Security of distributed Model Predictive Control under False Data injection

or How I Learned to Stop and Worry about Everything

Rafael Accácio NOGUEIRA

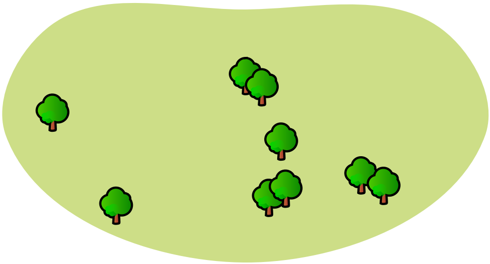
December 12, 2022

https://bit.ly/3g3S6X4

45 minutes !!!!
Good afternoon, thank you all for being here.I'm Rafael Accácio and I'm going to present my work on the security of distributed model predictive control under false data injection.

Context

Requirements evolve with time



These cyberphysical systems are the majority of the systems in our everyday lives.We can give example the traffic management,water distribution,electricity distribution, heat and cold and many more.But how to control those kinds of systems. Each has its own Dynamics and constraints, such comfort (Quality of service) or technical. Solution, mpc since we use models and it is easy to integrate the constraints.

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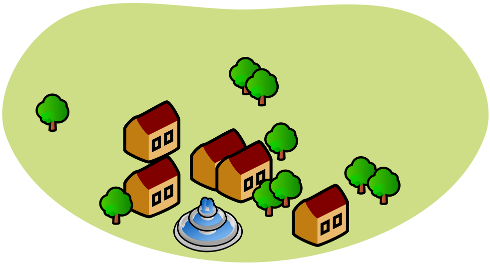

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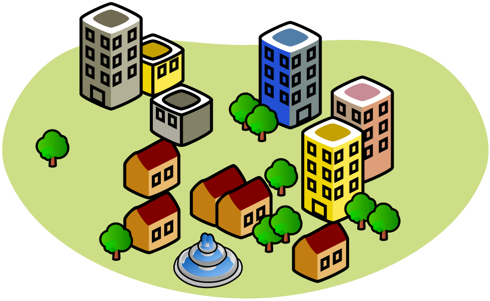

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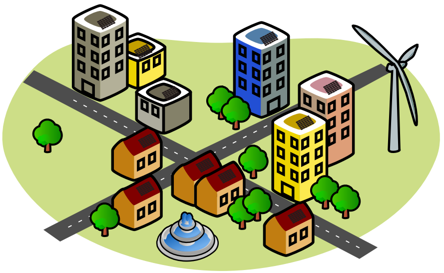

- Electricity Distribution System
- Heat distribution
- Water distribution
- Traffic management

  (include your problem here)

CentraleSupélec

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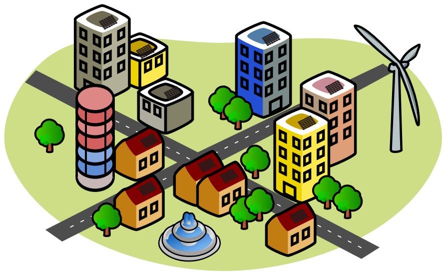

- Electricity Distribution System
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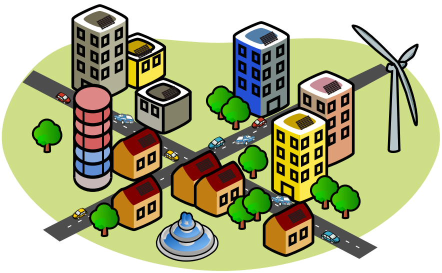

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- Multiple systems interacting
- Coupled by constraints
- Optimization objectives
  - Minimize energy consumption
  - Maximize user satisfaction
  - Follow a trajectory
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Model-based Predictive Control

Find best control sequence using predictions based on a model.

- Objective function to optimize
- System's Model (states and inputs)
- Other constraints to respect (QoS, technical restrictions, ...)

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For those who are not familiar with mpc. Mpc is the model based predictive controller.

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The objective is to find the best control sequence using predictions based on a model.

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When we say best,

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we mean optimal.

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$$
\begin{aligned}
\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad & J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k]) \\
\text{subject to} \quad & \left.\begin{aligned}
&\boldsymbol{x}[\xi|k] = f(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \\
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\end{aligned}\right\}
\begin{aligned}
&\forall \xi \in \{1, \dots, N\} \\
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\end{aligned}
$$

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So we need to solve an optimization problem.

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And we have the control sequence of u as the decision variable.

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which is calculated for a horizon N

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So, we need an objective function. For example follow a trajectory while minimizing the energy.

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A model of the system

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But we can also integrate some constraints, such QoS or technical restrictions

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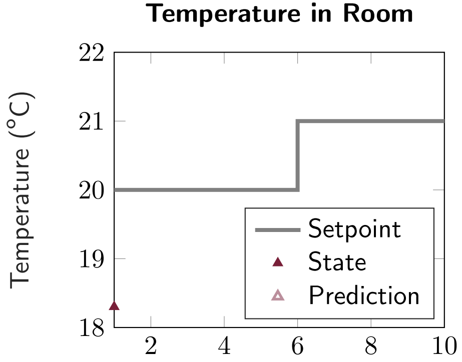
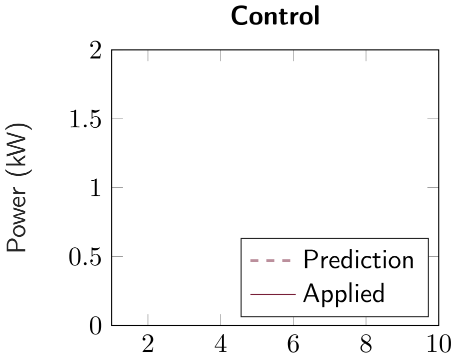
In a nutshell

Find optimal control sequence





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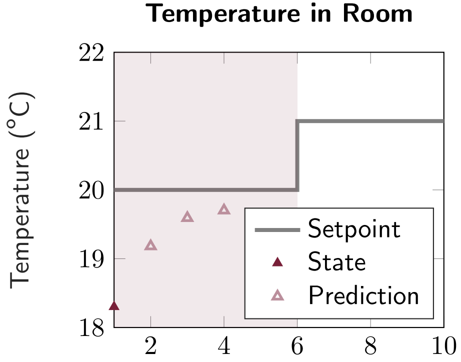
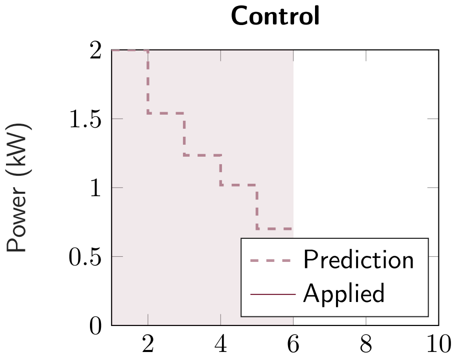
So, for example, if we may have a setpoint to follow

Find optimal control sequence





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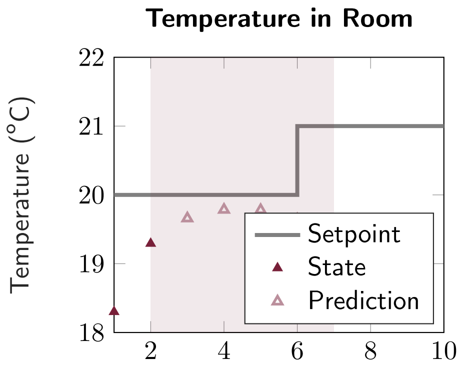
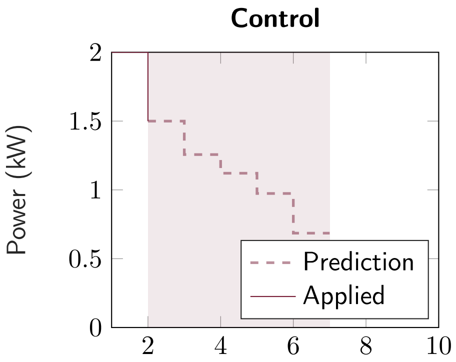
We find an optimal control sequence

Model Predictive Control

In a nutshell

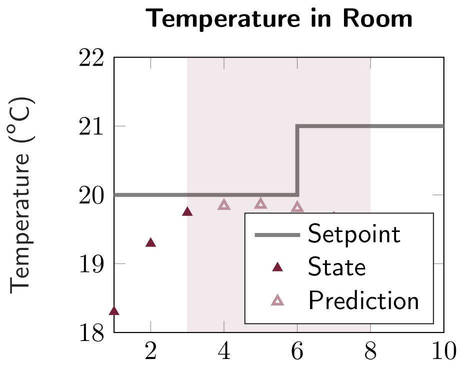
Find optimal control sequence, apply first element

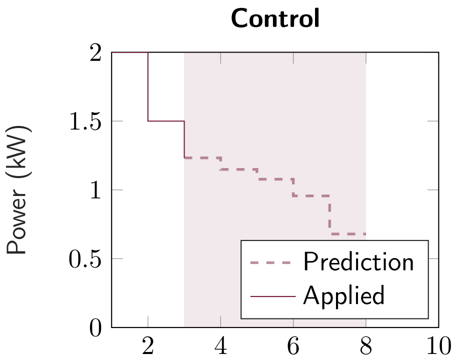




We apply only the first element

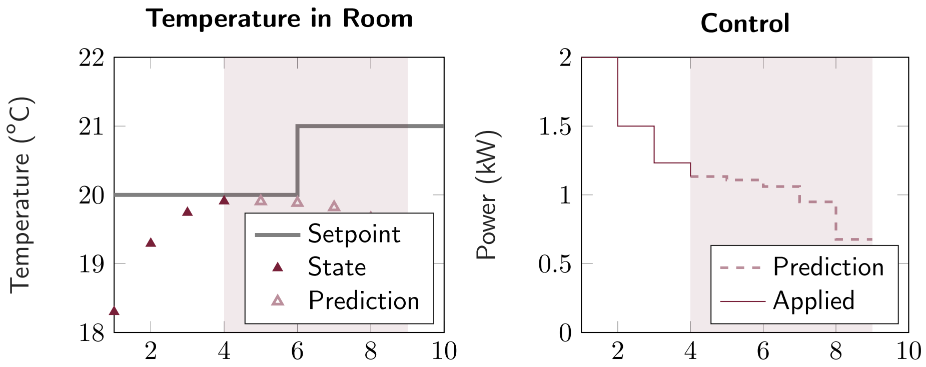
Find optimal control sequence, apply first element, rinse repeat





and then we repeat

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon



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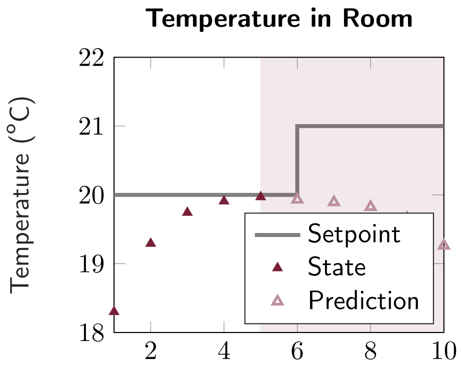
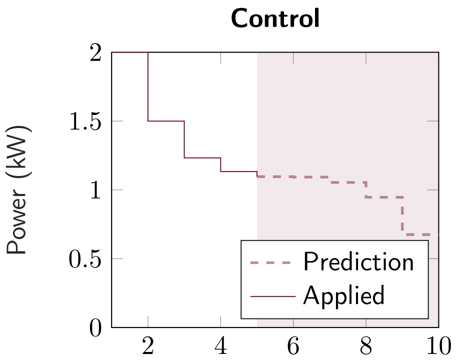
following what we call the receding horizon strategy. However this problem is not always straight-forward to solve, for some cases it can be easier.

Find optimal control sequence, apply first element, rinse repeat → Receding Horizon





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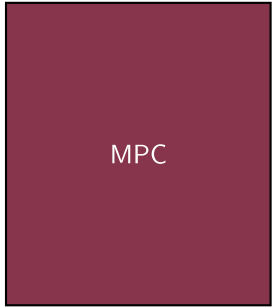
- Problem: Complexity depends on N, m, p and sizes of $\boldsymbol{x}$ and $\boldsymbol{u}$
- Solution: Divide and Conquer[1]



1

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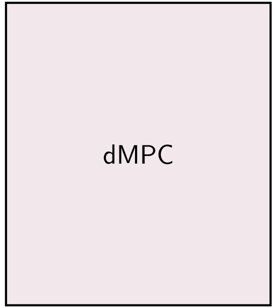
However, the solution will depend on the horizon, the number of constraints, and sizes of input and states, increasing the complexity of the calculation

Distributed Model Predictive Control

- Problem: Complexity depends on N, m, p and sizes of $\boldsymbol{x}$ and $\boldsymbol{u}$
- Solution: Divide and Conquer[1]



[1] *Distributed Model Predictive Control made easy*

A strategy to alleviate is to distribute the calculation whenever possible. And there are many ways to divide it as the book shows.

Distributed Model Predictive Control

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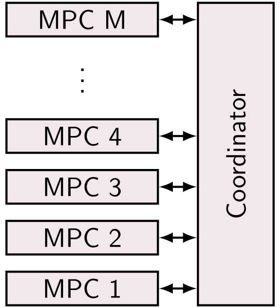

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Here we opt for a hierarchical strategy where we use multiple MPCs and an agent to coordinate and manage the coupling aspects of the problem.

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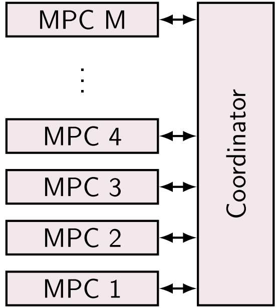

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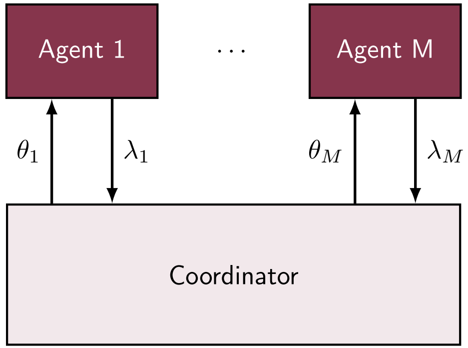
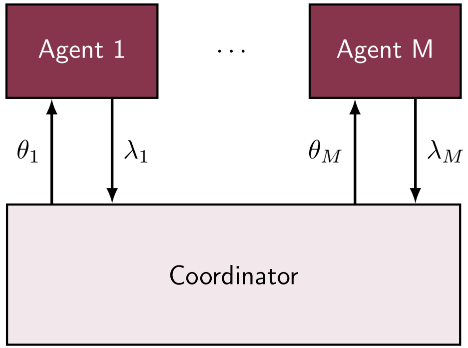


- Agents solve local problems
- Variables are updated

$\Big\}$ Until Convergence

Distributed Model Predictive Control

Optimization Frameworks



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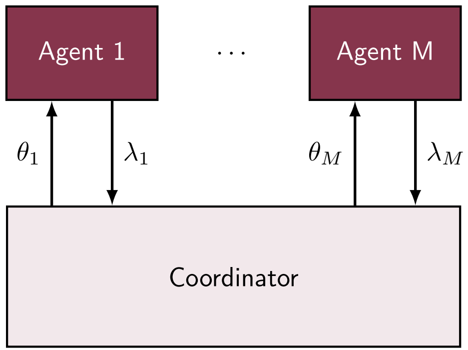

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Negotiation works if agents comply.
But what if some agents are ill-intentioned and attack the system?

- What are the consequences of an attack?
- Can we mitigate the effects?

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Distributed Model Predictive Control

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State of art

	Decomposition	Present vulnerabilities?	Resilient/Robust	Detection	Mitigation
[Vel+17a] [Mae+21]	Dual	Yes	Robust (Scenario)	NA	NA
[Vel+17b] [Vel+18]	Dual	Yes	Robust (f-robust)	NA	NA
[CMI18]	Jacobi-Gauß	Yes	–	–	–
[Ana+18] [Ana+19] [Ana+20]	Dual	Yes	Resilient	Analyt./Learn.	Disconnect (Robustness)
Our	Primal	Yes	Resilient	Active Analyt./Learn.	Data reconstruction

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Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition
2. Resilient Primal Decomposition-based dMPC for deprived systems
3. Resilient Primal Decomposition-based dMPC using Artificial Scarcity

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Outline

1. Vulnerabilities in distributed MPC based on Primal Decomposition
   - What is the Primal Decomposition?
   - How can an agent attack?
   - Consequences

Distributed Model Predictive Control

Primal Decomposition | Quantity Decomposition | Resource Allocation

Decompose original problem using primal problem

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An example of decomposition method is the Quantity decomposition where a semi-decomposable problem with a global coupling constraints can be decomposed into

Distributed Model Predictive Control

Primal Decomposition | Quantity Decomposition | Resource Allocation

Decompose original problem using primal problem

$$\begin{aligned}
&\underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} && J(\boldsymbol{x}[0|k], \boldsymbol{u}[0:N-1|k]) \\
&\text{subject to} && \left.\begin{aligned} \boldsymbol{x}[\xi|k] &= f(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \\ g_i(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) &\leqslant 0 \\ h_j(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) &= 0 \end{aligned}\right\} \begin{aligned} &\forall \xi \in \{1,\dots,N\} \\ &\forall i \in \{1,\dots,m\} \\ &\forall j \in \{1,\dots,p\} \end{aligned}
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Distributed Model Predictive Control

Decompose original problem using primal problem

$$\begin{aligned} \underset{\boldsymbol{u}[0:N-1|k]}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}}\sum_{\xi\in\mathcal{N}}\left[\|\boldsymbol{v}_i[\xi|k]\|^2_{Q_i} + \|\boldsymbol{u}_i[\xi-1|k]\|^2_{R_i}\right] \\ \text{subject to} \quad & \left.\begin{aligned} \boldsymbol{x}[\xi|k] &= f(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) \\ g_i(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) &\leqslant 0 \\ h_j(\boldsymbol{x}[\xi-1|k], \boldsymbol{u}[\xi-1|k]) &= 0 \end{aligned}\right\} \begin{aligned} &\forall \xi \in \{1,\dots,N\} \\ &\forall i \in \{1,\dots,m\} \\ &\forall j \in \{1,\dots,p\} \end{aligned} \end{aligned}$$

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Distributed Model Predictive Control

Primal Decomposition | Quantity Decomposition | Resource Allocation

E.g. $v_i = w_i - x_i$

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Decompose original problem using primal problem

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Distributed Model Predictive Control

Primal Decomposition | Quantity Decomposition | Resource Allocation

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$$\boldsymbol{\theta}[k]^{(p+1)} = \text{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)}\boldsymbol{\lambda}[k]^{(p)})$$

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Distributed Model Predictive Control

Primal Decomposition | Quantity Decomposition | Resource Allocation

Decompose original problem using primal problem $\mathcal{S} = \{\boldsymbol{\theta}[k] \mid I_c^M \boldsymbol{\theta}[k] \leq \boldsymbol{U}_{\max}\}$

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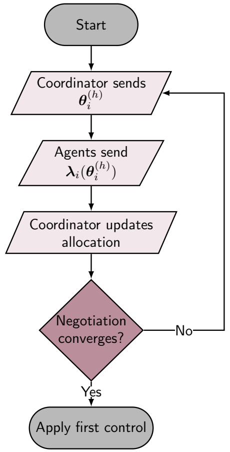
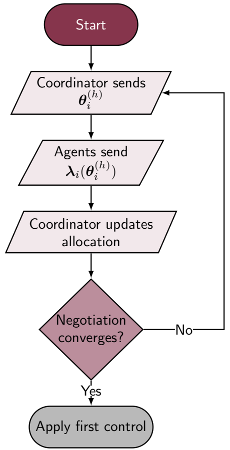
Quantity Decomposition | Resource Allocation



In a flowchart for a quantity decomposition based DMPC,

Quantity Decomposition | Resource Allocation



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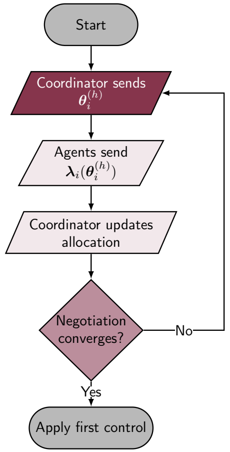

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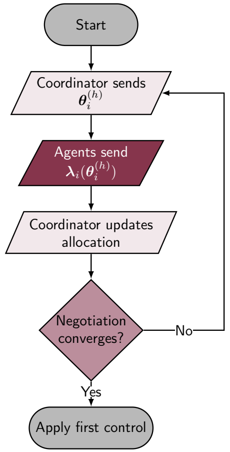
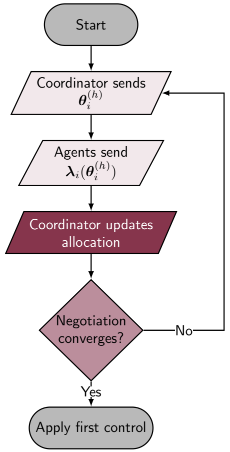
Quantity Decomposition | Resource Allocation



In a flowchart for a quantity decomposition based DMPC,the coordinator sends the allocation theta, the agents send the dual variable lambda,

Quantity Decomposition | Resource Allocation



In a flowchart for a quantity decomposition based DMPC,the coordinator sends the allocation theta, the agents send the dual variable lambda, the coordinator updates the allocation.

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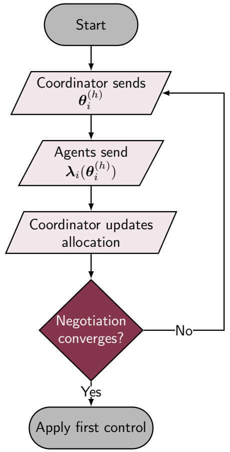
Quantity Decomposition | Resource Allocation



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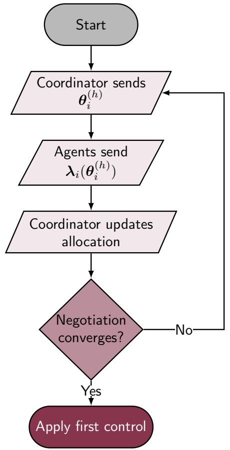
Quantity Decomposition | Resource Allocation



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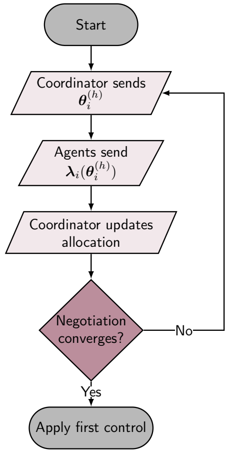
In a flowchart for a quantity decomposition based DMPC,the coordinator sends the allocation theta, the agents send the dual variable lambda, the coordinator updates the allocation. If negotiation converges, then the negotiation ends and each agent applies the first element of the control found

Quantity Decomposition | Resource Allocation



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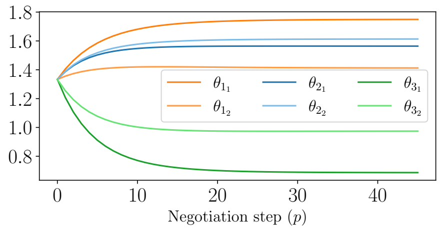

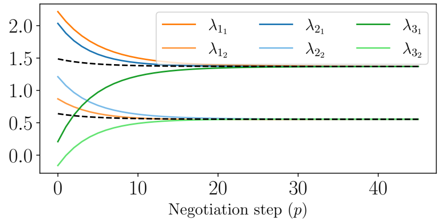

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How can a non-cooperative agent attack?

Literature

- [Vel+17a; CMI18] present some kinds of attacks
  - Objective function
    - Selfish Attack
    - Fake weights
    - Fake reference
  - Fake constraints
  - Liar agent (use different control)

Deception Attacks (False Data Injection)

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Deception Attacks
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How can a non-cooperative agent attack?

Our approach

- $\boldsymbol{\lambda}_i$ is the only interface with coordination
- $\boldsymbol{\lambda}_i$ depends on the parameters of the system
- Malicious agent sends a different $\boldsymbol{\lambda}_i$

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How can a non-cooperative agent attack?

Our approach

- $\boldsymbol{\lambda}_i$ is the only interface with coordination
- $\boldsymbol{\lambda}_i$ depends on the parameters of the system
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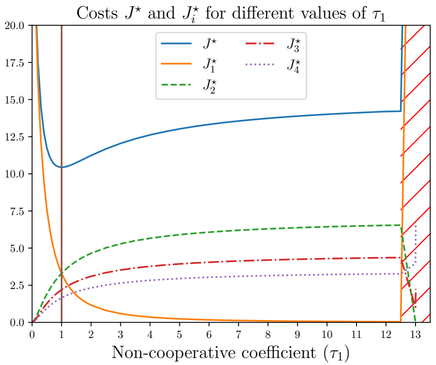

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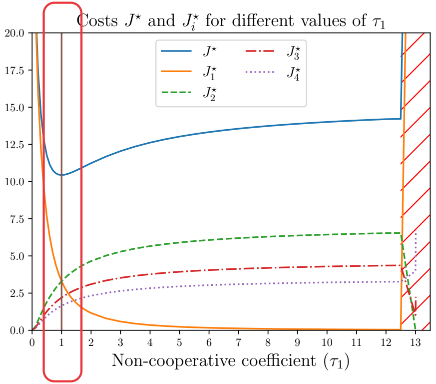

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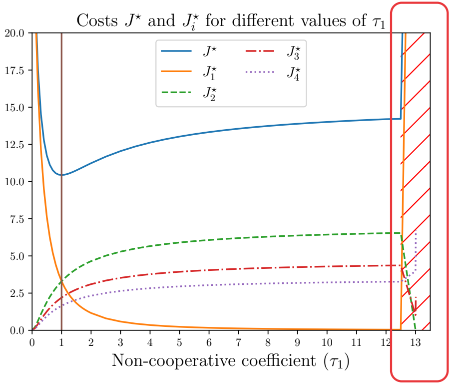

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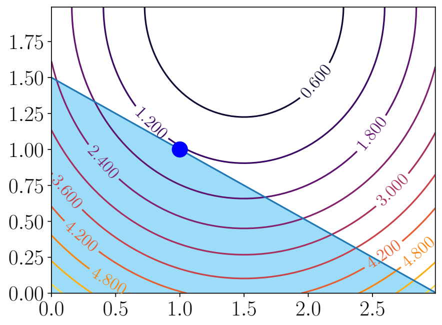

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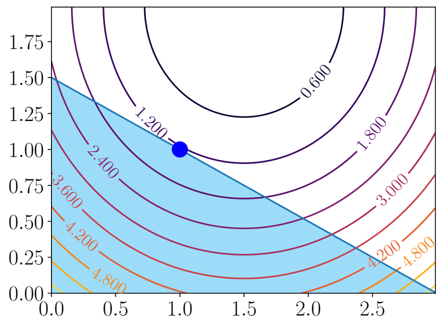

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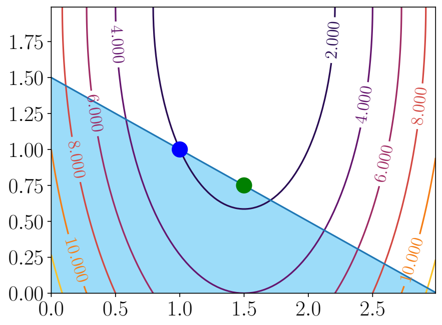

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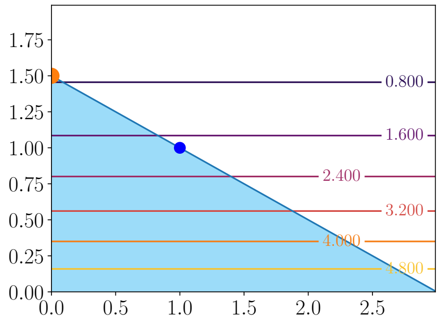

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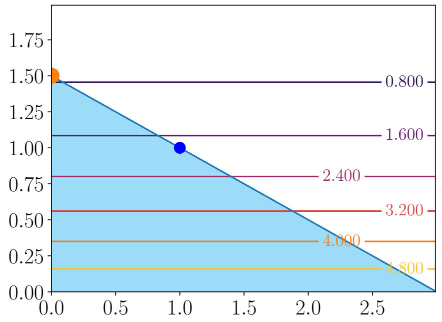

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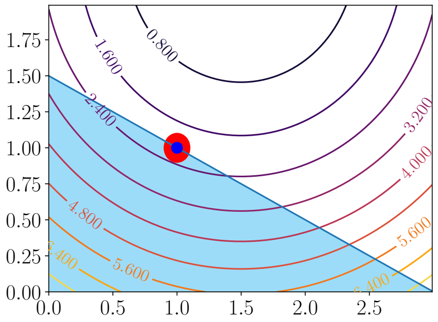

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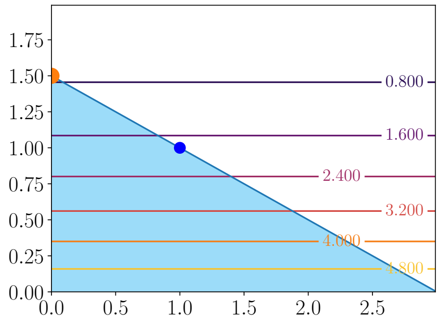

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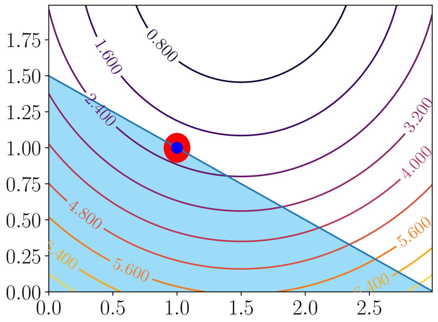

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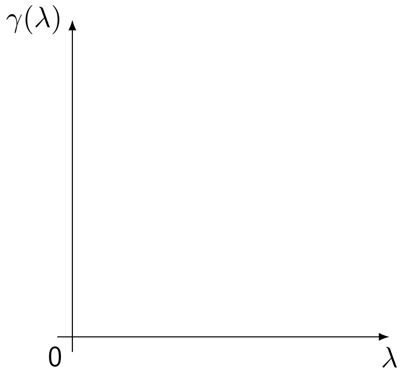

- $\lambda \geqslant 0$ means dissatisfaction
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  - $\gamma(\lambda) = 0 \Leftrightarrow \lambda = 0$

Assumptions

- Attacker is greedy $\gamma(\lambda) > \lambda$
- But not smart

  $\lambda_b > \lambda_a \rightarrow \gamma(\lambda_b) > \gamma(\lambda_a)$

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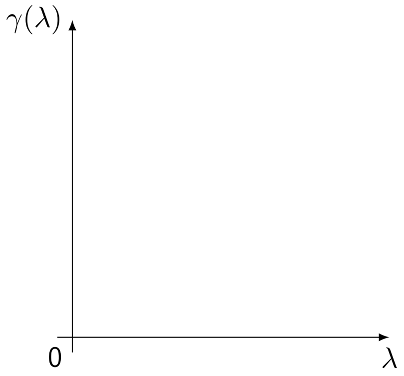

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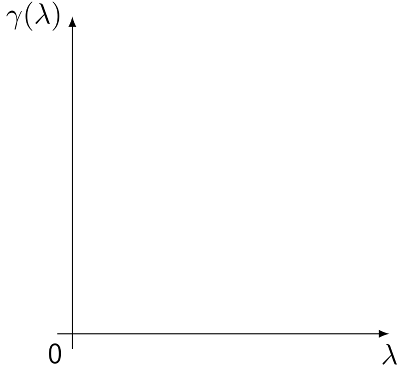

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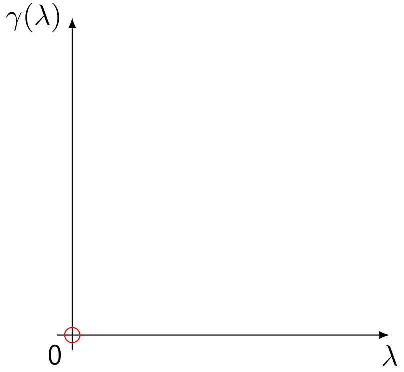

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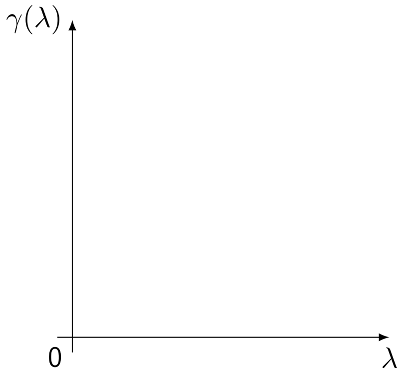

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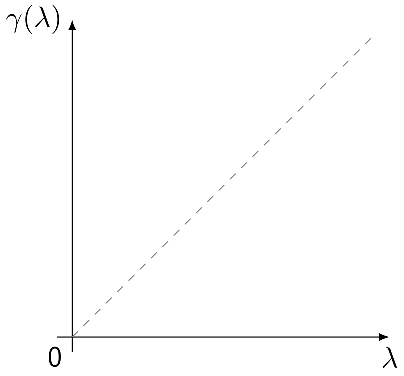

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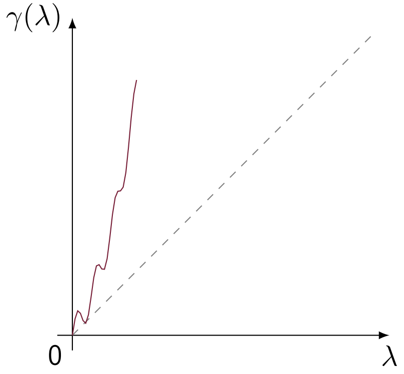

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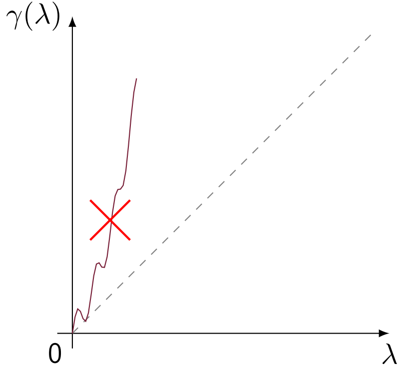

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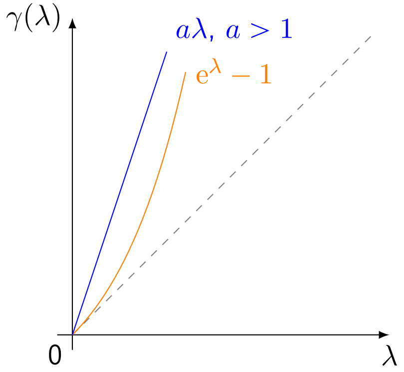

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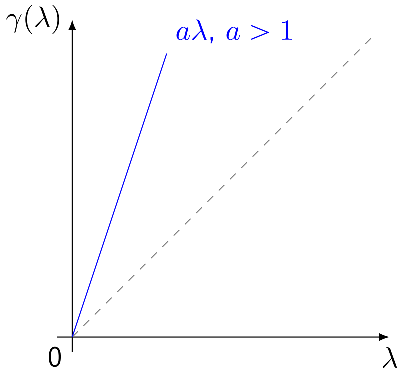

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Outline

2 Resilient Primal Decomposition-based dMPC for deprived systems
- Analyzing deprived systems
- Building an algorithm
- Applying mechanism

What are deprived systems?

$$
\begin{aligned}
\underset{\boldsymbol{U}_i[k]}{\text{minimize}} \quad & \frac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\
\text{subject to} \quad & \bar{\Gamma}_i\boldsymbol{U}_i[k] \preceq \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k]
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- Unconstrained solution $\mathring{\boldsymbol{U}}_i^\star[k] = -H_i^{-1}\boldsymbol{f}_i[k]$
- Deprived if constraints are active $\rightarrow \bar{\Gamma}_i\mathring{\boldsymbol{U}}_i^\star[k] > \boldsymbol{\theta}_i[k], \forall k$
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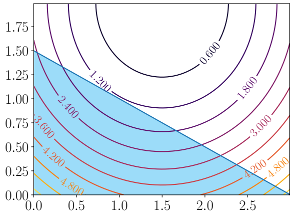

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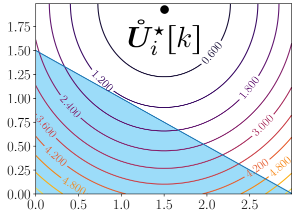

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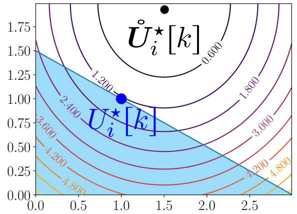

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Deprived Systems

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Analysis

- We can transform inequality constraints into equality ones[2]
- Solution is analytical and trivial.
  - If we solve for λ_i

$$\begin{aligned} \underset{\boldsymbol{U}_i[k]}{\text{minimize}} \quad & \tfrac{1}{2}\left\|\boldsymbol{U}_i[k]\right\|_{H_i}^2 + \boldsymbol{f}_i[k]^T\boldsymbol{U}_i[k] \\ \text{subject to} \quad & \bar{\Gamma}_i\boldsymbol{U}_i[k] \geqslant \boldsymbol{\theta}_i[k] : \boldsymbol{\lambda}_i[k] \end{aligned}$$

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where $P_i = (\bar{\Gamma}_i H_i^{-1}\bar{\Gamma}_i^T)^{-1}$ and $\boldsymbol{s}_i[k] = P_i\bar{\Gamma}_i H_i^{-1}\boldsymbol{f}_i[k]$.

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One can notice, that the solution embeds information not only of the objective function and the reference and state (presence of H_i and $\boldsymbol{f}_i[k]$), but also of the constraints (presence of $\bar{\Gamma}_i$ and $\boldsymbol{\theta}_i[k]$). So, changes in these parameters affect the resulting value of $\boldsymbol{\lambda}_i[k]$.

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Analysis (Continued)

$$\mathcal{S} = \{\boldsymbol{\theta}[k] \mid I_c^M \boldsymbol{\theta}[k] \leqslant \boldsymbol{U}_{\max}\}$$

$$\boldsymbol{\theta}[k]^{(p+1)} = \mathrm{Proj}^{\mathcal{S}}(\boldsymbol{\theta}[k]^{(p)} + \rho^{(p)} \boldsymbol{\lambda}[k]^{(p)})$$

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Deprived Systems

Under attack!

- Normal behavior

$$\boldsymbol{\lambda}_i[k] = -P_i\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i[k],$$

- Under attack

$$\tilde{\boldsymbol{\lambda}}_i = T_i[k]\boldsymbol{\lambda}_i = -T_iP_i\boldsymbol{\theta}_i[k] - T_i\boldsymbol{s}_i[k]$$

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Detection Mechanism

Assumption

We know nominal $\bar{P}_i$

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Attacker chooses $\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i) = T_i(k)\boldsymbol{\lambda}_i$
$-T_i(k)P_i\boldsymbol{\theta}_i - T_i(k)\boldsymbol{s}_i(k) \rightarrow -\tilde{P}_i\boldsymbol{\theta}_i - \tilde{\boldsymbol{s}}_i(k)$

- We can estimate[1] $\hat{P}_i$ and $\hat{\tilde{\boldsymbol{s}}}_i(k)$ such as:

$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i(\boldsymbol{\theta}_i)) = -\widehat{\tilde{P}_i}(k)\boldsymbol{\theta}_i - \hat{\tilde{\boldsymbol{s}}}_i(k)$$

- If $\hat{\tilde{P}}_i(k) \neq \bar{P}_i \rightarrow$ Attack

[1] Using Recursive Least Squares

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For the attack detection, let's assume we know the nominal P, called P bar

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In detail

- Error $E_i(k) = \|\widehat{\bar{P}}_i(k) - \bar{P}_i\|_F$
- Create threshold ϵ_P
- Indicator $d_i \in \{0, 1\}$ detects the attack in agent i.
- $d_i = 1$ if $E_i(k) > \epsilon_P$, 0 otherwise

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So, let's detail the detection mechanism.

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Estimating $\widehat{\widetilde{P}}_i[k]$

- We estimate $\hat{P}_i$ and $\widehat{\tilde{s}}_i(k)$ simultaneously using Recursive Least Squares
- Problem: Estimation during negotiation fails
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CentraleSupélec

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Mitigation mechanism

- Main idea: Reconstruct $\boldsymbol{\lambda}_i$ and use in negotiation

Assumption

We suppose $\tilde{\boldsymbol{\lambda}}_i = \mathbf{0}$ only if $\boldsymbol{\lambda}_i = \mathbf{0}$, which implies $T_i(k)$ invertible.

- Estimate the inverse of $T_i(k)$

$$\widehat{T_i(k)^{-1}} = \bar{P}_i \, \widehat{\widetilde{P}_i(k)^{-1}}$$

- Reconstruct $\boldsymbol{\lambda}_i$

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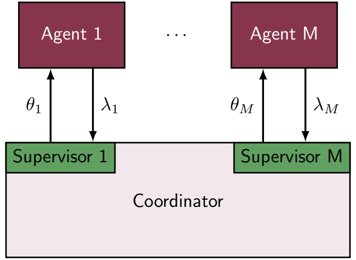
Complete Mechanism

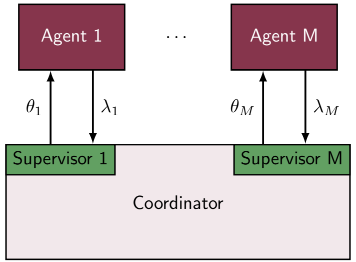


Two phases:

1. Detect which agents are non-cooperative
2. Reconstruct λ_i and use in negotiation

The complete mechanism is equivalent to add a supervisor for each agent inside the coordinator

Complete Mechanism



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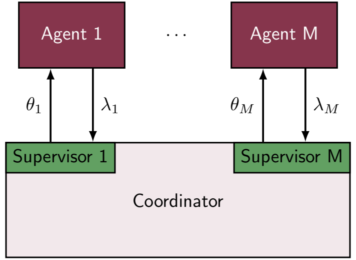

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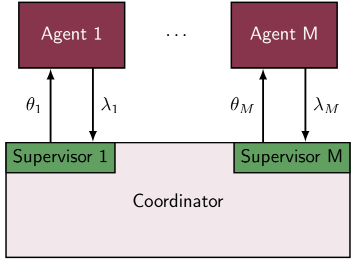

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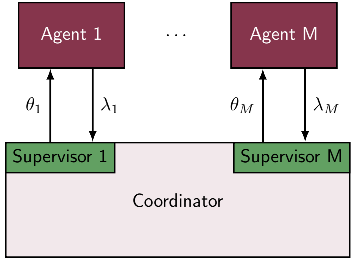

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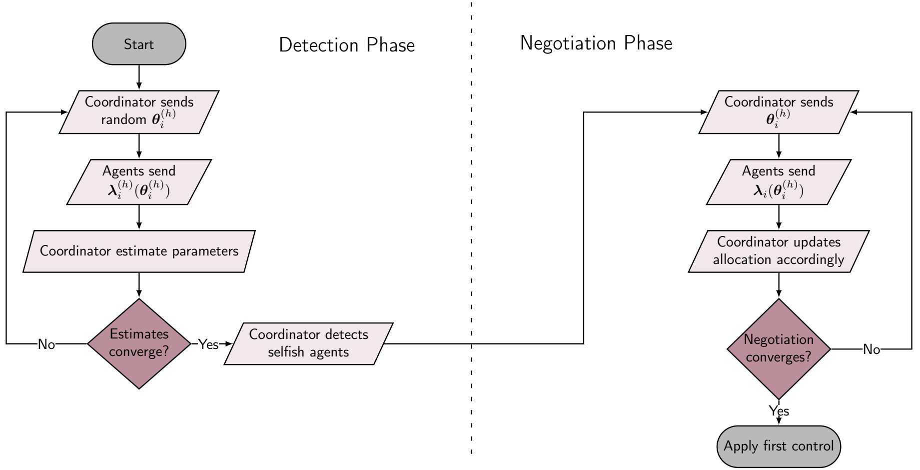
Complete algorithm

RPdMPC-DS

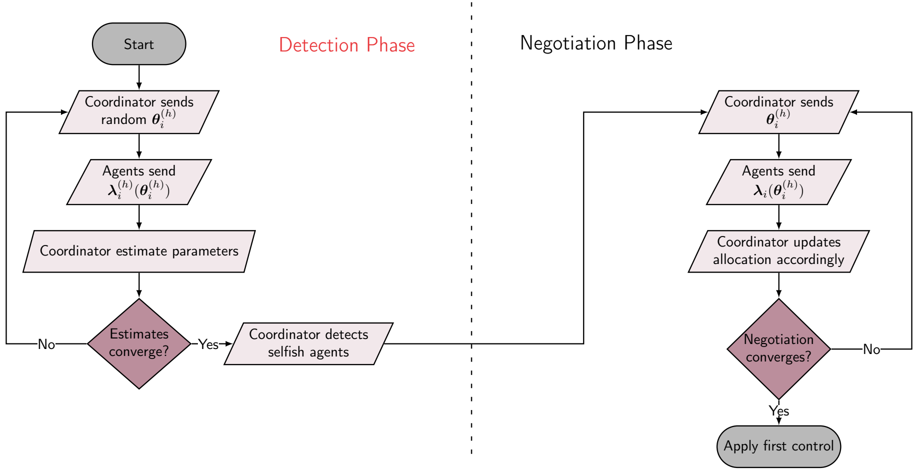


CentraleSupélec

Now, for the complete secure DMPC algorithm, as said it is divided into two phases

Complete algorithm

RPdMPC-DS



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The Detection phase

Complete algorithm

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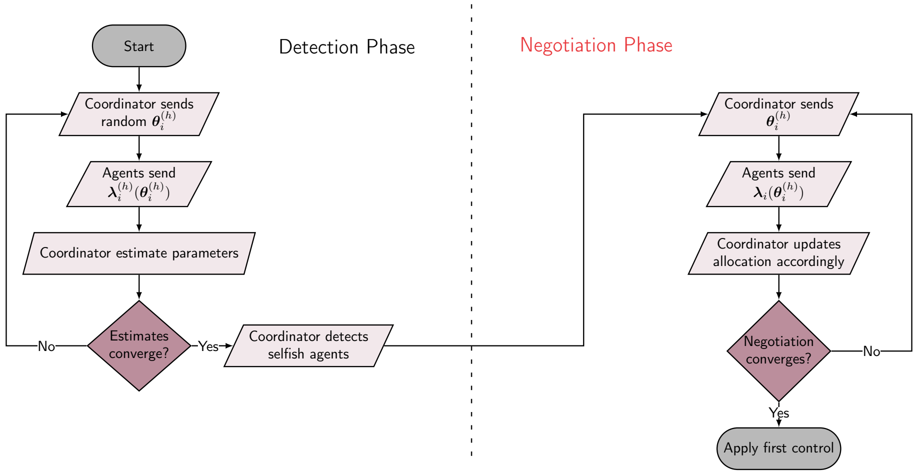

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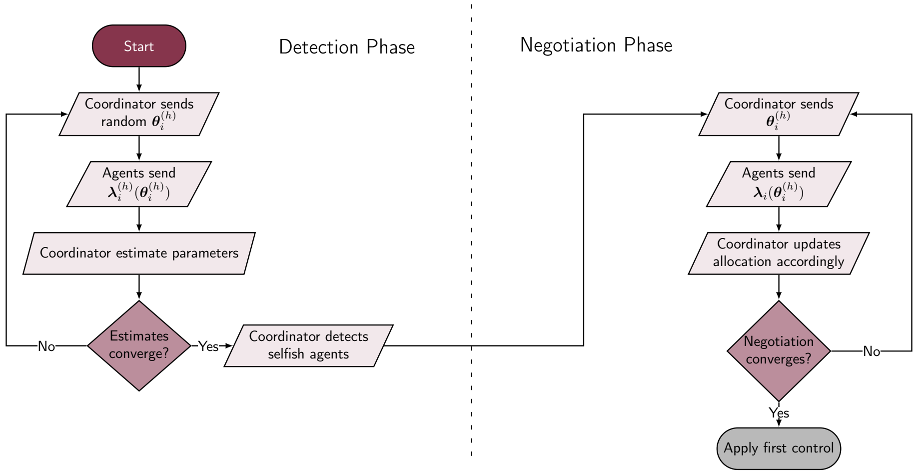

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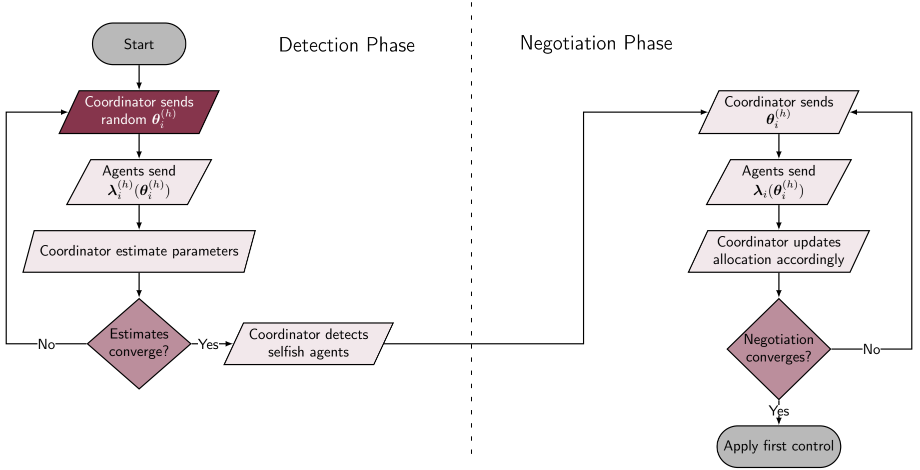

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Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
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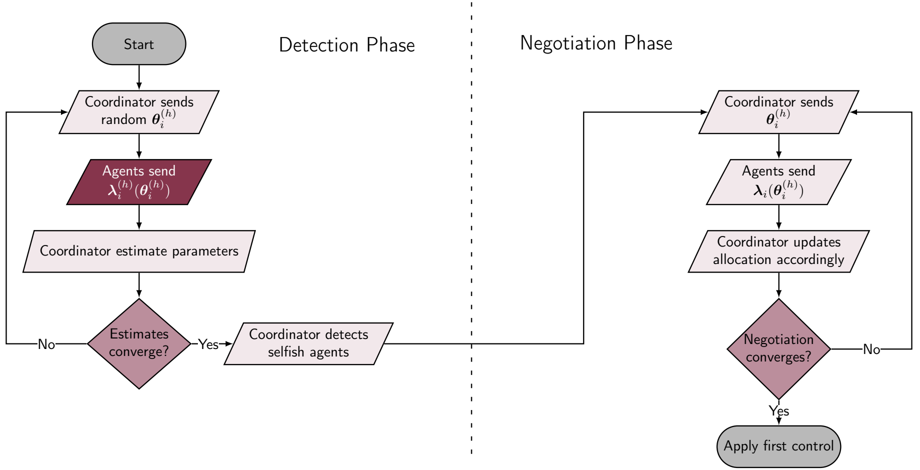
The coordinator sends random theta i

Complete algorithm

RPdMPC-DS



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Complete algorithm

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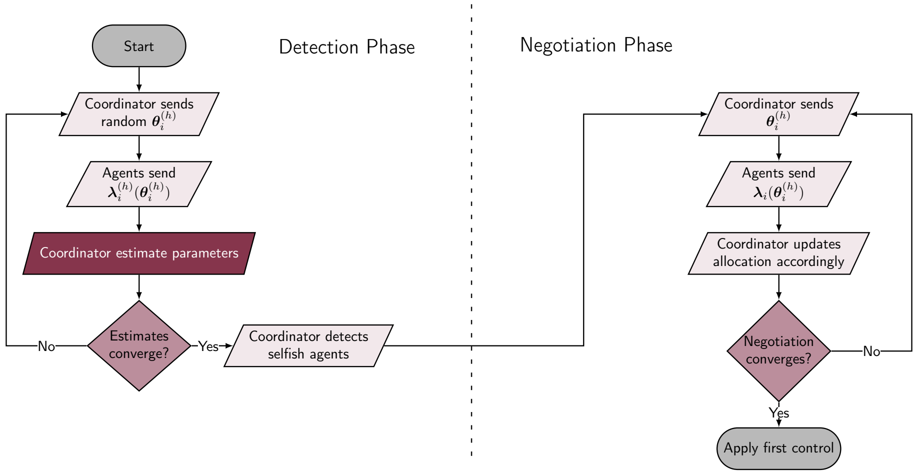

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The coordinator estimates the parameters P and s tilde

Complete algorithm

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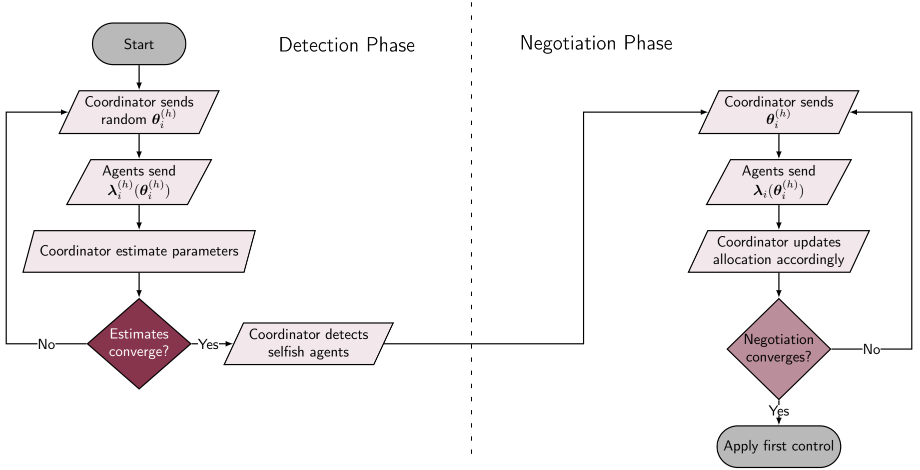

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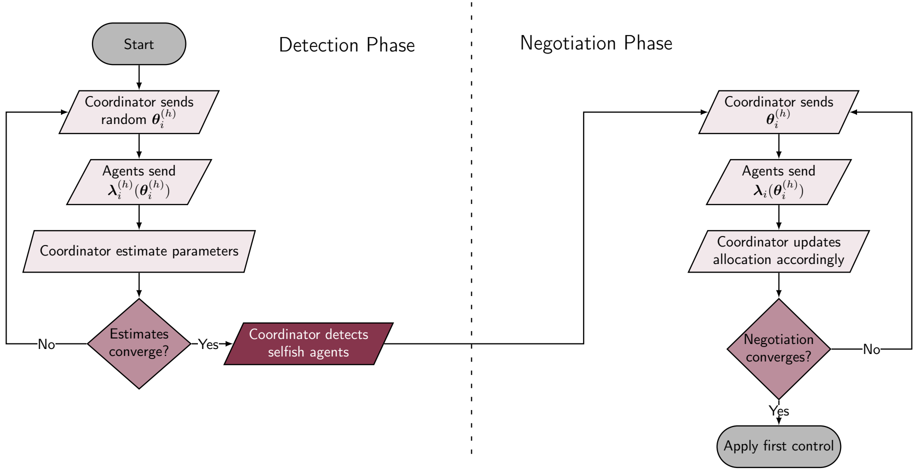

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The coordinator detects which agents are non-cooperative

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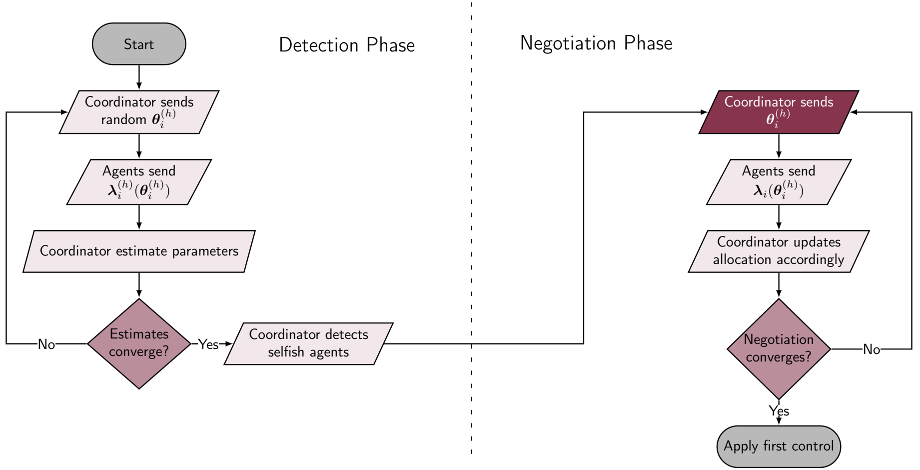

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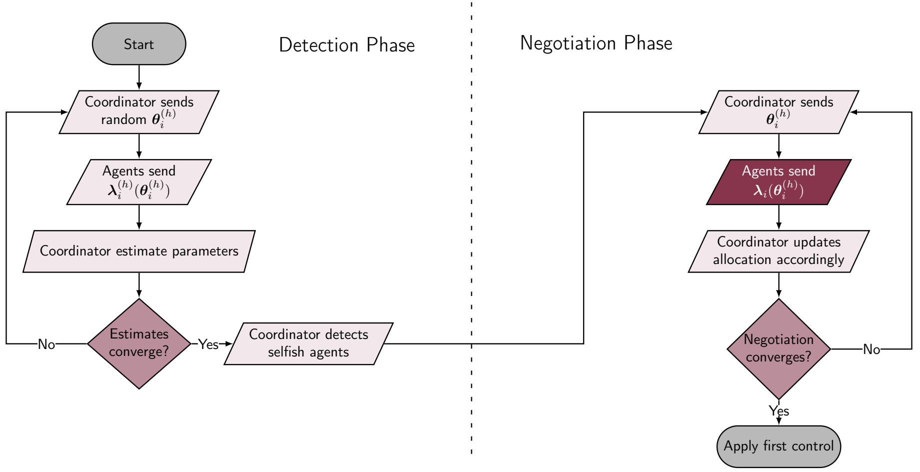

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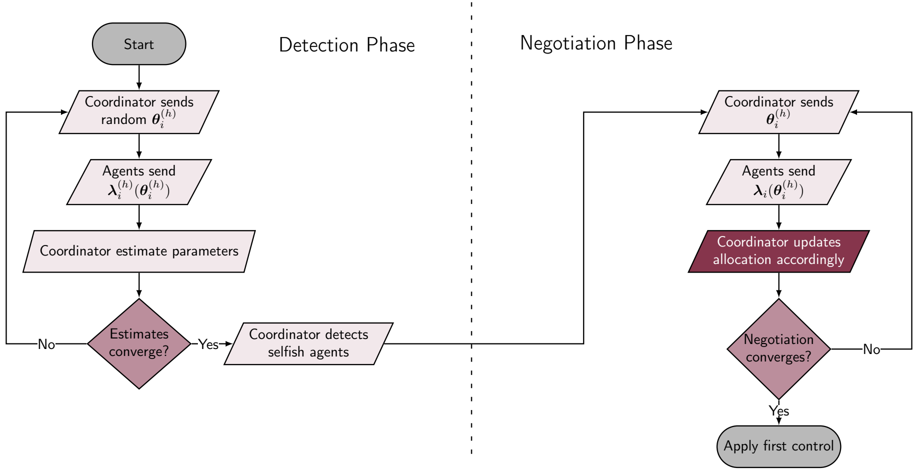

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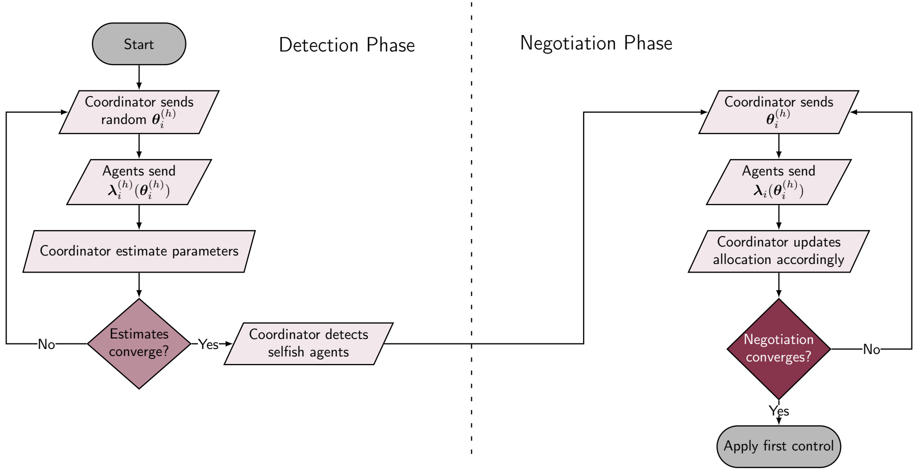

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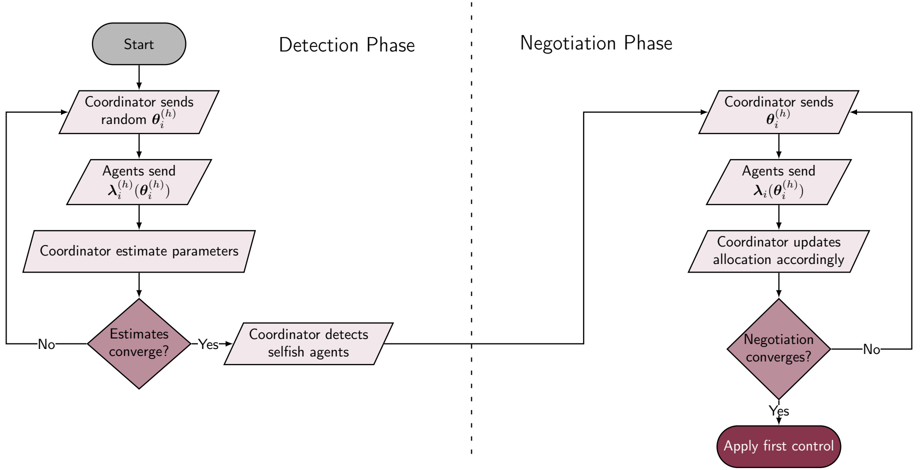

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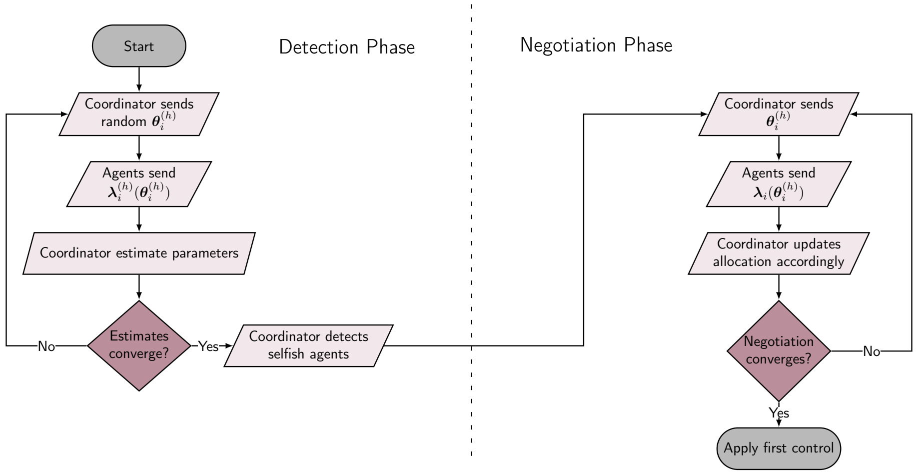

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Example

District Heating of 4 Distinct Houses Under Power Scarcity

- 4 distinct rooms modeled using 3R-2C
- Initial temperature under $20^o\mathrm{C}$
- Not enough power to achieve setpoint $\left(\sum_{i=1}^{4} \boldsymbol{u}_i(k) \leqslant 4\mathrm{kW}\right)$
- Simulated for a period of $5\mathrm{h}$
- ZOH $T_s = 0.25\mathrm{h}$
- 3 scenarios
  1. Nominal
  2. Agent I non cooperative from k>6 with T=4*I
  3. Similar but with secure algorithm

CentraleSupélec

We give an academic example of the temperature control of 4 distinct rooms under power scarcity

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The 4 rooms are distinct using the 3 resistor 2 capacitor model
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Results

Temporal

- N Nominal
- S Selflish behavior
- C selfish behavior with Correction

In the figure we see the air temperature and the estimation error for room 1

Results

Temporal

N Nominal

S Selflish behavior

C selfish behavior with Correction

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In the figure we see the air temperature and the estimation error for room 1
The nominal behavior is in orange, and as said it cannot reach the setpoint, in blue

Results

Temporal

N Nominal

S Selflish behavior

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When the room presents selfish behavior (in blue), it reduces its cost and get closer to the setpoint, we see that the attack increases the error

Results

Temporal

- N Nominal
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Results

Table 1: Comparison of costs J_i^N and J_G^N

Agent	Nominal	Selfish	Selfish + correction
I	103	64	104
II	73	91	73
III	100	123	101
IV	132	154	131
Global	408	442	409

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Now, if we compare the costs for each scenario we see how the cost of agent 1 decreases when it attacks, while the cost of other agents increase.

Results

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Outline

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Analysing

$$\boldsymbol{\lambda}_i[k] = \begin{cases} -P_i^{(0)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(0)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}_{\boldsymbol{\lambda}_i}^{n} \\ \vdots & \vdots \\ -P_i^{(2^{n_{\text{ineq}}}-1)}\boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^{(2^{n_{\text{ineq}}}-1)}[k], & \text{if } \boldsymbol{\theta}_i[k] \in \mathcal{R}_{\boldsymbol{\lambda}_i}^{2^{n_{\text{ineq}}}-1} \end{cases}. \quad (1)$$

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Artificial Scarcity

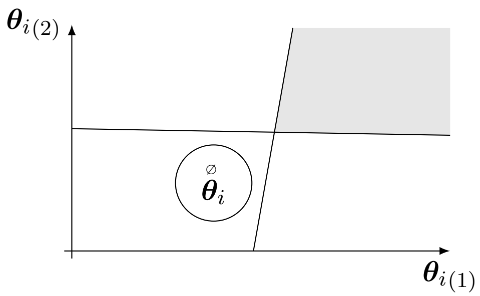


Figure 1: Ball $\mathcal{B}(\overset{\varnothing}{\boldsymbol{\theta}}_i, r)$.

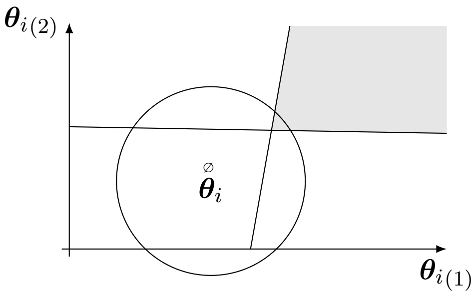


Figure 2: Ball $\mathcal{B}(\overset{\varnothing}{\boldsymbol{\theta}}_i, r)$ traversing zones.

Expectation Maximization

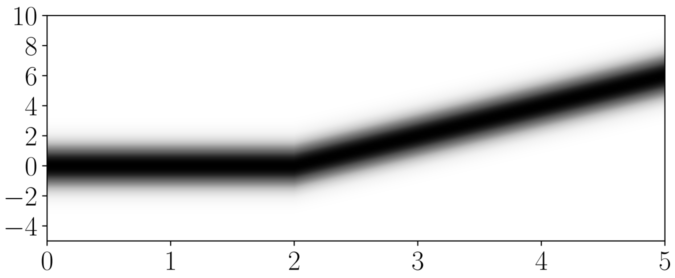


Figure 3: Gaussian Mixture for a 1D PWA function with 2 modes.

Expectation Maximization

Algorithm

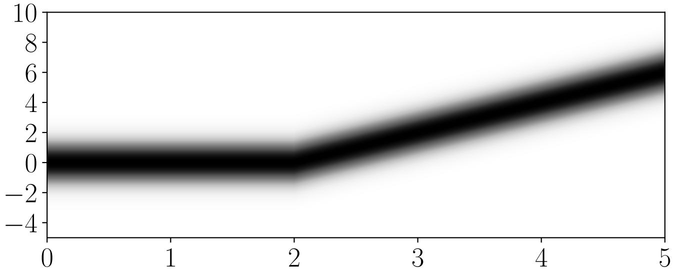
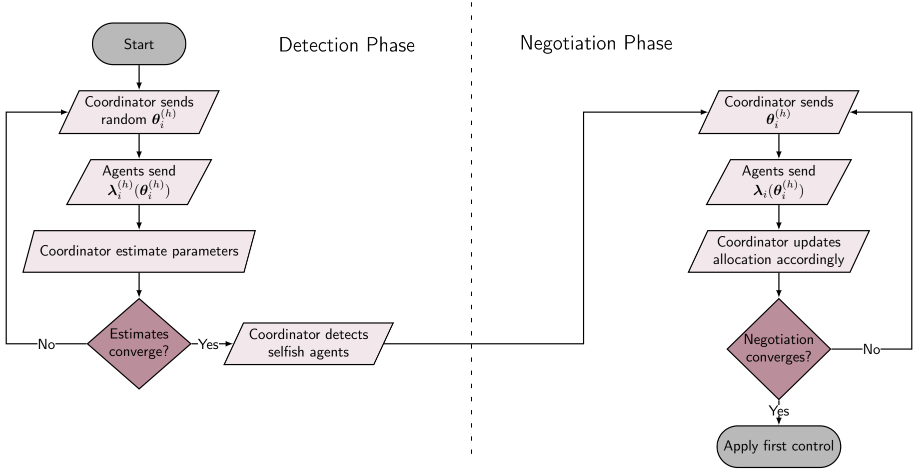


Figure 4: Gaussian Mixture for a 1D PWA function with 2 modes.

Complete algorithm

RPdMPC-AS



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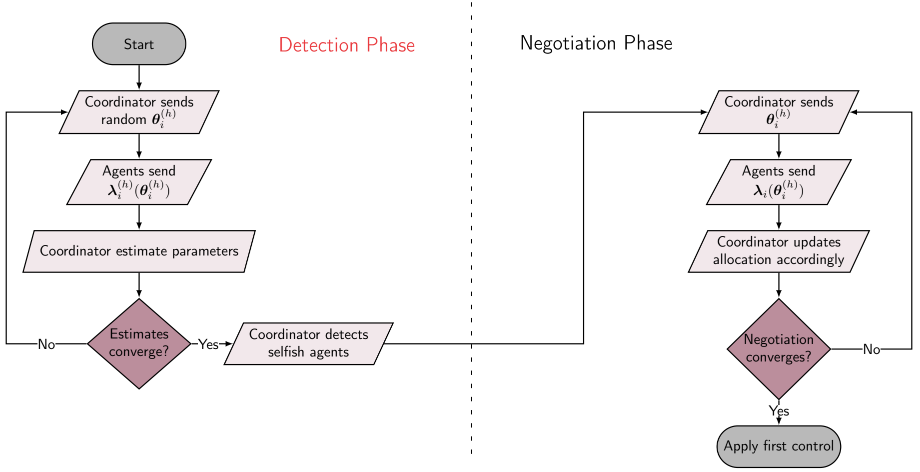
Now, for the complete secure DMPC algorithm, as said it is divided into two phases

Complete algorithm

RPdMPC-AS



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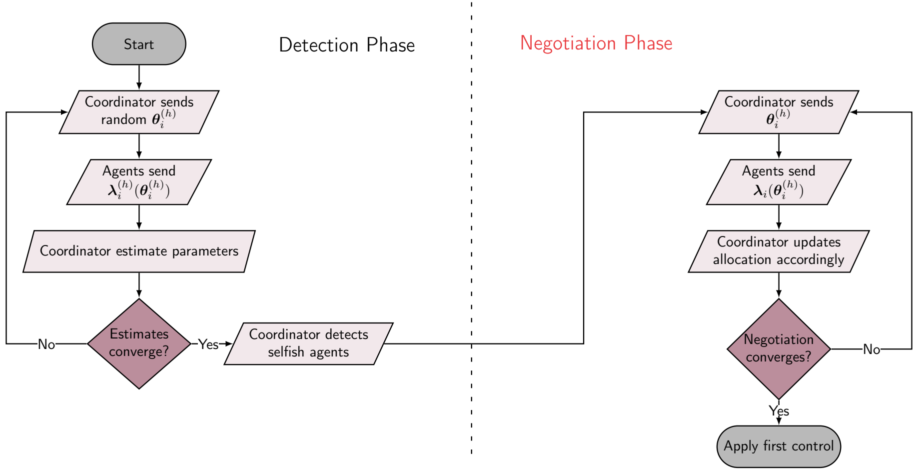
Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase

Complete algorithm

RPdMPC-AS



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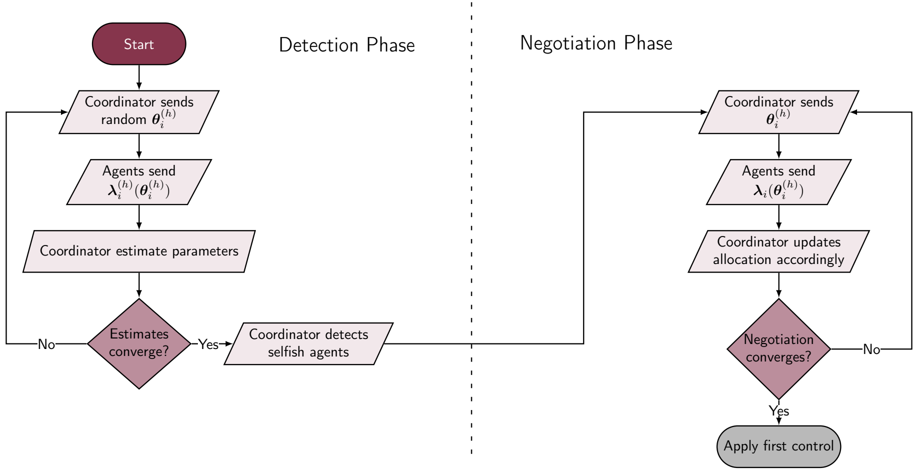
Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
and the negotiation phase

Complete algorithm

RPdMPC-AS



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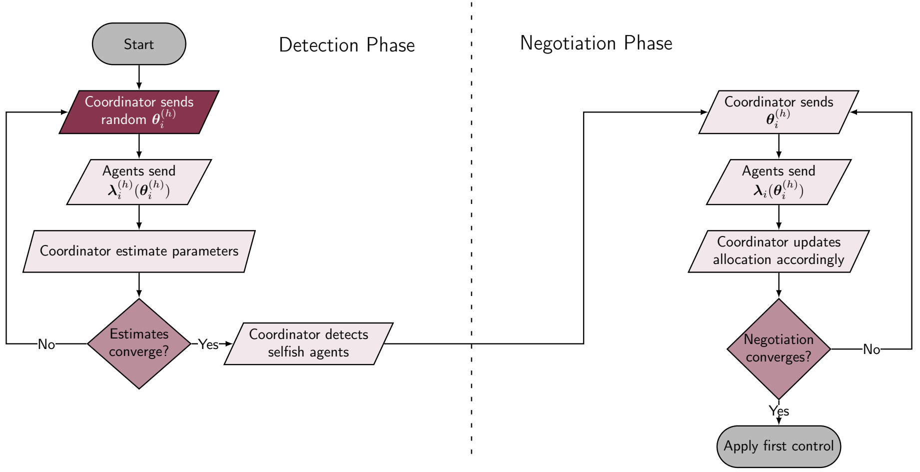
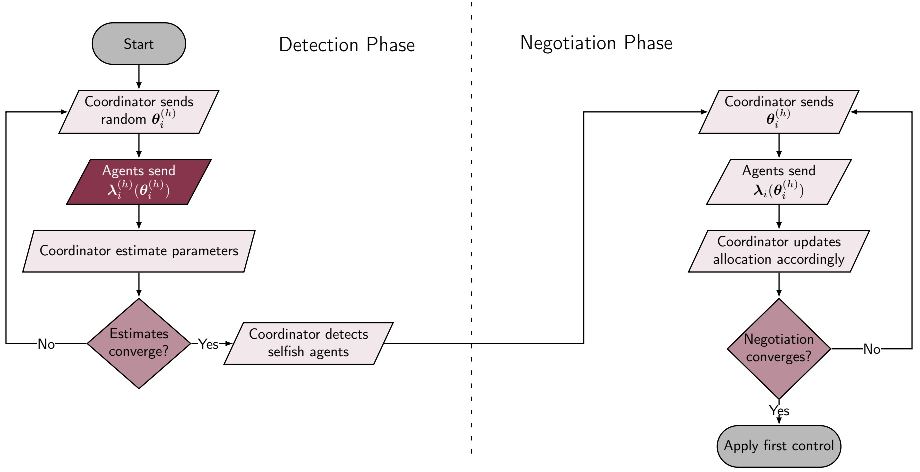


Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
and the negotiation phase
The coordinator sends random theta i

Complete algorithm

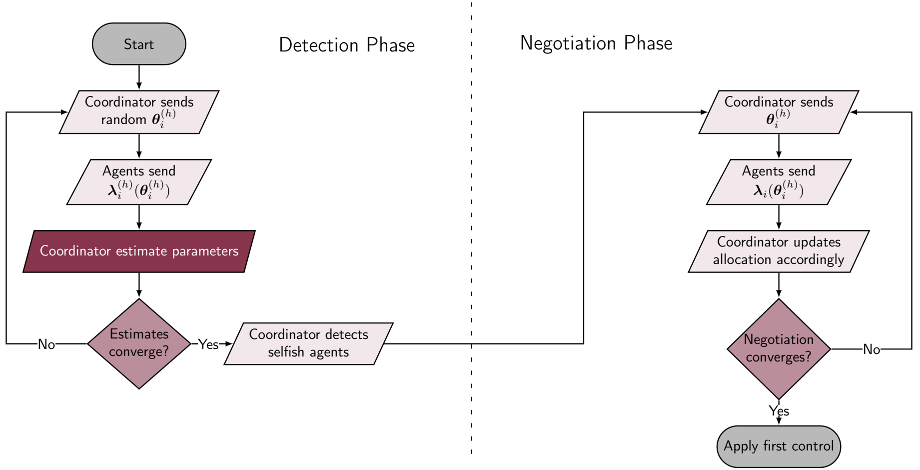
RPdMPC-AS



Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
and the negotiation phase
The coordinator sends random theta i
The agents send dual variable lambda i

Complete algorithm

RPdMPC-AS



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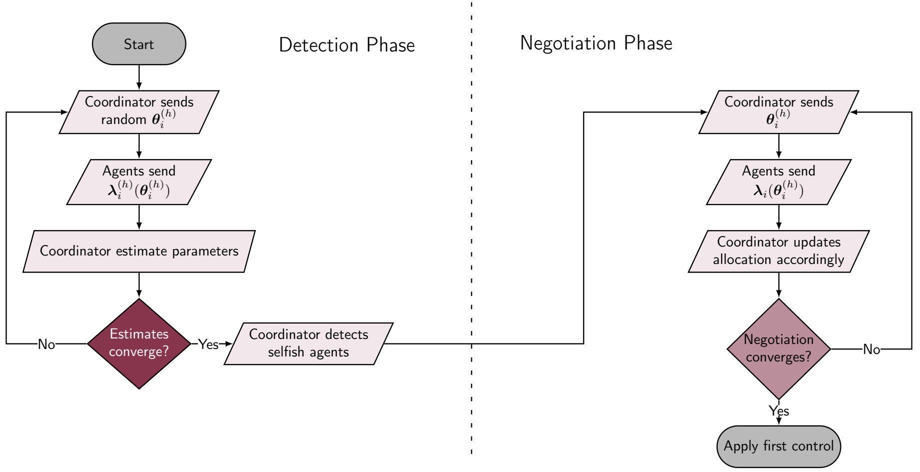
Now, for the complete secure DMPC algorithm, as said it is divided into two phases
The Detection phase
and the negotiation phase
The coordinator sends random theta i
The agents send dual variable lambda i
The coordinator estimates the parameters P and s tilde

Complete algorithm

RPdMPC-AS



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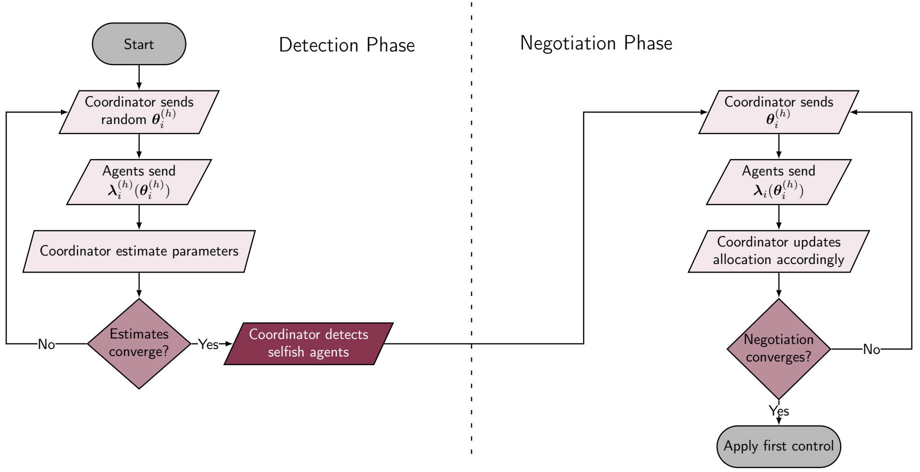
The Detection phase
and the negotiation phase
The coordinator sends random theta i
The agents send dual variable lambda i
The coordinator estimates the parameters P and s tilde
when the estimates converge

Complete algorithm

RPdMPC-AS



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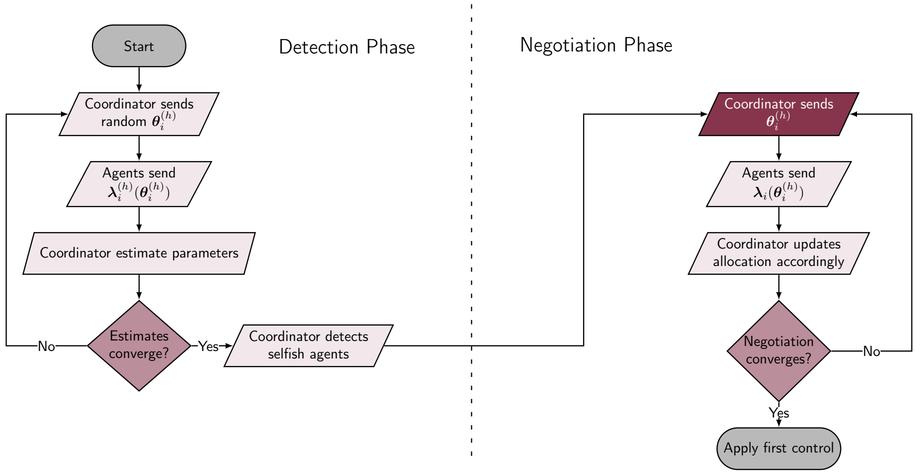
The Detection phase
and the negotiation phase
The coordinator sends random theta i
The agents send dual variable lambda i
The coordinator estimates the parameters P and s tilde
when the estimates converge
The coordinator detects which agents are non-cooperative

Complete algorithm

RPdMPC-AS



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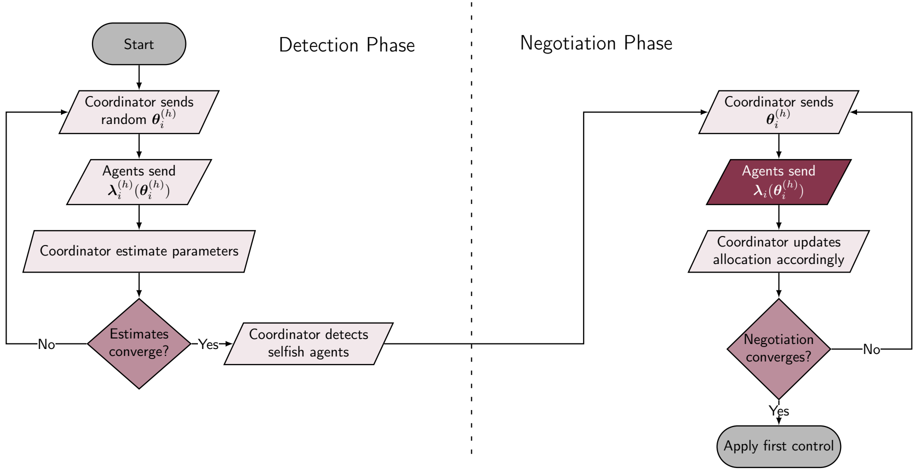
The Detection phase
and the negotiation phase
The coordinator sends random theta i
The agents send dual variable lambda i
The coordinator estimates the parameters P and s tilde
when the estimates converge
The coordinator detects which agents are non-cooperative
then the negotiation phase begins, the coordinator sends the theta i

Complete algorithm

RPdMPC-AS



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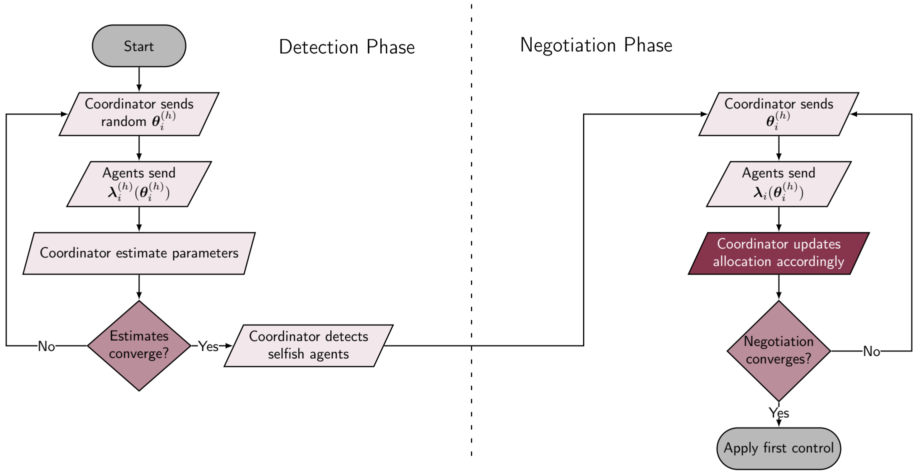

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Now, for the complete secure DMPC algorithm, as said it is divided into two phases
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The coordinator sends random theta i
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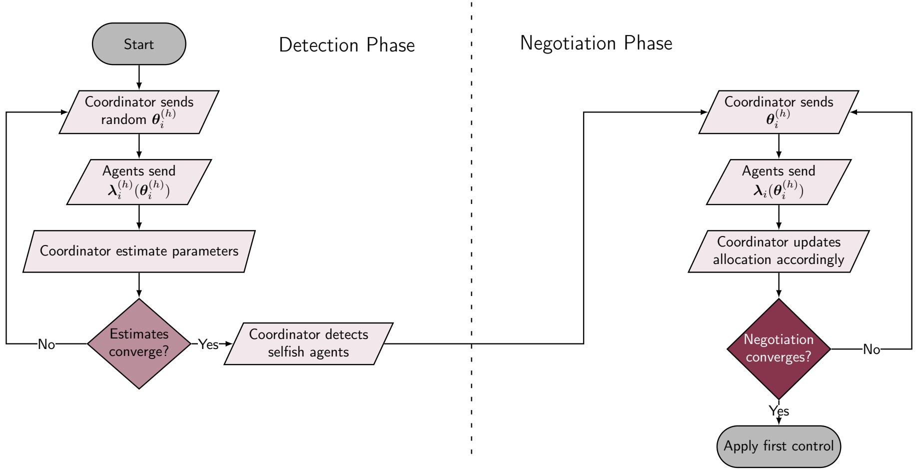
The coordinator detects which agents are non-cooperative
then the negotiation phase begins, the coordinator sends the theta i
the agents send the dual variable lambda i
and the coordinator updates the allocation accordingly using the reconstructed lambda or the one sent by the agent

Complete algorithm

RPdMPC-AS



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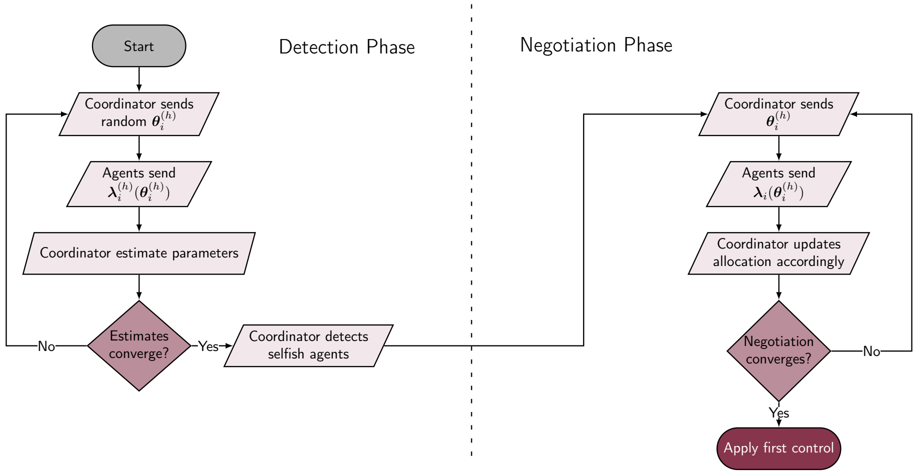

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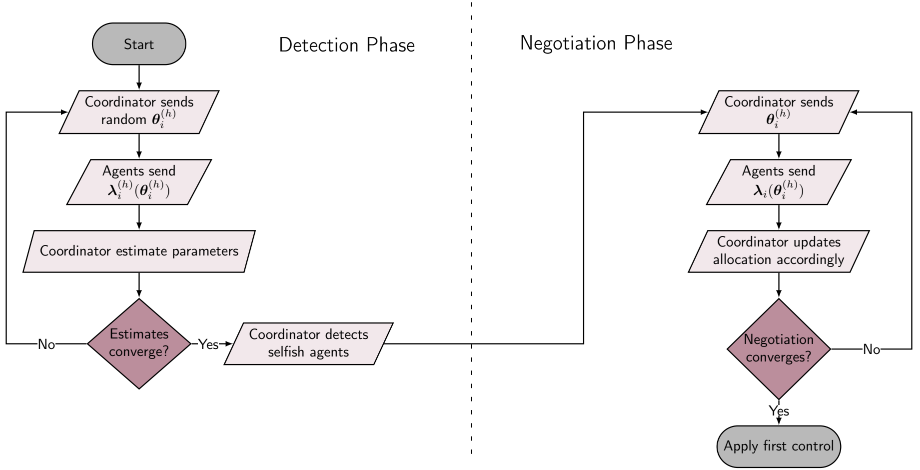
and once the negotiation converges
each agent applies the first control

Complete algorithm

RPdMPC-AS



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Thank you!

Repository
https://github.com/Accacio/thesis

Contact
rafael.accacio.nogueira@gmail.com

If you want to see the simulations of this paper we have a github repository, and if you want to send me an email about this paper or this presentation you can flash the QR code in the right. Thank you!

For Further Reading I

José M Maestre, Rudy R Negenborn, et al. *Distributed Model Predictive Control made easy*. Vol. 69. Springer, 2014. ISBN: 978-94-007-7005-8.

Wicak Ananduta et al. "Resilient Distributed Model Predictive Control for Energy Management of Interconnected Microgrids". In: *Optimal Control Applications and Methods* 41.1 (2020), pp. 146–169. DOI: 10.1002/oca.2534. URL: https://onlinelibrary.wiley.com/doi/pdf/10.1002/oca.2534.

José M. Maestre et al. "Scenario-Based Defense Mechanism Against Vulnerabilities in Lagrange-Based Dmpc". In: *Control Eng Pract* 114 (2021), p. 104879. ISSN: 0967-0661. DOI: 10.1016/j.conengprac.2021.104879.

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As a recommended reading I give a book about distributed MPC and an article with another secure dmpc algorithm based in another decomposition method. That's all

For Further Reading II

Pablo Velarde et al. "Vulnerabilities in Lagrange-Based Distributed Model Predictive Control". In: *Optimal Control Applications and Methods* 39.2 (Sept. 2018), pp. 601–621. DOI: 10.1002/oca.2368.

Wicak Ananduta et al. "Resilient Distributed Energy Management for Systems of Interconnected Microgrids". In: *2018 IEEE Conference on Decision and Control (CDC)*. 2018, pp. 3159–3164. DOI: 10.1109/CDC.2018.8619548.

Wicak Ananduta et al. "A Resilient Approach for Distributed MPC-Based Economic Dispatch in Interconnected Microgrids". In: *2019 18th European Control Conference (ECC)*. 2019, pp. 691–696. DOI: 10.23919/ECC.2019.8796208.

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For Further Reading III

P. Chanfreut, J. M. Maestre, and H. Ishii. “Vulnerabilities in Distributed Model Predictive Control based on Jacobi-Gauss Decomposition”. In: *2018 European Control Conference (ECC)*. June 2018, pp. 2587–2592. DOI: 10.23919/ECC.2018.8550239.

Pablo Velarde et al. “Scenario-based defense mechanism for distributed model predictive control”. In: *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*. IEEE. Dec. 2017, pp. 6171–6176. DOI: 10.1109/CDC.2017.8264590.

Pablo Velarde et al. “Vulnerabilities in Lagrange-Based DMPC in the Context of Cyber-Security”. In: *2017 IEEE International Conference on Autonomic Computing (ICAC)*. July 2017, pp. 215–220. DOI: 10.1109/ICAC.2017.53.

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Conditions

back

One way to ensure this, is to make the original constraint (??) to have at most as many rows as columns, i.e., $\# \boldsymbol{u}_{\max} \leqslant n_u$, although it may be a little restrictive.

θ dynamics

back

$$\boldsymbol{\theta}^{(p+1)} = \mathcal{A}_\theta \boldsymbol{\theta}^{(p)} + \mathcal{B}_\theta[k]$$

where

$$\mathcal{A}_\theta = \begin{bmatrix} I - \frac{M-1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \frac{1}{M}\rho^{(p)}P_1 & I - \frac{M-1}{M}\rho^{(p)}P_2 & \dots & \frac{1}{M}\rho^{(p)}P_M \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{M}\rho^{(p)}P_1 & \frac{1}{M}\rho^{(p)}P_2 & \dots & I - \frac{M-1}{M}\rho^{(p)}P_M \end{bmatrix}$$

$$\mathcal{B}_\theta[k] = \begin{bmatrix} -\frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \\ \vdots \\ \frac{1}{M}\rho^{(p)}\boldsymbol{s}_1[k] + \frac{1}{M}\rho^{(p)}\boldsymbol{s}_2[k] \cdots - \frac{M-1}{M}\rho^{(p)}\boldsymbol{s}_M[k] \end{bmatrix}$$

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