Security of distributed Model Predictive Control under False Data injection

Rafael Accácio NOGUEIRA

2022-12-07

https://bit.ly/3g3S6X4

45 minutes !!!!

Good afternoon, thank you all for being here.I'm Rafael Accácio and I'm going to present my work on the security of distributed model predictive control under false data injection.

Context

❝ Necessity is the mother of invention ❞

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“Necessity is the mother of invention”

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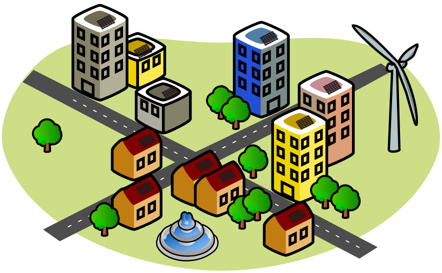

- Electricity Distribution System
- Heat distribution
- Water distribution
- Traffic management

  (include your problem here)

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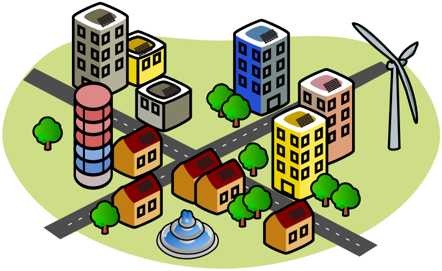

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- Multiple systems interacting
- Coupled by constraints
  - Technical/ Comfort
- Optimization objectives
  - Minimize energy consumption
  - Maximize user satisfaction
  - Follow a trajectory
- Solution → MPC

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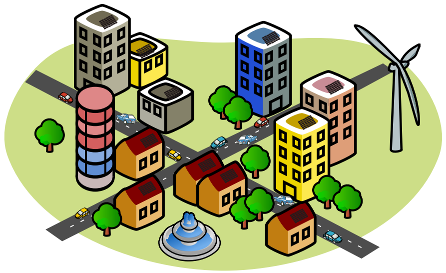

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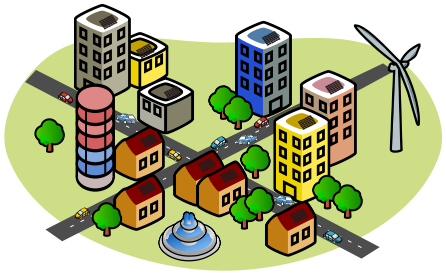

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Model-based Predictive Control

Find best control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence
  - Objective function to optimize
  - System's Model (states and inputs)
  - Other constraints to respect

For those who are not familiar with mpc. Mpc is the model based predictive controller.

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The objective is to find the best control sequence using predictions based on a model.

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When we say best,

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we mean optimal.

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So we need to solve an optimization problem.

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And we have the control sequence of u as the decision variable.

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which is calculated for a horizon N

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So, we need an objective function. For example follow a trajectory while minimizing the energy.

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A model of the system

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with its states

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and inputs

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But we can also integrate some constraints, such QoS or technical restrictions

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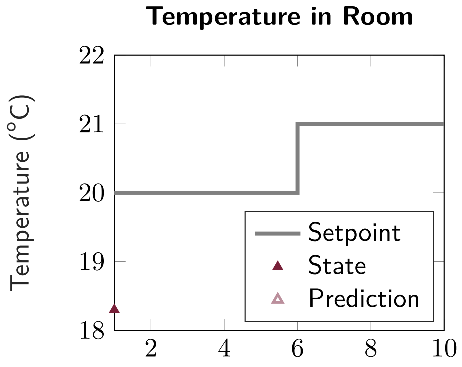
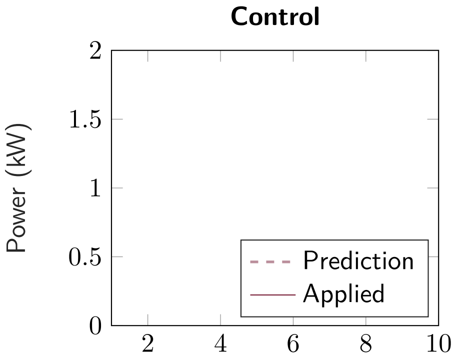
Find optimal control sequence



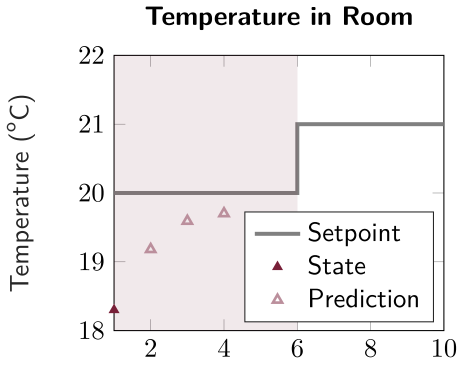


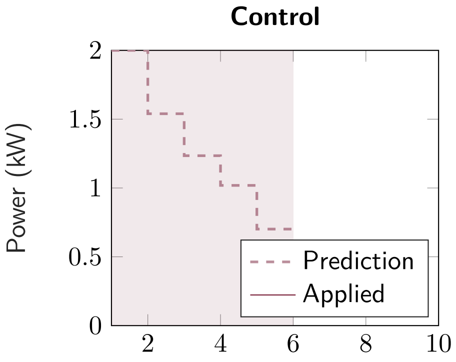
So, for example, if we may have a setpoint to follow

Model Predictive Control

In a nutshell

Find optimal control sequence



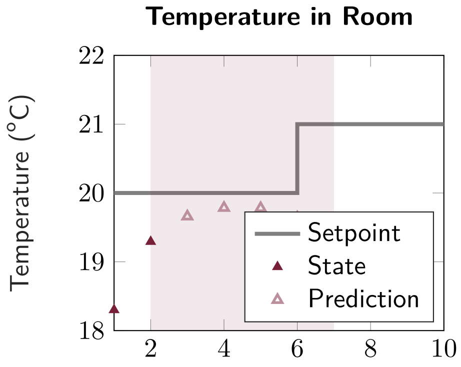


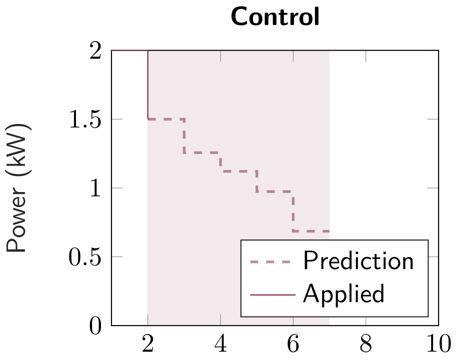
We find an optimal control sequence

Model Predictive Control

In a nutshell

Find optimal control sequence, apply first element





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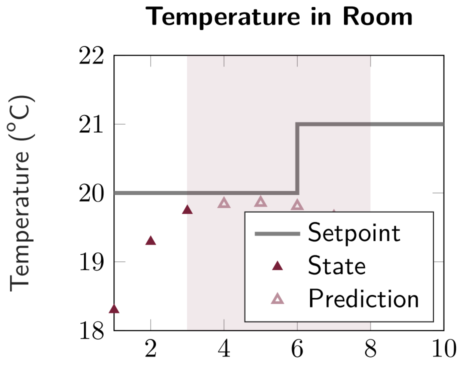
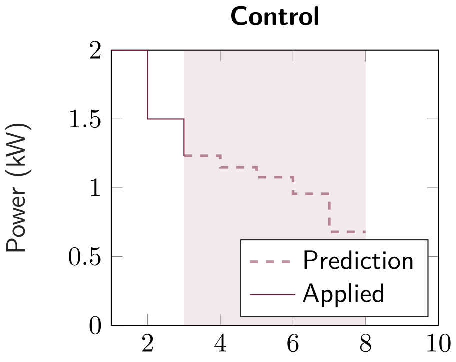
We apply only the first element

Model Predictive Control

In a nutshell

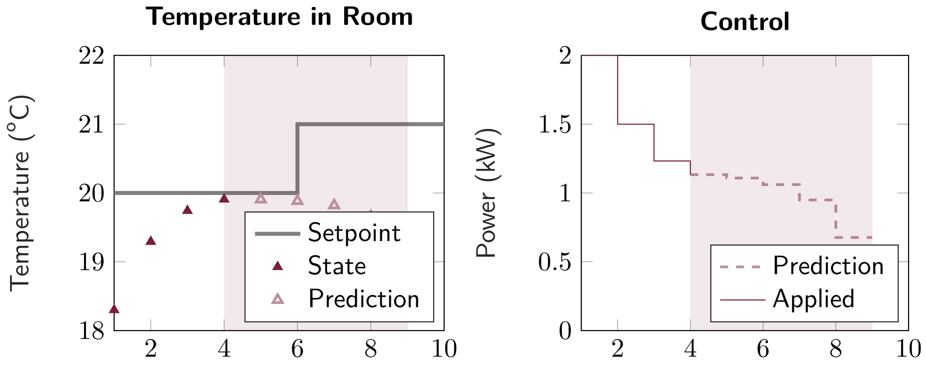
Find optimal control sequence, apply first element, rinse repeat





and then we repeat

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon



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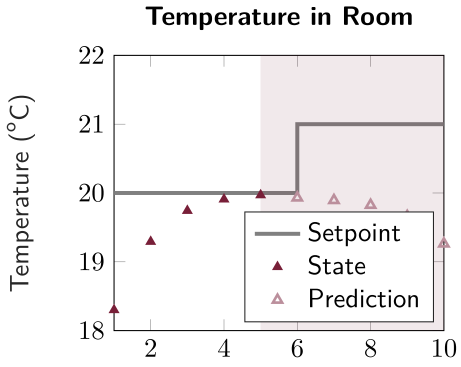
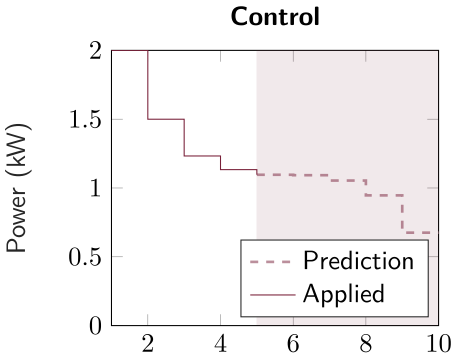
following what we call the receding horizon strategy.However this problem is not always straightforward to solve, for some cases it can be easier.

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





Model Predictive Control

Nothing is perfect

- Problems
  - Complexity of calculation
  - Topology (Geographical distribution)
  - Flexibility (Add/remove parts)
  - Privacy
- Solution: Divide and Conquer (distributed MPC)
  - Break calculation
  - Make Systems Communicate

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Distributed Model Predictive Control

It is about communication

- We break the MPC into multiple
- Make them Communicate
  - Many flavors to choose from

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However, the solution will depend on the horizon, the number of constraints, and sizes of input and states, increasing the complexity of the calculation

Distributed Model Predictive Control

It is about communication

- We break the MPC into multiple
- Make them Communicate
  - Many flavors to choose from

A strategy to alleviate is to distribute the calculation whenever possible. And there are many ways to divide it as the book shows.

It is about communication

- We break the MPC into multiple
- Make them Communicate
  - Many flavors to choose from[1]
    - Hierarchical/Anarchical
    - Sequential/Parallel
    - Synchronous/Asynchronous
    - Bidirectional/Unidirectional
    - ...

[1]

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Here we opt for a hierarchical strategy where we use multiple MPCs and an agent to coordinate and manage the coupling aspects of the problem.

Distributed Model Predictive Control

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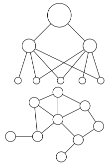

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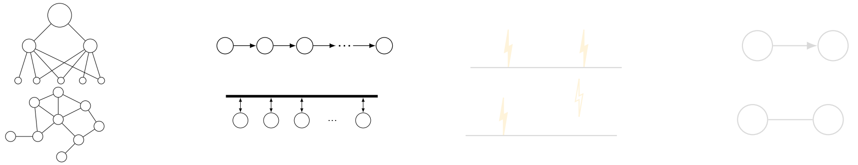

[1]

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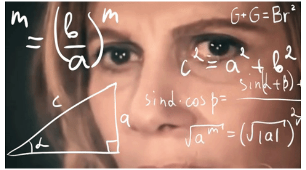

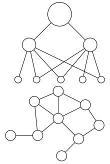

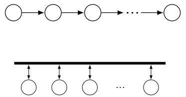

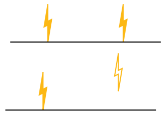

1

It is about communication

- We break the MPC into multiple
- Make them Communicate , But how?
  - Many flavors to choose from[1]
    - Hierarchical/Anarchical
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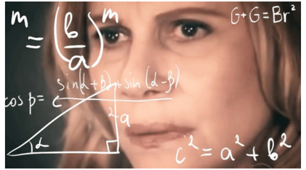

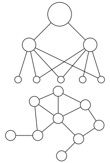

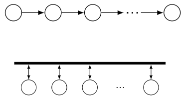

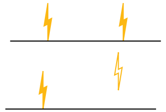

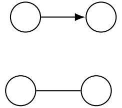

1

Distributed Model Predictive Control

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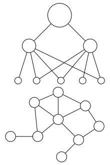

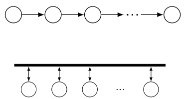

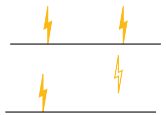

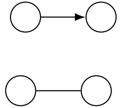

[1] Distributed Model Predictive Control made easy

CentraleSupélec

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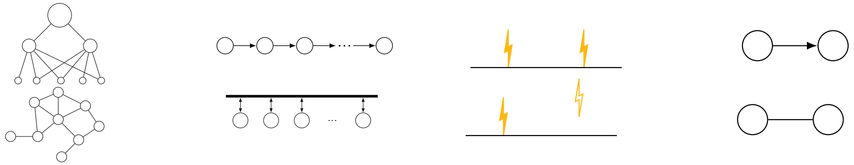

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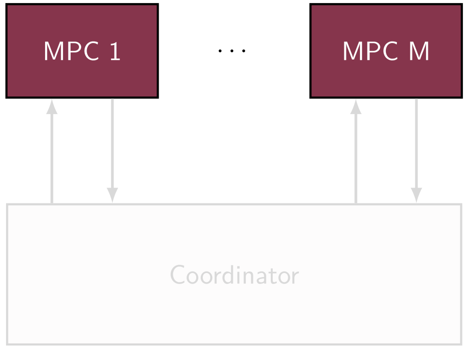
Distributed Model Predictive Control

Optimization Frameworks



- Coordinator → Hierarchical
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- No delay → Synchronous
- Agents solve local problems
- Variables are updated

Until Convergence

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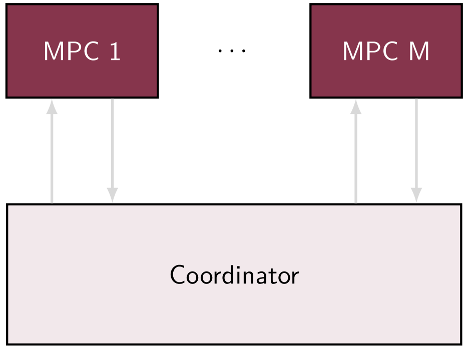

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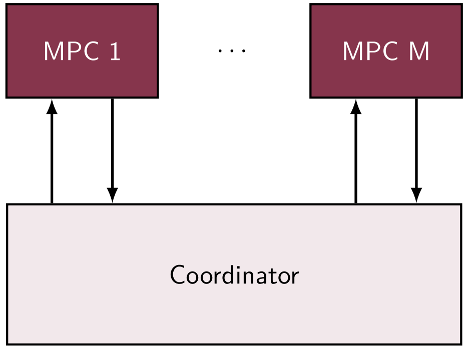

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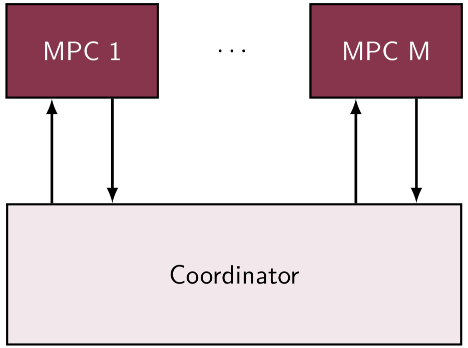

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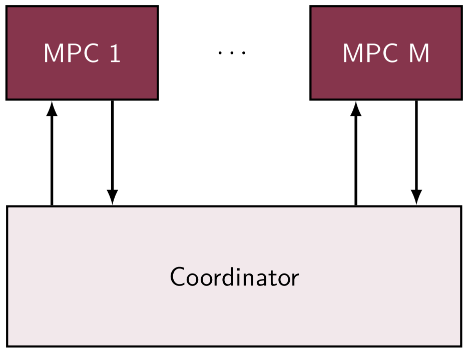

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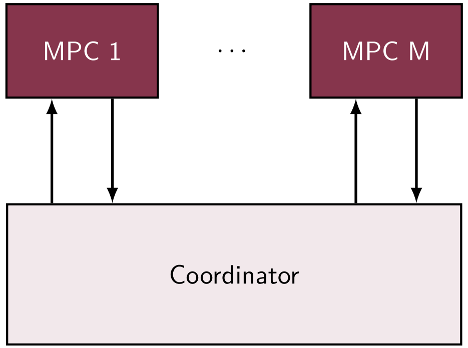

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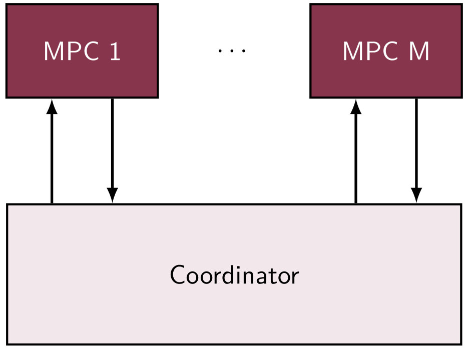

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CentraleSupélec

Negotiation works if agents comply.
But what if some agents are ill-intentioned and attack the system?

- How can an agent attack?
- What are the consequences of an attack?
- Can we mitigate the effects?

Let's have a preview!

CentraleSupélec

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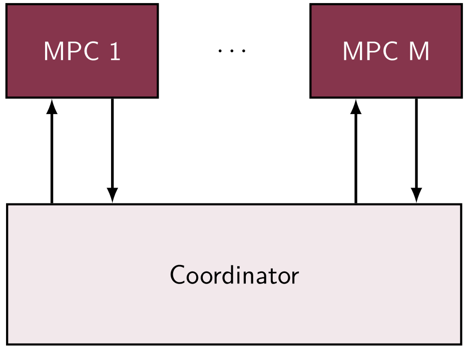

- [Vel+17a; CMI18] present attacks
  - Objective function
    - Selfish Attack
    - Fake weights
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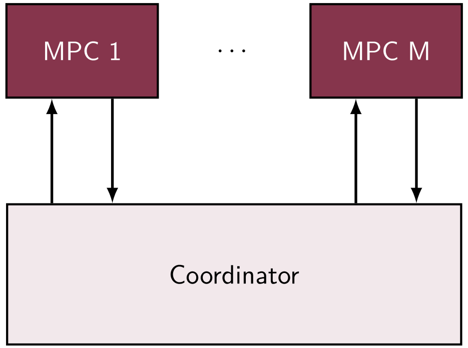
  - Liar agent

Deception Attacks

CentraleSupélec

How can a non-cooperative agent attack?

Literature



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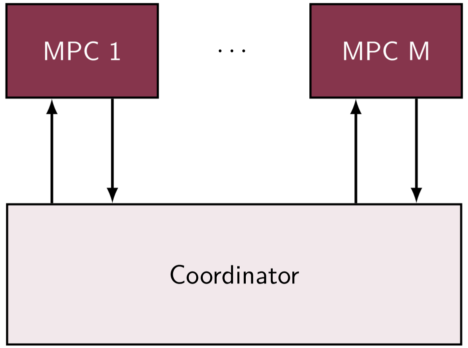

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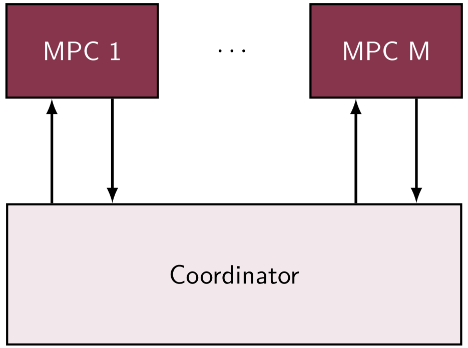

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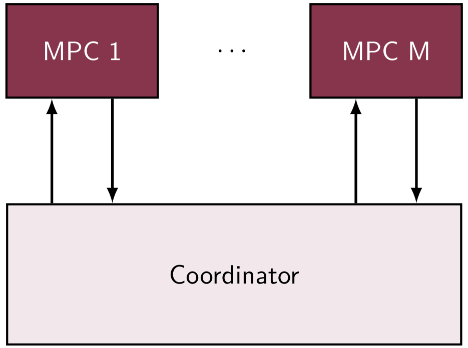

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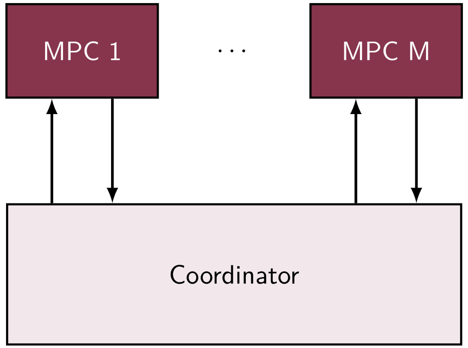

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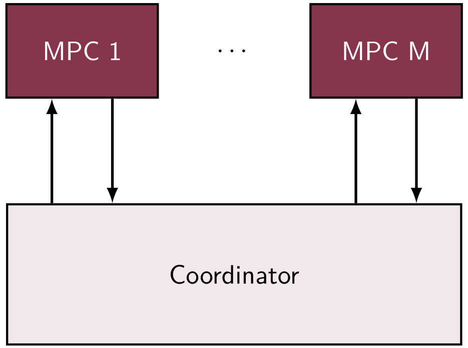

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Deception Attacks

(use control different from the agreed)

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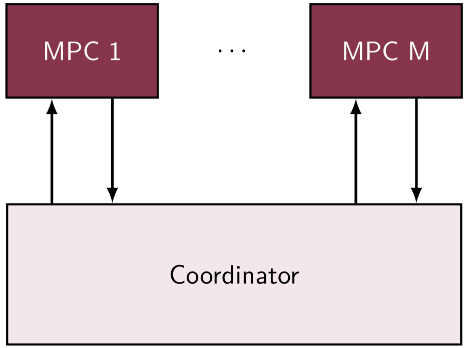

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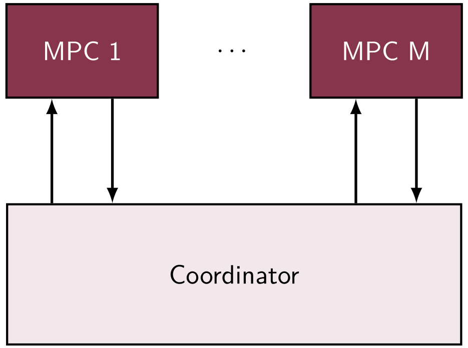

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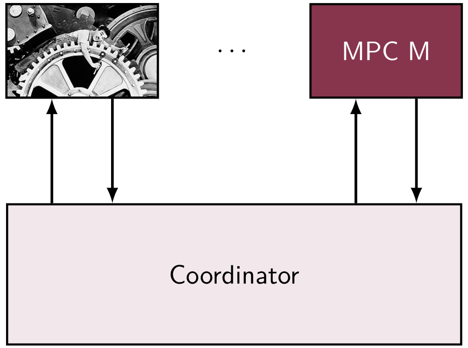

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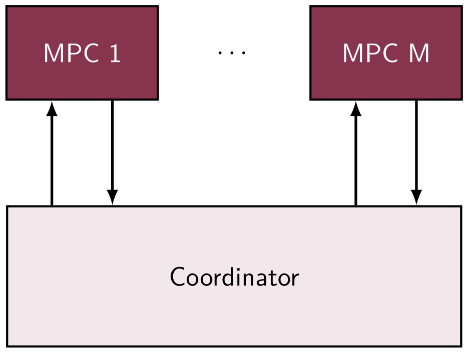
Deception Attacks (Internal change)



- We are in coordinator's shoes
- What matters is the interface
    - Attacker changes communication

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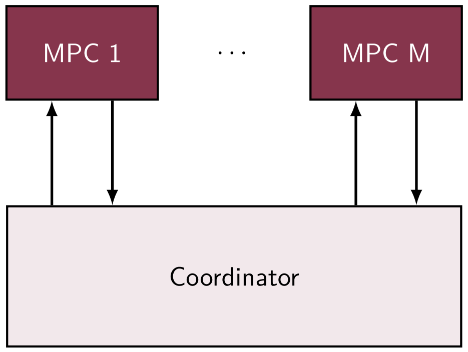
How can a non-cooperative agent attack?

Our approach



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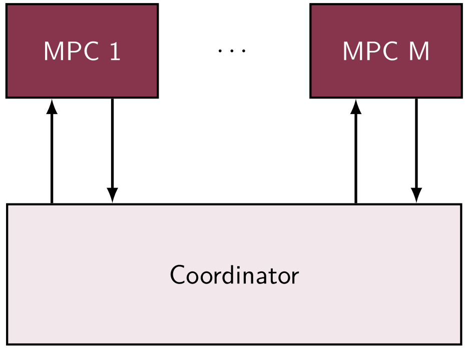

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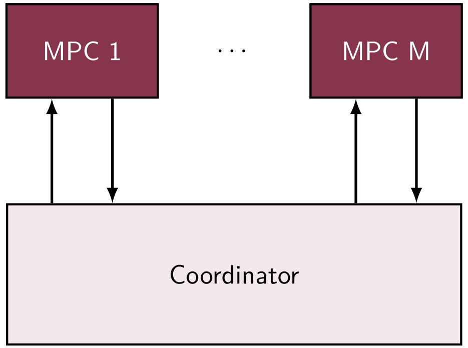

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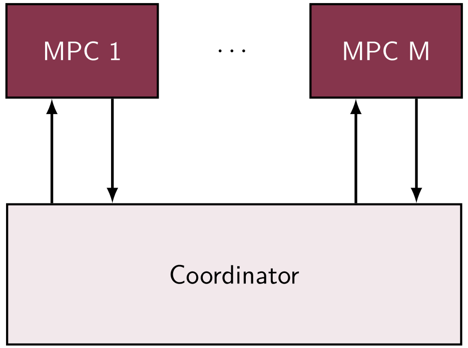

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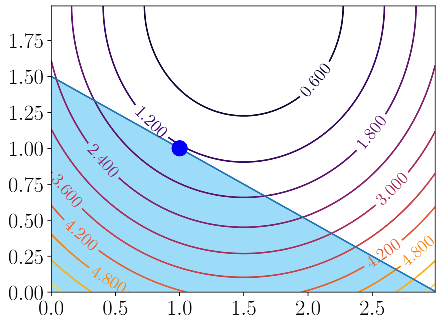
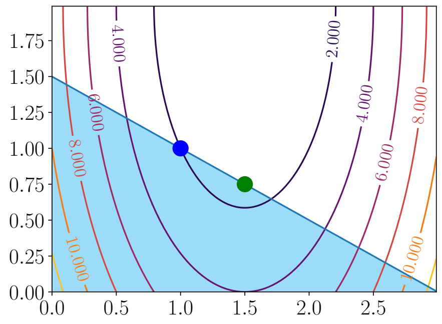
Consequence of an attack

- Attack modifies optimization problem
  - Optimum value is shifted



Original minimum.



Minimum after attack.

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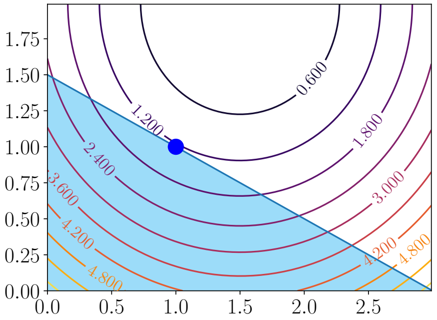

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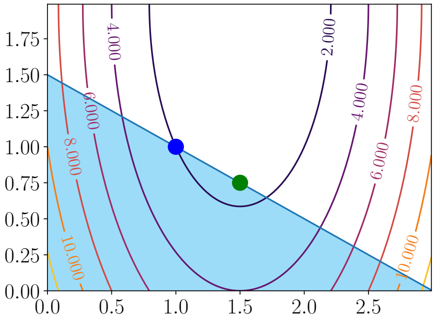

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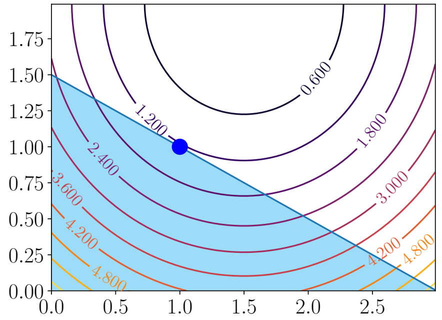

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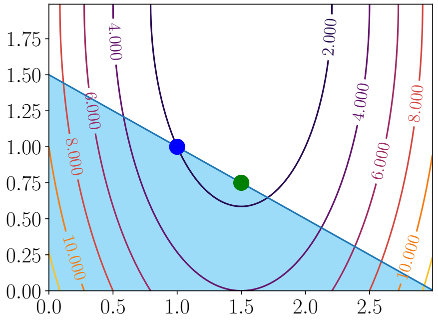

Minimum after attack.

Mitigating the effects

- We can recover by
  - Ignoring attacker
  - Recuperating original behavior (at least trying)

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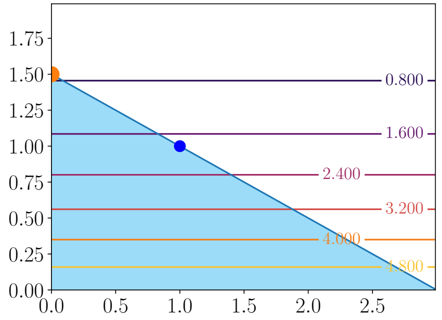

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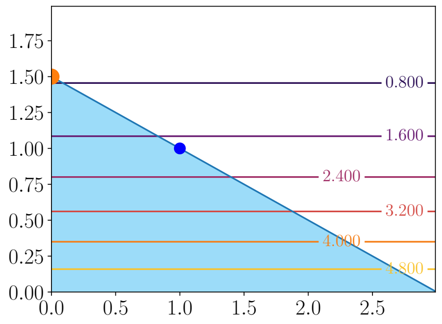

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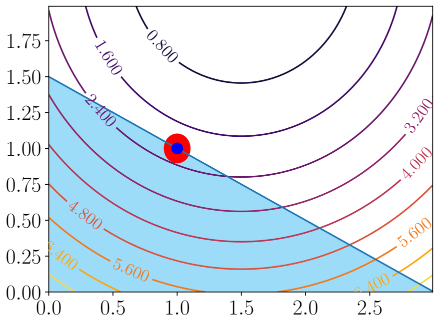

Recover original behavior.

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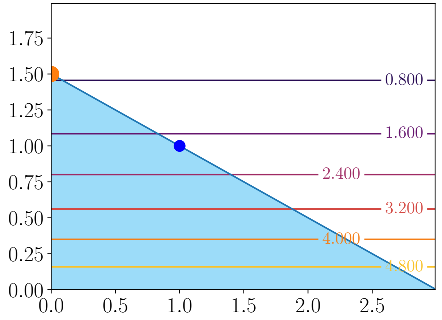

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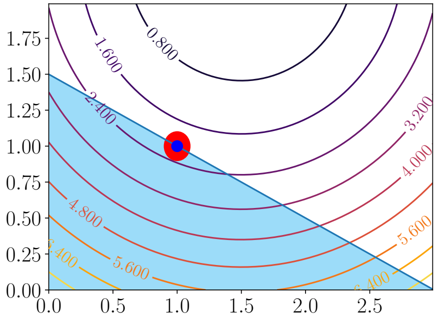

Recover original behavior.

Classification of mitigation techniques

- Passive (Robust) - 1 mode
- Active (Resilient) - 2 modes
  1. Detection/Isolation
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$\Big\{$ Attack free

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State of art

Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
[Vel+17a] [Mae+21]	Dual	Robust (Scenario)	NA	NA
[Vel+17b] [Vel+18]	Dual	Robust (f-robust)	NA	NA
[CMI18]	Jacobi-Gauß	–	–	–
[Ana+18] [Ana+19] [Ana+20]	Dual	Resilient	Analyt./Learn.	Disconnect (Robustness)
Our	Primal	Resilient	Active Analyt./Learn.	Data reconstruction

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State of art

Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
[Vel+17a] [Mae+21]	Dual	Robust (Scenario)	NA	NA
[Vel+17b] [Vel+18]	Dual	Robust (f-robust)	NA	NA
[CMI18]	Jacobi-Gauß	–	–	–
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Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition
2. Resilient Primal Decomposition-based dMPC for deprived systems
3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity

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To respond this this presentation is divided into 3 parts.First we present the decomposition and its vulnerabilites,

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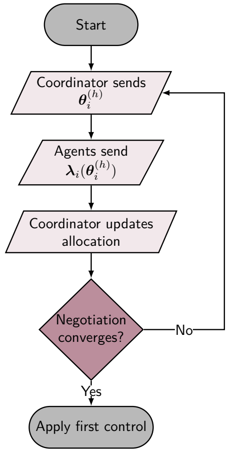
Outline

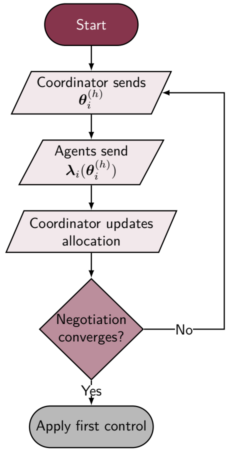
1. Vulnerabilities in distributed MPC based on Primal Decomposition
   - What is the Primal Decomposition?
   - How can an agent attack?
   - Consequences

Quantity Decomposition | Resource Allocation



In a flowchart for a quantity decomposition based DMPC,

Quantity Decomposition | Resource Allocation



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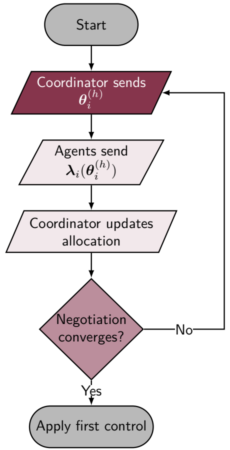
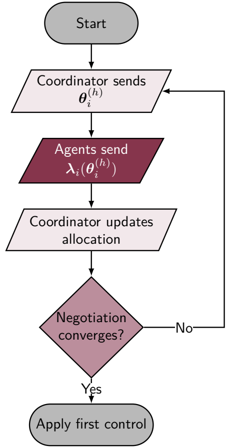


Allocation θ_i

In a flowchart for a quantity decomposition based DMPC,the coordinator sends the allocation theta,

Quantity Decomposition | Resource Allocation

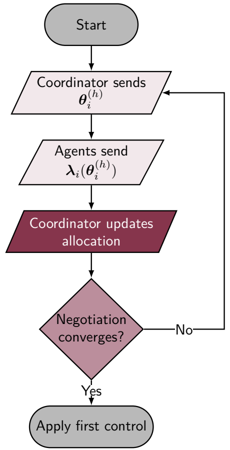


Allocation $\boldsymbol{\theta}_i$

Dissatisfaction $\boldsymbol{\lambda}_i$

In a flowchart for a quantity decomposition based DMPC,the coordinator sends the allocation theta, the agents send the dual variable lambda,

Quantity Decomposition | Resource Allocation



Allocation $\boldsymbol{\theta}_i$
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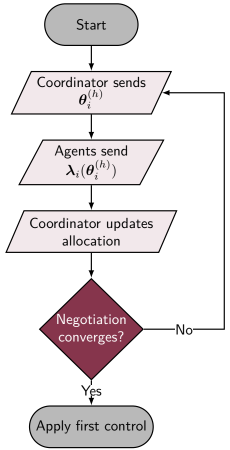
In a flowchart for a quantity decomposition based DMPC,the coordinator sends the allocation theta, the agents send the dual variable lambda, the coordinator updates the allocation.

Quantity Decomposition | Resource Allocation



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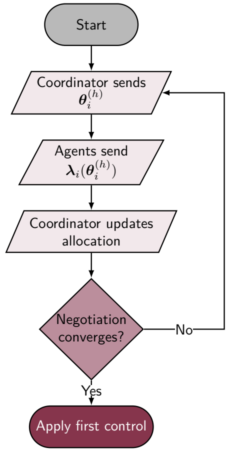
In a flowchart for a quantity decomposition based DMPC,the coordinator sends the allocation theta, the agents send the dual variable lambda, the coordinator updates the allocation. If negotiation converges,

Quantity Decomposition | Resource Allocation



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In a flowchart for a quantity decomposition based DMPC,the coordinator sends the allocation theta, the agents send the dual variable lambda, the coordinator updates the allocation. If negotiation converges, then the negotiation ends and each agent applies the first element of the control found

Primal Decomposition

or Quantity Decomposition | or Resource Allocation

- Objective is sum of local ones
- Constraints couple variables

$$\begin{aligned}\underset{\boldsymbol{u}_1,\dots,\boldsymbol{u}_M}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}} J_i(\boldsymbol{x}_i,\boldsymbol{u}_i)\\ \text{s.t.} \quad & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \preceq \boldsymbol{u}_{\mathsf{total}}\end{aligned}$$

$\downarrow$ For each $i \in \mathcal{M}$

1. Allocate $\boldsymbol{\theta}_i$ for each agent
2. They solve local problems and
3. Send dual variable $\boldsymbol{\lambda}_i$
4. Allocation is updated (respecting global constraint)

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$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

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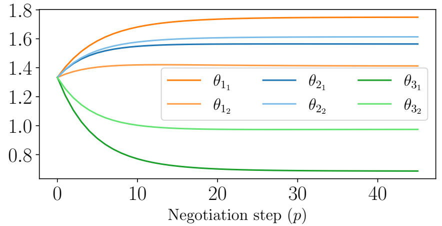
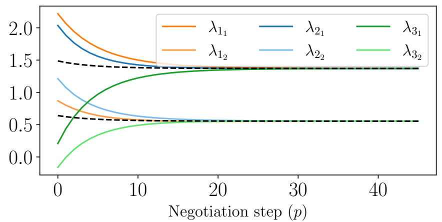
Quantity Decomposition | Resource Allocation

Until everybody is equally dissatisfied





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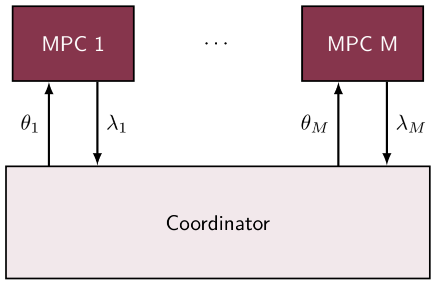
How can a non-cooperative agent attack?

Our approach



- $\boldsymbol{\lambda}_i$ is the only interface
- $\boldsymbol{\lambda}_i$ depends on local parameters
- Malicious agent modifies $\boldsymbol{\lambda}_i$

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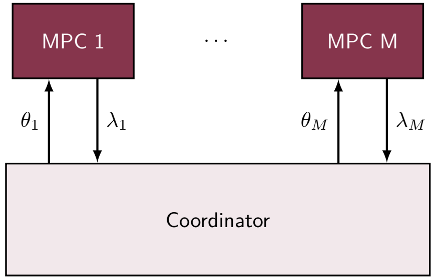

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How can an agent attack the system? Its only interface with the coordination is lambda

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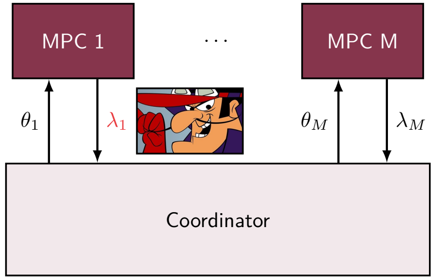

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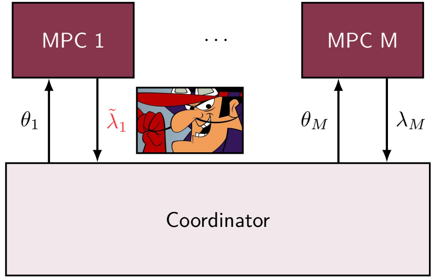

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$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$$

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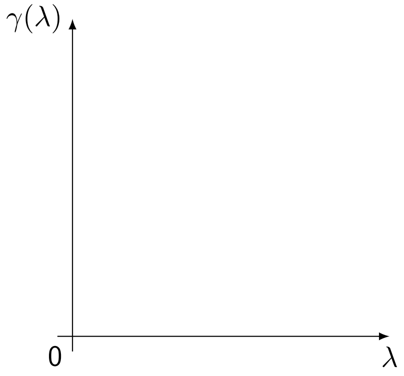
How does an agent lie?

Liar, Liar, Pants of fire



- $\lambda \geqslant 0$ means dissatisfaction
- $\lambda = 0$ means complete satisfaction

Assumptions

- *Attacker satisfied only if it really is* $\gamma(\lambda) = 0 \to \lambda = 0$
- *Attacker is greedy* $\gamma(\lambda) > \lambda$
- *Attack is monotonically increasing* $\lambda_b > \lambda_a \to \gamma(\lambda_b) > \gamma(\lambda_a)$

- Invertible
- If $\tilde{\lambda}_i = T_i[k]\lambda_i \to \exists T_i[k]^{-1}$

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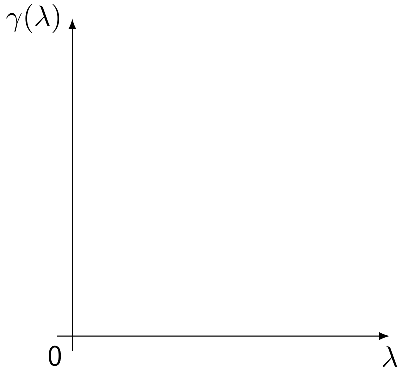

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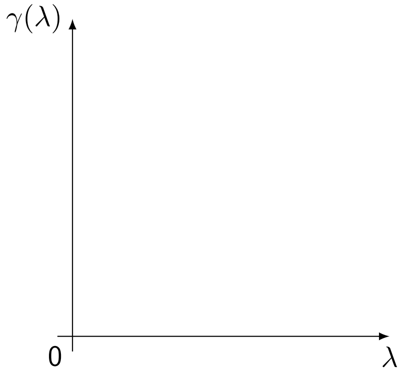

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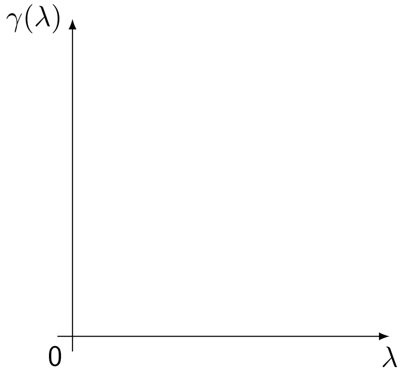

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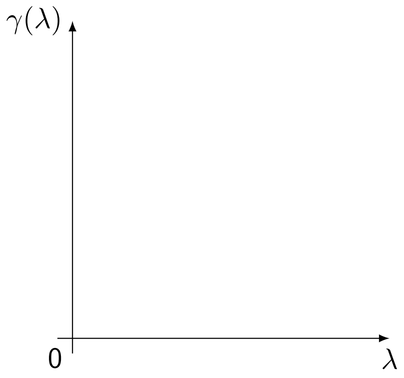
How does an agent lie?

Liar, Liar, Pants of fire



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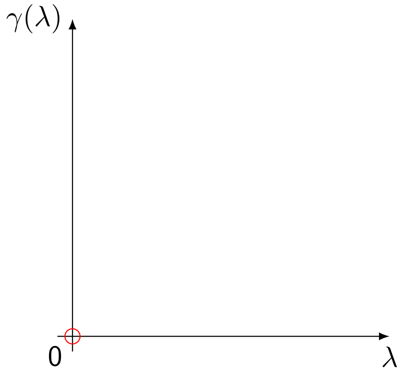

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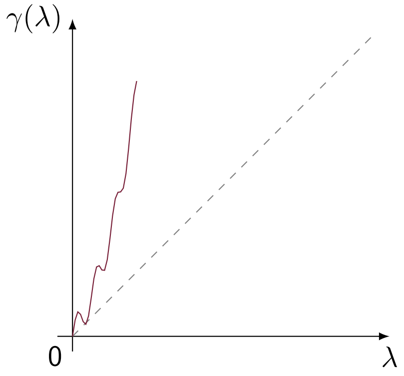

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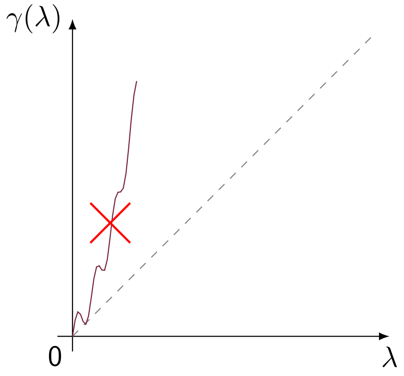

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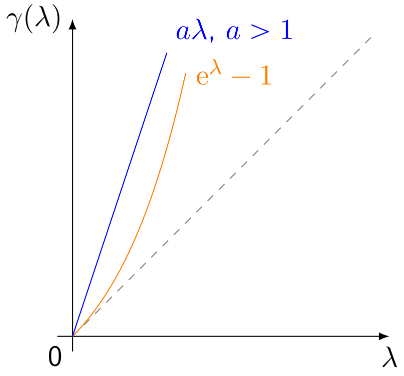

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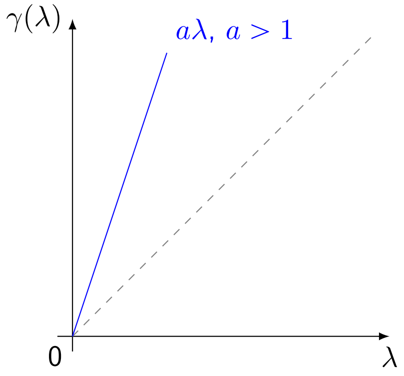

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Example

4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\lambda}_1 = \gamma_1(\lambda_1) = \tau_1 I \lambda_1$

- We can observe 3 things
  - Global minimum when $\tau_1 = 1$
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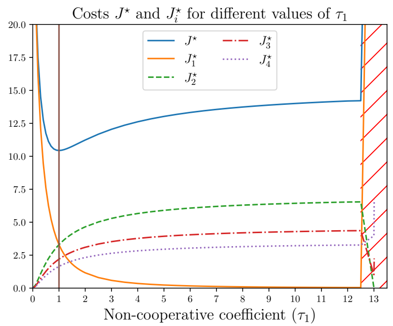

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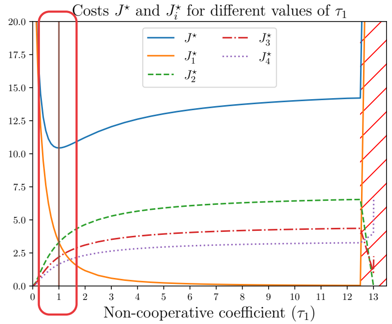

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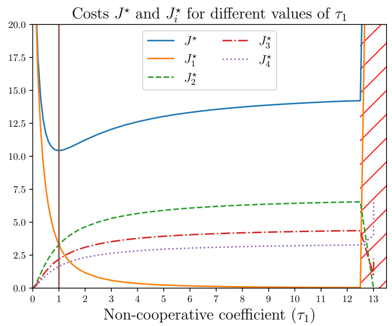

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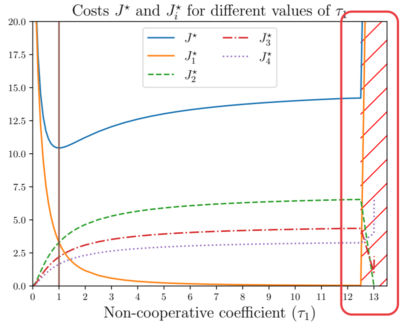

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Outline

2. Resilient Primal Decomposition-based dMPC for deprived systems
   - Analyzing deprived systems
   - Building an algorithm
   - Applying mechanism

What are deprived systems?

Systems whose optimal solution has all constraints active

- Unconstrained Solution $\mathring{\boldsymbol{U}}_i^\star[k]$
- $\bar{\Gamma}_i \mathring{\boldsymbol{U}}_i^\star[k] \geq \boldsymbol{\theta}_i[k] \rightarrow$ Scarcity
  - Solution projected onto boundary
  - Same as with equality constraints[2]

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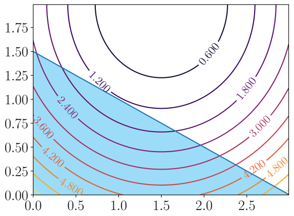

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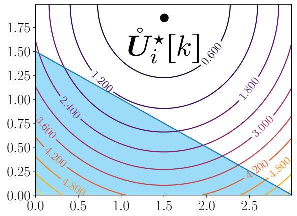

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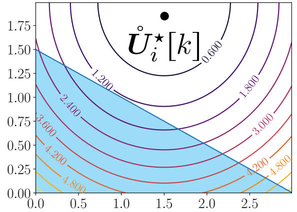

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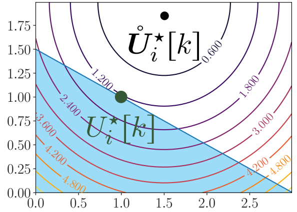

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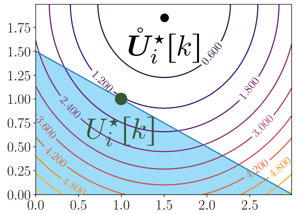

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Deprived Systems

But why?

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(local parameters unknown by coordinator) $\left\{ \begin{array}{l} \bullet\ P_i \text{ is time invariant} \\ \bullet\ \boldsymbol{s}_i[k] \text{ is time variant} \end{array} \right.$

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Deprived Systems

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  - Affine solution

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Detection Mechanism

Assumption

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- Ok, but how can we estimate $\hat{\bar{P}}_i[k]$?

[1] Using Recursive Least Squares for example

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For the attack detection, let's assume we know the nominal P, called P bar

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For the attack detection, let's assume we know the nominal P, called P bar
And we also assume the attacker chooses a linear attack where the lambda sent is equal to a matrix T times the original lambda which yields the formula shown

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Estimating $\widehat{\widetilde{P}}_i[k]$

- We need to estimate $\widehat{\widetilde{P}}_i[k]$ and $\widehat{s}_i[k]$ simultaneously
- Challenge: Estimation during negotiation fails
  - Update function couples θ_i^p and $\lambda_i^p \rightarrow$ low input excitation
- Solution: Send a random[3] sequence to increase excitation.

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Classification of mitigation techniques

- Active (Resilient)
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Mitigation mechanism

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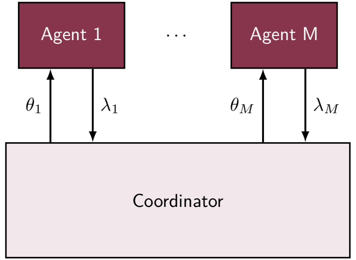
Complete Mechanism



- Supervise exchanges by inquiring the agents
- Estimate how they will behave

Two Phases

1. Detect which agents are non-cooperative
2. Reconstruct λ_i and use in negotiation

CentraleSupélec

The complete mechanism is equivalent to add a supervisor for each agent inside the coordinator

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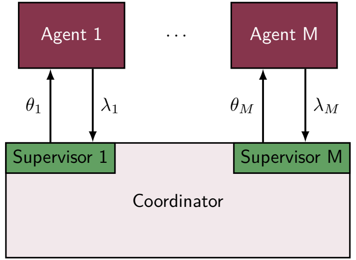

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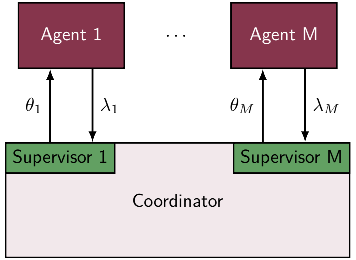

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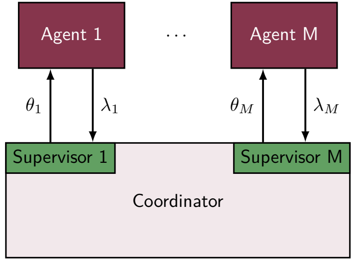

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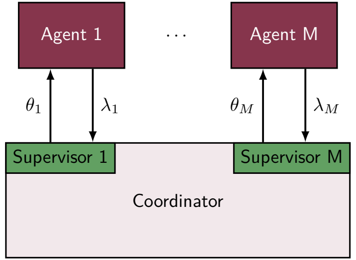

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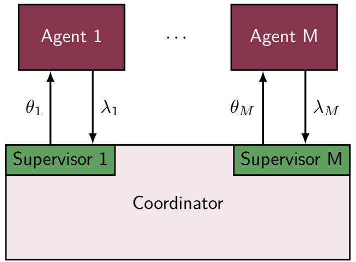

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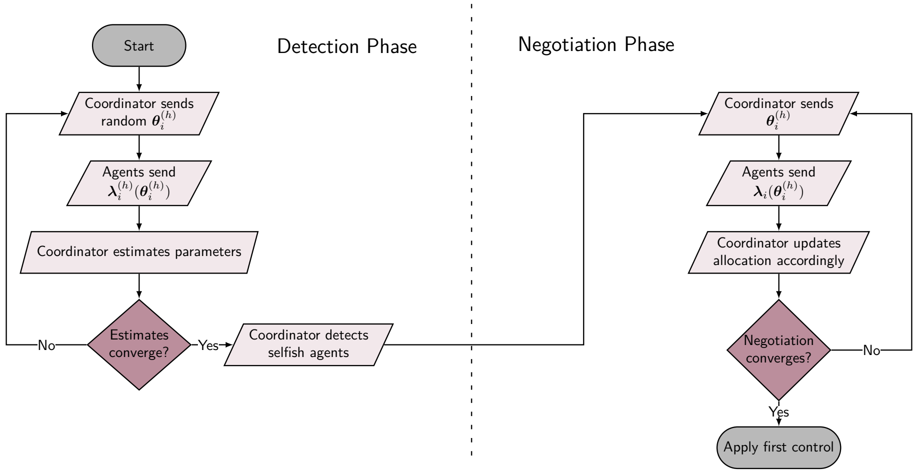
Complete algorithm

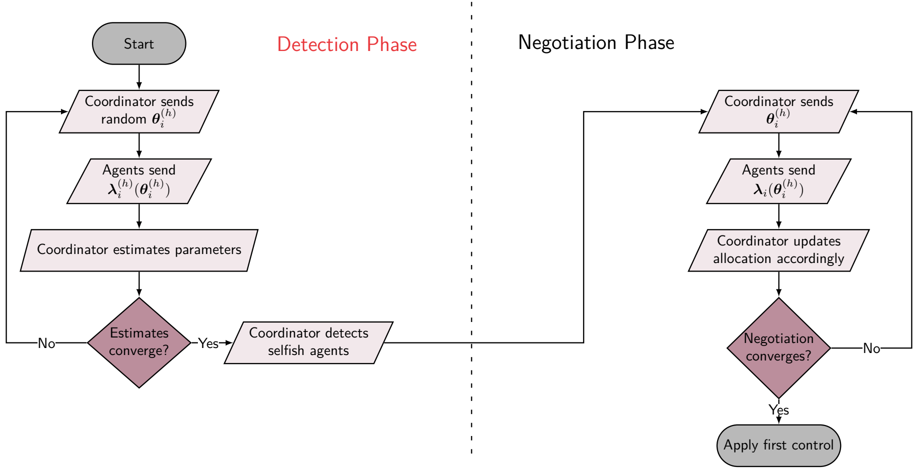
RPdMPC-DS



Now, for the complete secure DMPC algorithm, as said it is divided into two phases

Complete algorithm

RPdMPC-DS

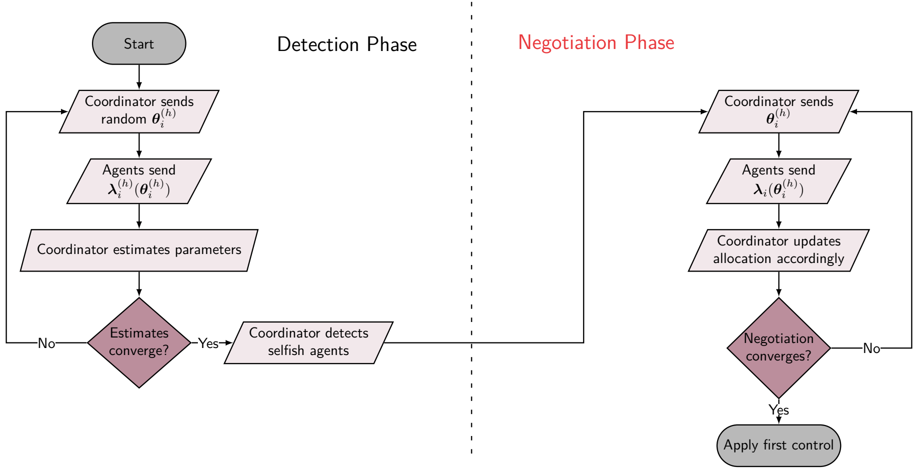


CentraleSupélec

Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase

Complete algorithm

RPdMPC-DS



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The Detection phase
and the negotiation phase

Complete algorithm

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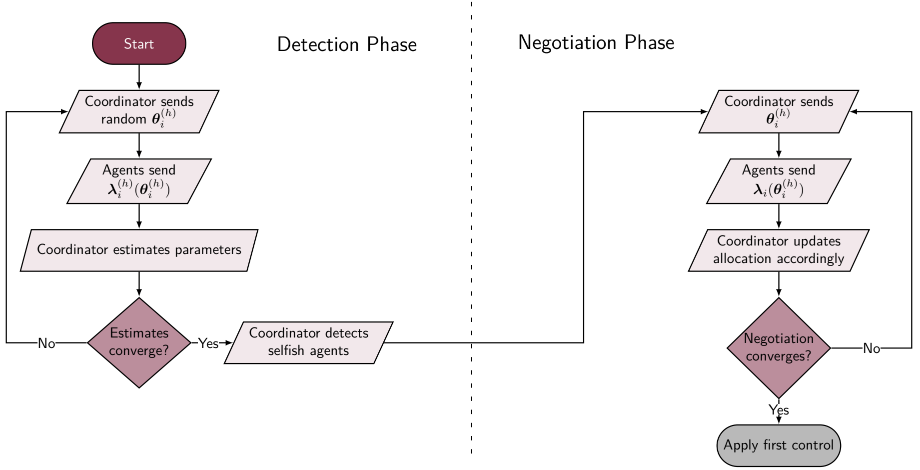

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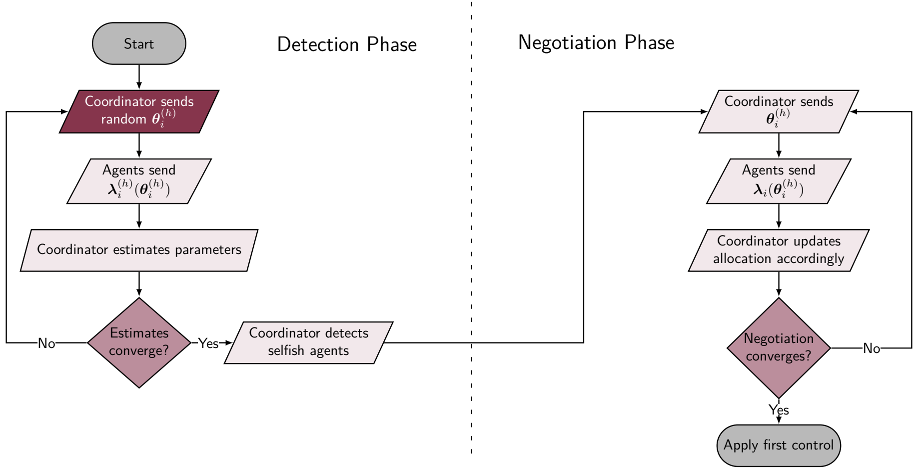
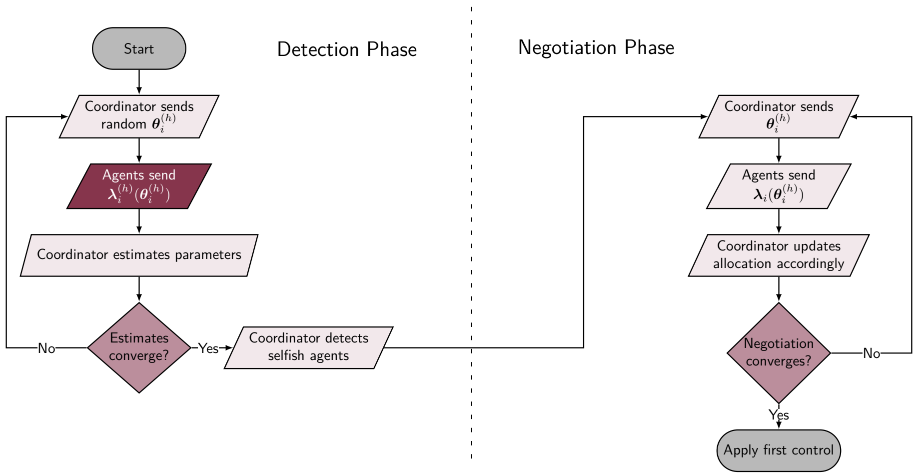


Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
and the negotiation phase
The coordinator sends random theta i

Complete algorithm

RPdMPC-DS



Now, for the complete secure DMPC algorithm, as said it is divided into two phases
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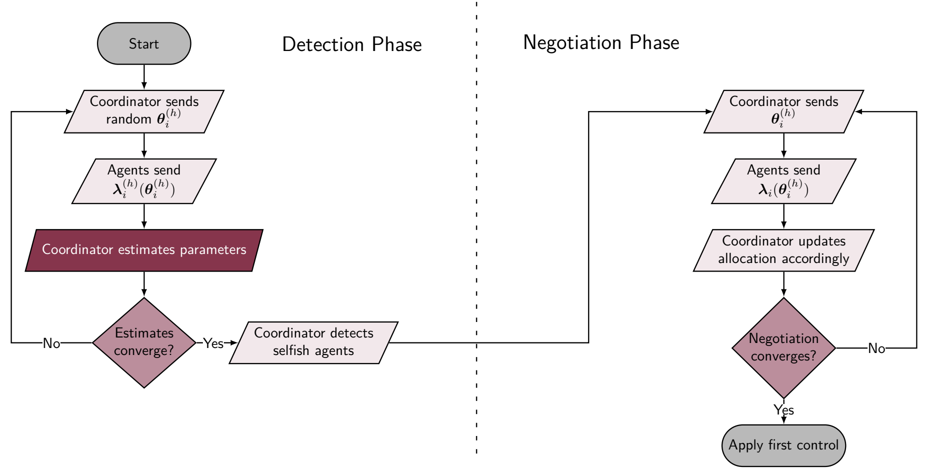
Complete algorithm

RPdMPC-DS



Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
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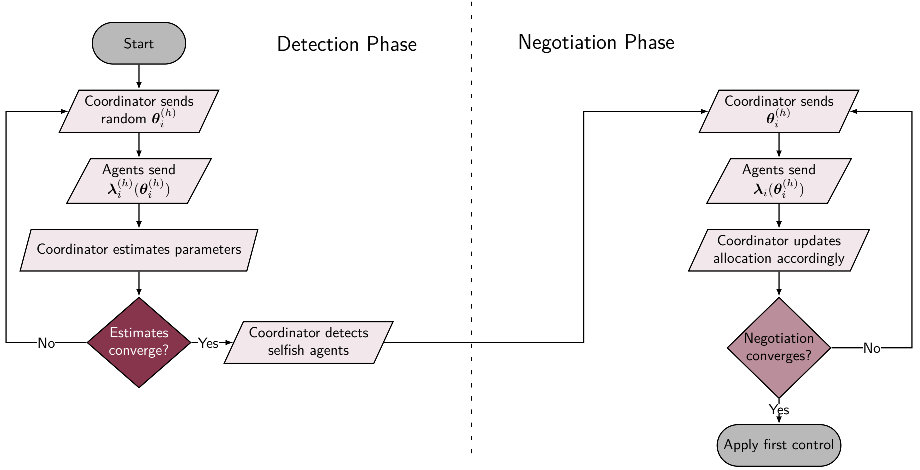
The coordinator sends random theta i
The agents send dual variable lambda i
The coordinator estimates the parameters P and s tilde

Complete algorithm

RPdMPC-DS



CentraleSupélec

Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
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The coordinator sends random theta i
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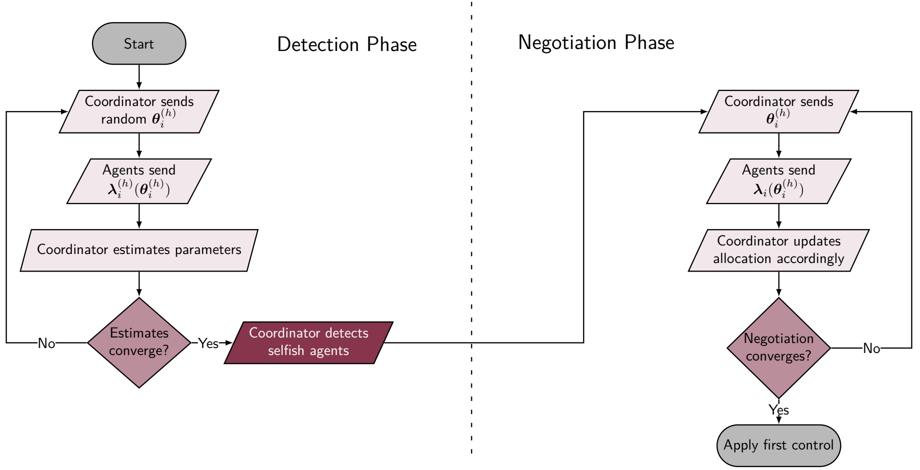
The coordinator estimates the parameters P and s tilde
when the estimates converge

Complete algorithm

RPdMPC-DS



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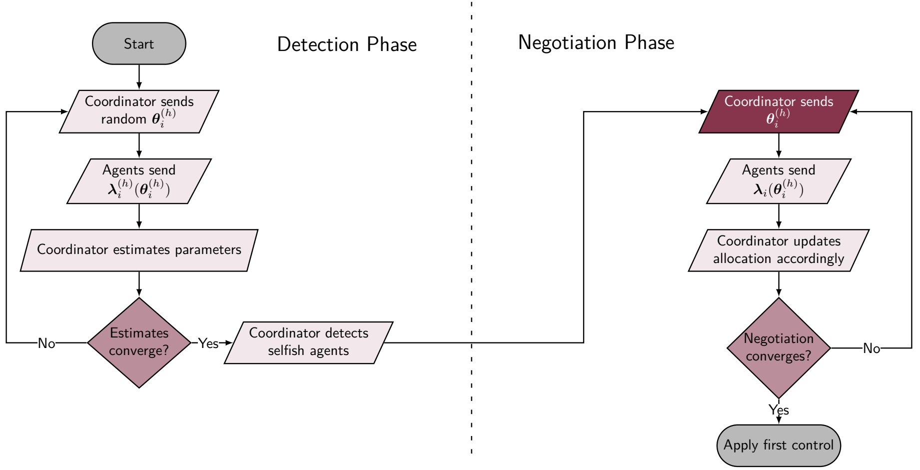
The coordinator detects which agents are non-cooperative

Complete algorithm

RPdMPC-DS



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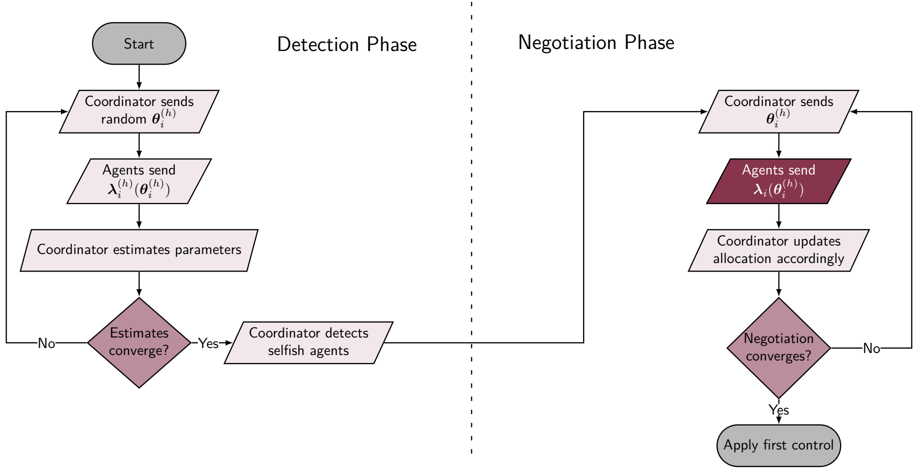
then the negotiation phase begins, the coordinator sends the theta i

Complete algorithm

RPdMPC-DS



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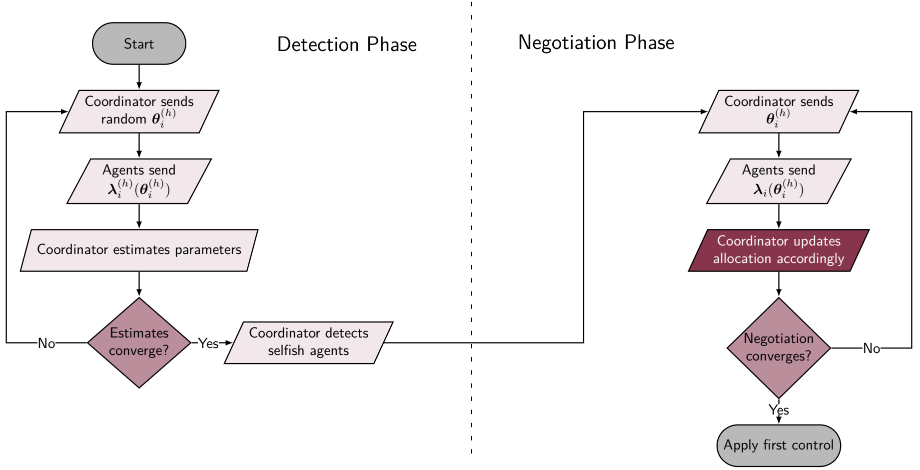

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and the coordinator updates the allocation accordingly using the reconstructed lambda or the one sent by the agent

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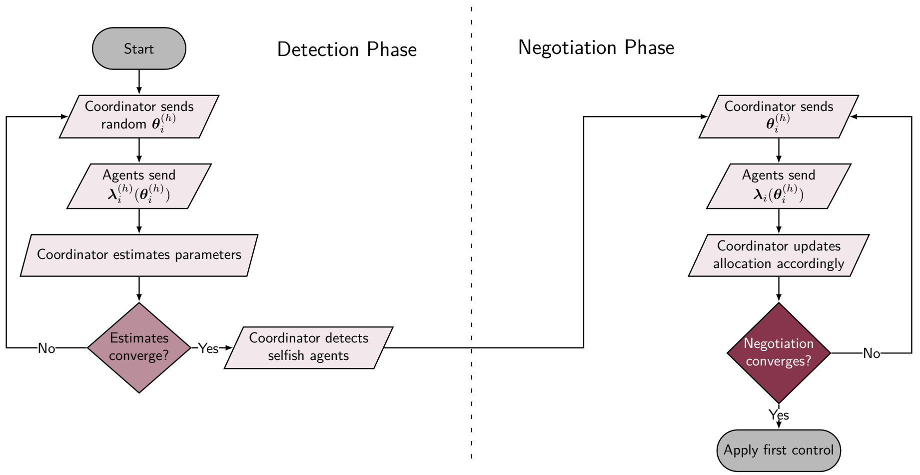

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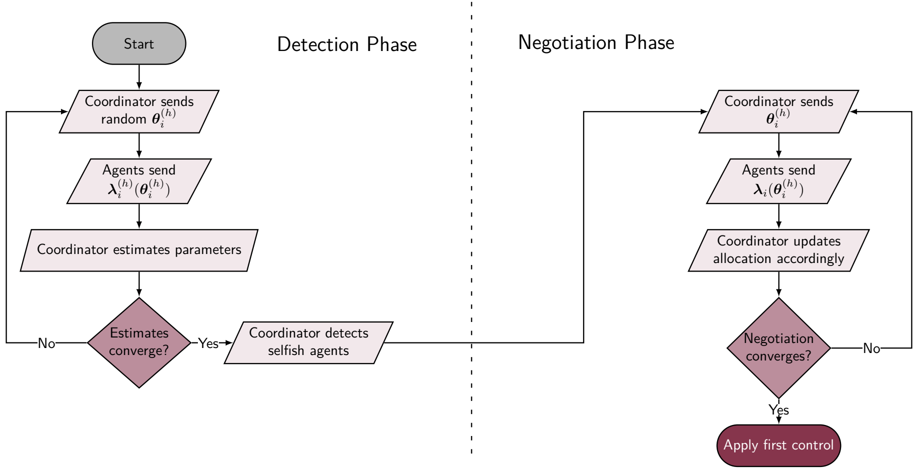

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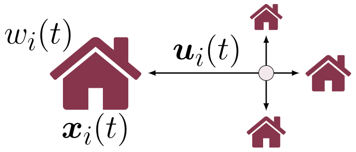
the agents send the dual variable lambda i
and the coordinator updates the allocation accordingly using the reconstructed lambda or the one sent by the agent
and once the negotiation converges
each agent applies the first control

Example

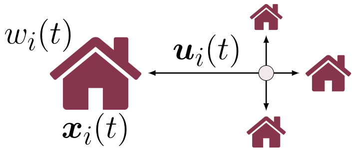


District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)
- Not enough power
- Period of $5\mathrm{h}$
- 3 scenarios
  - N Nominal
  - C Agent I cheats (dMPC)
  - S Agent I cheats (RPdMPC-DS)

We give an academic example of the temperature control of 4 distinct rooms under power scarcity

Example



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CentraleSupélec

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The 4 rooms are distinct using the 3 resistor 2 capacitor model

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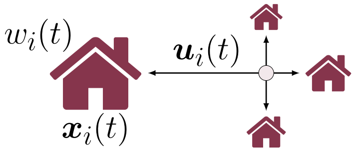

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We give an academic example of the temperature control of 4 distinct rooms under power scarcity
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the initial temperature of all rooms is under 20 degrees celsius, which is the final setpoint

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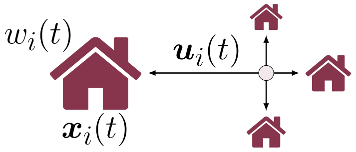

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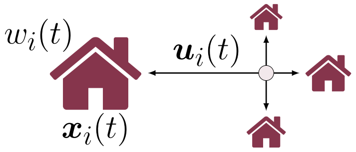
CentraleSupélec

We give an academic example of the temperature control of 4 distinct rooms under power scarcity
The 4 rooms are distinct using the 3 resistor 2 capacitor model
the initial temperature of all rooms is under 20 degrees celsius, which is the final setpoint
But they are under power scarcity that prevents them from reaching the setpoint

Example



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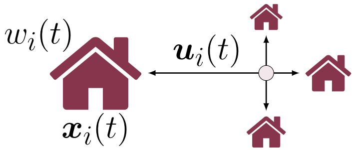

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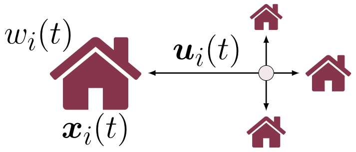
we simulate for a period of 5h
with sampling time of 15 minutes

Example



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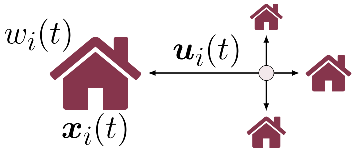
3 scenarios are simulated, the nominal, one where agent 1 presents non cooperative behavior from k greater than 6, and another with the selfish behavior and the secure mechanism activated

Example



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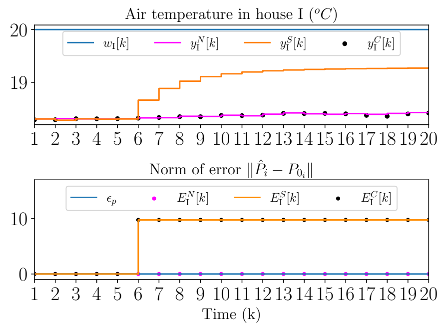
Results

Temporal



Temperature in house I.
Error $E_I(k)$.
N Nominal, S Selfish, C Corrected

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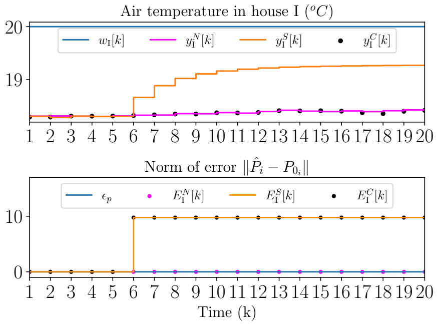
In the figure we see the air temperature and the estimation error for house 1

Results

Temporal



Temperature in house I.
Error $E_I(k)$.
Ⓝ Nominal, Ⓢ Selfish, Ⓒ Corrected

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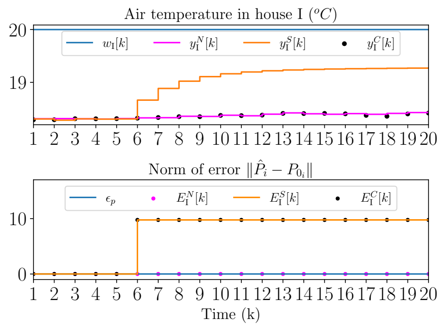
The nominal behavior is in magenta, and as said it cannot reach the setpoint, in blue

Results

Temporal



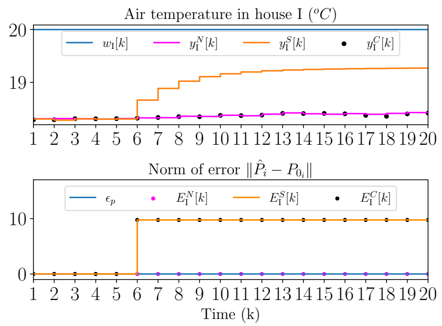
Temperature in house I.
Error $E_I(k)$.
N Nominal, S Selfish, C Corrected

- S Temperature close to reference
- C Temperature close to N

When the room presents selfish behavior (in orange), it reduces its cost and get closer to the setpoint, we see that the attack increases the error

Results

Temporal



Temperature in house I.
Error $E_I(k)$.
Ⓝ Nominal, Ⓢ Selfish, Ⓒ Corrected

- Ⓢ Temperature close to reference
- Ⓒ Temperature close to Ⓝ

Now, for the case with correction (the black dots), even if it attacks the system, the temperature is close to its nominal value

Results

Costs

Objective functions J_i (% error)

Agent	Scenario N	Scenario S	Scenario C
I	299.5	$190.8\ (-36.3)$	$301.0\ (0.0)$
II	192.4	$234.1\ (21.7)$	$191.4\ (-0.5)$
III	305.9	$359.1\ (17.4)$	$305.9\ (-0.0)$
IV	297.5	$349.9\ (17.6)$	$297.2\ (-0.1)$
Global	1095.3	$1133.9\ (3.5)$	$1095.5\ (0.0)$

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Now, if we compare the costs for each scenario we see how the cost of agent 1 decreases when it attacks, while the cost of other agents increase.

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Costs

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Now, if we compare the costs for each scenario we see how the cost of agent 1 decreases when it attacks, while the cost of other agents increase.
When the secure algorithm is activated the costs are very close to the original ones. So

Outline

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Relaxing scarcity assumption

- Let's relax the scarcity assumption
- And add some local constraints
- Similarly we have the local problems and update

$$
\begin{aligned}
\underset{\boldsymbol{U}[k]}{\text{minimize}} \quad & \tfrac{1}{2}\|\boldsymbol{U}[k]\|_H^2 + \boldsymbol{f}[k]^T \boldsymbol{U}[k] \\
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\end{aligned}
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\end{aligned}
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\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})
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Analyzing System

Solution for $\lambda_i[k]$

Instead of having

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

Now we have

$$\boldsymbol{\lambda}_i[k] = \begin{cases} -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^n_{\boldsymbol{\lambda}_i} \\ \vdots & \vdots \\ -P_i^{(2^{n_{\text{ineq}}}-1)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(2^{n_{\text{ineq}}}-1)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^{2^{n_{\text{ineq}}}-1}_{\boldsymbol{\lambda}_i} \end{cases}$$

Increasingly Sparse

For n_{ineq} constraints $\rightarrow 2^{n_{\text{ineq}}}$ permutations

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Analyzing System

Solution for $\lambda_i[k]$

Instead of having

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

Now we have

$$\boldsymbol{\lambda}_i[k] = \begin{cases} -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^n_{\boldsymbol{\lambda}_i} \\ \quad \vdots & \quad \vdots \\ -P_i^{(2^{n_{\text{ineq}}}-1)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(2^{n_{\text{ineq}}}-1)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^{2^{n_{\text{ineq}}}-1}_{\boldsymbol{\lambda}_i} \end{cases}$$

Increasingly Sparse

For n_{ineq} constraints $\longrightarrow$ $2^{n_{\text{ineq}}}$ permutations

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For n_{ineq} constraints $\rightarrow 2^{n_{\text{ineq}}}$ permutations

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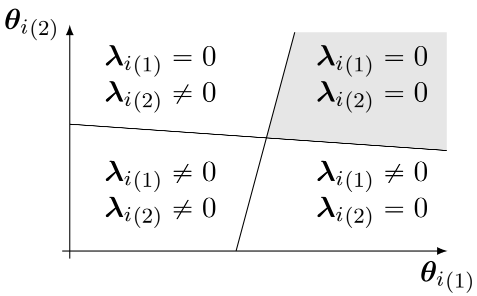
Analyzing System

Solution for $\lambda_i[k]$ (Continued)



Two constraints partitioning $\boldsymbol{\theta}_i$ solution space.

Analyzing System

Negotiation

$$\mathrm{Proj}^{\mathbb{S}}(\boldsymbol{\theta}[k]^{(p)}+\rho^{(p)}\boldsymbol{\lambda}[k]^{(p)}) = \underset{\boldsymbol{x}}{\mathrm{argmin}} \left\{ \begin{array}{ll} \underset{\boldsymbol{U}[k]}{\text{minimize}} & \left\| \boldsymbol{x} - \boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)} \right\| \\ \text{subject to} & I_c^M \boldsymbol{x} \leq \boldsymbol{U}_{\max} : \boldsymbol{\mu} \end{array} \right\}$$

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Analyzing System

Negotiation (Continued)

$$\boldsymbol{\theta}[k]^{(p+1)} = \begin{cases} \boldsymbol{x}_0 + I_c^{M\,(0)} \left[-P_{\boldsymbol{\mu}}^{(0)} \boldsymbol{U}_{max} + \boldsymbol{s}_{\boldsymbol{\mu}}^{(0)}[k] \right], & \text{if } \boldsymbol{x}_0 \in \mathcal{R}_{\boldsymbol{\mu}}^{0} \\ \quad\vdots & \quad\vdots \\ \boldsymbol{x}_0, & \text{if } \boldsymbol{x}_0 \in \mathcal{R}_{\boldsymbol{\mu}}^{2^c-1} \end{cases}$$

$$\text{where } \boldsymbol{x}_0 = \boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)}$$

Analyzing System

Conclusion

$$\bar{\epsilon} = \overbrace{2^{c}}^{\text{regions in projection}} \times \underbrace{2^{n_{\text{ineq}}} \times \cdots \times 2^{n_{\text{ineq}}}}_{M\times\text{regions in each } \boldsymbol{\lambda}_i} = 2^{c+Mn_{\text{ineq}}}$$

Mitigation

- Ideal

$$\widehat{T_i[k]^{-1}} = \bar{P}_i \widehat{\widehat{P}_i[k]}^{-1}$$

- $P_i^{(0)}$ only invertible

$$\widehat{T_i[k]^{-1}} = \bar{P}_i^{(0)} \widehat{\widehat{P}_i^{(0)}[k]}^{-1}$$

  - But how to force scarcity? Artificial Scarcity

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Artificial Scarcity

Who is it? Who is it?

- Since we control θ_i let's choose one where all constraints are active

Artificial Scarcity

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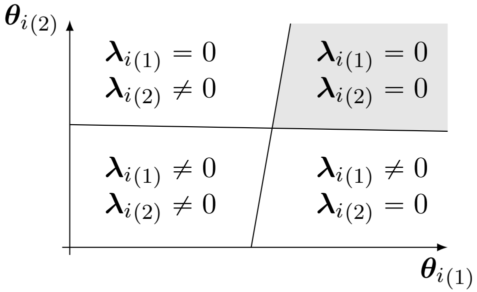

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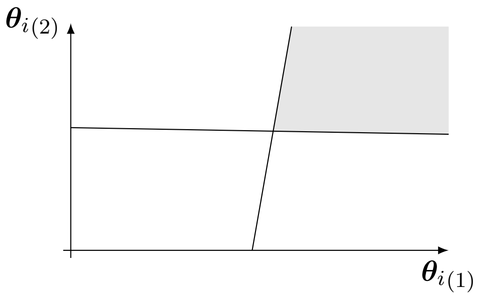

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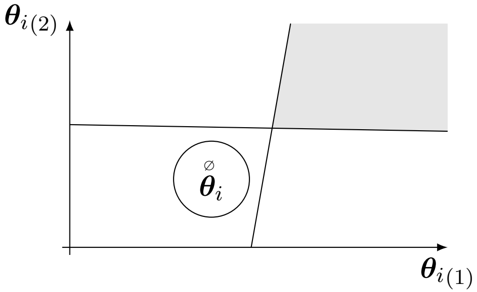

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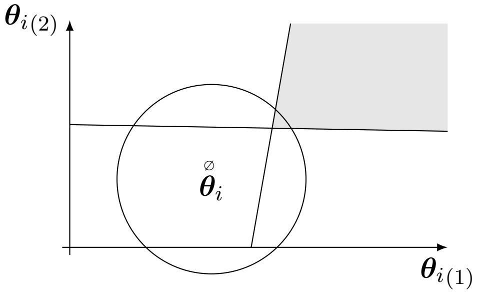

Expectation Maximization

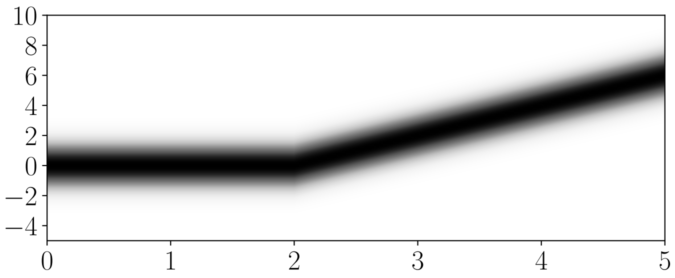

Figure 1: Gaussian Mixture for a 1D PWA function with 2 modes.

Expectation Maximization

Algorithm

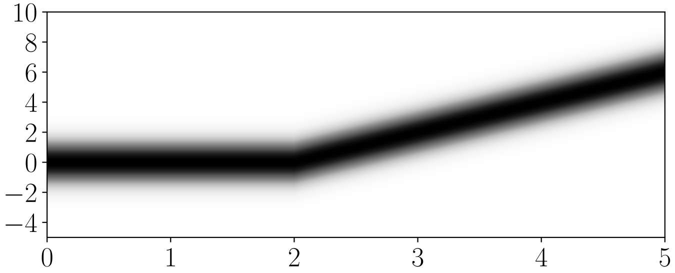

Figure 2: Gaussian Mixture for a 1D PWA function with 2 modes.

Detection

Same same

- Error $E_i^{(0)}[k] = \left\| \widehat{\bar{P}}_i^{(0)}[k] - \bar{P}_i^{(0)} \right\|_F$
- Create threshold $\epsilon_{P_i^{(0)}}$
- Indicator $\eth_i \in \{0, 1\}$ detects the attack in agent i.
- $\eth_i^{(0)} = \mathbb{1}_{\{E_i^{(0)}[k] \geq \epsilon_{P_i^{(0)}}\}}$

So, let's detail the detection mechanism.

Detection

Same same

- Error $E_i^{(0)}[k] = \left\| \widehat{\widetilde{P}}_i^{(0)}[k] - \bar{P}_i^{(0)} \right\|_F$
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First we calculate the norm of the error between the estimated P and the nominal. (Here we use the Frobenius norm)

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So, let's detail the detection mechanism.
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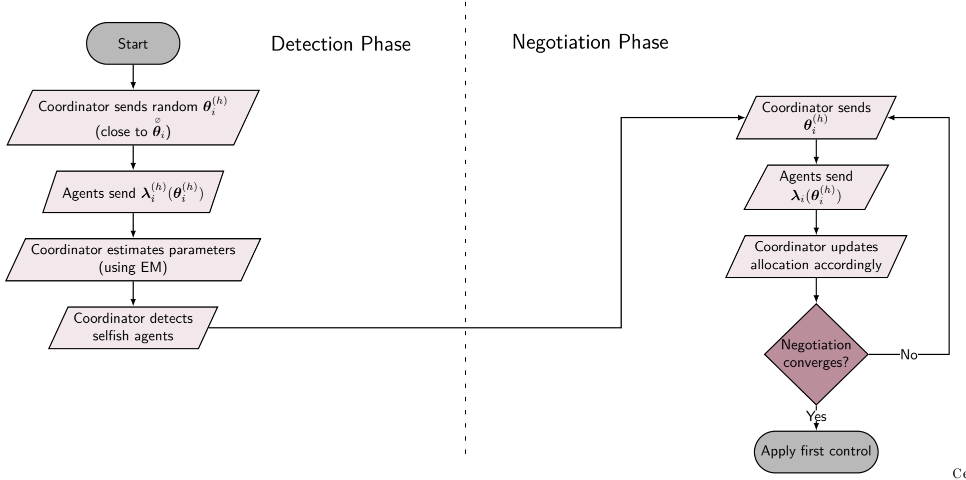
Then, we create a threshold epsilon p
and finally we create an indicator d i for the attack of agent i
It is equal to 1 if the error is greater than the threshold, indicating an attack or 0 otherwise

Complete algorithm

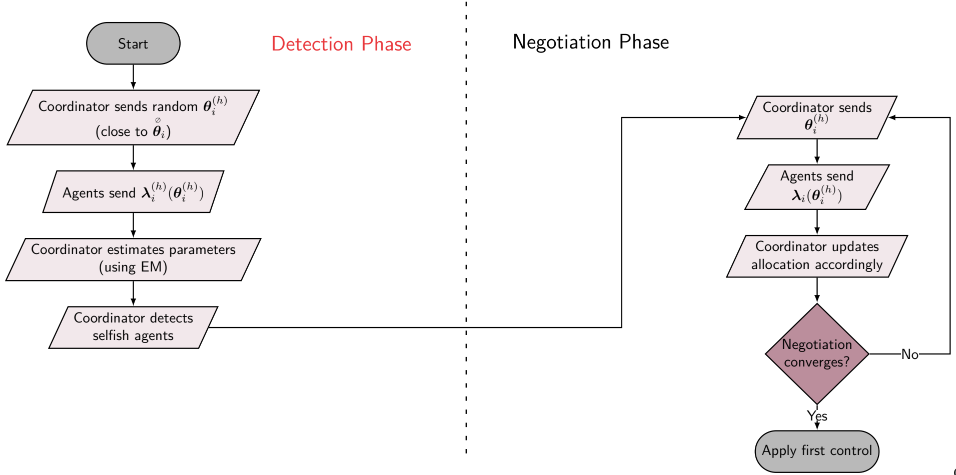
RPdMPC-AS Refaire



Now, for the complete secure DMPC algorithm, as said it is divided into two phases

Complete algorithm

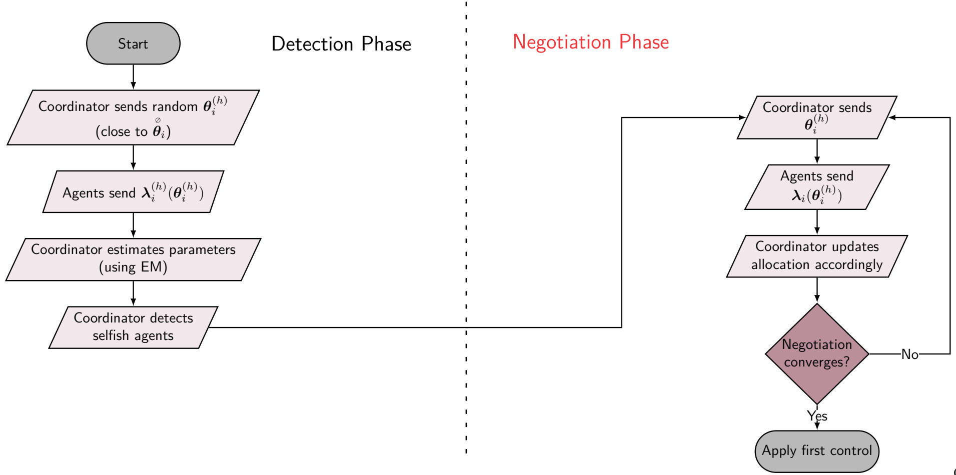
RPdMPC-AS Refaire



Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase

Complete algorithm

RPdMPC-AS Refaire



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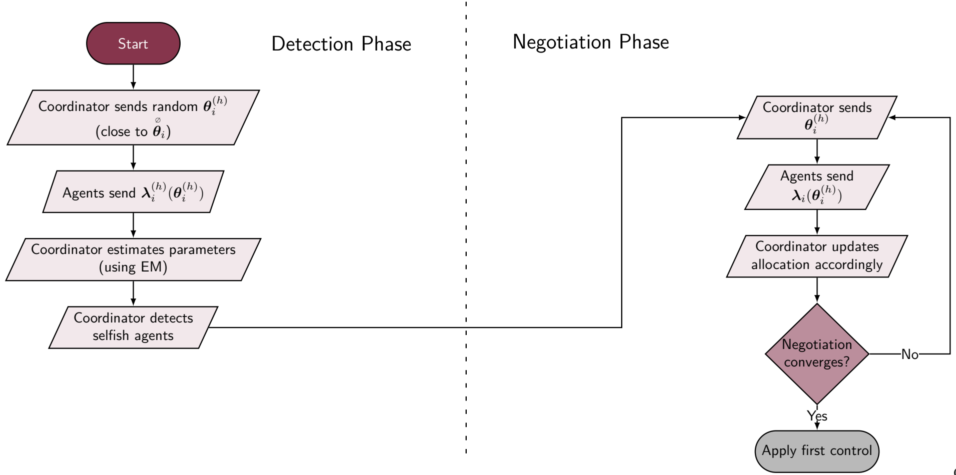
Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
and the negotiation phase

Complete algorithm

RPdMPC-AS Refaire



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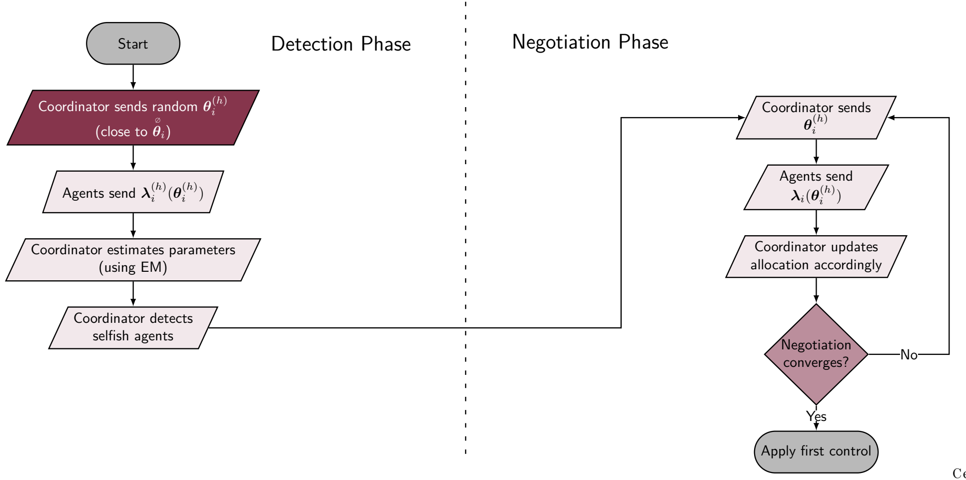
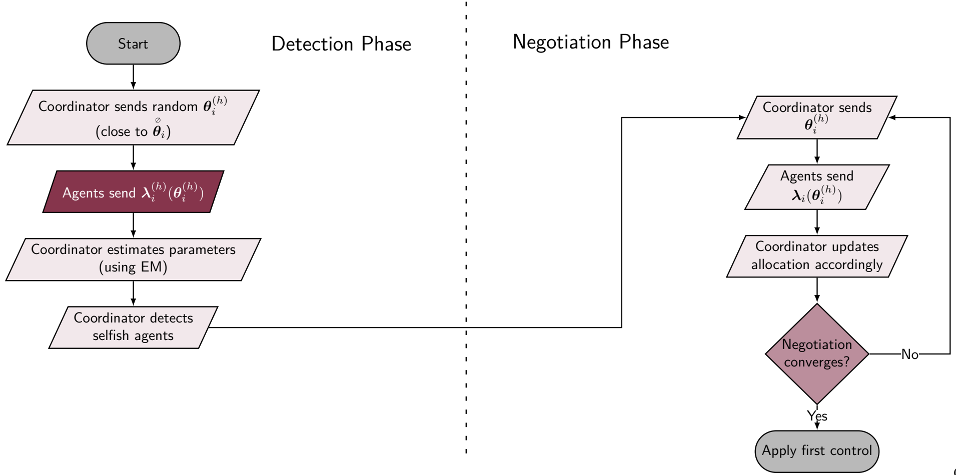


Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
and the negotiation phase
The coordinator sends random theta i

Complete algorithm

RPdMPC-AS Refaire



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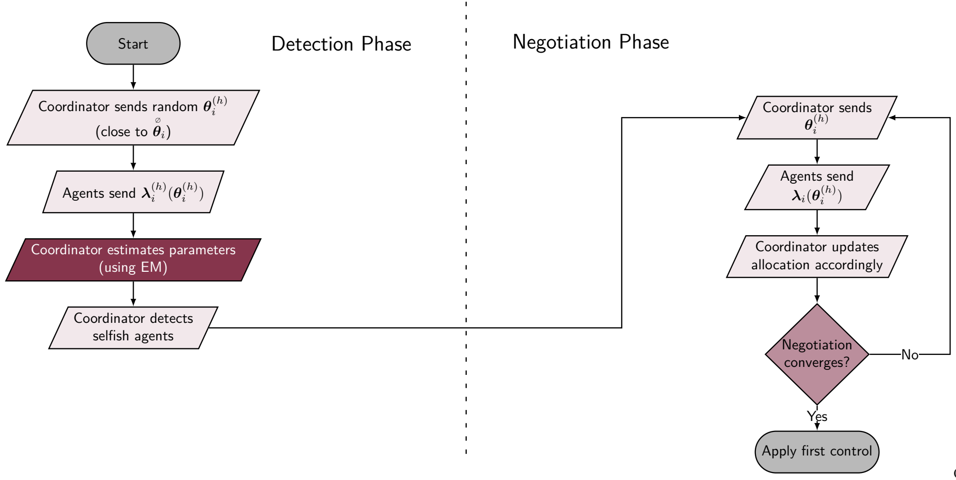
Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
and the negotiation phase
The coordinator sends random theta i
The agents send dual variable lambda i

Complete algorithm

RPdMPC-AS Refaire



Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
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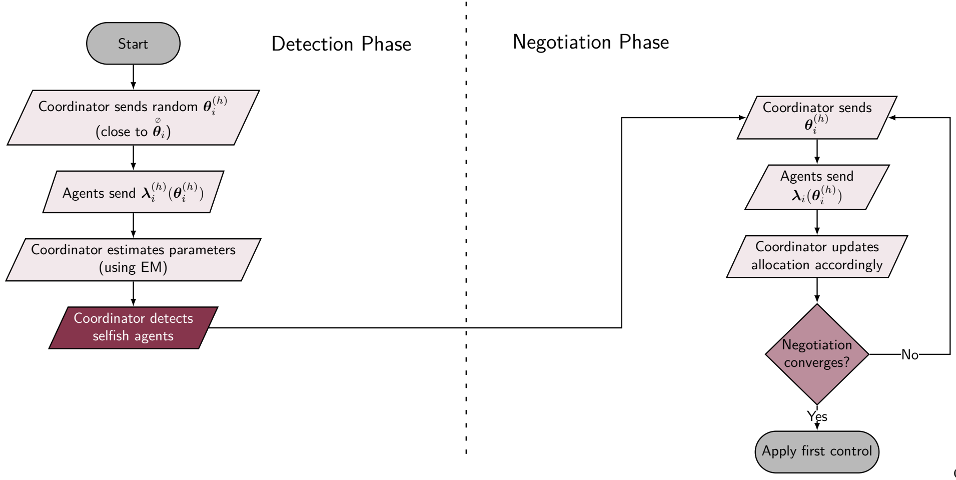
The coordinator sends random theta i
The agents send dual variable lambda i
The coordinator estimates the parameters P and s tilde

Complete algorithm

RPdMPC-AS Refaire



Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
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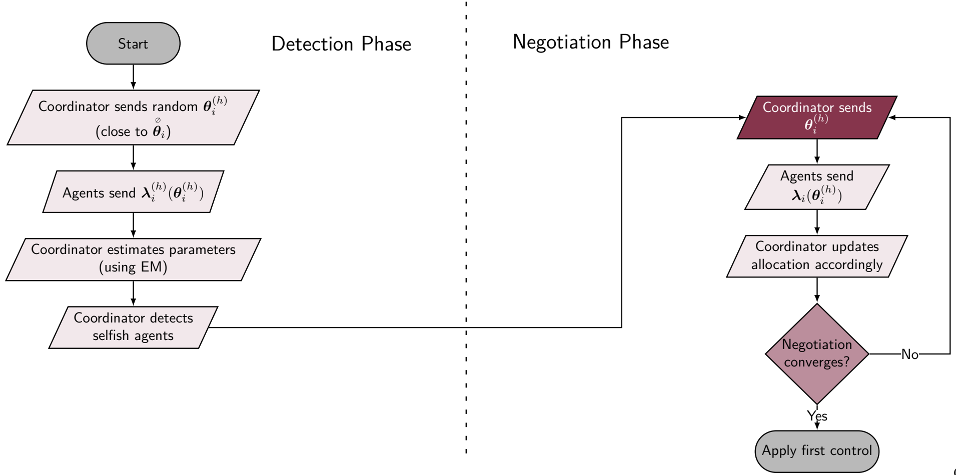
The coordinator sends random theta i
The agents send dual variable lambda i
The coordinator estimates the parameters P and s tilde
when the estimates converge

Complete algorithm

RPdMPC-AS Refaire



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The coordinator sends random theta i
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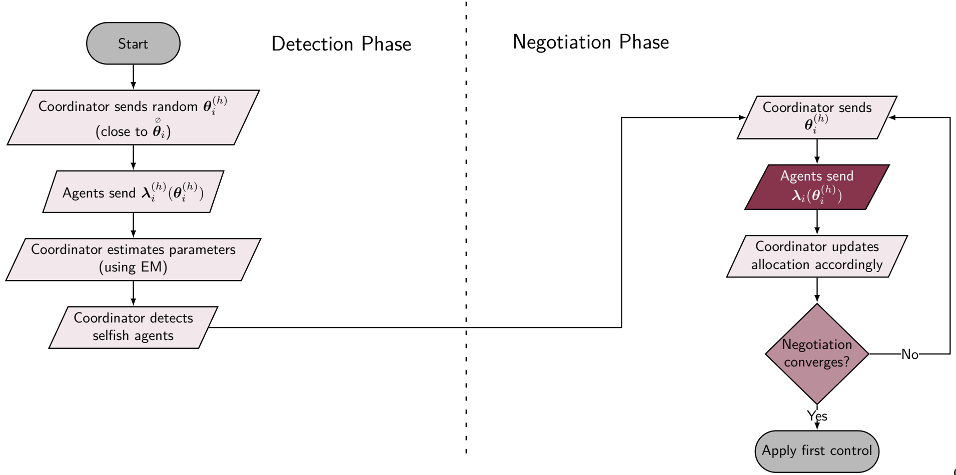
when the estimates converge
The coordinator detects which agents are non-cooperative

Complete algorithm

RPdMPC-AS Refaire



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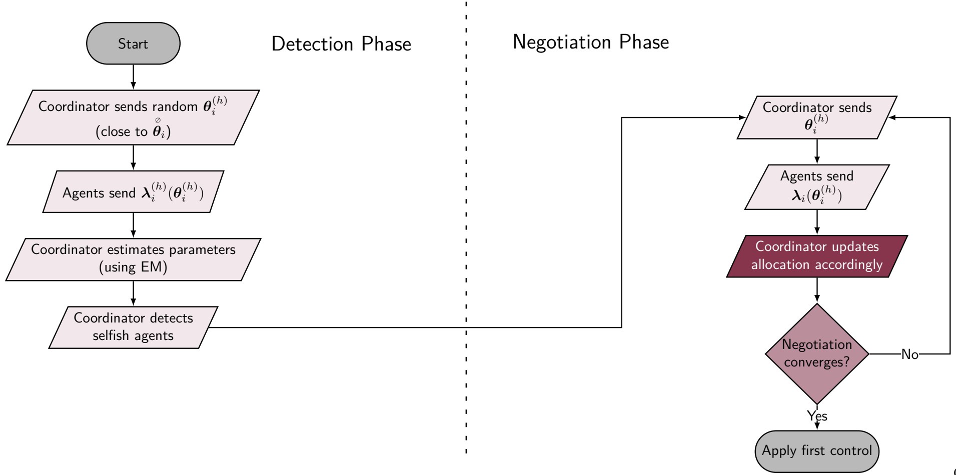
when the estimates converge
The coordinator detects which agents are non-cooperative
then the negotiation phase begins, the coordinator sends the theta i

Complete algorithm

RPdMPC-AS Refaire



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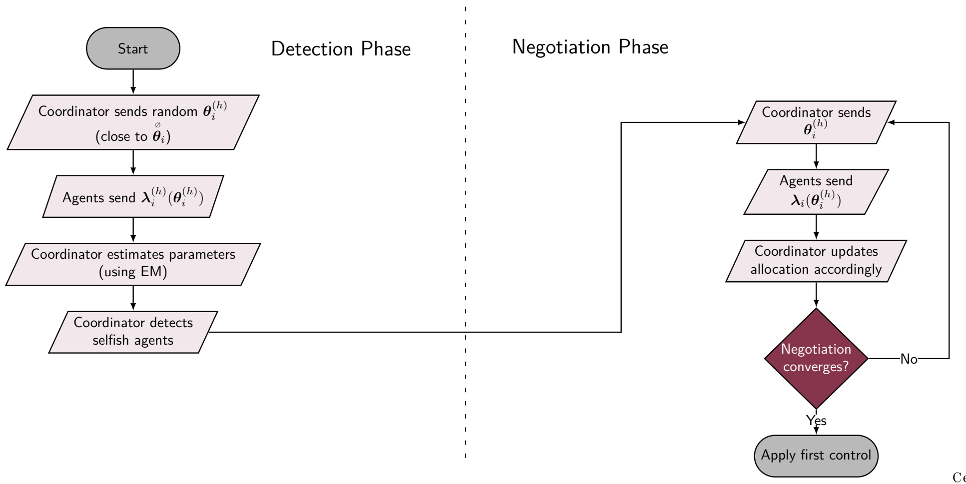

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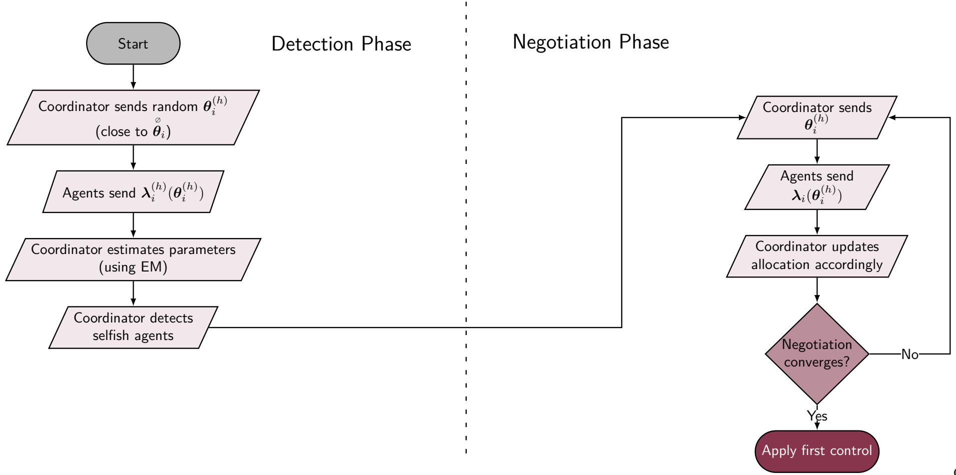
then the negotiation phase begins, the coordinator sends the theta i
the agents send the dual variable lambda i
and the coordinator updates the allocation accordingly using the reconstructed lambda or the one sent by the agent

Complete algorithm

RPdMPC-AS Refaire



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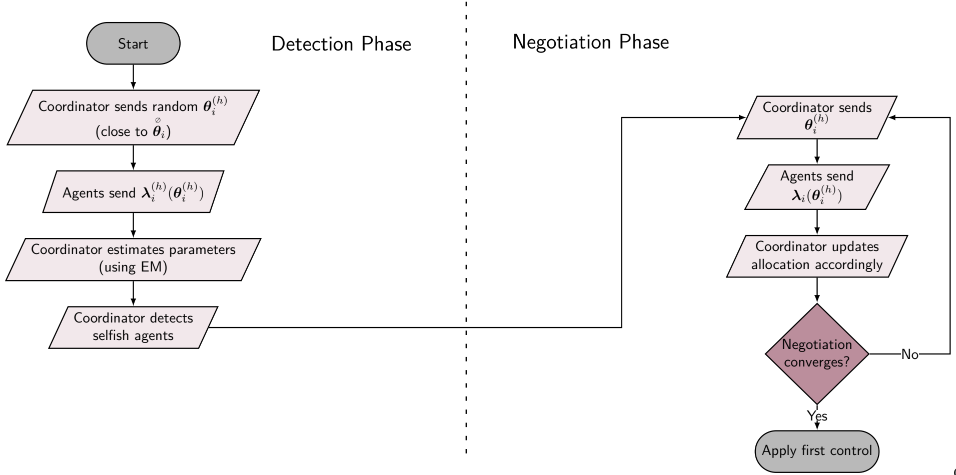

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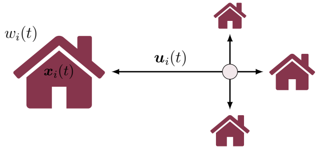
The coordinator detects which agents are non-cooperative
then the negotiation phase begins, the coordinator sends the theta i
the agents send the dual variable lambda i
and the coordinator updates the allocation accordingly using the reconstructed lambda or the one sent by the agent
and once the negotiation converges
each agent applies the first control

Example



$\left(\sum_{i=1}^{4} \boldsymbol{u}_i(k) \leqslant 4\text{kW}\right)$

District Heating Network (4 Houses)

- Houses modeled using 3R-2C
- Not enough power
- Period of 5h ($T_s = 0.25\text{h}$)
- 3 scenarios
  - N Nominal
  - C Agent I cheats (dMPC)
  - S Agent I cheats (RPdMPC-AS)
- $T_I = \begin{bmatrix} 14.43288267 & 0. & 0. & 0. \\ 0. & 13.4590903 & 0. & 0. \\ 0. & 0. & 6.93065061 & 0. \\ 0. & 0. & 0. & 3.4447393 \end{bmatrix}$

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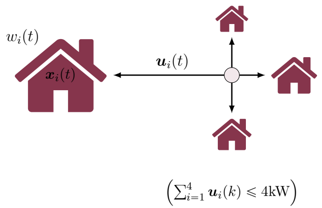
Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C
- ~~Not enough power~~ (Change $(\boldsymbol{x}_0, \boldsymbol{w}_0)$)
- Period of 5h ($T_s = 0.25\text{h}$)
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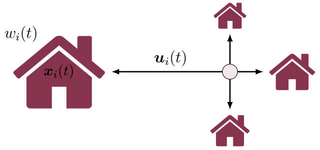
Example



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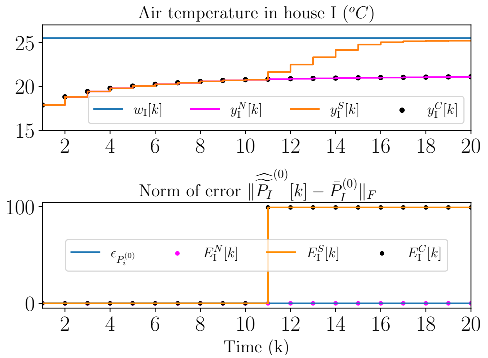
Results

Temporal



Temperature in house I and the variable $E_I(k)$ for different scenarios.
N Nominal, S Selfish behavior, C Selfish + Correction

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In the figure we see the air temperature and the estimation error for room 1

Results

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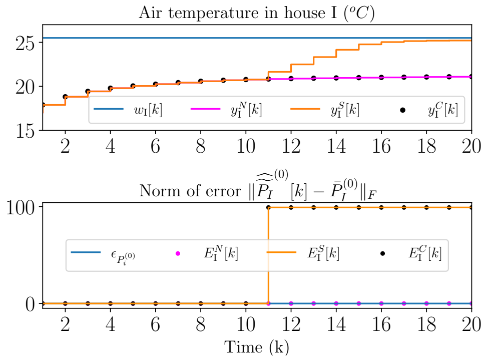

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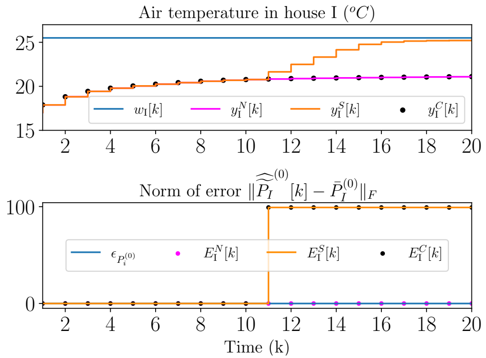

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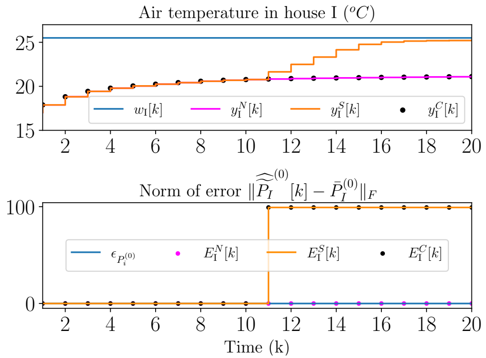

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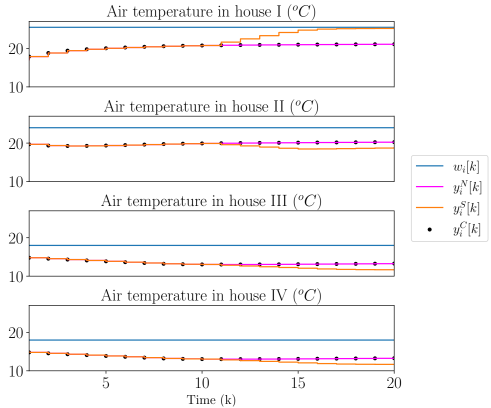
Now, for the case with correction (the red dots), even if it attacks the system, the temperature is close to its nominal value

Results

Temporal (Continued)



Air temperature in all houses for different scenarios.
N Nominal, S Selfish behavior, C Selfish + Correction

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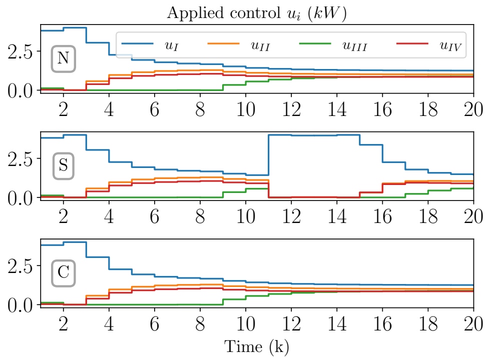
Results

Control



Control applied in all houses for different scenarios.
N Nominal, S Selfish behavior, C Selfish + Correction

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Results

Costs

Objective functions J_i (% error).

Agent	Scenario N	Scenario S	Scenario C
I	19868.2	12618.5 (-36.5)	19868.2 (-0.0)
II	13784.5	18721.1 (35.8)	13784.5 (0.0)
III	17276.0	22324.9 (29.2)	17276.1 (0.0)
IV	10086.0	13872.4 (37.5)	10086.0 (0.0)
Global	61014.7	67536.9 (10.7)	61014.7 (-0.0)

Summary

- Vulnerabilities of Primal decomposition dMPC
- Resilient strategy for 2 kinds of systems
  - Deprived systems (where demand is greater than total resources)
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Future Directions

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- Partial reconstruction of cheating matrix
- Resilient strategy with soft constraints
- Recursive EM (or alternative)
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Thank you!

Repository
https://github.com/Accacio/thesis

Contact
rafael.accacio.nogueira@gmail.com

If you want to see the simulations of this paper we have a github repository, and if you want to send me an email about this paper or this presentation you can flash the QR code in the right. Thank you!