Security of distributed Model Predictive Control under False Data injection

Rafael Accácio NOGUEIRA

2022-12-08

https://bit.ly/3g3S6X4

IETR

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Context

“Necessity is the mother of invention”

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"Necessity is the mother of invention"

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- Electricity Distribution System
- Heat distribution
- Water distribution
- Traffic management

  (include your problem here)

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- Coupled by constraints
  - Technical/ Comfort
- Optimization objectives
  - Minimize energy consumption
  - Maximize user satisfaction
  - Follow a trajectory
- Solution → MPC

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Find best control sequence using predictions based on a model.

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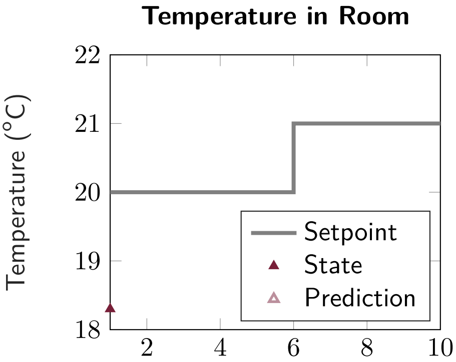
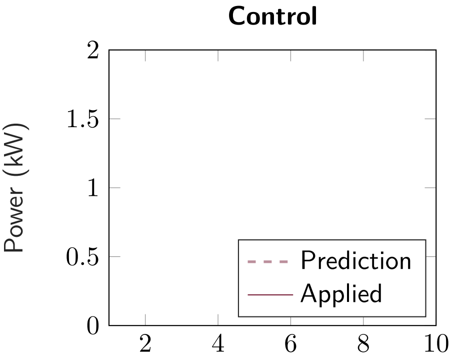
In a nutshell

Find optimal control sequence





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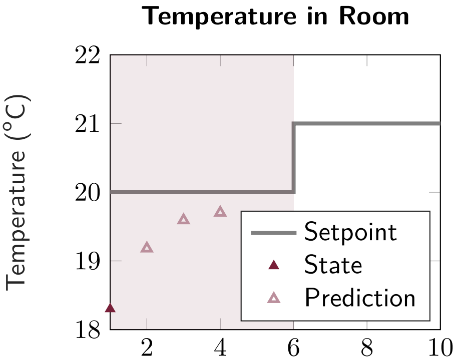
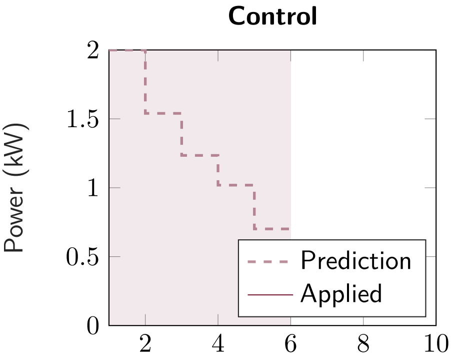
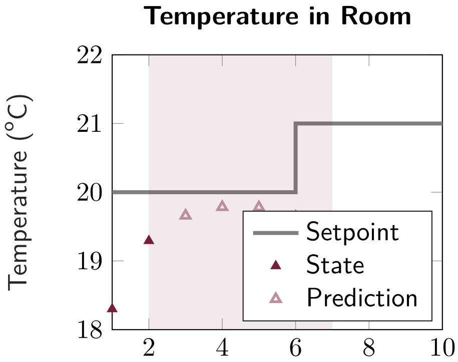
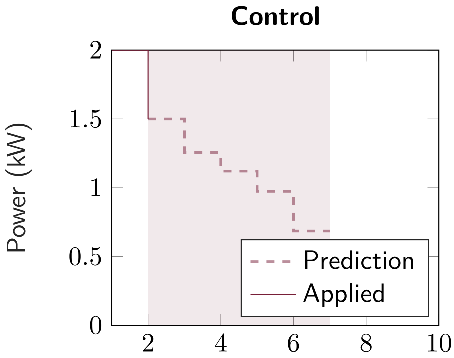




In a nutshell

Find optimal control sequence, apply first element

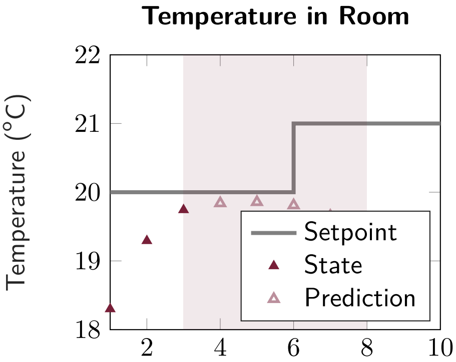


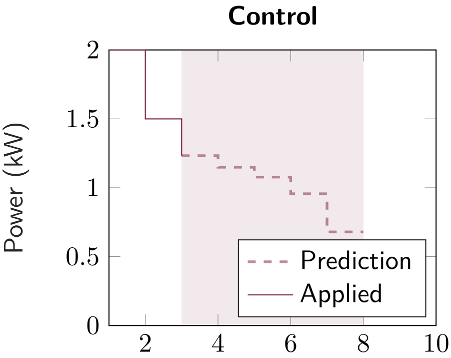


Model Predictive Control

In a nutshell

Find optimal control sequence, apply first element, rinse repeat





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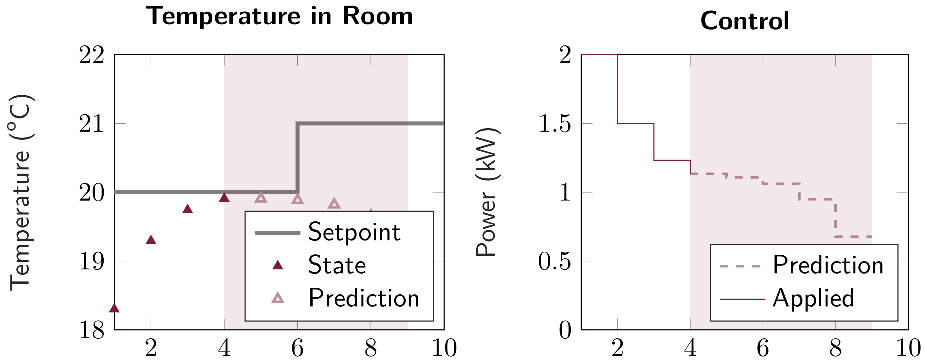
Find optimal control sequence, apply first element, rinse repeat → Receding Horizon



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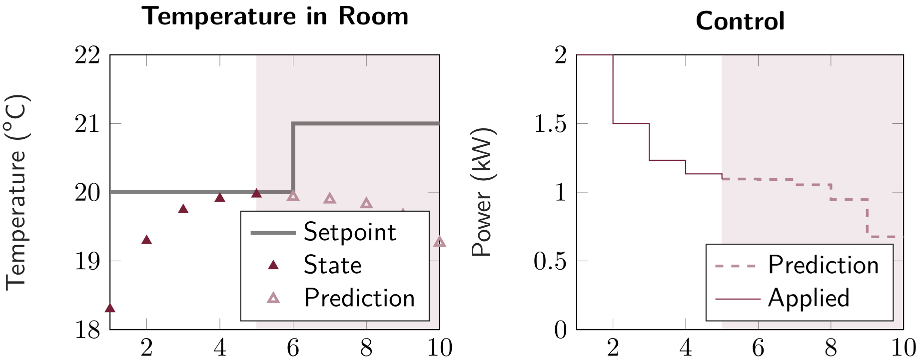

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Nothing is perfect

- Problems
  - Complexity of calculation
  - Topology (Geographical distribution)
  - Flexibility (Add/remove parts)
  - Privacy
- Solution: Divide and Conquer (distributed MPC)
  - Break calculation
  - Make Systems Communicate

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Distributed Model Predictive Control

It is about communication

- We break the MPC into multiple
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  - Many flavors to choose from[1]
    - Hierarchical/Anarchical
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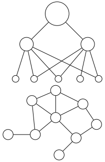
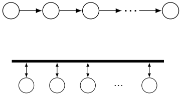

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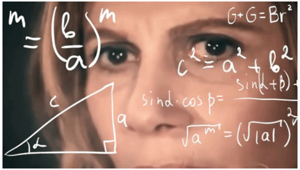

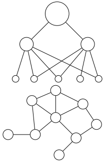

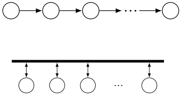

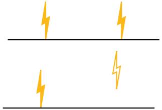

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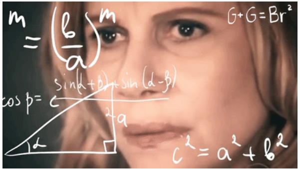

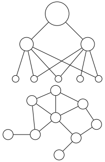

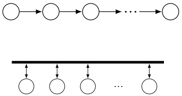

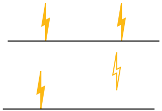

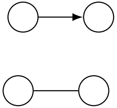

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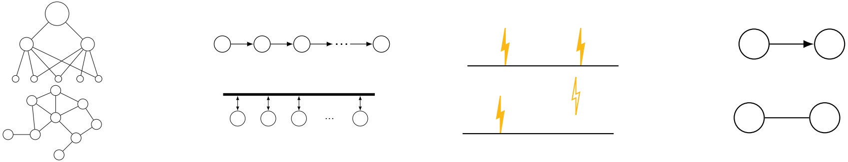
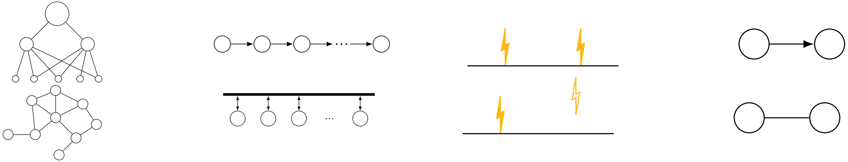


[1] Distributed Model Predictive Control made easy

Distributed Model Predictive Control

It is about communication

- We break the MPC into multiple
- Make them Communicate , But how?
  - Many flavors to choose from[1]
    - Hierarchical/Anarchical
    - Sequential/Parallel
    - Synchronous/Asynchronous
    - Bidirectional/Unidirectional
    - ...



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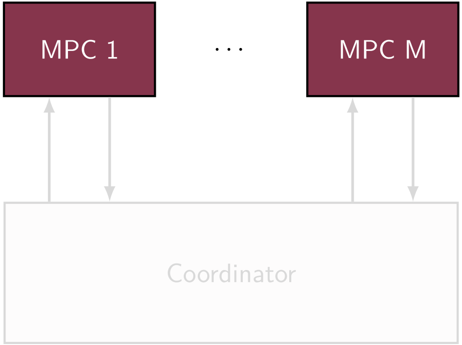

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- Bidirectional
- No delay → Synchronous
- Agents solve local problems
- Variables are updated

Until Convergence

Distributed Model Predictive Control

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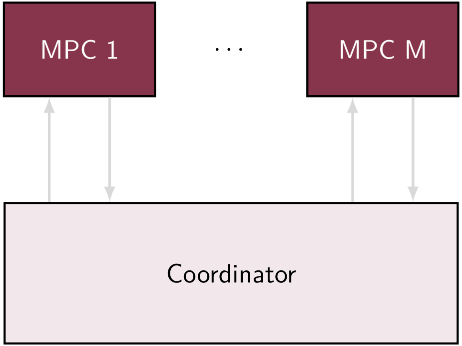

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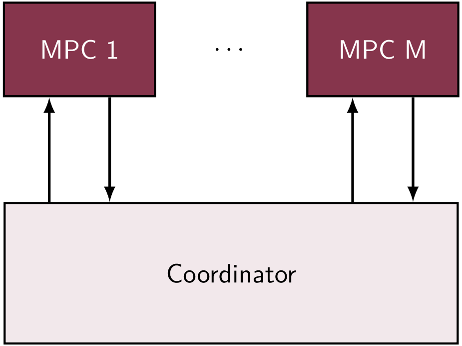

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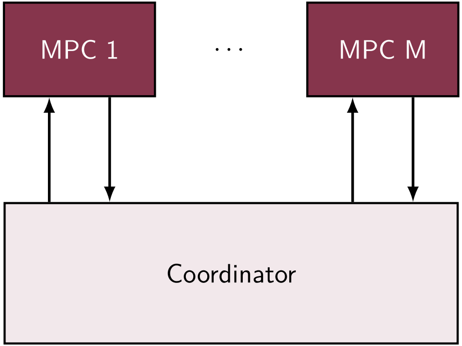

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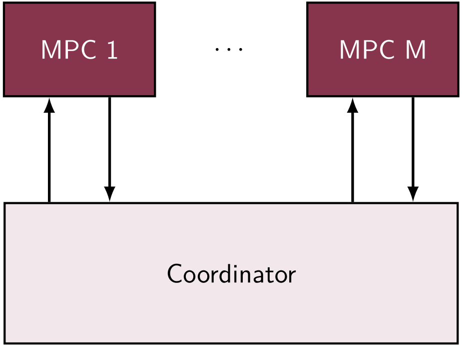

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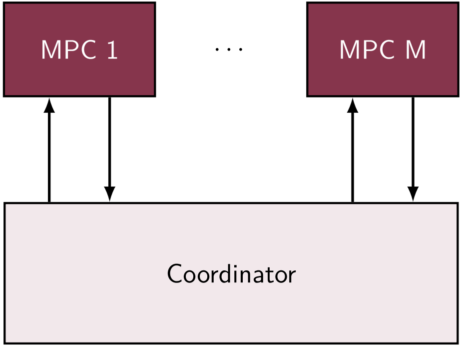

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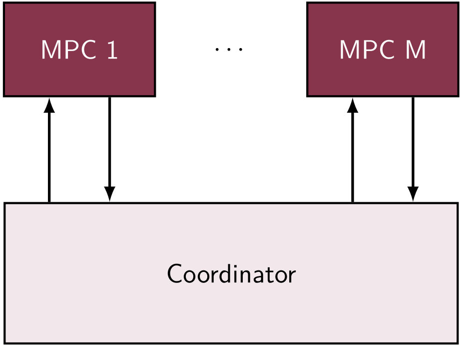

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Negotiation works if agents comply.

But what if some agents are ill-intentioned and attack the system?

- How can an agent attack?
- What are the consequences of an attack?
- Can we mitigate the effects?

Let's have a preview!

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How can a non-cooperative agent attack?

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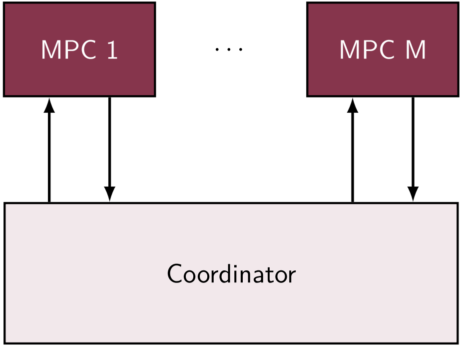

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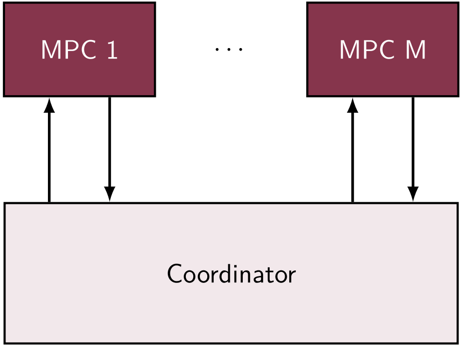

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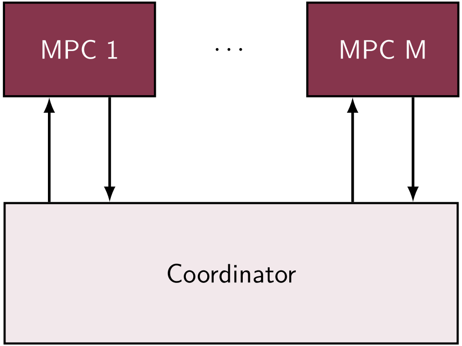

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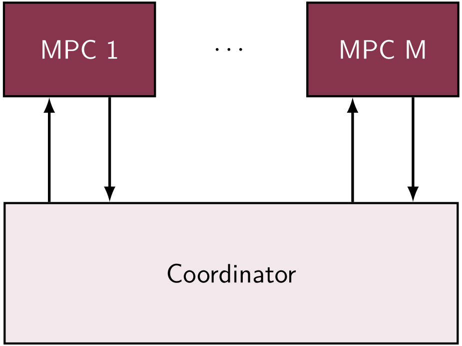

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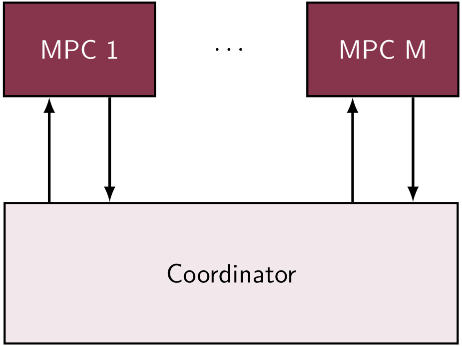

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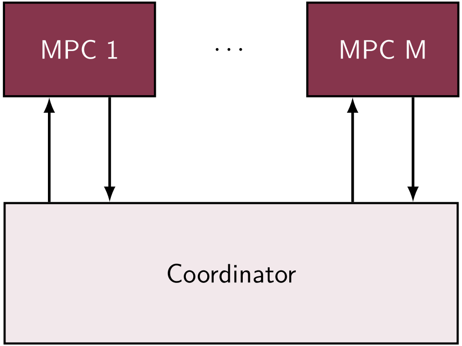

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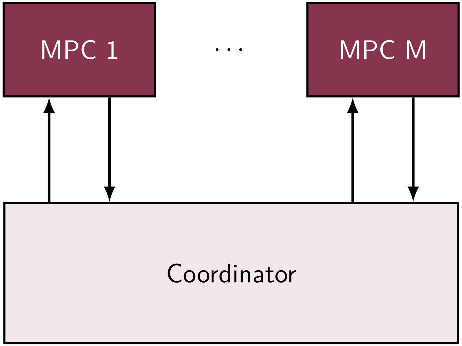

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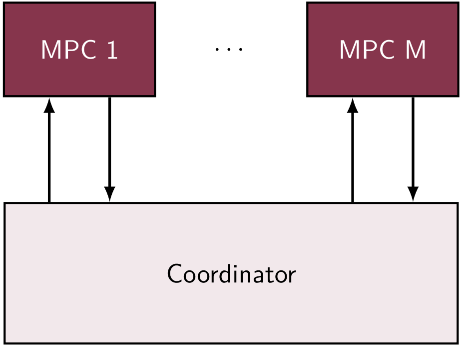

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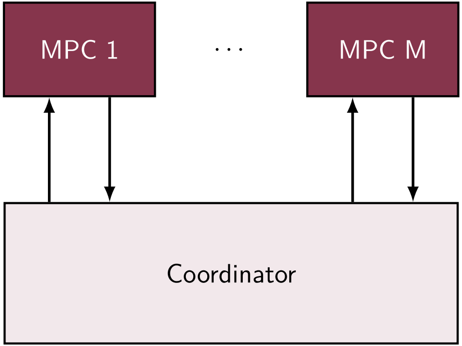

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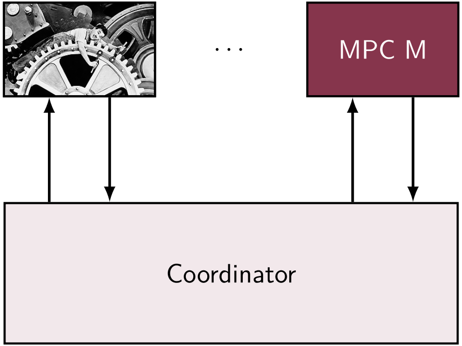

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Deception Attacks (Internal change)

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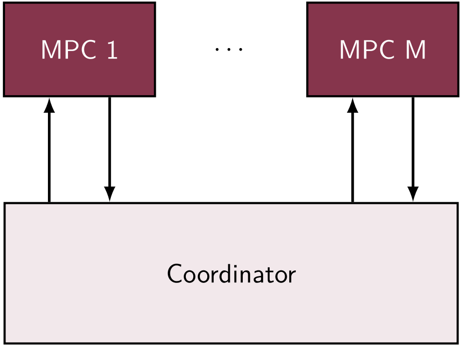
How can a non-cooperative agent attack?

Our approach



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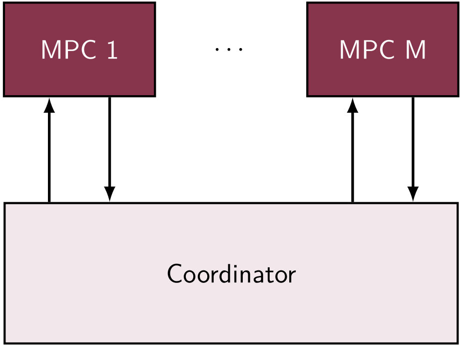

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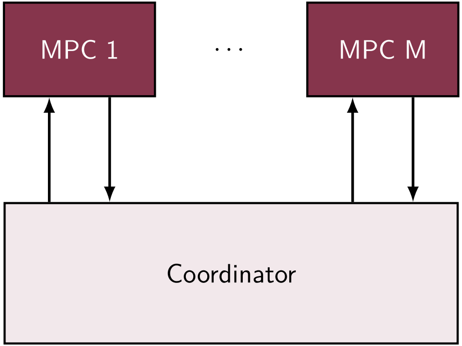

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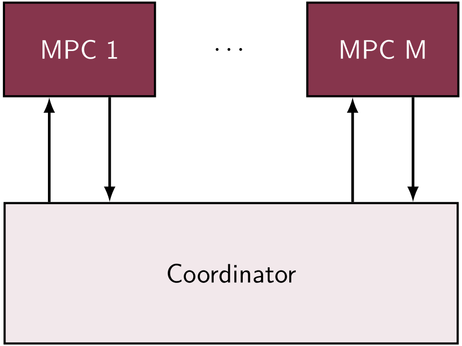

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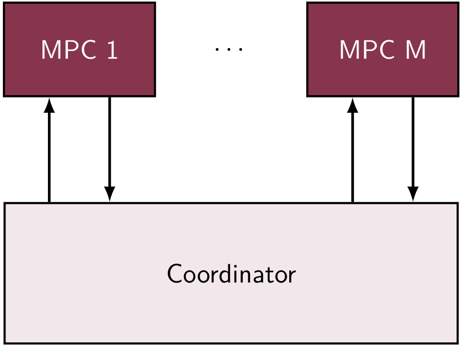

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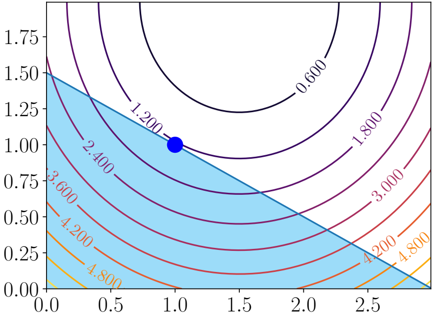
Consequence of an attack

- Attack modifies optimization problem
    - Optimum value is shifted



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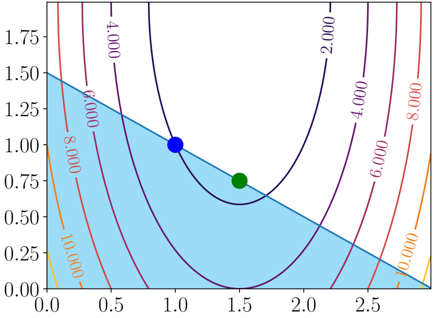

Minimum after attack.

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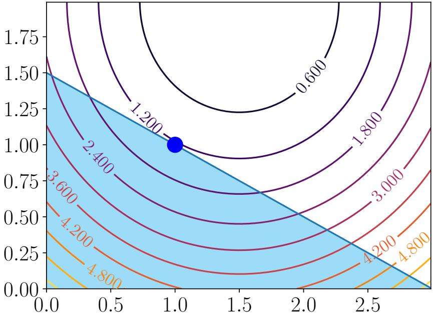

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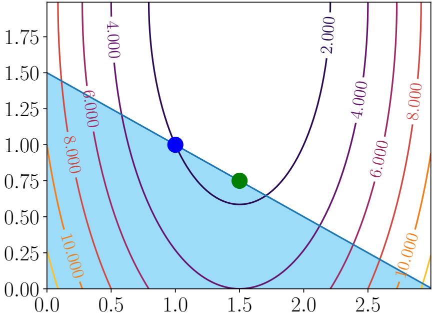

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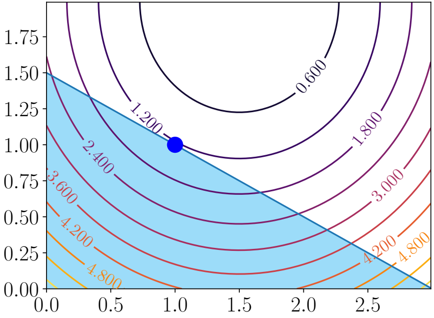

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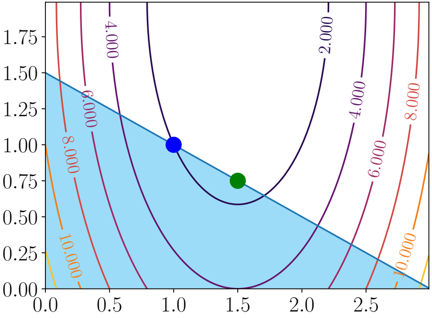

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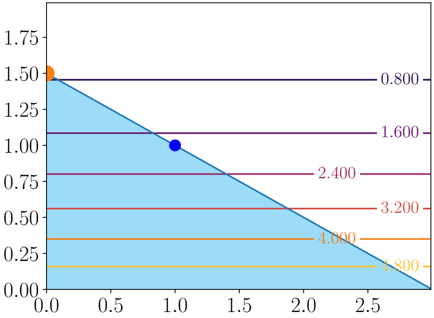

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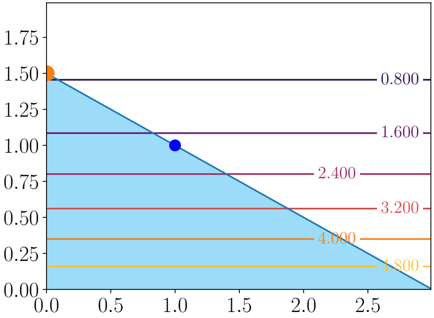

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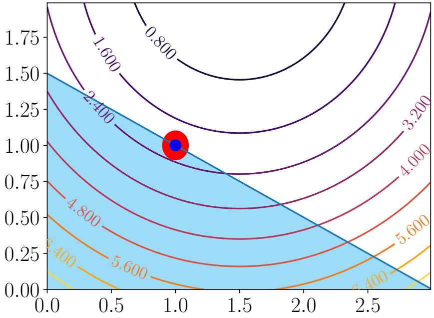

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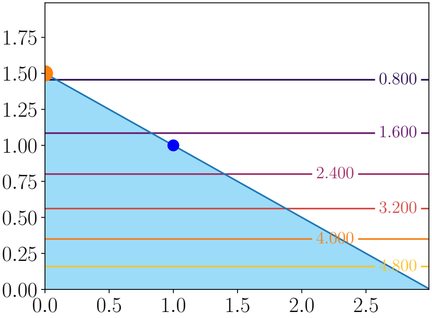

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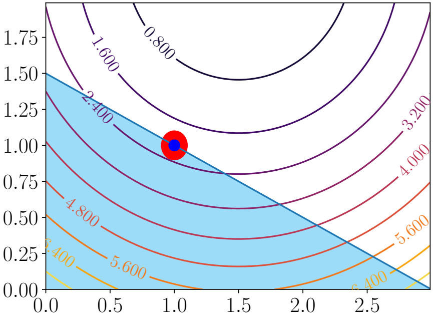

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Classification of mitigation techniques

- Passive (Robust) - 1 mode
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State of art

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	Decomposition	Resilient/Robust	Detection	Mitigation
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[CMI18]	Jacobi-Gauß	–	–	–
[Ana+18] [Ana+19] [Ana+20]	Dual	Resilient	Analyt./Learn.	Disconnect (Robustness)
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Outline

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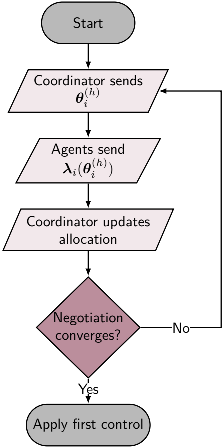
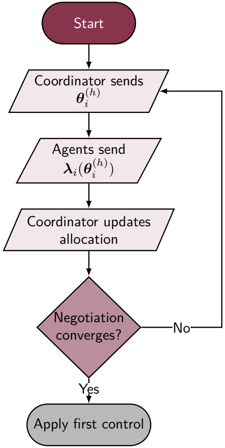
Outline

Quantity Decomposition | Resource Allocation



Quantity Decomposition | Resource Allocation



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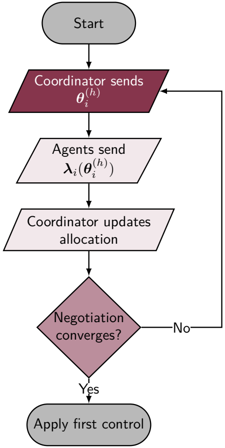
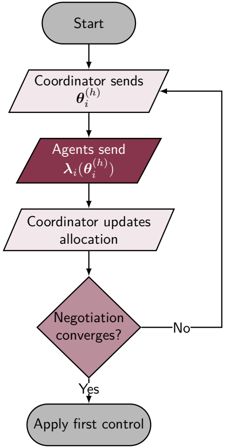


Allocation $\boldsymbol{\theta}_i$

Quantity Decomposition | Resource Allocation



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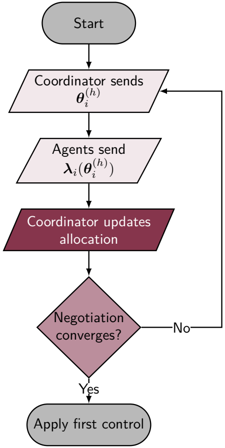

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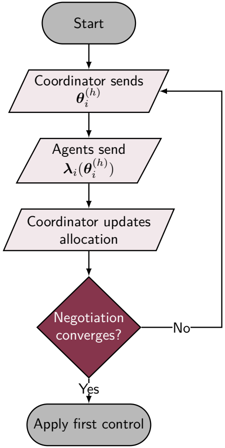

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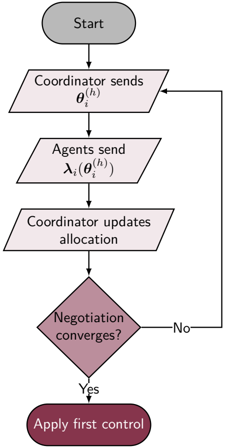

Allocation $\boldsymbol{\theta}_i$

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Primal Decomposition

or Quantity Decomposition | or Resource Allocation

- Objective is sum of local ones
- Constraints couple variables

$$\begin{aligned} \underset{\boldsymbol{u}_1,\dots,\boldsymbol{u}_M}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}} J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{u}_{\text{total}} \end{aligned}$$

For each $i \in \mathcal{M}$

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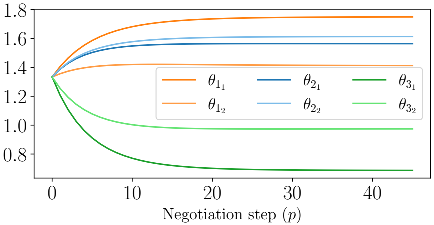
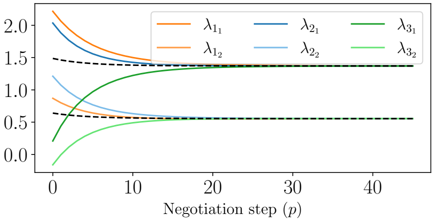
Quantity Decomposition | Resource Allocation

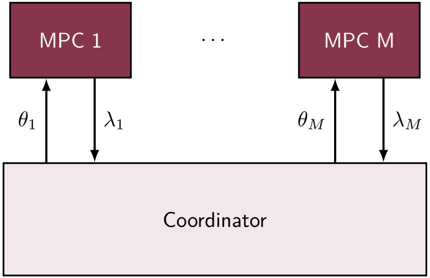
Until everybody is equally dissatisfied





How can a non-cooperative agent attack?

Our approach



- $\boldsymbol{\lambda}_i$ is the only interface
- $\boldsymbol{\lambda}_i$ depends on local parameters
- Malicious agent modifies $\boldsymbol{\lambda}_i$

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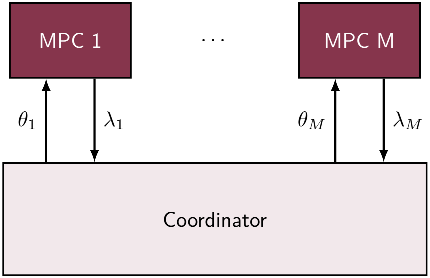

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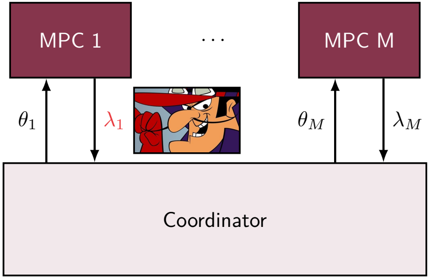

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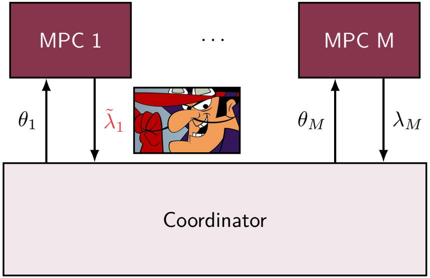

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$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$$

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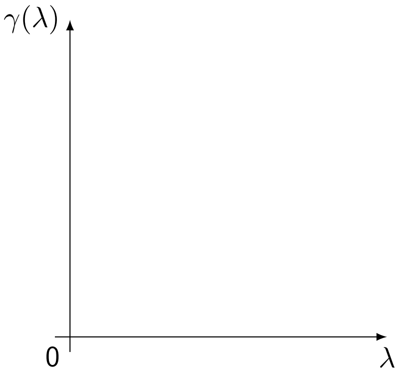
How does an agent lie?

Liar, Liar, Pants of fire



- $\lambda \geqslant 0$ means dissatisfaction
- $\lambda = 0$ means complete satisfaction

Assumptions

- *Attacker satisfied only if it really is* $\gamma(\lambda) = 0 \rightarrow \lambda = 0$
- *Attacker is greedy* $\gamma(\lambda) > \lambda$
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- Invertible
- If $\tilde{\lambda}_i = T_i[k]\lambda_i \rightarrow \exists T_i[k]^{-1}$

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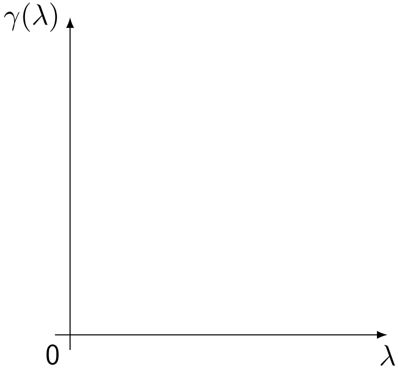

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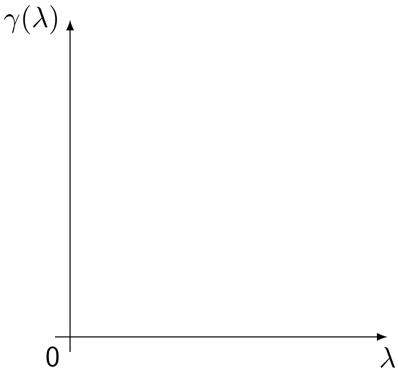

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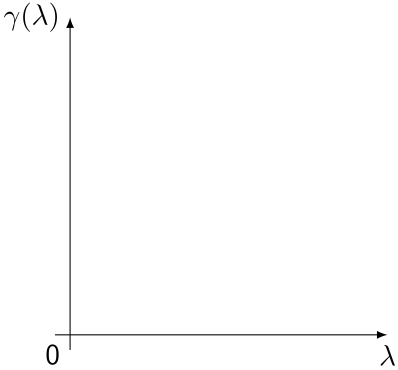

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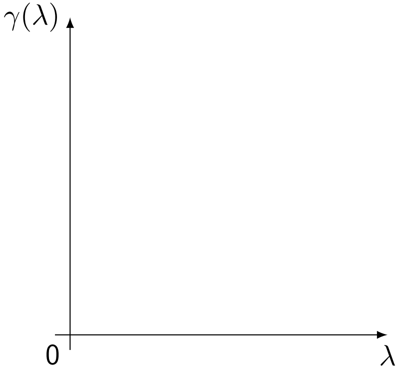

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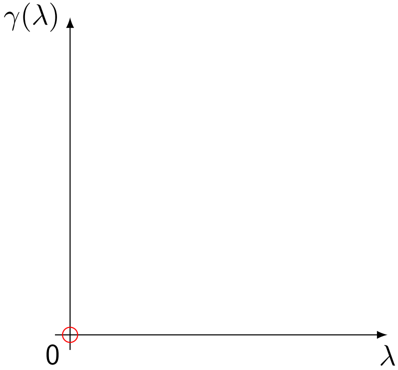

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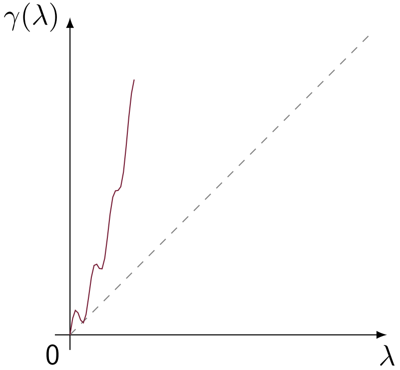

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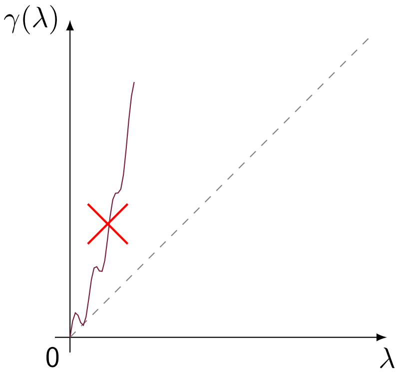

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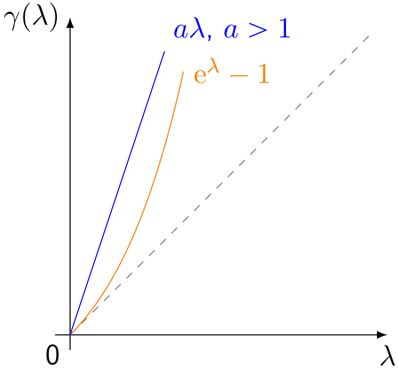

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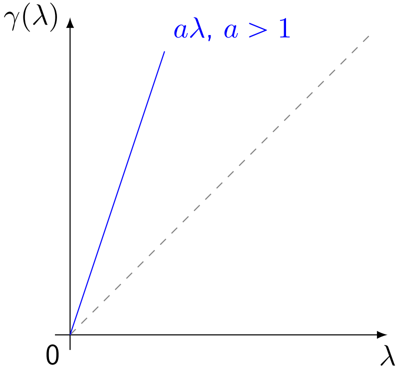

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- If $\tilde{\boldsymbol{\lambda}}_i = T_i[k]\boldsymbol{\lambda}_i \rightarrow \exists T_i[k]^{-1}$

Example

4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\lambda}_1 = \gamma_1(\lambda_1) = \tau_1 I \lambda_1$

- We can observe 3 things
  - Global minimum when $\tau_1 = 1$
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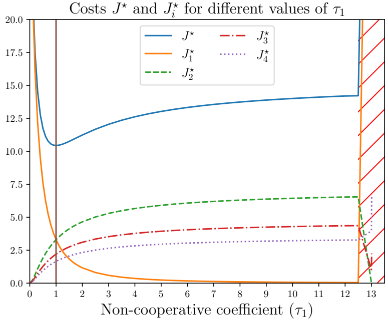

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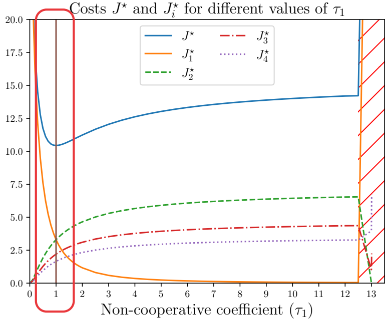

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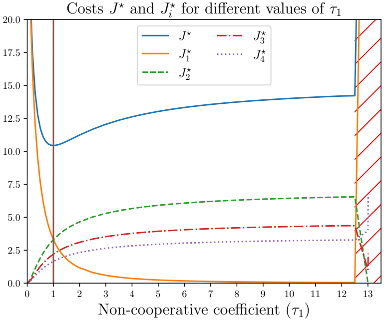

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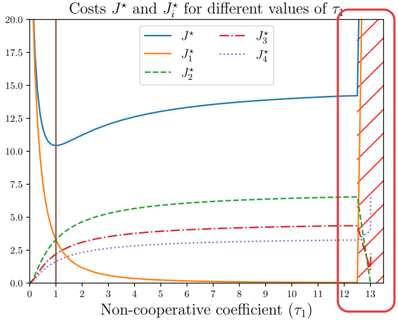

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Outline

2. Resilient Primal Decomposition-based dMPC for deprived systems
    - Analyzing deprived systems
    - Building an algorithm
    - Applying mechanism

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What are deprived systems?

Systems whose optimal solution has all constraints active

- Unconstrained Solution $\mathring{\boldsymbol{U}}_i^\star[k]$
- $\bar{\Gamma}_i \mathring{\boldsymbol{U}}_i^\star[k] \geq \boldsymbol{\theta}_i[k] \rightarrow$ Scarcity
  - Solution projected onto boundary
  - Same as with equality constraints[2]

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T \boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i \boldsymbol{U}_i[k] \leq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

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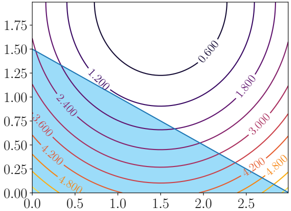

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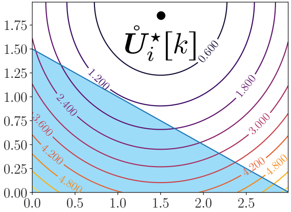

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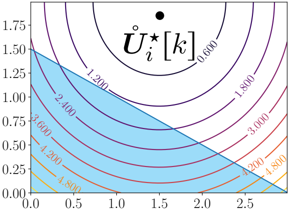

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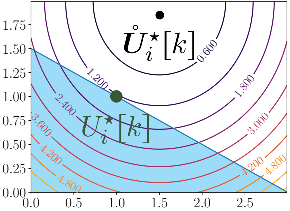

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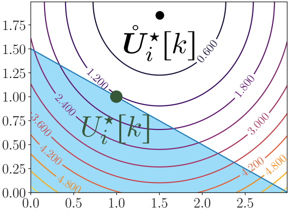

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Assumptions

- *Quadratic local problems*
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- Solution is analytical and affine

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(local parameters unknown by coordinator) $\begin{cases} \bullet\ P_i \text{ is time invariant} \\ \bullet\ \boldsymbol{s}_i[k] \text{ is time variant} \end{cases}$

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Under attack!

- Normal behavior
  - Affine solution

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

- Under attack $\rightarrow \tilde{\boldsymbol{\lambda}}_i = T_i[k]\boldsymbol{\lambda}_i$
  - Parameters modified

$$\tilde{\boldsymbol{\lambda}}_i = -\tilde{P}_i[k]\boldsymbol{\theta}_i[k] - \tilde{\boldsymbol{s}}_i[k]$$

- But wait! P_i is not supposed to change!
- Change $\rightarrow$ Probably an Attack! Let's take advantage of this!

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Detection Mechanism

Assumption

We know nominal $\bar{P}_i$

- If we estimate[1] $\hat{P}_i[k]$ and $\widehat{\tilde{s}}_i[k]$ such as:

$$\tilde{\boldsymbol{\lambda}}_i = -\widehat{\tilde{P}}_i[k]\boldsymbol{\theta}_i - \widehat{\tilde{\boldsymbol{s}}}_i[k]$$

- If $\left\|\widehat{\tilde{P}}_i[k] - \bar{P}_i\right\|_F > \epsilon_P \rightarrow$ Attack
- Ok, but how can we estimate $\widehat{\tilde{P}}_i[k]$?

[1] Using Recursive Least Squares for example

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Estimating $\widehat{\widetilde{P}}_i[k]$

- We estimate $\widehat{\widetilde{P}}_i[k]$ and $\widehat{\widetilde{s}}_i[k]$ simultaneously using RLS
- Challenge: Online estimation during negotiation fails
  - Update function couples θ_i^p and λ_i^p $\rightarrow$ low input excitation
- Solution: Send a random[3] sequence to increase excitation until convergence.

[3] A random signal has persistent excitation of any order ( )

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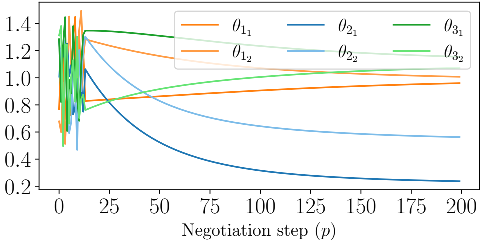

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Classification of mitigation techniques

- Active (Resilient)
  1. Detection/Isolation ✔
  2. Mitigation ?

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Mitigation mechanism

Reconstructing $\boldsymbol{\lambda}_i$

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  - We can recover $T_i[k]^{-1}$

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- Choose adequate version for coordination

$$\overset{\text{mod}}{\boldsymbol{\lambda}}_i = \begin{cases} \overset{\text{rec}}{\boldsymbol{\lambda}}_i, & \text{if attack detected} \\ \tilde{\boldsymbol{\lambda}}_i, & \text{otherwise} \end{cases}$$

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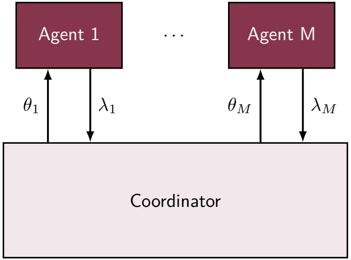
Complete Mechanism

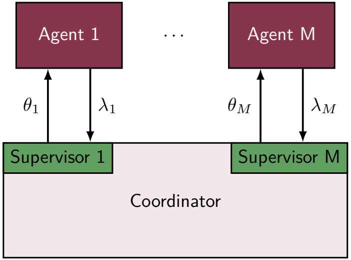


- Supervise exchanges by inquiring the agents
- Estimate how they will behave

Two Phases

1. Detect which agents are non-cooperative
2. Reconstruct λ_i and use in negotiation

Complete Mechanism



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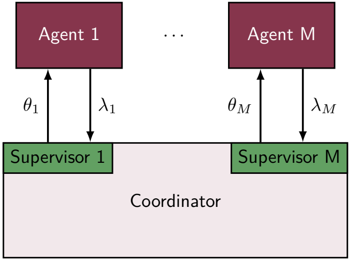

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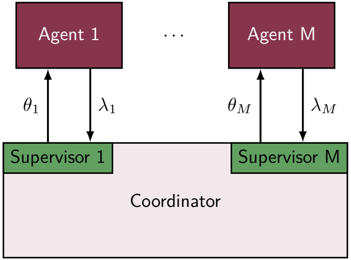

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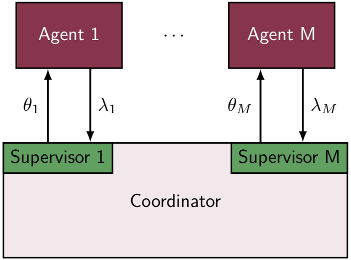

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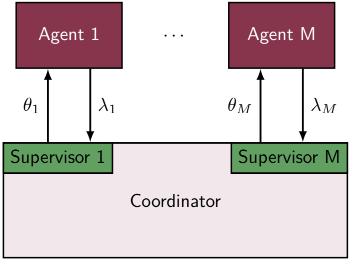

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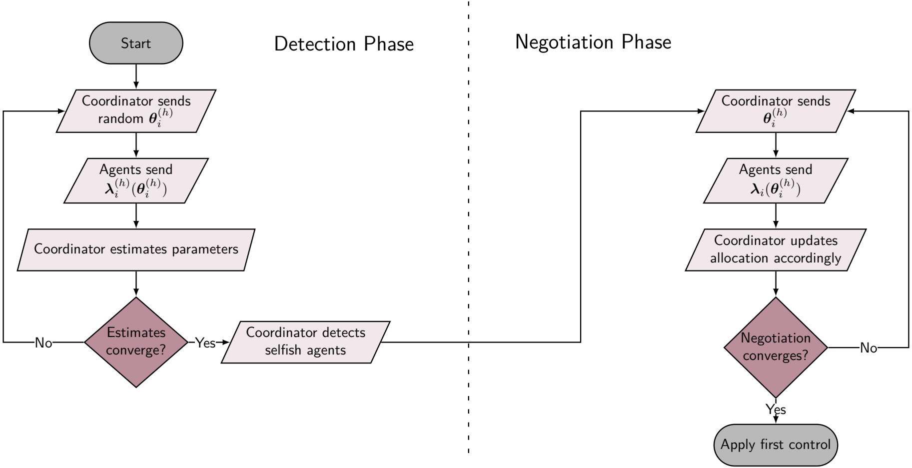

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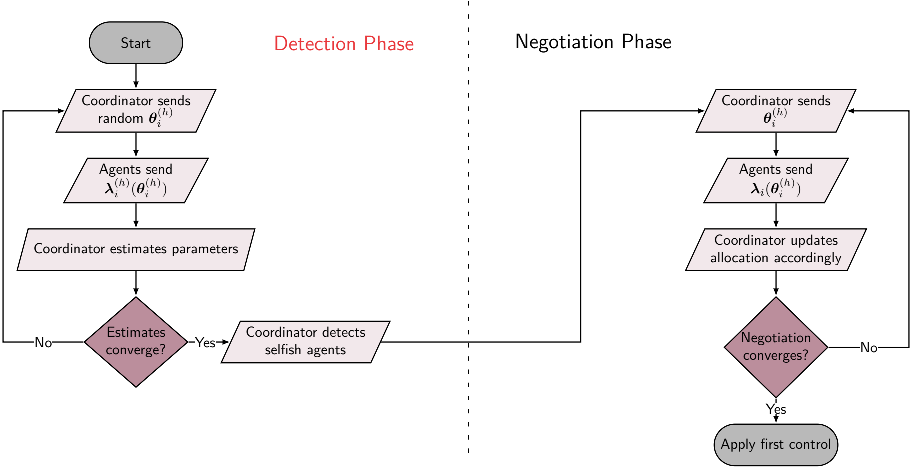

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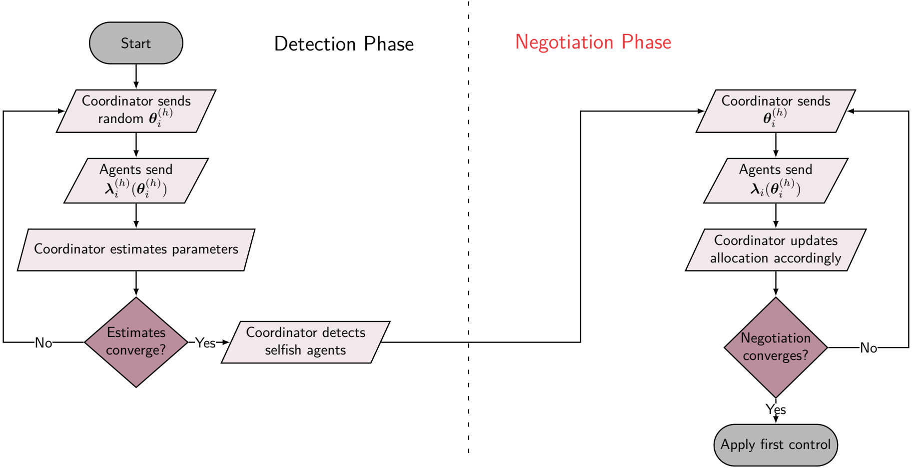

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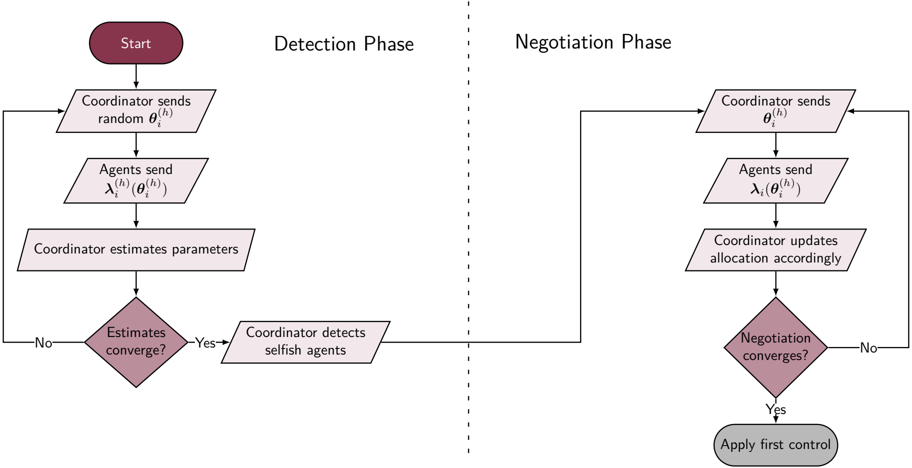

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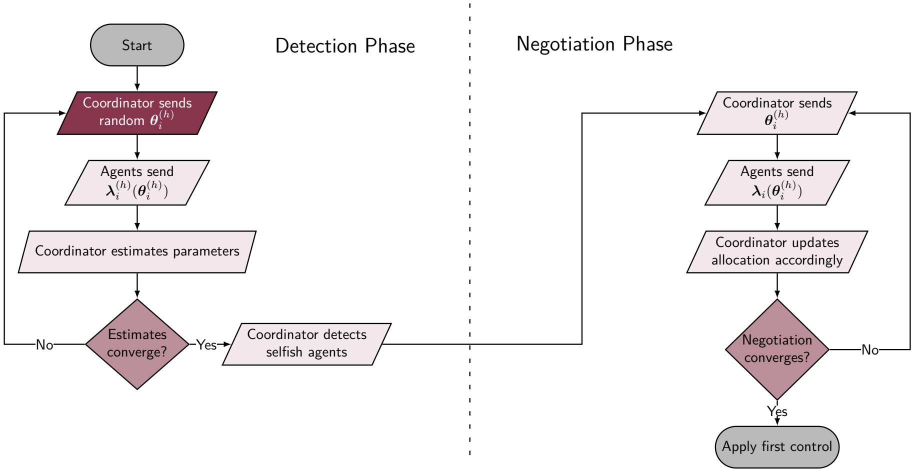

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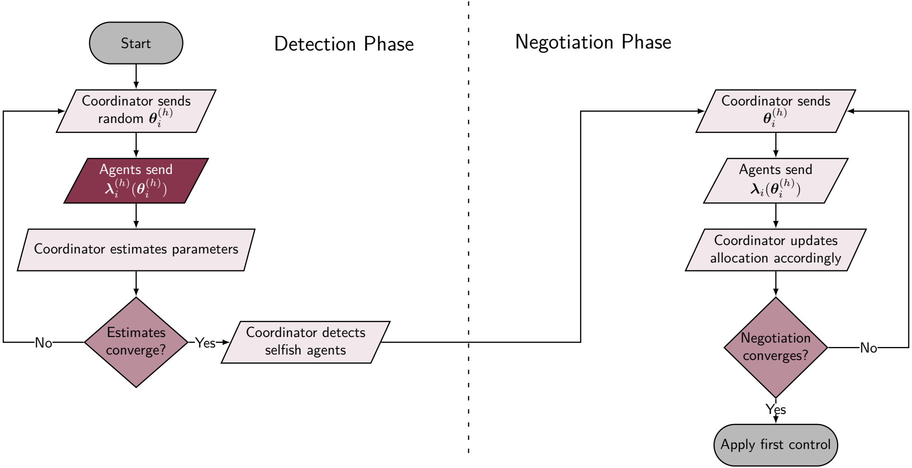

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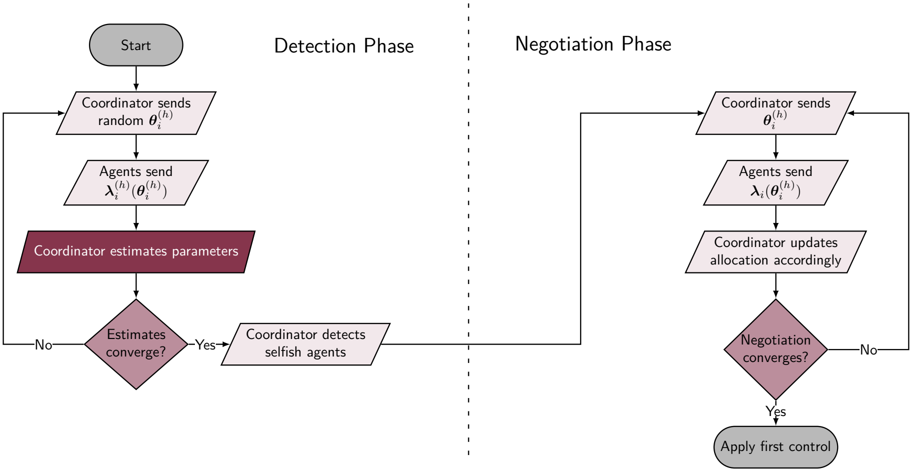

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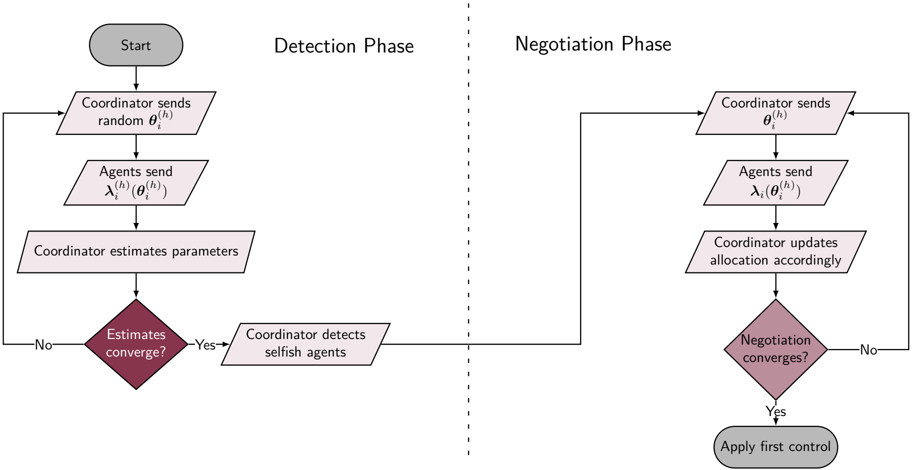

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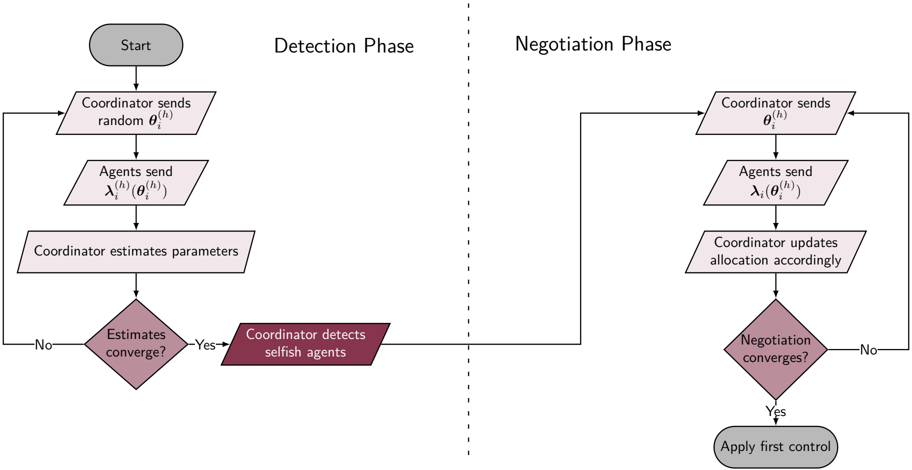

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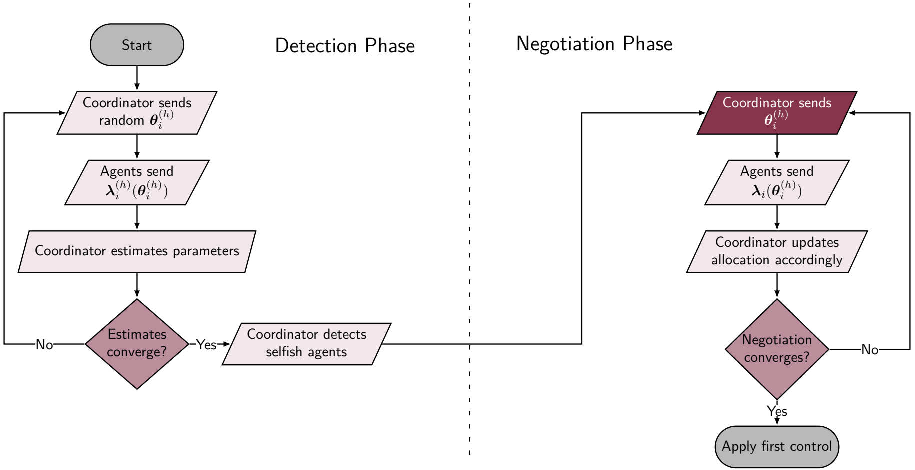

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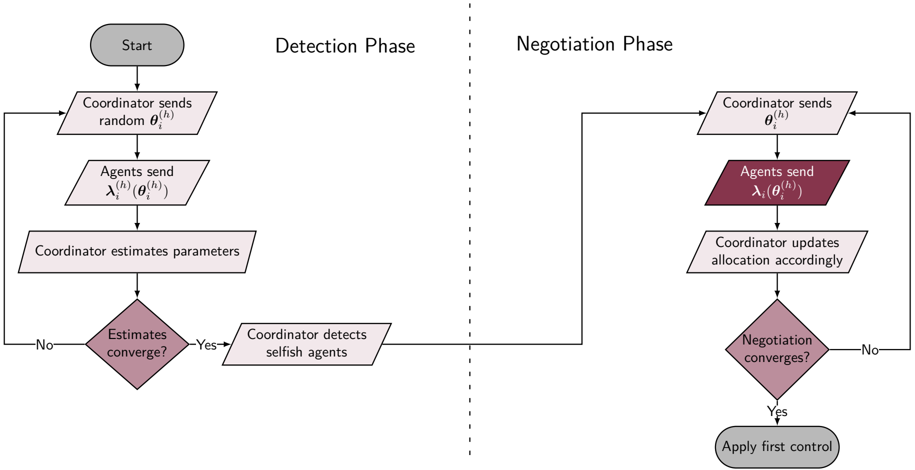

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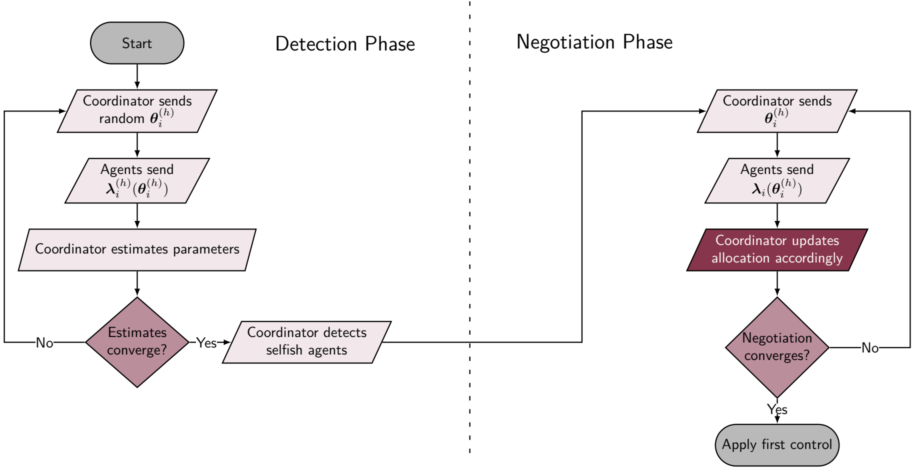

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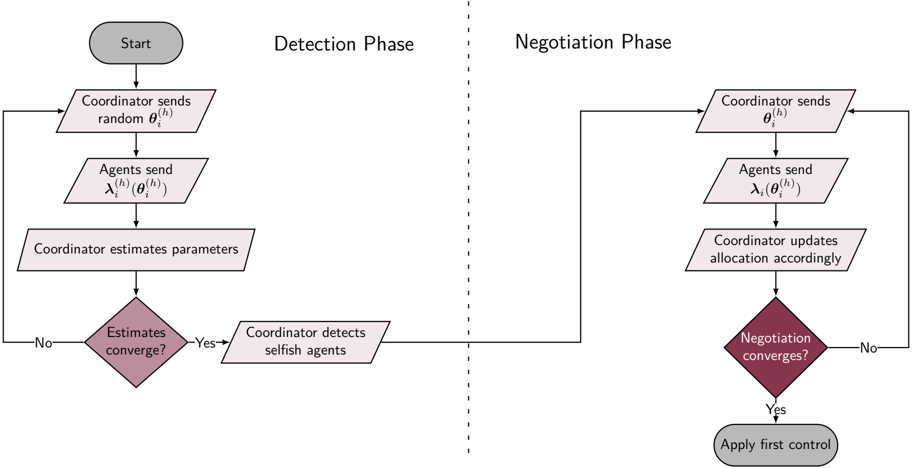

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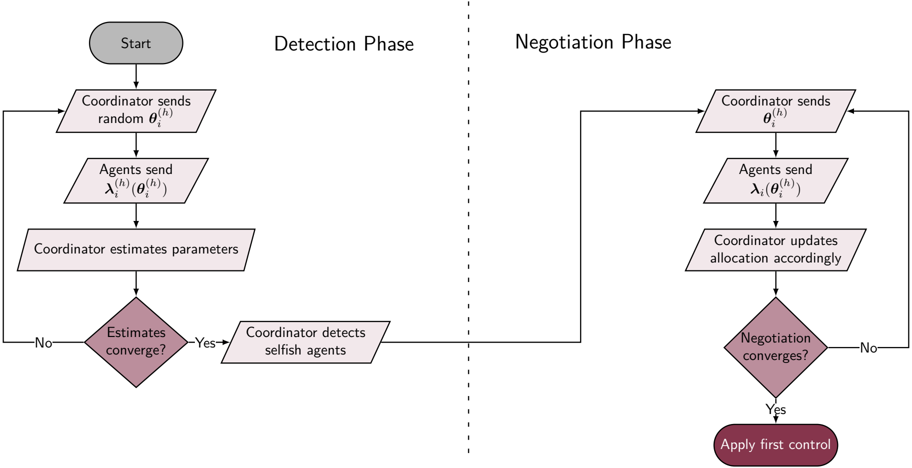

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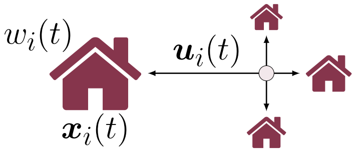

District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)
- Not enough power
- Period of 5h
- 3 scenarios
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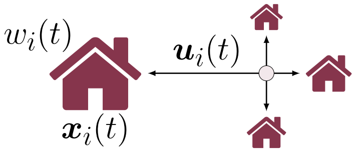

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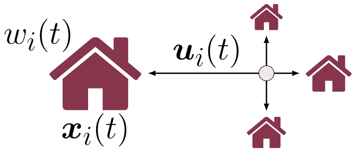

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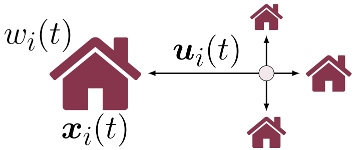

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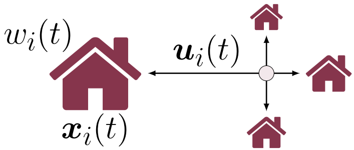

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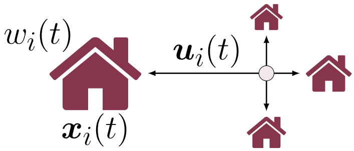

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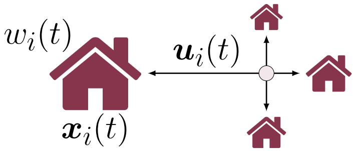

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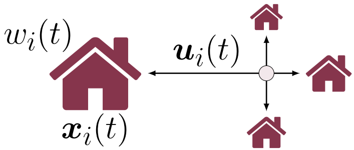

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Results

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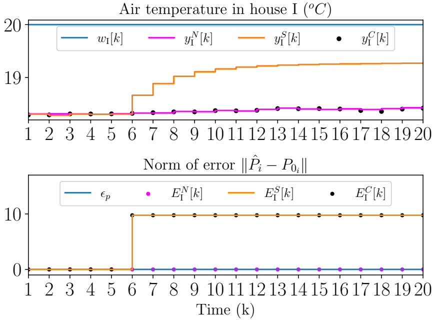

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Error $E_I(k)$.
N Nominal, S Selfish, C Corrected

Results

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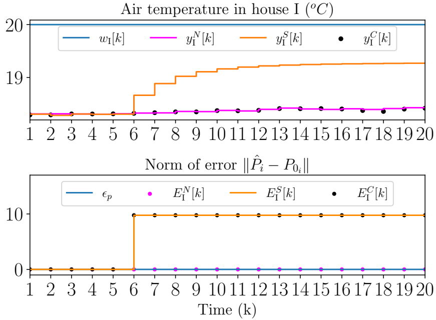
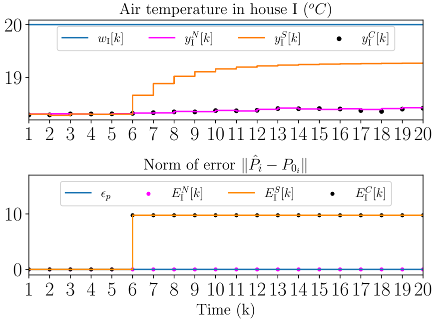


Temperature in house I.
Error $E_I(k)$.
N Nominal, S Selfish, C Corrected

- Agent starts cheating in $k = 6$
- S Agent increases its comfort
- C Restablish behavior close to N

Results

Temporal



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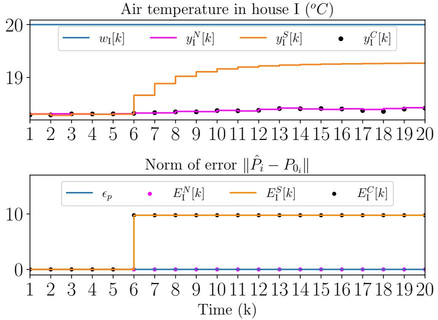

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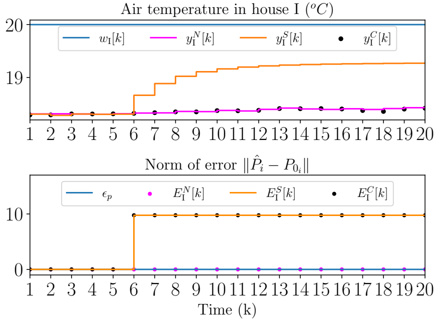

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Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.3	0.503
II	21.671	-0.547
III	17.387	-0.004
IV	17.626	-0.09
Global	3.526	0.016

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Outline

3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity
   - Relaxing some assumptions
   - Adapting the algorithm
   - Applying mechanism

Relaxing scarcity assumption

- Systems are not completely deprived
  - We can't change our constraints to equality ones anymore
  - Nor use the simpler update equation

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\|\boldsymbol{U}_i[k]\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i\boldsymbol{U}_i[k] \preceq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \text{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

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Analyzing System

Solution for $\lambda_i[k]$

Instead of having one single affine solution

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

Now, we may have multiple (Piecewise affine function)

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Still the $P_i^{(n)}$ are time independent

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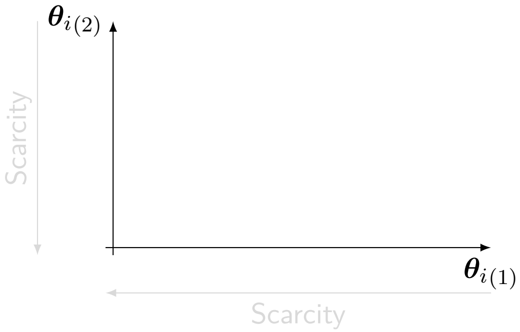

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Analyzing System

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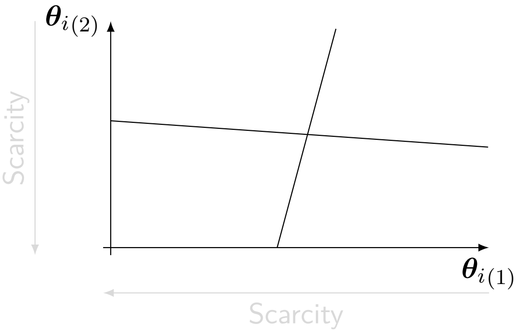

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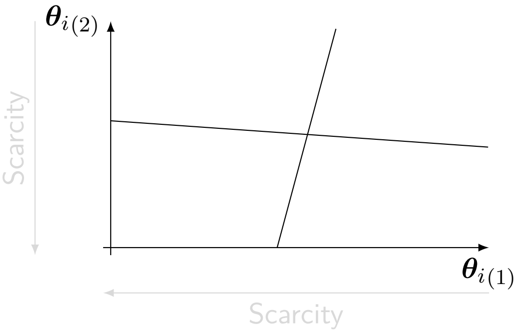

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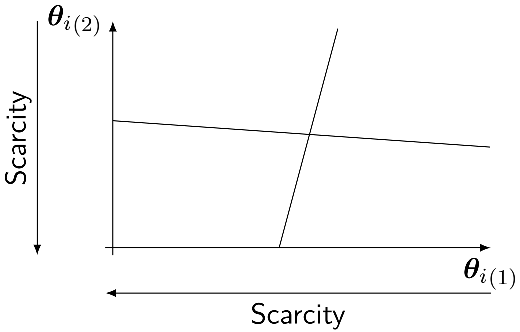

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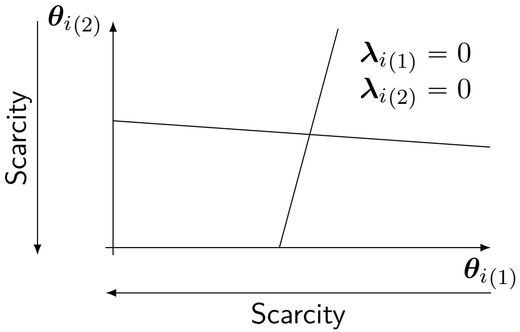

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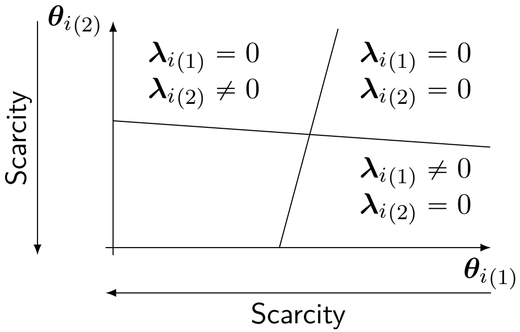

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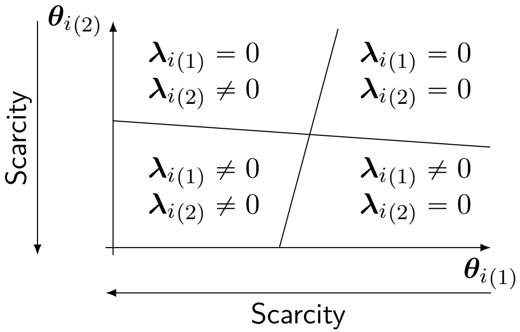

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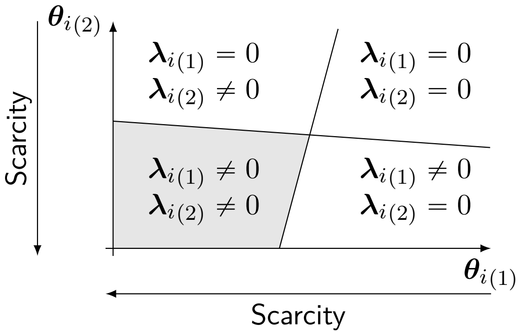

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CentraleSupélec

Artificial Scarcity

Who is it? Who is it?

Assumption

We known a point $\overset{\varnothing}{\theta}_i$ which activates all constraints[4]

[4] If we have local constraints, we suppose this point respects them.

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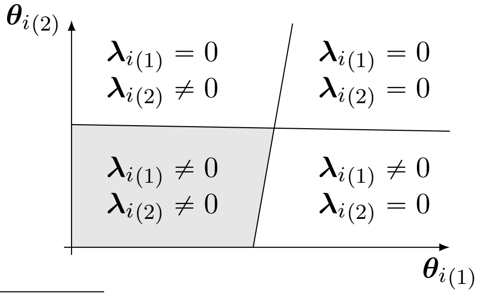

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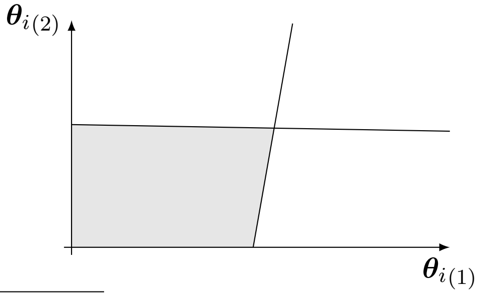

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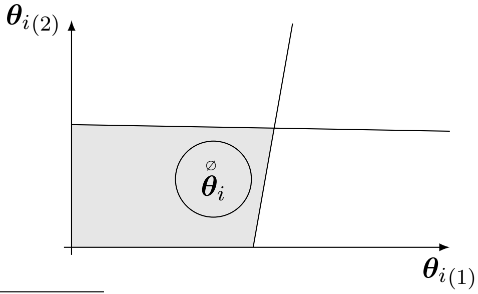

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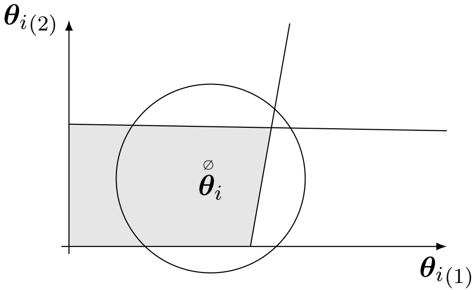
Assumption

We known a point $\overset{\varnothing}{\boldsymbol{\theta}}_i$ which activates all constraints[4]



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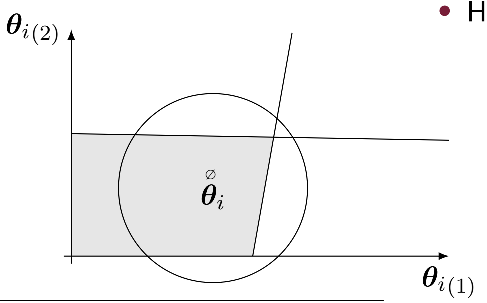

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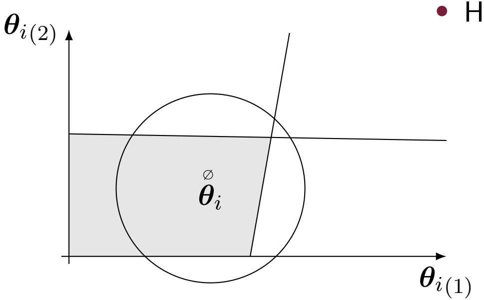

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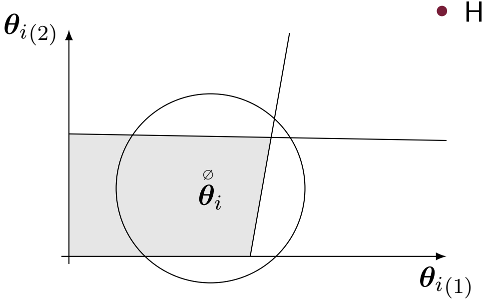

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- Iterative method to estimate parameters of multimodal models[5]
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Detection and Mitigation

Same same, but different

Assumption

We know nominal $\bar{P}_i^{(0)}$

- Detection

$$\left\| \widehat{\widetilde{P}}_i^{(0)}[k] - \bar{P}_i^{(0)} \right\|_F \geqslant \epsilon_{P_i^{(0)}}$$

- Mitigation

$$\widehat{T_i[k]}^{-1} = \bar{P}_i^{(0)} \widehat{\widetilde{P}}_i^{(0)}[k]^{-1}.$$

$$\overset{\text{rec}}{\boldsymbol{\lambda}_i} = \widehat{T_i[k]}^{-1} \tilde{\boldsymbol{\lambda}}_i.$$

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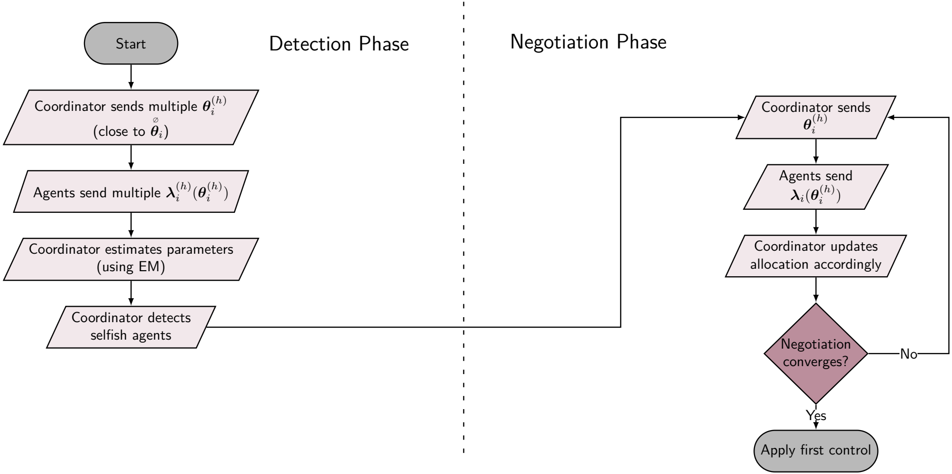
RPdMPC-AS



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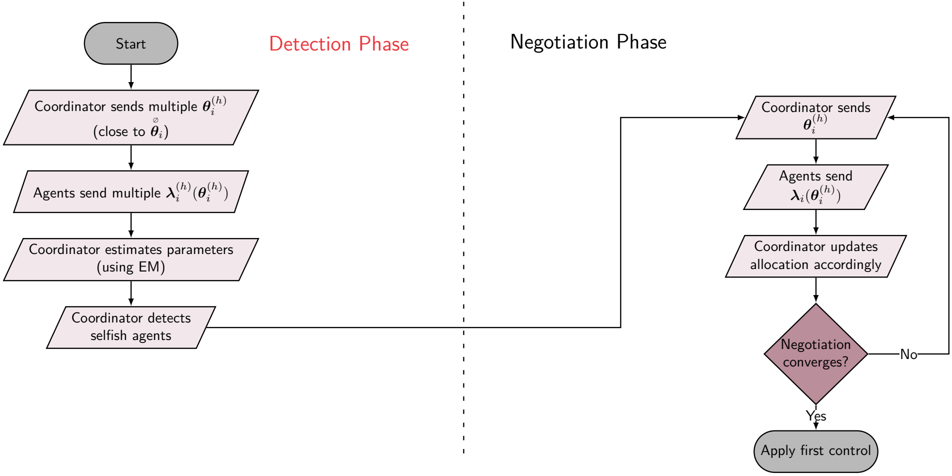

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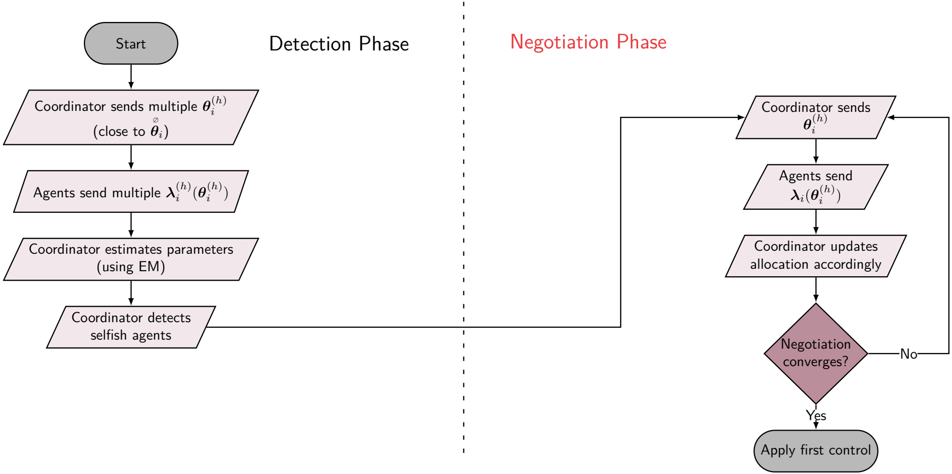

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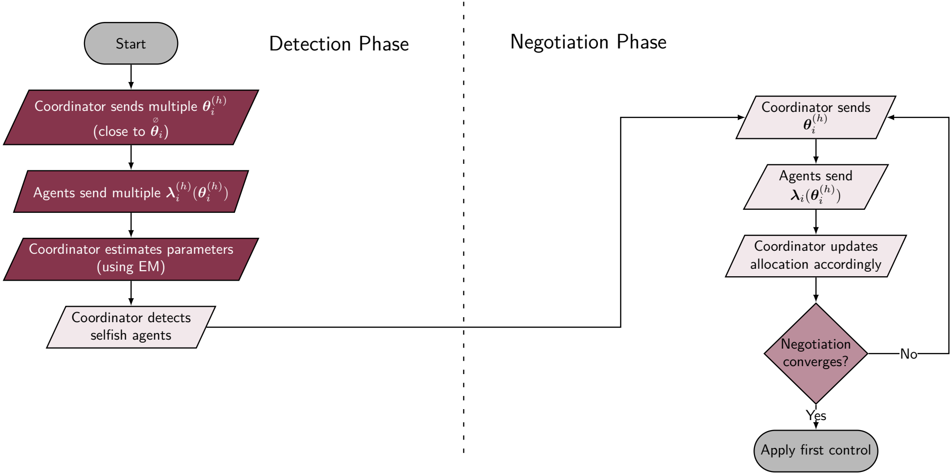

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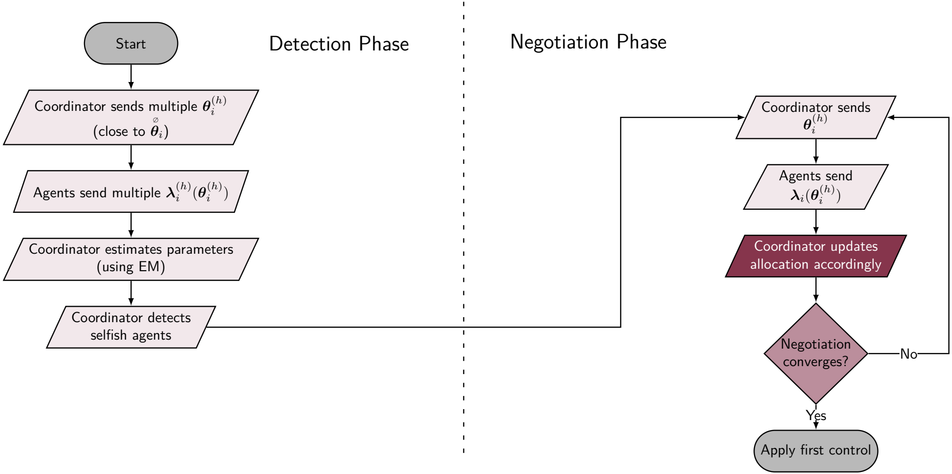
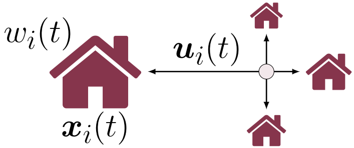


Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C
- Not enough power
- Period of $5\mathrm{h}$ ($T_s = 0.25\mathrm{h}$)
- 3 scenarios
  - N Nominal
  - C Agent I cheats (dMPC)
  - S Agent I cheats (RPdMPC-AS)

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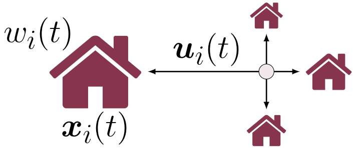

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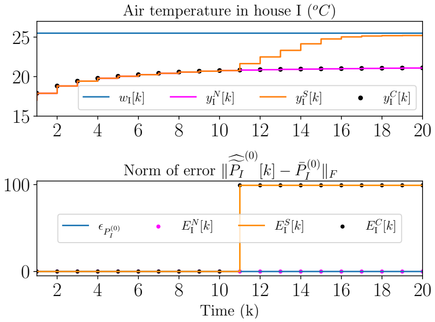
Temporal



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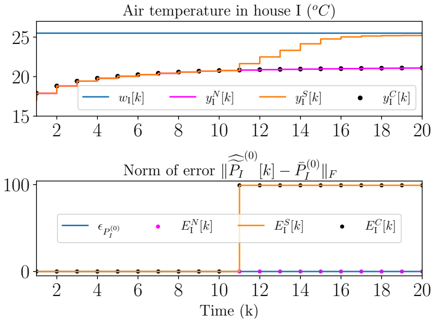

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Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.489	-0.0
II	35.813	0.0
III	29.225	0.0
IV	37.541	0.0
Global	10.689	-0.0

Too good to be true!

It's a kind of magic!

- Unfortunately EM is not magic
  - Slow convergence
  - Dependency on initialization
    - No guarantees of achieving global optimum
- Some "solutions":
  - Force some parameters to converge faster (case dependant)
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Outline

4 Conclusion

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Main takeaways

- How can an agent attack? ✓
  - Attacker can change the communication to receive more ressources.
- What are the consequences of an attack? ✓
  - Suboptimality and maybe instability
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  - Yes! By exploring the scarcity of the systems!

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  - We found some parameters that are constant when there is no cheating
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- Study of robustness + noise
- Resilient strategy with soft constraints
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Thank you!

Repository
https://github.com/Accacio/thesis

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