Security of distributed Model Predictive Control under False Data injection

Rafael Accácio NOGUEIRA

2022-12-07

https://bit.ly/3g3S6X4

IETR

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Context

❝ Necessity is the mother of invention ❞

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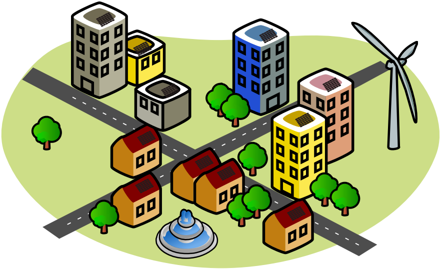

- Electricity Distribution System
- Heat distribution
- Water distribution
- Traffic management
  (include your problem here)

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- Multiple systems interacting
- Coupled by constraints
  - Technical/ Comfort
- Optimization objectives
  - Minimize energy consumption
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  - Follow a trajectory
- Solution → MPC

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Find best control sequence using predictions based on a model.

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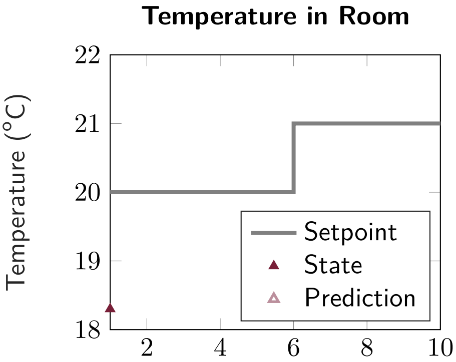
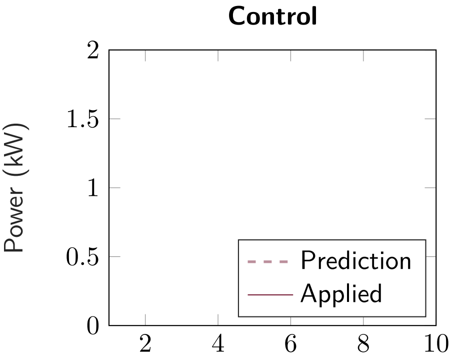
Model Predictive Control

In a nutshell

Find optimal control sequence





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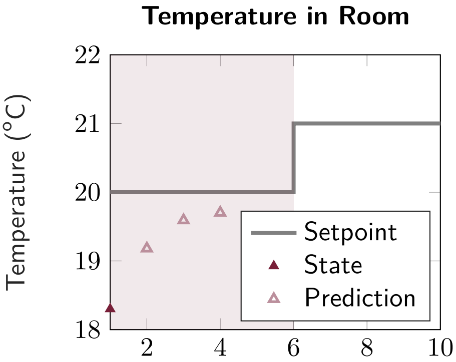
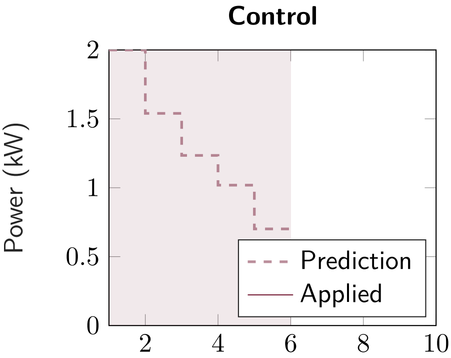
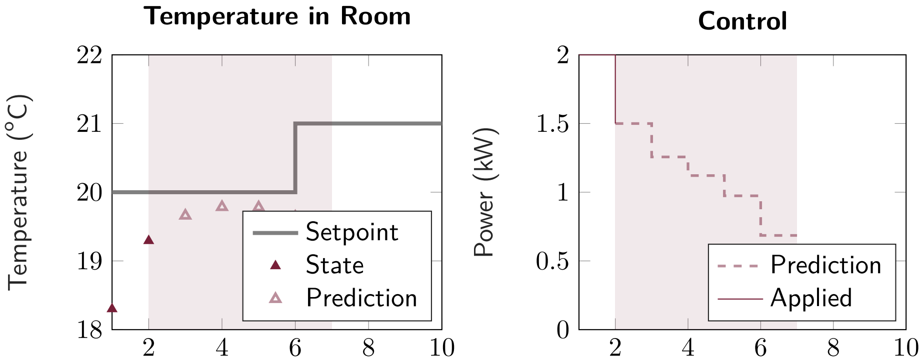




In a nutshell

Find optimal control sequence, apply first element



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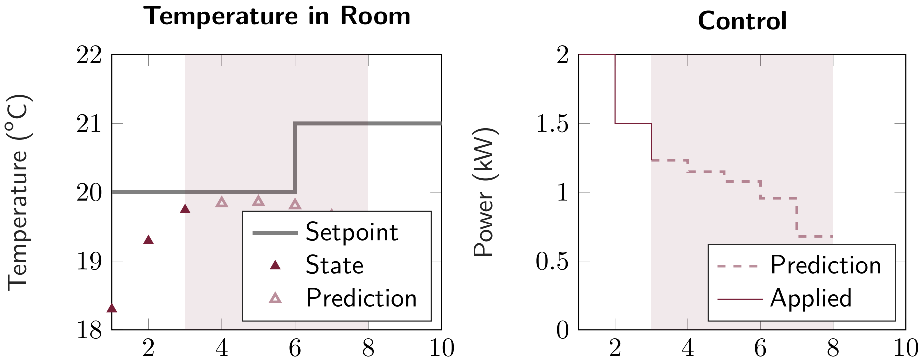
In a nutshell

Find optimal control sequence, apply first element, rinse repeat



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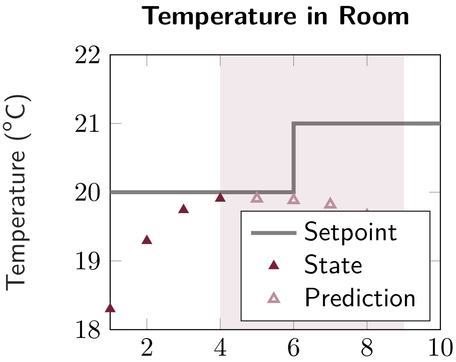
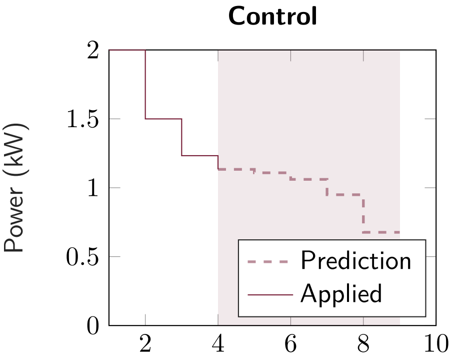
In a nutshell

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





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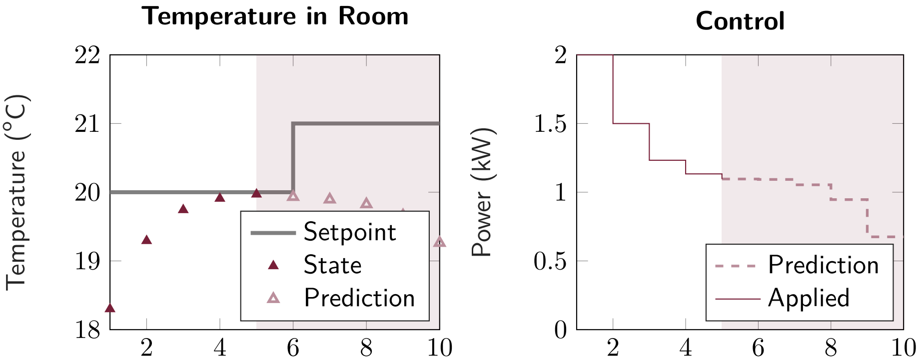

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Nothing is perfect

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It is about communication

- We break the MPC into multiple
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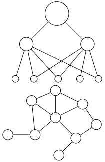

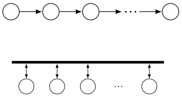

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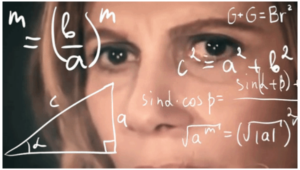

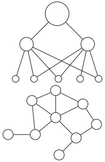

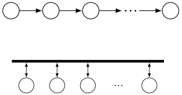

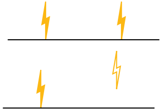

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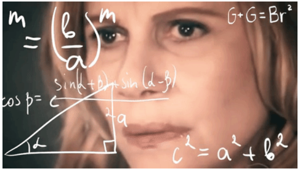

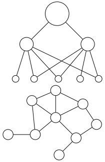

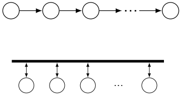

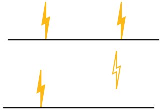

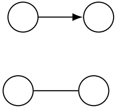

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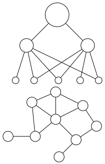
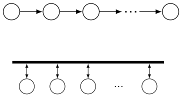
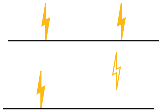
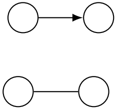






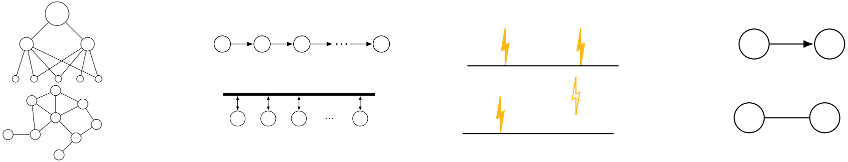


[1] Distributed Model Predictive Control made easy

Distributed Model Predictive Control

It is about communication

- We break the MPC into multiple
- Make them Communicate , But how?
  - Many flavors to choose from[1]
    - Hierarchical/Anarchical
    - Sequential/Parallel
    - Synchronous/Asynchronous
    - Bidirectional/Unidirectional
    - ...



[1] Distributed Model Predictive Control made easy

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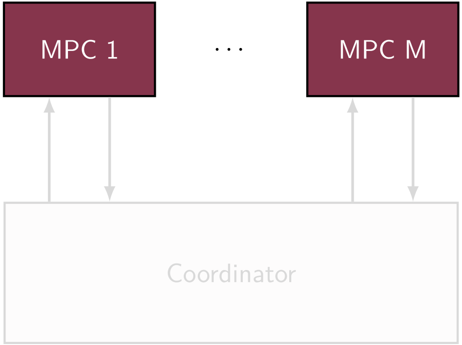
Distributed Model Predictive Control

Optimization Frameworks

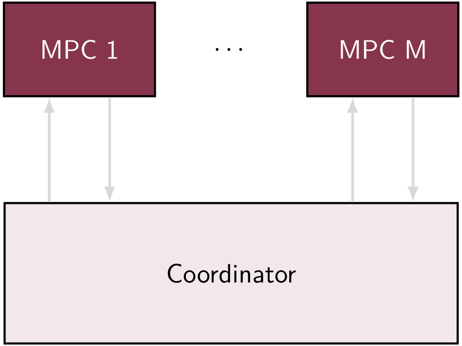


- Coordinator → Hierarchical
- Bidirectional
- No delay → Synchronous
- Agents solve local problems
- Variables are updated

Until Convergence

Distributed Model Predictive Control

Optimization Frameworks



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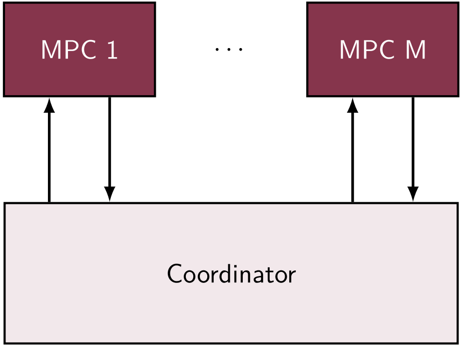

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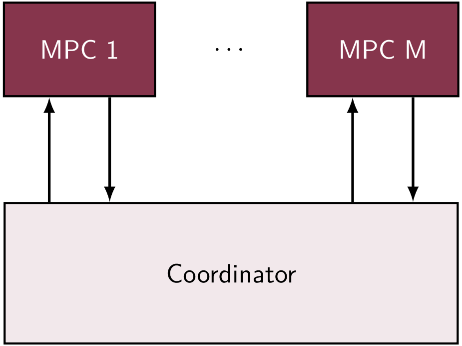

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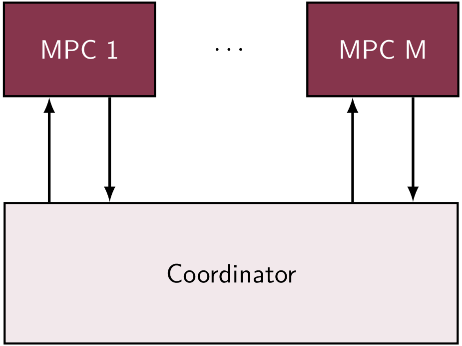

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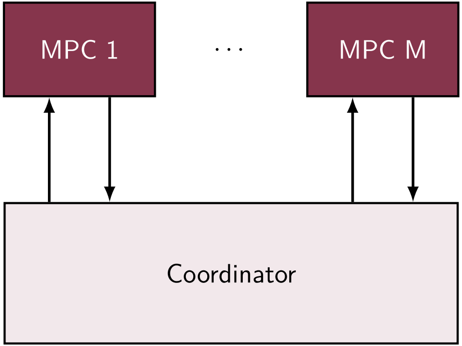

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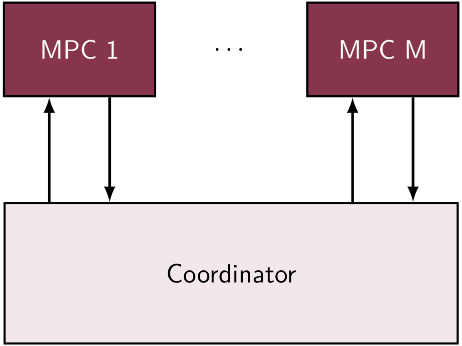

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Distributed Model Predictive Control

Negotiation works if agents comply.

But what if some agents are ill-intentioned and attack the system?

- How can an agent attack?
- What are the consequences of an attack?
- Can we mitigate the effects?

Let's have a preview!

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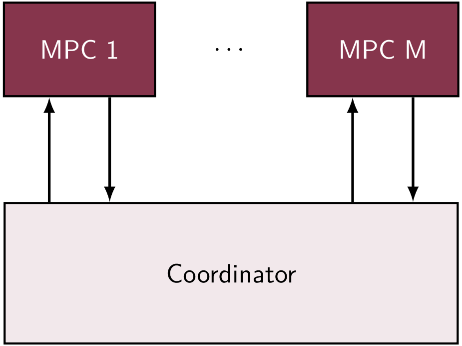
How can a non-cooperative agent attack?

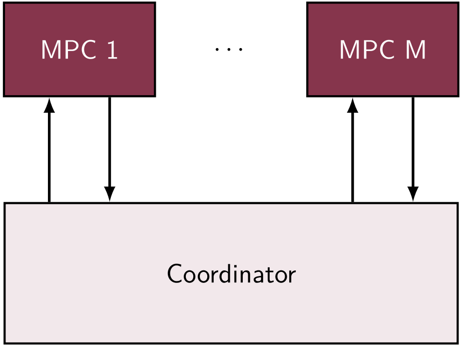


- [Vel+17a; CMI18] present attacks
  - Objective function
    - Selfish Attack
    - Fake weights
    - Fake reference
  - Fake constraints
  - Liar agent

Deception Attacks

How can a non-cooperative agent attack?

Literature



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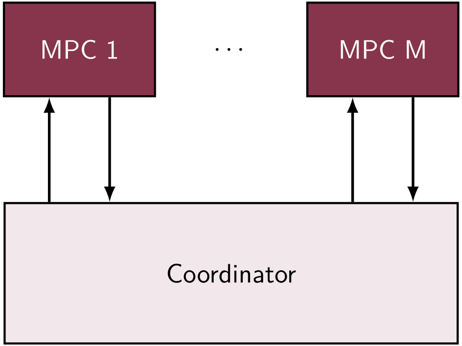

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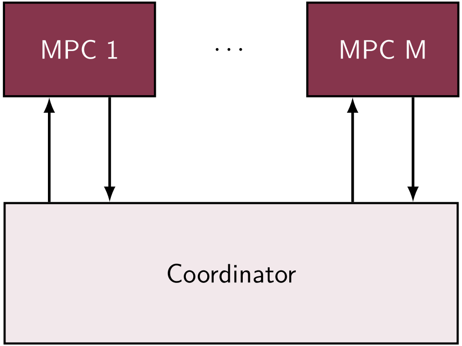

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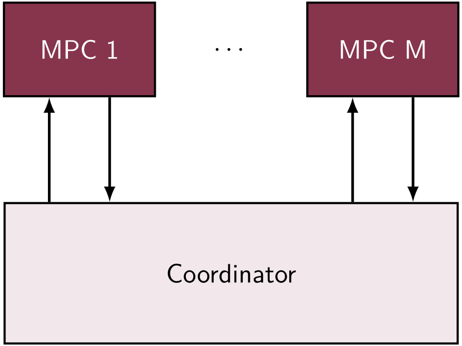

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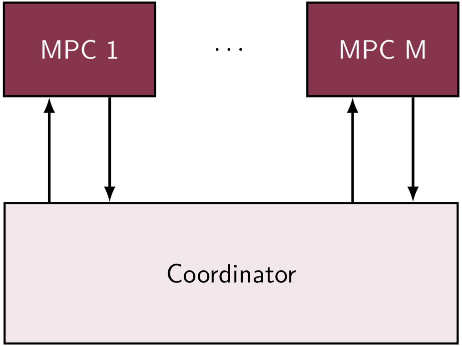

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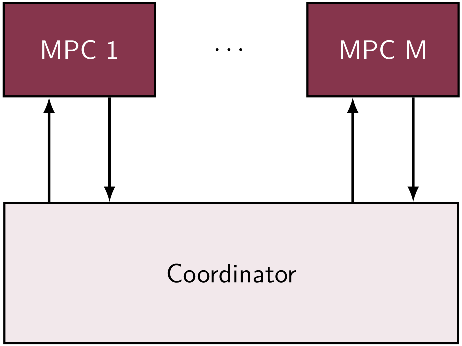

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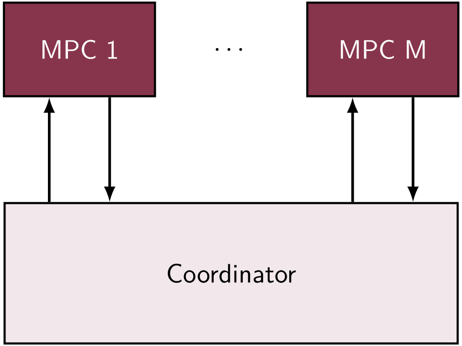

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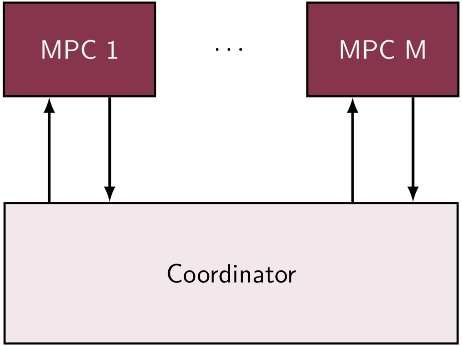

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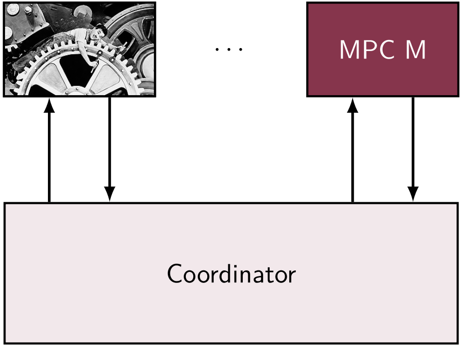

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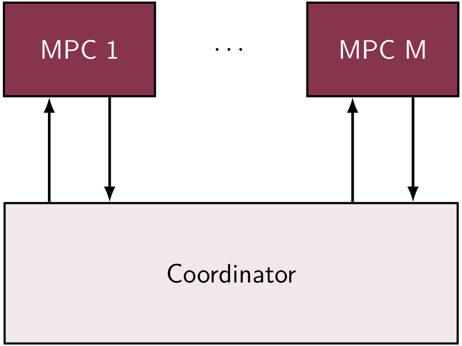
Deception Attacks (Internal change)

How can a non-cooperative agent attack?

Our approach



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- What matters is the interface
  - Attacker changes communication

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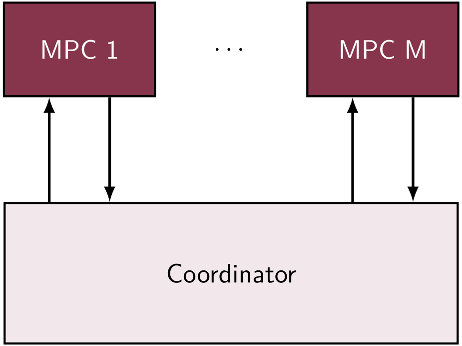

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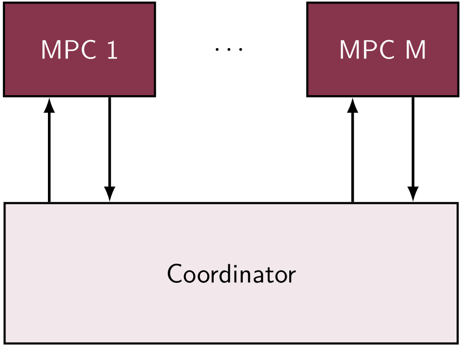

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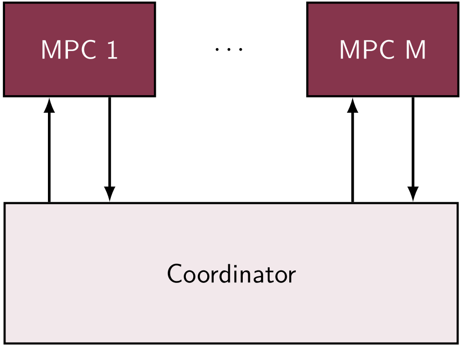

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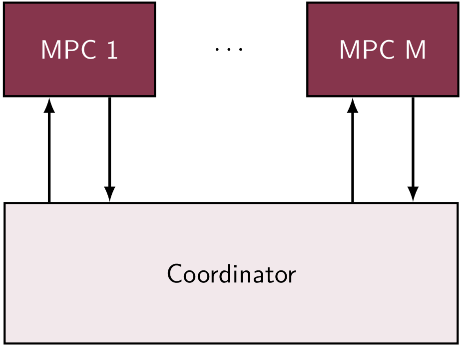

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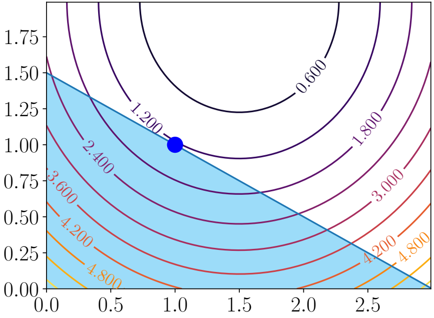
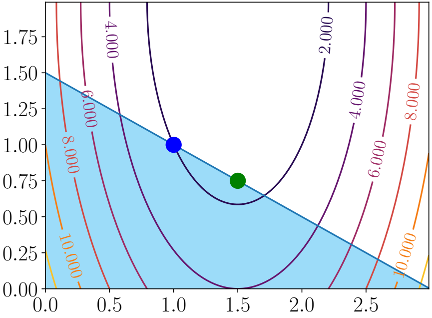
Consequence of an attack

- Attack modifies optimization problem
  - Optimum value is shifted



Original minimum.



Minimum after attack.

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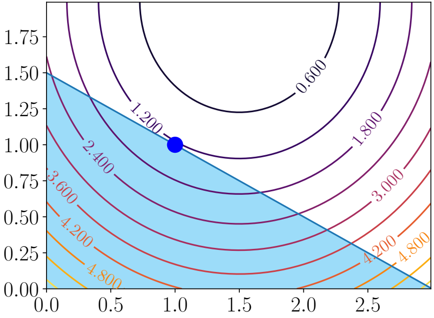

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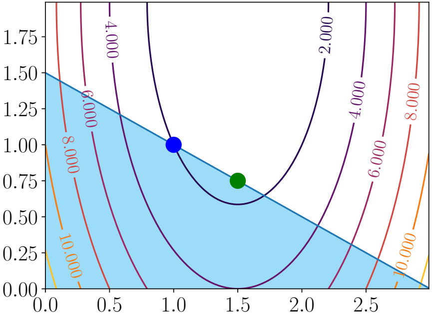

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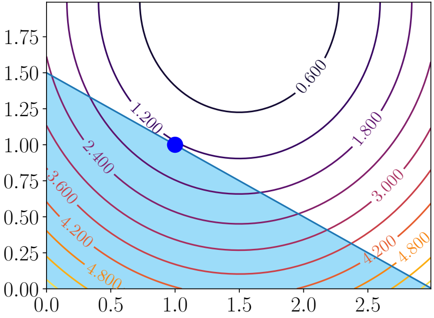

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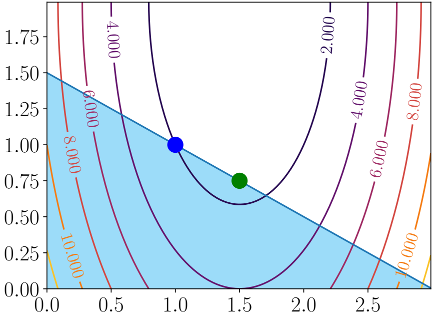

Minimum after attack.

Mitigating the effects

- We can recover by
  - Ignoring attacker
  - Recuperating original behavior (at least trying)

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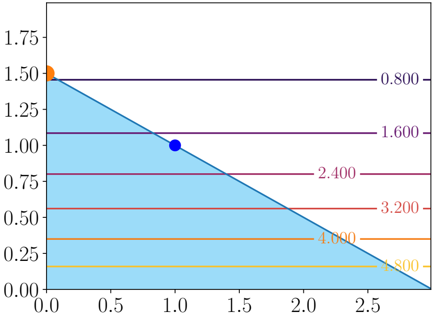

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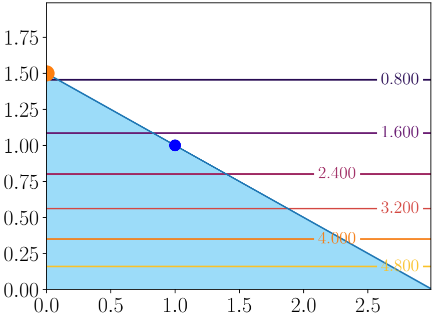

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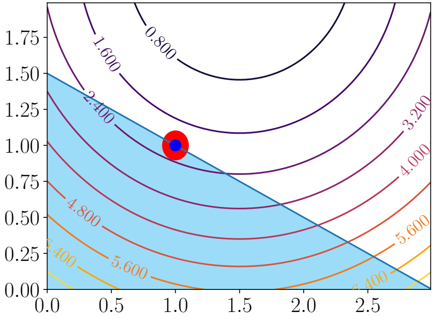

Recover original behavior.

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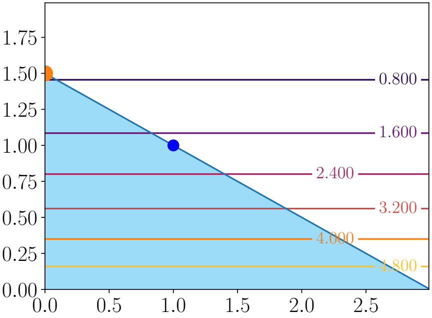

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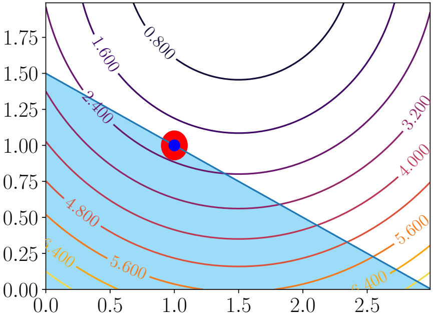

Recover original behavior.

Classification of mitigation techniques

- Passive (Robust) - 1 mode
- Active (Resilient) - 2 modes
  1. Detection/Isolation
  2. Mitigation

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Attack free

Classification of mitigation techniques

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State of art

Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
[Vel+17a] [Mae+21]	Dual	Robust (Scenario)	NA	NA
[Vel+17b] [Vel+18]	Dual	Robust (f-robust)	NA	NA
[CMI18]	Jacobi-Gauß	–	–	–
[Ana+18] [Ana+19] [Ana+20]	Dual	Resilient	Analyt./Learn.	Disconnect (Robustness)
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[Ana+18] [Ana+19] [Ana+20]	Dual	Resilient	Analyt./Learn.	Disconnect (Robustness)
Our	Primal	Resilient	Active Analyt./Learn.	Data reconstruction

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Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
[Vel+17a] [Mae+21]	Dual	Robust (Scenario)	NA	NA
[Vel+17b] [Vel+18]	Dual	Robust (f-robust)	NA	NA
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State of art

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Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition
2. Resilient Primal Decomposition-based dMPC for deprived systems
3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity

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Outline

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Primal Decomposition

or Quantity Decomposition | or Resource Allocation

- Objective is sum of local ones
- Constraints couple variables

$$\begin{aligned} \underset{\boldsymbol{u}_1,\ldots,\boldsymbol{u}_M}{\text{minimize}} \quad & \sum_{i \in \mathcal{M}} J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{u}_{\text{total}} \end{aligned}$$

For each $i \in \mathcal{M}$

1. Allocate a part for each agent
2. They solve local problems and
3. communicate how dissatisfied
4. Allocation is updated (respecting global constraint)

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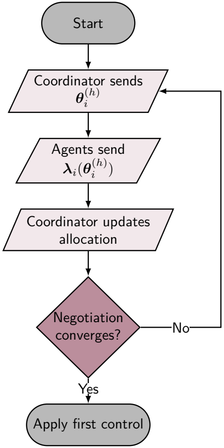
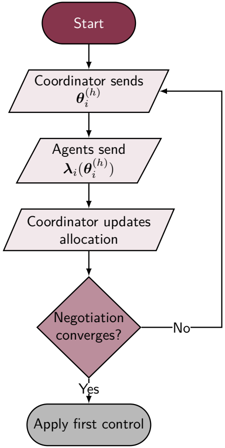
Quantity Decomposition | Resource Allocation



Quantity Decomposition | Resource Allocation



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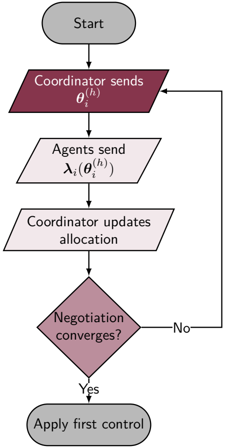
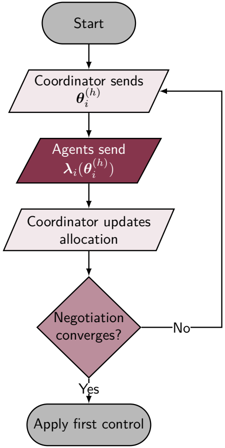


Allocation $\boldsymbol{\theta}_i$

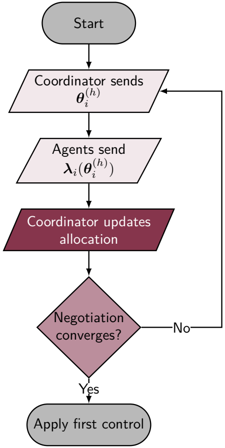
Quantity Decomposition | Resource Allocation



Allocation $\boldsymbol{\theta}_i$

Dissatisfaction $\boldsymbol{\lambda}_i$

Quantity Decomposition | Resource Allocation



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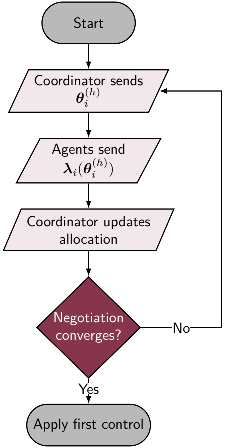

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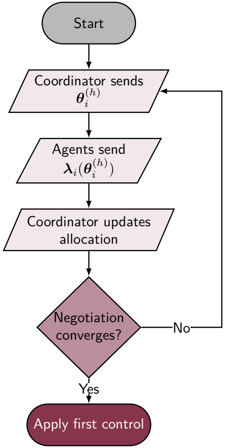

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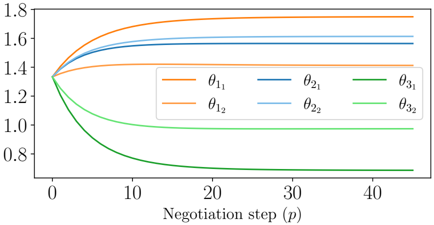
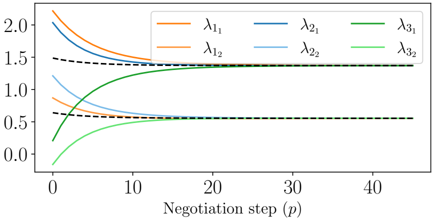
Quantity Decomposition | Resource Allocation

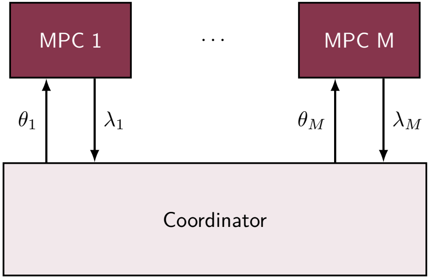
Until everybody is equally dissatisfied





How can a non-cooperative agent attack?

Our approach



- $\boldsymbol{\lambda}_i$ is the only interface
- $\boldsymbol{\lambda}_i$ depends on local parameters
- Malicious agent modifies $\boldsymbol{\lambda}_i$

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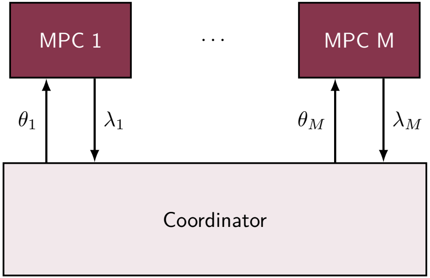

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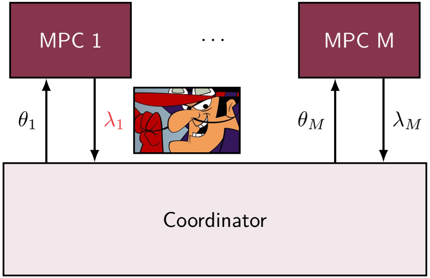

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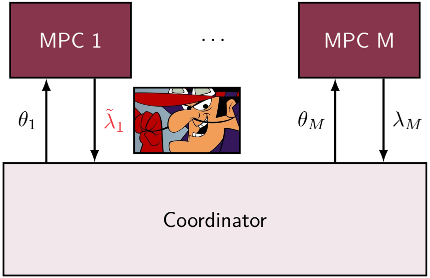

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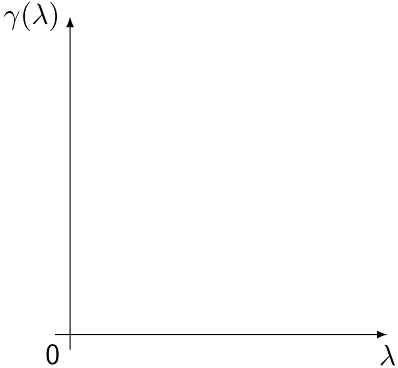
$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$$

How does an agent lie?

Liar, Liar, Pants of fire



- $\lambda \geqslant 0$ means dissatisfaction
- $\lambda = 0$ means complete satisfaction

Assumptions

- *Attacker is not naïve*
  $\gamma(\lambda) = 0 \rightarrow \lambda = 0$
- *Attacker is greedy* $\gamma(\lambda) > \lambda$
- *Really greedy*
  $\lambda_b > \lambda_a \rightarrow \gamma(\lambda_b) > \gamma(\lambda_a)$

- Invertible
- If linear $\rightarrow \exists T^{-1}$

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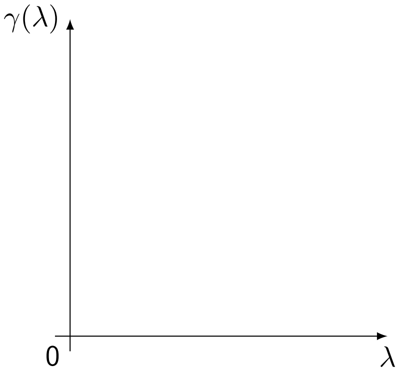

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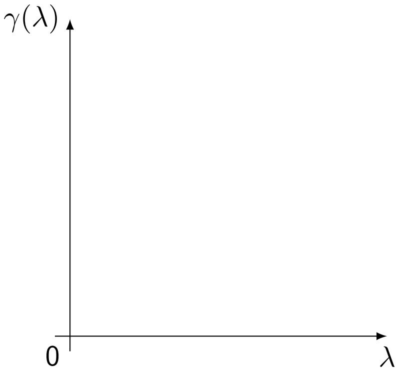

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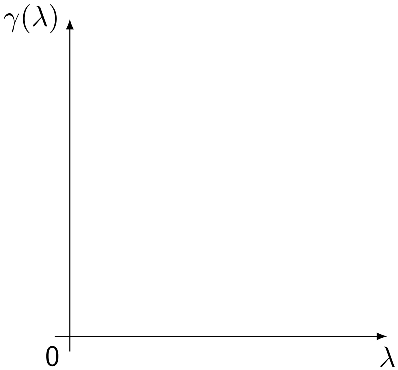

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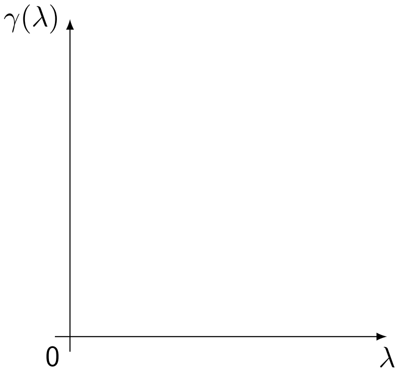

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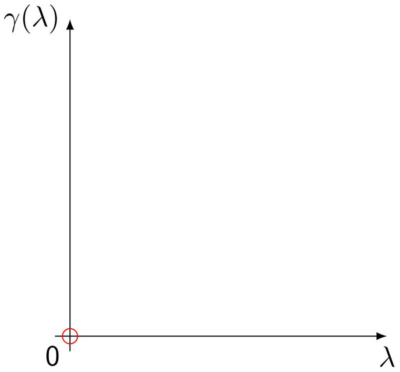

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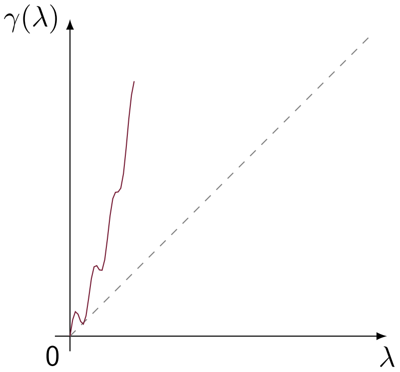

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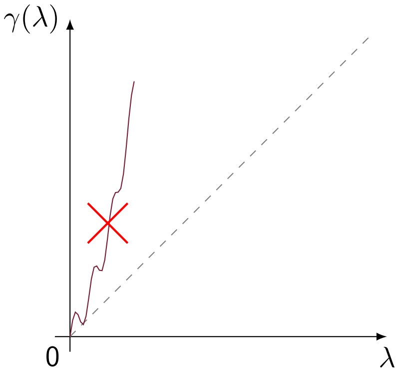

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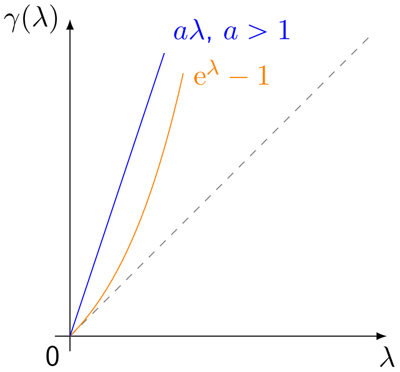

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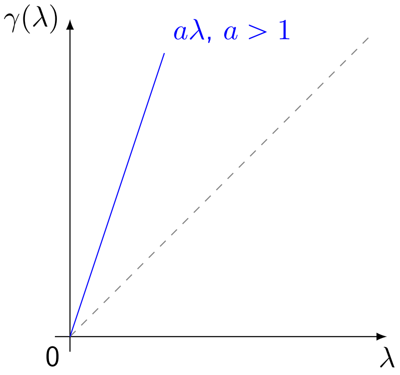

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Example

4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\lambda}_1 = \gamma_1(\lambda_1) = \tau_1 I \lambda_1$

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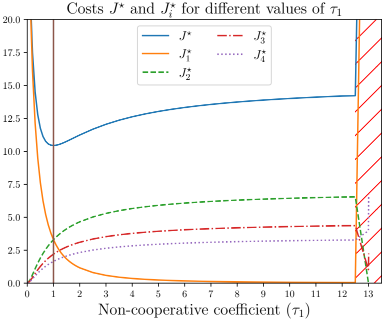

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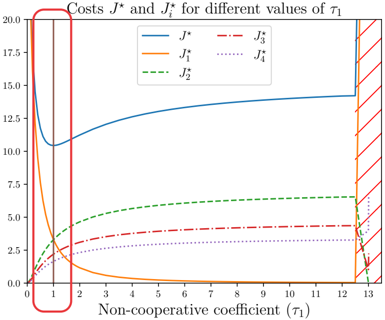

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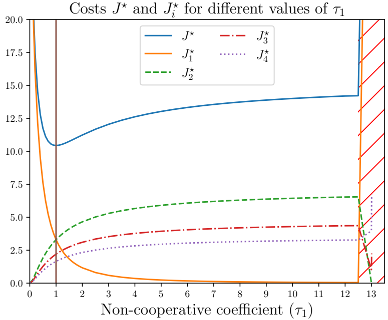

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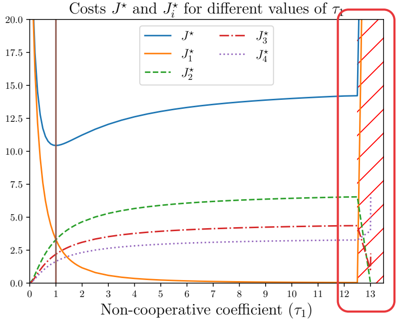

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- Yes! (At least in some cases)
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Outline

What are deprived systems?

TL;DR: Systems where all constraints are active

- Unconstrained Solution $\mathring{\boldsymbol{U}}_i^\star[k]$
- All constraints active = Scarcity
  - Solution projected onto boundary
  - Same as with equality constraints[2]

$$\begin{aligned} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} \quad & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\ \text{subject to} \quad & \bar{\Gamma}_i\boldsymbol{U}_i[k] \leq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{aligned}$$

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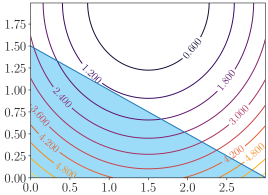

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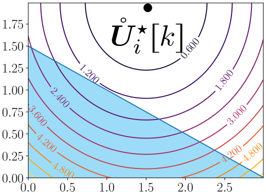

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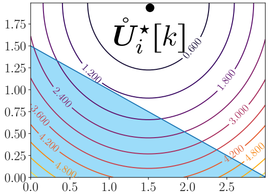

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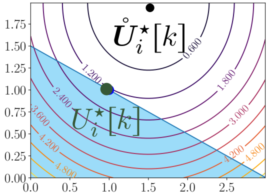

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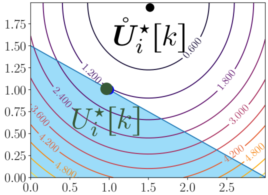

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But why?

- No Scarcity
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  - → No incentive to cheat
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  - → Consensus/Compromise
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Analysis

Assumptions

- *Quadratic local problems*
- *Scarcity*

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- Solution is analytical

(local parameters unknown by coordinator) $\left\{ \begin{array}{l} \bullet\ P_i \text{ is time invariant} \\ \bullet\ \boldsymbol{s}_i[k] \text{ is time variant} \end{array} \right.$

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Analysis (Negotiation)

- Update expression also becomes analytical
- If we input $\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$ → DTS with forced input

$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

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see here

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Deprived Systems

Under attack!

- Normal behavior
  - Affine solution

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

- Under attack $\rightarrow \tilde{\boldsymbol{\lambda}}_i = T_i[k]\boldsymbol{\lambda}_i$
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- New dynamics (Possibly Unstable)

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Detection Mechanism

Assumption

We know nominal $\bar{P}_i$

- If we estimate[1] $\hat{P}_i[k]$ and $\widehat{\tilde{\boldsymbol{s}}}_i[k]$ such as:

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- If $\left\|\widehat{\tilde{P}}_i[k] - \bar{P}_i\right\|_F > \epsilon_P \rightarrow$ Attack
- Ok, but how can we estimate $\widehat{\tilde{P}}_i[k]$?

[1] Using Recursive Least Squares for example

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Estimating $\hat{\tilde{P}}_i[k]$

- We need to estimate $\hat{\tilde{P}}_i[k]$ and $\hat{\tilde{s}}_i[k]$ simultaneously
- Challenge: Estimation during negotiation fails
  - Update function couples θ_i^p and λ_i^p → low input excitation
- Solution: Send a random[3] sequence to increase excitation.

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Classification of mitigation techniques

- Active (Resilient)
  1. Detection/Isolation ✔
  2. Mitigation ?

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Mitigation mechanism

Reconstructing $\boldsymbol{\lambda}_i$

- We now have $\widehat{\tilde{P}}_i[k]$
  - Since $\tilde{P}_i[k] = T_i[k]\bar{P}_i$
  - We can recover $T_i[k]^{-1}$

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$$\overset{\text{mod}}{\boldsymbol{\lambda}}_i = \begin{cases} \overset{\text{rec}}{\boldsymbol{\lambda}}_i, & \text{if attack detected} \\ \tilde{\boldsymbol{\lambda}}_i, & \text{otherwise} \end{cases}$$

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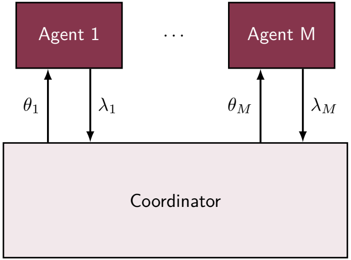
Complete Mechanism

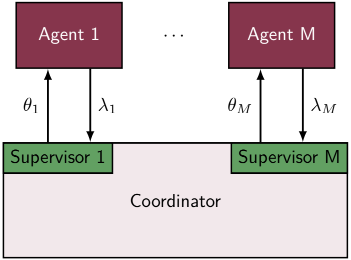


- Supervise exchanges by inquiring the agents
- Estimate how they will behave

Two Phases

1. Detect which agents are non-cooperative
2. Reconstruct λ_i and use in negotiation

Complete Mechanism



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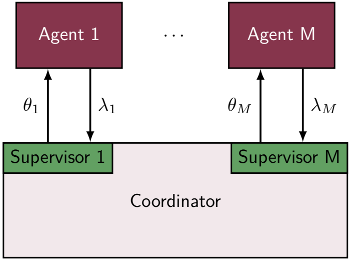

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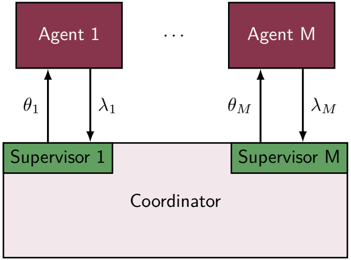

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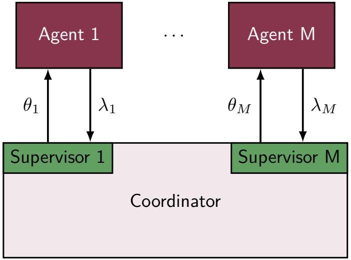

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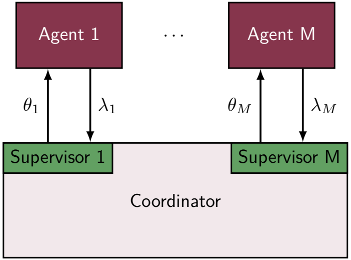

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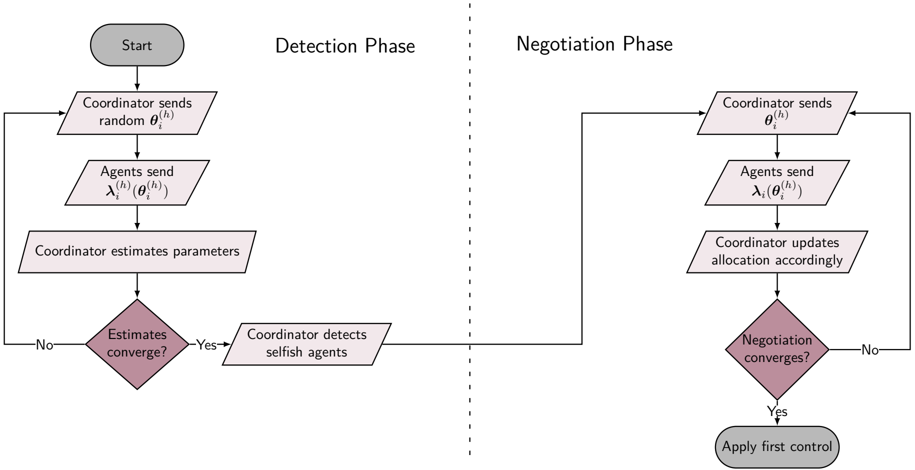
Complete algorithm

RPdMPC-DS



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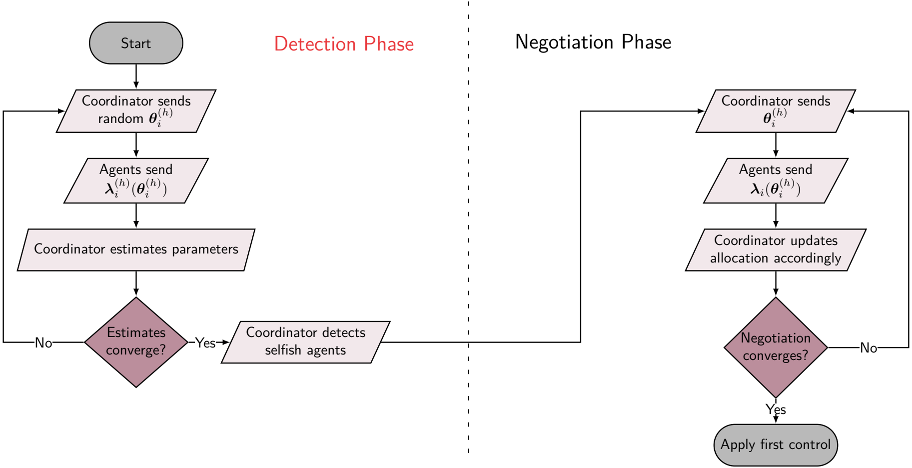

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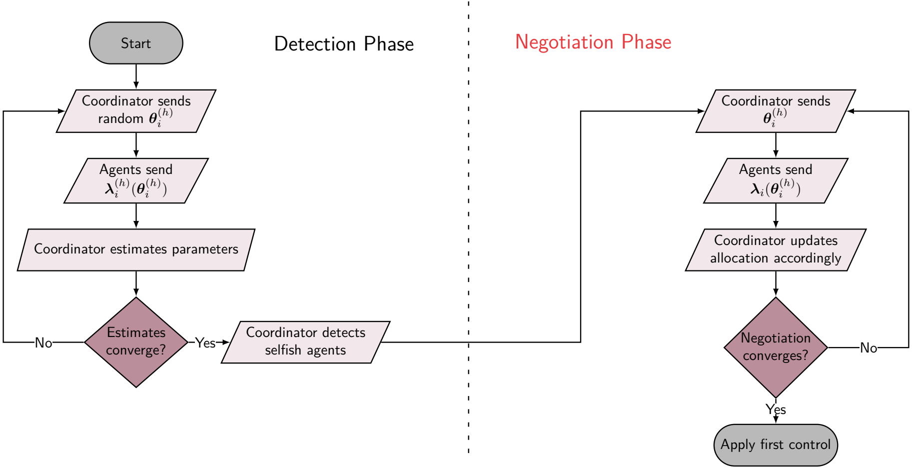

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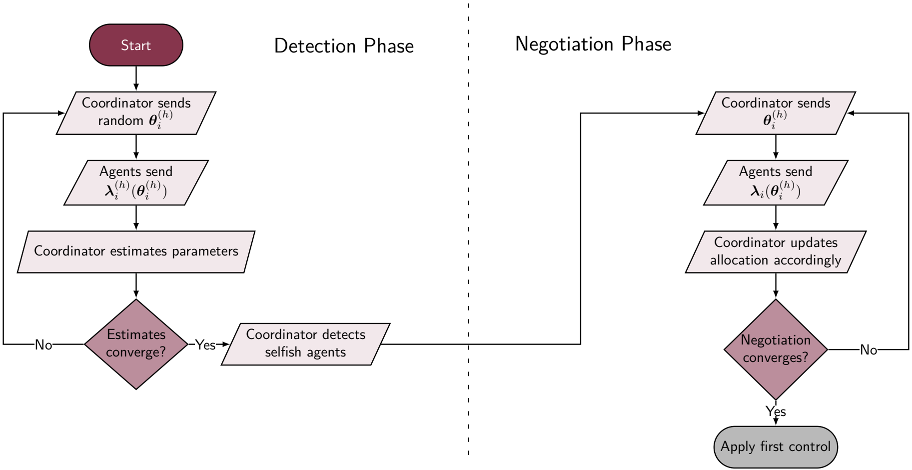

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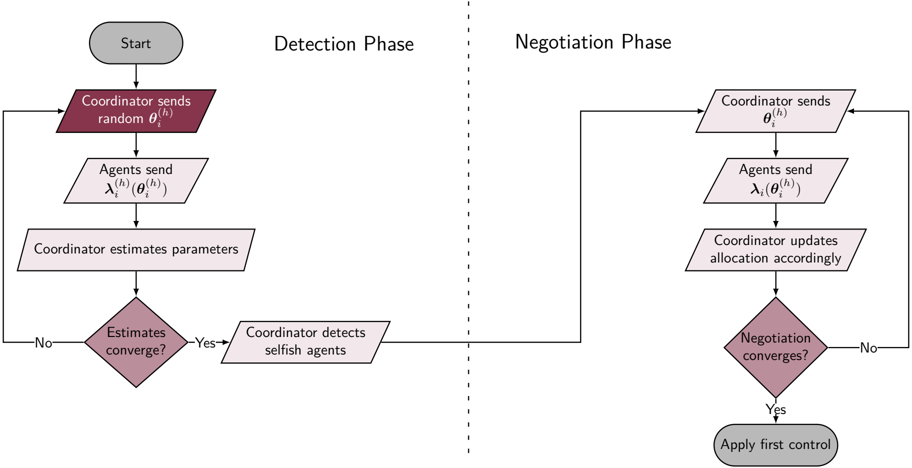

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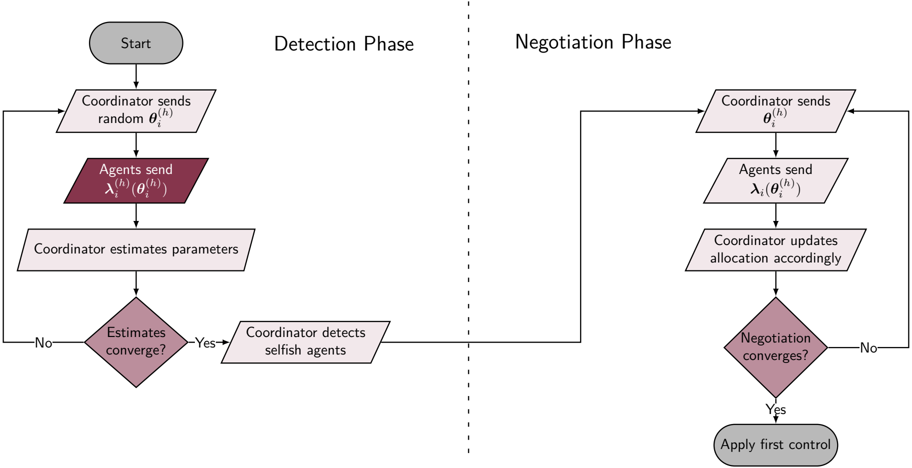

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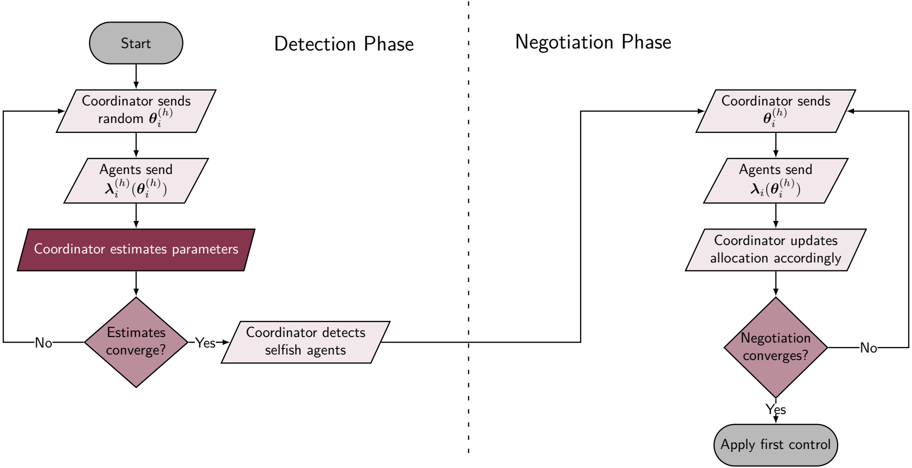

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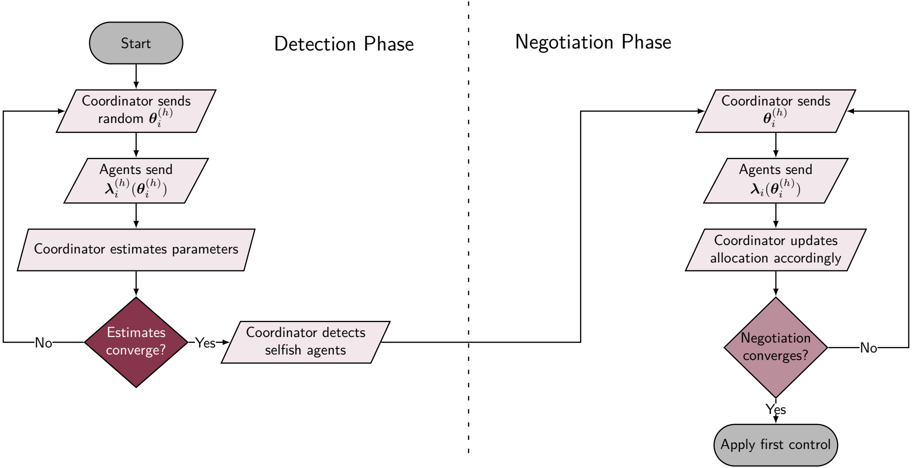

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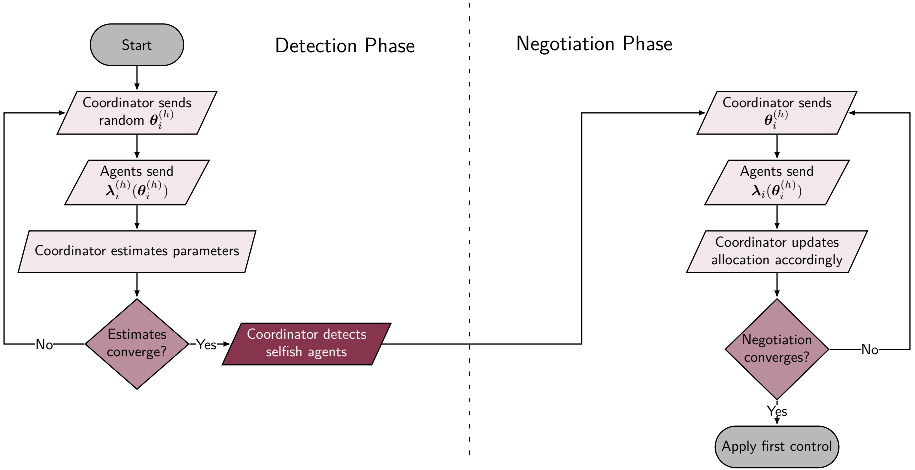

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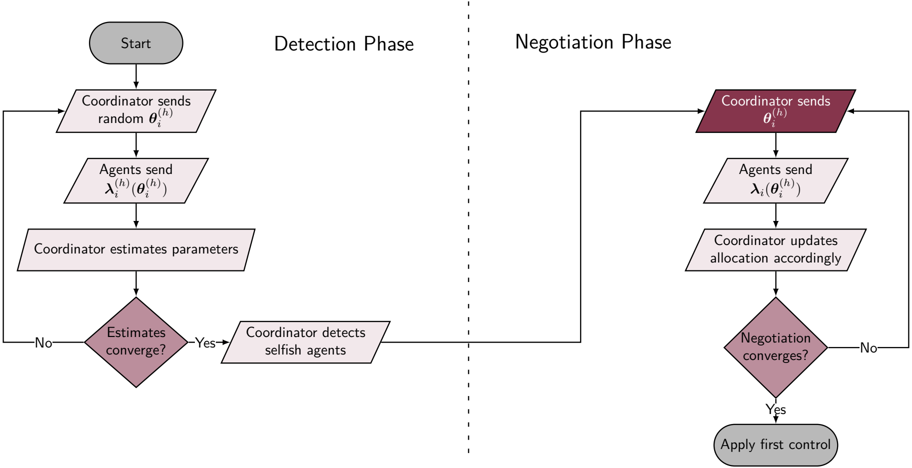

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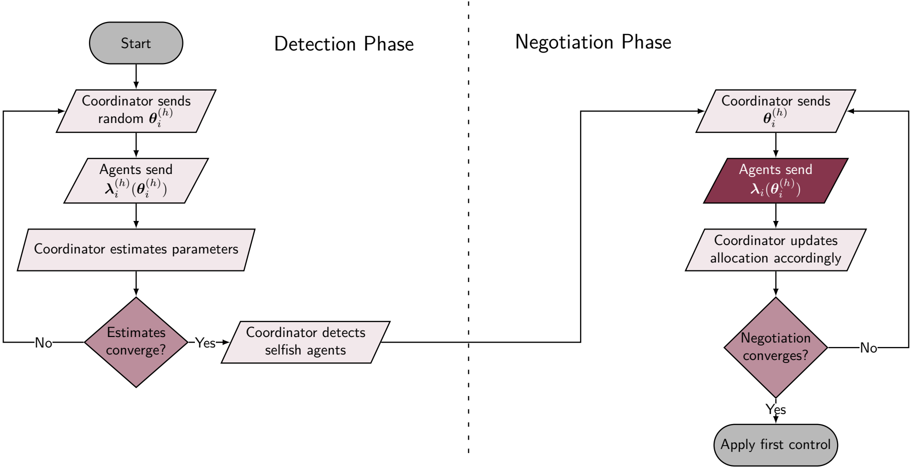

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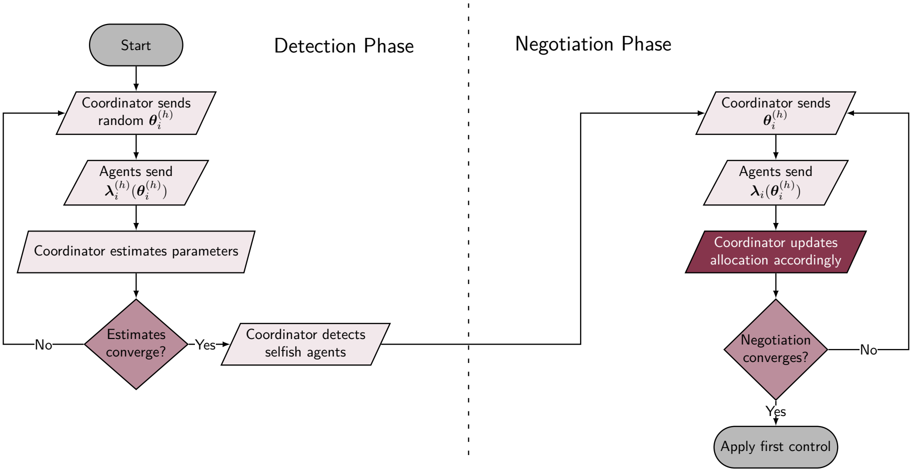

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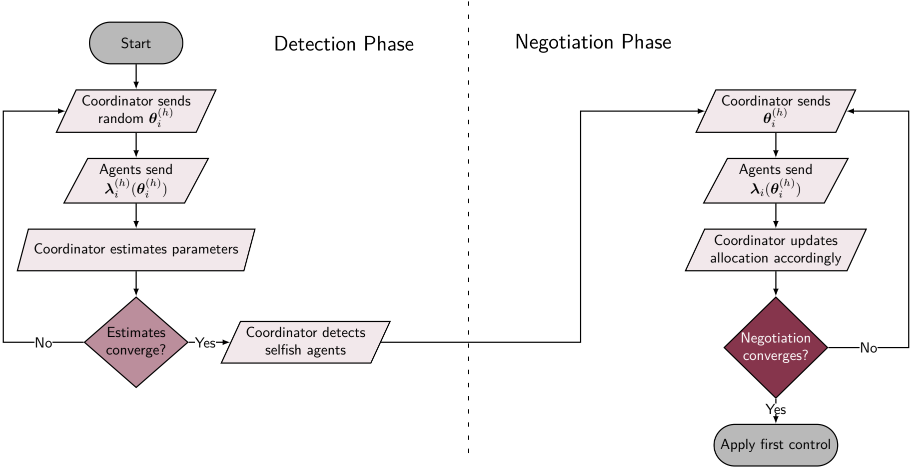

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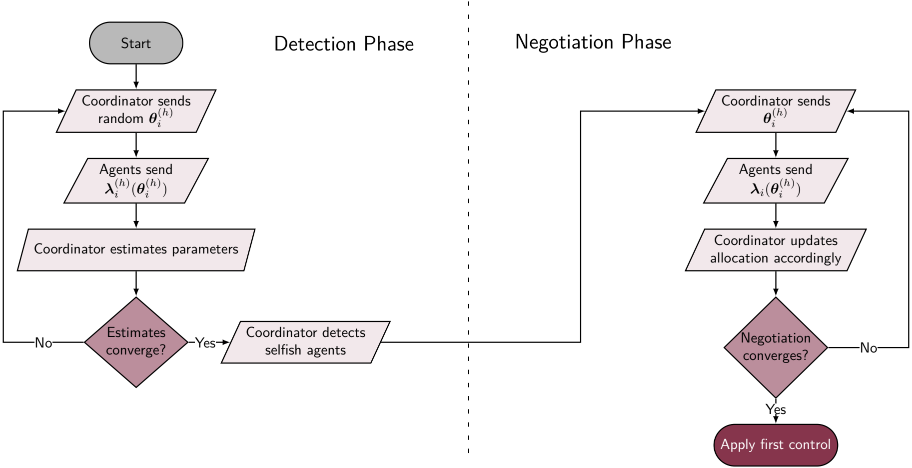
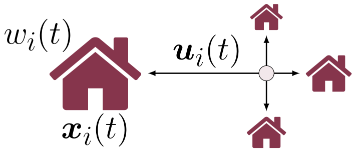


Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)
- Not enough power
- Period of $5\mathrm{h}$
- 3 scenarios
  - N Nominal
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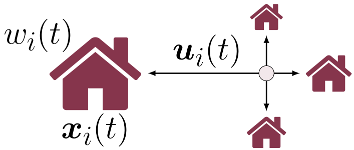

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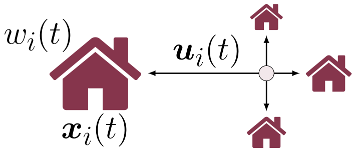

District Heating Network (4 Houses)

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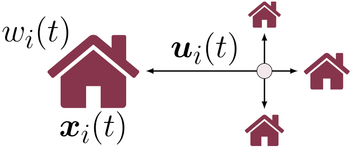
Example



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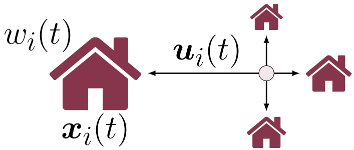

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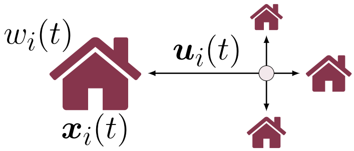

District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)
- Not enough power
- Period of $5\mathrm{h}$
- 3 scenarios
  - N Nominal
  - C Agent I cheats (dMPC)
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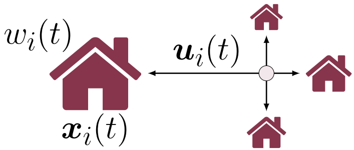

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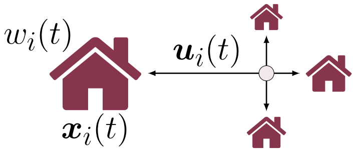

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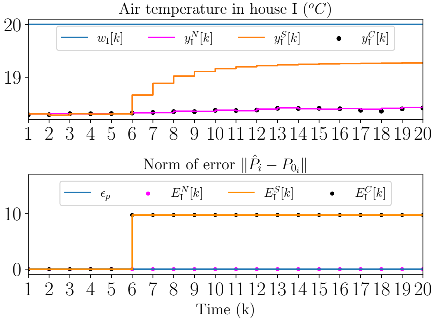
Temporal



Temperature in house I.
Error $E_I(k)$.
N Nominal, S Selfish, C Corrected

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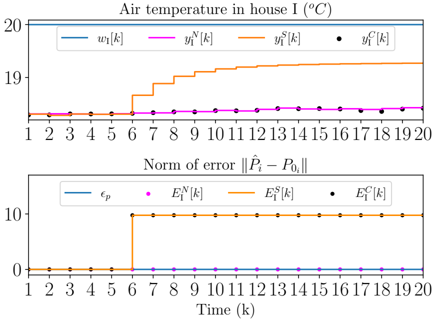

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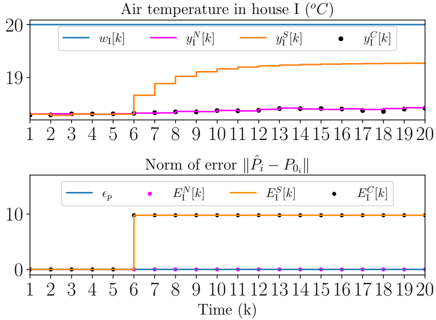

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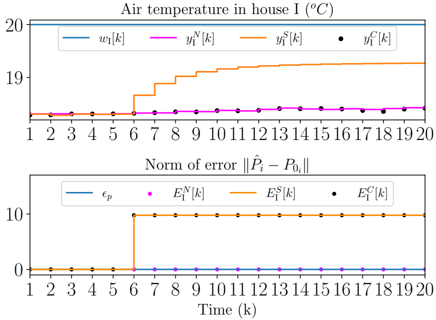

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Results

Costs

Objective functions J_i (% error)

Agent	Scenario N	Scenario S	Scenario C
I	299.5	190.8 (−36.3)	301.0 (0.0)
II	192.4	234.1 (21.7)	191.4 (−0.5)
III	305.9	359.1 (17.4)	305.9 (−0.0)
IV	297.5	349.9 (17.6)	297.2 (−0.1)
Global	1095.3	1133.9 (3.5)	1095.5 (0.0)

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Outline

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Relaxing scarcity assumption

- Let's relax the scarcity assumption
- And add some local constraints
- Similarly we have the local problems and update

$$\begin{aligned}\underset{\boldsymbol{U}[k]}{\text{minimize}} \quad & \frac{1}{2}\left\|\boldsymbol{U}[k]\right\|_H^2 + \boldsymbol{f}[k]^T\boldsymbol{U}[k] \\ \text{subject to} \quad & \bar{\Gamma}\boldsymbol{U}[k] \leq \boldsymbol{U}_{\max} \\ & \boldsymbol{U}[k] \in \mathcal{U}\end{aligned}$$

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$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

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Analyzing System

Solution for $\lambda_i[k]$

Instead of having

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

Now we have

$$\boldsymbol{\lambda}_i[k] = \begin{cases} -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^{n}_{\boldsymbol{\lambda}_i} \\ \vdots & \vdots \\ -P_i^{\left(2^{n_{\text{ineq}}}-1\right)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{\left(2^{n_{\text{ineq}}}-1\right)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^{2^{n_{\text{ineq}}}-1}_{\boldsymbol{\lambda}_i} \end{cases}$$

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For n_{ineq} constraints $\longrightarrow 2^{n_{\text{ineq}}}$ permutations

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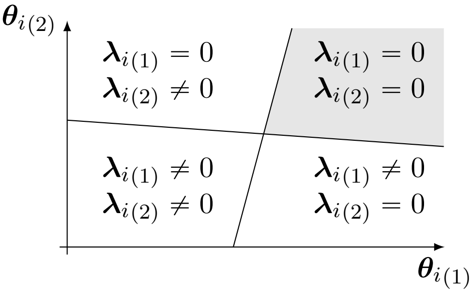
Analyzing System

Solution for $\lambda_i[k]$ (Continued)



Two constraints partitioning $\boldsymbol{\theta}_i$ solution space.

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Analyzing System

Negotiation

$$\mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)}+\rho^{(p)}\boldsymbol{\lambda}[k]^{(p)}) = \underset{\boldsymbol{x}}{\operatorname{argmin}} \left\{ \begin{array}{ll} \underset{\boldsymbol{U}[k]}{\text{minimize}} & \left\|\boldsymbol{x} - \boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)}\right\| \\ \text{subject to} & I_c^M \boldsymbol{x} \preceq \boldsymbol{U}_{\max} : \boldsymbol{\mu} \end{array} \right\}$$

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Analyzing System

Negotiation (Continued)

$$\boldsymbol{\theta}[k]^{(p+1)} = \begin{cases} \boldsymbol{x}_0 + I_c^{M(0)}\left[-P_{\boldsymbol{\mu}}^{(0)}\boldsymbol{U}_{max} + \boldsymbol{s}_{\boldsymbol{\mu}}^{(0)}[k]\right], & \text{if } \boldsymbol{x}_0 \in \mathcal{R}_{\boldsymbol{\mu}}^{0} \\ \vdots & \vdots \\ \boldsymbol{x}_0, & \text{if } \boldsymbol{x}_0 \in \mathcal{R}_{\boldsymbol{\mu}}^{2^c-1} \end{cases}$$

$$\text{where } \boldsymbol{x}_0 = \boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)}$$

Analyzing System

Conclusion

$$\bar{\epsilon} = \overbrace{2^{c}}^{\text{regions in projection}} \times \underbrace{2^{n_{\text{ineq}}} \times \cdots \times 2^{n_{\text{ineq}}}}_{M\times\text{regions in each }\boldsymbol{\lambda}_i} = 2^{c+Mn_{\text{ineq}}}$$

Mitigation

- Ideal

$$\widehat{T_i[k]^{-1}} = \bar{P}_i \widehat{\tilde{P}}_i[k]^{-1}$$

- $P_i^{(0)}$ only invertible

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CentraleSupélec

Artificial Scarcity

Who is it? Who is it?

- Since we control $\boldsymbol{\theta}_i$ let's choose one where all constraints are active

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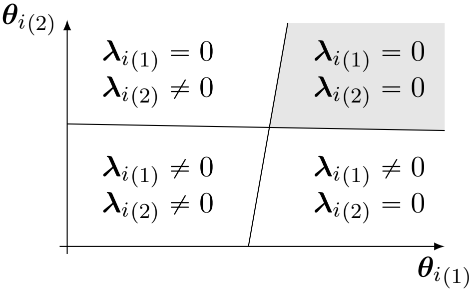

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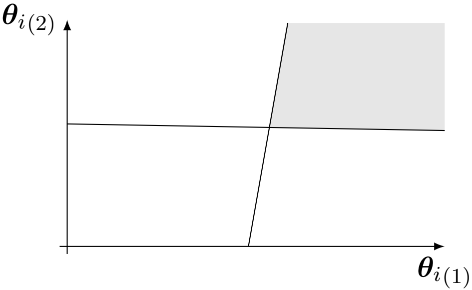

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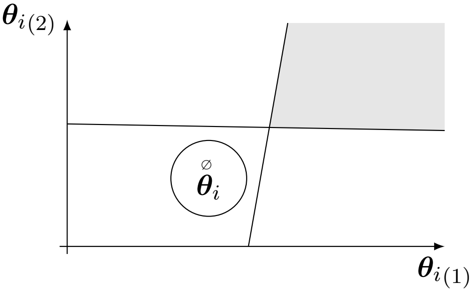

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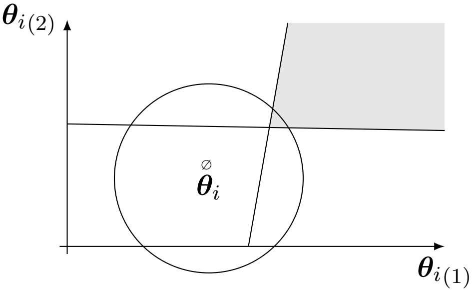

Expectation Maximization

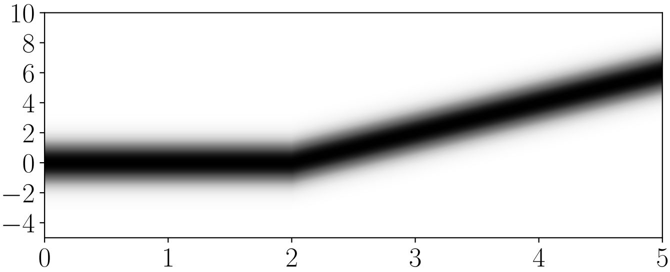

Figure 1: Gaussian Mixture for a 1D PWA function with 2 modes.

Expectation Maximization

Algorithm

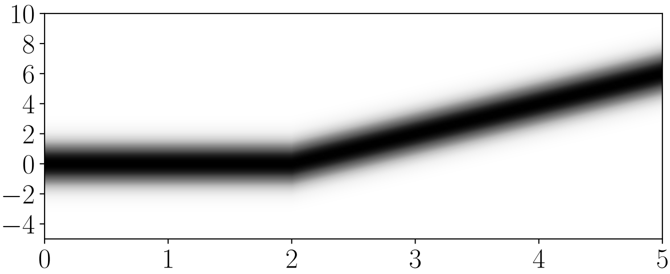

Figure 2: Gaussian Mixture for a 1D PWA function with 2 modes.

Detection

Same same

- Error $E_i^{(0)}[k] = \left\| \widehat{\widetilde{P}}_i^{(0)}[k] - \bar{P}_i^{(0)} \right\|_F$
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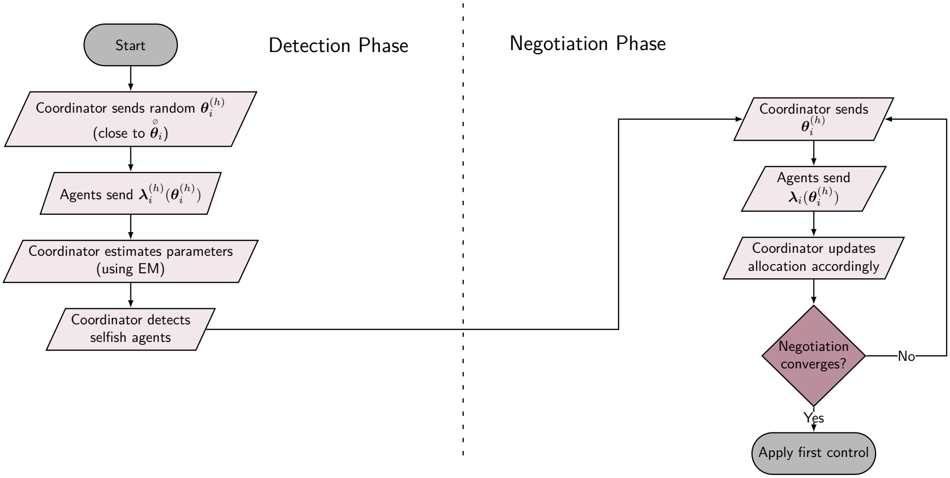
Complete algorithm

RPdMPC-AS Refaire



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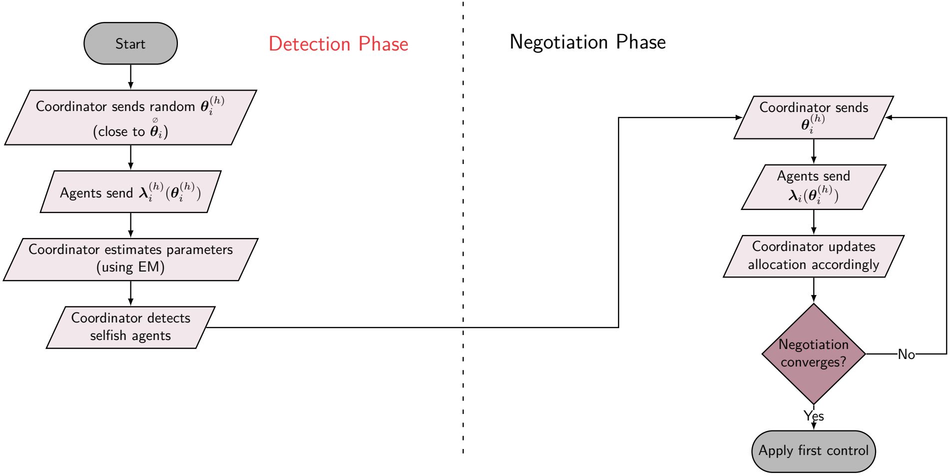

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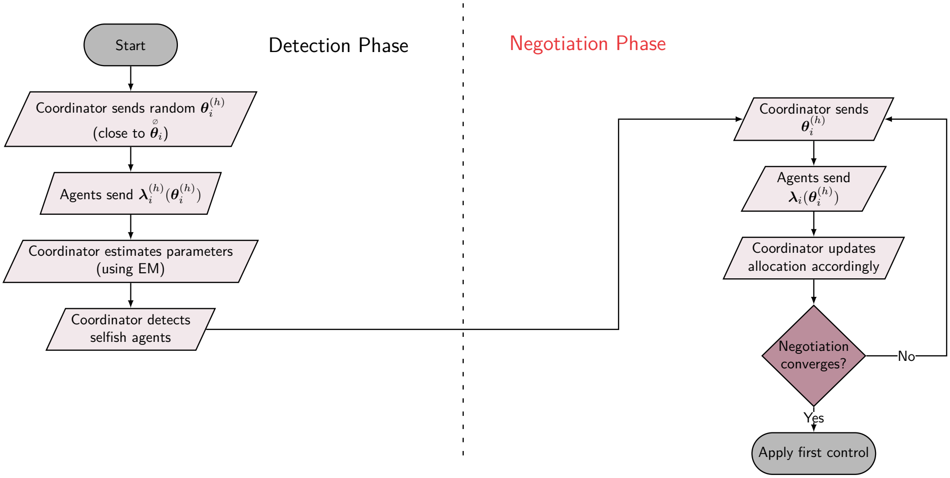

Complete algorithm

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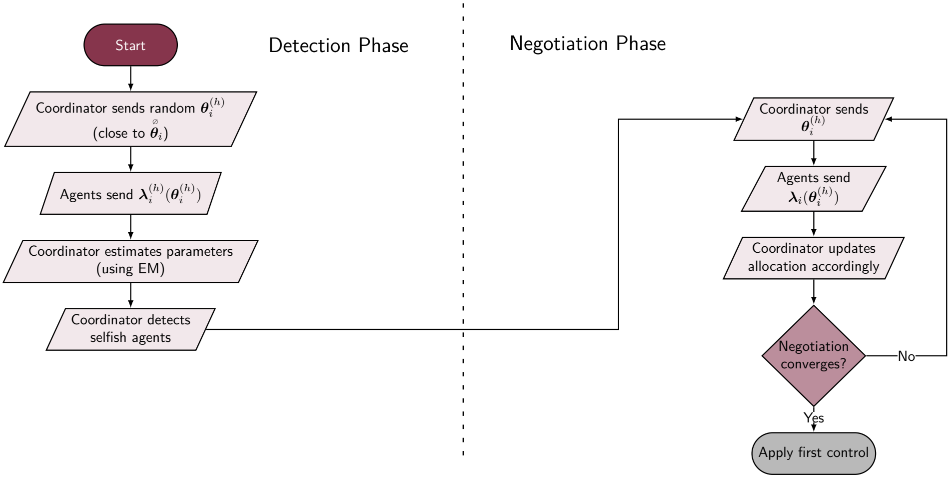

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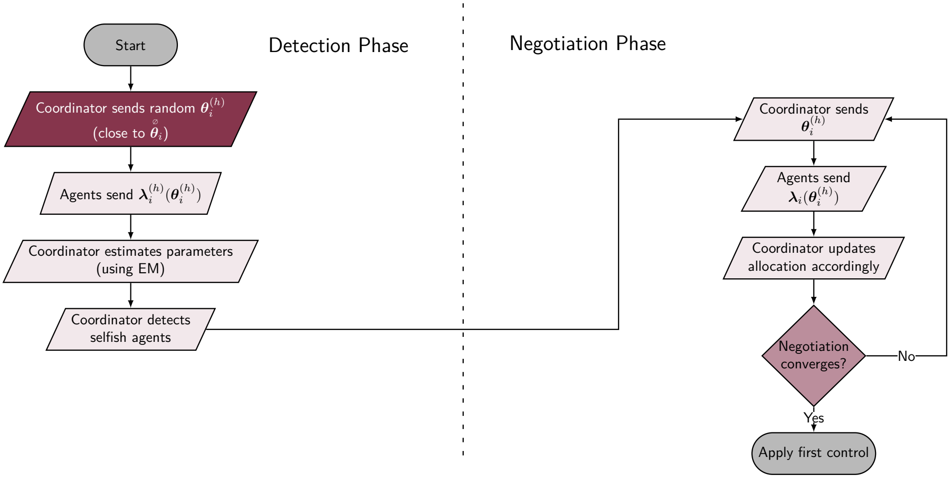

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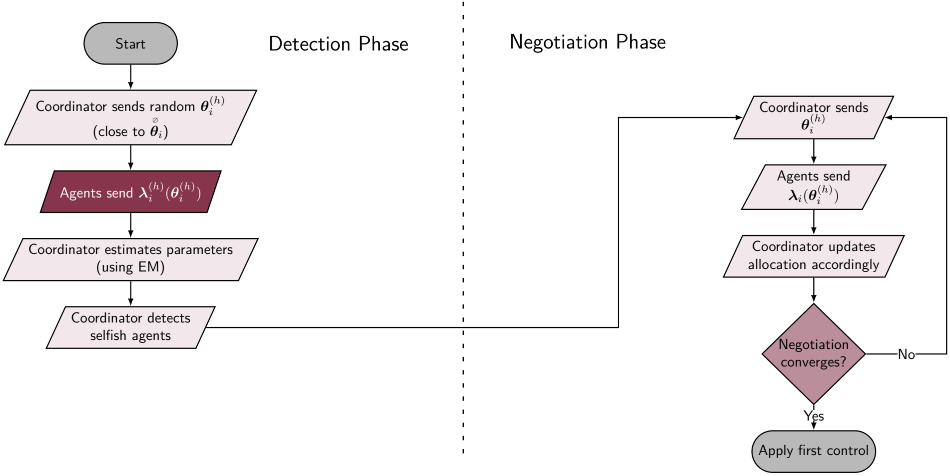

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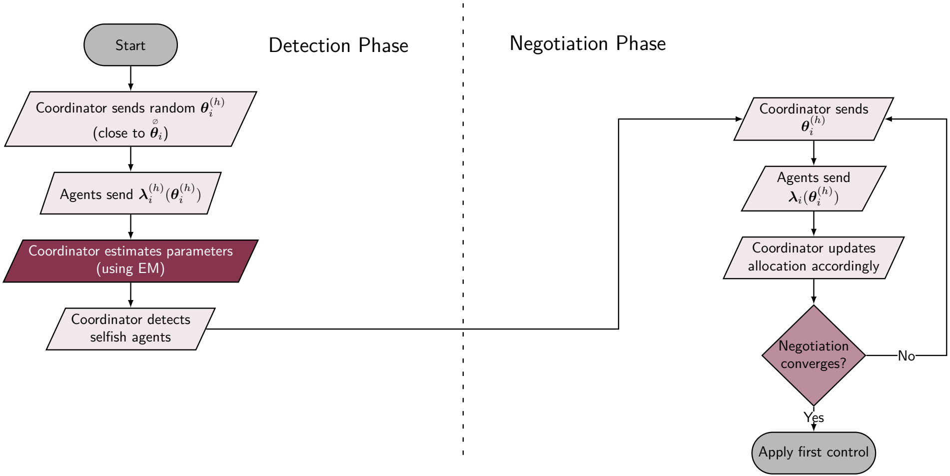

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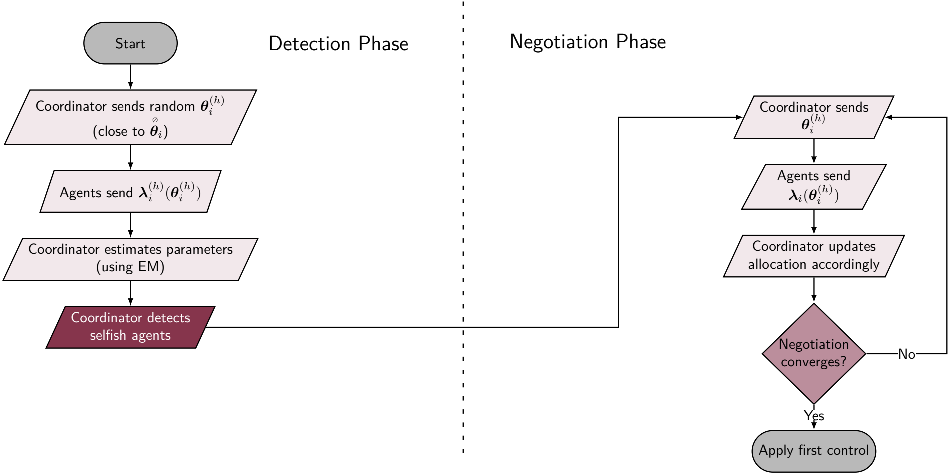

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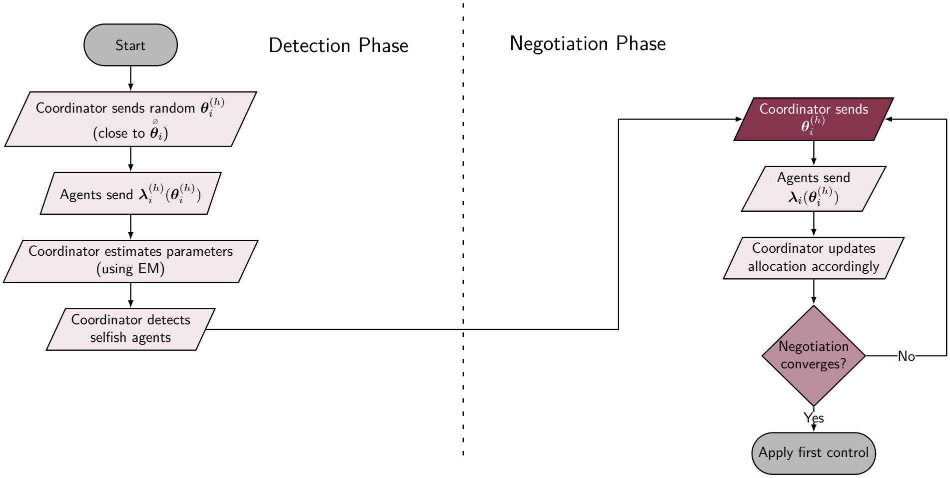

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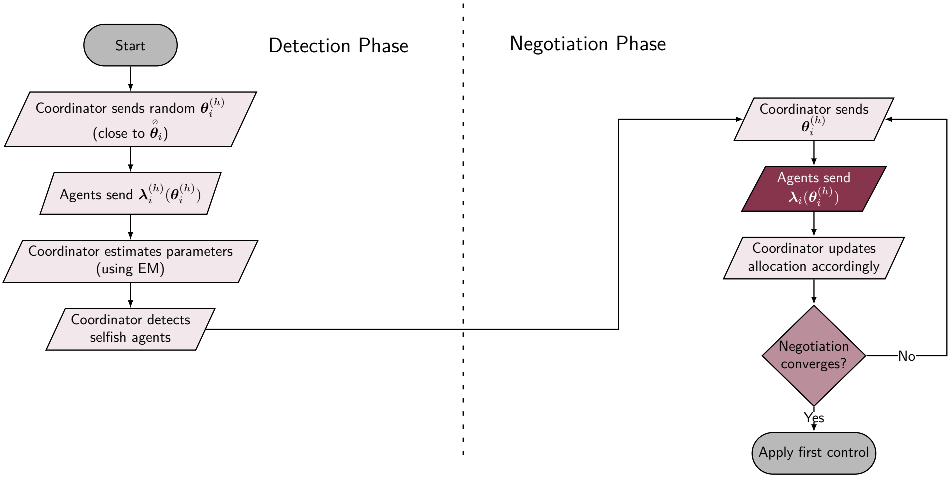

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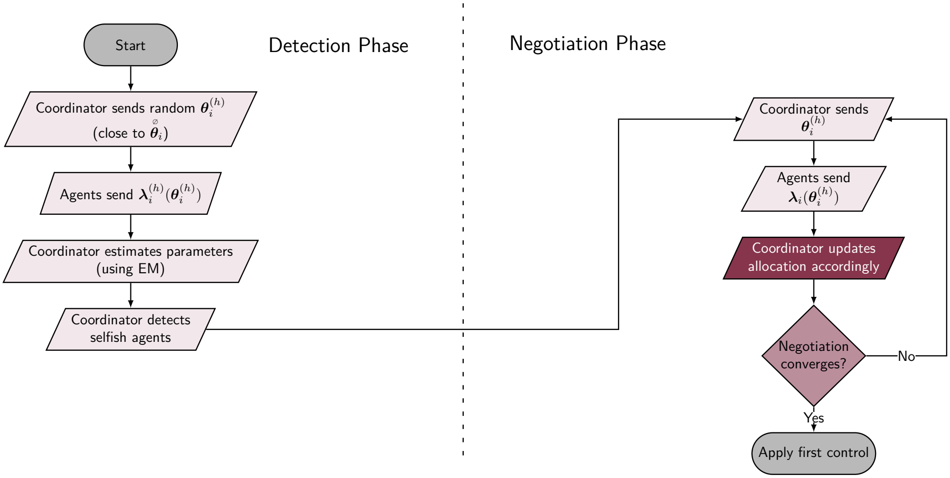

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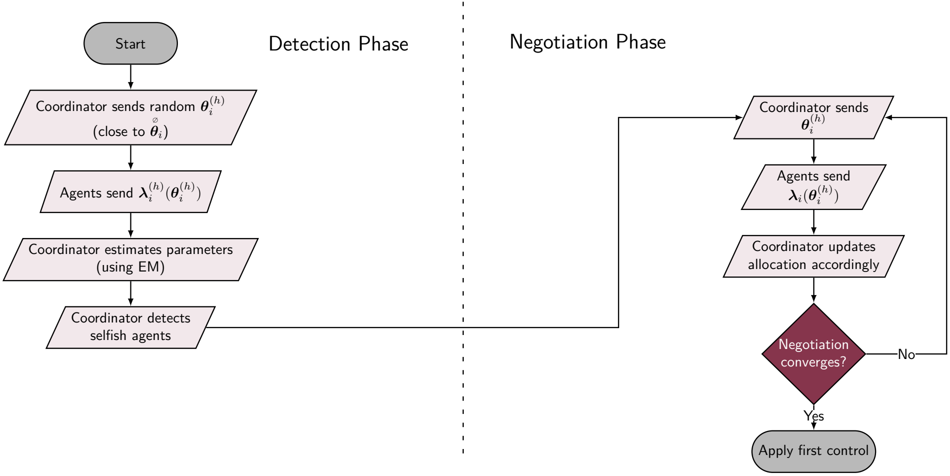

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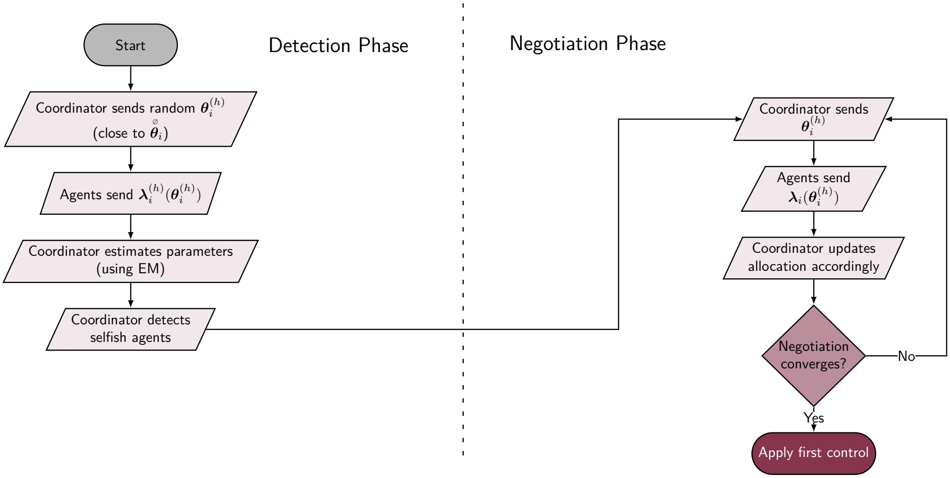

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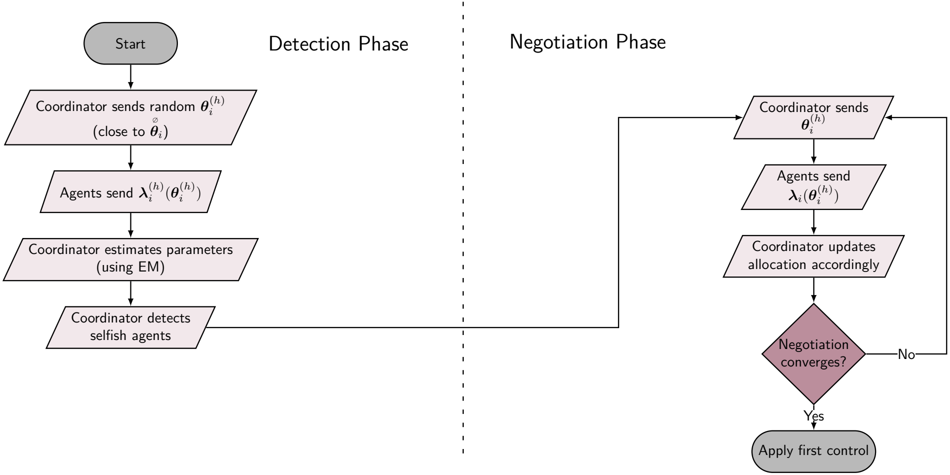
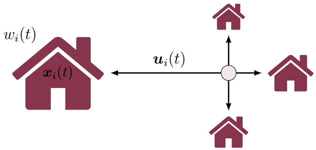


Example

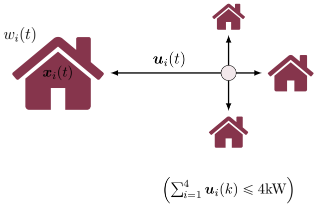


$$\left(\sum_{i=1}^{4} \boldsymbol{u}_i(k) \leqslant 4\text{kW}\right)$$

District Heating Network (4 Houses)

- Houses modeled using 3R-2C
- Not enough power
- Period of 5h ($T_s = 0.25\text{h}$)
- 3 scenarios
  - (N) Nominal
  - (C) Agent I cheats (dMPC)
  - (S) Agent I cheats (RPdMPC-AS)
- $T_I = \begin{bmatrix} 14.43288267 & 0. & 0. & 0. \\ 0. & 13.4590903 & 0. & 0. \\ 0. & 0. & 6.93065061 & 0. \\ 0. & 0. & 0. & 3.4447393 \end{bmatrix}$

Example



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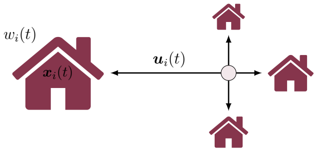

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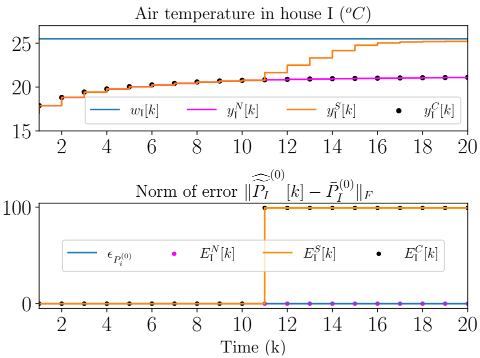
Results

Temporal



Temperature in house I and the variable $E_I(k)$ for different scenarios.
N Nominal, S Selfish behavior, C Selfish + Correction

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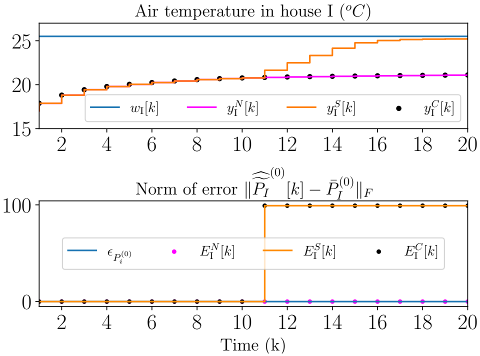
Results

Temporal



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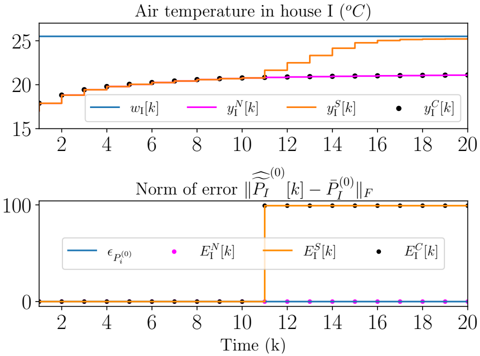

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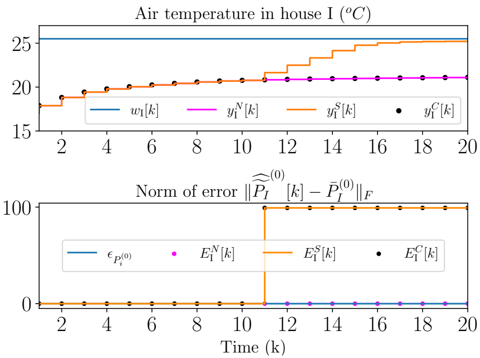

Temperature in house I and the variable $E_I(k)$ for different scenarios.
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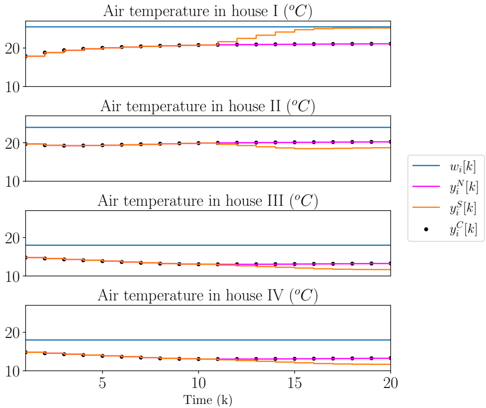
Results

Temporal (Continued)

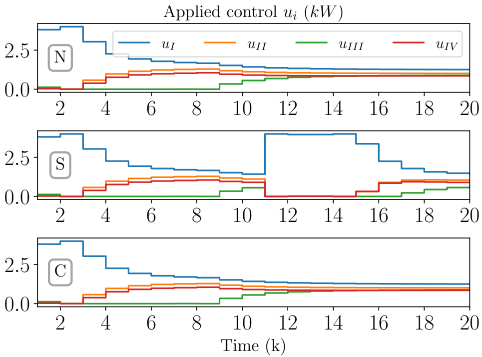


Air temperature in all houses for different scenarios.

N Nominal, S Selfish behavior, C Selfish + Correction

Results

Control



Control applied in all houses for different scenarios.
N Nominal, S Selflish behavior, C Selfish + Correction

Results

Costs

Objective functions J_i (% error).

Agent	Scenario N	Scenario S	Scenario C
I	19868.2	12618.5 (−36.5)	19868.2 (−0.0)
II	13784.5	18721.1 (35.8)	13784.5 (0.0)
III	17276.0	22324.9 (29.2)	17276.1 (0.0)
IV	10086.0	13872.4 (37.5)	10086.0 (0.0)
Global	61014.7	67536.9 (10.7)	61014.7 (−0.0)

Summary

- Vulnerabilities of Primal decomposition dMPC
- Resilient strategy for 2 kinds of systems
  - Deprived systems (where demand is greater than total resources)
  - Systems with possible artificial scarcity (sensible optimal demand)

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- Partial reconstruction of cheating matrix
- Resilient strategy with soft constraints
- Recursive EM (or alternative)
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Thank you!

Repository
https://github.com/Accacio/thesis

Contact
rafael.accacio.nogueira@gmail.com

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