Security of distributed Model Predictive Control under False Data injection

Rafael Accácio NOGUEIRA

2022-12-09

IETR

https://bit.ly/3g3S6X4

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Context

"Necessity is the mother of invention"

Context

"Necessity is the mother of invention"

Context

❝Necessity is the mother of invention❞

- Electricity Distribution System
- Heat distribution
- Water distribution
- Traffic management

  (include your problem here)

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- Multiple systems interacting
- Coupled by constraints
  - Technical/ Comfort
- Optimization objectives
  - Minimize energy consumption
  - Maximize user satisfaction
  - Follow a trajectory
- Solution → MPC

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Find best control sequence using predictions based on a model.

- We need an optimization problem
  - Decision variable is the control sequence
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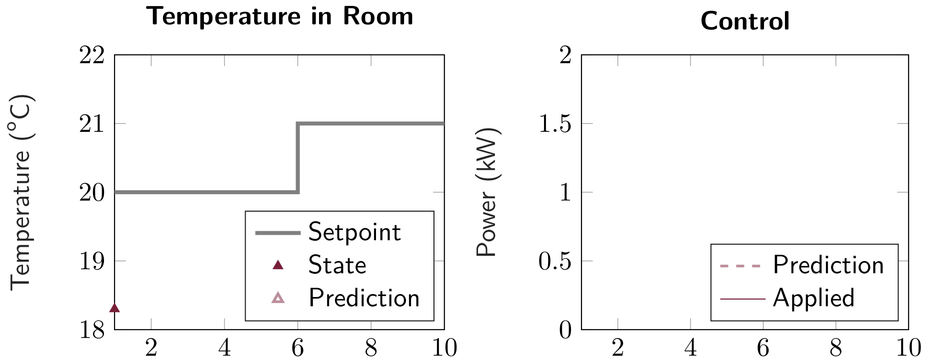
In a nutshell

Find optimal control sequence



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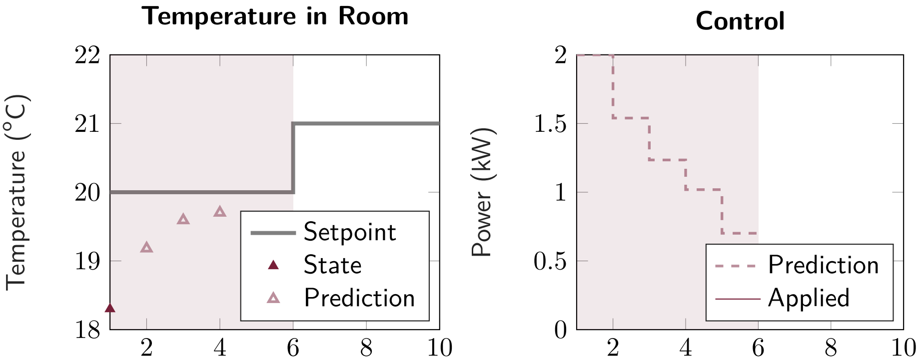
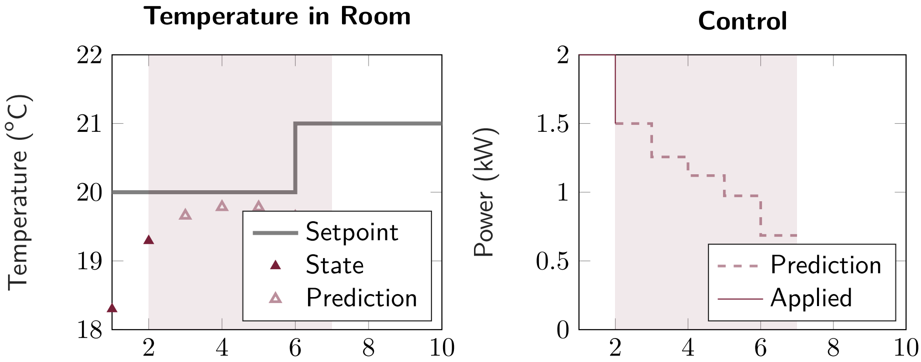


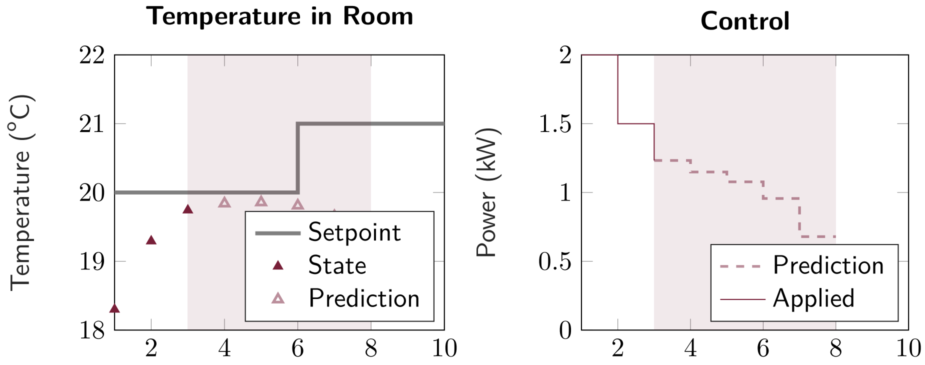
Model Predictive Control

In a nutshell

Find optimal control sequence, apply first element

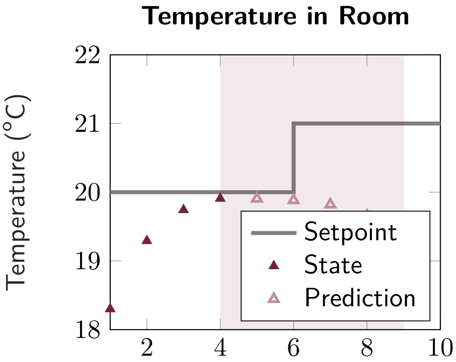


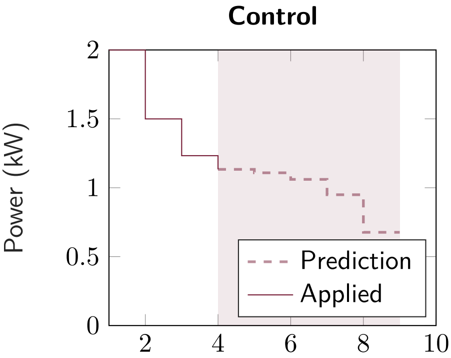
Find optimal control sequence, apply first element, rinse repeat



In a nutshell

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





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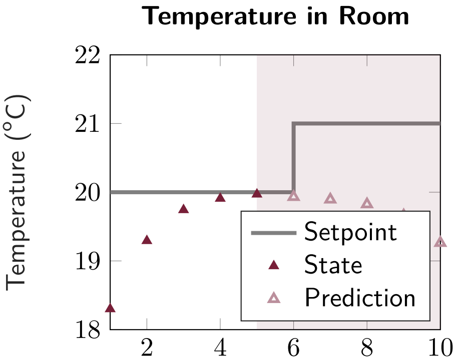

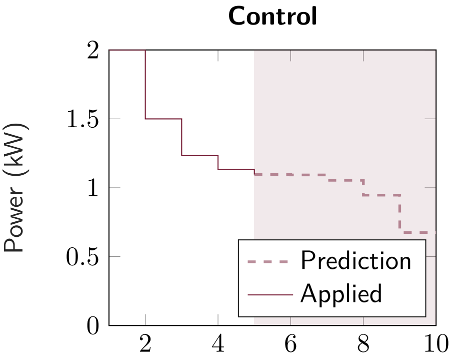

Model Predictive Control

Nothing is perfect

- Problems
  - Complexity of calculation
  - Topology (Geographical distribution)
  - Flexibility (Add/remove parts)
  - Privacy
- Solution: Divide and Conquer (distributed MPC)
  - Break calculation
  - Make Systems Communicate

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Distributed Model Predictive Control

It is about communication

- We break the MPC into multiple
- Make them Communicate
  - Many flavors to choose from[1]
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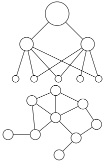

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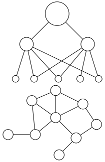

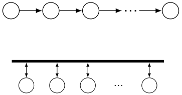

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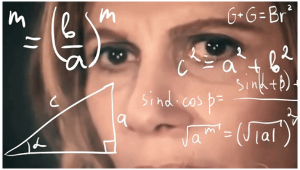

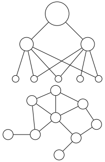

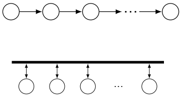

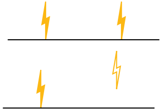

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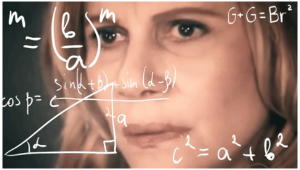

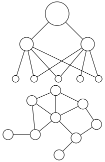

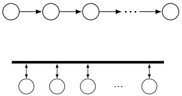

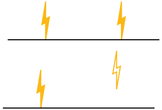

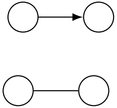

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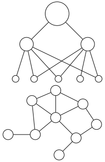
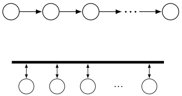
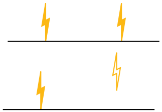
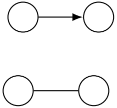

[1] Distributed Model Predictive Control made easy

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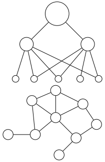
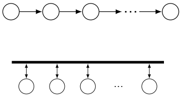
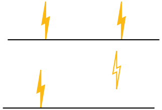
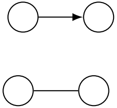
Distributed Model Predictive Control

It is about communication

- We break the MPC into multiple
- Make them Communicate , But how?
  - Many flavors to choose from[1]
    - Hierarchical/Anarchical
    - Sequential/Parallel
    - Synchronous/Asynchronous
    - Bidirectional/Unidirectional
    - ...

[1] Distributed Model Predictive Control made easy

Distributed Model Predictive Control

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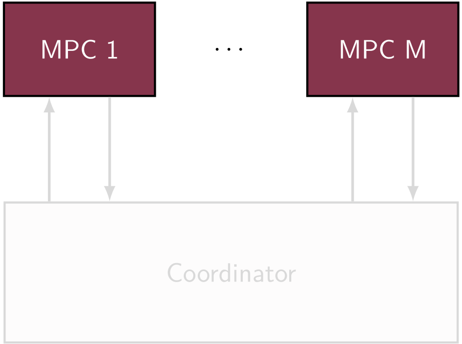

- Coordinator → Hierarchical
- Bidirectional
- No delay → Synchronous
- Agents solve local problems
- Variables are updated

Until Convergence

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Distributed Model Predictive Control

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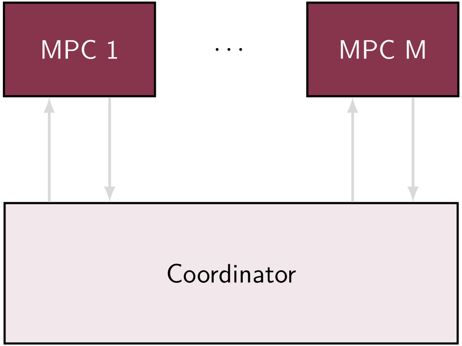

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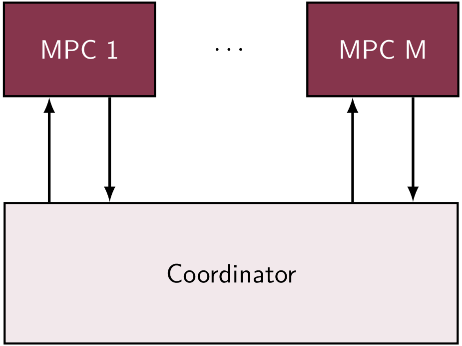

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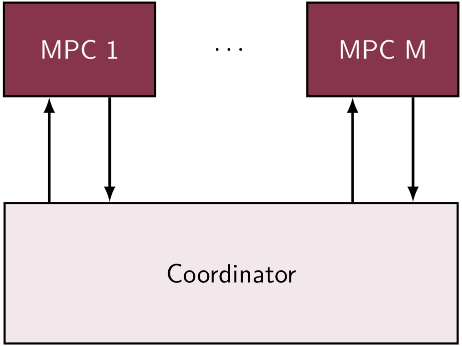

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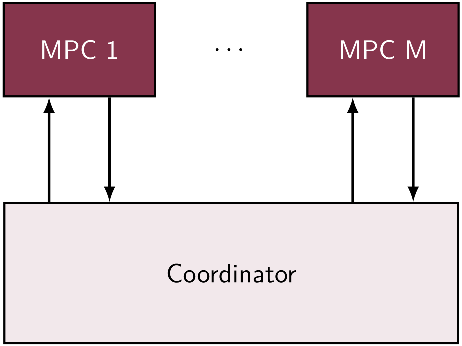

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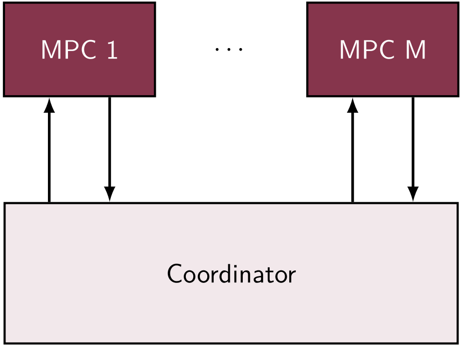

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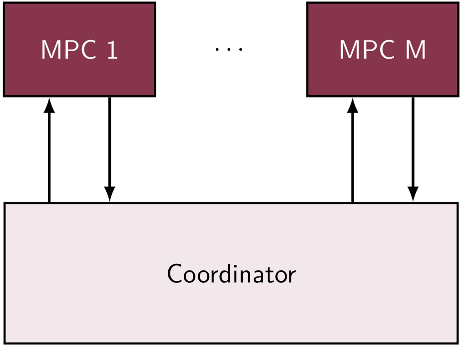

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Distributed Model Predictive Control

Negotiation works if agents comply.
But what if some agents are ill-intentioned and attack the system?

- How can an agent attack?
- What are the consequences of an attack?
- Can we mitigate the effects?

Let's have a preview!

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How can a non-cooperative agent attack?

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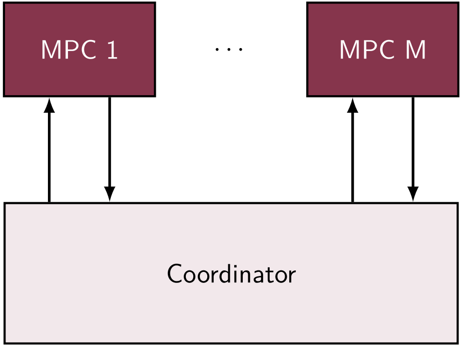

- [Vel+17a; CMI18] present attacks
  - Objective function
    - Selfish Attack
    - Fake weights
    - Fake reference
  - Fake constraints
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Deception Attacks

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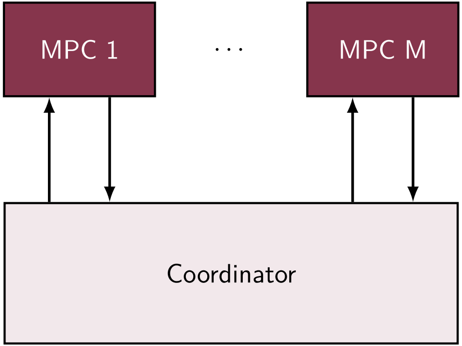

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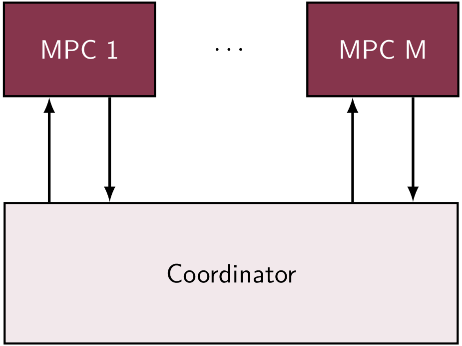

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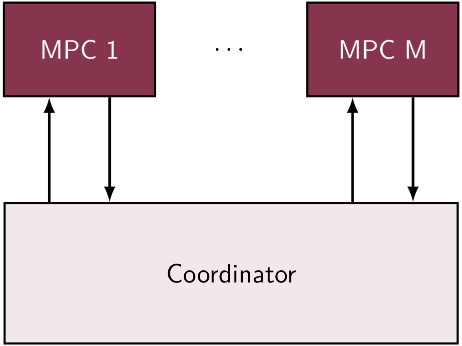

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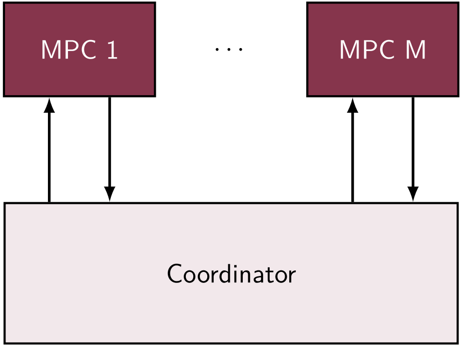

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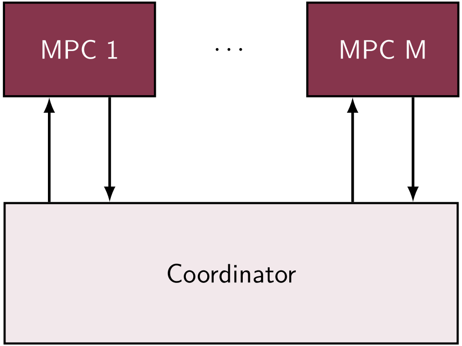

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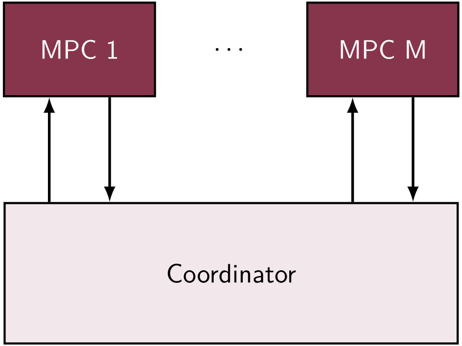

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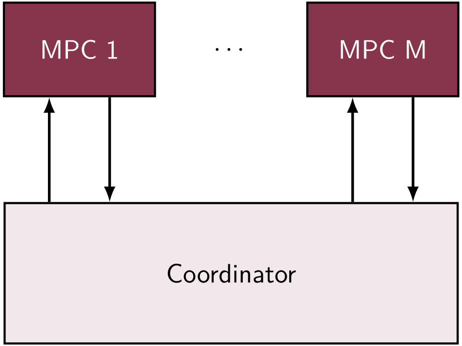

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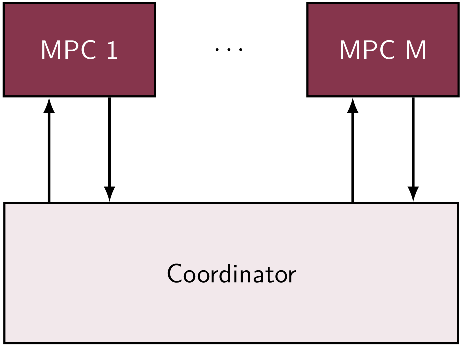

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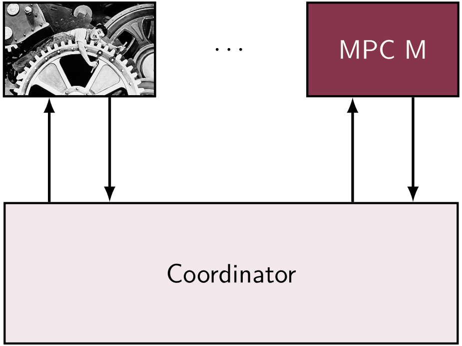

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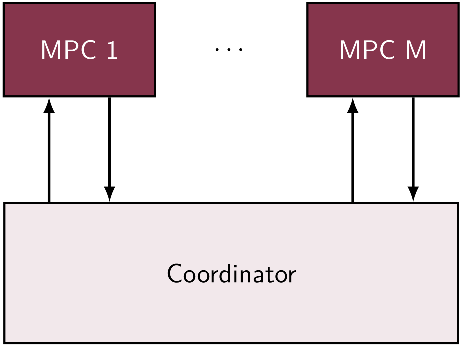
Deception Attacks (Internal change)

How can a non-cooperative agent attack?

Our approach



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- What matters is the interface
  - Attacker changes communication
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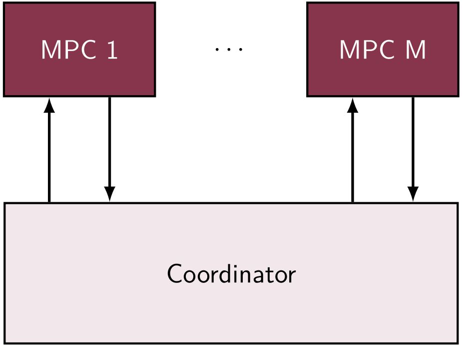

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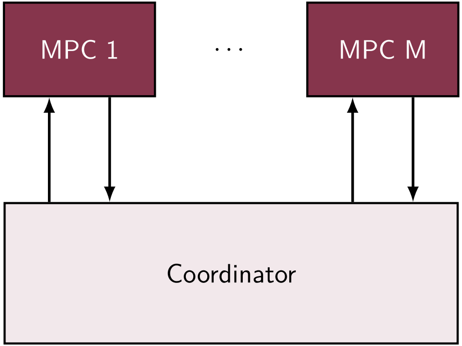

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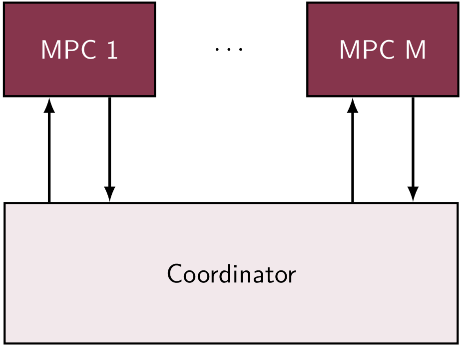

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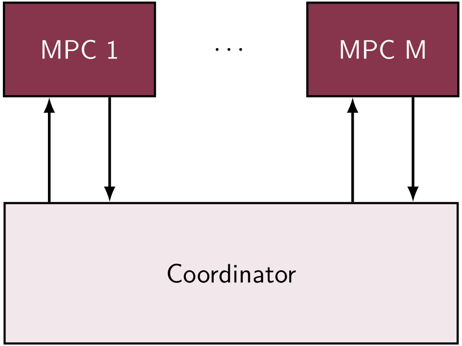

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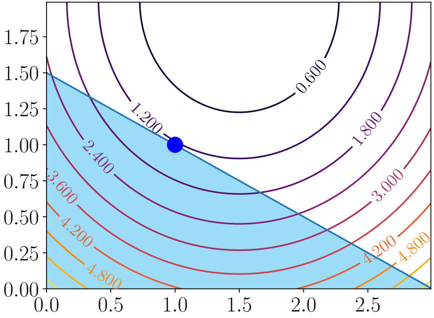
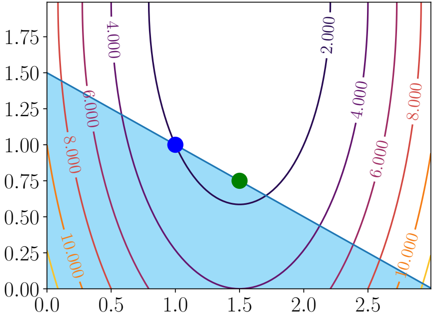
Consequence of an attack

- Attack modifies optimization problem
  - Optimum value is shifted



Original minimum.



Minimum after attack.

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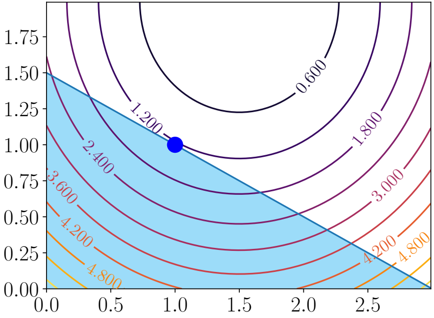

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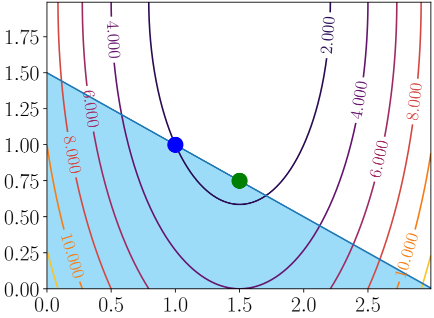

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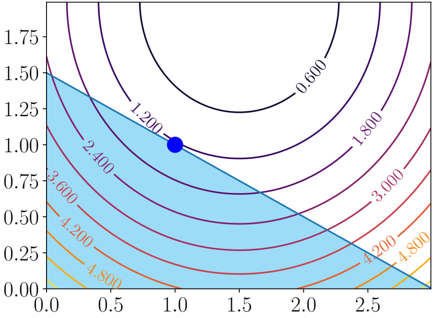

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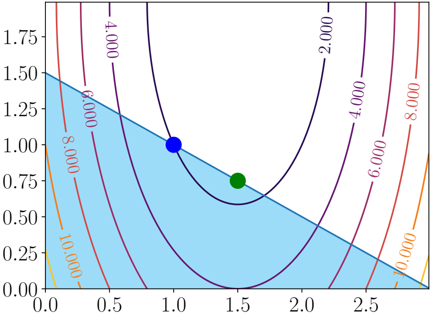

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Mitigating the effects

- We can recover by
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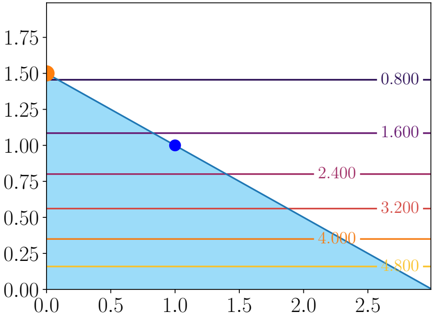

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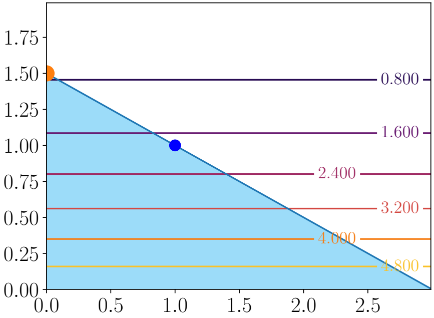

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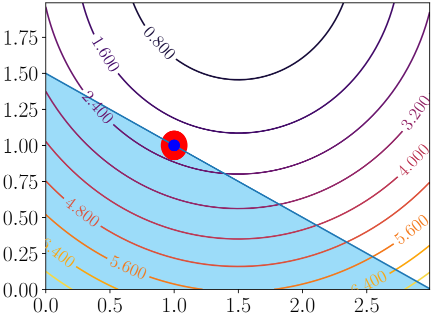

Recover original behavior.

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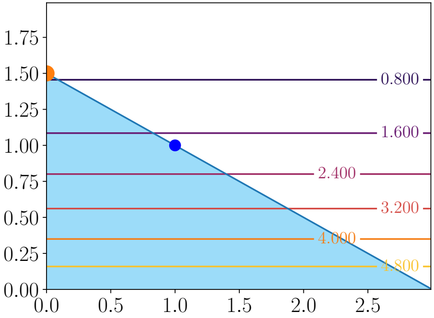

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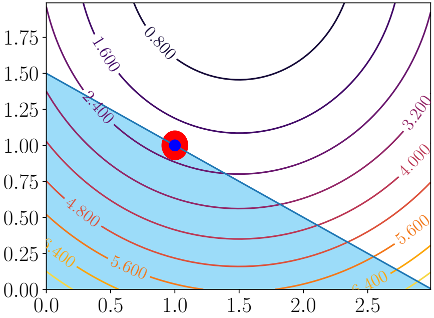

Recover original behavior.

Classification of mitigation techniques

- Passive (Robust) - 1 mode
- Active (Resilient) - 2 modes
  1. Detection/Isolation
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$\Bigg\{$ Attack free

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State of art

Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
[Vel+17a] [Mae+21]	Dual	Robust (Scenario)	NA	NA
[Vel+17b] [Vel+18]	Dual	Robust (f-robust)	NA	NA
[CMI18]	Jacobi-Gauß	–	–	–
[Ana+18] [Ana+19] [Ana+20]	Dual	Resilient	Analyt./Learn.	Disconnect (Robustness)
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State of art

Security dMPC

	Decomposition	Resilient/Robust	Detection	Mitigation
[Vel+17a] [Mae+21]	Dual	Robust (Scenario)	NA	NA
[Vel+17b] [Vel+18]	Dual	Robust (f-robust)	NA	NA
[CMI18]	Jacobi-Gauß	–	–	–
[Ana+18] [Ana+19] [Ana+20]	Dual	Resilient	Analyt./Learn.	Disconnect (Robustness)
Our	Primal	Resilient	Active Analyt./Learn.	Data reconstruction

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Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition
2. Resilient Primal Decomposition-based dMPC for deprived systems
3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity
4. Conclusion

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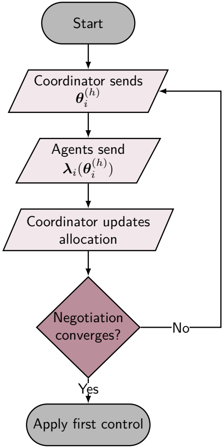
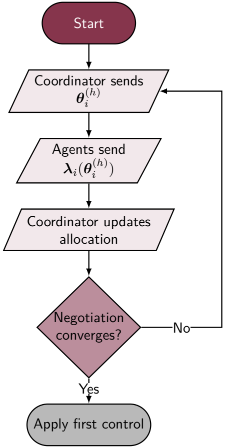
Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition
   - What is the Primal Decomposition?
   - How can an agent attack?
   - Consequences

Quantity Decomposition | Resource Allocation



Quantity Decomposition | Resource Allocation



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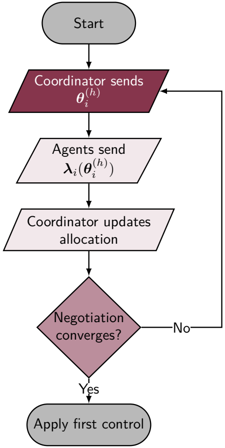
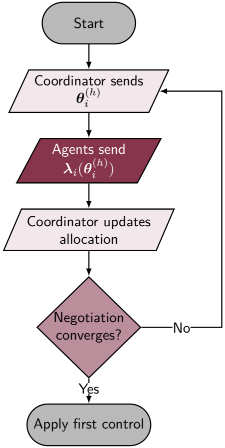


Allocation $\boldsymbol{\theta}_i$

Quantity Decomposition | Resource Allocation



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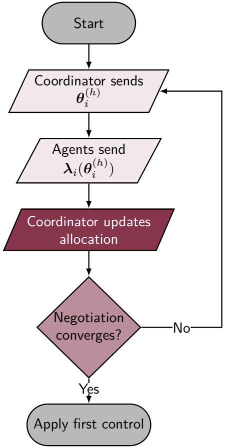

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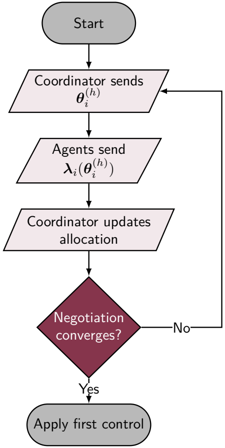

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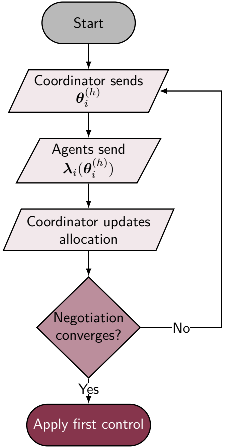

Allocation $\boldsymbol{\theta}_i$

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Primal Decomposition

or Quantity Decomposition | or Resource Allocation

- Objective is sum of local ones
- Constraints couple variables

$$\begin{aligned} \underset{\boldsymbol{u}_1,\dots,\boldsymbol{u}_M}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}} J_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{M}} \boldsymbol{h}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \preceq \boldsymbol{u}_{\text{total}} \end{aligned}$$

$\downarrow$ For each $i \in \mathcal{M}$

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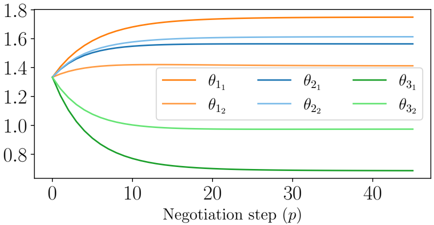
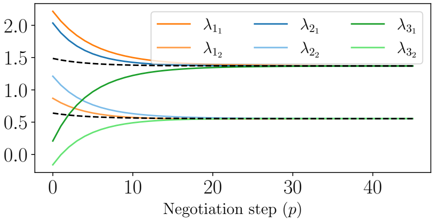
Quantity Decomposition | Resource Allocation

Until everybody is equally dissatisfied





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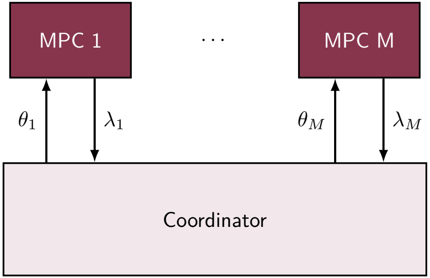
How can a non-cooperative agent attack?

Our approach



- $\boldsymbol{\lambda}_i$ is the only interface
- $\boldsymbol{\lambda}_i$ depends on local parameters
- Malicious agent modifies $\boldsymbol{\lambda}_i$

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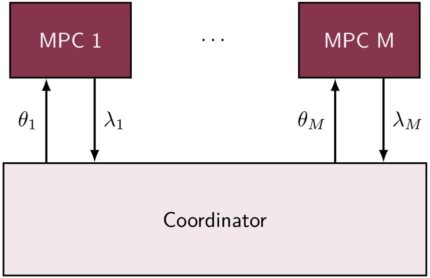

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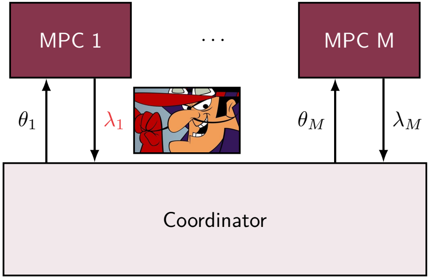

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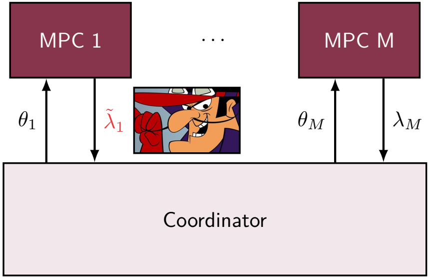

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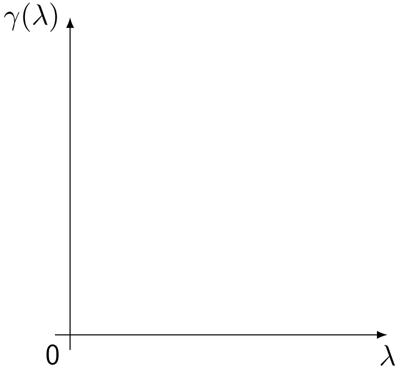
$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i)$$

How does an agent lie?

Liar, Liar, Pants of fire



- $\lambda \geqslant 0$ means dissatisfaction
- $\lambda = 0$ means complete satisfaction

Assumptions

- *Attacker satisfied only if it really is* $\gamma(\lambda) = 0 \rightarrow \lambda = 0$
- *Attacker is greedy* $\gamma(\lambda) > \lambda$
- *Attack is monotonically increasing* $\lambda_b > \lambda_a \rightarrow \gamma(\lambda_b) > \gamma(\lambda_a)$

- Invertible
- If $\tilde{\lambda}_i = T_i[k]\lambda_i \rightarrow \exists T_i[k]^{-1}$

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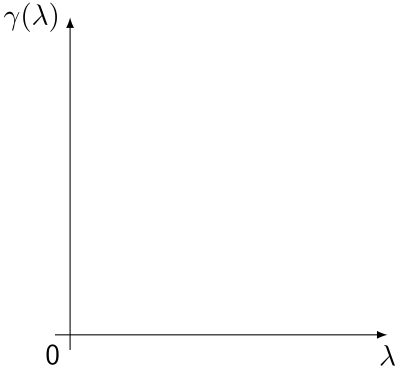

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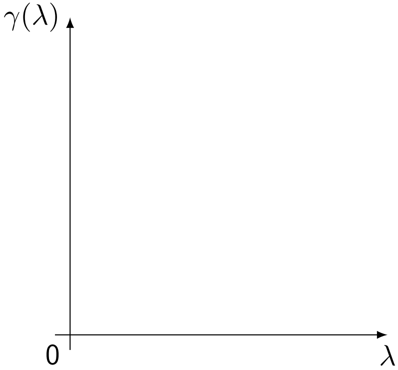

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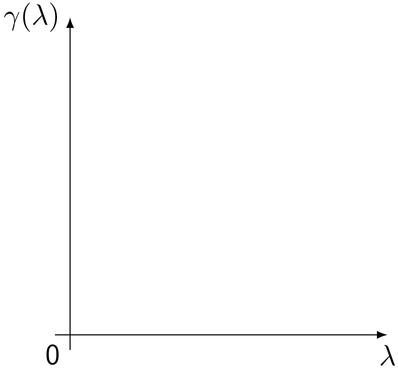

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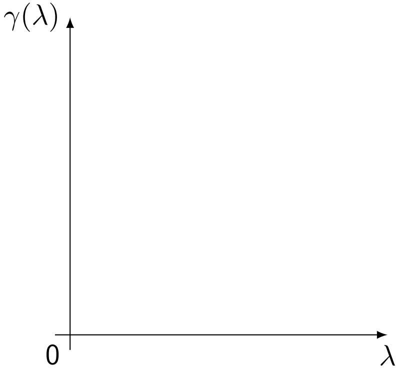

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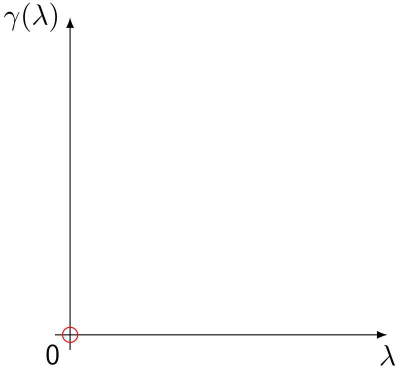

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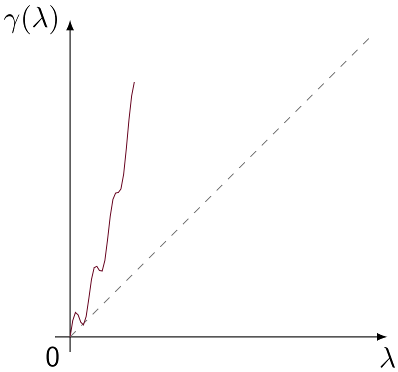

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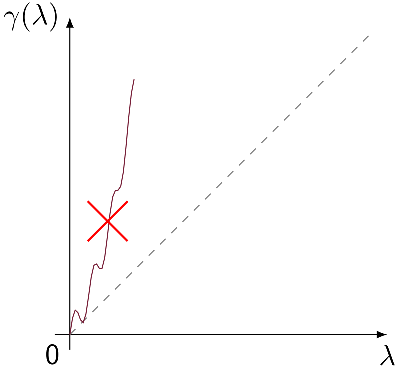

- $\lambda \geqslant 0$ means dissatisfaction
- $\lambda = 0$ means complete satisfaction

Assumptions

- *Attacker satisfied only if it really is*
  - $\gamma(\lambda) = 0 \rightarrow \lambda = 0$
- *Attacker is greedy* $\gamma(\lambda) > \lambda$
- *Attack is monotonically increasing*
  $\lambda_b > \lambda_a \rightarrow \gamma(\lambda_b) > \gamma(\lambda_a)$

- Invertible
- If $\tilde{\boldsymbol{\lambda}}_i = T_i[k]\boldsymbol{\lambda}_i \rightarrow \exists T_i[k]^{-1}$

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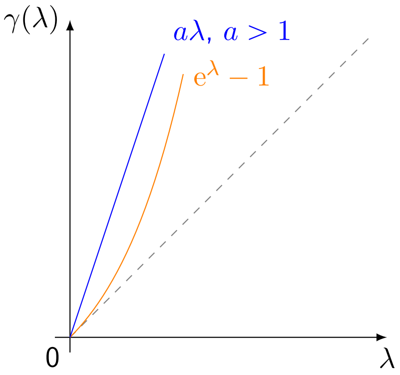

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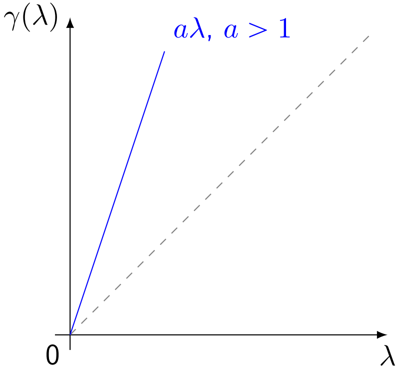

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Example

4 distinct agents

- Agent 1 is non-cooperative
- It uses $\tilde{\lambda}_1 = \gamma_1(\lambda_1) = \tau_1 I \lambda_1$

- We can observe 3 things
  - Global minimum when $\tau_1 = 1$
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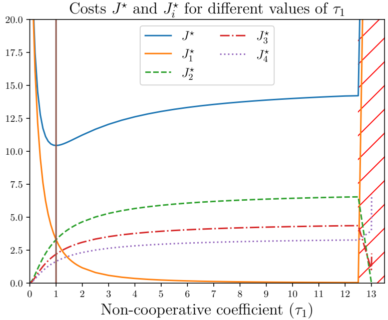

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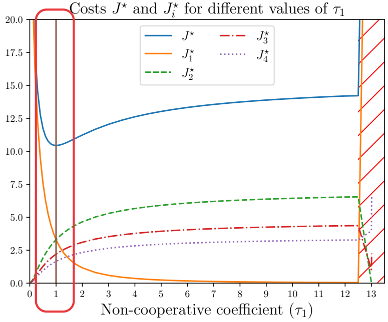

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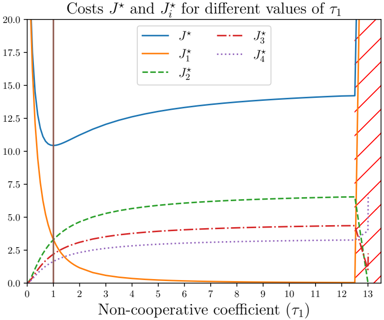

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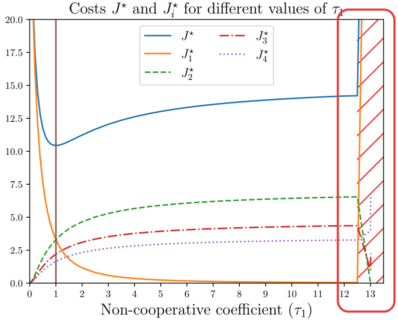

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Outline

2. Resilient Primal Decomposition-based dMPC for deprived systems
    - Analyzing deprived systems
    - Building an algorithm
    - Applying mechanism

What are deprived systems?

Systems whose optimal solution has all constraints active

- Unconstrained Solution $\mathring{\boldsymbol{U}}_i^\star[k]$
- $\bar{\Gamma}_i\mathring{\boldsymbol{U}}_i^\star[k] \geq \boldsymbol{\theta}_i[k] \rightarrow$ Scarcity
  - Solution projected onto boundary
  - Same as with equality constraints[2]

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i\boldsymbol{U}_i[k] \leq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

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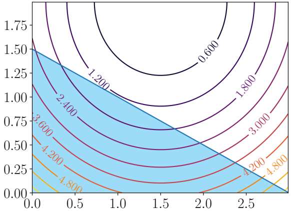

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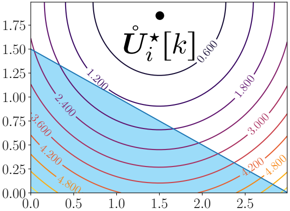

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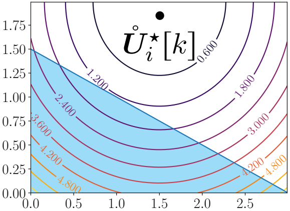

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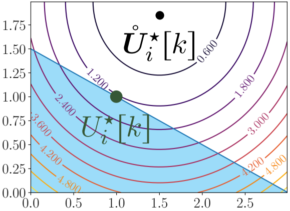

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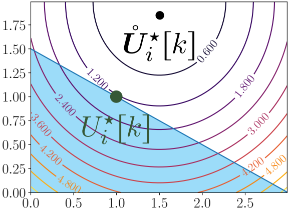

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- No Scarcity
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- *Quadratic local problems*
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- Solution is analytical and affine

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Detection Mechanism

Assumption

We know nominal $\bar{P}_i$

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- Ok, but how can we estimate $\widehat{\tilde{P}}_i[k]$?

[1] Using Recursive Least Squares for example

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- We estimate $\hat{\tilde{P}}_i[k]$ and $\hat{\tilde{s}}_i[k]$ simultaneously using RLS
- Challenge: Online estimation during negotiation fails
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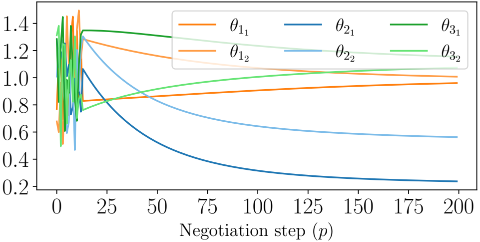

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Classification of mitigation techniques

- Active (Resilient)
  1. Detection/Isolation ✔
  2. Mitigation ?

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Mitigation mechanism

Reconstructing $\boldsymbol{\lambda}_i$

- Now, we have $\widehat{\tilde{P}}_i[k]$
  - Since $\tilde{P}_i[k] = T_i[k]\bar{P}_i$
  - We can recover $T_i[k]^{-1}$

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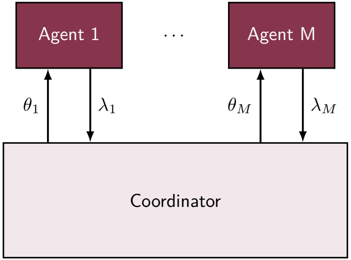
Complete Mechanism

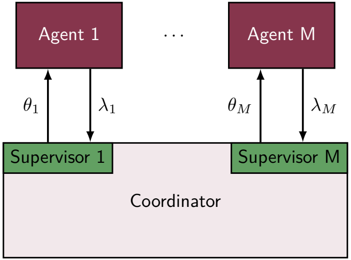


- Supervise exchanges by inquiring the agents
- Estimate how they will behave

Two Phases

1. Detect which agents are non-cooperative
2. Reconstruct λ_i and use in negotiation

Complete Mechanism



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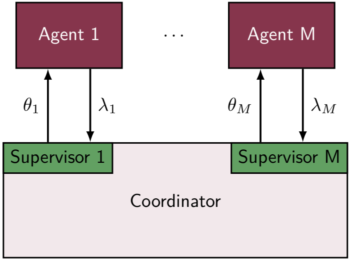

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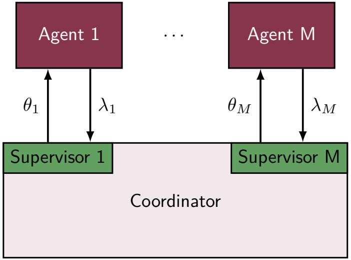

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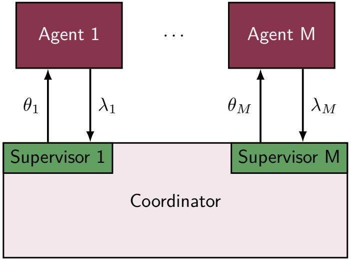

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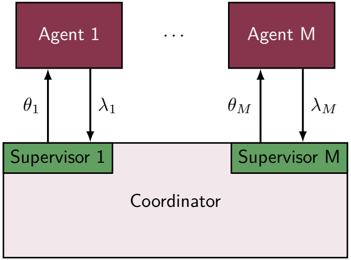

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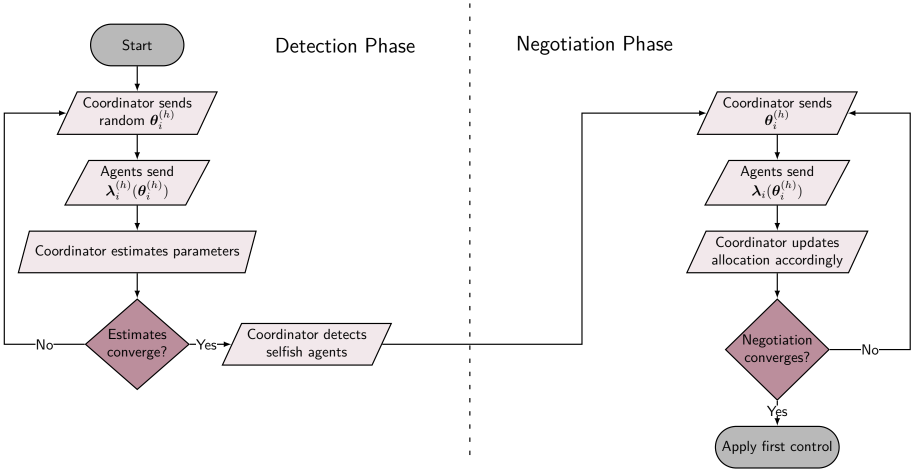

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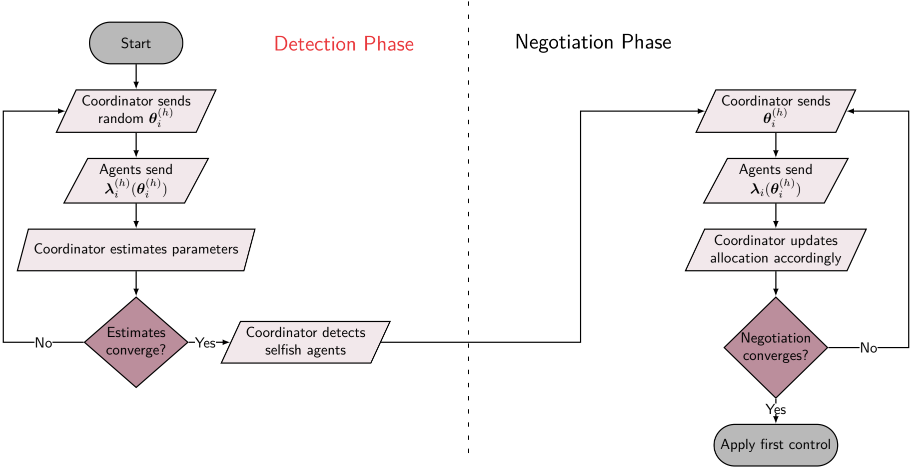

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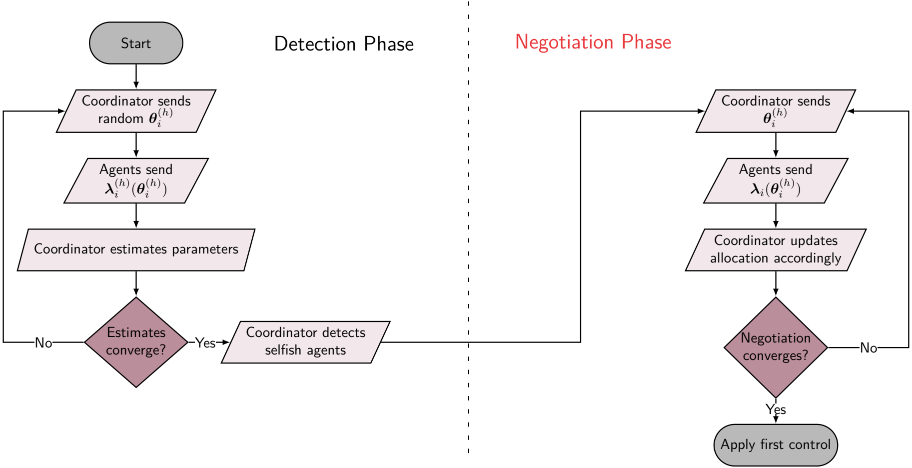

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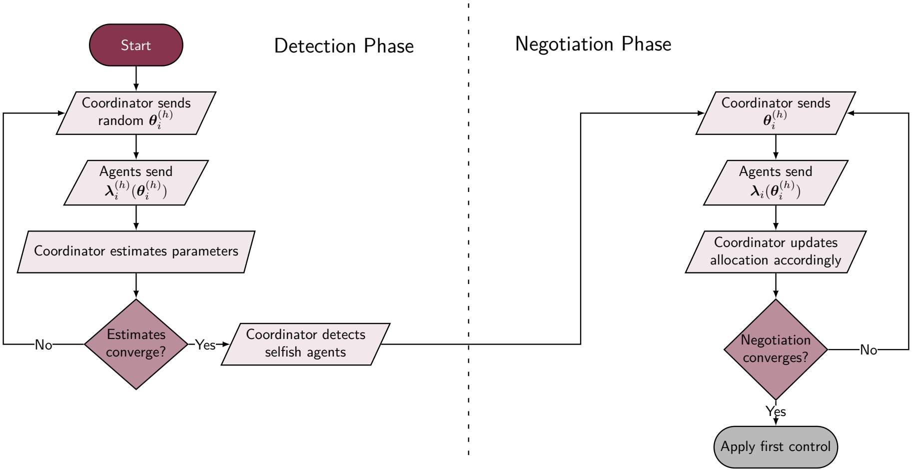

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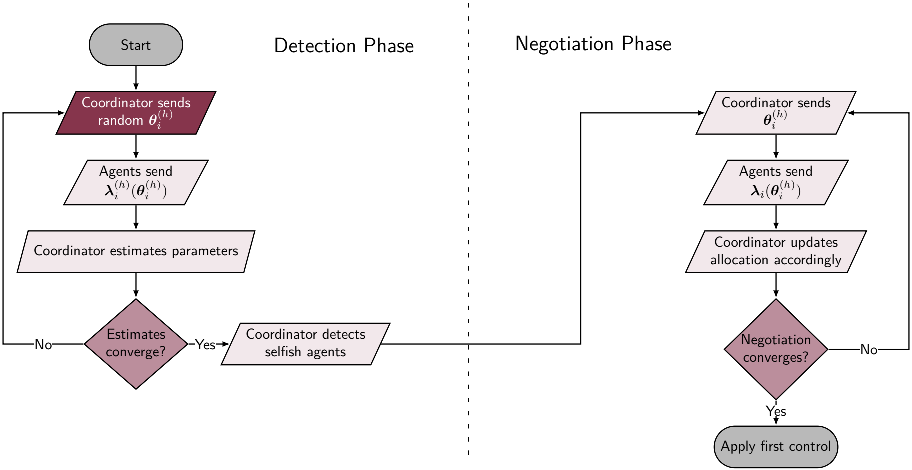

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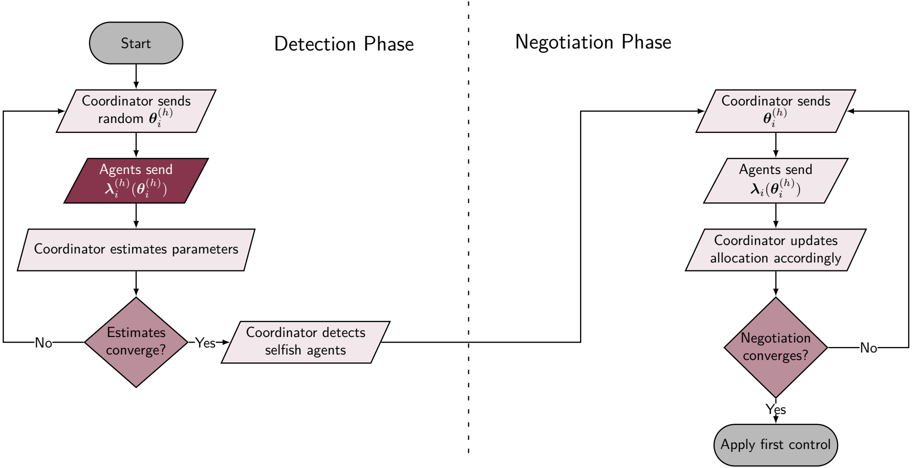

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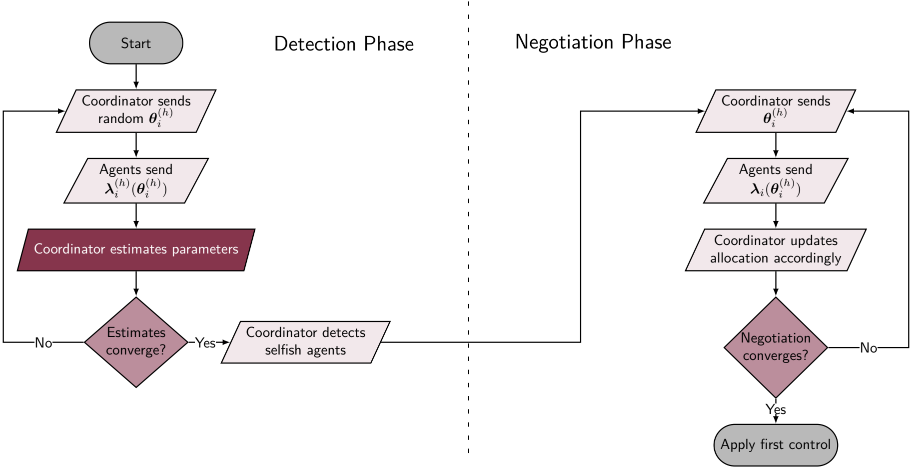

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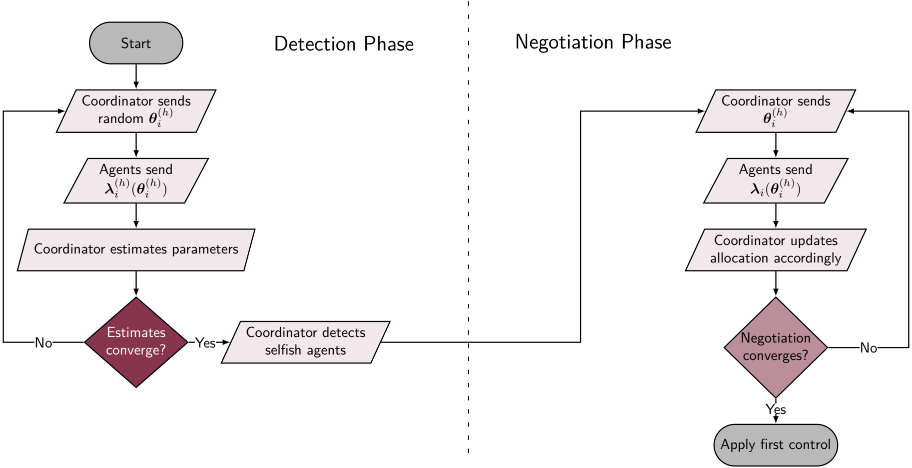

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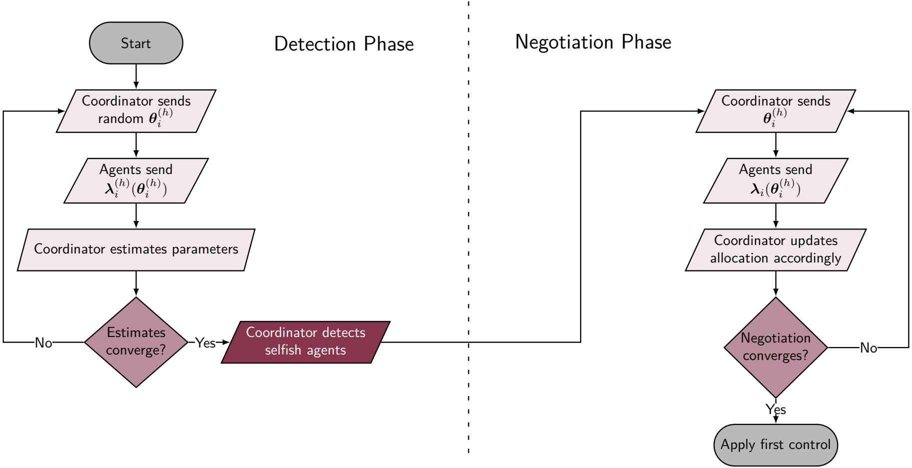

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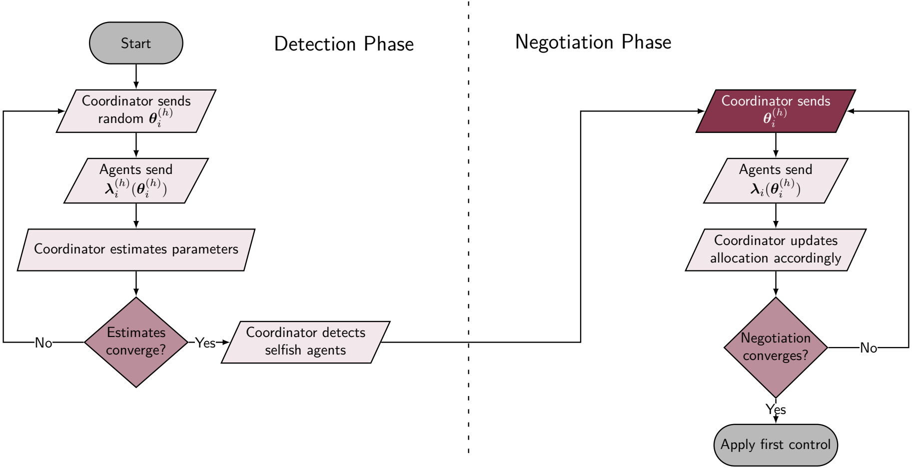

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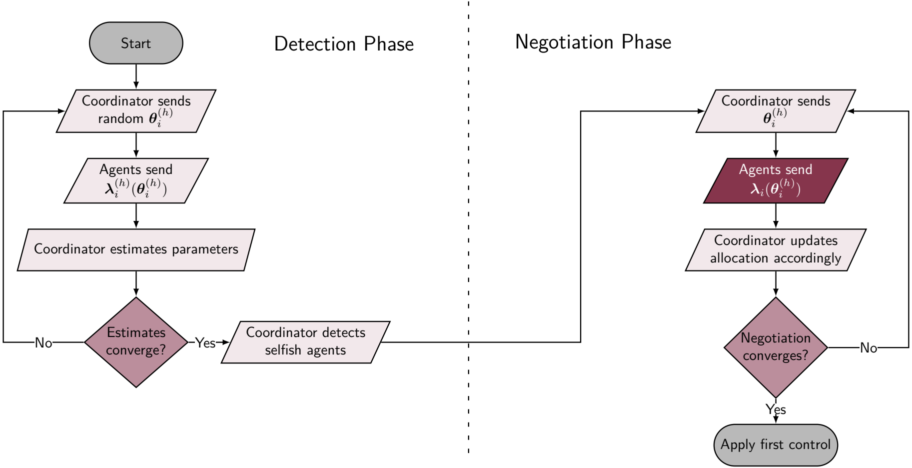

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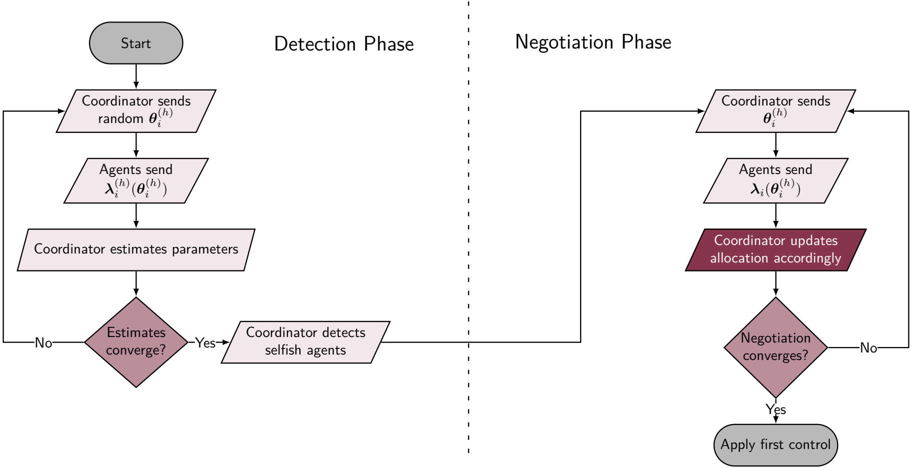

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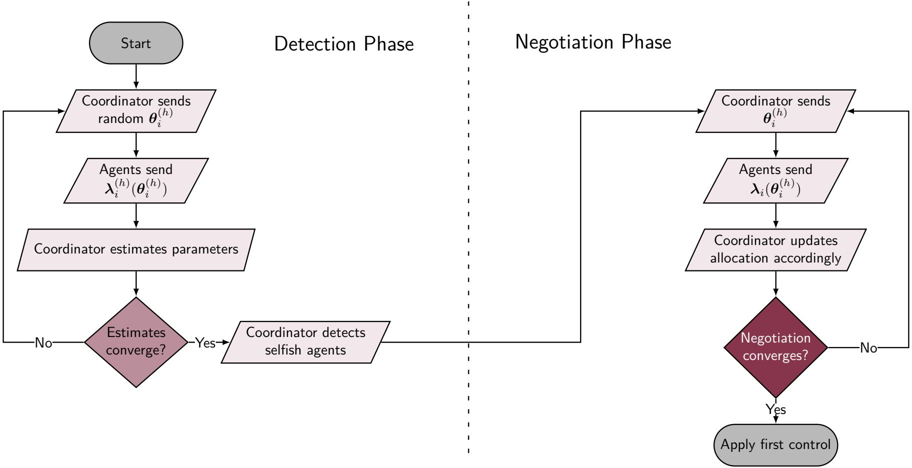

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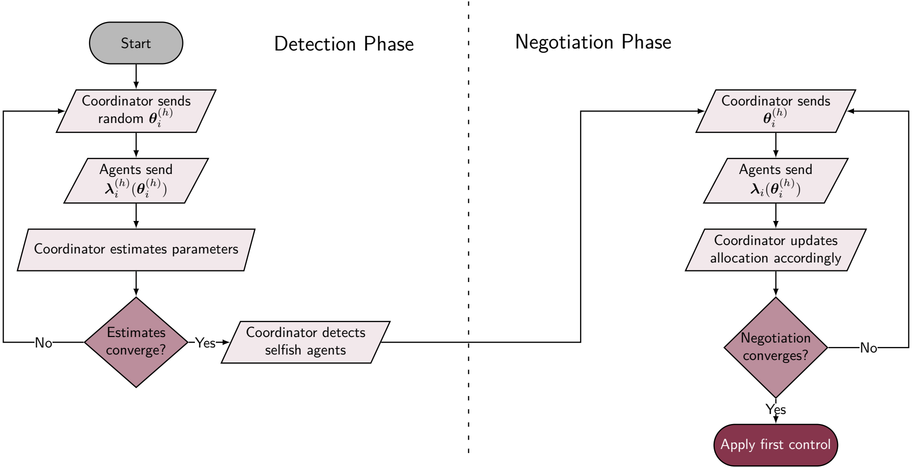

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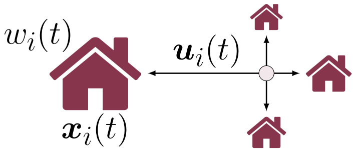

District Heating Network (4 Houses)

- Houses modeled using 3R-2C (monozone)
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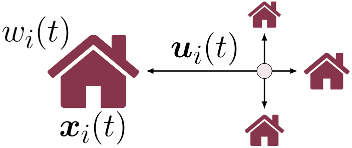

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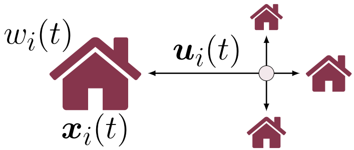

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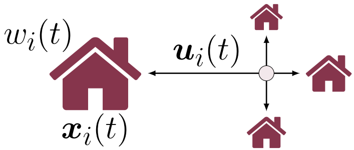

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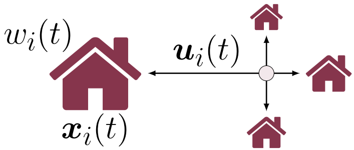

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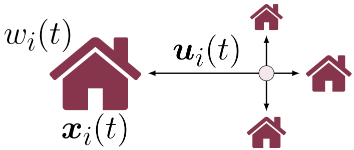

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- Houses modeled using 3R-2C (monozone)
- Not enough power
- Period of 5h
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  - C Agent I cheats (dMPC)
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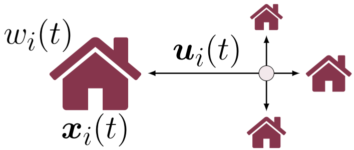

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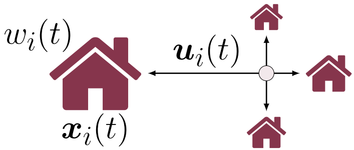

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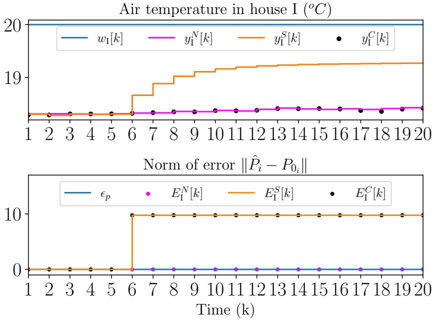

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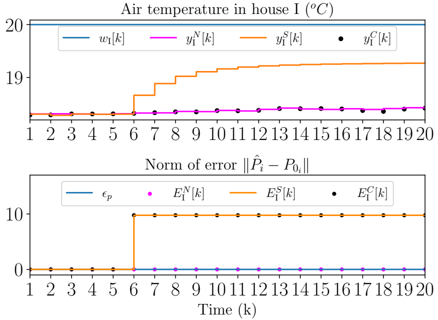

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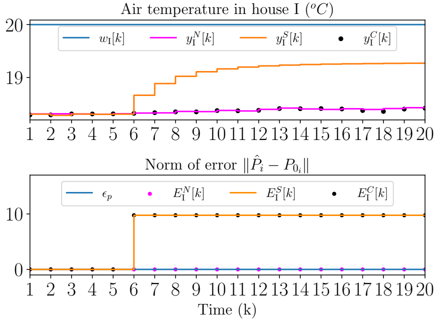

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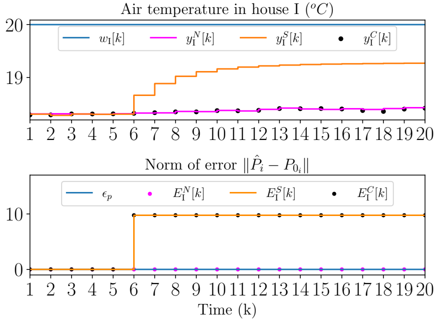

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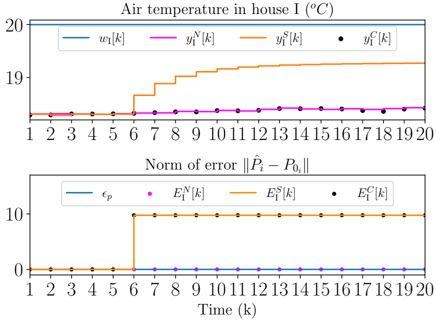

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Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.3	0.503
II	21.671	-0.547
III	17.387	-0.004
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Outline

3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity
   - Relaxing some assumptions
   - Adapting the algorithm
   - Applying mechanism

Relaxing scarcity assumption

- Systems are not completely deprived
  - We can't change our constraints to equality ones anymore
  - Nor use the simpler update equation

$$\begin{array}{ll} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\ \text{subject to} & \bar{\Gamma}_i\boldsymbol{U}_i[k] \leq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{array}$$

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Analyzing System

Solution for $\lambda_i[k]$

Instead of having one single affine solution

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k]$$

Now, we may have multiple (Piecewise affine function)

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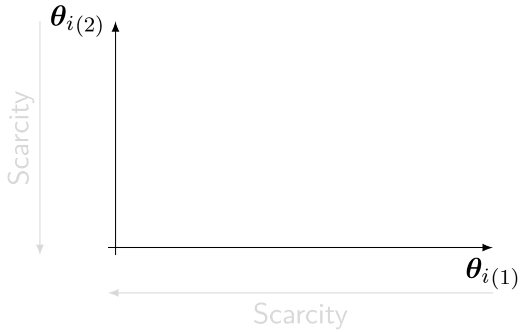

Separation surfaces depend on state and local parameters.
Unknown by the coordinator.

Analyzing System

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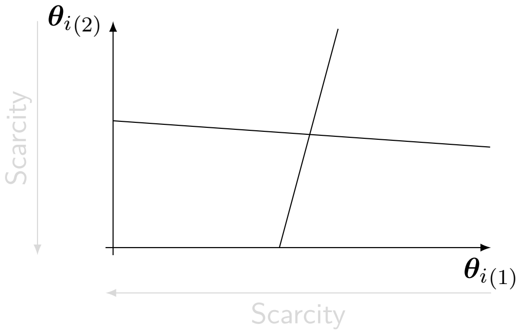

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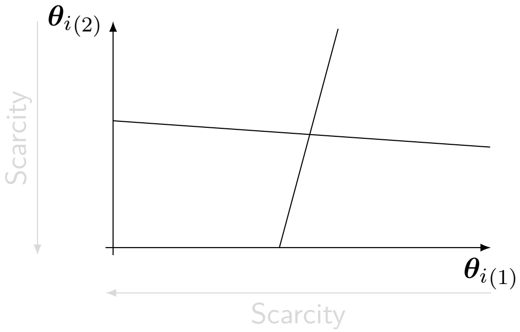

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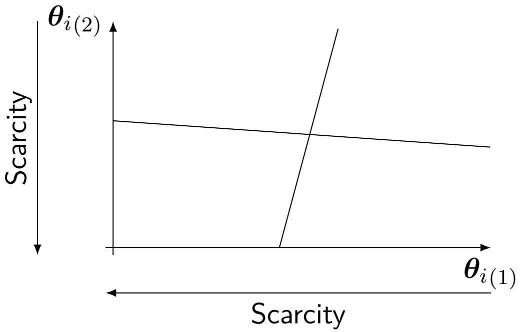

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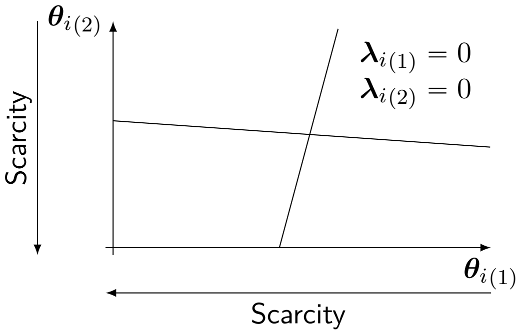

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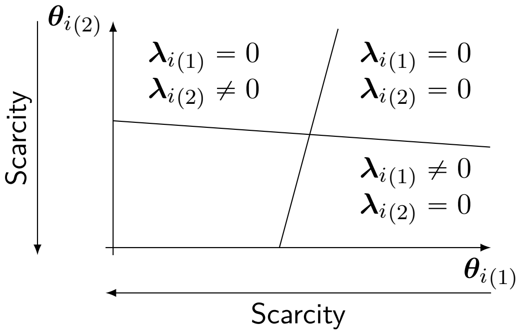

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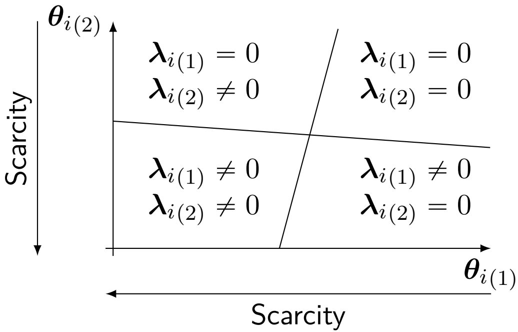

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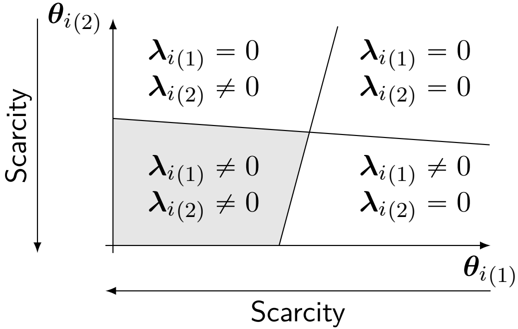

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Analyzing System

Under attack!

$$\tilde{\boldsymbol{\lambda}}_i[k] = T_i[k]\boldsymbol{\lambda}_k$$

Parameters are modified. But not the regions' limits

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- Problem: We don't known in which region $\boldsymbol{\theta}_i$ is
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Parameters are modified. But not the regions' limits

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- If we can estimate $\widetilde{P}_i^{(0)}$ we can use same strategy than before
- Problem: We don't known in which region $\boldsymbol{\theta}_i$ is
- Solution: Let's force it using Artificial Scarcity

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Analyzing System

Under attack!

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Artificial Scarcity

Who is it? Who is it?

Assumption

We known a point $\overset{\varnothing}{\theta}_i$ which activates all constraints[4]

[4] If we have local constraints, we suppose this point respects them.

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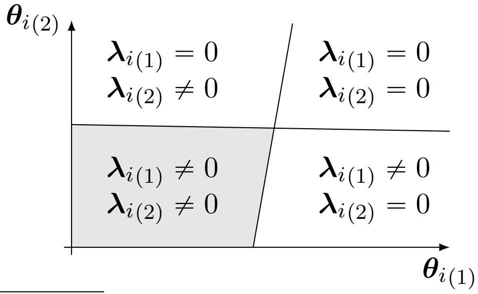

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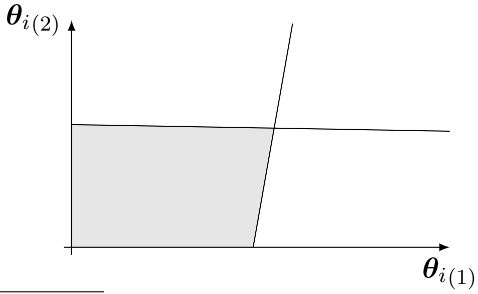

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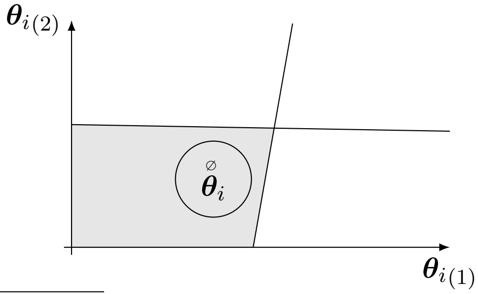

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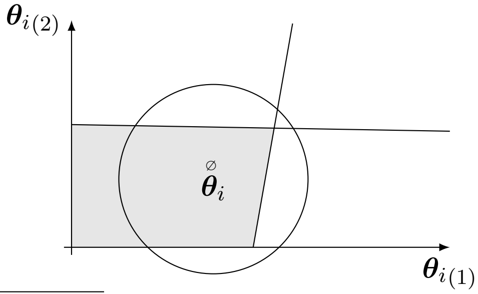

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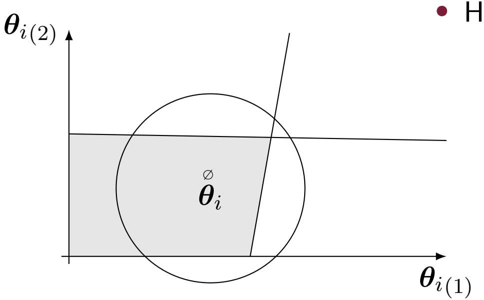

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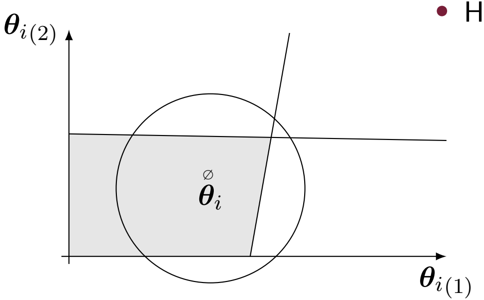

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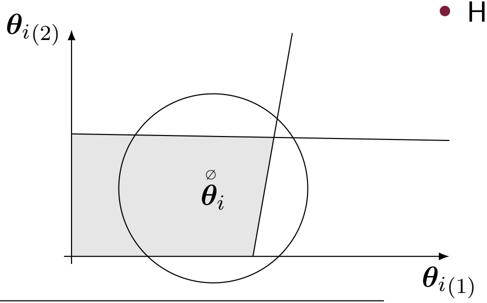

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- Iterative method to estimate parameters of multimodal models[5]
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Detection and Mitigation

Same same, but different

Assumption

We know nominal $\bar{P}_i^{(0)}$

- Detection

$$\left\| \widehat{\bar{P}}_i^{(0)}[k] - \bar{P}_i^{(0)} \right\|_F \geqslant \epsilon_{P_i^{(0)}}$$

- Mitigation

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$$\lambda_i^{\text{rec}} = \widehat{T_i[k]^{-1}} \tilde{\lambda}_i.$$

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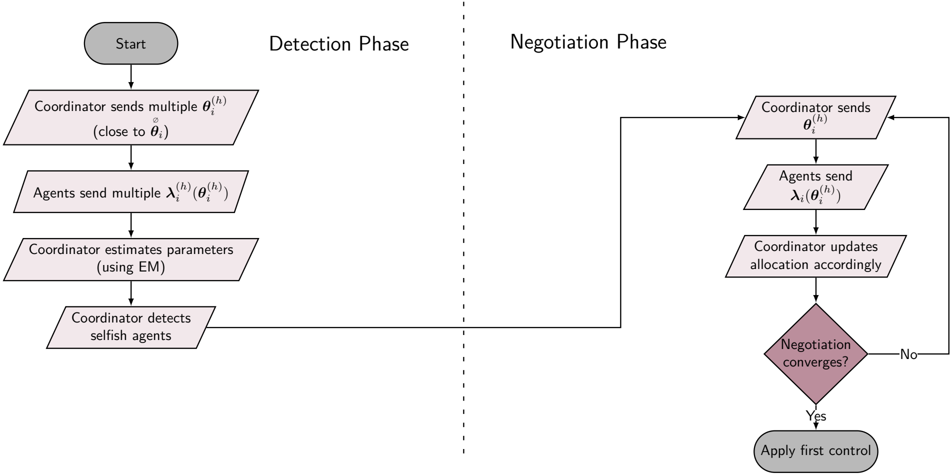
Complete algorithm

RPdMPC-AS



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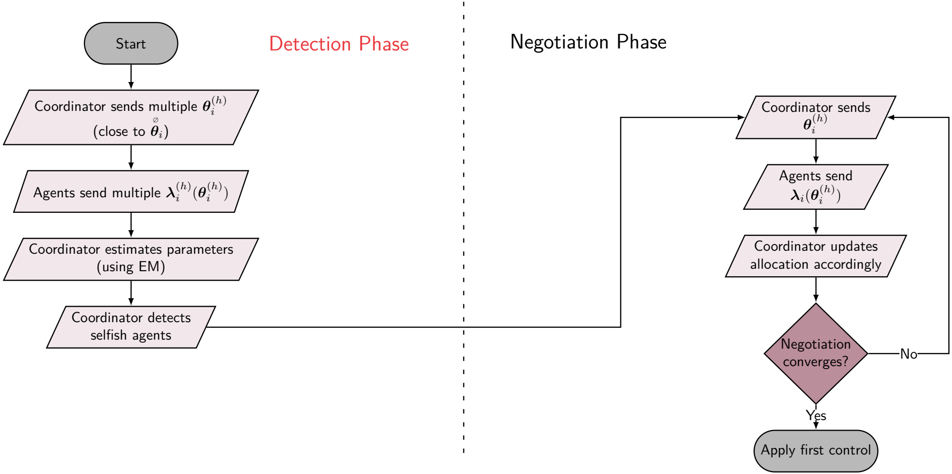

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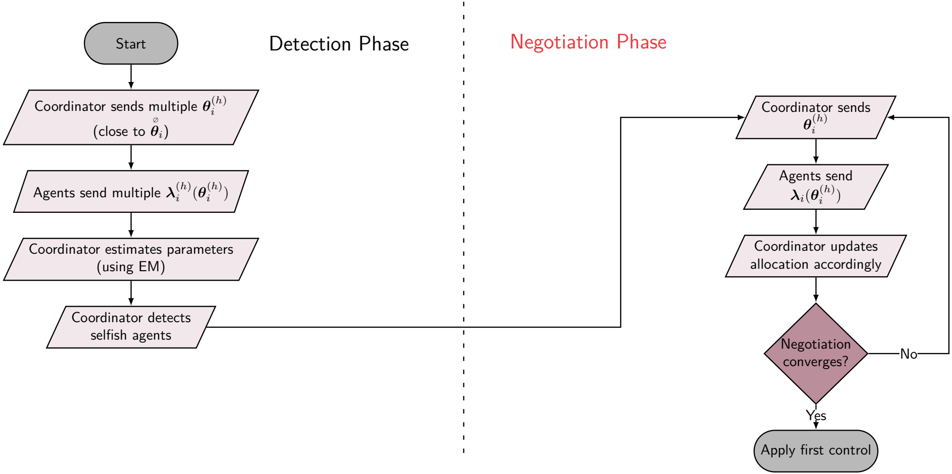

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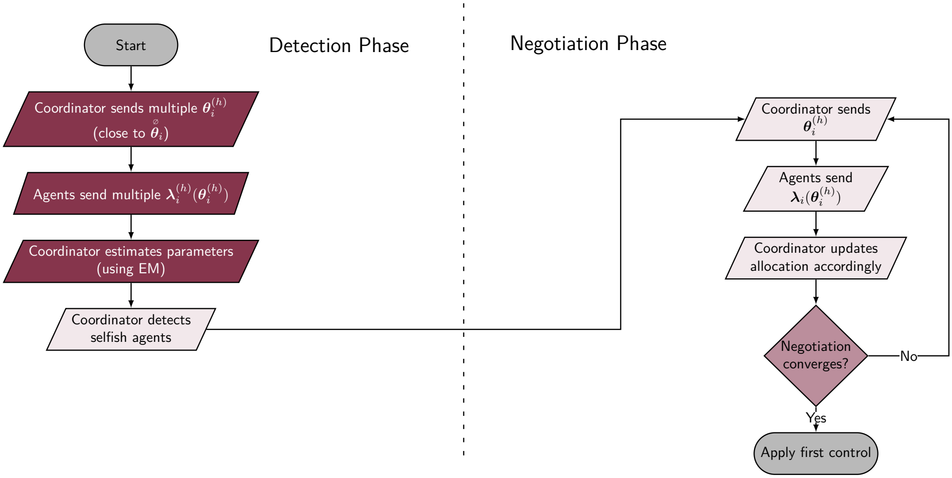

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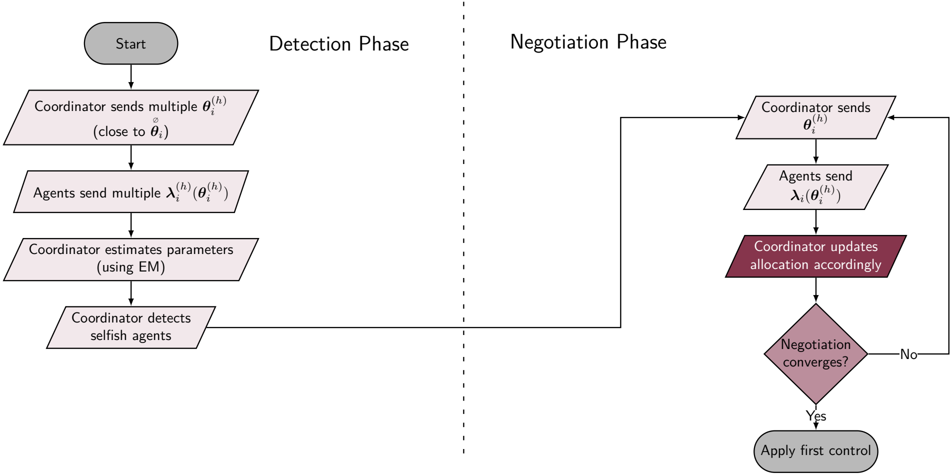

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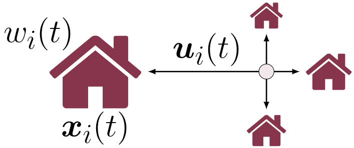

District Heating Network (4 Houses)

- Houses modeled using 3R-2C
- Not enough power
- Period of $5\mathrm{h}$ ($T_s = 0.25\mathrm{h}$)
- 3 scenarios
  - N Nominal
  - C Agent I cheats (dMPC)
  - S Agent I cheats (RPdMPC-AS)

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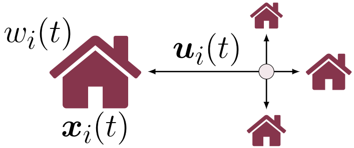
Example



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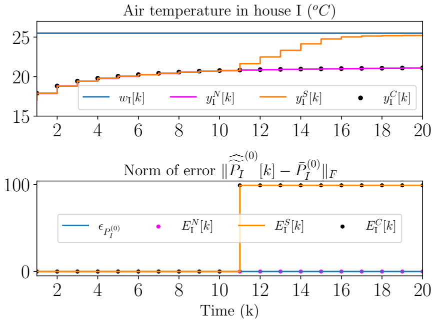
Results

Temporal



Temperature in house I.
Error $E_I(k)$.
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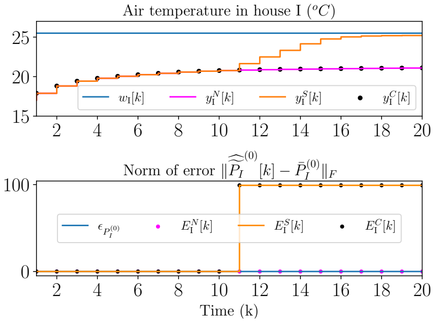

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Results

Costs

Objective functions J_i (Normalized error %)

Agent	Selfish	Corrected
I	-36.489	-0.0
II	35.813	0.0
III	29.225	0.0
IV	37.541	0.0
Global	10.689	-0.0

Too good to be true!

It's a kind of magic!

- Unfortunately EM is not magic
  - Slow convergence
  - Dependency on initialization
    - No guarantees of achieving global optimal
- Some "solutions":
  - Force some parameters to converge faster (case dependant)
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Outline

Conclusion

Main takeaways

- How can an agent attack? ✓
  - Attacker can change the communication to receive more ressources.
- What are the consequences of an attack? ✓
  - Suboptimality and maybe instability
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  - Yes! By exploring the scarcity of the systems!

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- Insights from the analysis of the solutions of the optimization problems
  - We found some parameters that are constant when there is no cheating
  - The same parameters change when system is attacked
- Exploiting the solution, we find how to invert the cheating function
  - Straightforward if system is scarce
  - If not scarce we try to force it artifically
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Open question Future Directions

- Partial/incremental reconstruction of cheating matrix
- Study of robustness + noise
- Resilient strategy with soft constraints
- Recursive EM (or alternative)
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Thank you!

Repository
https://github.com/Accacio/thesis

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For Further Reading I

K.J. Åström and B. Wittenmark. Adaptive Control. Addison-Wesley series in electrical and computer engineering: Control engineering. Addison-Wesley, 1989. ISBN: 9780201097207. DOI: 10.1007/978-3-662-08546-2_24.

José M Maestre, Rudy R Negenborn, et al. Distributed Model Predictive Control made easy. Vol. 69. Springer, 2014. ISBN: 978-94-007-7005-8.

Wicak Ananduta et al. "Resilient Distributed Model Predictive Control for Energy Management of Interconnected Microgrids". In: Optimal Control Applications and Methods 41.1 (2020), pp. 146–169. DOI: 10.1002/oca.2534. URL: https://onlinelibrary.wiley.com/doi/pdf/10.1002/oca.2534.

For Further Reading II

José M. Maestre et al. “Scenario-Based Defense Mechanism Against Vulnerabilities in Lagrange-Based Dmpc”. In: Control Eng Pract 114 (2021), p. 104879. ISSN: 0967-0661. DOI: 10.1016/j.conengprac.2021.104879.

Pablo Velarde et al. “Vulnerabilities in Lagrange-Based Distributed Model Predictive Control”. In: Optimal Control Applications and Methods 39.2 (Sept. 2018), pp. 601–621. DOI: 10.1002/oca.2368.

Wicak Ananduta et al. “Resilient Distributed Energy Management for Systems of Interconnected Microgrids”. In: 2018 IEEE Conference on Decision and Control (CDC). 2018, pp. 3159–3164. DOI: 10.1109/CDC.2018.8619548.

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For Further Reading III

Wicak Ananduta et al. "A Resilient Approach for Distributed MPC-Based Economic Dispatch in Interconnected Microgrids". In: 2019 18th European Control Conference (ECC). 2019, pp. 691–696. DOI: 10.23919/ECC.2019.8796208.

P. Chanfreut, J. M. Maestre, and H. Ishii. "Vulnerabilities in Distributed Model Predictive Control based on Jacobi-Gauss Decomposition". In: 2018 European Control Conference (ECC). June 2018, pp. 2587–2592. DOI: 10.23919/ECC.2018.8550239.

Pablo Velarde et al. "Scenario-based defense mechanism for distributed model predictive control". In: 2017 IEEE 56th Annual Conference on Decision and Control (CDC). IEEE. Dec. 2017, pp. 6171–6176. DOI: 10.1109/CDC.2017.8264590.

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For Further Reading IV

Pablo Velarde et al. "Vulnerabilities in Lagrange-Based DMPC in the Context of Cyber-Security". In: 2017 IEEE International Conference on Autonomic Computing (ICAC). July 2017, pp. 215–220. DOI: 10.1109/ICAC.2017.53.

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Conditions

◂ back

One way to ensure this, is to make the original constraint (**??**) to have at most as many rows as columns, i.e., $\# \boldsymbol{u}_{\mathrm{max}} \leqslant n_u$, although it may be a little restrictive.

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θ dynamics

back

$$\boldsymbol{\theta}^{(p+1)} = \mathcal{A}_\theta \boldsymbol{\theta}^{(p)} + \mathcal{B}_\theta[k]$$

where

$$\mathcal{A}_\theta = \begin{bmatrix} I - \frac{M-1}{M}\rho^{(p)} P_1 & \frac{1}{M}\rho^{(p)} P_2 & \dots & \frac{1}{M}\rho^{(p)} P_M \\ \frac{1}{M}\rho^{(p)} P_1 & I - \frac{M-1}{M}\rho^{(p)} P_2 & \dots & \frac{1}{M}\rho^{(p)} P_M \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{M}\rho^{(p)} P_1 & \frac{1}{M}\rho^{(p)} P_2 & \dots & I - \frac{M-1}{M}\rho^{(p)} P_M \end{bmatrix}$$

$$\mathcal{B}_\theta[k] = \begin{bmatrix} -\frac{M-1}{M}\rho^{(p)} \boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)} \boldsymbol{s}_2[k] \dots - \frac{1}{M}\rho^{(p)} \boldsymbol{s}_M[k] \\ \frac{1}{M}\rho^{(p)} \boldsymbol{s}_1[k] - \frac{M-1}{M}\rho^{(p)} \boldsymbol{s}_2[k] \dots - \frac{1}{M}\rho^{(p)} \boldsymbol{s}_M[k] \\ \vdots \\ \frac{1}{M}\rho^{(p)} \boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)} \boldsymbol{s}_2[k] \dots - \frac{M-1}{M}\rho^{(p)} \boldsymbol{s}_M[k] \end{bmatrix}$$

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