Security of distributed Model Predictive Control under False Data injection

or How I Learned to Stop and Worry about Everything

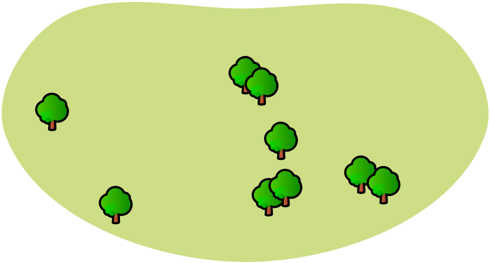
Rafael Accácio NOGUEIRA

December 12, 2022

https://bit.ly/3g3S6X4

Context

Requirements evolve with time



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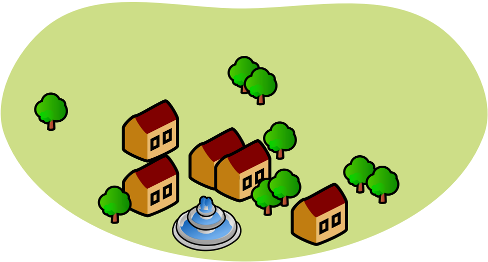

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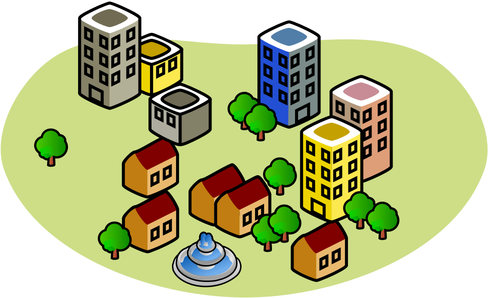

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- Electricity Distribution System
- Heat distribution
- Water distribution
- Traffic management

(include your problem here)

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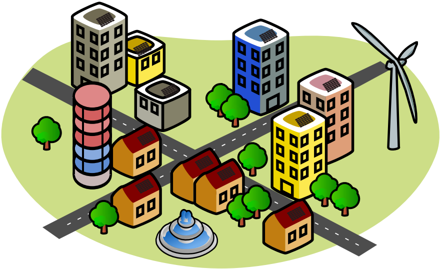

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- Multiple systems interacting
- Coupled by constraints
- Optimization objectives
  - Minimize energy consumption
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- Solution → MPC

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Find best control sequence using predictions based on a model.

- Objective function to optimize
- System's Model (states and inputs)
- Other constraints to respect (QoS, technical restrictions, ...)

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In a nutshell

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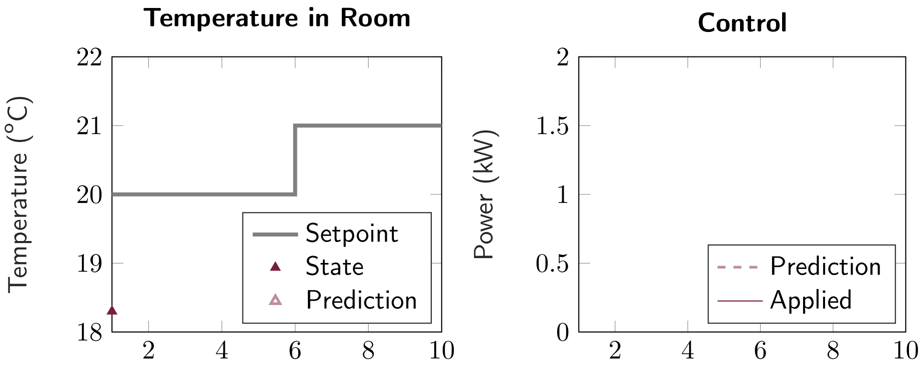

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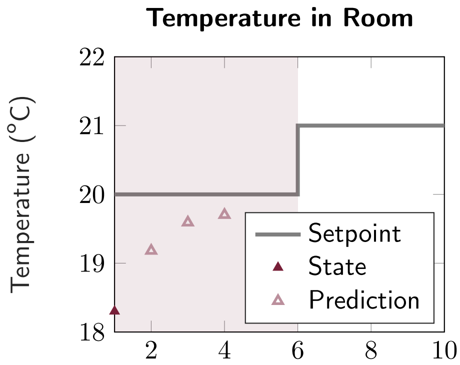
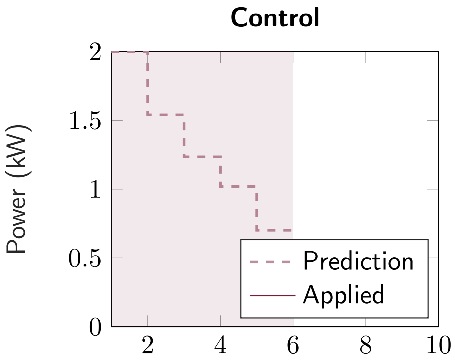
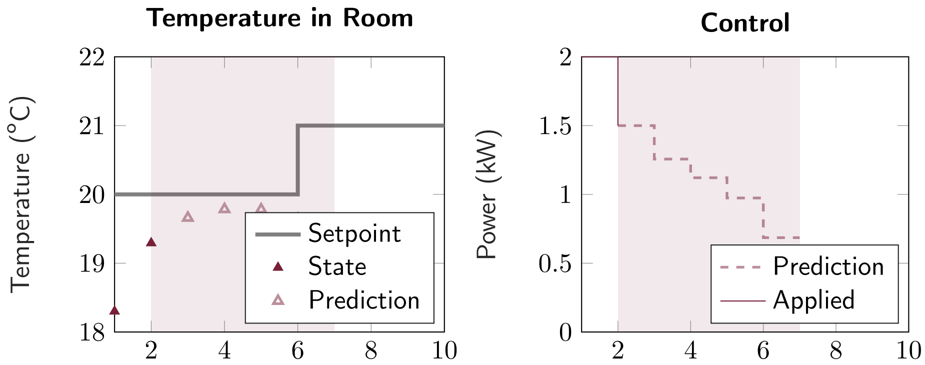




Model Predictive Control

In a nutshell

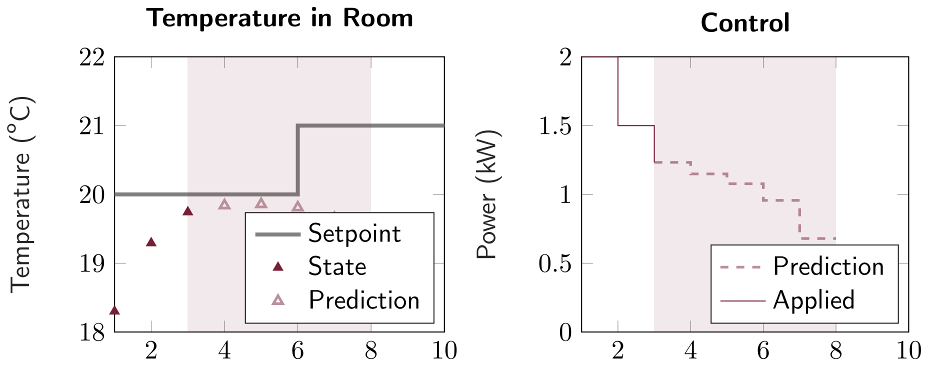
Find optimal control sequence, apply first element



Model Predictive Control

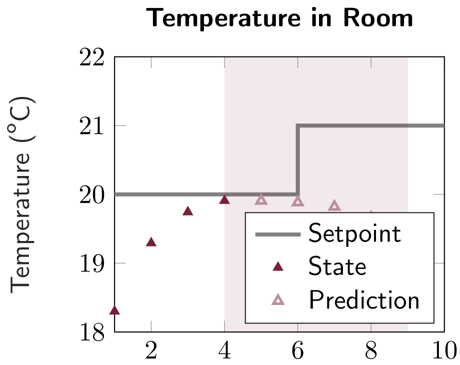
In a nutshell

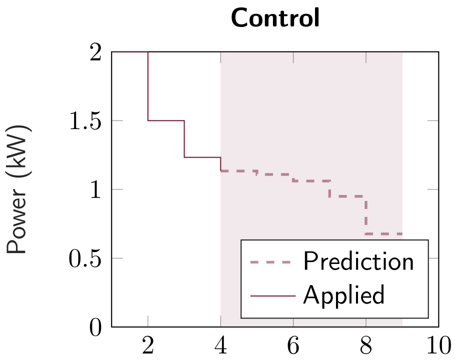
Find optimal control sequence, apply first element, rinse repeat



In a nutshell

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





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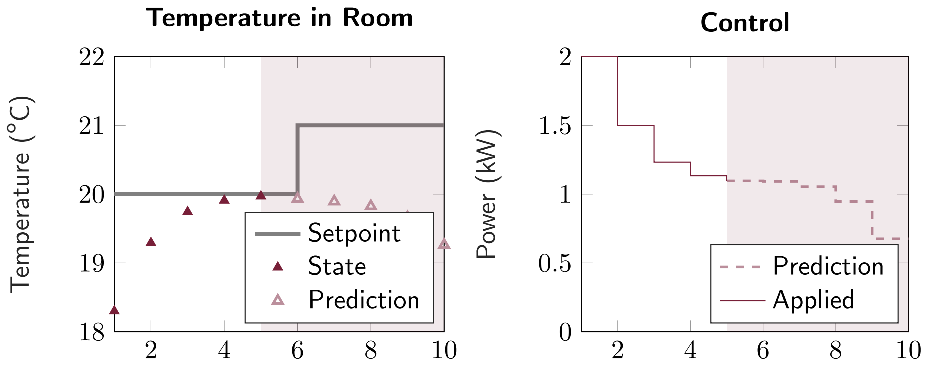

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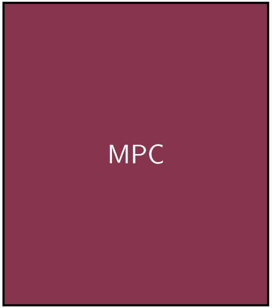

1

Distributed Model Predictive Control

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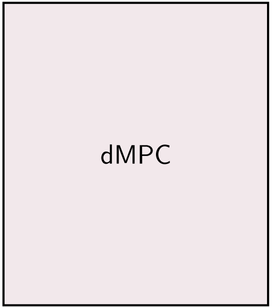

[1] *Distributed Model Predictive Control made easy*

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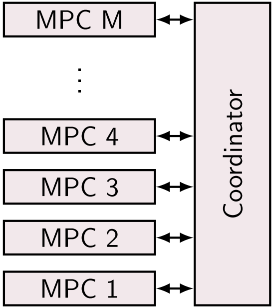

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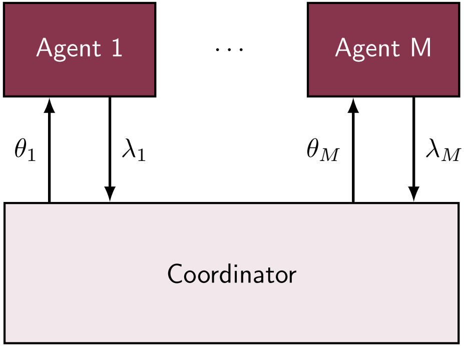

- Agents solve local problems
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$\left.\right\}$ Until Convergence

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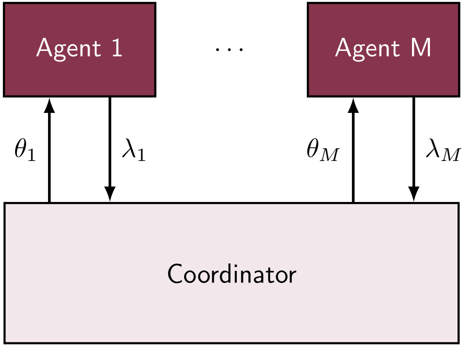

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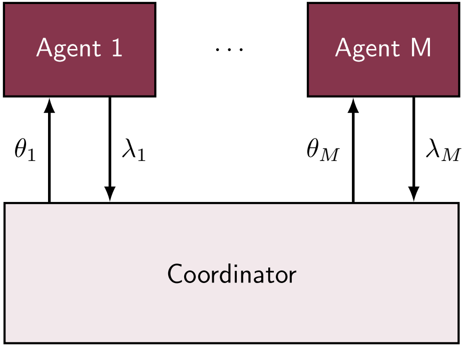

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Negotiation works if agents comply.

But what if some agents are ill-intentioned and attack the system?

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State of art

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[Vel+17a] [Mae+21]	Dual	Yes	Robust (Scenario)	NA	NA
[Vel+17b] [Vel+18]	Dual	Yes	Robust (f-robust)	NA	NA
[CMI18]	Jacobi-Gauß	Yes	–	–	–
[Ana+18] [Ana+19] [Ana+20]	Dual	Yes	Resilient	Analyt./Learn.	Disconnect (Robustness)
Our	Primal	Yes	Resilient	Active Analyt./Learn.	Data reconstruction

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1. Vulnerabilities in distributed MPC based on Primal Decomposition
2. Resilient Primal Decomposition-based dMPC for deprived systems
3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity

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Distributed Model Predictive Control

Primal Decomposition | Quantity Decomposition | Resource Allocation

Decompose original problem using primal problem

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E.g. $v_i = w_i - x_i$

Decompose original problem using primal problem

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\text{subject to} \quad & \left.\begin{array}{c} \boldsymbol{x}[\xi|k] = A\boldsymbol{x}[\xi-1|k] + B\boldsymbol{u}[\xi-1|k] \\ g_i(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \leqslant 0 \\ h_j(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) = 0 \end{array}\right\} \begin{array}{l} \forall \xi \in \{1,\dots,N\} \\ \forall i \in \{1,\dots,m\} \\ \forall j \in \{1,\dots,p\} \end{array}
\end{aligned}$$

Distributed Model Predictive Control

Primal Decomposition | Quantity Decomposition | Resource Allocation

Decompose original problem using primal problem

$$\begin{aligned}
\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}}\sum_{\xi\in\mathcal{N}}\left[\|\boldsymbol{v}_i[\xi|k]\|^2_{Q_i}+\|\boldsymbol{u}_i[\xi-1|k]\|^2_{R_i}\right] \\
\text{subject to} \quad & \left.\begin{aligned}\boldsymbol{x}[\xi|k] = A_i\boldsymbol{x}[\xi-1|k]+B_i\boldsymbol{u}[\xi-1|k] \\ \sum_{i\in\mathcal{M}}\Gamma_i\boldsymbol{u}_i[\xi|k]\leqslant\boldsymbol{u}_{\max}\end{aligned}\right\} \forall\xi\in\mathcal{N}
\end{aligned}$$

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Distributed Model Predictive Control

Primal Decomposition | Quantity Decomposition | Resource Allocation

Decompose original problem using primal problem

$$\begin{aligned} \underset{\boldsymbol{U}_1[k],\dots,\boldsymbol{U}_M[k]}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}} \left[\frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|^2_{H_i} + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k]\right] \\ \text{subject to} \quad & \sum_{i\in\mathcal{M}} \left[\bar{\Gamma}_i\boldsymbol{U}_i[k]\right] \preceq \boldsymbol{U}_{\mathsf{max}} \end{aligned}$$

Distributed Model Predictive Control

Decompose original problem using primal problem

$$\begin{aligned}
&\underset{\boldsymbol{U}_1[k]}{\text{minimize}} && \frac{1}{2}\left\|\boldsymbol{U}_1[k]\right\|_{H_1}^2 + \boldsymbol{f}_1[k]^T\boldsymbol{U}_1[k] \\
&\text{subject to} && \bar{\Gamma}_1\boldsymbol{U}_1[k] \preceq \boldsymbol{\theta}_1[k] : \boldsymbol{\lambda}_1[k] \\
& && \vdots \\
&\underset{\boldsymbol{U}_M[k]}{\text{minimize}} && \frac{1}{2}\left\|\boldsymbol{U}_M[k]\right\|_{H_M}^2 + \boldsymbol{f}_M[k]^T\boldsymbol{U}_M[k] \\
&\text{subject to} && \bar{\Gamma}_M\boldsymbol{U}_M[k] \preceq \boldsymbol{\theta}_M[k] : \boldsymbol{\lambda}_M[k]
\end{aligned}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

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Distributed Model Predictive Control

Decompose original problem using primal problem

$$\mathcal{S} = \{\boldsymbol{\theta}[k] \mid I_c^M \boldsymbol{\theta}[k] \preceq \boldsymbol{U}_{\max}\}$$

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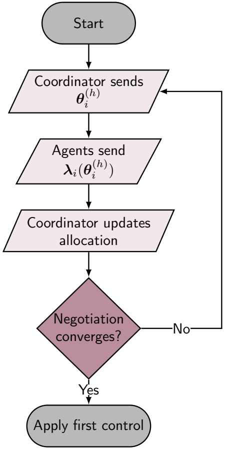
Quantity Decomposition | Resource Allocation



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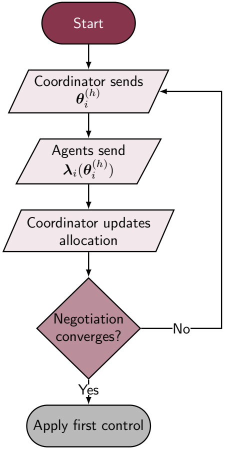

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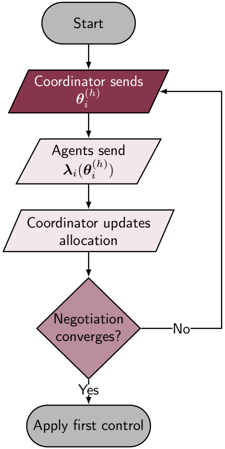

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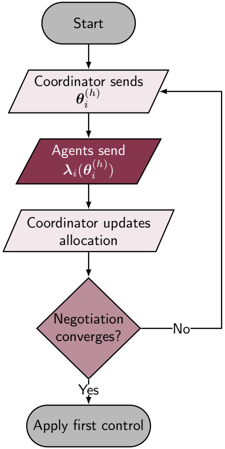

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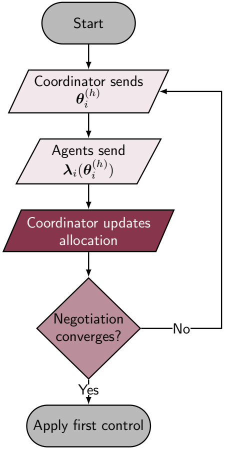

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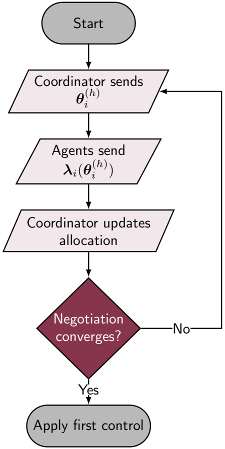

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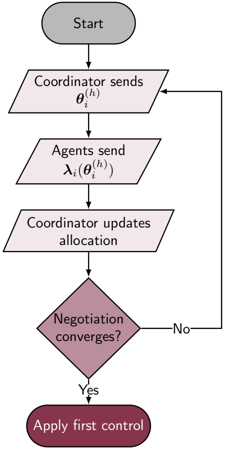

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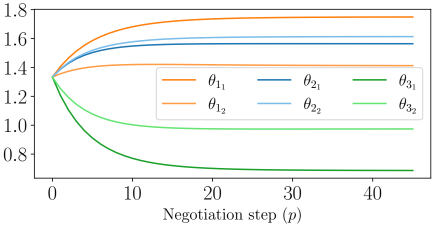

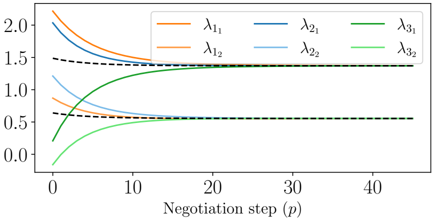

How can a non-cooperative agent attack?

Literature

- [Vel+17a; CMI18] present some kinds of attacks
  - Objective function
    - Selfish Attack
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Deception Attacks (False Data Injection)

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Our approach

- $\boldsymbol{\lambda}_i$ is the only interface with coordination
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Example

Let's suppose $\gamma_i(\boldsymbol{\lambda}_i) = T_i\boldsymbol{\lambda}_i$

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4 distinct agents

- Agent 1 is non-cooperative
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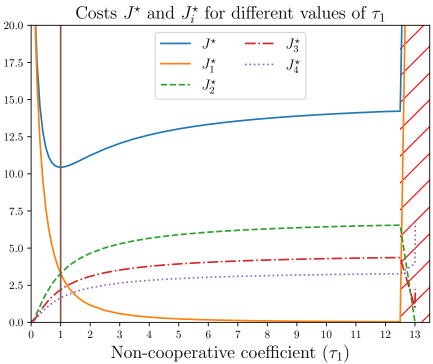

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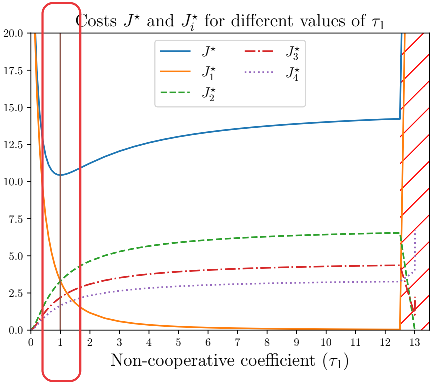

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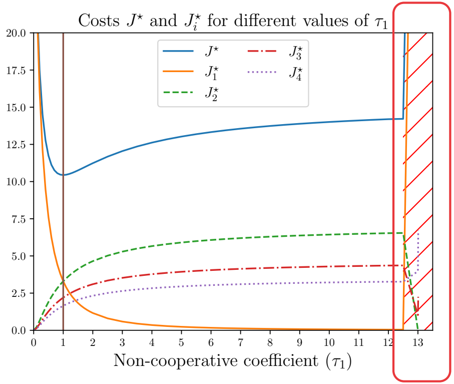

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There are vulnerabilities

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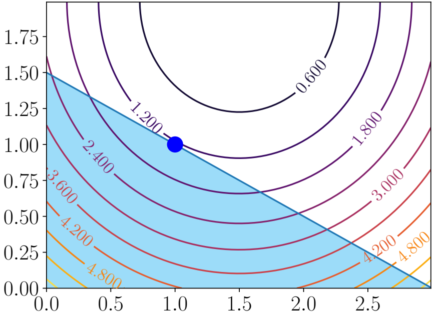

Original minimum.

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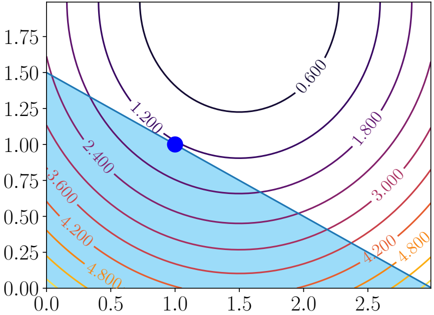

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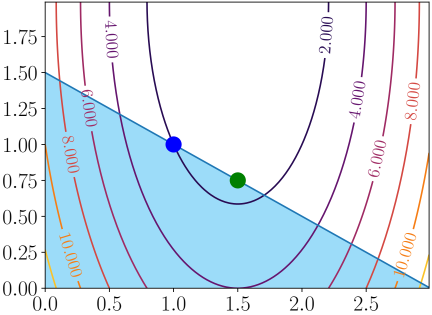

Minimum after attack.

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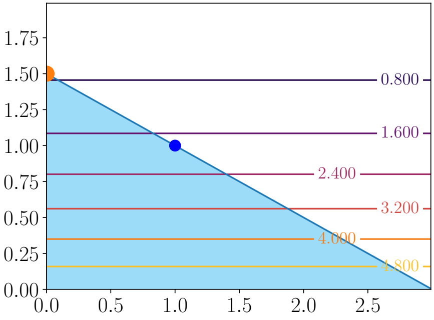

Ignore attacker.

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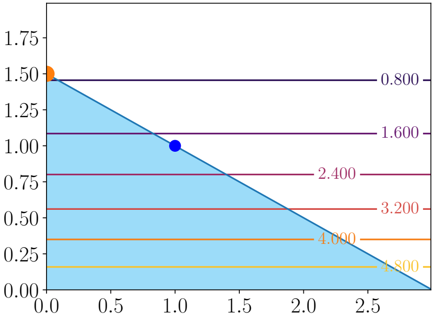

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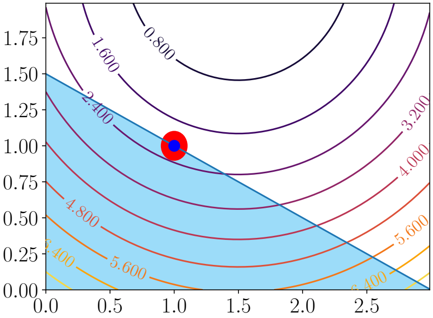

Recover original behavior.

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Mitigating

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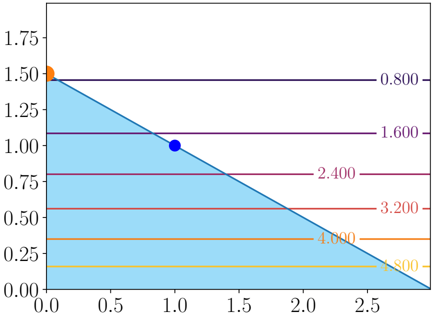

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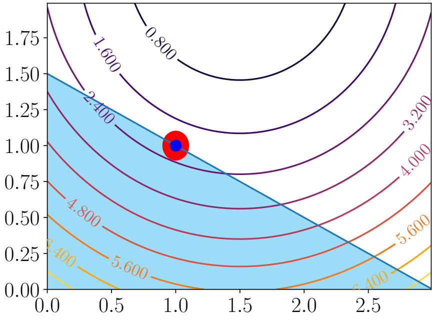

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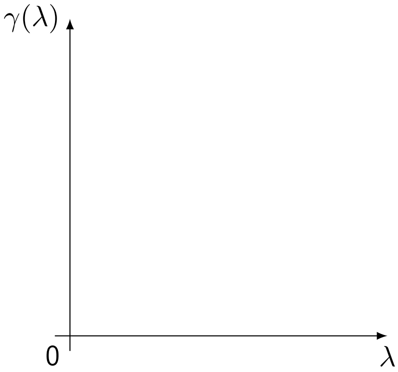
Unidimensional Case



- $\lambda \geqslant 0$ means dissatisfaction
- $\lambda = 0$ means complete satisfaction
  - $\gamma(\lambda) = 0 \Leftrightarrow \lambda = 0$

Assumptions

- Attacker is greedy $\gamma(\lambda) > \lambda$
- But not smart

  $\lambda_b > \lambda_a \rightarrow \gamma(\lambda_b) > \gamma(\lambda_a)$

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Is $\gamma_i(\boldsymbol{\lambda}_i)$ invertible?

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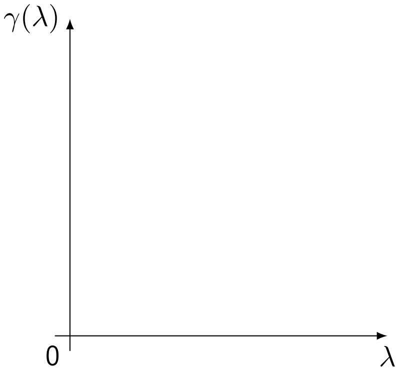

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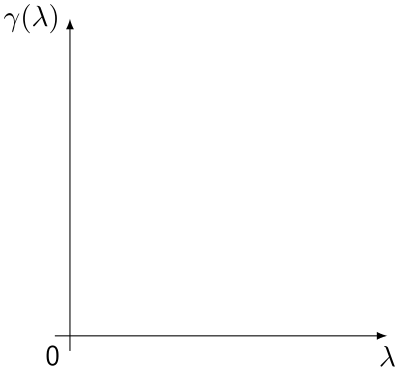

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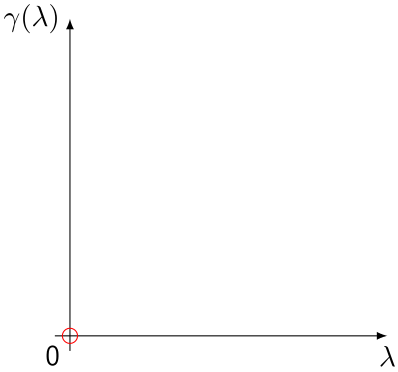

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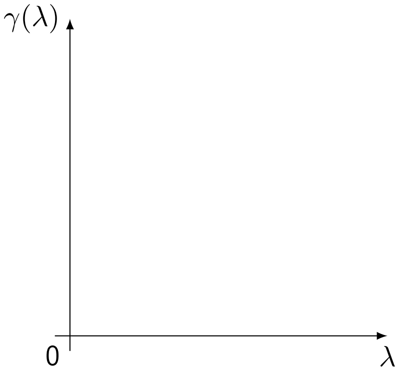

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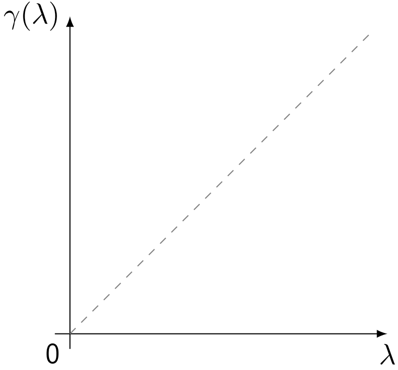

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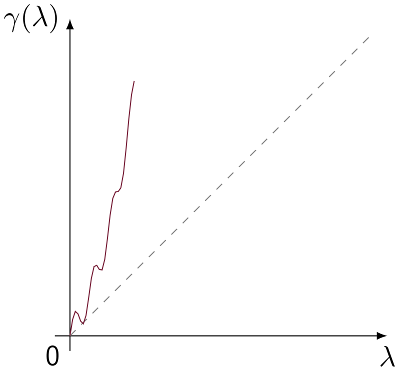

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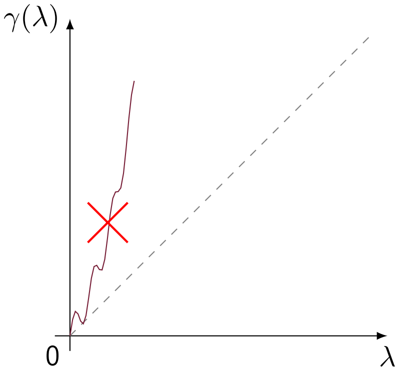

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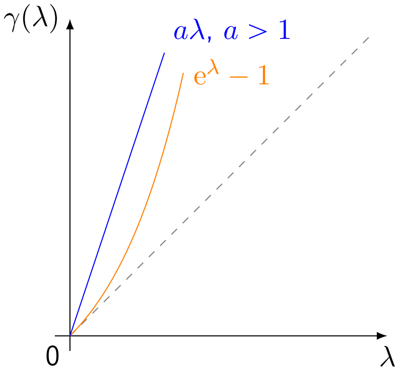

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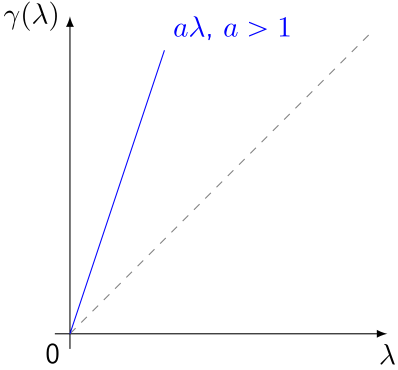

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Is $\gamma_i(\boldsymbol{\lambda}_i)$ invertible?

Linear Multidimensional Case

- $\gamma(\lambda) = T\lambda$
- $T\lambda = 0 \Leftrightarrow \lambda = 0 \rightarrow \exists T^{-1}$
- Each agent could choose different T_i each time

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Strategy

- Recover original behavior
  - Invert effects of function $\gamma_i(\boldsymbol{\lambda}_i)$ $\rightarrow$ Estimate $T_i[k]^{-1}$
- But how? Analyzing the system

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Outline

What are deprived systems?

$$\begin{aligned} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} \quad & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T \boldsymbol{U}_i[k] \\ \text{subject to} \quad & \bar{\Gamma}_i \boldsymbol{U}_i[k] \preceq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{aligned}$$

- Unconstrained solution $\mathring{\boldsymbol{U}}_i^\star[k] = -H_i^{-1}\boldsymbol{f}_i[k]$
- Deprived if constraints are active $\rightarrow \bar{\Gamma}_i \mathring{\boldsymbol{U}}_i^\star[k] \succ \boldsymbol{\theta}_i[k], \forall k$
  - Solution projected onto boundaries (equality constraints)

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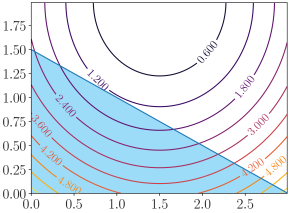

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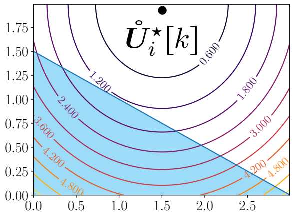

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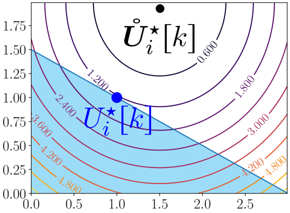

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Deprived Systems

But why?

- No Scarcity → All constraints satisfied → $\lambda_i = 0$ → No coordination needed
- Scarcity → Competition → Consensus/Compromise (or cheating)

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Deprived Systems

Analysis

- We can transform inequality constraints into equality ones[2]
- Solution is analytical and trivial.
  - If we solve for λ_i

$$
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$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k],$$

where $P_i = (\bar{\Gamma}_i H_i^{-1}\bar{\Gamma}_i^T)^{-1}$ and $\boldsymbol{s}_i[k] = P_i\bar{\Gamma}_i H_i^{-1}\boldsymbol{f}_i[k]$.

[2] Under some conditions

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Deprived Systems

Analysis (Continued)

$$\mathcal{S} = \{\boldsymbol{\theta}[k] \mid I_c^M \boldsymbol{\theta}[k] \leqslant \boldsymbol{U}_{\max}\}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

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$$\boldsymbol{\theta}^{(p+1)} = \mathcal{A}_\theta\boldsymbol{\theta}^{(p)} + \mathcal{B}_\theta[k]$$ see here

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Deprived Systems

Under attack!

- Normal behavior

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- Under attack

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Detection Mechanism

Assumption

We know nominal $\bar{P}_i$

Assumption

Attacker chooses $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i) = T_i(k)\boldsymbol{\lambda}_i$
$-T_i(k)P_i\boldsymbol{\theta}_i - T_i(k)\boldsymbol{s}_i(k) \rightarrow -\tilde{P}_i\boldsymbol{\theta}_i - \tilde{\boldsymbol{s}}_i(k)$

- We can estimate[1] $\hat{P}_i$ and $\widehat{\tilde{\boldsymbol{s}}}_i(k)$ such as:

$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i(\boldsymbol{\theta}_i)) = -\widehat{\tilde{P}_i}(k)\boldsymbol{\theta}_i - \widehat{\tilde{\boldsymbol{s}}}_i(k)$$

- If $\widehat{\tilde{P}}_i(k) \neq \bar{P}_i \rightarrow$ Attack

[1] Using Recursive Least Squares

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$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i(\boldsymbol{\theta}_i)) = -\widehat{\tilde{P}_i}(k)\boldsymbol{\theta}_i - \hat{\tilde{\boldsymbol{s}}}_i(k)$$

- If $\hat{\tilde{P}}_i(k) \neq \bar{P}_i \rightarrow$ Attack

[1] Using Recursive Least Squares

Detection Mechanism

Assumption

We know nominal $\bar{P}_i$

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Detection

In detail

- Error $E_i(k) = \|\widehat{\tilde{P}}_i(k) - \bar{P}_i\|_F$
- Create threshold ϵ_P
- Indicator $d_i \in \{0, 1\}$ detects the attack in agent i.
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Estimating $\widehat{\widetilde{P}}_i[k]$

- We estimate $\hat{P}_i$ and $\widehat{\widetilde{\boldsymbol{s}}}_i(k)$ simultaneously using Recursive Least Squares
- Problem: Estimation during negotiation fails
  - Consecutive $\boldsymbol{\lambda}_i^p$ and $\boldsymbol{\theta}_i^p$ are linearly dependent $\rightarrow$ low input excitation
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Mitigation mechanism

- Main idea: Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

Assumption

We suppose $\tilde{\boldsymbol{\lambda}}_i = \mathbf{0}$ only if $\boldsymbol{\lambda}_i = \mathbf{0}$, which implies $T_i(k)$ invertible.

- Estimate the inverse of $T_i(k)$

$$\widehat{T_i(k)^{-1}} = \bar{P}_i\, \widehat{\tilde{P}_i}(k)^{-1}$$

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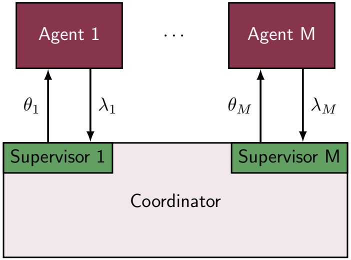
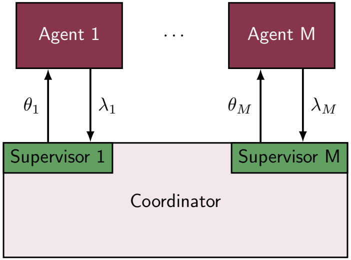
Complete Mechanism



Two phases:

1. Detect which agents are non-cooperative
2. Reconstruct λ_i and use in negotiation

Complete Mechanism



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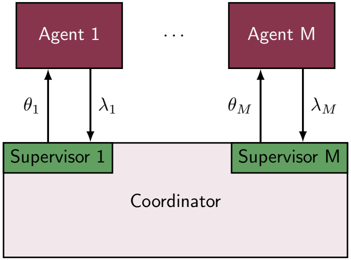

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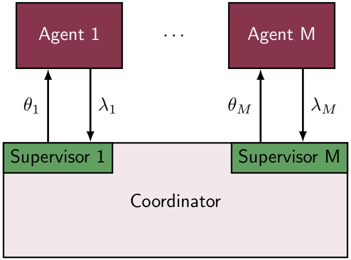

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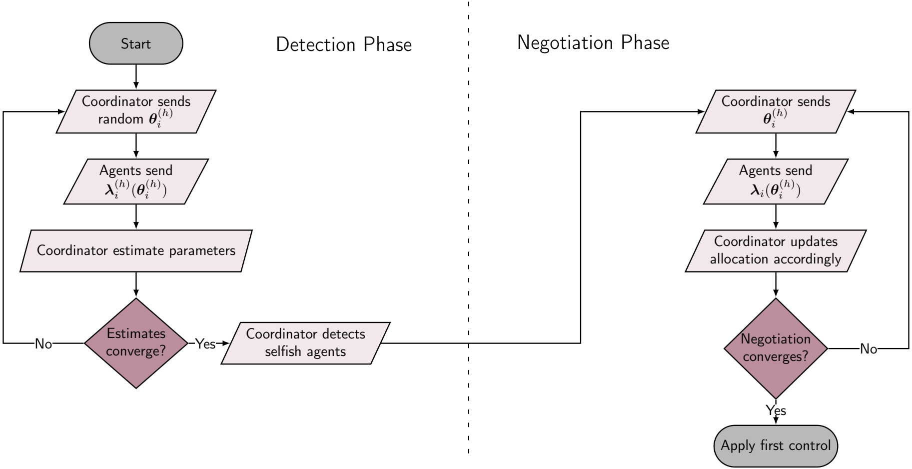
Complete algorithm

RPdMPC-DS



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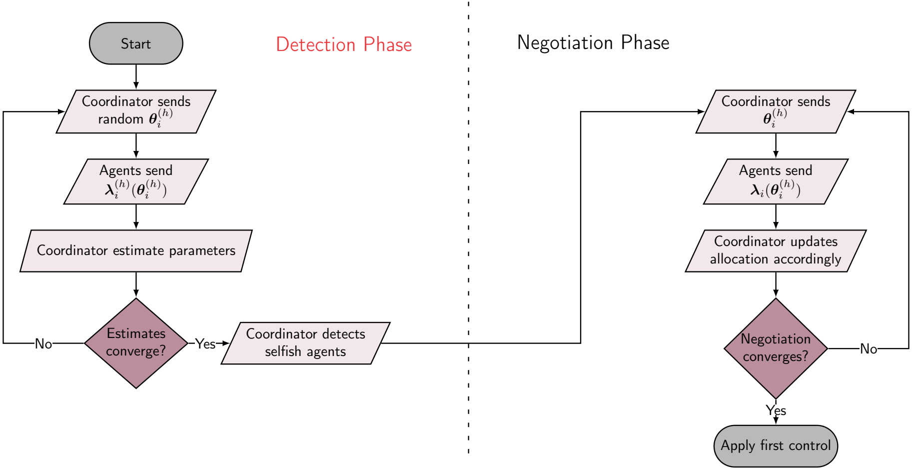

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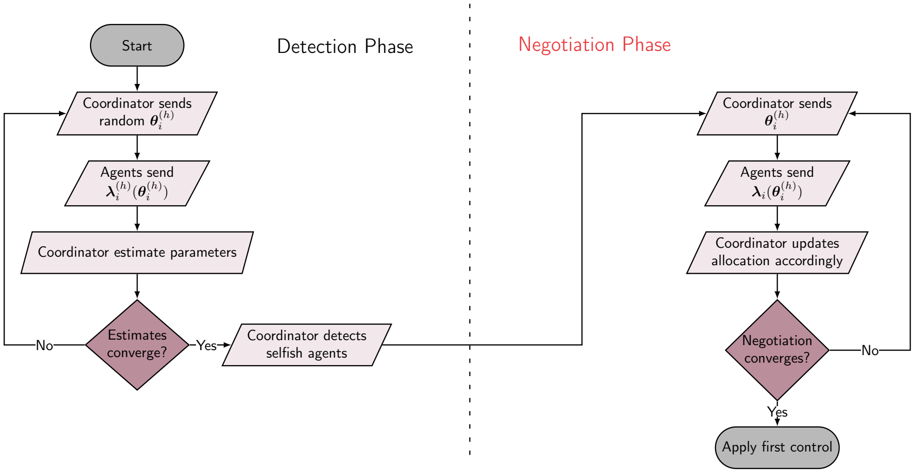

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Complete algorithm

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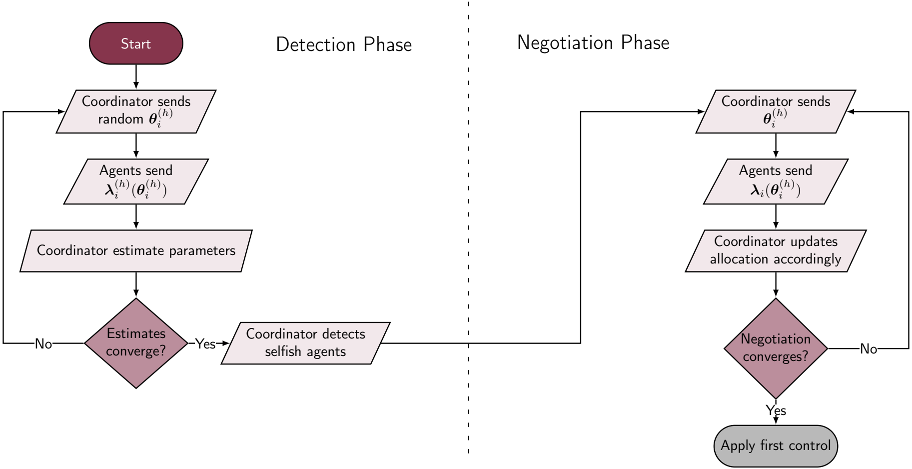

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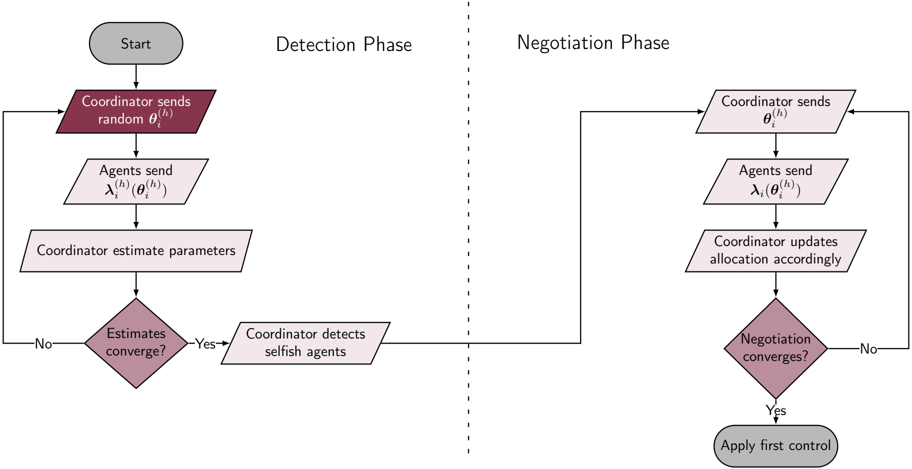

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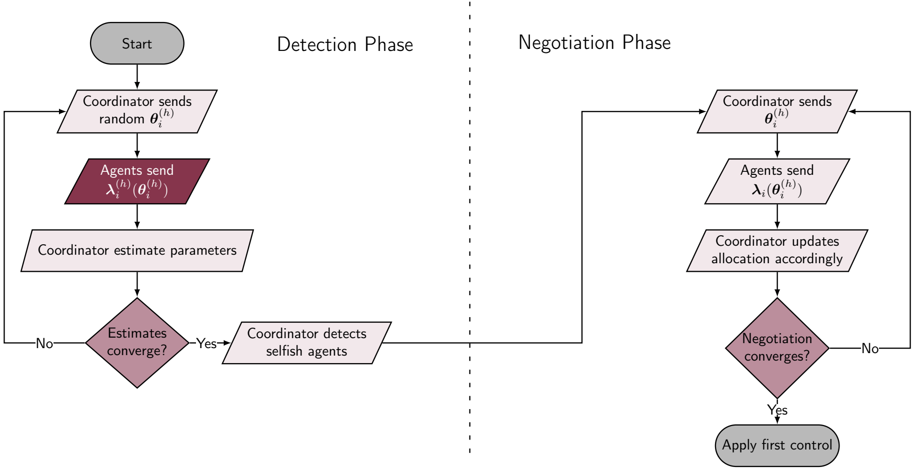

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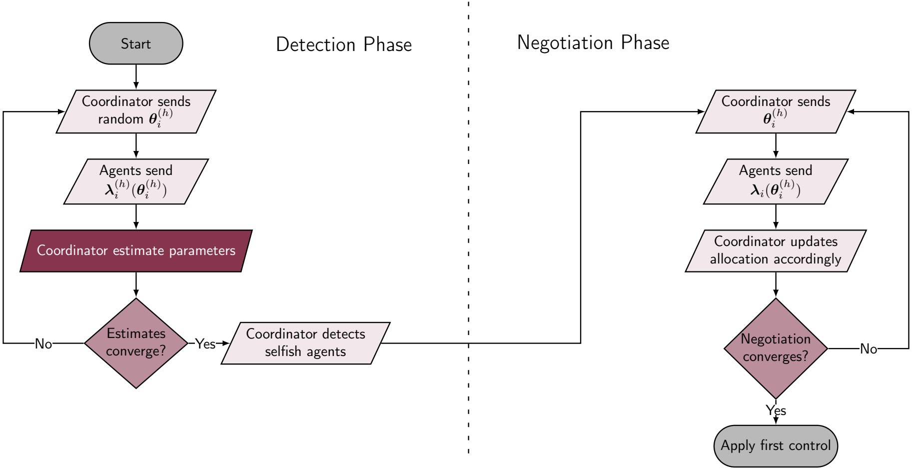

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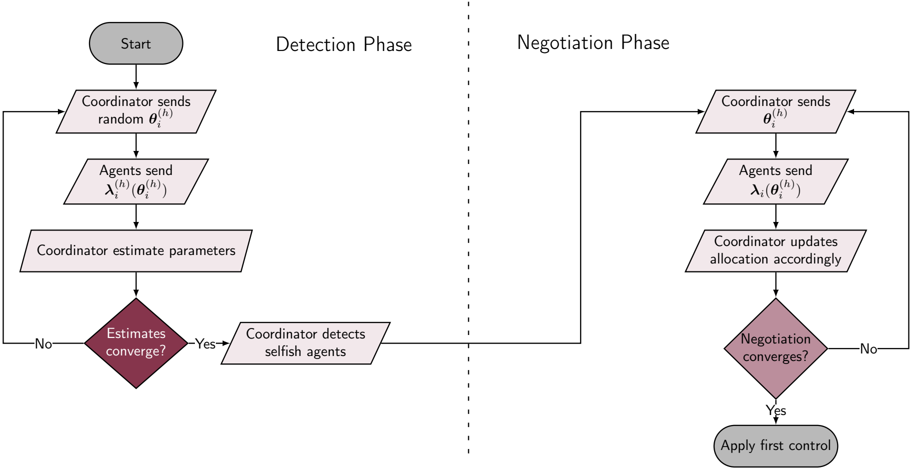

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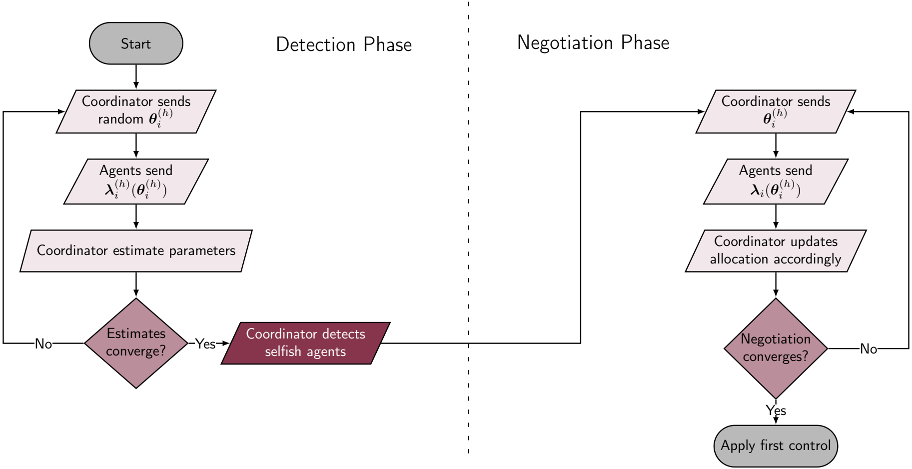

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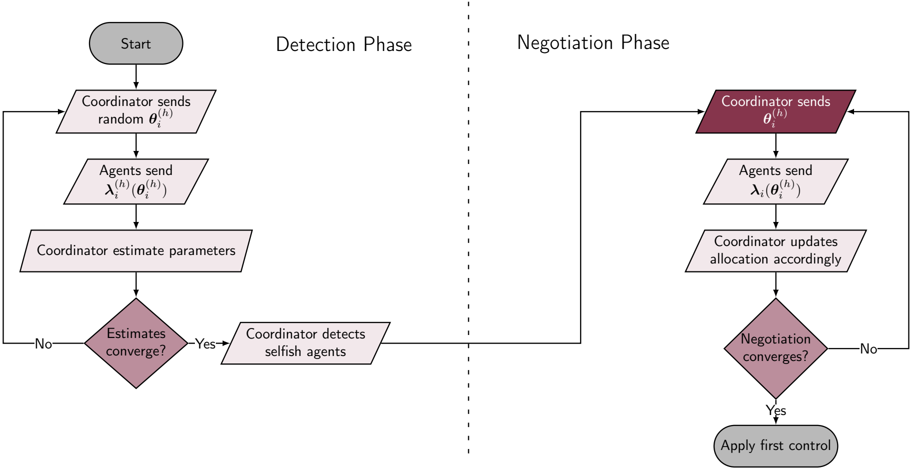

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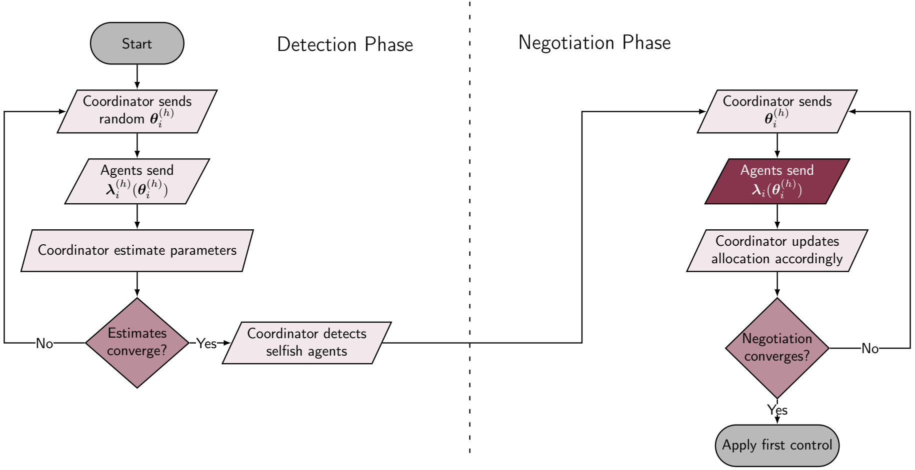

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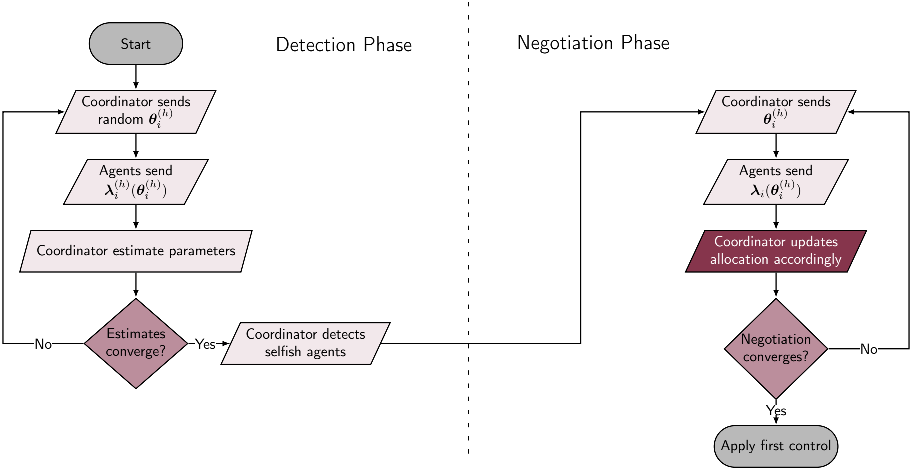

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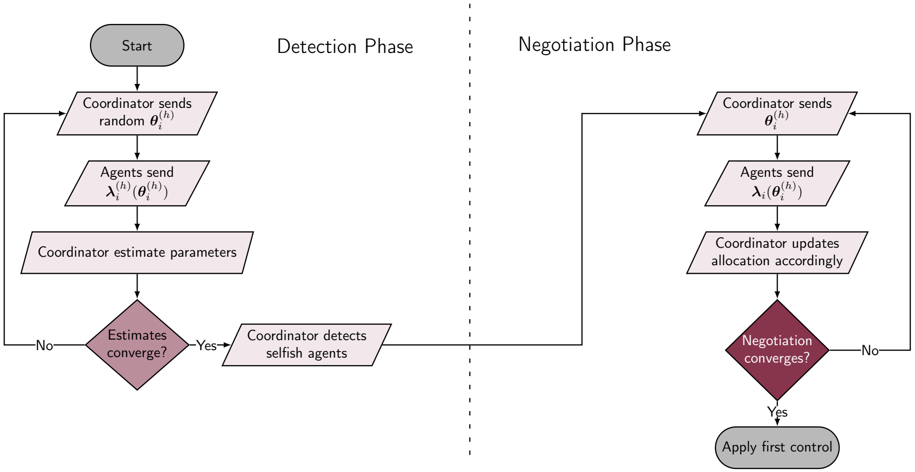

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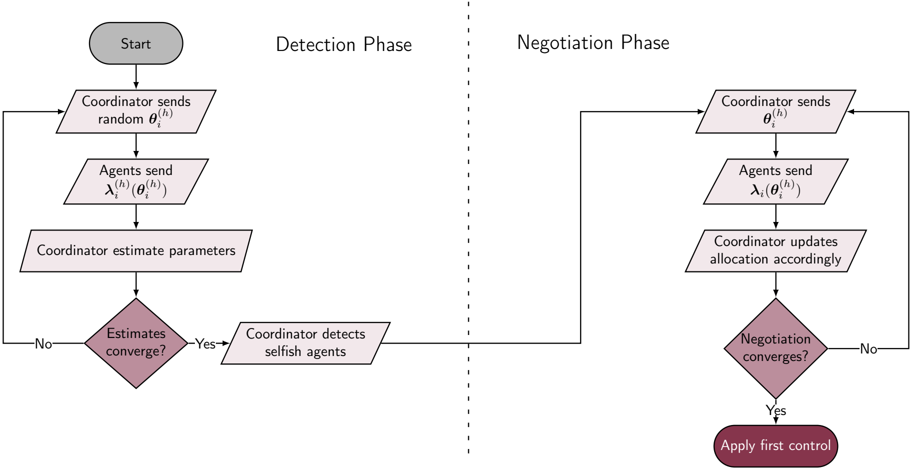

Example

District Heating of 4 Distinct Houses Under Power Scarcity

- 4 distinct rooms modeled using 3R-2C
- Initial temperature under 20^oC
- Not enough power to achieve setpoint $\left(\sum_{i=1}^{4} \boldsymbol{u}_i(k) \leqslant 4\text{kW}\right)$
- Simulated for a period of 5h
- ZOH $T_s = 0.25\text{h}$
- 3 scenarios
  1. Nominal
  2. Agent I non cooperative from k>6 with T=4*I
  3. Similar but with secure algorithm

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Results

Temporal

- N Nominal
- S Selflish behavior
- C selfish behavior with Correction

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Results

Table 1: Comparison of costs J_i^N and J_G^N

Agent	Nominal	Selfish	Selfish + correction
I	103	64	104
II	73	91	73
III	100	123	101
IV	132	154	131
Global	408	442	409

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Outline

3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity
    - Relaxing some assumptions
    - Adapting the algorithm
    - Results

Analysing

$$\boldsymbol{\lambda}_i[k] = \begin{cases} -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}_{\boldsymbol{\lambda}_i}^{n} \\ \vdots & \vdots \\ -P_i^{\left(2^{n_{\text{ineq}}}-1\right)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{\left(2^{n_{\text{ineq}}}-1\right)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}_{\boldsymbol{\lambda}_i}^{2^{n_{\text{ineq}}}-1} \end{cases}. \tag{1}$$

Artificial Scarcity

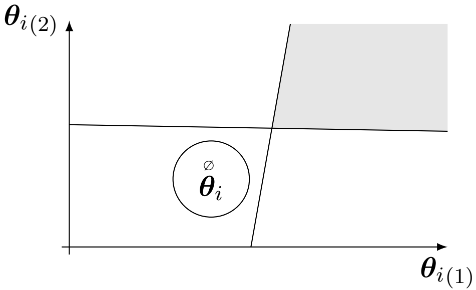

Figure 1: Ball $\mathcal{B}(\overset{\varnothing}{\boldsymbol{\theta}}_i, r)$.

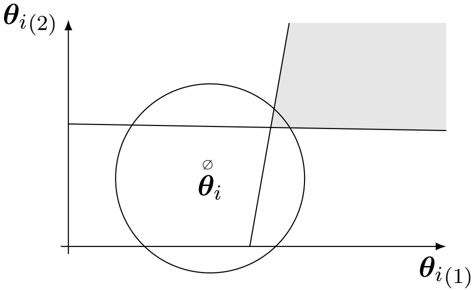

Figure 2: Ball $\mathcal{B}(\overset{\varnothing}{\boldsymbol{\theta}}_i, r)$ traversing zones.

Expectation Maximization

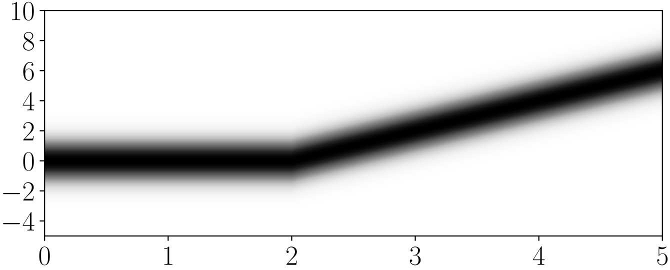

Figure 3: Gaussian Mixture for a 1D PWA function with 2 modes.

Expectation Maximization

Algorithm

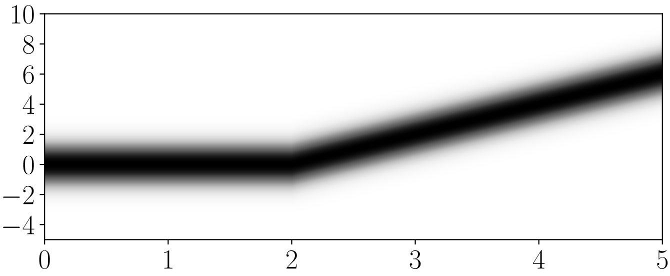

Figure 4: Gaussian Mixture for a 1D PWA function with 2 modes.

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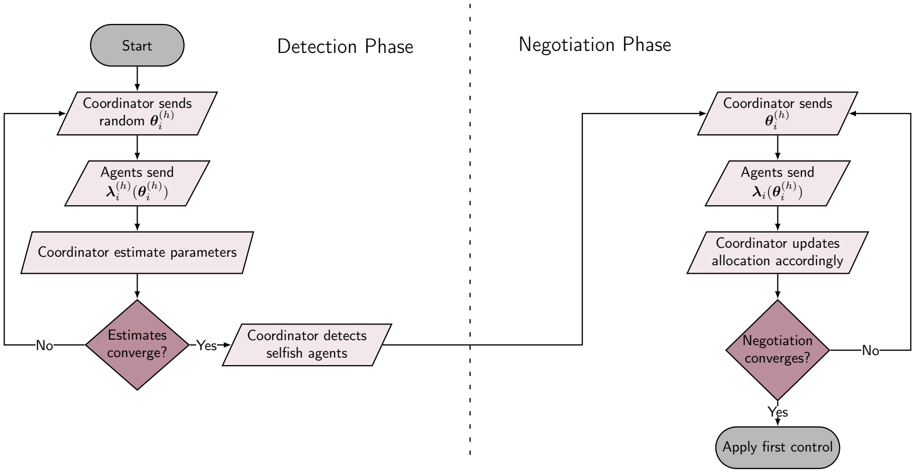
RPdMPC-AS



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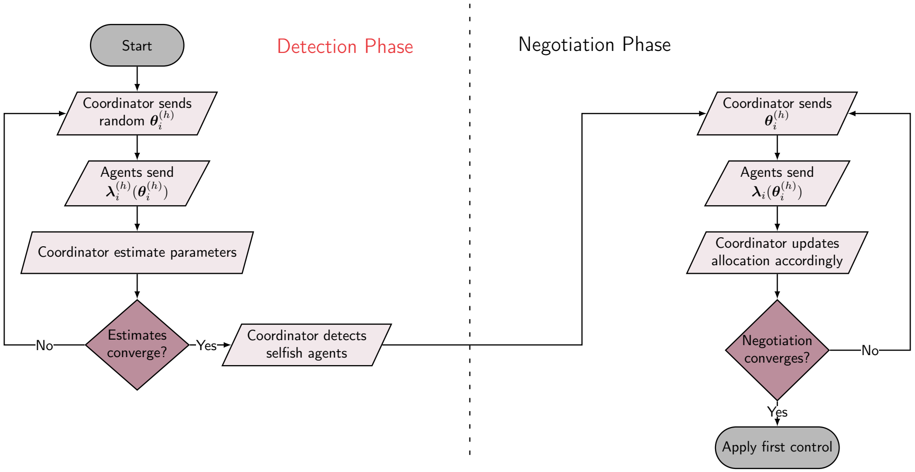

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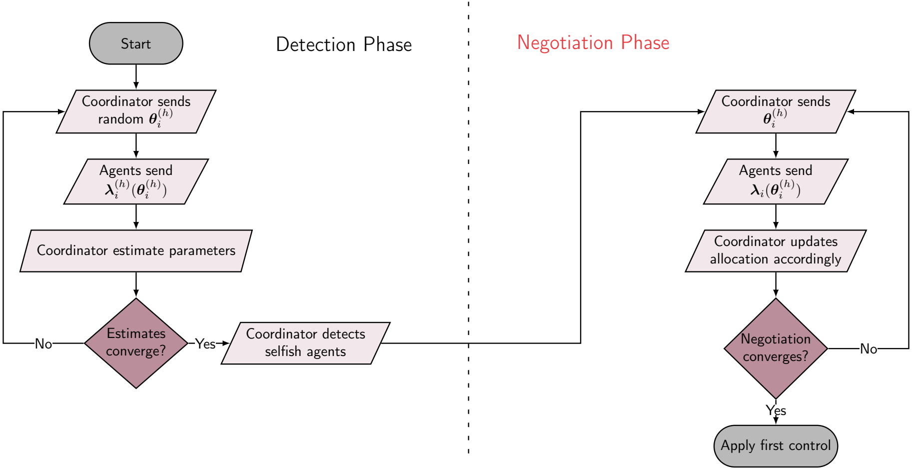

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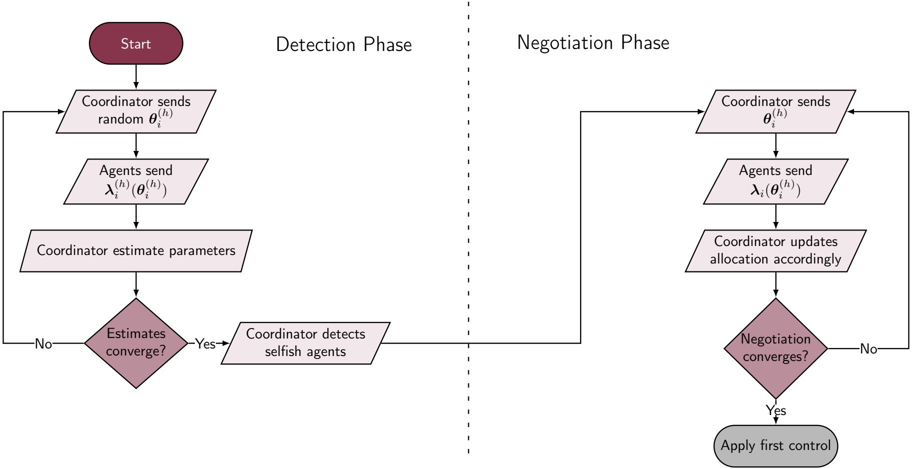

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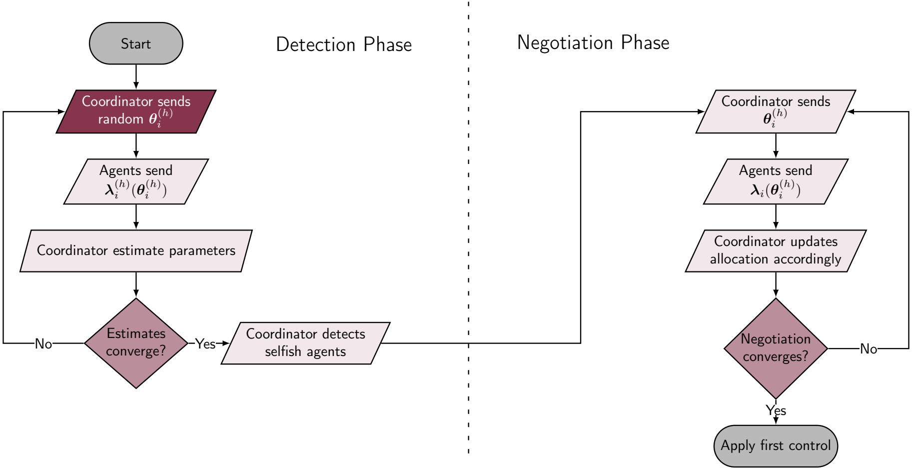

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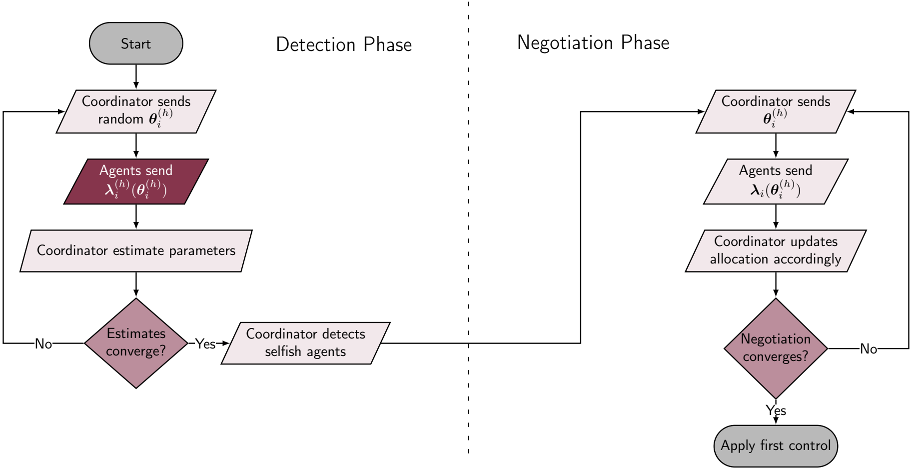

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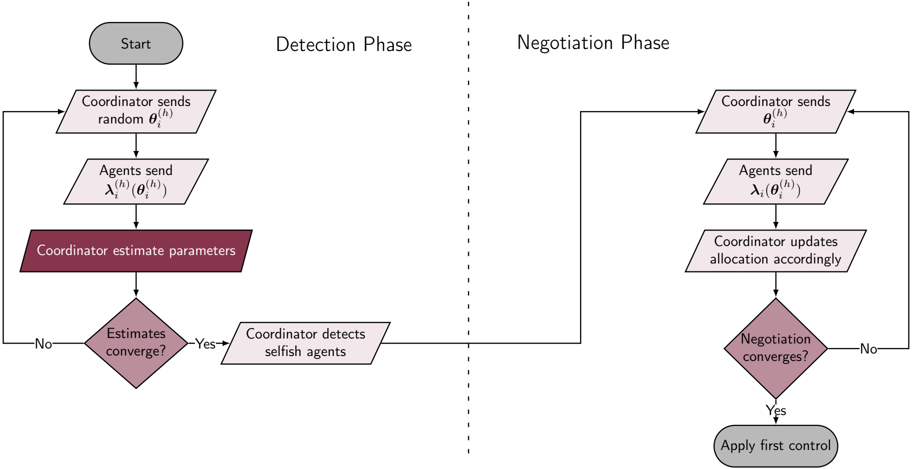

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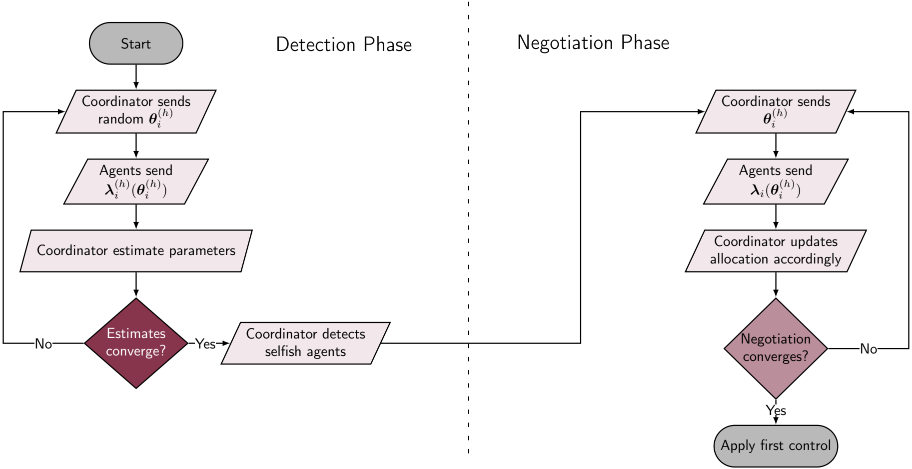

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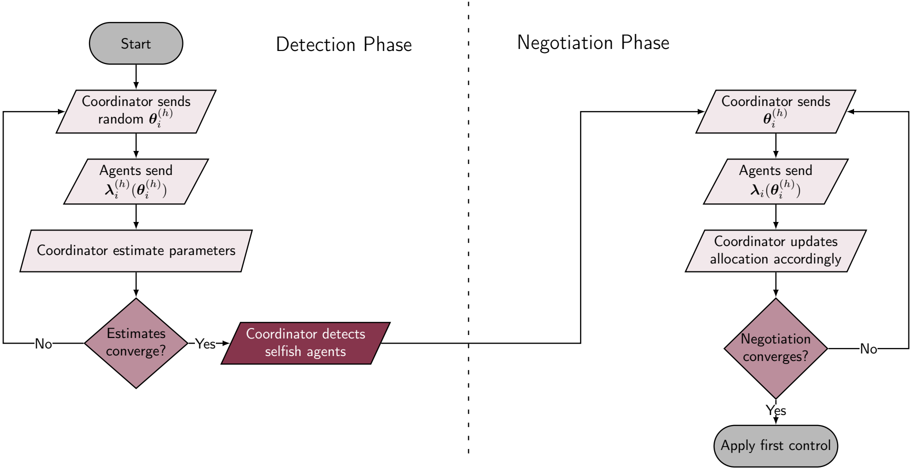

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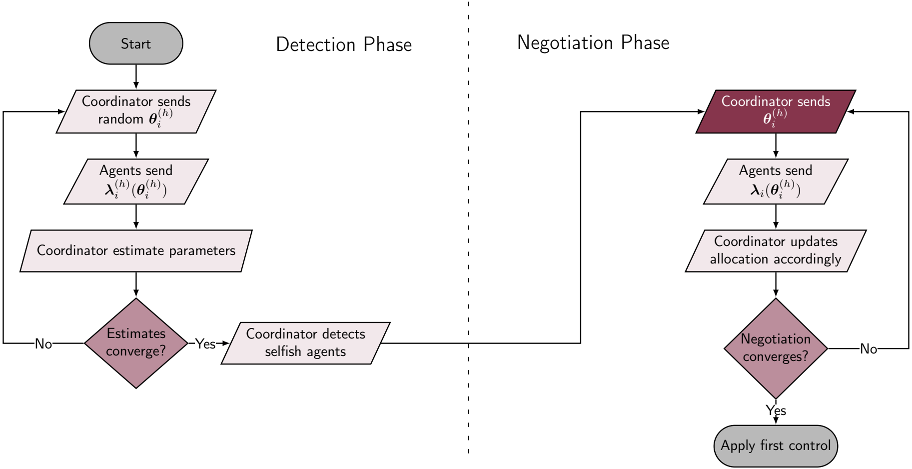

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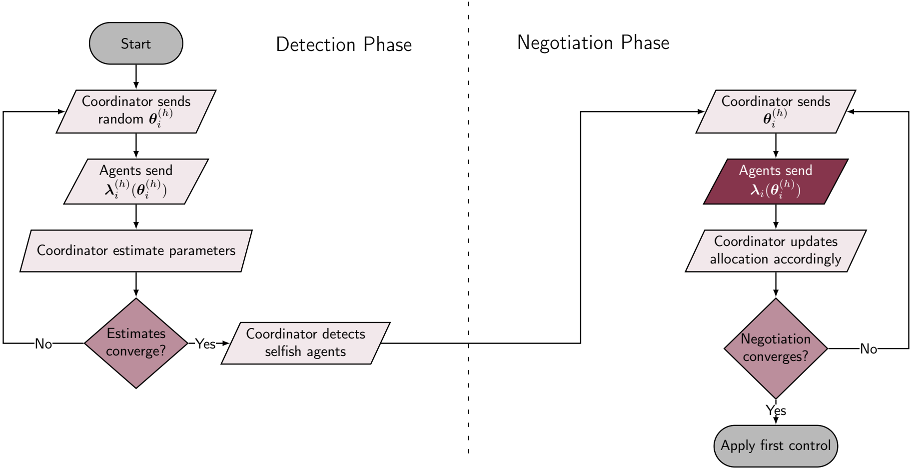

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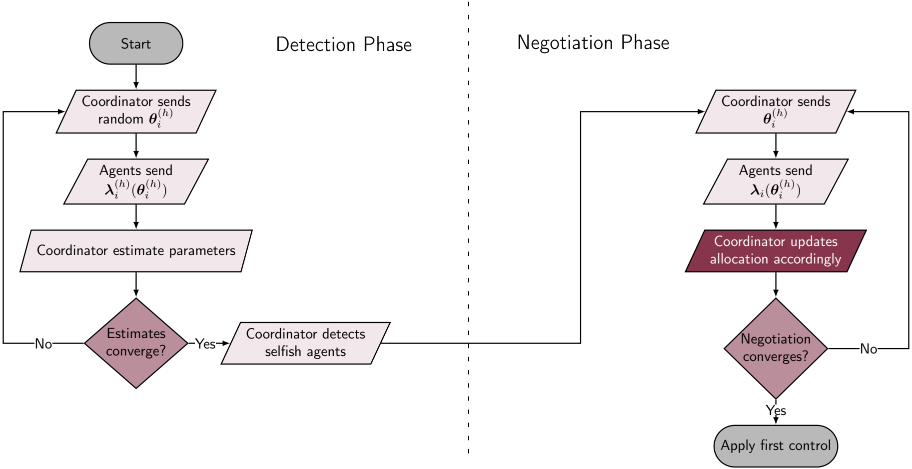

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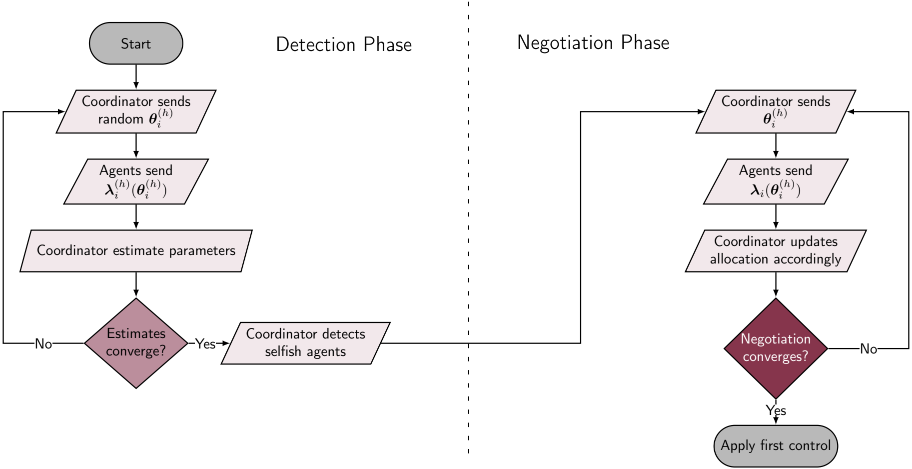
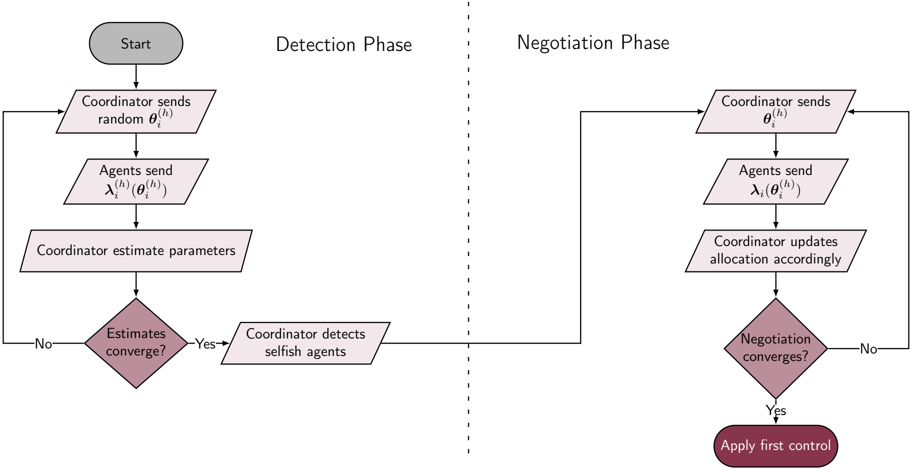
Complete algorithm

RPdMPC-AS



Complete algorithm

RPdMPC-AS



Thank you!

Repository
https://github.com/Accacio/thesis

Contact
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For Further Reading I

José M Maestre, Rudy R Negenborn, et al. *Distributed Model Predictive Control made easy*. Vol. 69. Springer, 2014. ISBN: 978-94-007-7005-8.

Wicak Ananduta et al. "Resilient Distributed Model Predictive Control for Energy Management of Interconnected Microgrids". In: *Optimal Control Applications and Methods* 41.1 (2020), pp. 146–169. DOI: 10.1002/oca.2534. URL: https://onlinelibrary.wiley.com/doi/pdf/10.1002/oca.2534.

José M. Maestre et al. "Scenario-Based Defense Mechanism Against Vulnerabilities in Lagrange-Based Dmpc". In: *Control Eng Pract* 114 (2021), p. 104879. ISSN: 0967-0661. DOI: 10.1016/j.conengprac.2021.104879.

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For Further Reading II

Pablo Velarde et al. "Vulnerabilities in Lagrange-Based Distributed Model Predictive Control". In: *Optimal Control Applications and Methods* 39.2 (Sept. 2018), pp. 601–621. DOI: 10.1002/oca.2368.

Wicak Ananduta et al. "Resilient Distributed Energy Management for Systems of Interconnected Microgrids". In: *2018 IEEE Conference on Decision and Control (CDC)*. 2018, pp. 3159–3164. DOI: 10.1109/CDC.2018.8619548.

Wicak Ananduta et al. "A Resilient Approach for Distributed MPC-Based Economic Dispatch in Interconnected Microgrids". In: *2019 18th European Control Conference (ECC)*. 2019, pp. 691–696. DOI: 10.23919/ECC.2019.8796208.

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For Further Reading III

- P. Chanfreut, J. M. Maestre, and H. Ishii. "Vulnerabilities in Distributed Model Predictive Control based on Jacobi-Gauss Decomposition". In: *2018 European Control Conference (ECC)*. June 2018, pp. 2587–2592. DOI: 10.23919/ECC.2018.8550239.
- Pablo Velarde et al. "Scenario-based defense mechanism for distributed model predictive control". In: *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*. IEEE. Dec. 2017, pp. 6171–6176. DOI: 10.1109/CDC.2017.8264590.
- Pablo Velarde et al. "Vulnerabilities in Lagrange-Based DMPC in the Context of Cyber-Security". In: *2017 IEEE International Conference on Autonomic Computing (ICAC)*. July 2017, pp. 215–220. DOI: 10.1109/ICAC.2017.53.

Conditions

◂ back

One way to ensure this, is to make the original constraint (??) to have at most as many rows as columns, i.e., $\#\boldsymbol{u}_{\max} \leqslant n_u$, although it may be a little restrictive.

θ dynamics

back

$$\boldsymbol{\theta}^{(p+1)} = \mathcal{A}_\theta \boldsymbol{\theta}^{(p)} + \mathcal{B}_\theta[k]$$

where

$$\mathcal{A}_\theta = \begin{bmatrix} I - \frac{M-1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \frac{1}{M}\rho^{(p)}P_1 & I - \frac{M-1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & I - \frac{M-1}{M}\rho^{(p)}P_M \end{bmatrix}$$

$$\mathcal{B}_\theta[k] = \begin{bmatrix} -\frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \vdots \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \end{bmatrix}$$

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