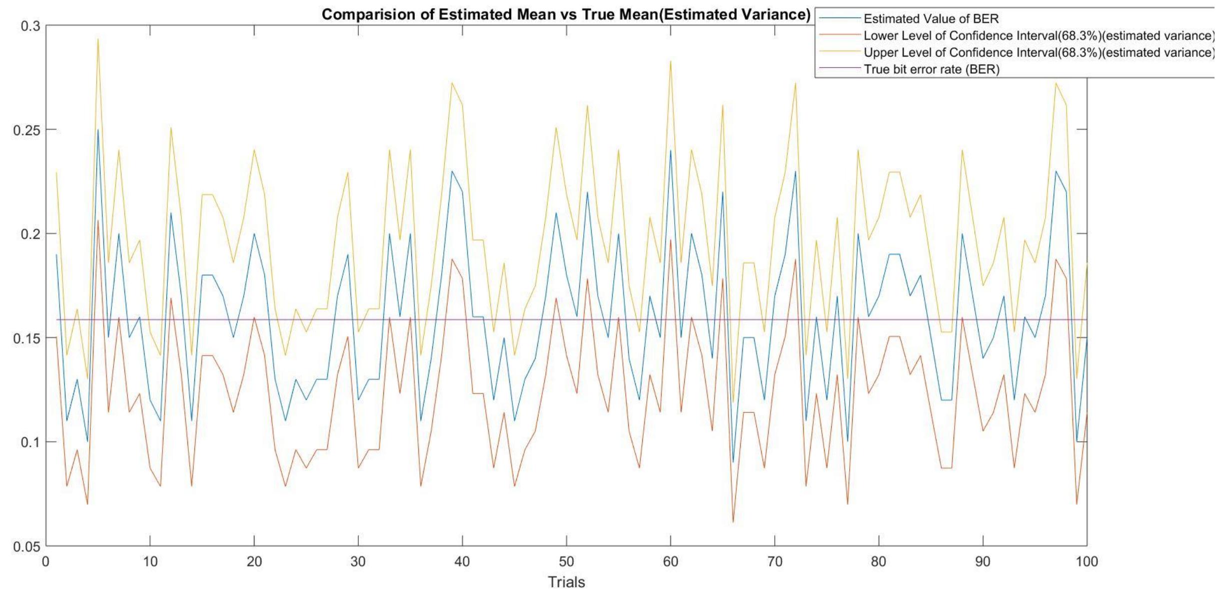


Comparision of Estimated Mean vs True Mean(Estimated Variance)
Estimated Value of BER
Lower Level of Confidence Interval(68.3%)(estimated variance)
Upper Level of Confidence Interval(68.3%)(estimated variance)
True bit error rate (BER)
Trials

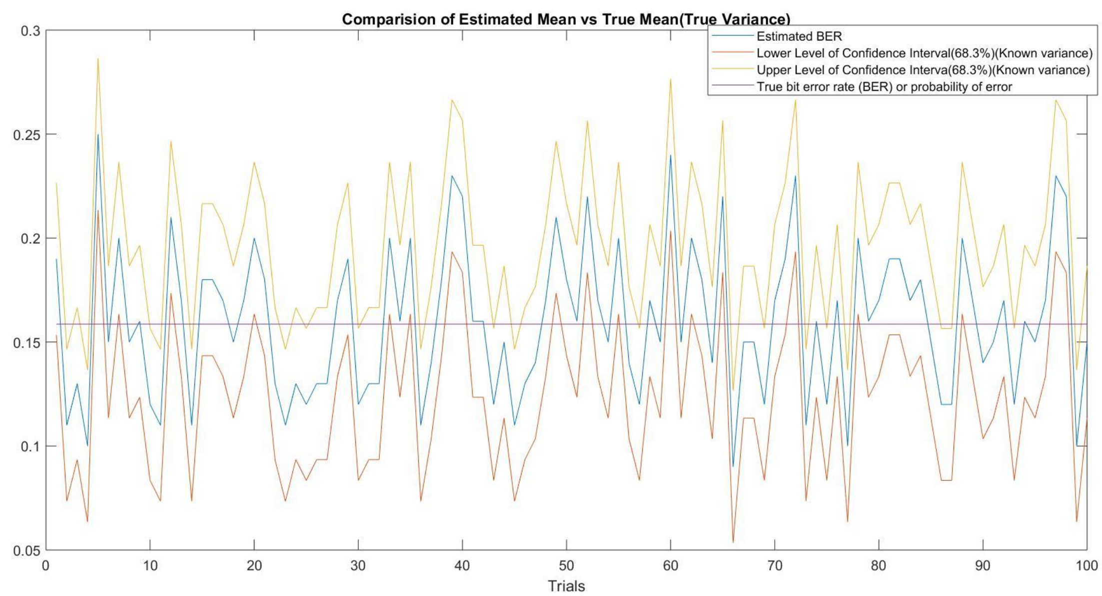


Comparision of Estimated Mean vs True Mean(True Variance)
Estimated BER
Lower Level of Confidence Interval(68.3%)(Known variance)
Upper Level of Confidence Interva(68.3%)(Known variance)
True bit error rate (BER) or probability of error
Trials

B) N = 100

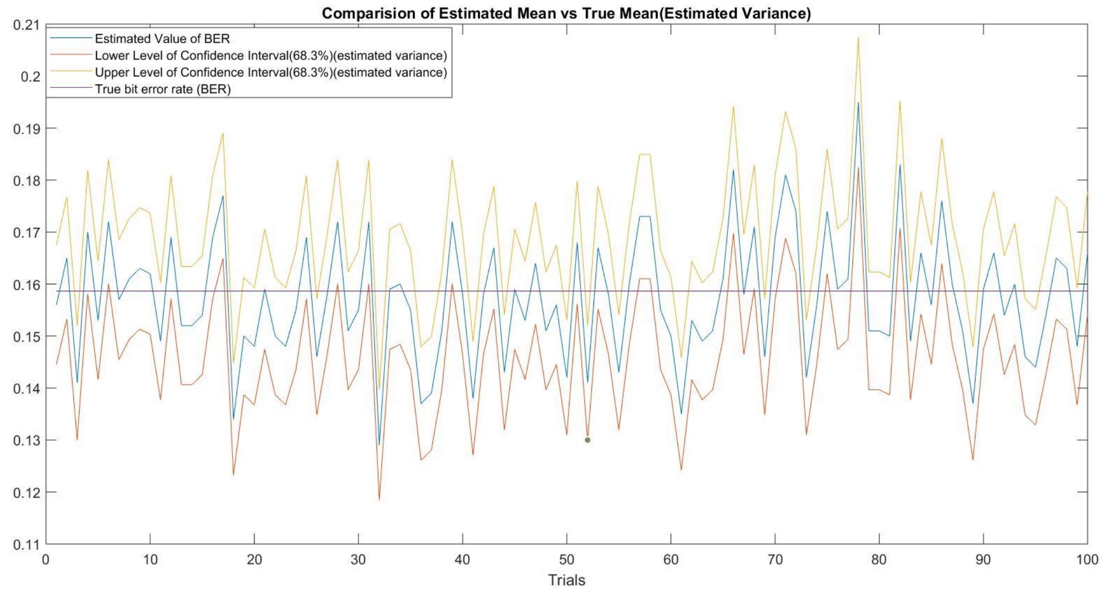


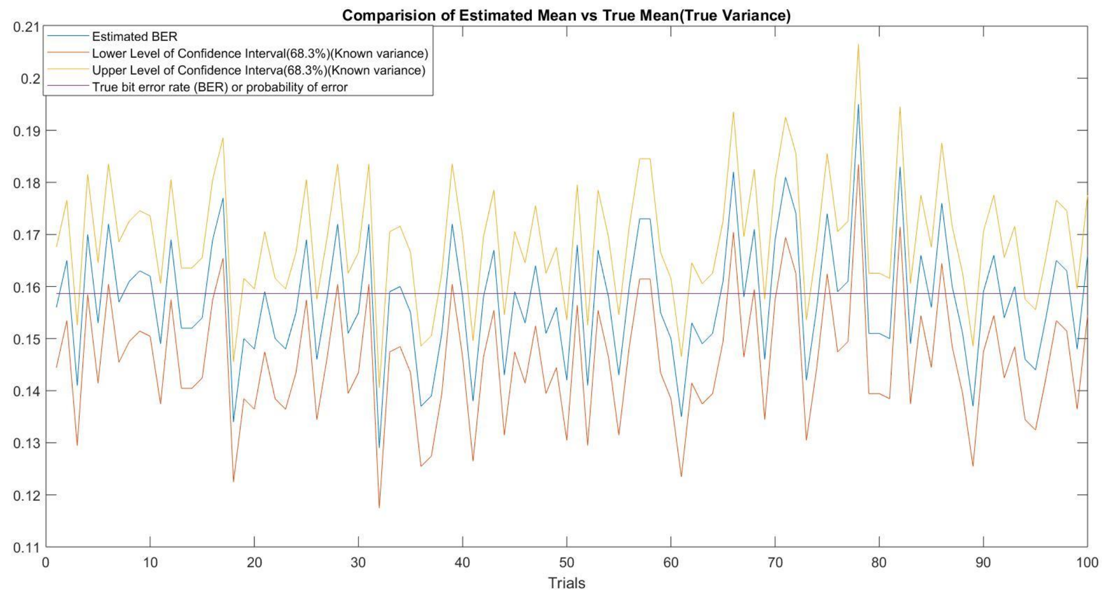
Comparision of Estimated Mean vs True Mean(Estimated Variance)
Estimated Value of BER
Lower Level of Confidence Interval(68.3%)(estimated variance)
Upper Level of Confidence Interval(68.3%)(estimated variance)
True bit error rate (BER)
0.3
0.25
0.2
0.15
0.1
0.05
0
10
20
30
40
50
60
70
80
90
100
Trials



Comparision of Estimated Mean vs True Mean(True Variance)
Estimated BER
Lower Level of Confidence Interval(68.3%)(Known variance)
Upper Level of Confidence Interva(68.3%)(Known variance)
True bit error rate (BER) or probability of error
0.3
0.25
0.2
0.15
0.1
0.05
0
10
20
30
40
50
60
70
80
90
100
Trials

C) N = 1000





For n = 10, estimated mean case there are lot of case where we can’t even estimate the confidence interval as the data seems to converge to zero. This may be happening because of lack of availability of enough sample points. As it doesn’t seem to happen for n =100 or n = 1000. This implies that this type of problem can be avoided by using larger datasets.

CONCLUSION:

With this project I was able to learn about the practical implementation of confidence interval in the field of Digital Communication. To determine the quality of digital communication system we measure its Bit Error Rate or probability of bit error. To find bit error we can use the mean estimator in the system and it will give us a proper result as long as we have taken a large enough sample size. The most important take away was that with larger datasets our confidence intervals grow even more certain.

APPENDIX:

```
clc;
clear variables;
close all;
E_b = input('Enter the value of power of Signal(E_b):
');
N_0 = input('Enter the value for power of Noice(N_0) :
');
n = input('Enter the number of bits: ');
m = input('Enter the number of trials: ');
B = input('Enter the bit value(restricted to -1,1): ');
while B~=-1 && B~=1
    B = input('You did not enter the right value,
Please re-enter: ');
end
bit_error_rate = 0.5*erfc(sqrt(E_b/N_0));
var_actual = bit_error_rate.*(1-bit_error_rate);
%generate the recieved values
rng(592,'twister');
Noice = randn(n,m);
R = sqrt(E_b)*B + sqrt(N_0/2).*Noice;
if B<0
    R = -1.*R;
end
X = zeros(n,m);
%Measuring the error
for i = 1:n
    for j = 1:m
        if R(i,j)<0
```

```
            X(i,j)=1;
        end
    end
end
%estimating the mean
mean_estimate = mean(X);
%Calculating the interval for mean with known variance
delta = sqrt(var_actual)/(sqrt(n));
mean_upper_limit_KV = mean_estimate + delta;
mean_lower_limit_KV = mean_estimate - delta;
%Calculating the interval for mean with unknown
variance
std_estimate = std(X);
deltaunk = std_estimate./(sqrt(n));
mean_upper_limit_unk = mean_estimate + deltaunk;
mean_lower_limit_unk = mean_estimate - deltaunk;
%Counting fraction
count_KV = 0;
countunk = 0;
for i = 1:m
    if bit_error_rate<=mean_upper_limit_KV(1,i) &&
bit_error_rate>=mean_lower_limit_KV(1,i)
        count_KV = count_KV + 1;
    end
    if bit_error_rate<=mean_upper_limit_unk(1,i) &&
bit_error_rate>=mean_lower_limit_unk(1,i)
        countunk = countunk + 1;
    end
end
fraction_known_variance = (count_KV/m);
fraction_unknown_variance = (countunk/m);
%Plotting all the outcomes
figure();
p1=plot(1:100, mean_estimate(1:100));
l1='Estimated BER';
hold on
p2=plot(1:100, mean_lower_limit_KV(1:100));
```

```
l2='Lower Level of Confidence Interval(68.3%)(Known
variance)';
p3=plot(1:100, mean_upper_limit_KV(1:100));
l3='Upper Level of Confidence Interva(68.3%)(Known
variance)';
p4=plot(1:100, bit_error_rate*ones(1,100));
l4='True bit error rate (BER) or probability of error';
hold off
legend([p1 p2 p3 p4], {l1, l2, l3, l4});
xlabel('Trials');
title('Comparision of Estimated Mean vs True Mean(True
Variance)');
figure();
p5=plot(1:100, mean_estimate(1:100));
l5='Estimated Value of BER';
hold on
p6=plot(1:100, mean_lower_limit_unk(1:100));
l6='Lower Level of Confidence Interval(68.3%)(estimated
variance)';
p7=plot(1:100, mean_upper_limit_unk(1:100));
l7='Upper Level of Confidence Interval(68.3%)(estimated
variance)';
p8=plot(1:100, bit_error_rate*ones(1,100));
l8='True bit error rate (BER)';
hold off
legend([p5 p6 p7 p8], {l5, l6, l7, l8});
xlabel('Trials');
title('Comparision of Estimated Mean vs True
Mean(Estimated Variance)');
```

REFERENCES:

1. https://en.wikipedia.org/wiki/Bit_error_rate
2. Probability and Random Processes for Electrical and Computer Engineers – John A. Gubner