Security of distributed Model Predictive Control under False Data injection

Rafael Accácio NOGUEIRA

2022-12-09

https://bit.ly/3g3S6X4

Context

“Necessity is the mother of invention”

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❝ Necessity is the mother of invention ❞

- Electricity Distribution System
- Heat distribution
- Water distribution
- Traffic management

  (include your problem here)

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- Multiple systems interacting
- Coupled by constraints
  - Technical/ Comfort
- Optimization objectives
  - Minimize energy consumption
  - Maximize user satisfaction
  - Follow a trajectory
- Solution → MPC

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Find best control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence
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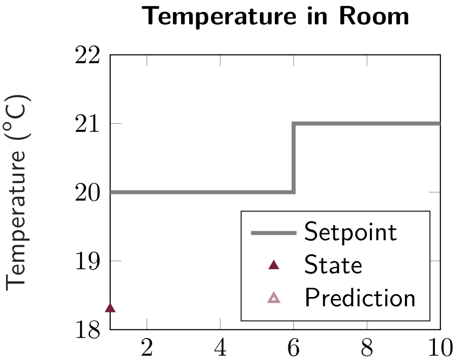
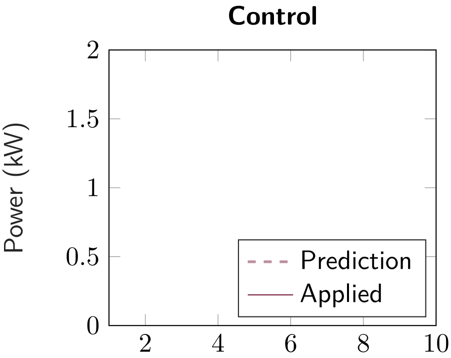
In a nutshell

Find optimal control sequence





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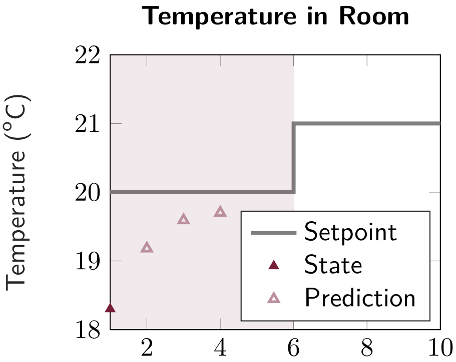
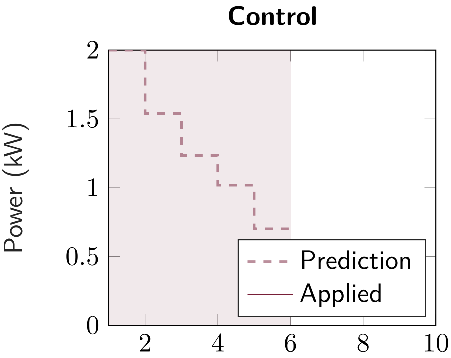
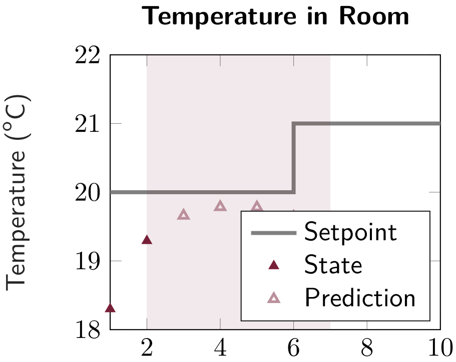
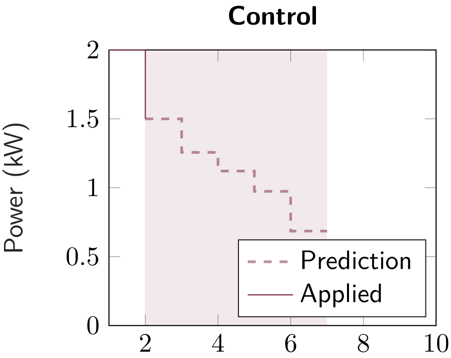




In a nutshell

Find optimal control sequence, apply first element

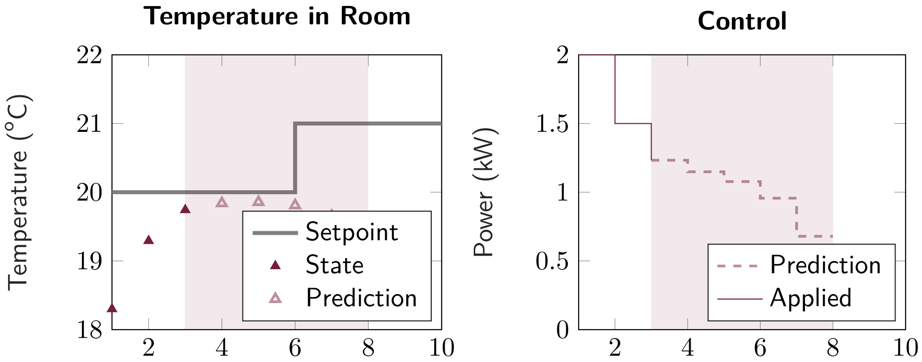




Model Predictive Control

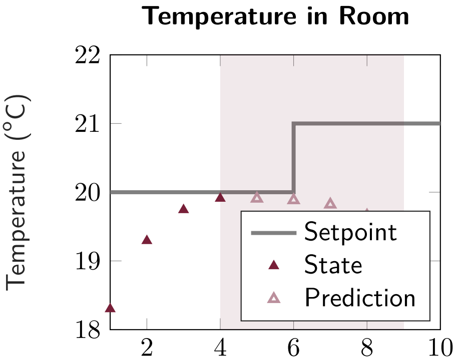
In a nutshell

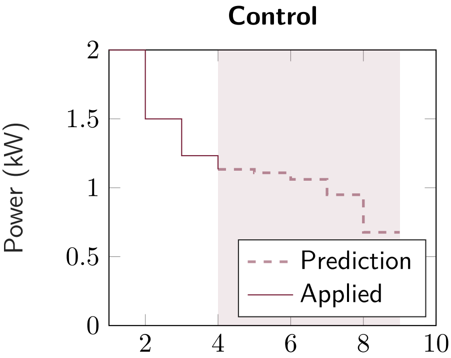
Find optimal control sequence, apply first element, rinse repeat



In a nutshell

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





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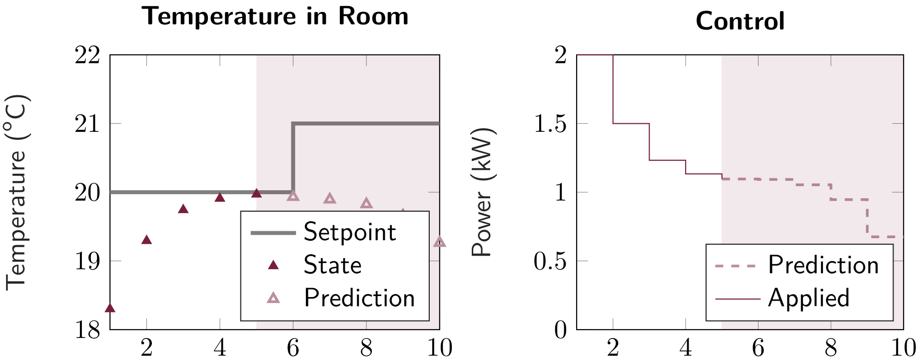

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Nothing is perfect

- Problems
  - Complexity of calculation
  - Topology (Geographical distribution)
  - Flexibility (Add/remove parts)
  - Privacy
- Solution: Divide and Conquer (distributed MPC)
  - Break calculation
  - Make systems communicate

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Distributed Model Predictive Control

It is about communication

- We break the MPC into multiple
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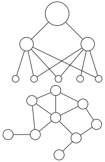

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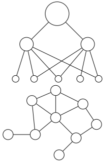

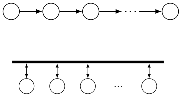

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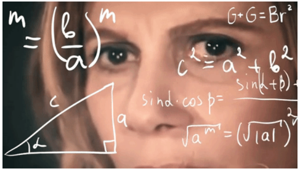

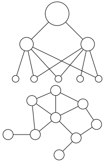

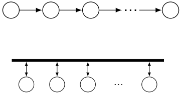

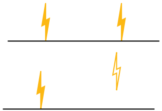

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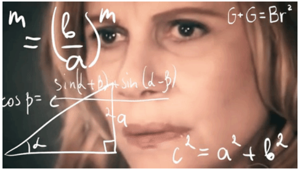

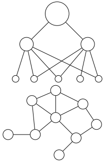

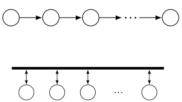

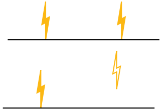

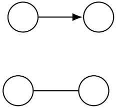

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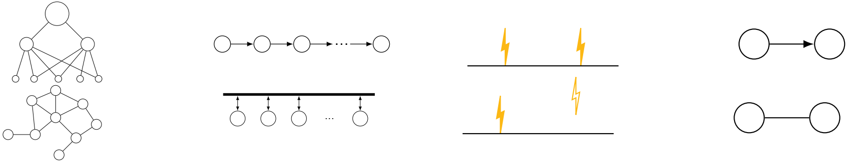

[1] Distributed Model Predictive Control made easy

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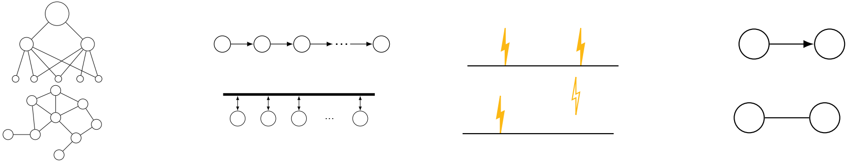
It is about communication

- We break the MPC into multiple
- Make them communicate, But how?
  - Many flavors to choose from[1]
    - Hierarchical/Anarchical
    - Sequential/Parallel
    - Synchronous/Asynchronous
    - Bidirectional/Unidirectional
    - ...



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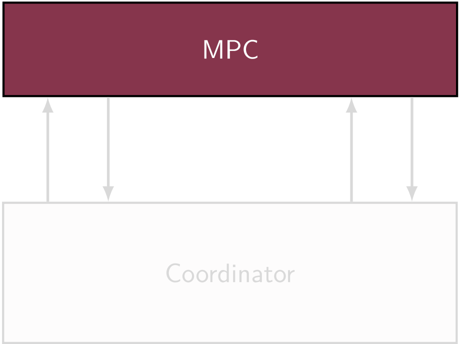
Distributed Model Predictive Control

Communication Frameworks



- Coordinator → Hierarchical
- Bidirectional
- No delay → Synchronous
- Agents solve local problems
- Variables are updated

Until Convergence

Distributed Model Predictive Control

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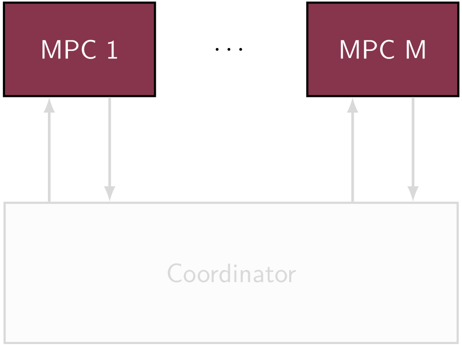

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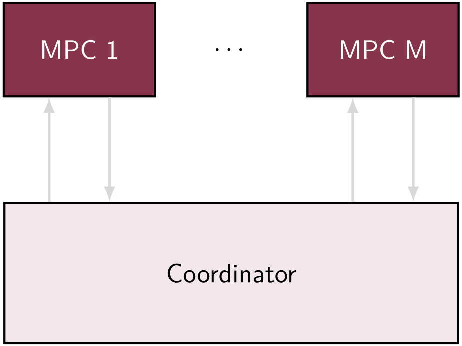

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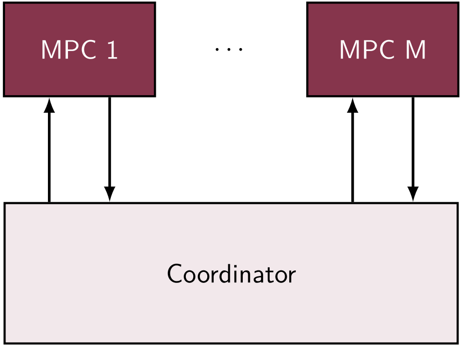

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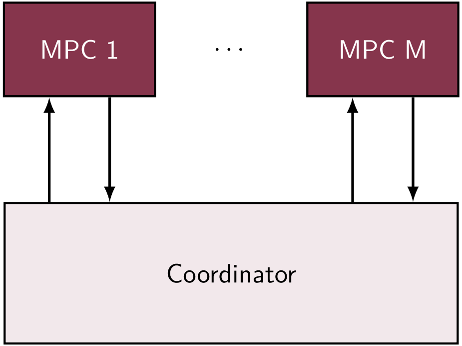

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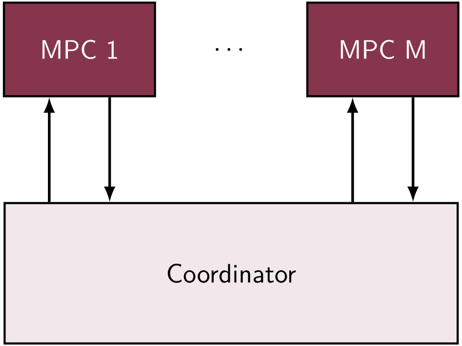

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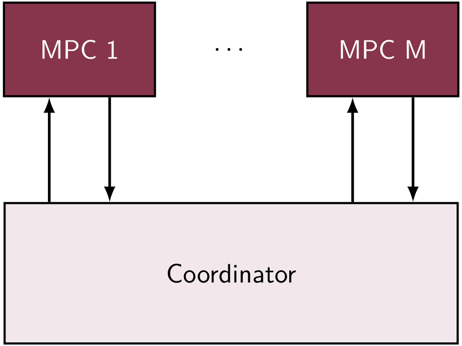

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Distributed Model Predictive Control

Negotiation works if agents comply.

But what if some agents are ill-intentioned and attack the system?

- How can an agent attack?
- What are the consequences of an attack?
- Can we mitigate the effects?

Let's have a preview!

CentraleSupélec

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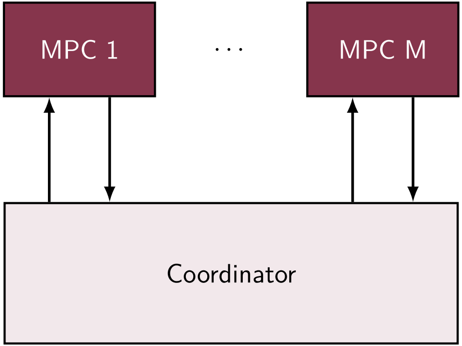
How can a non-cooperative agent attack?

Literature



- [Vel+17a; CMI18] present attacks
  - Objective function
    - Selfish Attack
    - Fake weights
    - Fake reference
  - Fake constraints
  - Liar agent

Deception Attacks

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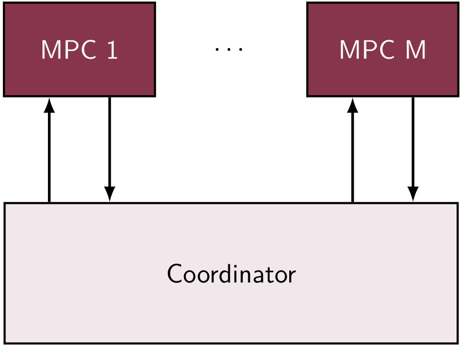

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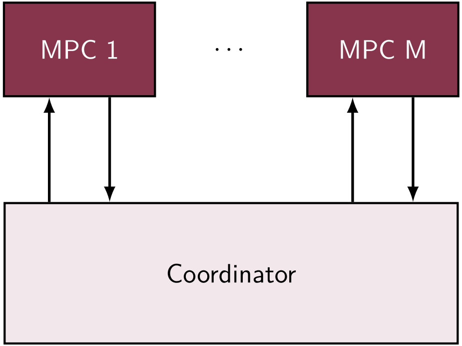

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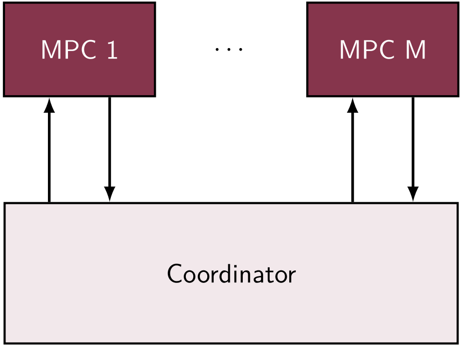

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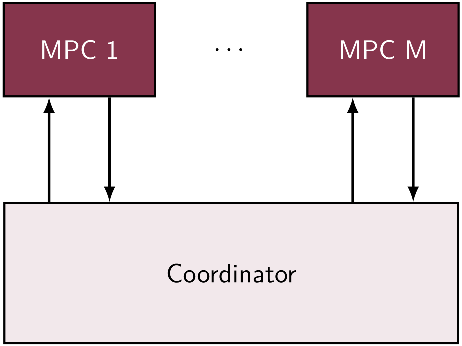

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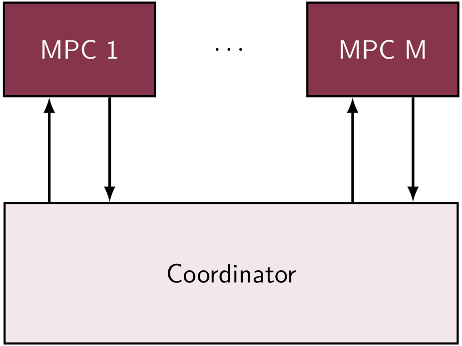

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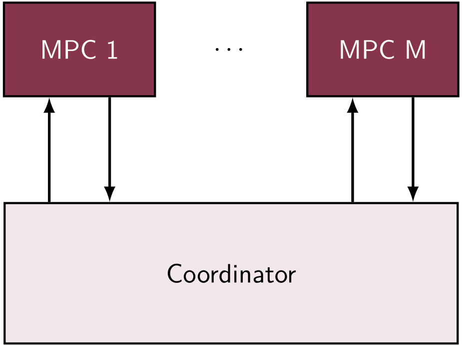

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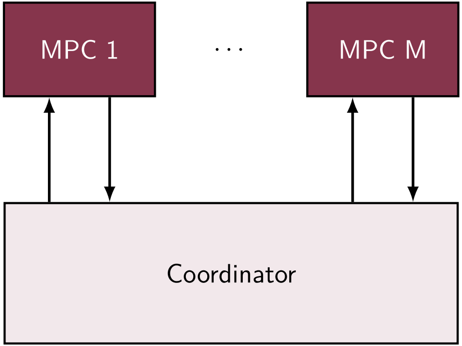

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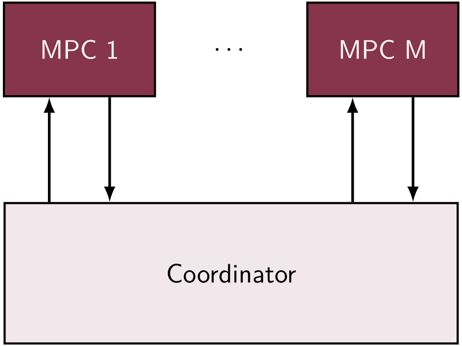

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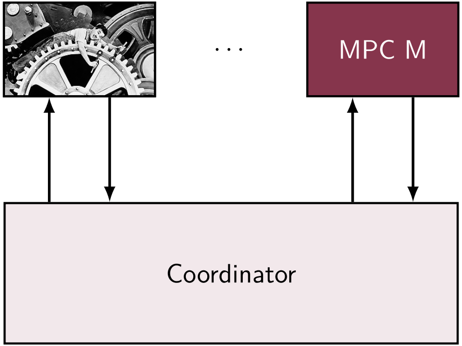

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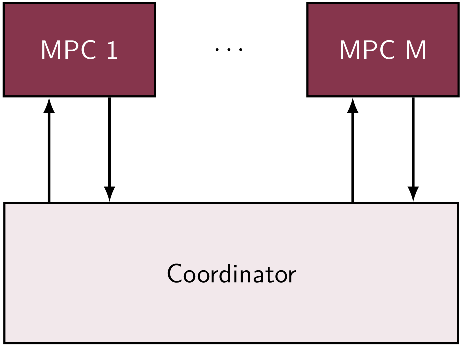
Deception Attacks (Internal change)

How can a non-cooperative agent attack?

Our approach



- We are in coordinator's shoes
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    - False Data Injection

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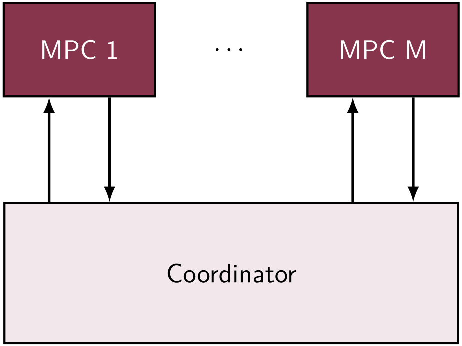

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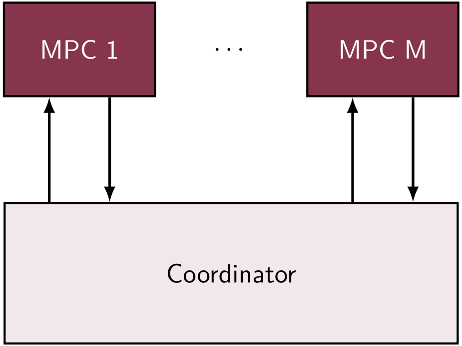

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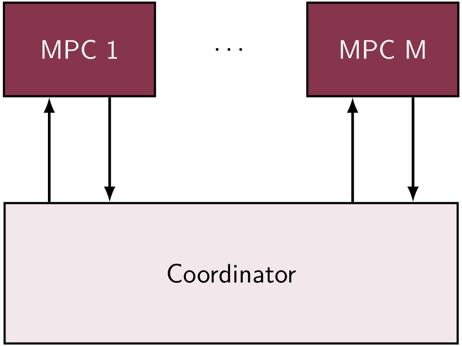

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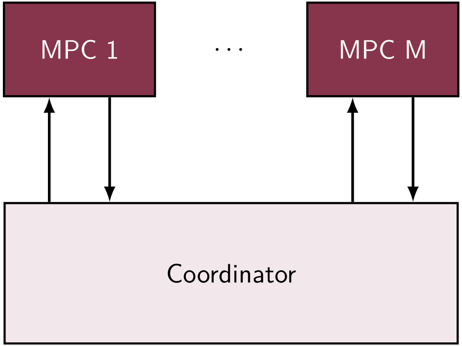

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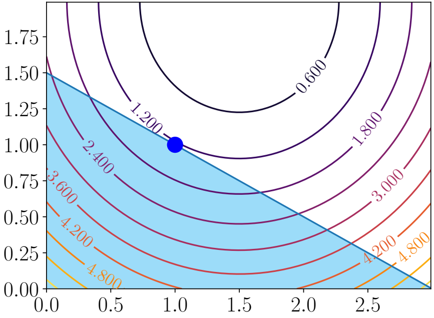
Consequence of an attack

- Attack modifies optimization problem
  - Optimum value is shifted



Original minimum.

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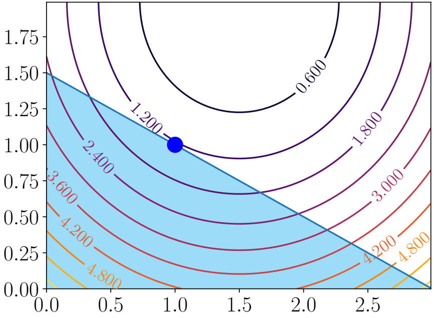

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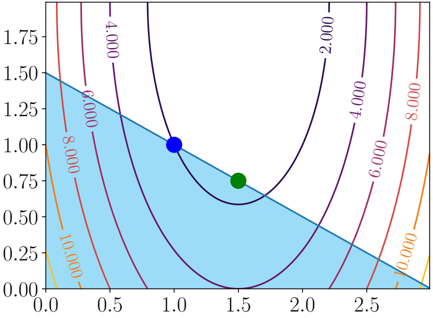

Minimum after attack.

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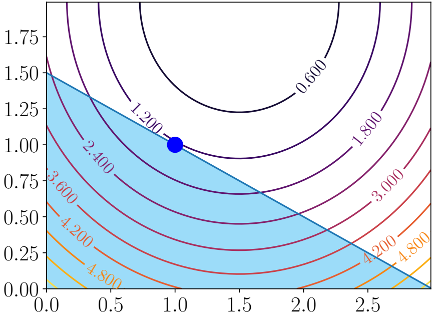

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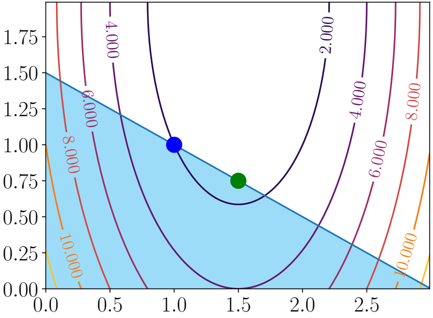

Minimum after attack.

Mitigating the effects

- We can recover by
  - Ignoring attacker
  - Recuperating original behavior (at least trying)

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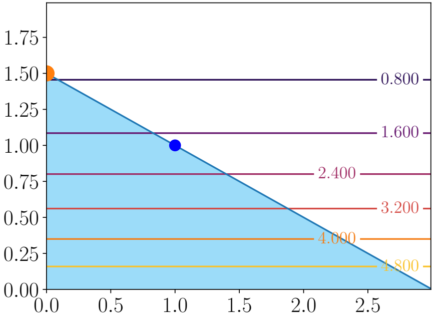

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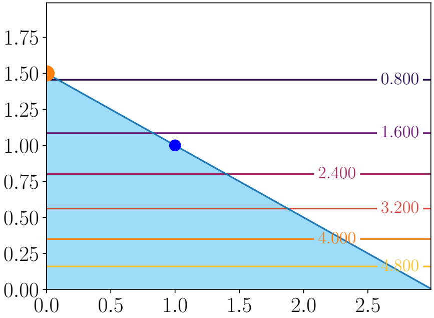

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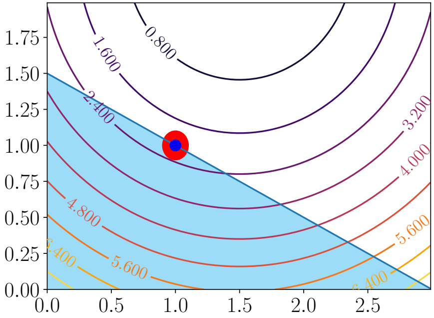

Recover original behavior.

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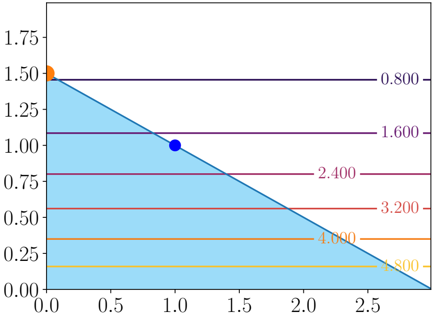

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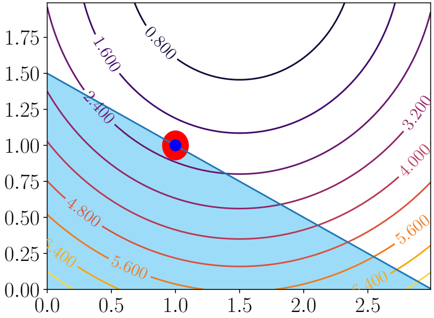

Recover original behavior.

Classification of mitigation techniques

- Passive (Robust) - 1 mode
- Active (Resilient) - 2 modes
  1. Detection/Isolation
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State of art

Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
[Vel+17a] [Mae+21]	Dual	Robust (Scenario)	NA	NA
[Vel+17b] [Vel+18]	Dual	Robust (f-robust)	NA	NA
[CMI18]	Jacobi-Gauß	–	–	–
[Ana+18] [Ana+19] [Ana+20]	Dual	Resilient	Analyt./Learn.	Disconnect (Robustness)
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Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition
2. Resilient Primal Decomposition-based dMPC for deprived systems
3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity
4. Conclusion

- 1 and 2 yielded [NBG21] (SysTol'21)
- 3 yielded [Nog+22] (NecSys'22)

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3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity
4. Conclusion

- 1 and 2 yielded [NBG21] (SysTol'21)
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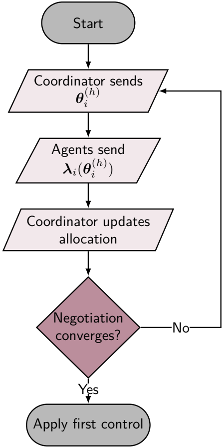
Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition
   - What is the Primal Decomposition?
   - How can an agent attack?
   - Consequences

Primal Decomposition

or Quantity Decomposition | or Resource Allocation



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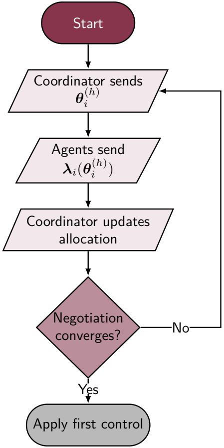

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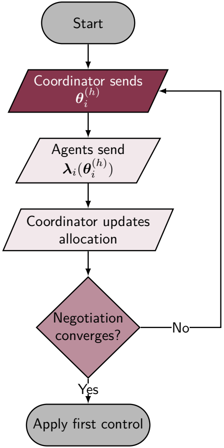

Allocation $\boldsymbol{\theta}_i$

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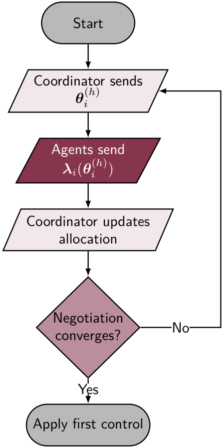

Allocation $\boldsymbol{\theta}_i$

Dissatisfaction $\boldsymbol{\lambda}_i$

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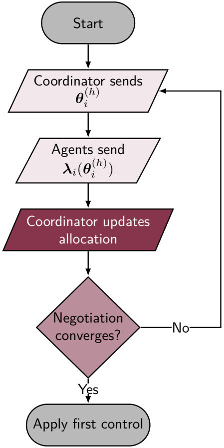

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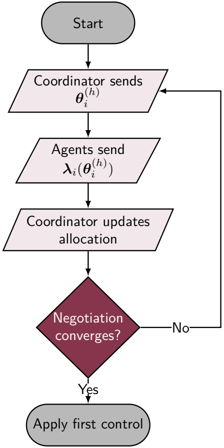

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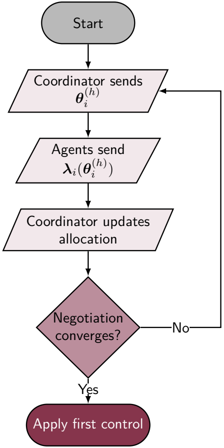

Allocation $\boldsymbol{\theta}_i$

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Primal Decomposition

In detail

- Objective is sum of local ones
- Constraints couple variables

$$\begin{array}{ll} \underset{\boldsymbol{u}_1,\dots,\boldsymbol{u}_M}{\text{minimize}} & \sum_{i\in\mathcal{M}} J_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \\ \text{s.t.} & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \preceq \boldsymbol{u}_{\text{total}} \end{array}$$

For each $i \in \mathcal{M}$

1. Allocate $\boldsymbol{\theta}_i$ for each agent
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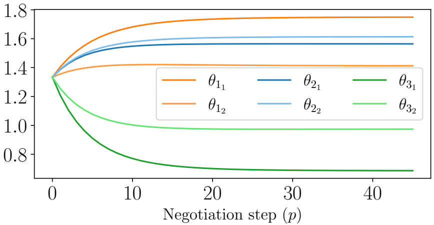
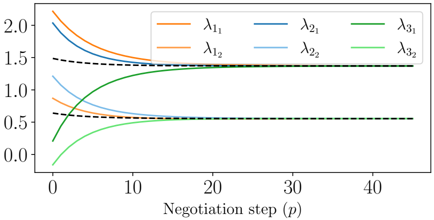
Example

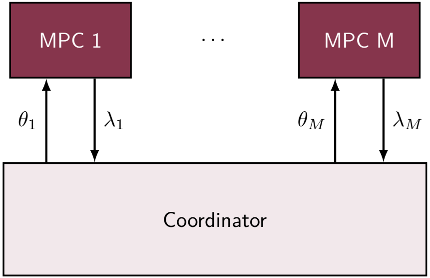
Until everybody is equally dissatisfied





How can a non-cooperative agent attack?

Our approach



- $\boldsymbol{\lambda}_i$ is the only interface
- $\boldsymbol{\lambda}_i$ depends on local parameters
- Malicious agent modifies $\boldsymbol{\lambda}_i$

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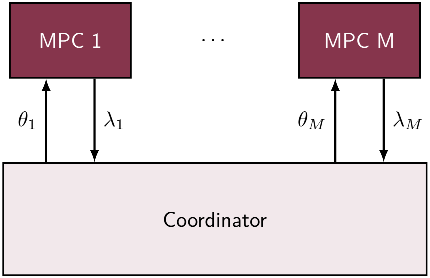

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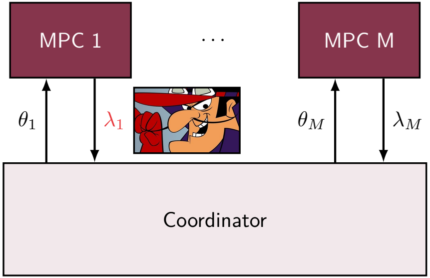

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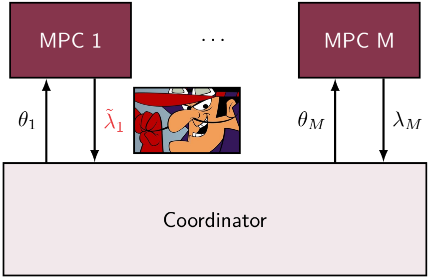

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$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$$

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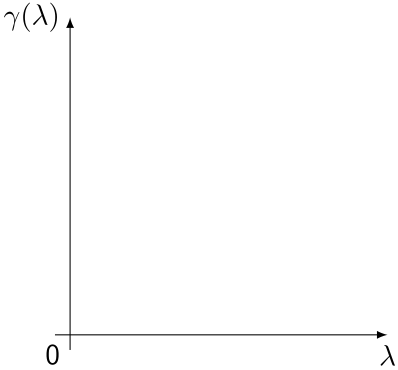
How does an agent lie?

Liar, Liar, Pants of fire



- $\lambda \geqslant 0$ means dissatisfaction
- $\lambda = 0$ means complete satisfaction

Assumptions

- *Attacker satisfied only if it really is*
  $\gamma(\lambda) = 0 \rightarrow \lambda = 0$
- *Attacker is greedy* $\gamma(\lambda) > \lambda$
- *Attack is monotonically increasing*
  $\lambda_b > \lambda_a \rightarrow \gamma(\lambda_b) > \gamma(\lambda_a)$

- Invertible
- If $\tilde{\lambda}_i = T_i[k]\lambda_i \rightarrow \exists T_i[k]^{-1}$

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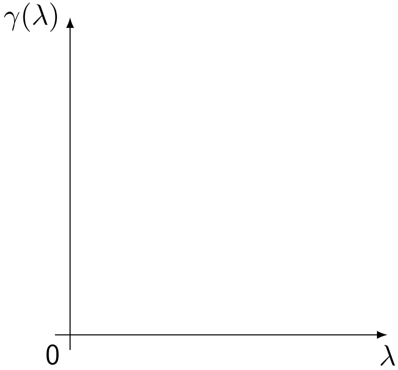

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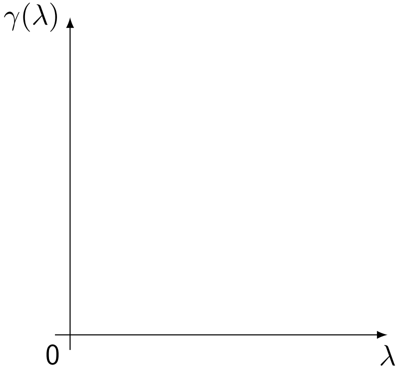

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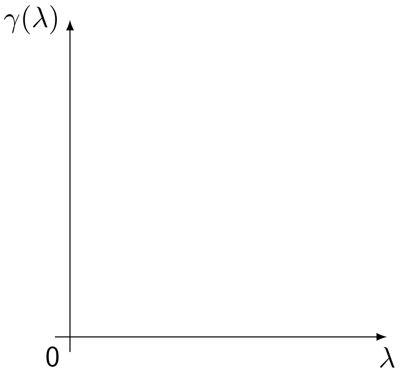

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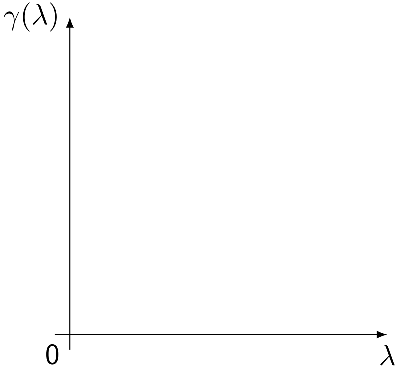

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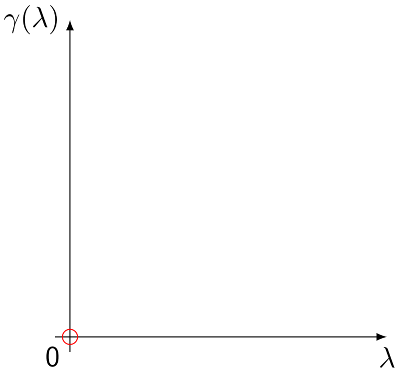

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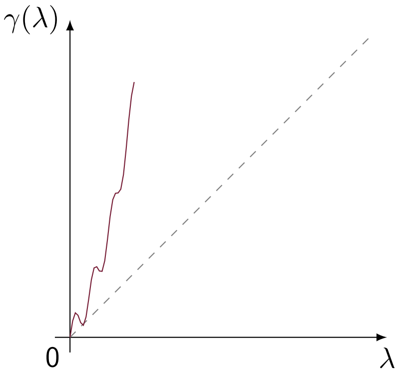

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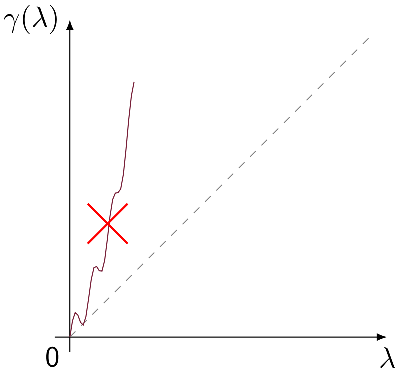

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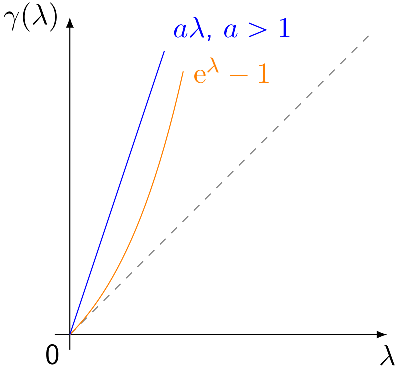

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- If $\tilde{\boldsymbol{\lambda}}_i = T_i[k]\boldsymbol{\lambda}_i \rightarrow \exists T_i[k]^{-1}$

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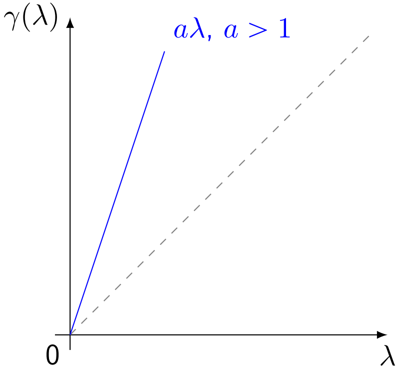
How does an agent lie?

Liar, Liar, Pants of fire



- $\lambda \geqslant 0$ means dissatisfaction
- $\lambda = 0$ means complete satisfaction

Assumptions

- *Attacker satisfied only if it really is*
  - $\gamma(\lambda) = 0 \rightarrow \lambda = 0$
- *Attacker is greedy* $\gamma(\lambda) > \lambda$
- *Attack is monotonically increasing* $\lambda_b > \lambda_a \rightarrow \gamma(\lambda_b) > \gamma(\lambda_a)$

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Example

4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\lambda}_1 = \gamma_1(\lambda_1) = \tau_1 I \lambda_1$

- We can observe 3 things
  - Global minimum when $\tau_1 = 1$
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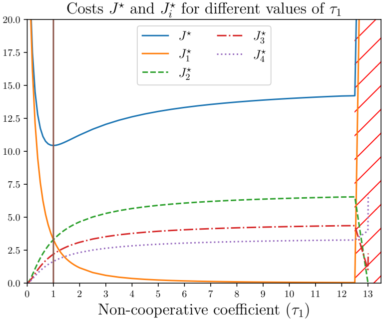

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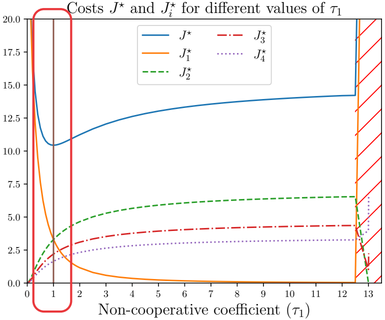

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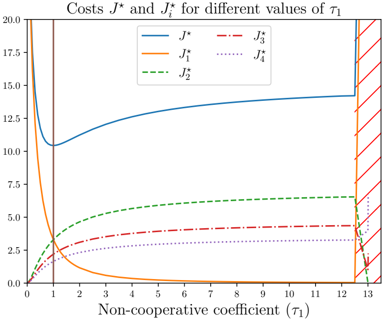

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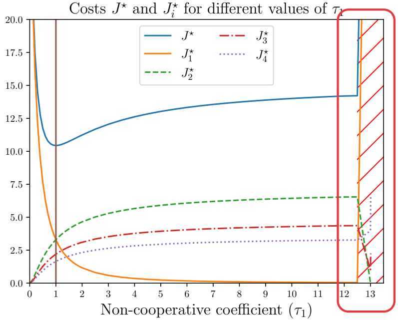

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Outline

2. Resilient Primal Decomposition-based dMPC for deprived systems
    - Analyzing deprived systems
    - Building an algorithm
    - Applying mechanism

What are deprived systems?

Systems whose optimal solution has all constraints active

- Unconstrained Solution $\mathring{\boldsymbol{U}}_i^\star[k]$
- $\bar{\Gamma}_i \mathring{\boldsymbol{U}}_i^\star[k] \geq \boldsymbol{\theta}_i[k] \rightarrow$ Scarcity
  - Solution projected onto boundary
  - Same as with equality constraints[2]

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T \boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i \boldsymbol{U}_i[k] \leq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

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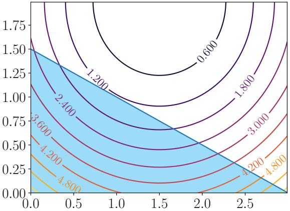

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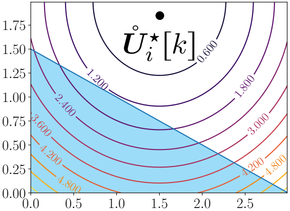

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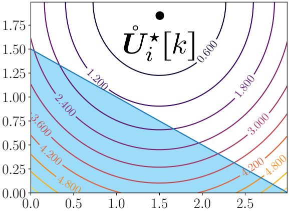

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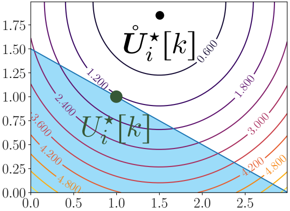

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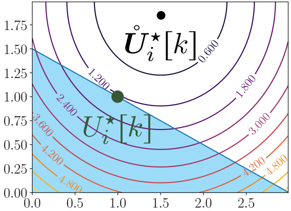

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Assumptions

- *Quadratic local problems*
- *Scarcity*

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(local parameters unknown by coordinator) $\left\{ \begin{array}{l} \bullet\ P_i \text{ is time invariant} \\ \bullet\ \boldsymbol{s}_i[k] \text{ is time variant} \end{array} \right.$

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  - Affine solution

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  - Affine solution

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Assumption

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[1] Using Recursive Least Squares for example

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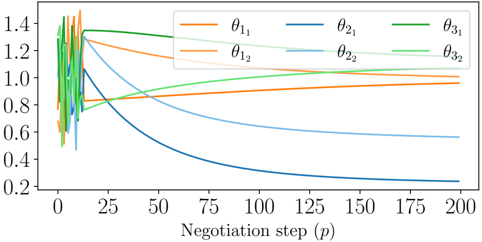

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Classification of mitigation techniques

- Active (Resilient)
  1. Detection/Isolation ✔
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Mitigation mechanism

Reconstructing $\boldsymbol{\lambda}_i$

- Now, we have $\widehat{\tilde{P}}_i[k]$
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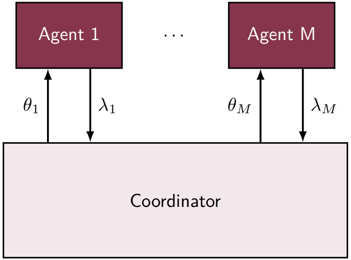

- Supervise exchanges by inquiring the agents
- Estimate how they will behave

Two Phases

1. Detect which agents are non-cooperative
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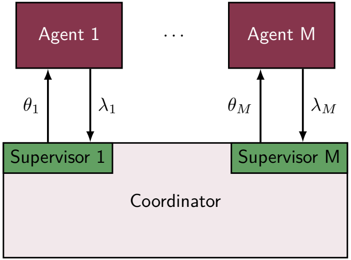

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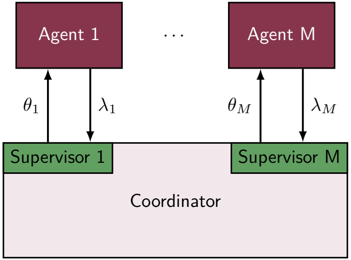

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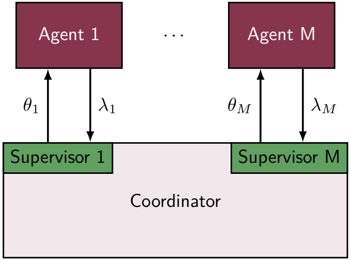

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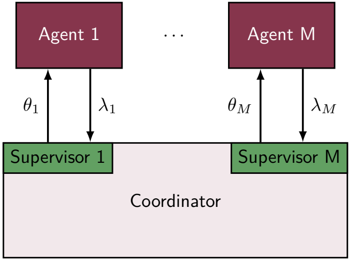

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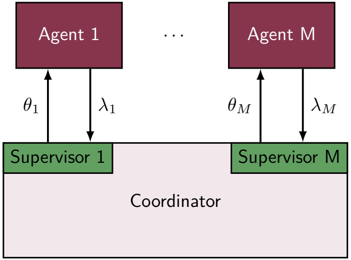

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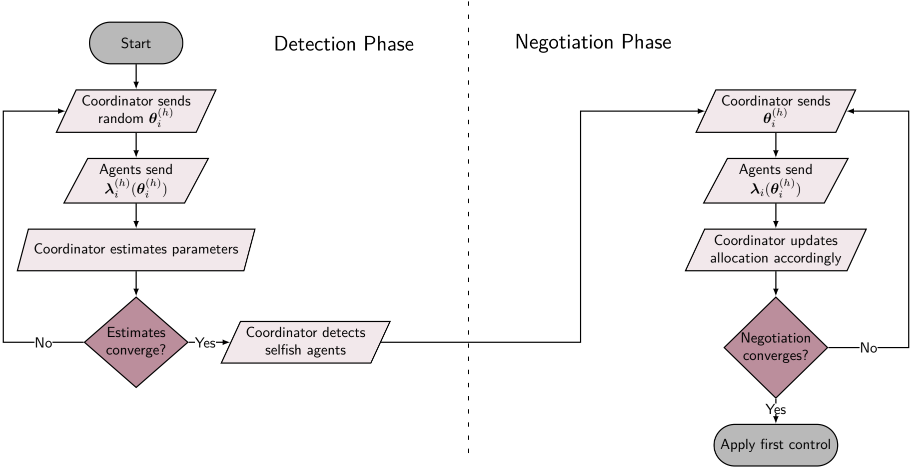

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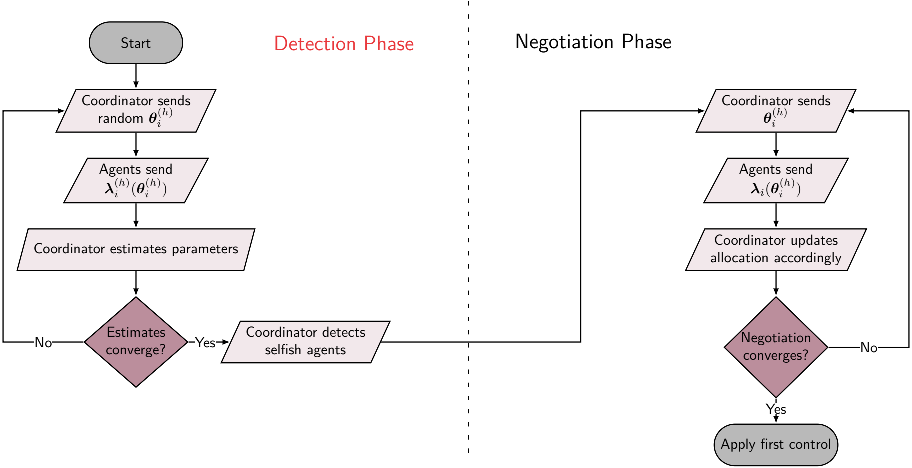

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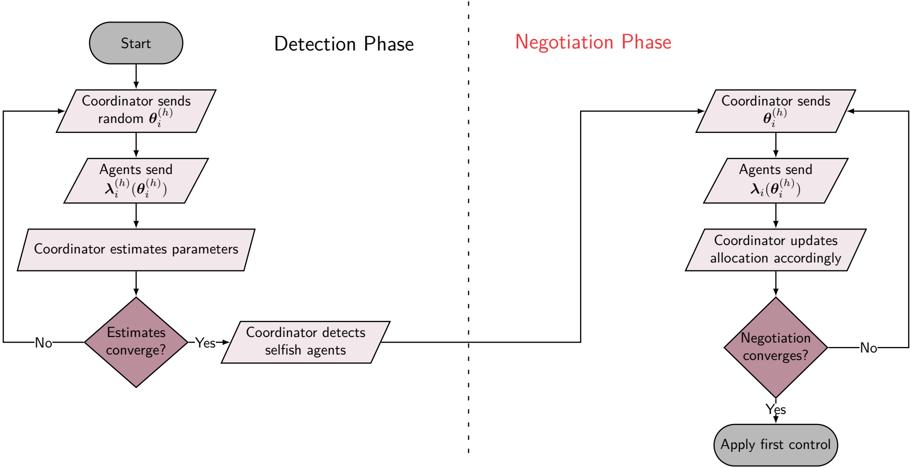

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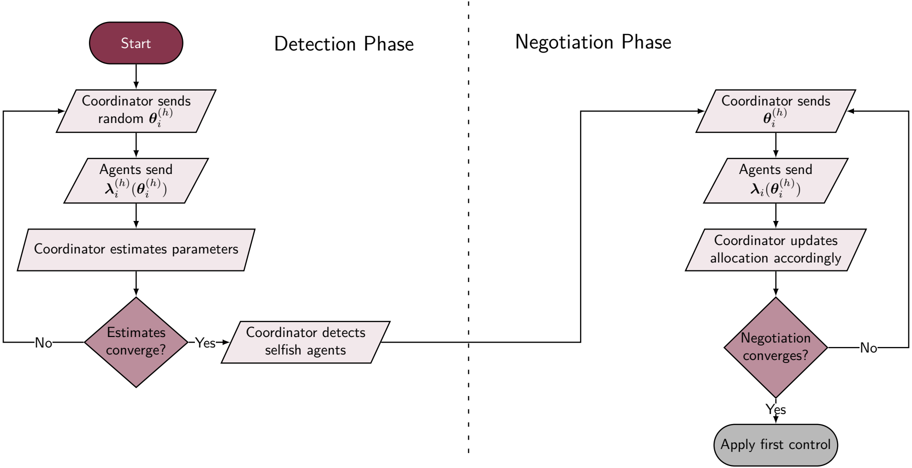

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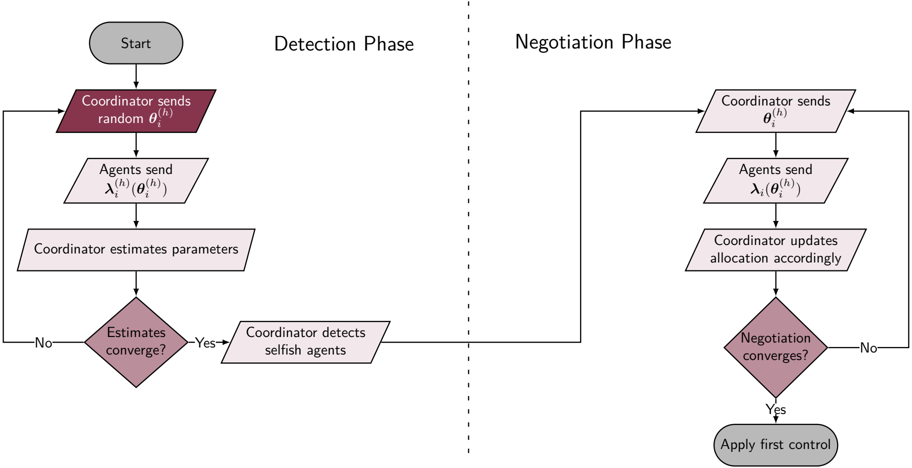

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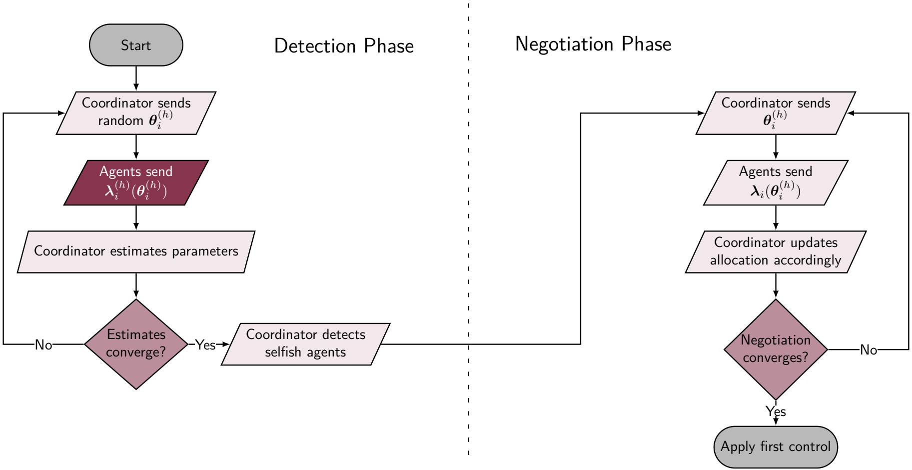

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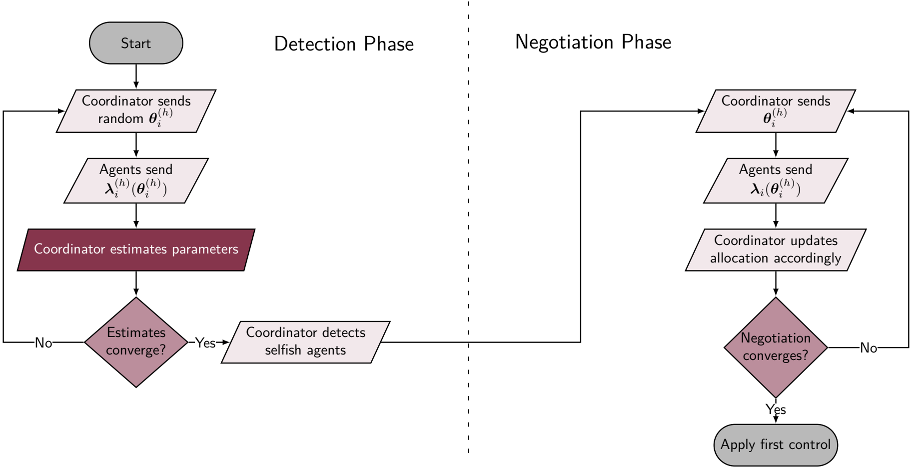

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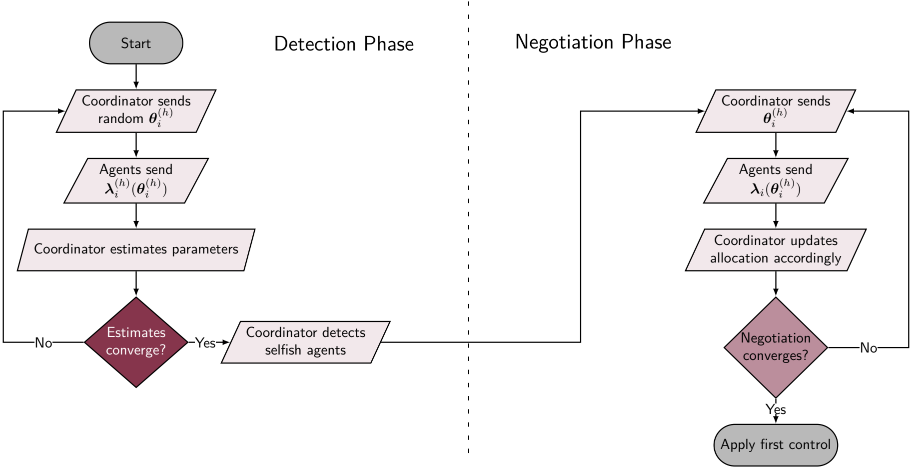

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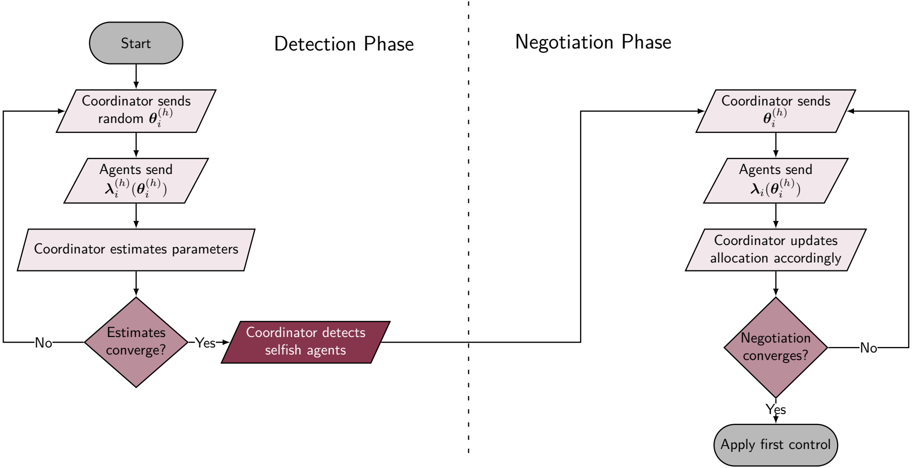

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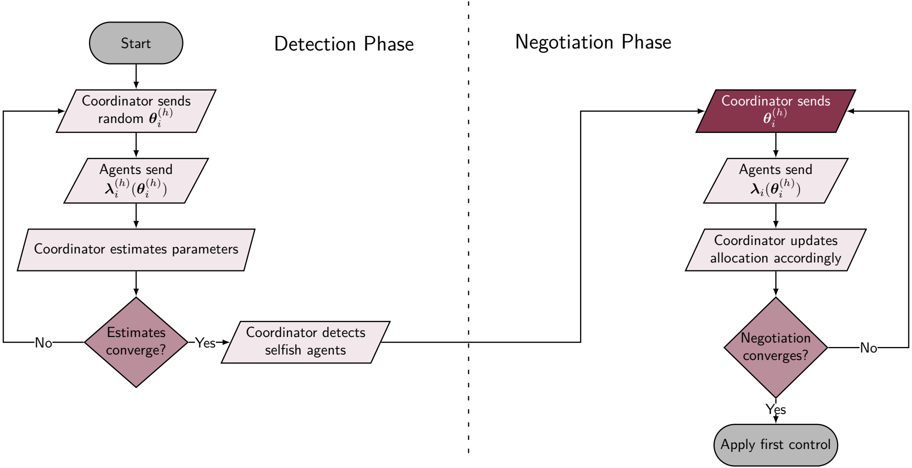

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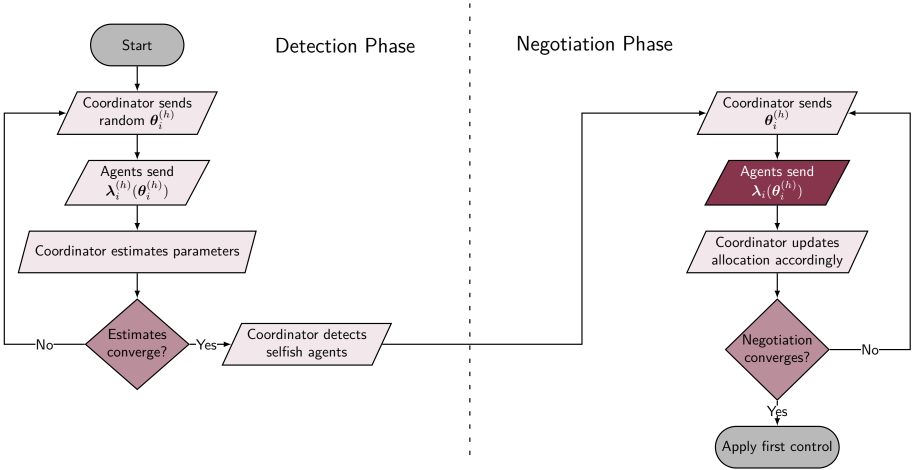

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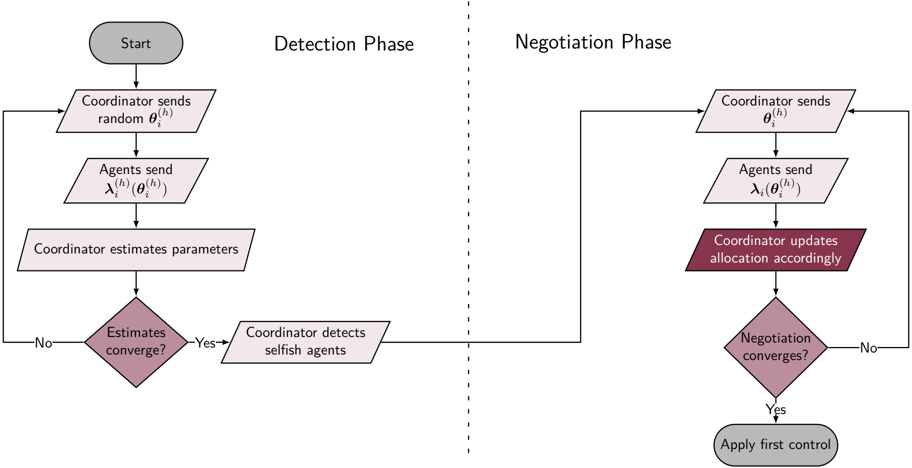

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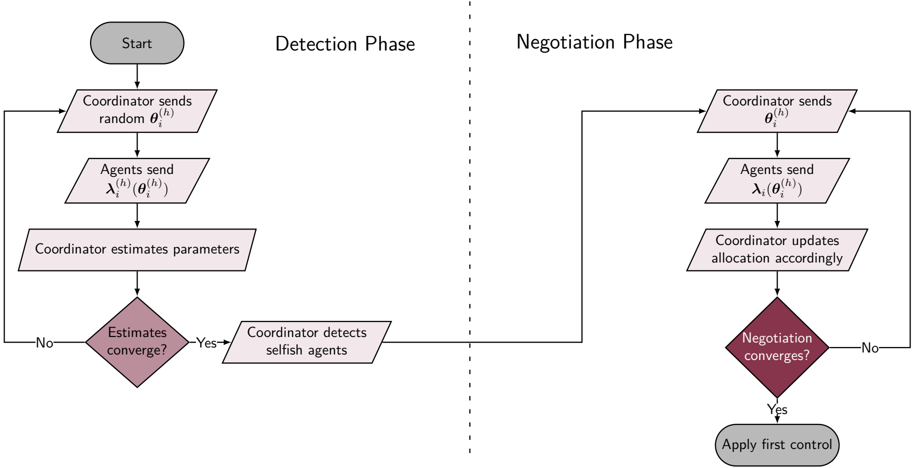

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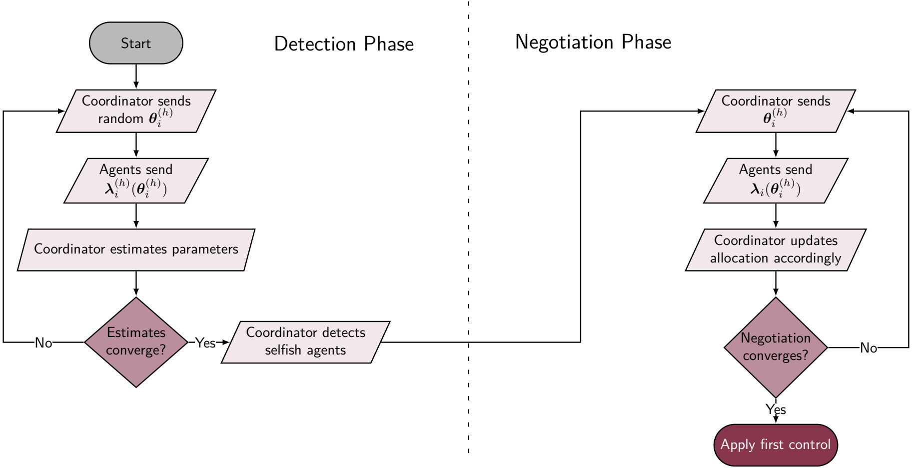

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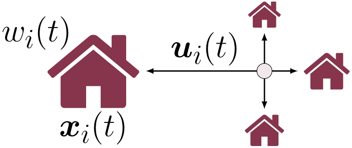

District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)
- Not enough power
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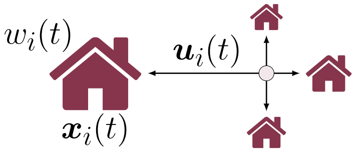

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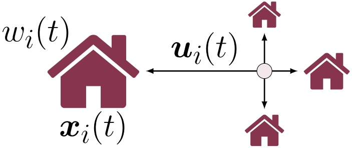

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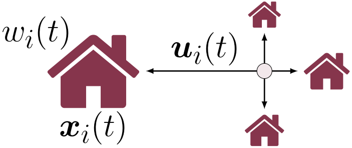

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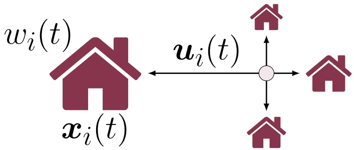

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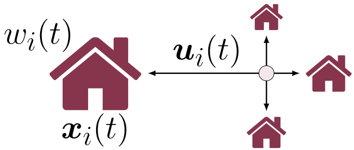

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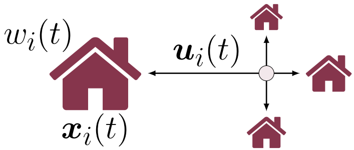

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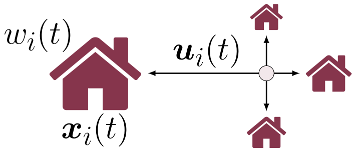
Example

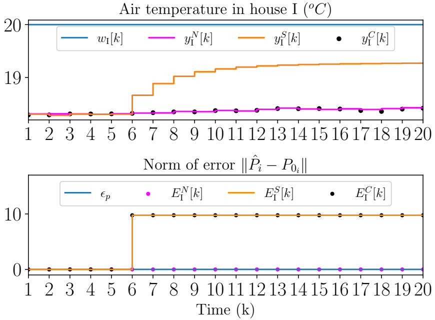


District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)
- Not enough power
- Period of $5\mathrm{h}$
- 3 scenarios
  - N Nominal
  - C Agent I cheats (dMPC)
  - S Agent I cheats (RPdMPC-DS)

Results

Temporal



Temperature in house I.

Error $E_I(k)$.

N Nominal, S Selflish, C Corrected

Results

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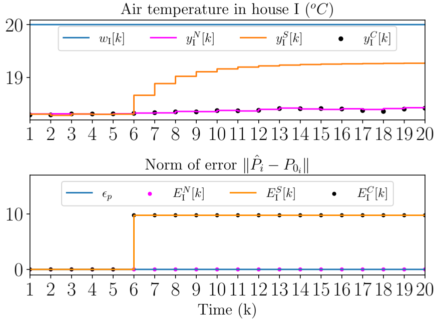

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- Agent starts cheating in $k = 6$
- S Agent increases its comfort
- C Restablish behavior close to N

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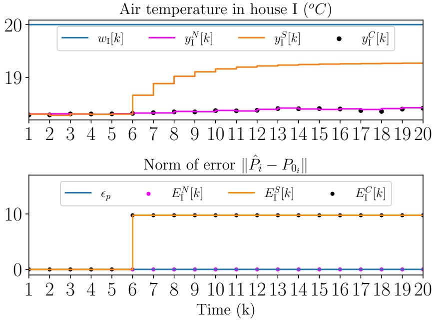

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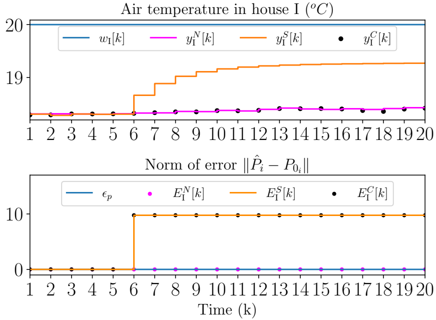

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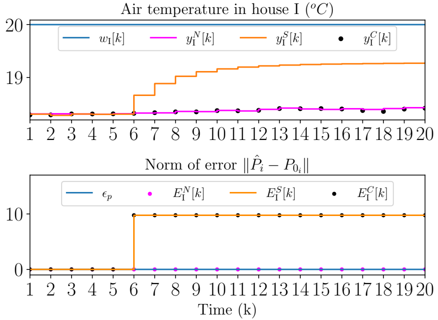

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Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.3	0.503
II	21.671	-0.547
III	17.387	-0.004
IV	17.626	-0.09
Global	3.526	0.016

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Outline

CentraleSupélec

Relaxing scarcity assumption

- Systems are not completely deprived
  - We can't change our constraints to equality ones anymore
  - Nor use the simpler update equation

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i\boldsymbol{U}_i[k] \leq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

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Analyzing System

Solution for $\lambda_i[k]$

Instead of having one single affine solution

$$\boldsymbol{\lambda}_i[k] = -P_i \boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

Now, we may have multiple (Piecewise affine function)

$$\boldsymbol{\lambda}_i[k] = \begin{cases} -P_i^{(0)} \boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^{0}_{\boldsymbol{\lambda}_i} \\ \vdots & \vdots \\ -P_i^{(2^{n_{\text{ineq}}}-1)} \boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(2^{n_{\text{ineq}}}-1)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^{2^{n_{\text{ineq}}}-1}_{\boldsymbol{\lambda}_i} \end{cases}$$

Still the $P_i^{(n)}$ are time independent

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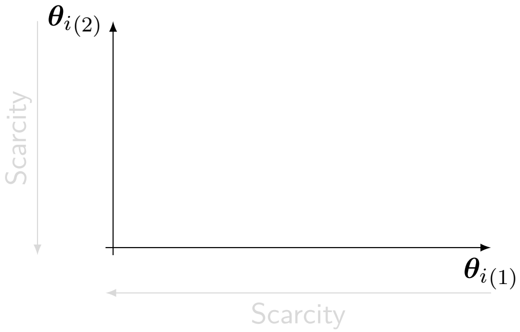

Separation surfaces depend on state and local parameters.
Unknown by the coordinator.

Analyzing System

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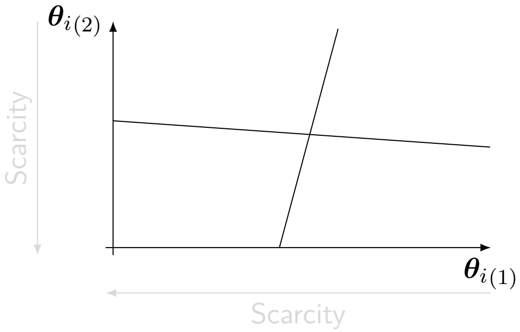

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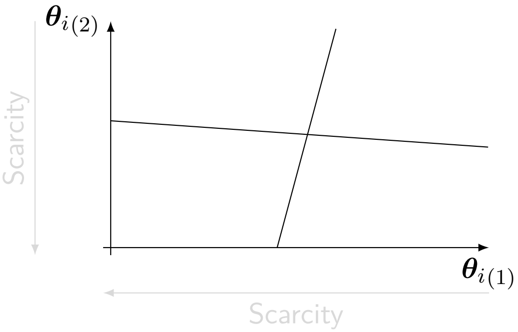

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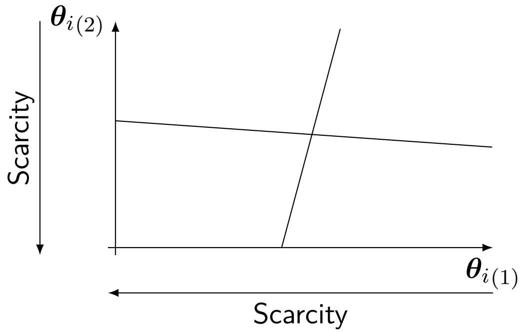

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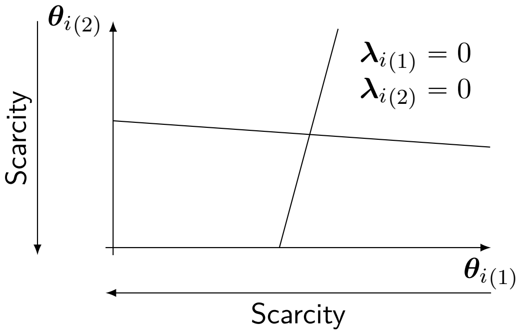

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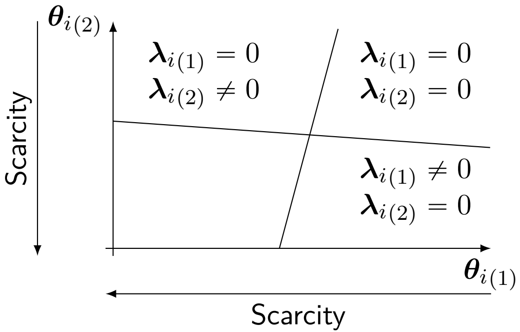

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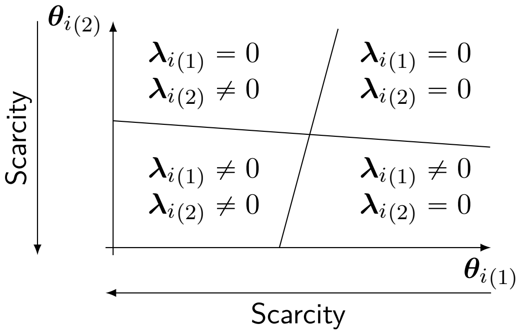

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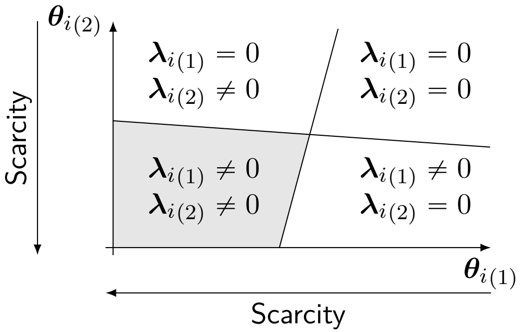

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Analyzing System

Under attack!

$$\tilde{\boldsymbol{\lambda}}_i[k] = T_i[k]\boldsymbol{\lambda}_k$$

Parameters are modified. But not the regions' limits

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- If we can estimate $\widetilde{P}_i^{(0)}$ we can use same strategy than before
- Problem: We don't known in which region $\boldsymbol{\theta}_i$ is
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$$\tilde{\boldsymbol{\lambda}}_i[k] = \begin{cases} -\widetilde{P}_i^{(0)}\boldsymbol{\theta}_i[k] - \widetilde{\boldsymbol{s}}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^0 \\ \vdots & \vdots \\ -\widetilde{P}_i^{(2^{n_{\text{ineq}}}-1)}\boldsymbol{\theta}_i[k] - \widetilde{\boldsymbol{s}}_i^{(2^{n_{\text{ineq}}}-1)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}_{\boldsymbol{\lambda}_i}^{2^{n_{\text{ineq}}}-1} \end{cases}$$

- If we can estimate $\widetilde{P}_i^{(0)}$ we can use same strategy than before
- Problem: We don't known in which region $\boldsymbol{\theta}_i$ is
- Solution: Let's force it using Artificial Scarcity

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Artificial Scarcity

Who is it? Who is it?

Assumption

We know a point $\overset{\varnothing}{\boldsymbol{\theta}}_i$ which activates all constraints[4]

[4] If we have local constraints, we suppose this point respects them.

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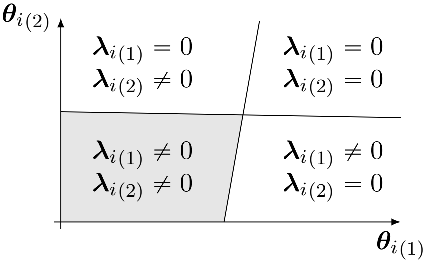

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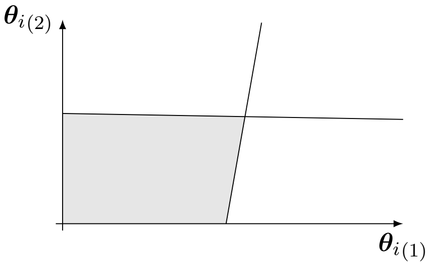

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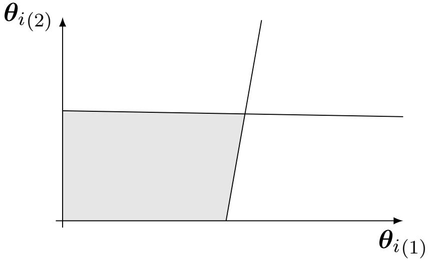

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- How to known the radius?
  - We don't.
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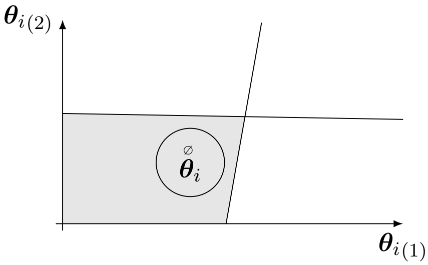

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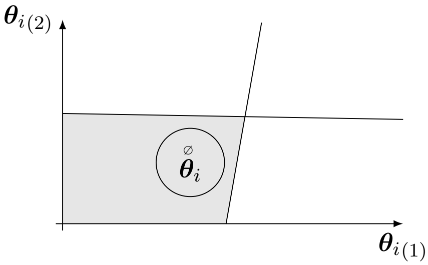

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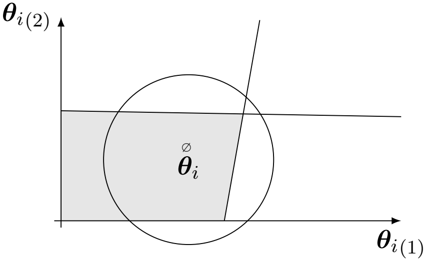

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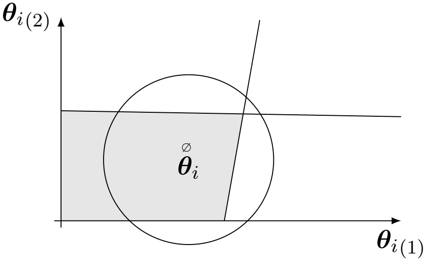

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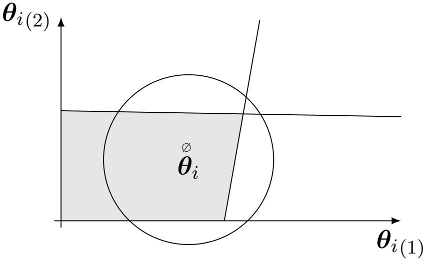

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- Iterative method to estimate parameters of multimodal models[5]
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Detection and Mitigation

Same same, but different

Assumption

We know nominal $\bar{P}_i^{(0)}$

- Detection

$$\left\| \widehat{\widetilde{P}}_i^{(0)}[k] - \bar{P}_i^{(0)} \right\|_F \geqslant \epsilon_{P_i^{(0)}}$$

- Mitigation

$$\widehat{T_i[k]^{-1}} = \bar{P}_i^{(0)} \widehat{\widetilde{P}}_i^{(0)}[k]^{-1}.$$

$$\overset{\text{rec}}{\boldsymbol{\lambda}_i} = \widehat{T_i[k]^{-1}} \tilde{\boldsymbol{\lambda}}_i.$$

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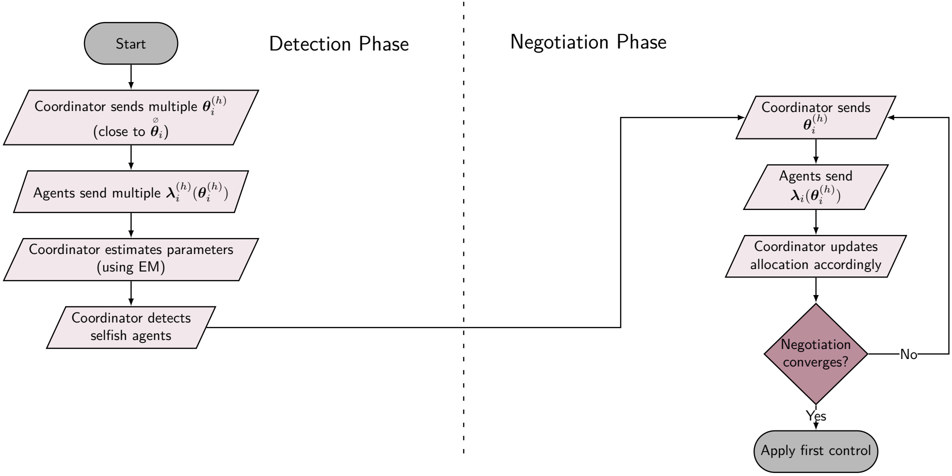
Complete algorithm

RPdMPC-AS



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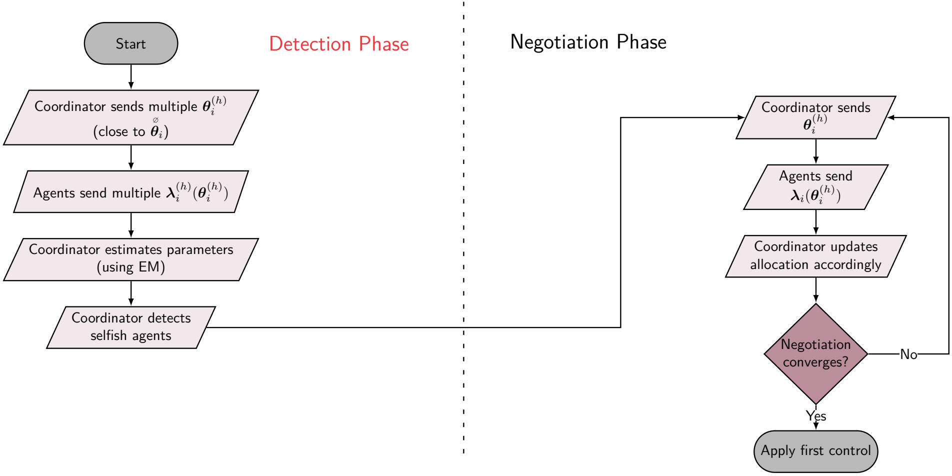

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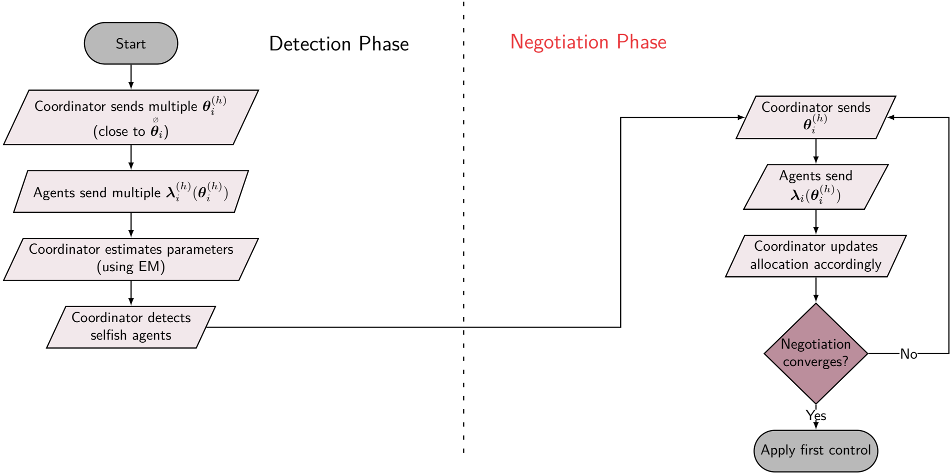

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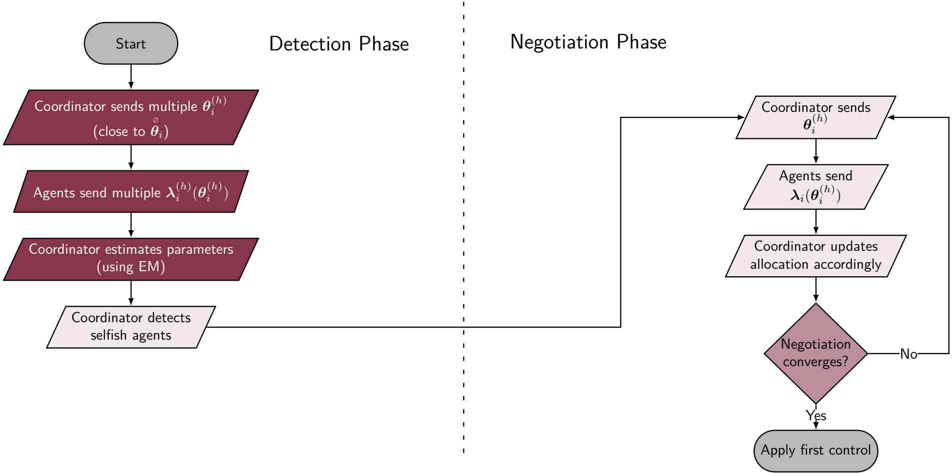

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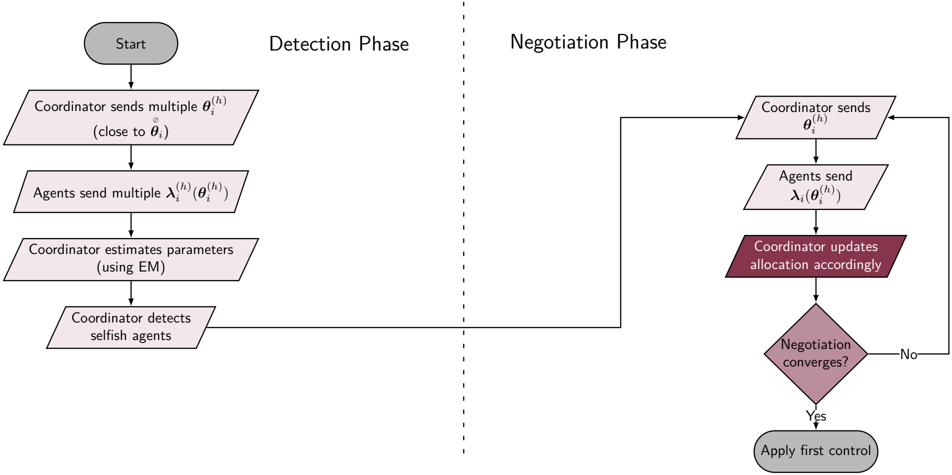
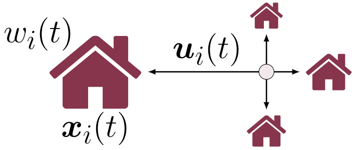


Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C
- Not enough power
- Period of 5h $(T_s = 0.25\text{h})$
- 3 scenarios
  - N Nominal
  - C Agent I cheats (dMPC)
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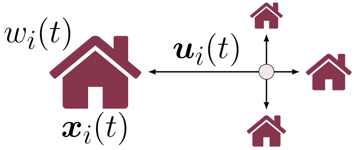

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Results

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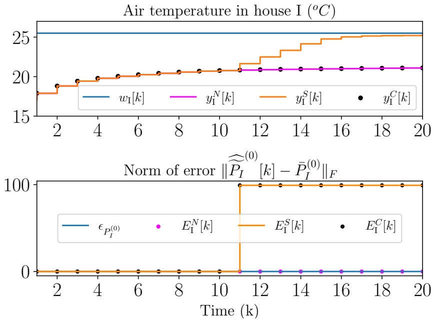

Temperature in house I.

Error $E_I(k)$.

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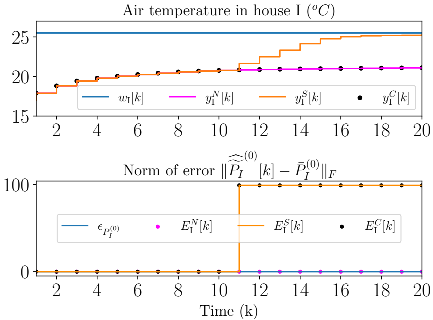

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Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.489	$-4.12e-05$
II	35.813	$1.74e-05$
III	29.225	$2.14e-05$
IV	37.541	$1.73e-05$
Global	10.689	$-6e-07$

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Too good to be true!

It's a kind of magic!

- Unfortunately EM is not magic
  - Slow convergence
  - Dependency on initialization
    - No guarantees of achieving global optimal
- Some "solutions":
  - Force some parameters to converge faster (case dependant)
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Main takeaways

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  - Attacker can change the communication to receive more ressources.
- What are the consequences of an attack? ✓
  - Suboptimality and maybe instability
- Can we mitigate the effects? ✓
  - Yes! By exploring the scarcity of the systems!

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- Insights from the analysis of the solutions of the optimization problems
  - We found some parameters that are constant when there is no cheating
  - The same parameters change when system is attacked
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  - Straightforward if system is scarce
  - If not scarce we try to force it artifically
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Open question Future Directions

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Questions? Comments?

Repository
https://github.com/Accacio/thesis

Contact
rafael.accacio.nogueira@gmail.com

For Further Reading I

K.J. Åström and B. Wittenmark. Adaptive Control. Addison-Wesley series in electrical and computer engineering: Control engineering. Addison-Wesley, 1989. ISBN: 9780201097207. DOI: 10.1007/978-3-662-08546-2_24.

José M Maestre, Rudy R Negenborn, et al. Distributed Model Predictive Control made easy. Vol. 69. Springer, 2014. ISBN: 978-94-007-7005-8.

Wicak Ananduta et al. “Resilient Distributed Model Predictive Control for Energy Management of Interconnected Microgrids”. In: Optimal Control Applications and Methods 41.1 (2020), pp. 146–169. DOI: 10.1002/oca.2534. URL: https://onlinelibrary.wiley.com/doi/pdf/10.1002/oca.2534.

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José M. Maestre et al. “Scenario-Based Defense Mechanism Against Vulnerabilities in Lagrange-Based Dmpc”. In: Control Eng Pract 114 (2021), p. 104879. ISSN: 0967-0661. DOI: 10.1016/j.conengprac.2021.104879.

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Wicak Ananduta et al. “Resilient Distributed Energy Management for Systems of Interconnected Microgrids”. In: 2018 IEEE Conference on Decision and Control (CDC). 2018, pp. 3159–3164. DOI: 10.1109/CDC.2018.8619548.

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Rafael Accácio Nogueira, Romain Bourdais, and Hervé Guéguen. "Detection and Mitigation of Corrupted Information in Distributed Model Predictive Control Based on Resource Allocation". In: 2021 5th Conference on Control and Fault-Tolerant Systems (SysTol). 2021, pp. 329–334. DOI: 10.1109/SysTol52990.2021.9595927.

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For Further Reading V

Pablo Velarde et al. “Vulnerabilities in Lagrange-Based DMPC in the Context of Cyber-Security”. In: 2017 IEEE International Conference on Autonomic Computing (ICAC). July 2017, pp. 215–220. DOI: 10.1109/ICAC.2017.53.

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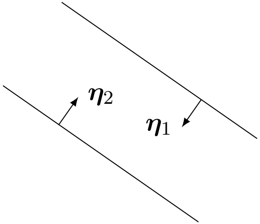
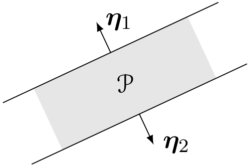
Conditions

back

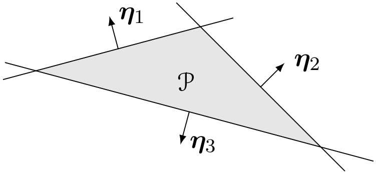
One way to ensure this, is to make the original constraints to form a cone.



No intersection



$\langle^{\boldsymbol{\eta}_2}_{\boldsymbol{\eta}_1} = 180^o$



A 3-sided polyhedron.

θ dynamics

◂ back

$$\boldsymbol{\theta}^{(p+1)} = \mathcal{A}_\theta \boldsymbol{\theta}^{(p)} + \mathcal{B}_\theta[k]$$

where

$$\mathcal{A}_\theta = \begin{bmatrix} I - \frac{M-1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \frac{1}{M}\rho^{(p)}P_1 & I - \frac{M-1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & I - \frac{M-1}{M}\rho^{(p)}P_M \end{bmatrix}$$

$$\mathcal{B}_\theta[k] = \begin{bmatrix} -\frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \vdots \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \end{bmatrix}$$