Q2. We need to apply Linear Discriminant analysis for $p = 1$ to find the probability that the company will issue dividends.

Step 1: Developing the equation of probability.

We know, from Bayes theorem

$$P_K(x) = \frac{\pi_k \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\left(\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}}{\sum_{l=1}^{K} \pi_l \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\left(\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}} \quad -(1)$$

In this expression, we know that the value of the denominator is $P(x)$, which is independent of 'K' and can be treated as a constant. Combining the constants together, we get:

$$P_K(x) = C \cdot \pi_k \cdot e^{-\left(\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}$$

Taking $\log_e()$ on both sides, we get:

$$\ln(P_K(x)) = \ln(C) + \ln(\pi_k) - \frac{1}{2\sigma^2}(x-\mu_k)^2$$

$$= \ln(C) + \ln(\pi_k) - \frac{x^2}{2\sigma^2} + x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2}$$

$$= \ln(c) + \ln(\pi_k) - \frac{x^2}{2\sigma^2} + x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2}$$

$$= \underbrace{\ln(c) - \frac{x^2}{2\sigma^2}}_{\text{constants w.r.t 'k'}} + \ln(\pi_k) + x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2}$$

$$\ln(P_k(x)) = c' + \underbrace{\ln(\pi_k) + x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2}}_{\text{Expression for } \delta_k(x)} \quad - (2)$$

Expression for $\delta_k(x)$ which we maximize with respect to different 'k' to see which class the obs. belongs to.

This expression is important as we cannot directly evaluate $\delta_k(x)$ to find $P_k(x)$, we must also calculate the c' term to get the true probability.

Calculating c':

$$c' = \frac{-x^2}{2\sigma^2} + \ln\left(\frac{\frac{1}{\sqrt{2\pi}\sigma}}{\pi_0 \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\left(\frac{1}{2\sigma^2}(x-\mu_0)^2\right)} + \pi_1 \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\left(\frac{1}{2\sigma^2}(x-\mu_1)^2\right)}}\right)$$

where $x = 4$, $\pi_0 = 0.2$, $\pi_1 = 0.8$

$\sigma^2 = 36$, $\sigma = 6$, $\mu_0 = 0$, $\mu_1 = 10$

(PTO →)

Substituting the values, we get:

$$c' = \frac{-(4)^2}{2(36)} + \ln\left(\frac{\cancel{\frac{1}{\sqrt{2\pi}\times 6}}}{\cancel{\frac{1}{\sqrt{2\pi}\times 6}}\left(0.2\times e^{-\left(\frac{1}{2\times 36}(4-0)^2\right)} + 0.8\times e^{-\left(\frac{1}{2\times 36}(4-10)^2\right)}\right)} \right)$$

Removing common terms

$$= \frac{-2}{9} + \ln\left(\frac{1}{0.2\times e^{-(4^2/72)} + 0.8\times e^{-(6^2/72)}} \right)$$

$$= \frac{-2}{9} + 0.437928$$

$$c' = 0.21570 \quad - (3)$$

Evaluating $\delta_1(x)$

$$\delta_1(x) = \ln(0.8) + 4\times\frac{10}{36} - \frac{10^2}{2(36)}$$

$$\delta_1(x) = -0.50092 \quad - (4)$$

Thus, substituting (3), (4) in (2), we get

$$\ln(P_1(x)) = 0.21570 - 0.50092 = -0.28522$$

$$\Rightarrow P_1(x) = 0.75184 \simeq 75.2\,\%$$

This is the probability with which the company will issue a dividend. (for $X = 4$)