Security of distributed Model Predictive Control under False Data Injection

Rafael Accácio NOGUEIRA

2022-12-12

https://bit.ly/3g3S6X4

IETR

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Context

“Necessity is the mother of invention”

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Context

❝ Necessity is the mother of invention ❞

- Electricity Distribution System

Context

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- Electricity Distribution System
- Heat distribution
- Water distribution

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- Electricity Distribution System
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- Traffic management

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  (include your problem here)

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- Multiple systems interacting

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- Coupled by constraints

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  - Technical/ Comfort

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“Necessity is the mother of invention”

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❝Necessity is the mother of invention❞

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  - Minimize energy consumption

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  - Minimize energy consumption
  - Maximize user satisfaction

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“Necessity is the mother of invention”

- Multiple systems interacting
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  - Technical/ Comfort
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  - Minimize energy consumption
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  - Follow a trajectory

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- Multiple systems interacting
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- Optimization objectives
  - Minimize energy consumption
  - Maximize user satisfaction
  - Follow a trajectory
- Solution → MPC

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Model-based Predictive Control

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Find best control sequence using predictions based on a model.

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- We need an optimization problem

$$\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k])$$

Model-based Predictive Control

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence

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Find optimal control sequence using predictions based on a model.

- We need an optimization problem
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Model-based Predictive Control

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence (Over horizon N)
  - Objective function to optimize
  - System's Model (states and inputs)

$$\begin{aligned}
\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad & J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k]) \\
\text{subject to} \quad & \left.\begin{aligned} \boldsymbol{x}[\xi|k] = f(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \\ \\ \end{aligned}\right\} \forall \xi \in \{1, \dots, N\}
\end{aligned}$$

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\boldsymbol{x}[\xi|k] = f(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \\
g_i(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \leqslant 0 \\
h_j(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) = 0
\end{aligned}\right\}
\begin{aligned}
&\forall \xi \in \{1, \dots, N\} \\
&\forall i \in \{1, \dots, m\} \\
&\forall j \in \{1, \dots, p\}
\end{aligned}
\end{aligned}
$$

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Model-based Predictive Control

Find optimal control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence (Over horizon N)
  - Objective function to optimize
  - System's Model (states and inputs)
  - Other constraints to respect (QoS, technical restrictions, ...)

$$
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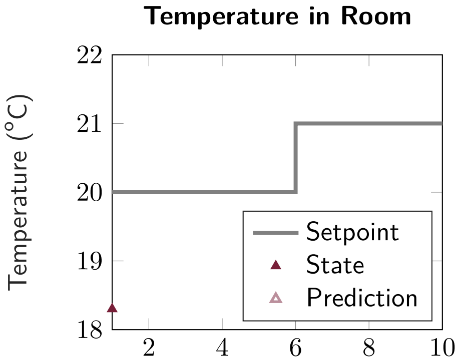
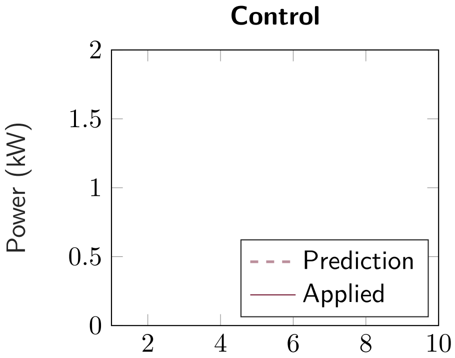
Model Predictive Control

In a nutshell

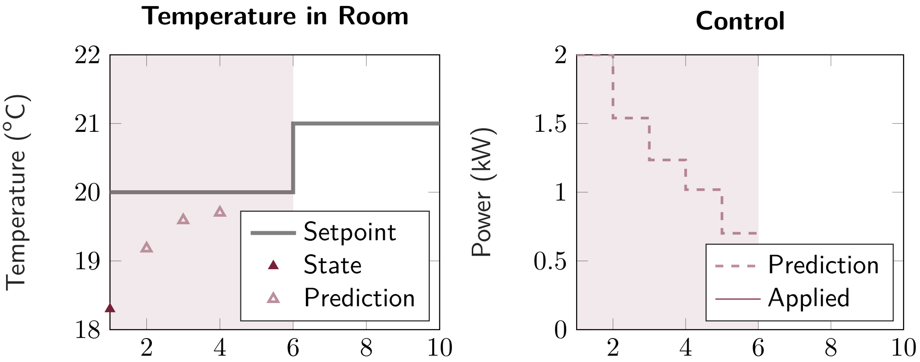




Model Predictive Control

In a nutshell

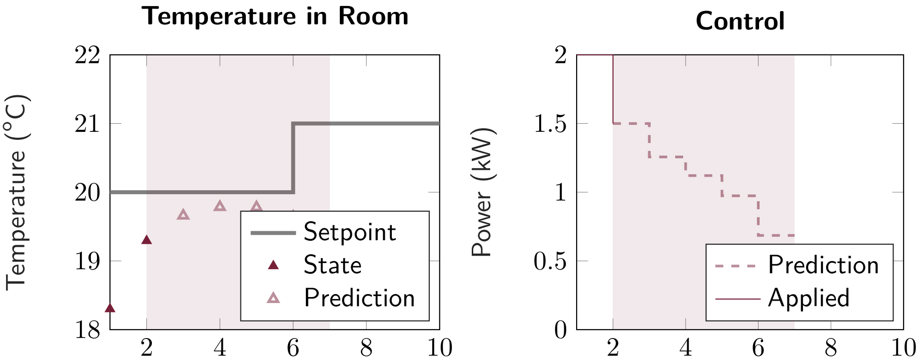
Find optimal control sequence



Model Predictive Control

In a nutshell

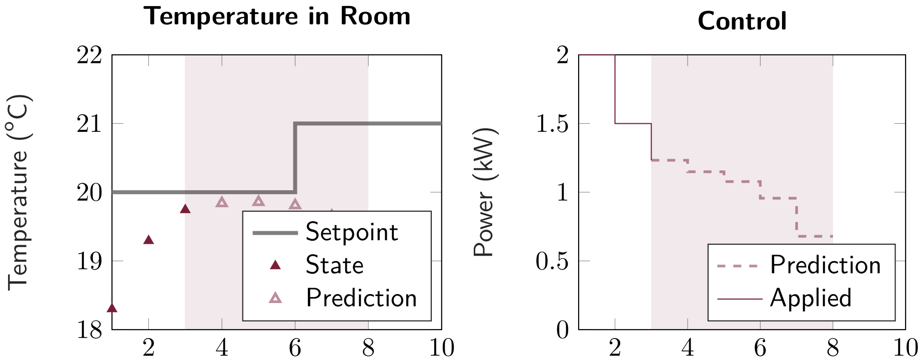
Find optimal control sequence, apply first element



Model Predictive Control

In a nutshell

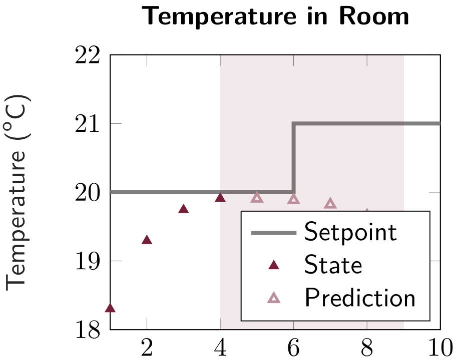
Find optimal control sequence, apply first element, rinse repeat

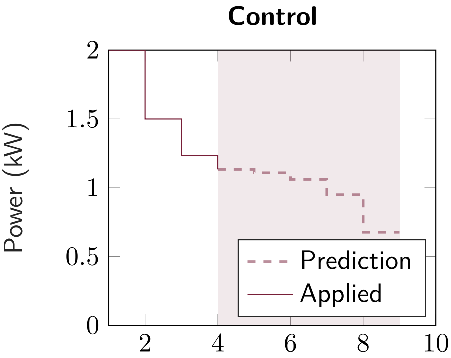


Model Predictive Control

In a nutshell

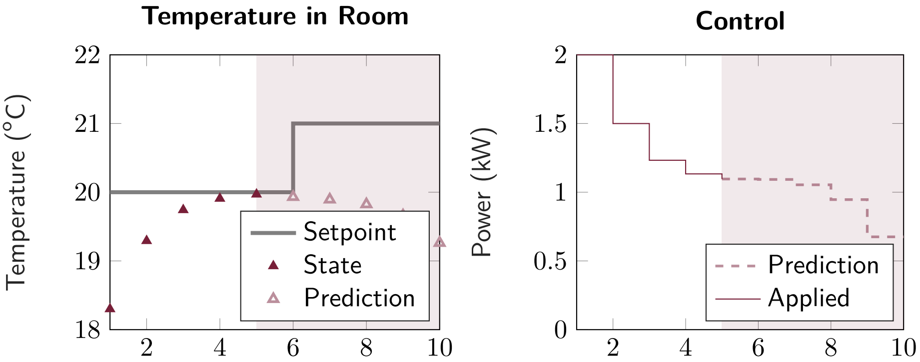
Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





In a nutshell

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon



Model Predictive Control

Nothing is perfect

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- Solution: Divide and Conquer (distributed MPC)

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- Solution: Divide and Conquer (distributed MPC)
  - Break calculation
  - Make agents communicate

Distributed Model Predictive Control

It is about communication

- We break the MPC into multiple
- Make agents communicate.

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It is about communication

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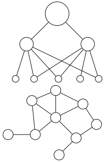

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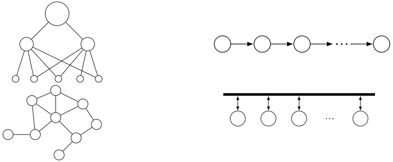

Distributed Model Predictive Control

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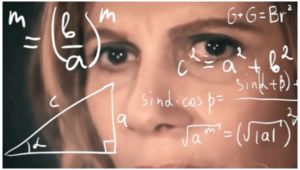

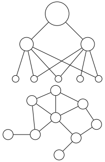

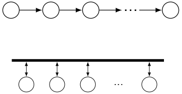

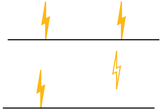

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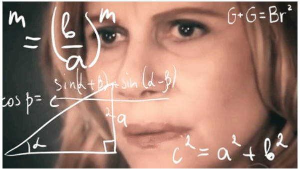

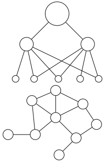

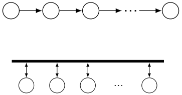

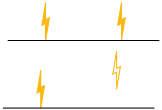

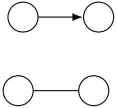

Distributed Model Predictive Control

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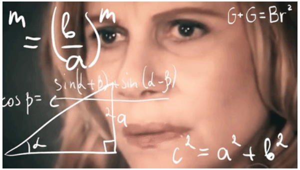

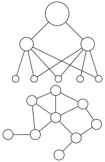

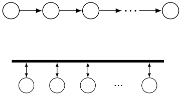

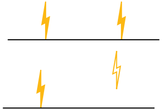

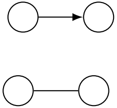

[1] Distributed Model Predictive Control made easy

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Distributed Model Predictive Control

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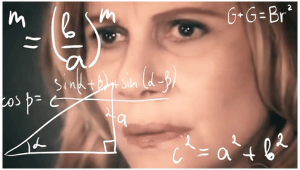

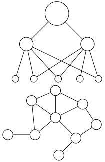

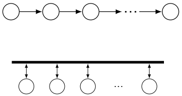

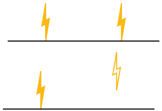

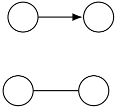

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Distributed Model Predictive Control

Communication Frameworks

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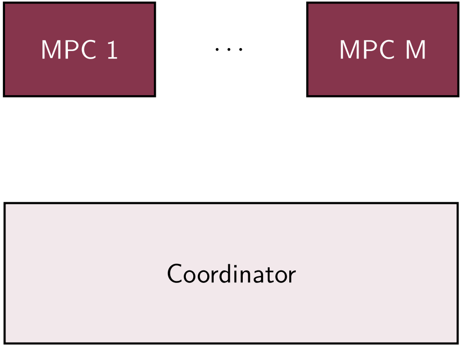
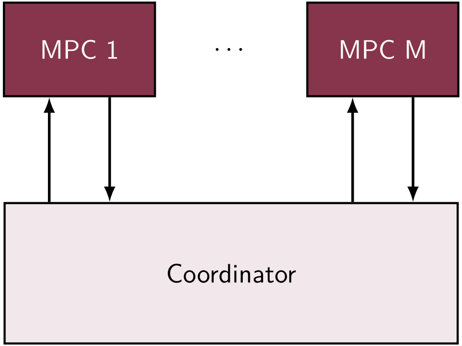
Communication Frameworks



- Coordinator → Hierarchical

Distributed Model Predictive Control

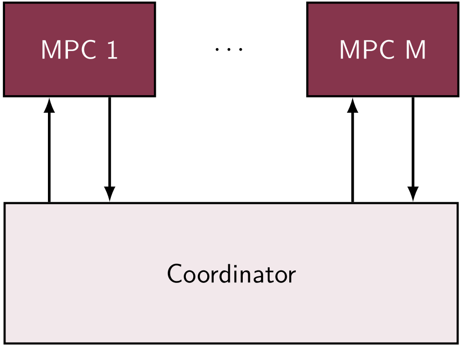
Communication Frameworks



- Coordinator → Hierarchical
- Bidirectional

Distributed Model Predictive Control

Communication Frameworks



- Coordinator → Hierarchical
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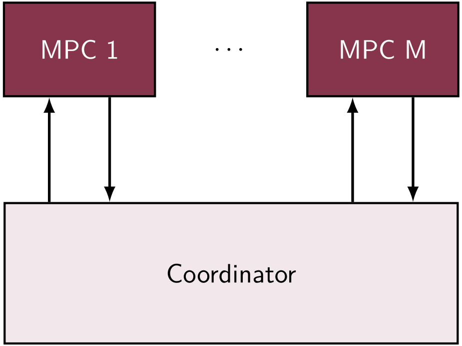
- No delay → Synchronous

Distributed Model Predictive Control

Communication Frameworks



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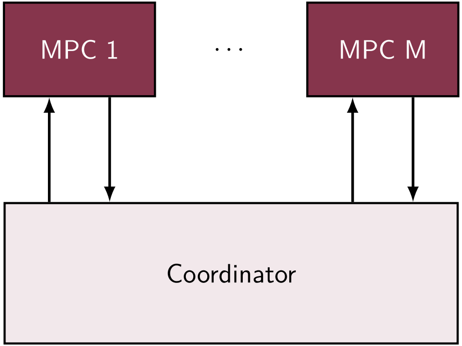
- Agents solve local problems

Distributed Model Predictive Control

Communication Frameworks



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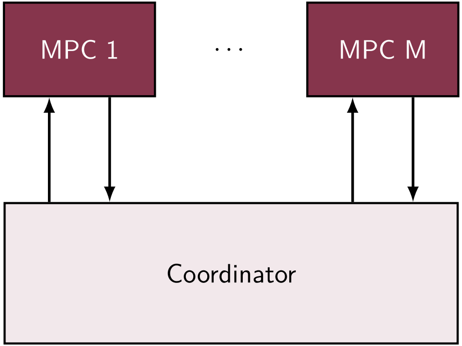

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} Until Convergence

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Distributed Model Predictive Control

Negotiation works if agents comply.

Distributed Model Predictive Control

Negotiation works if agents comply.
But what if some agents are ill-intentioned and attack the system?

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- How can an agent attack?
- What are the consequences of an attack?
- Can we mitigate the effects?

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But what if some agents are ill-intentioned and attack the system?

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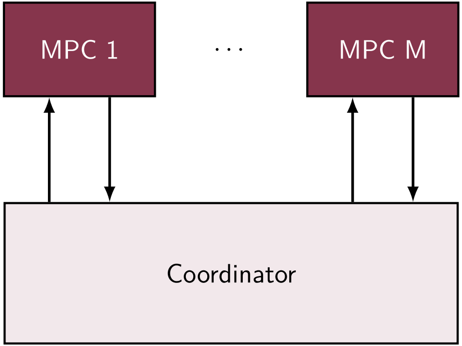
Let's have a preview!

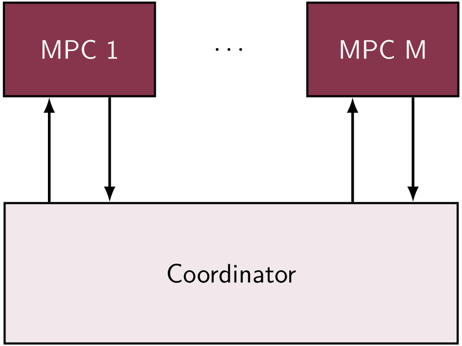
How can a non-cooperative agent attack?

Literature



How can a non-cooperative agent attack?

Literature



- [Vel+17a; CMI18] present attacks

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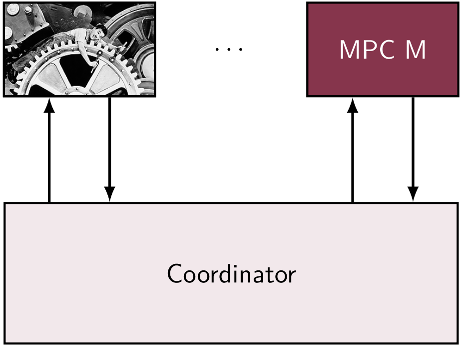

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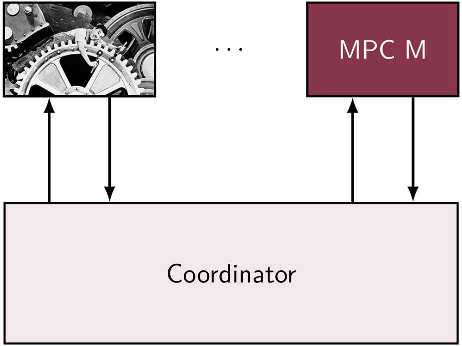
How can a non-cooperative agent attack?

Literature



- [Vel+17a; CMI18] present attacks
  - Fake objective function
  - Fake constraints
  - Use different control

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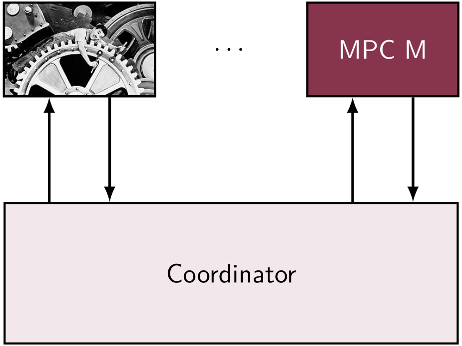

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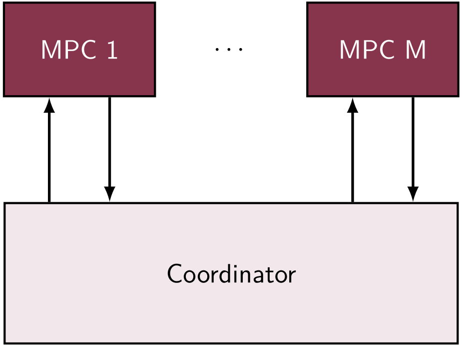
Deception Attacks

How can a non-cooperative agent attack?

Our approach



- We are in coordinator's shoes

How can a non-cooperative agent attack?

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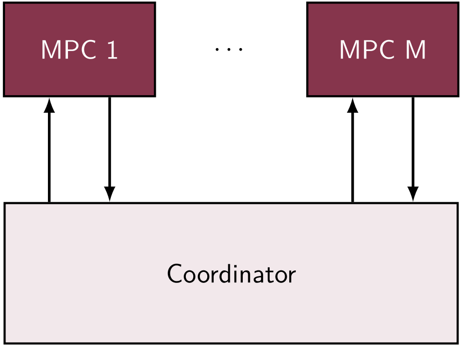

- We are in coordinator's shoes
- What matters is the interface

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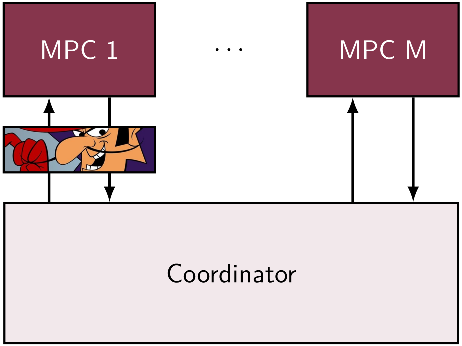

- We are in coordinator's shoes
- What matters is the interface
    - Attacker changes communication

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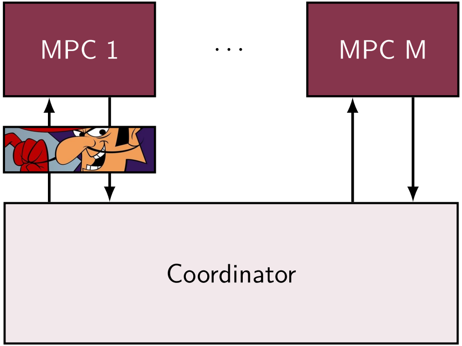

- We are in coordinator's shoes
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    - False Data Injection

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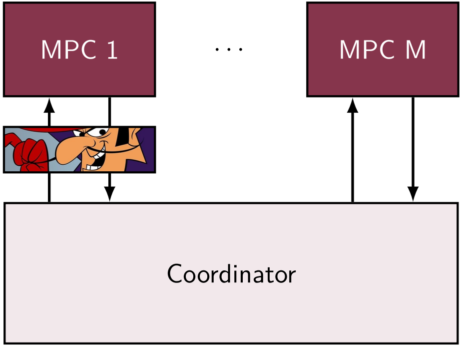

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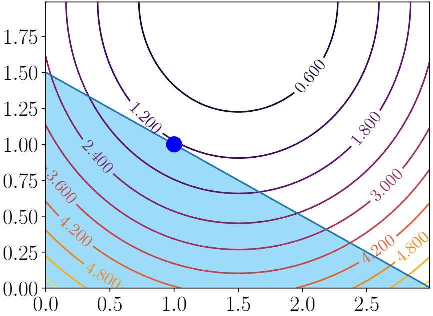
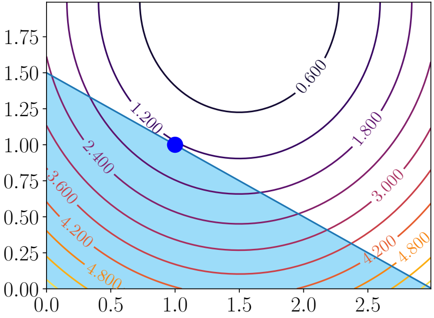
Consequence of an attack



Original minimum.

Consequence of an attack

- Attack modifies optimization problem



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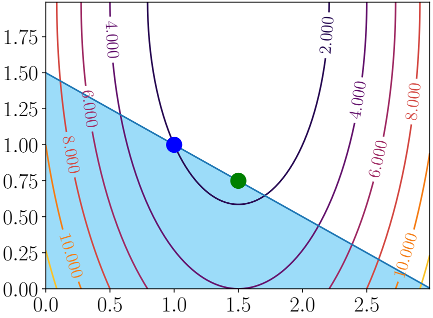
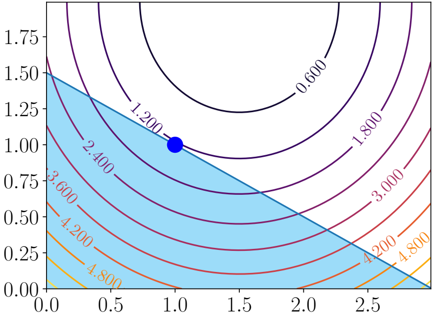


Minimum after attack.

Consequence of an attack

- Attack modifies optimization problem
  - Optimum value is shifted



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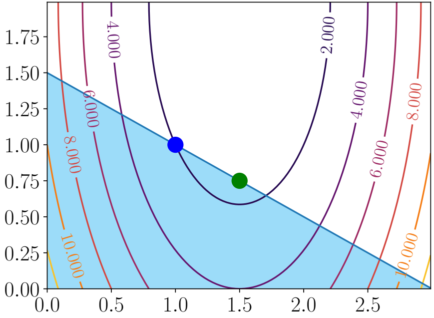

Minimum after attack.

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Mitigating the effects

Mitigating the effects

- We can recover by

Mitigating the effects

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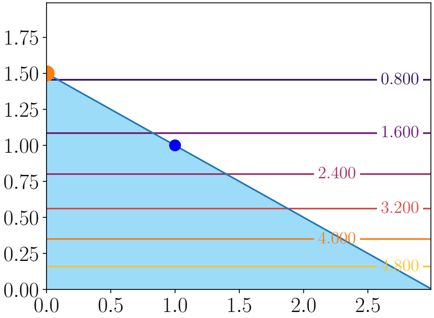

Ignore attacker.

Mitigating the effects

- We can recover by
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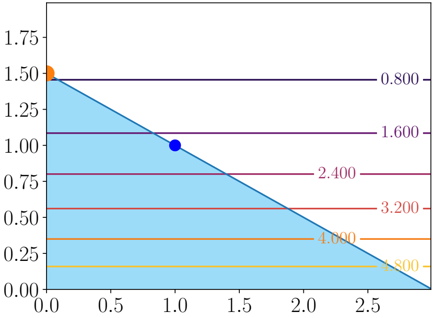
  - Recuperating original behavior (at least trying)



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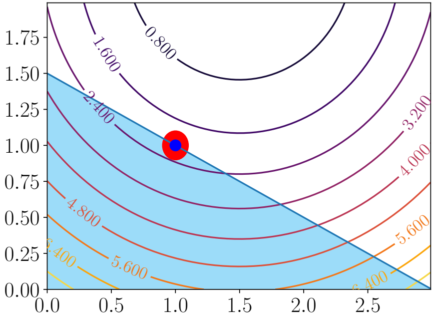

Recover original behavior.

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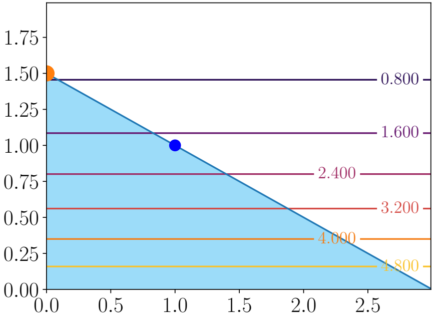

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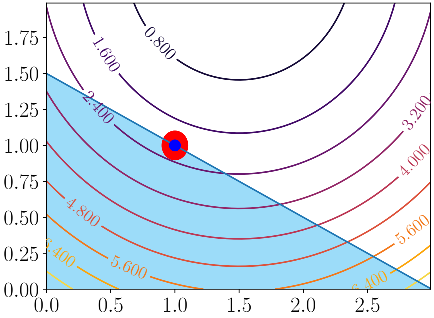

Recover original behavior.

Classification of mitigation techniques

Passive (Robust)

Active (Resilient)

Classification of mitigation techniques

Passive (Robust)

- 1 mode

Active (Resilient)

- 2 modes

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Classification of mitigation techniques

Passive (Robust)

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Active (Resilient)

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  1. Attack free
  2. When attack is detected

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State of art

Security dMPC

	Decomposition	Resilient/Robust
[Vel+17a] [Mae+21]	Dual	Robust (Scenario)
[Vel+17b] [Vel+18]	Dual	Robust (f-robust)
[CMI18]	Jacobi-Gauß	–
[Ana+18] [Ana+19] [Ana+20]	Dual	Resilient

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Our	Primal	Resilient

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State of art

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Our	Primal	Resilient	Active Analyt./Learn.	Data reconstruction

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State of art

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Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition

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2. Resilient Primal Decomposition-based dMPC for deprived systems

Outline

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1. Vulnerabilities in distributed MPC based on Primal Decomposition
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4. Conclusion

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1. Vulnerabilities in distributed MPC based on Primal Decomposition
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- 1 and 2 yielded [NBG21] (SysTol'21)

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1. Vulnerabilities in distributed MPC based on Primal Decomposition
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Simon Leglaive
AIMAC Team

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Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition
   - What is the Primal Decomposition?
   - How can an agent attack?
   - Consequences

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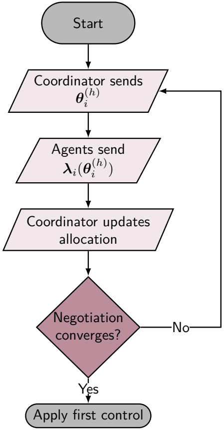
Primal Decomposition

or Quantity Decomposition | or Resource Allocation



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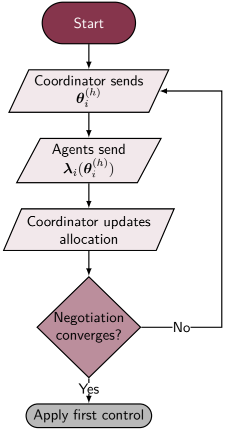

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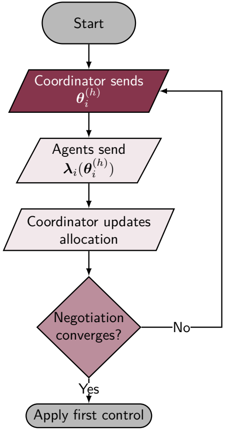

Allocation $\boldsymbol{\theta}_i$

Primal Decomposition

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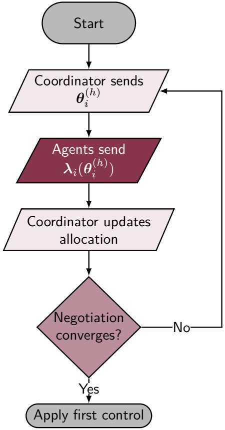

Allocation $\boldsymbol{\theta}_i$

Dissatisfaction $\boldsymbol{\lambda}_i$

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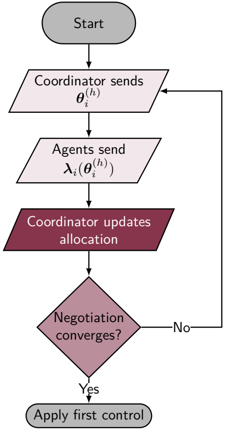

Allocation $\boldsymbol{\theta}_i$

Dissatisfaction $\boldsymbol{\lambda}_i$

Update $\boldsymbol{\theta}_i^{+} = f_i(\boldsymbol{\theta}_i, \boldsymbol{\lambda}_i)$

Primal Decomposition

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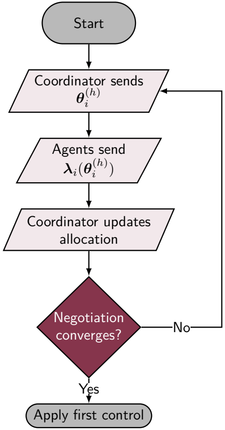

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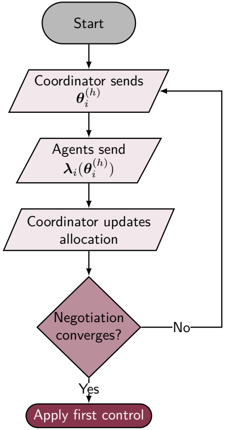

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Primal Decomposition

In detail

$$\begin{aligned} \underset{\boldsymbol{u}_1,\dots,\boldsymbol{u}_M}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}} J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{u}_{\text{total}} \end{aligned}$$

Primal Decomposition

In detail

- Objective is sum of local ones

$$\begin{array}{ll} \underset{\boldsymbol{u}_1,\ldots,\boldsymbol{u}_M}{\text{minimize}} & \sum_{i\in\mathcal{M}} J_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \\ \text{s.t.} & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \preceq \boldsymbol{u}_{\text{total}} \end{array}$$

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Primal Decomposition

In detail

- Objective is sum of local ones
- Constraints couple variables

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$$\Big\downarrow \text{ For each } i \in \mathcal{M}$$

$$\begin{aligned} \underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \\ \text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i \end{aligned}$$

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Primal Decomposition

In detail

- Objective is sum of local ones
- Constraints couple variables

1. Allocate $\boldsymbol{\theta}_i$ for each agent

$$\begin{aligned} \underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i \end{aligned}$$

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Primal Decomposition

In detail

- Objective is sum of local ones
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1. Allocate $\boldsymbol{\theta}_i$ for each agent
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$$\begin{aligned} \underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{aligned}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)}$$

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Primal Decomposition

In detail

- Objective is sum of local ones
- Constraints couple variables

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3. Send dual variable $\boldsymbol{\lambda}_i$
4. Allocation is updated (respecting global constraint)

$$\begin{aligned} \underset{\boldsymbol{u}_i}{\text{minimize}} \quad & J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \\ \text{s. t.} \quad & \boldsymbol{h}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \preceq \boldsymbol{\theta}_i : \boldsymbol{\lambda}_i \end{aligned}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \text{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)})$$

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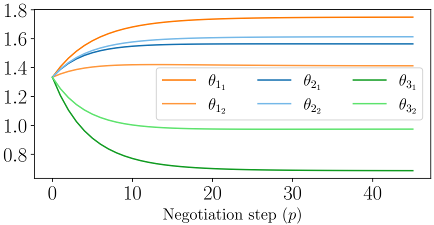
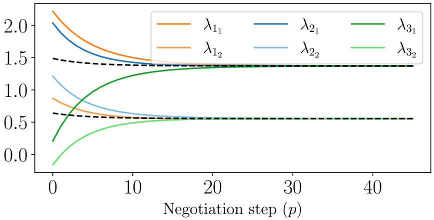
Example

Until everybody is equally dissatisfied





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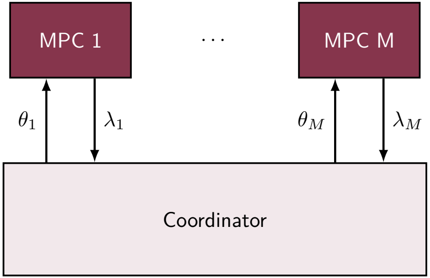
How can a non-cooperative agent attack?

Our approach



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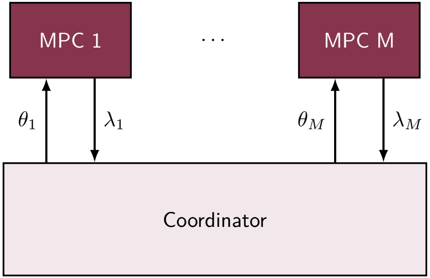

- $\boldsymbol{\lambda}_i$ is the only interface
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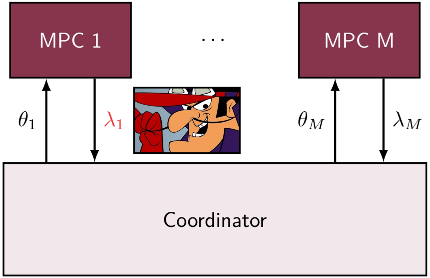
How can a non-cooperative agent attack?

Our approach



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- Malicious agent modifies $\boldsymbol{\lambda}_i$

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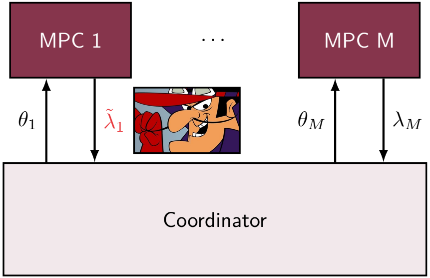

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- $\boldsymbol{\lambda}_i$ depends on local parameters
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$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$$

How does an agent lie?

Liar, Liar, Pants of fire

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Liar, Liar, Pants of fire

- $\lambda \geqslant 0$ means dissatisfaction

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Assumptions

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Assumptions

- *Same attack during negotiation*

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  - $\gamma(\lambda) = 0 \rightarrow \lambda = 0$

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- Attack is invertible

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- Attack is invertible $\rightarrow \exists T_i[k]^{-1}$

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How does an agent lie?

Liar, Liar, Pants of fire

- $\lambda \geqslant 0$ means dissatisfaction
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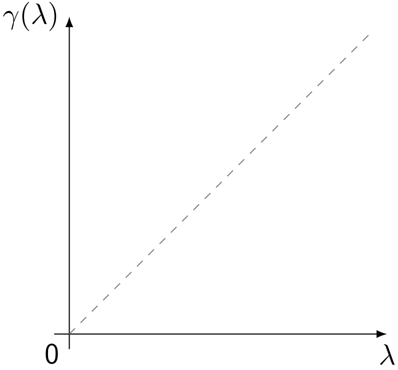

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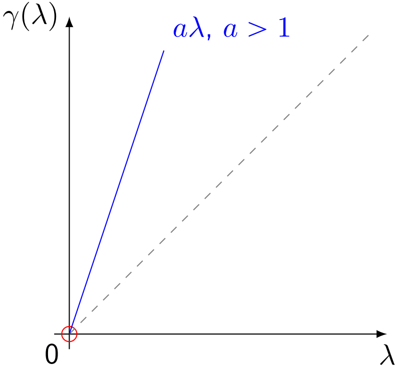

Example

Example

4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\boldsymbol{\lambda}}_1 = \gamma_1(\boldsymbol{\lambda}_1) = \tau_1 I \boldsymbol{\lambda}_1$

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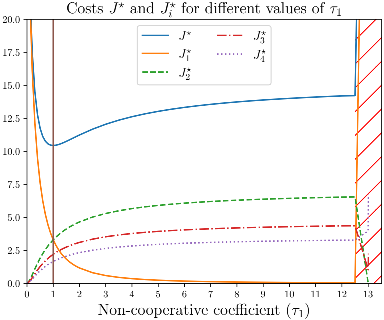

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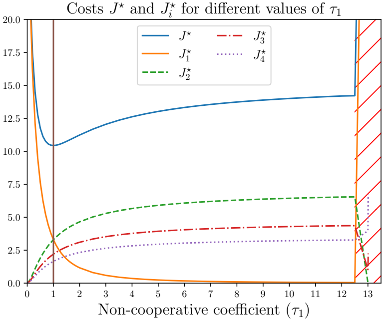

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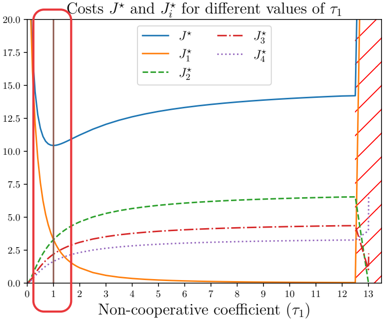

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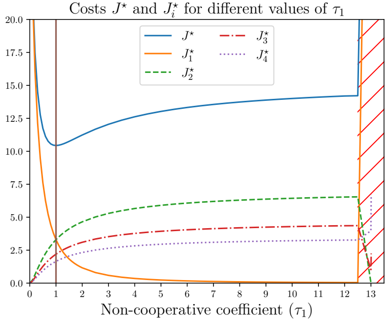

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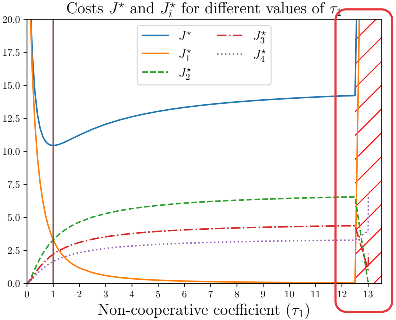

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  - All collapses if too greedy

- But can we mitigate these effects?

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- But can we mitigate these effects?
- Yes!

- But can we mitigate these effects?
- Yes! (At least in some cases)

Outline

2 Resilient Primal Decomposition-based dMPC for deprived systems
- Analyzing deprived systems
- Building an algorithm
- Applying mechanism

What are deprived systems?

What are deprived systems?

Systems whose optimal solution has all constraints active

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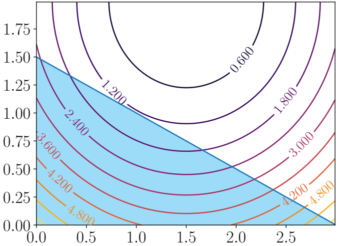
Systems whose optimal solution has all constraints active



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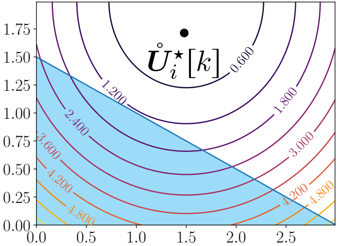

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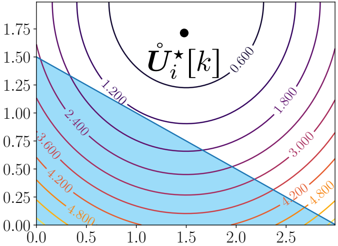

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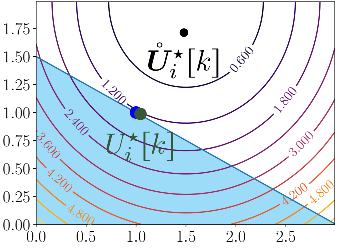

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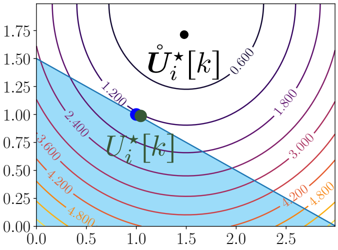
  - Same as with equality constraints[2]



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[2] If system can have all constraints active simultaneously see here

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Deprived Systems

Analysis

Assumptions

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- *Quadratic local problems*

Deprived Systems

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Deprived Systems

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$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

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(local parameters unknown by coordinator) $\left\{ \right.$

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Deprived Systems

Under attack!

- Normal behavior

Deprived Systems

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Deprived Systems

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$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

- But wait!

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- But wait! P_i is not supposed to change!
- Change $\rightarrow$ Probably an Attack! Let's take advantage of this!

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Detection Mechanism

Detection Mechanism

- We estimate[3] $\hat{P}_i[k]$ and $\widehat{\tilde{\boldsymbol{s}}}_i[k]$ such as:

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[3] Using Recursive Least Squares for example

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- If $\left\|\widehat{\tilde{P}}_i[k] - \bar{P}_i\right\|_F > \epsilon_P \rightarrow$ Attack
- Ok, but how can we estimate $\widehat{\tilde{P}}_i[k]$?

[3] Using Recursive Least Squares for example

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Estimating $\widehat{\tilde{P}}_i[k]$

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- Solution: Send a random[4] sequence to increase excitation until convergence.

[4] A random signal causes persistent excitation of any order (Adaptive Control)

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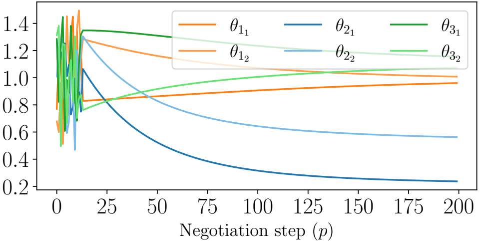

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Classification of mitigation techniques

- Active (Resilient)
  1. Detection/Isolation ✔
  2. Mitigation

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Mitigation mechanism

Reconstructing $\boldsymbol{\lambda}_i$

- Now, we have $\widehat{\tilde{P}}_i[k]$

Mitigation mechanism

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Mitigation mechanism

Reconstructing $\boldsymbol{\lambda}_i$

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$$\widehat{T_i[k]^{-1}} = P_i \widehat{\tilde{P}}_i[k]^{-1}$$

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Mitigation mechanism

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- Reconstruct $\boldsymbol{\lambda}_i$

$$\overset{\text{rec}}{\boldsymbol{\lambda}}_i = -\bar{P}_i \boldsymbol{\theta}_i - \widehat{T_i[k]^{-1}} \widehat{\tilde{\boldsymbol{s}}}_i[k]$$

Mitigation mechanism

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- Choose adequate version for coordination

$$\overset{\text{mod}}{\boldsymbol{\lambda}}_i = \begin{cases} \overset{\text{rec}}{\boldsymbol{\lambda}}_i, & \text{if attack detected} \\ \tilde{\boldsymbol{\lambda}}_i, & \text{otherwise} \end{cases}$$

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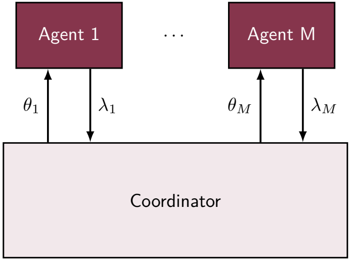
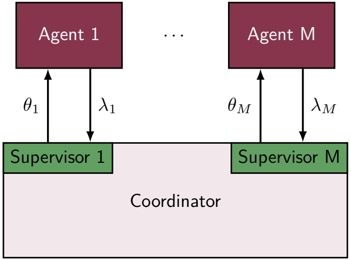
Complete Mechanism

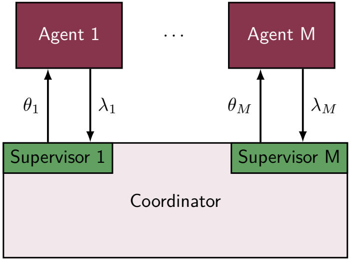


Complete Mechanism



- Supervise exchanges by inquiring the agents

Complete Mechanism



- Supervise exchanges by inquiring the agents
- Estimate how they will behave

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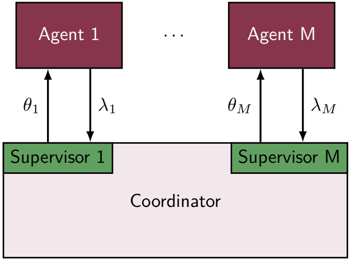

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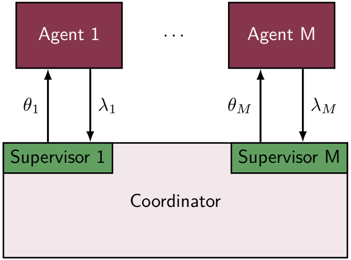
Two Phases

Complete Mechanism



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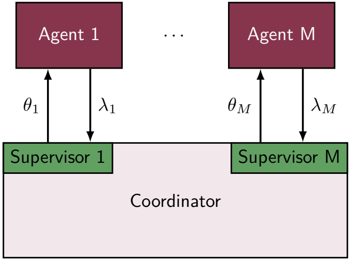
Two Phases

1. Detect which agents are non-cooperative

Complete Mechanism



- Supervise exchanges by inquiring the agents
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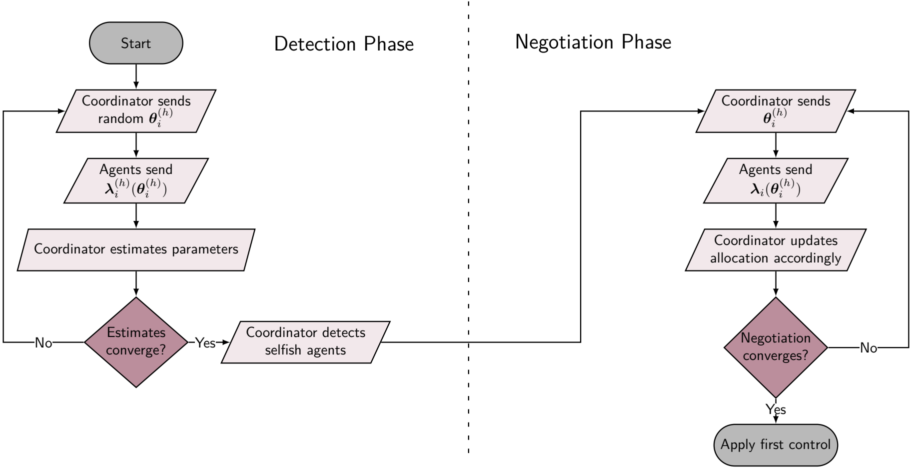
Two Phases

1. Detect which agents are non-cooperative
2. Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

Complete algorithm

RPdMPC-DS



Complete algorithm

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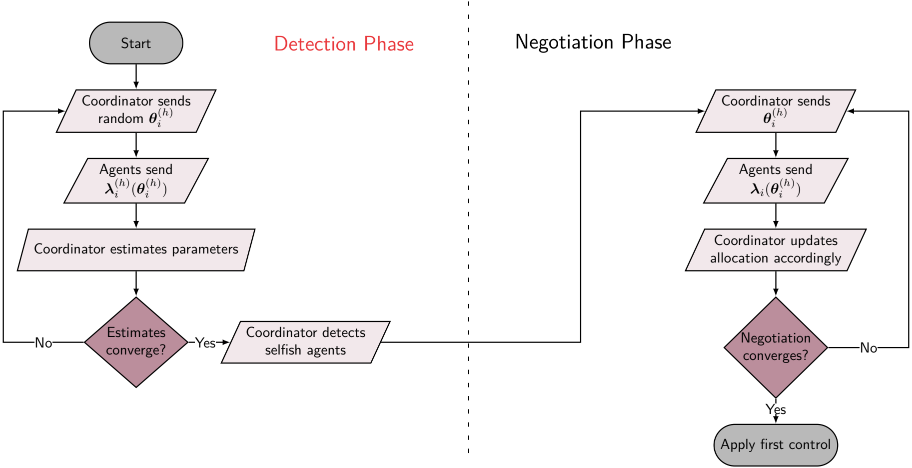

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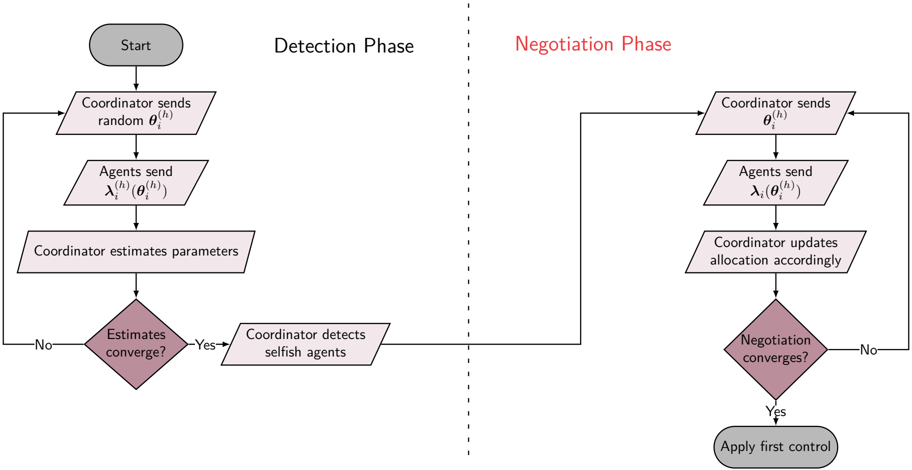

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Complete algorithm

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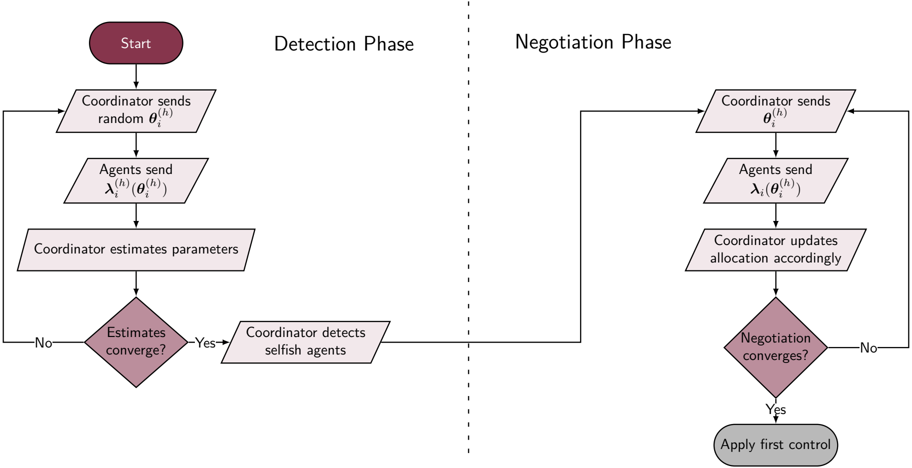

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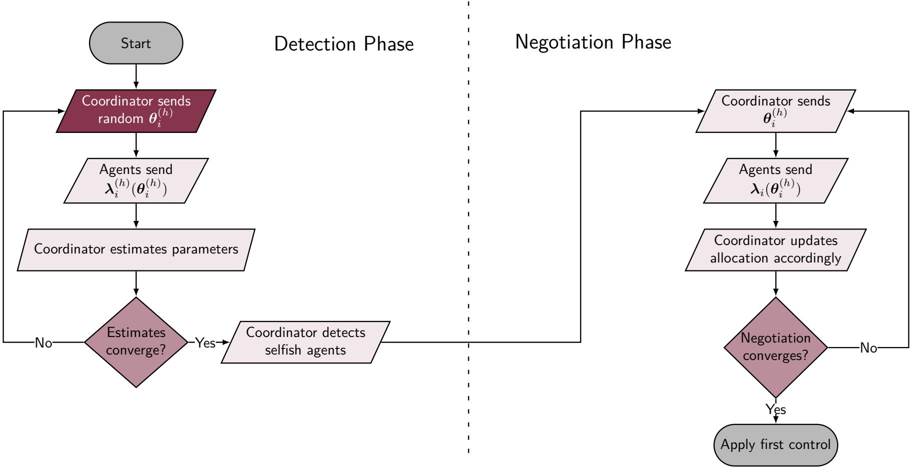

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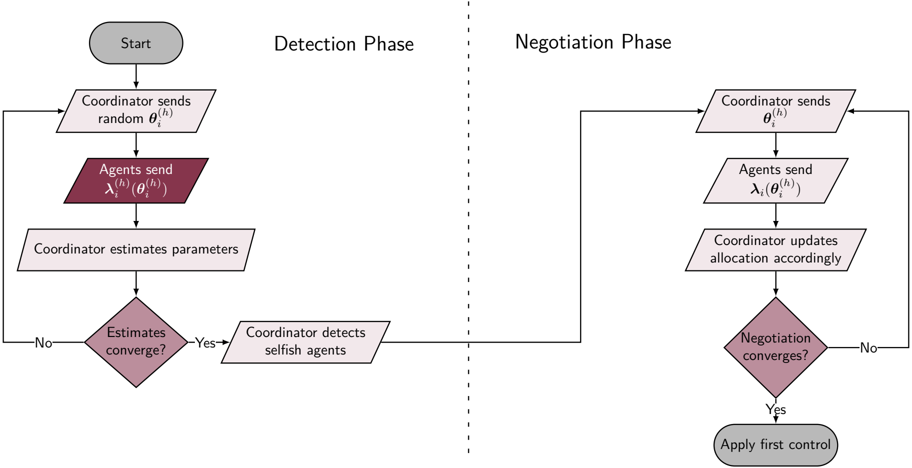

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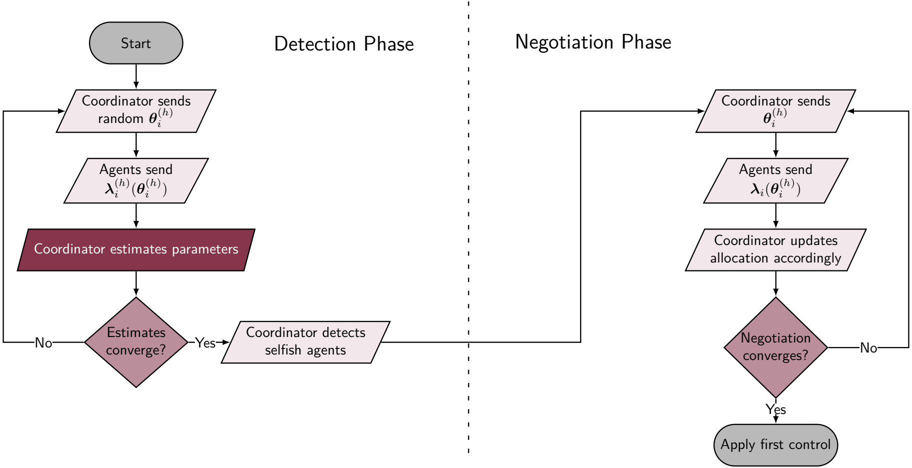

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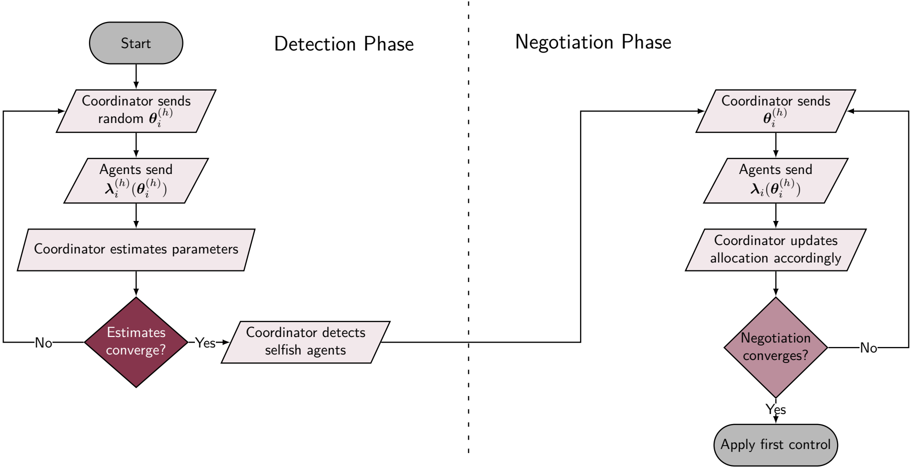

CentraleSupélec

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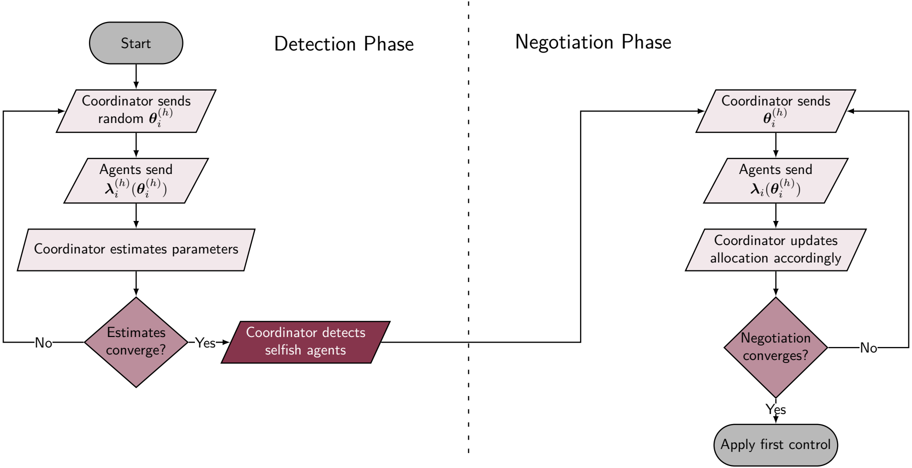

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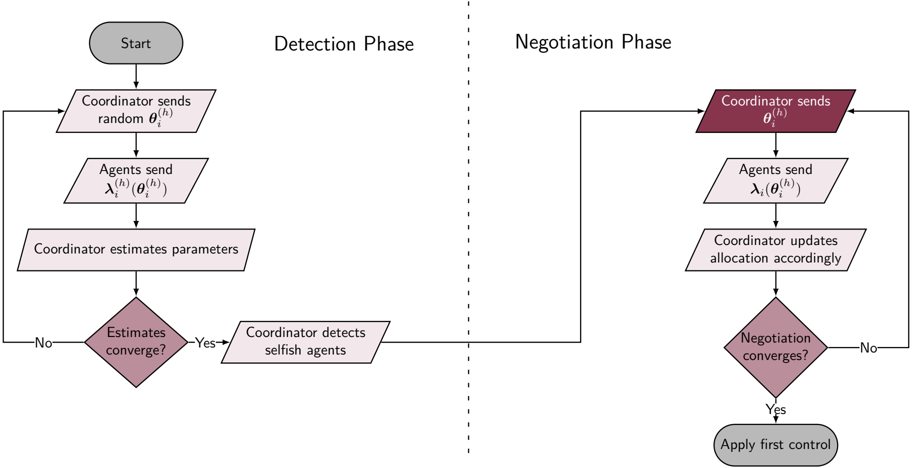

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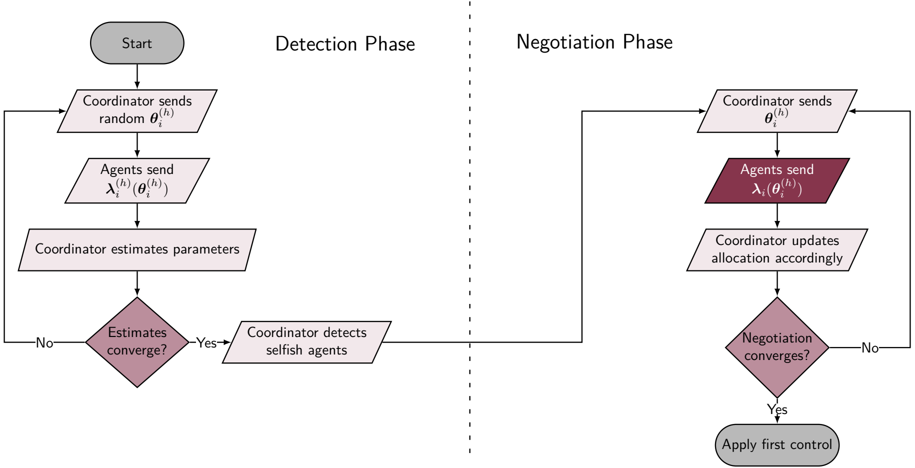

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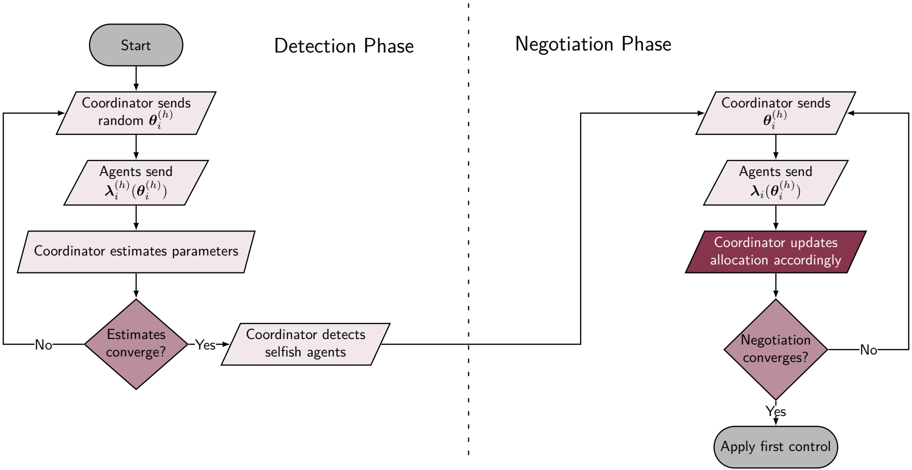

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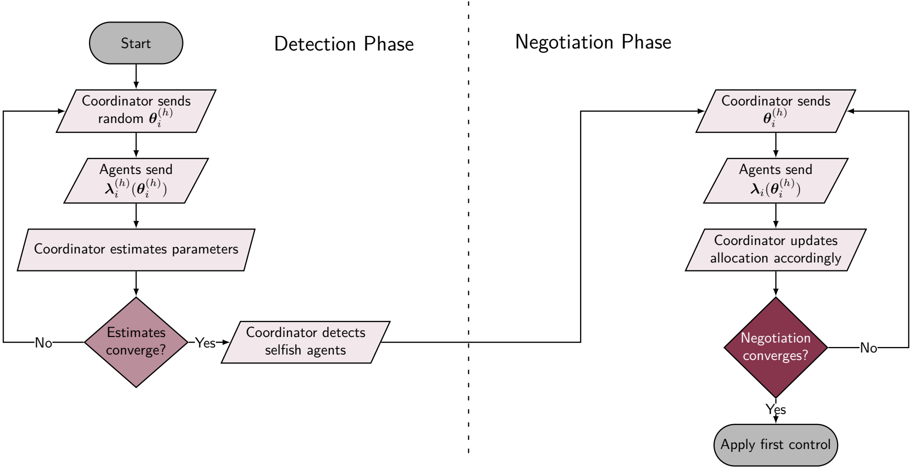

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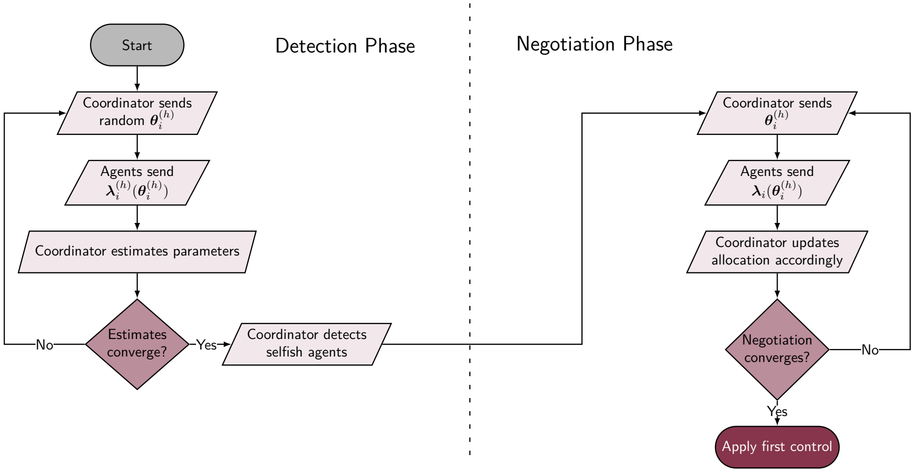
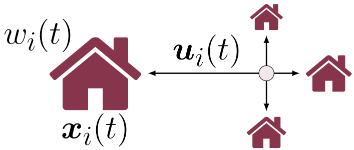
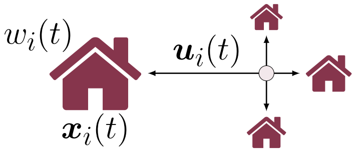


Example



District Heating Network (4 Houses)

Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)

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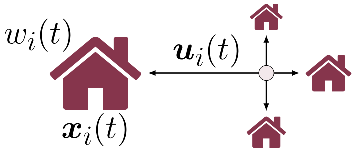

District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)
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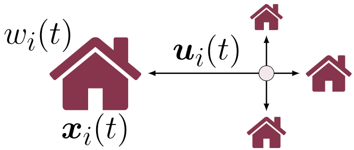
Example



District Heating Network (4 Houses)

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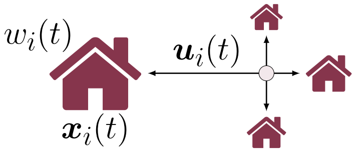

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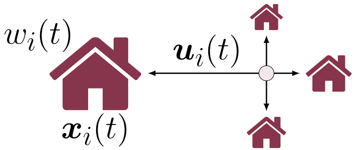
Example



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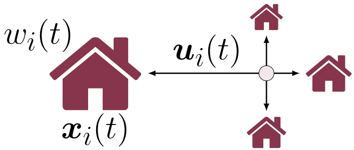

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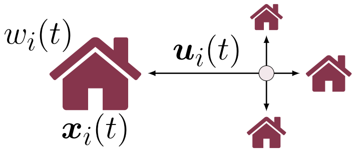

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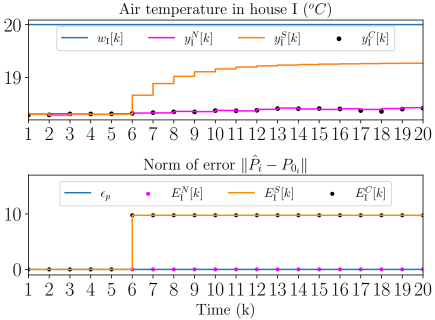
  - S Agent I cheats (RPdMPC-DS)

Results

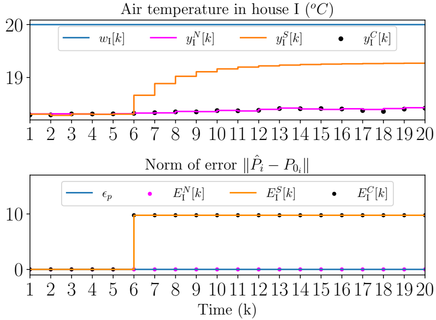
Temporal



Temperature in house I.
Error $E_I(k)$.
N Nominal, S Selfish, C Corrected

Results

Temporal



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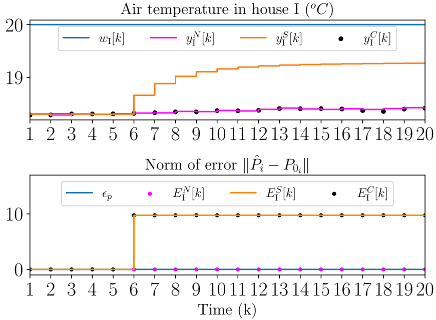

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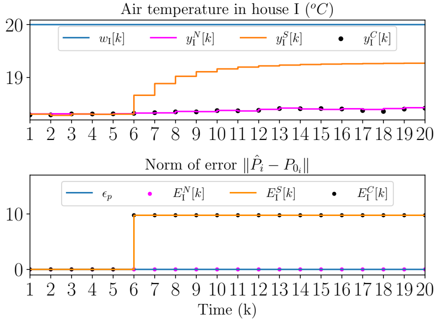

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Results

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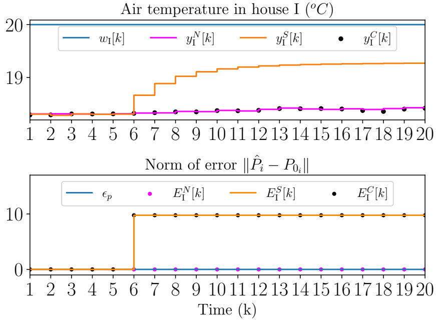

Temperature in house I.
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- S Agent increases its comfort
- C Restablish behavior close to N

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Results

Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.3	0.503
II	21.671	-0.547
III	17.387	-0.004
IV	17.626	-0.09
Global	3.526	0.016

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Outline

3 Resilient Primal Decomposition-based dMPC using Artificial Scarcity
- Relaxing some assumptions
- Adapting the algorithm
- Applying mechanism

Relaxing scarcity assumption

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- Systems are not completely deprived

Relaxing scarcity assumption

- Systems are not completely deprived
  - We can't change our constraints to equality ones anymore

$$\begin{aligned} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} \quad & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T \boldsymbol{U}_i[k] \\ \text{subject to} \quad & \bar{\Gamma}_i \boldsymbol{U}_i[k] \preceq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{aligned}$$

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Relaxing scarcity assumption

- Systems are not completely deprived
  - We can't change our constraints to equality ones anymore
  - Nor use the simpler update equation

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i\boldsymbol{U}_i[k] \preceq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

Analyzing System

Solution for $\lambda_i[k]$

Instead of having one single affine solution

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

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Now, we may have multiple (Piecewise affine function)

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Still the $P_i^{(n)}$ are time independent

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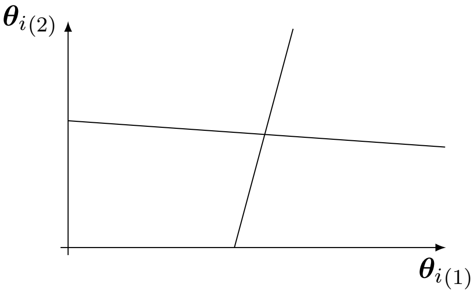
Analyzing System

Solution for $\lambda_i[k]$ (Continued)

Analyzing System

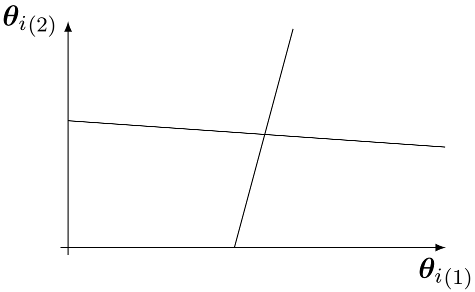
Solution for $\lambda_i[k]$ (Continued)



Separation surfaces depend on state and local parameters.

Analyzing System

Solution for $\lambda_i[k]$ (Continued)



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Unknown by the coordinator.

Analyzing System

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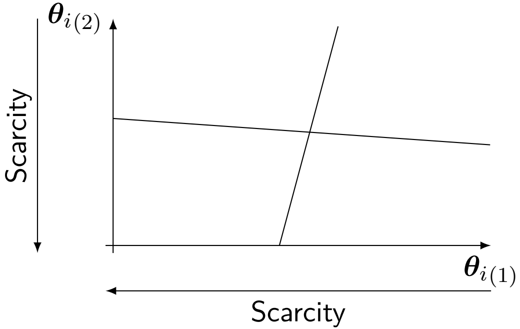

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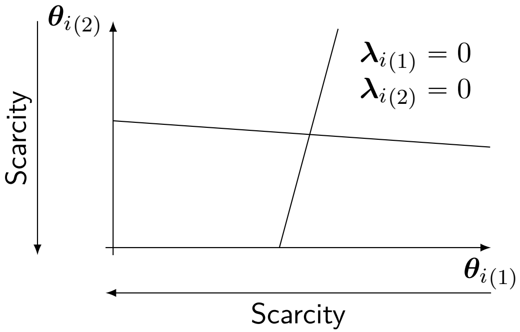

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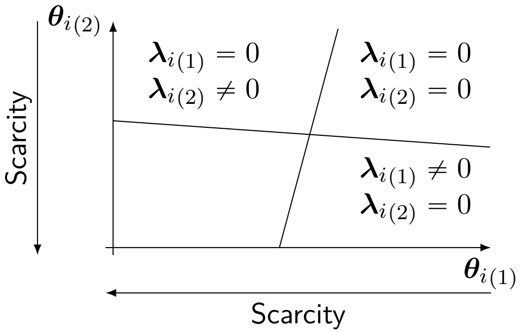

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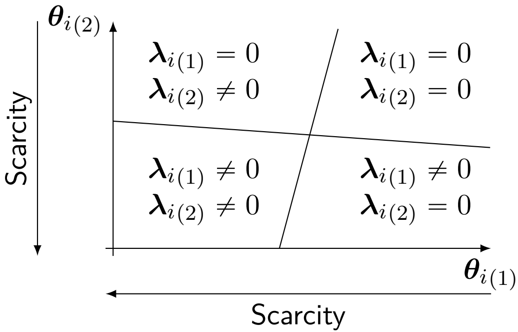

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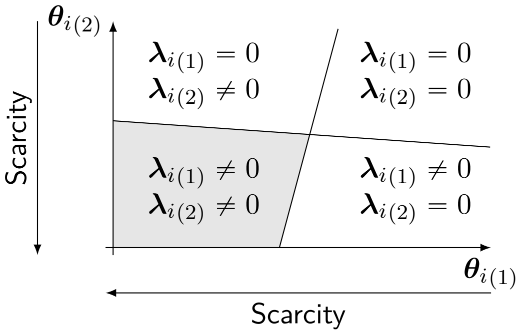

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$\uparrow$ Scarcity $\downarrow$ Sparsity

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Scarcity $\uparrow$ Sparsity $\downarrow$

All constraints active $\quad -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k] \quad \rightarrow \quad -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$

Analyzing System

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None constraints active $\quad -P_i^{\left(2^{n_{\text{ineq}}}-1\right)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{\left(2^{n_{\text{ineq}}}-1\right)}[k] \rightarrow \mathbf{0}$

Analyzing System

Solution for $\lambda_i[k]$ (Continued) Still?

$$\boldsymbol{\lambda}_i[k] = \begin{cases} -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^0_{\boldsymbol{\lambda}_i} \\ \vdots & \vdots \\ -P_i^{(2^{n_\text{ineq}}-1)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(2^{n_\text{ineq}}-1)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^{2^{n_\text{ineq}}-1}_{\boldsymbol{\lambda}_i} \end{cases}$$

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Assumptions

The region $\mathcal{R}^0_{\boldsymbol{\lambda}_i} \neq \varnothing$ and we known a point $\overset{\varnothing}{\boldsymbol{\theta}}_i \in \mathcal{R}^0_{\boldsymbol{\lambda}_i}$

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Analyzing System

Under attack!

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$$\tilde{\boldsymbol{\lambda}}_i[k] = T_i[k]\boldsymbol{\lambda}_k$$

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Parameters are modified.

$$\tilde{\boldsymbol{\lambda}}_i[k] = \begin{cases} -\widetilde{P}_i^{(0)}\boldsymbol{\theta}_i[k] - \widetilde{\boldsymbol{s}}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}^0 \\ \vdots & \vdots \\ -\widetilde{P}_i^{(2^{n_{\text{ineq}}}-1)}\boldsymbol{\theta}_i[k] - \widetilde{\boldsymbol{s}}_i^{(2^{n_{\text{ineq}}}-1)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}_{\boldsymbol{\lambda}_i}^{2^{n_{\text{ineq}}}-1} \end{cases}$$

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Under attack!

$$\tilde{\boldsymbol{\lambda}}_i[k] = T_i[k]\boldsymbol{\lambda}_k$$

Parameters are modified. But not the regions' limits

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- If we can estimate $\widetilde{P}_i^{(0)}$ we can use same strategy than before

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- If we can estimate $\widetilde{P}_i^{(0)}$ we can use same strategy than before
- Problem: We don't know in which region $\boldsymbol{\theta}_i$ is
- Solution: Let's force it using Artificial Scarcity

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Artificial Scarcity

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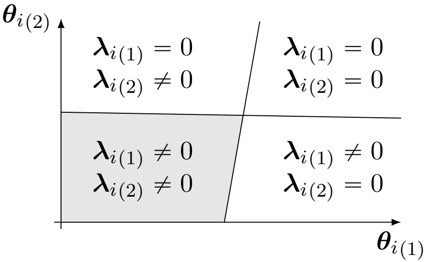

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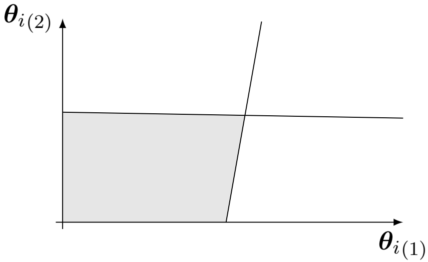

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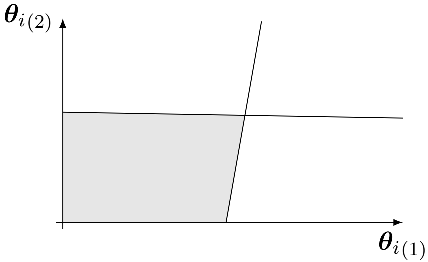

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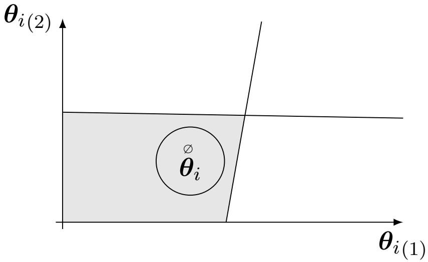

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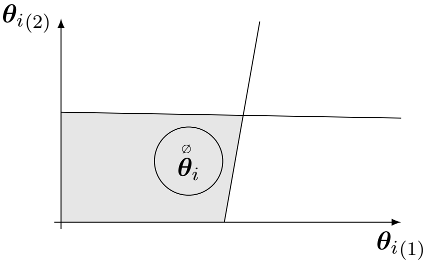

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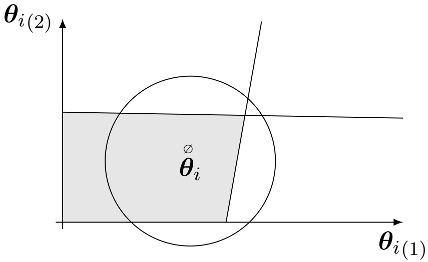

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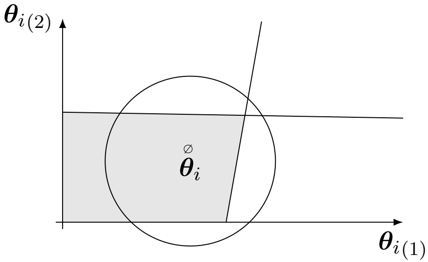

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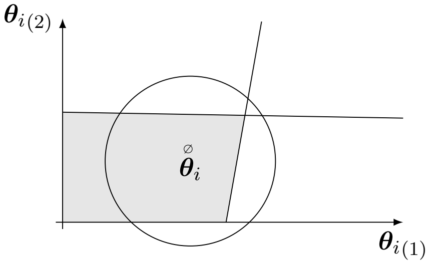

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- How do we known the radius?
  - Unfortunately we don't.
- How to estimate $\widehat{\widetilde{P}}_i^{(0)}[k]$ nonetheless?

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Enter Expectation Maximization

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- Iterative method to estimate parameters of multimodal models

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[6] Such as our PWA function after using some tricks

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- We recover the associated parameter, i.e., $\widehat{\widetilde{P}}_i^{(0)}[k]$

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Detection and Mitigation

Same same, but different

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Assumption

We estimate nominal $\bar{P}_i^{(0)}$ *from attack free negotiation*

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$$\left\| \widehat{\widetilde{P}}_i^{(0)}[k] - \bar{P}_i^{(0)} \right\|_F \geqslant \epsilon_{P_i^{(0)}}$$

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$$\widehat{T_i[k]^{-1}} = \bar{P}_i^{(0)} \widehat{\widetilde{P}}_i^{(0)}[k]^{-1}.$$

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$$\overset{\text{rec}}{\boldsymbol{\lambda}}_i = \widehat{T_i[k]^{-1}} \tilde{\boldsymbol{\lambda}}_i.$$

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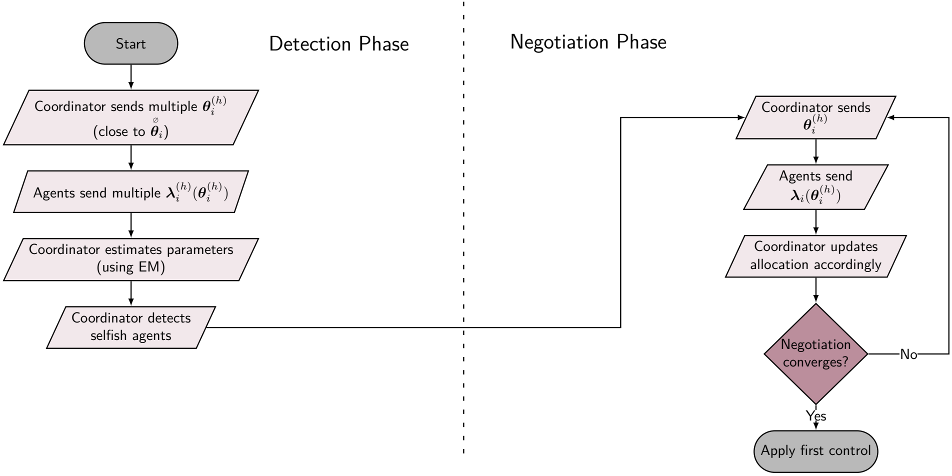
Complete algorithm

RPdMPC-AS



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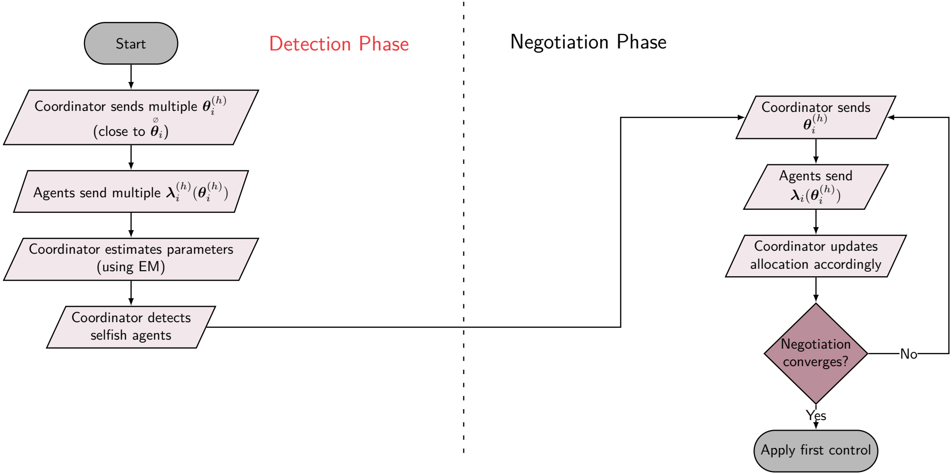

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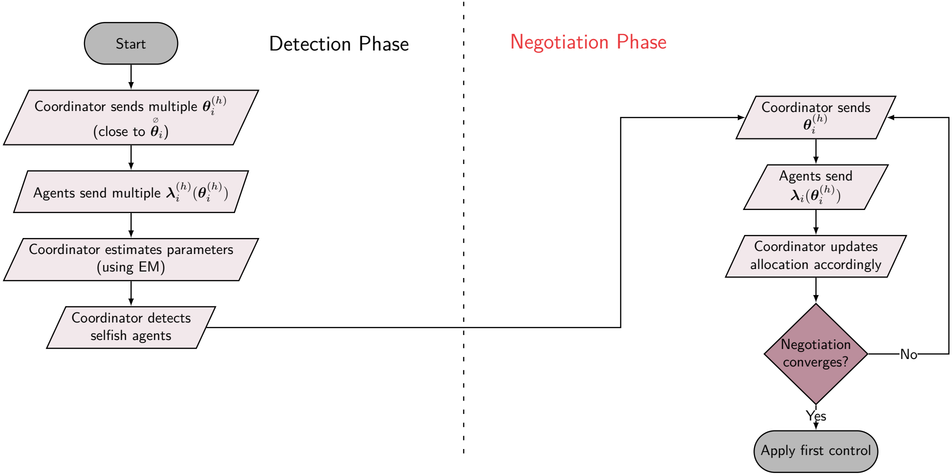

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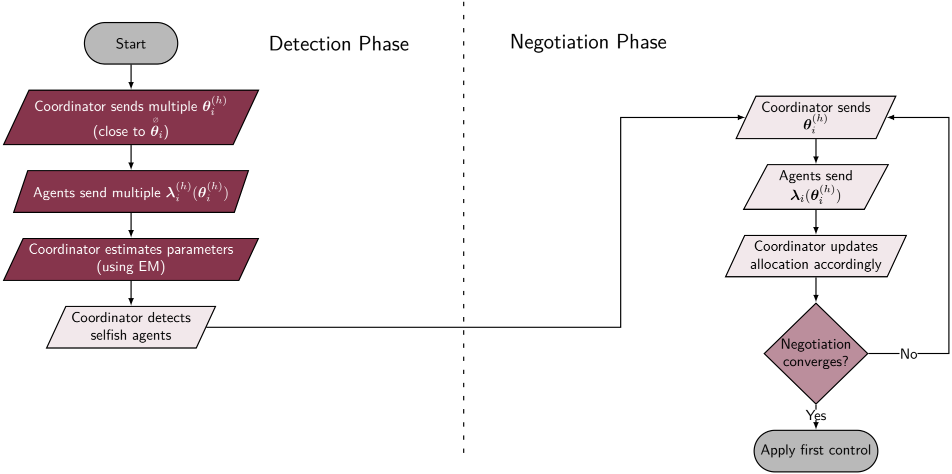

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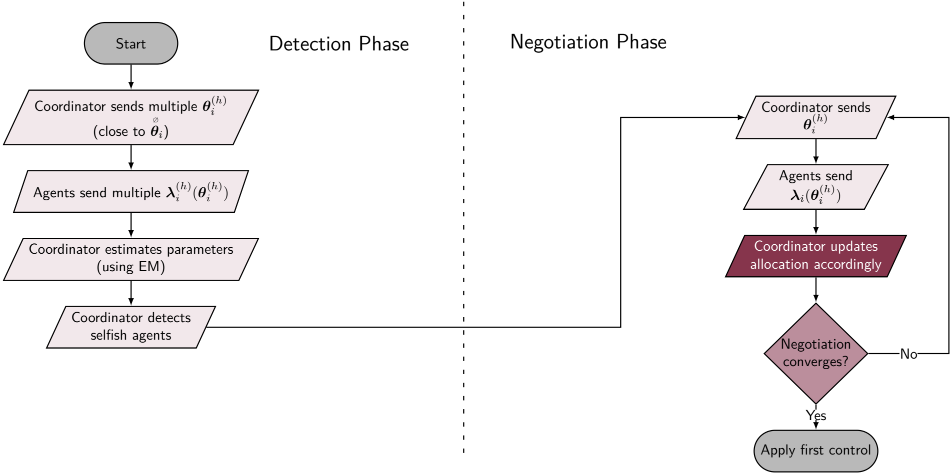
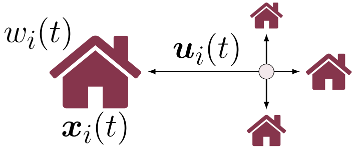


Example



District Heating Network (4 Houses)

- Houses modeled using 3R-2C
- Not enough power
- Period of 5h $(T_s = 0.25h \rightarrow k = \{1 : 20\})$
- 3 scenarios
  - N Nominal
  - C Agent I cheats (dMPC)
  - S Agent I cheats (RPdMPC-AS)

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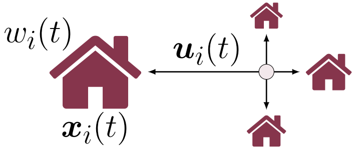

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Results

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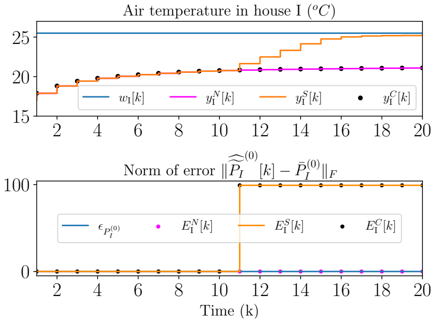

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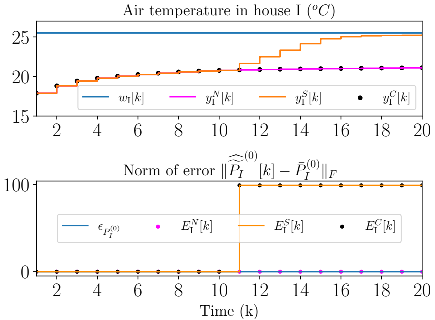

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Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.489	$-4.12e-05$
II	35.813	$1.74e-05$
III	29.225	$2.14e-05$
IV	37.541	$1.73e-05$
Global	10.689	$-6e-07$

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  - Associate with other methods of the same family

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Outline

4. Conclusion

Conclusion

Main takeaways

- How can an agent attack?
- What are the consequences of an attack?
- Can we mitigate the effects?

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    - Suboptimality and maybe instability
- Can we mitigate the effects? ✔
    - Yes! By exploring the scarcity of the systems!

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Conclusion

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- Insights from the analysis of the solutions of the optimization problems:
  - Sensibilities are constant when there is no cheating
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- Exploiting the scarcity of resources, we find how to invert the attack
  - Straightforward if system is completely deprived

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Conclusion

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    - Sensibilities are constant when there is no cheating
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Open question Future Directions

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Questions? Comments?

Repository

https://github.com/Accacio/thesis

Contact

rafael.accacio.nogueira@gmail.com

For Further Reading I

K.J. Åström and B. Wittenmark. Adaptive Control. Addison-Wesley series in electrical and computer engineering: Control engineering. Addison-Wesley, 1989. ISBN: 9780201097207. DOI: 10.1007/978-3-662-08546-2_24.

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For Further Reading II

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For Further Reading III

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Rafael Accácio Nogueira, Romain Bourdais, and Hervé Guéguen. "Detection and Mitigation of Corrupted Information in Distributed Model Predictive Control Based on Resource Allocation". In: 2021 5th Conference on Control and Fault-Tolerant Systems (SysTol). 2021, pp. 329–334. DOI: 10.1109/SysTol52990.2021.9595927.

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Pablo Velarde et al. "Vulnerabilities in Lagrange-Based DMPC in the Context of Cyber-Security". In: 2017 IEEE International Conference on Autonomic Computing (ICAC). July 2017, pp. 215–220. DOI: 10.1109/ICAC.2017.53.

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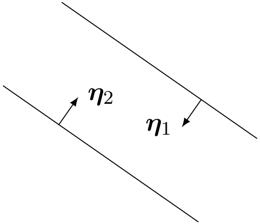
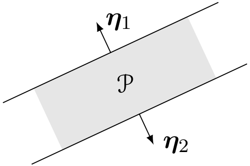
Conditions

back

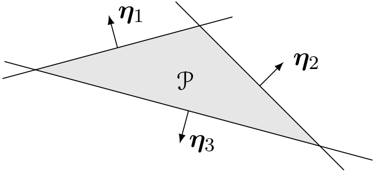
One way to ensure this, is to make the original constraints to form a cone.



No intersection



$\langle_{\boldsymbol{\eta}_1}^{\boldsymbol{\eta}_2} = 180^o$



A 3-sided polyhedron.

θ dynamics

back

$$\boldsymbol{\theta}^{(p+1)} = \mathcal{A}_\theta \boldsymbol{\theta}^{(p)} + \mathcal{B}_\theta[k]$$

where

$$\mathcal{A}_\theta = \begin{bmatrix} I - \frac{M-1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \frac{1}{M}\rho^{(p)}P_1 & I - \frac{M-1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & I - \frac{M-1}{M}\rho^{(p)}P_M \end{bmatrix}$$

$$\mathcal{B}_\theta[k] = \begin{bmatrix} -\frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \vdots \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \end{bmatrix}$$

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