



Introduction to Scientific Computation

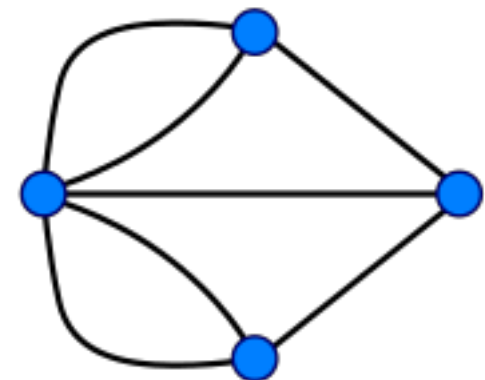
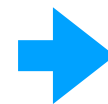
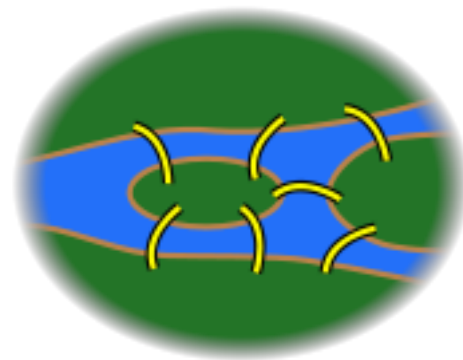
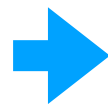
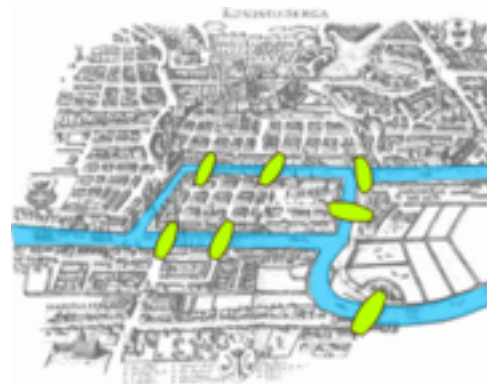
Lecture 5

Fall 2018

Graphs

Graph

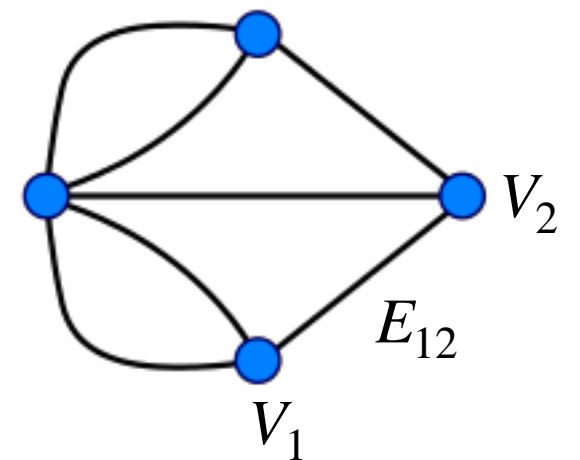
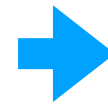
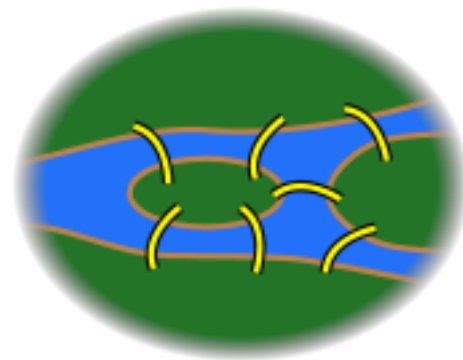
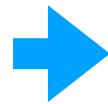
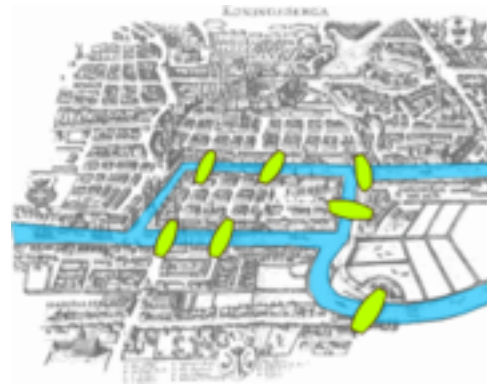
Seven Bridges of Königsberg



Leonhard Euler, 1736

Graph

$$G :< V, E >$$



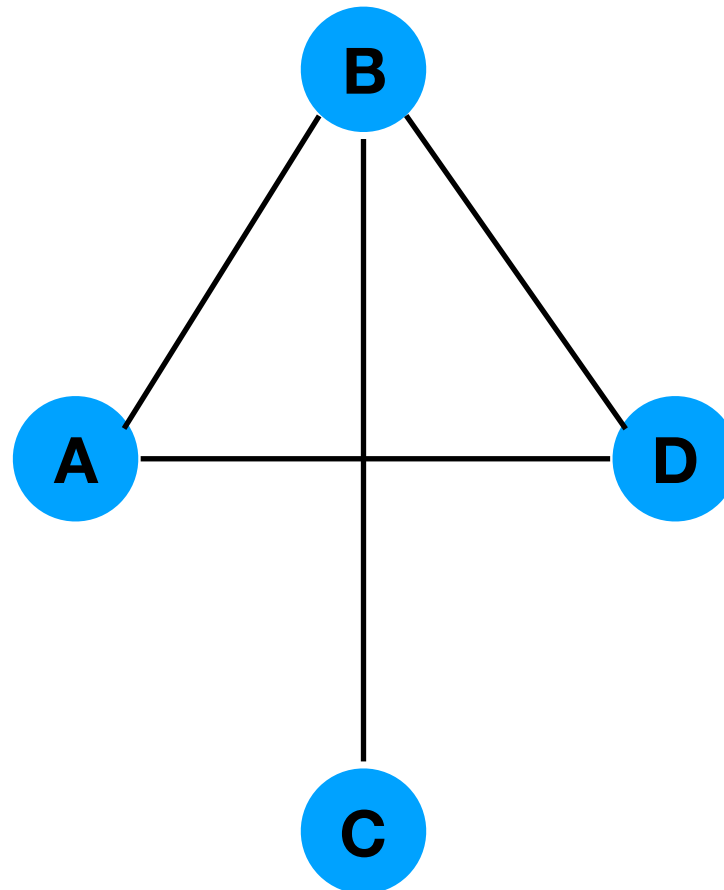
V node in the graph

E edge in the graph, the connection between two nodes

Applications

- Road networks
- Electronic circuits
- Telecommunication networks
- Social networks
- Any relationships ...

Types

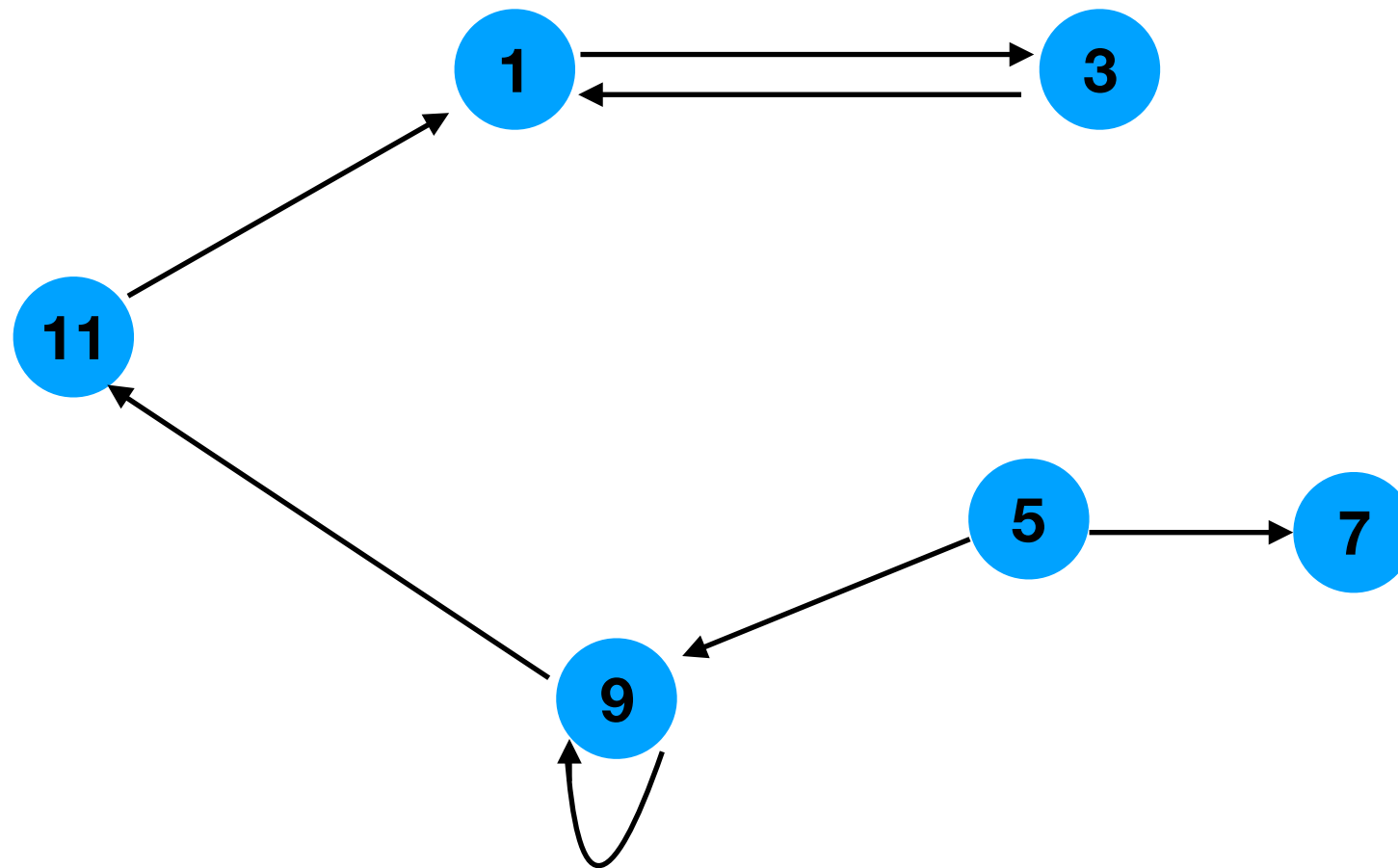


Undirected G

$$V = \{A, B, C, D\}$$

$$E = \{AB, BD, AD, BC\}$$

Types

Directed G

$$V = \{1, 3, 5, 7, 9, 11\}$$

$$E = \{(1, 3), (3, 1), (5, 7), (5, 9), (5, 7), (5, 9), (9, 9), (9, 11), (11, 1)\}$$

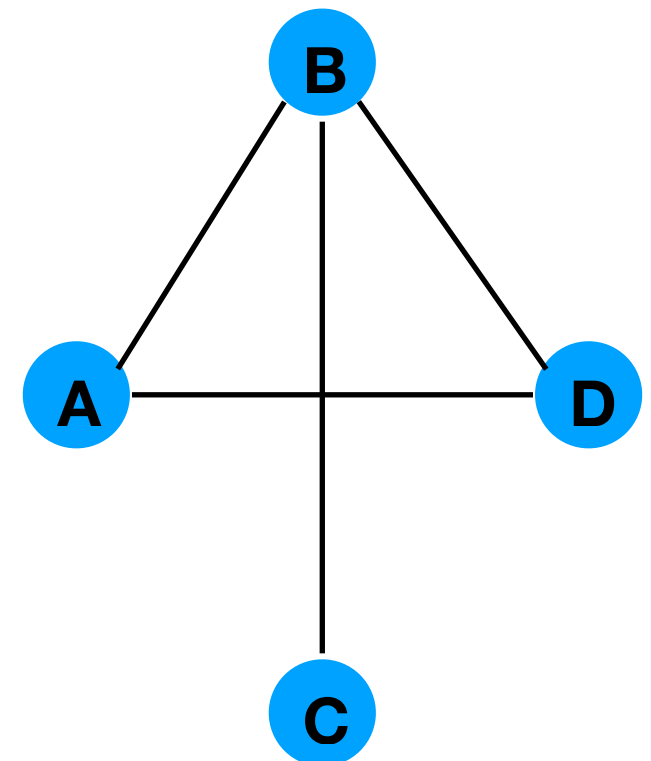
Definitions

Adjacency - two vertices are called **adjacent** if they are connected by edge

Path - the sequence of vertices which connects two nodes in a graph

Complete graph - every vertex is connected to every other vertex

Wighted graph - graph with the values assigned to edges



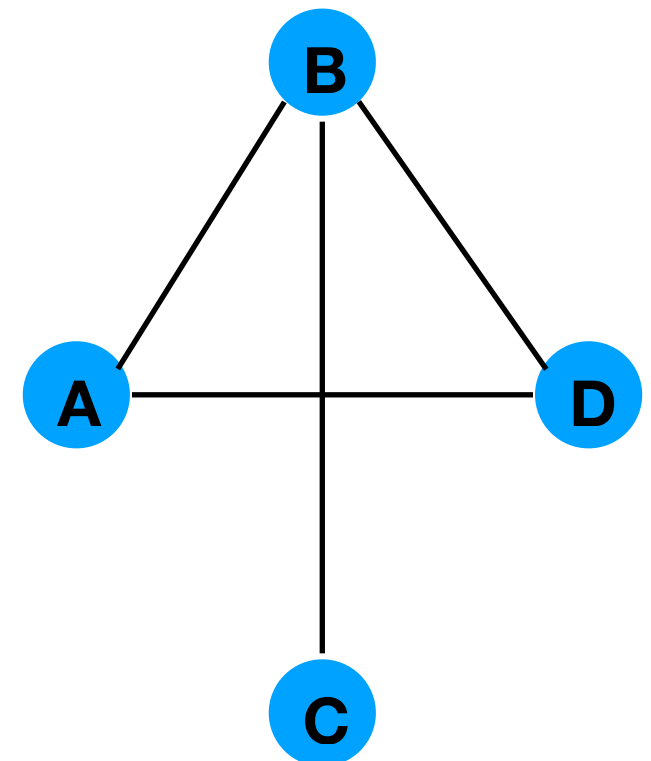
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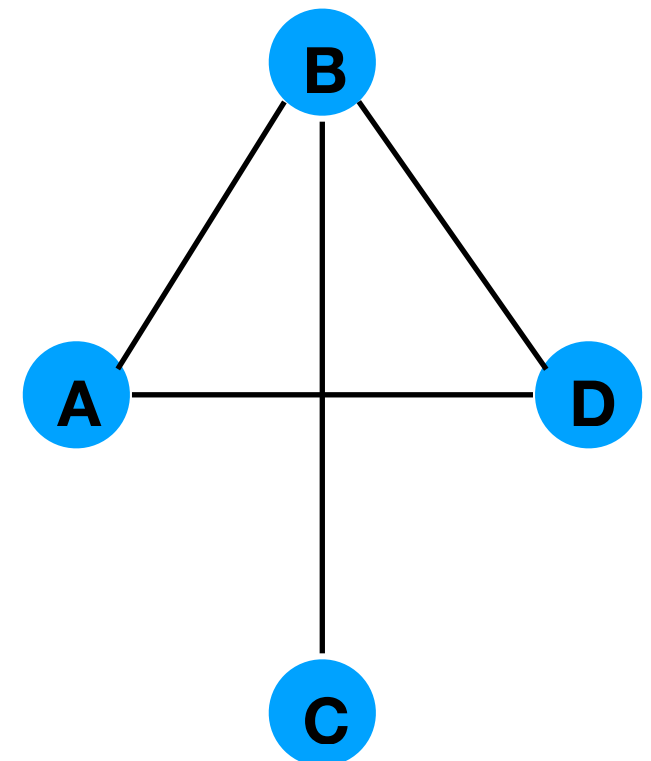
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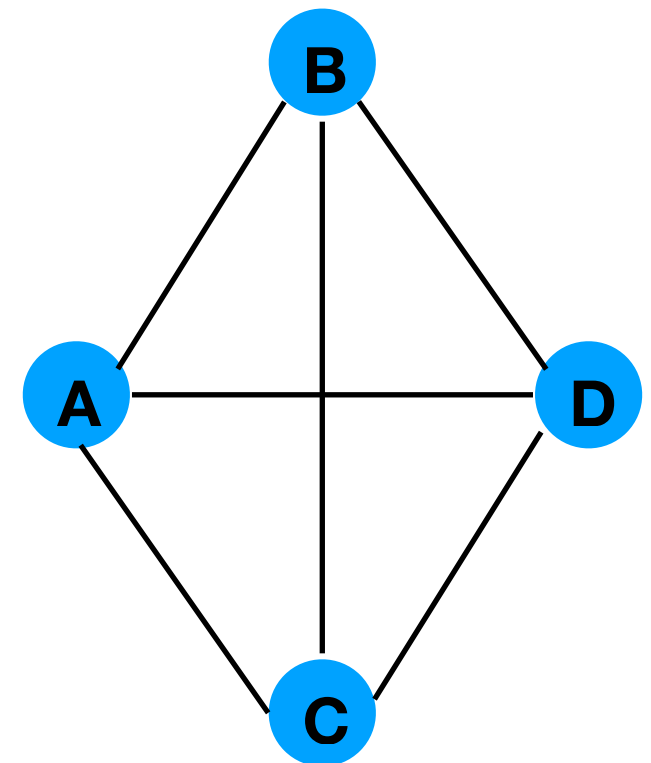
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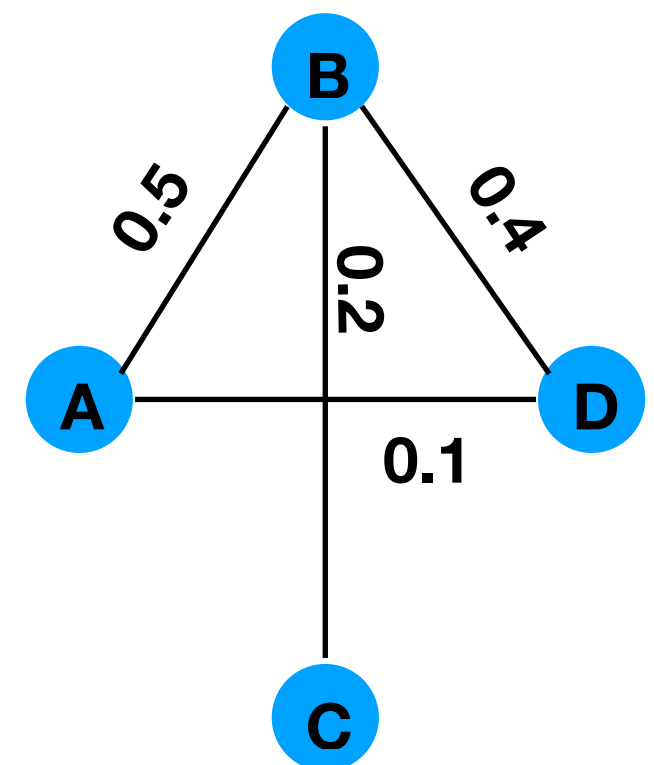
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Definitions

Quiz: how many edges exist in a complete graph ?

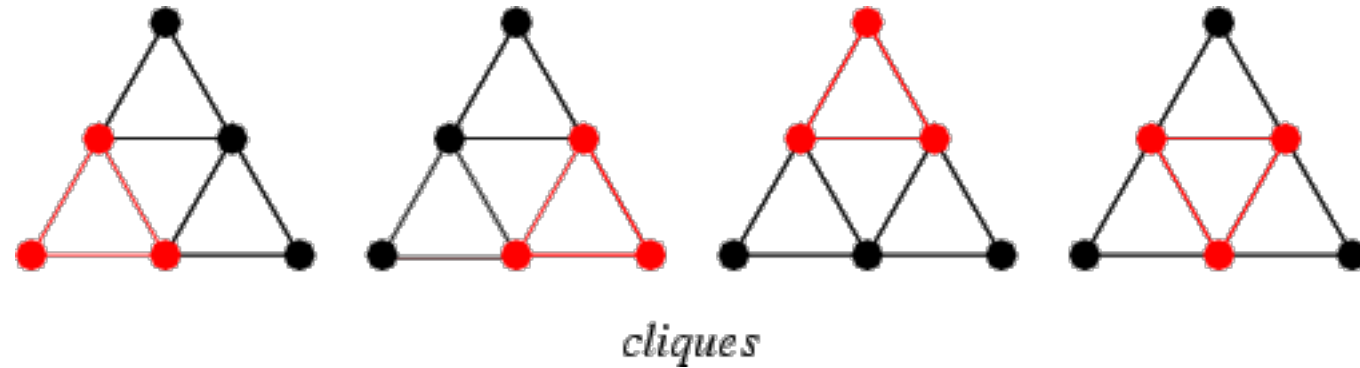
Definitions

Quiz: how many edges exist in a complete graph ?

Answer:
$$\frac{N^2 - N}{2}$$

Definitions

Clique - complete subgraph



Euler trail (path) - the path in a finite graph which visits every edge exactly once

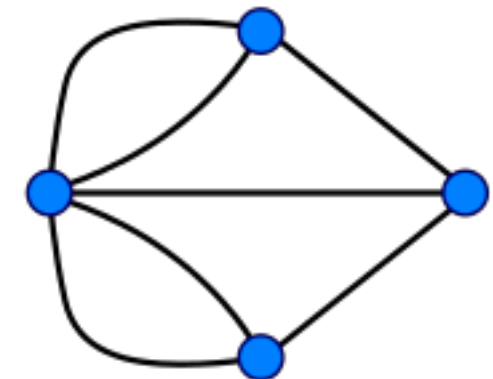
Degree $d(V)$ - number of edges incident to V

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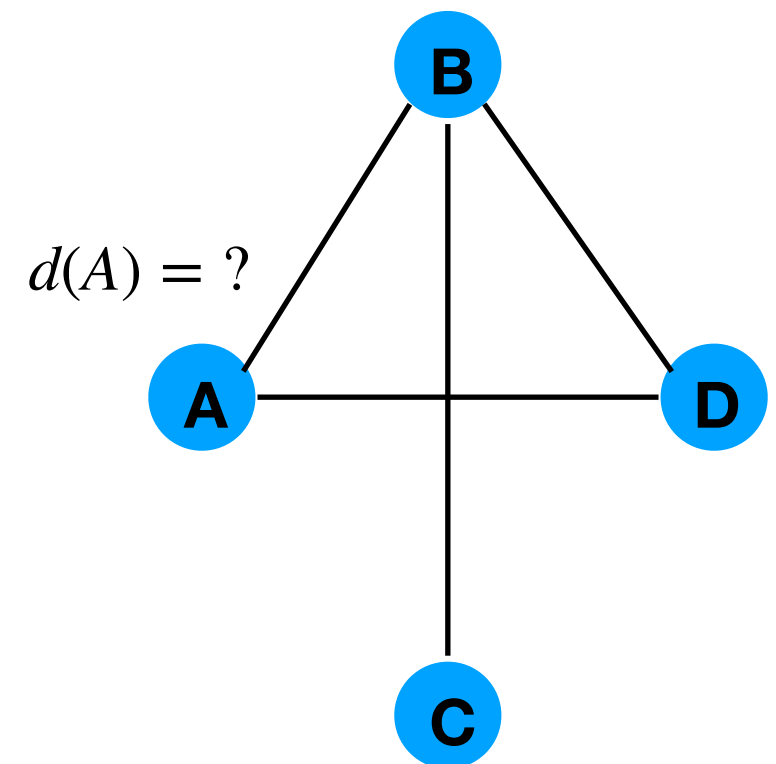


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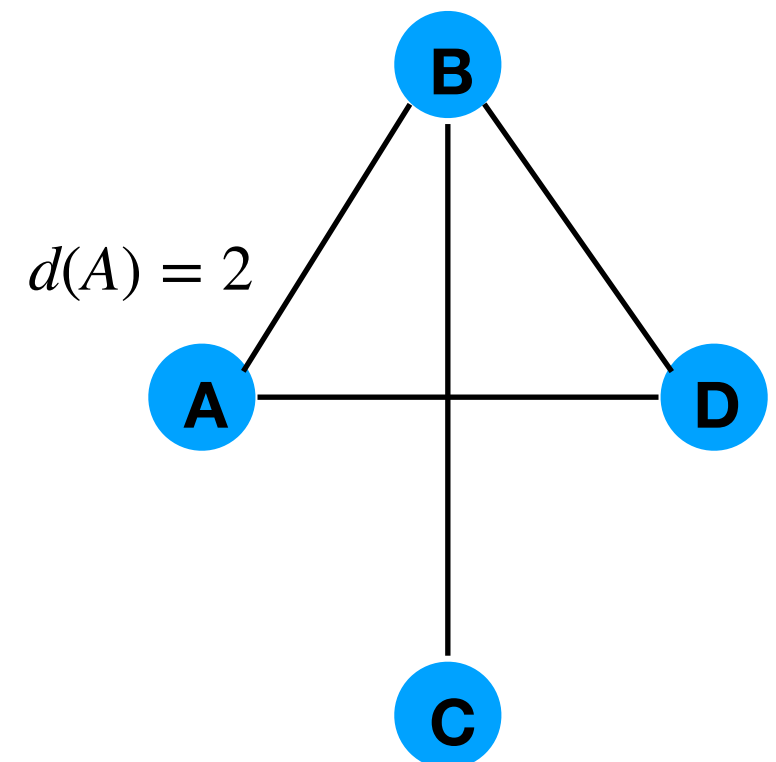


Definitions

Clique - complete subgraph

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Definitions

Handshaking lemma

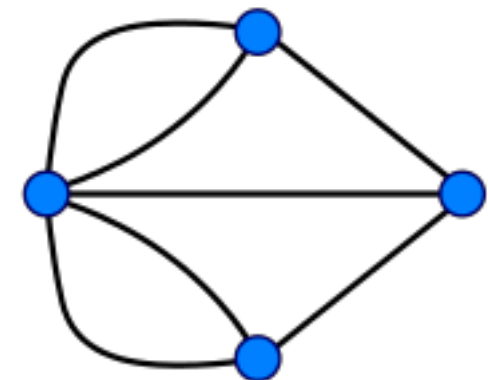
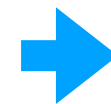
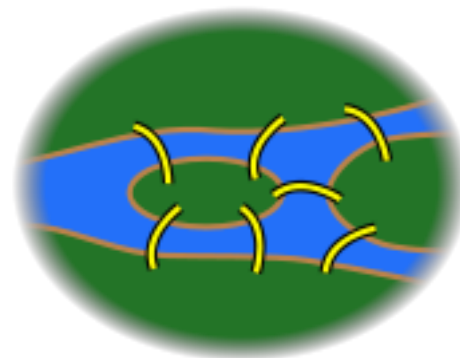
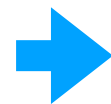
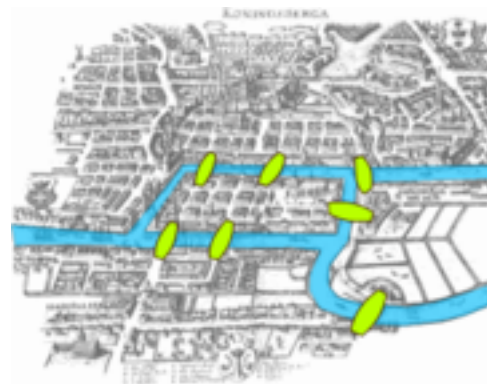
$$G = \langle V, E \rangle$$

$$\sum_{u \in V} d(u) = 2 |E|$$

in a party of people some of whom shake hands, an even number of people must have shaken an odd number of other people's hands.

Graph

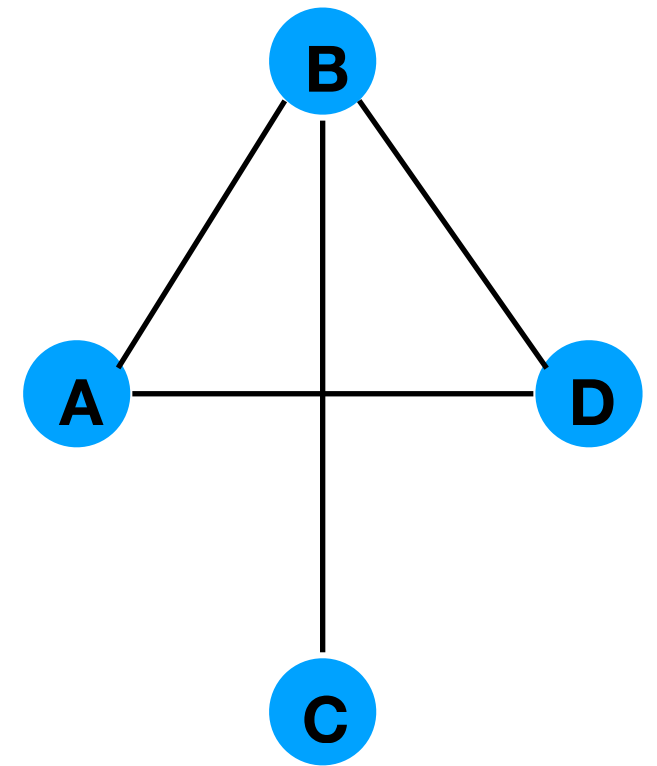
Back to Seven Bridges of Königsberg



An undirected graph has an Euler path if and only if exactly zero or two vertices have odd degree, and all of its vertices with nonzero degree belong to a single connected component.

Leonhard Euler, 1736

CS representation



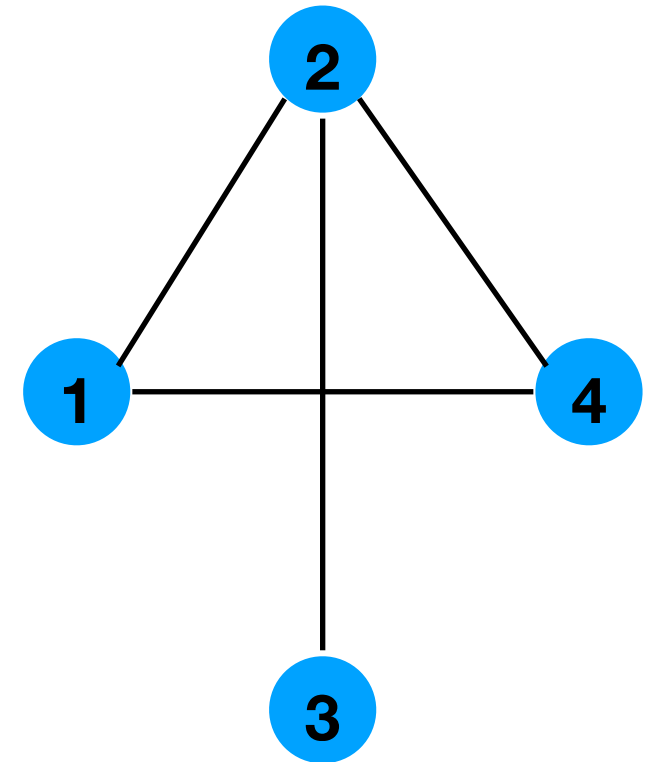
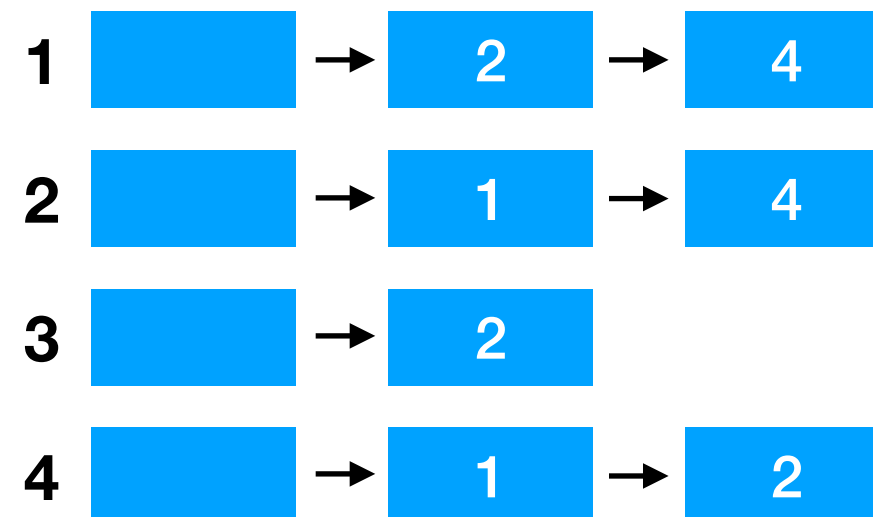
CS representation

$$G = \langle V, E \rangle$$

Adjacency matrix

	1	2	3	4
1	0	1	0	1
2	1	0	1	1
3	0	1	0	0
4	1	1	0	0

Adjacency list



CS representation

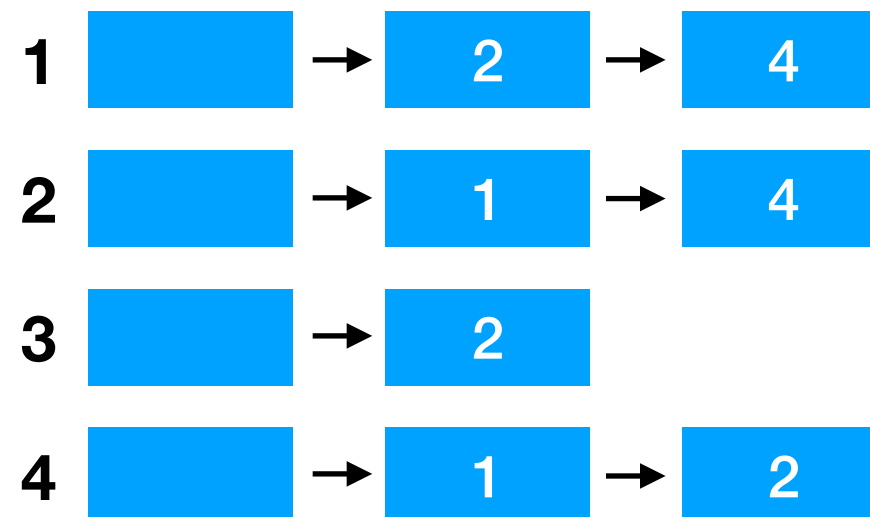
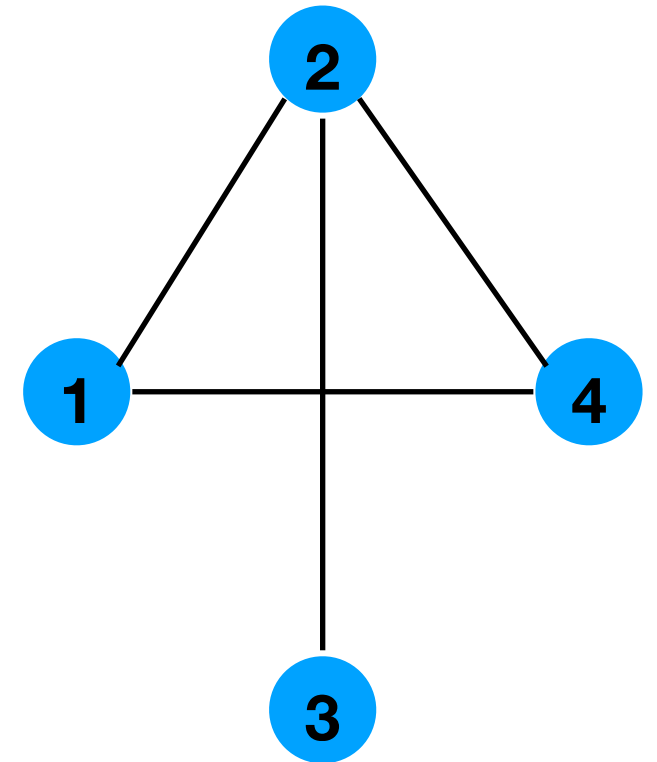
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Adjacency matrix

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Directed: N^2 Undirected: N^2

Adjacency list

Directed: $N + M$ Undirected: $N + 2M$ 

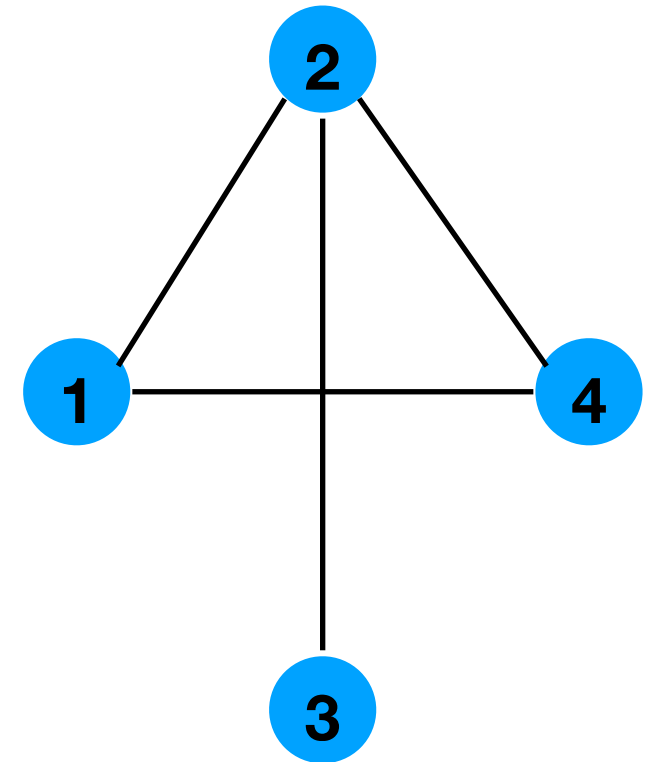
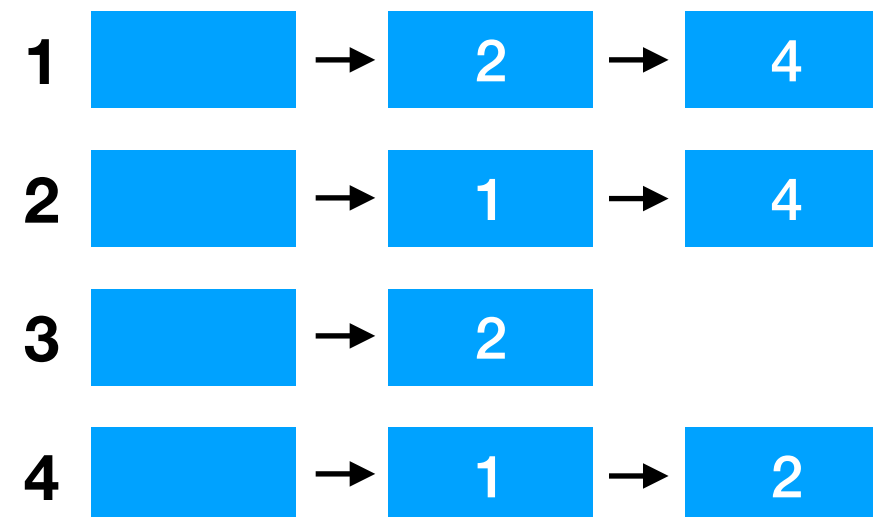
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Adjacency list



Quiz: What is better to test if an edge is in the graph ?

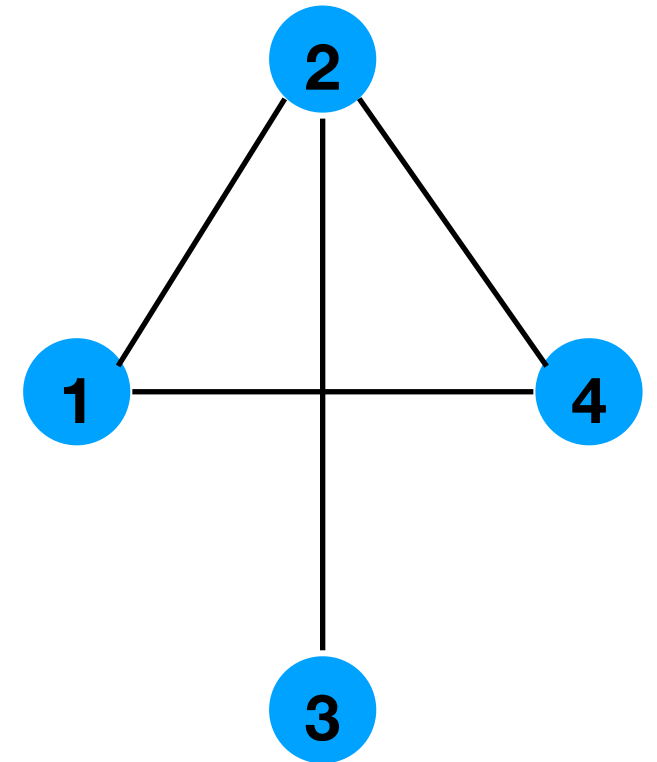
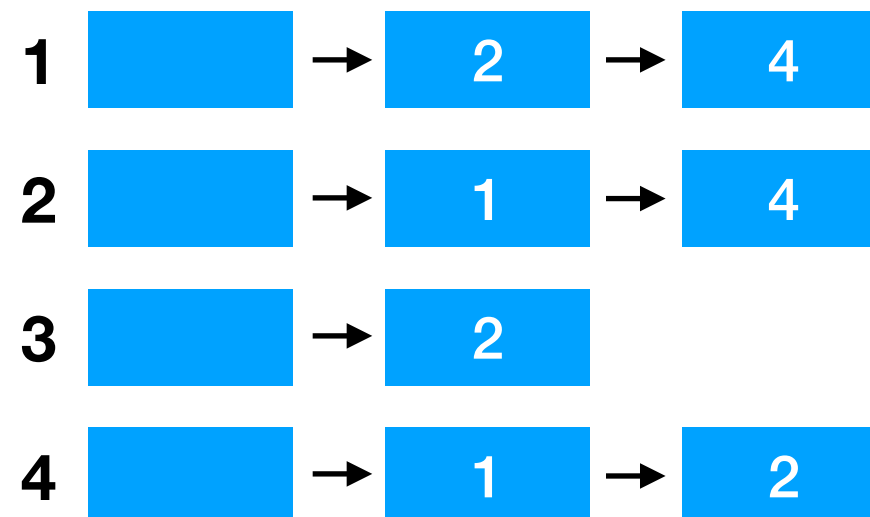
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Adjacency list



Quiz: What is better to test if an edge is in the graph ?

Answer: matrix

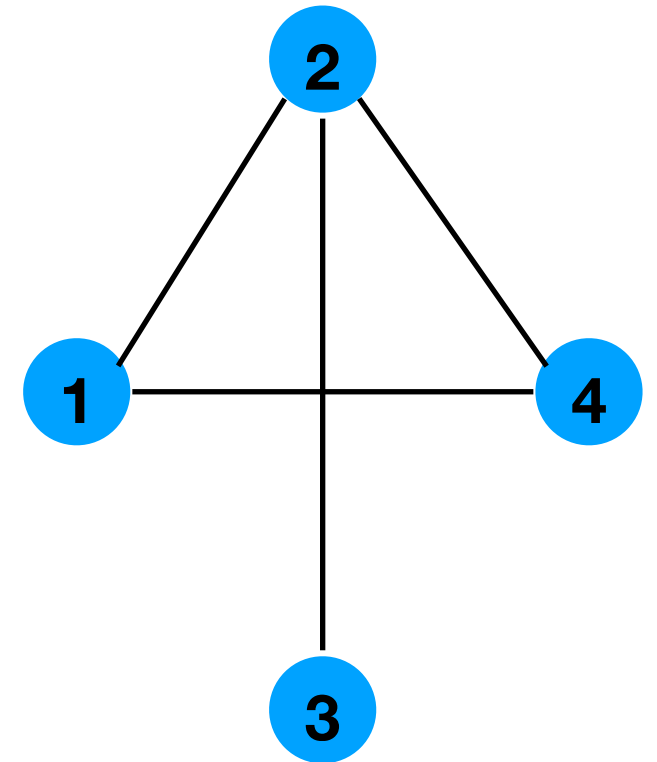
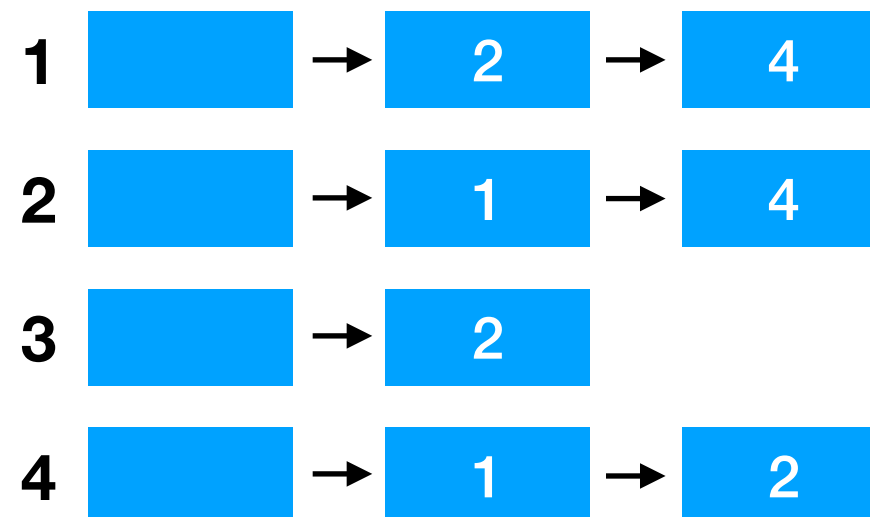
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Adjacency list



Quiz: What is faster to find the degree of vertex ?

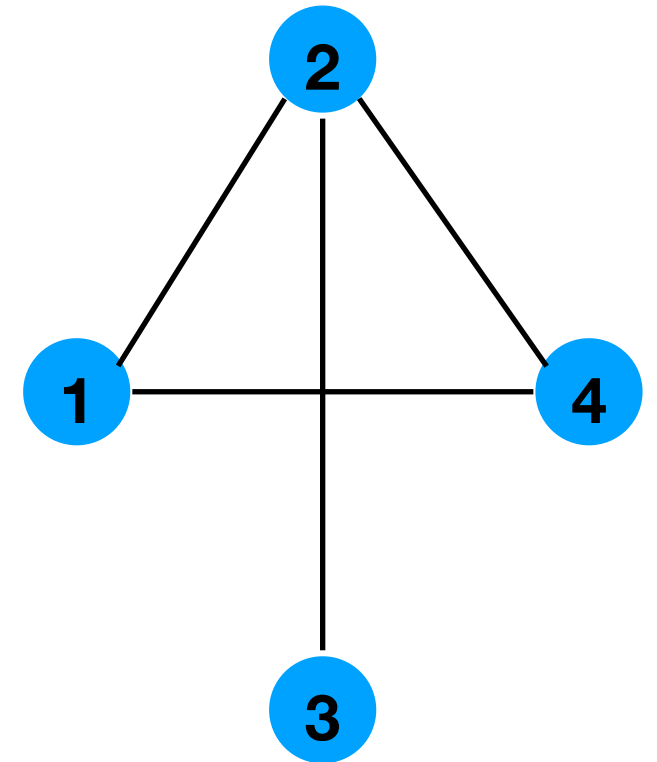
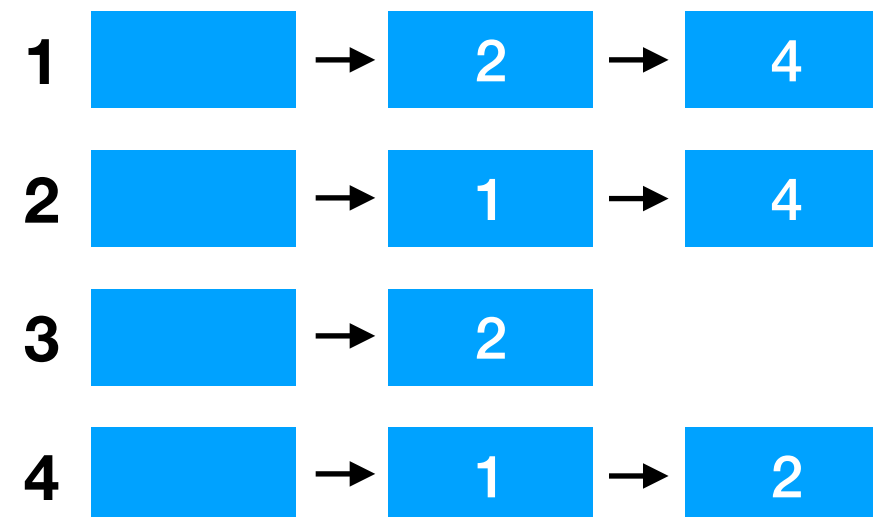
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Adjacency list



Quiz: What is better to test if an edge is in the graph ?

Answer: list

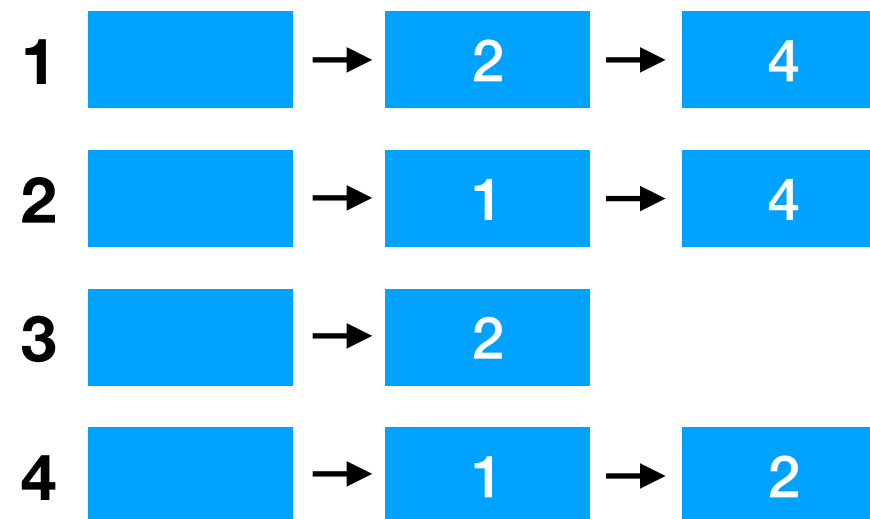
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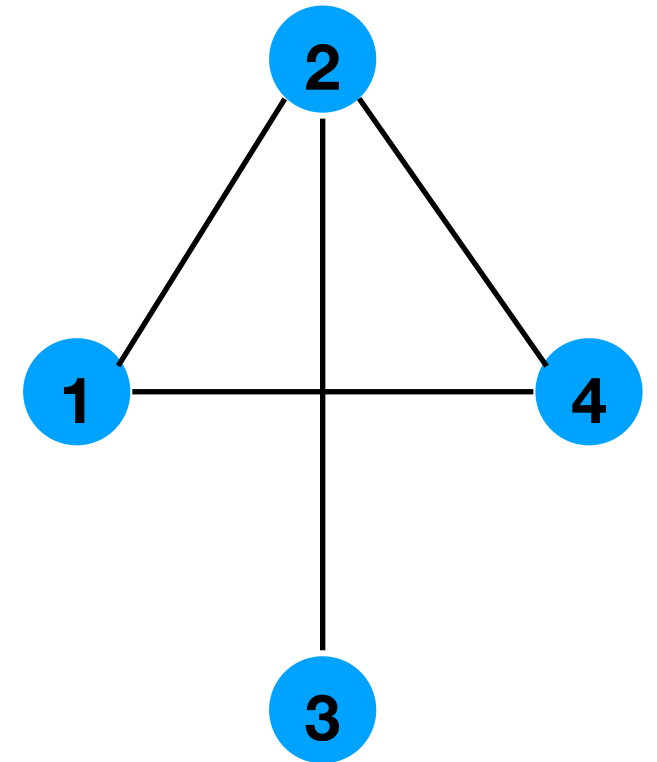
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Adjacency list



$$M + N \textbf{ VS } N^2$$



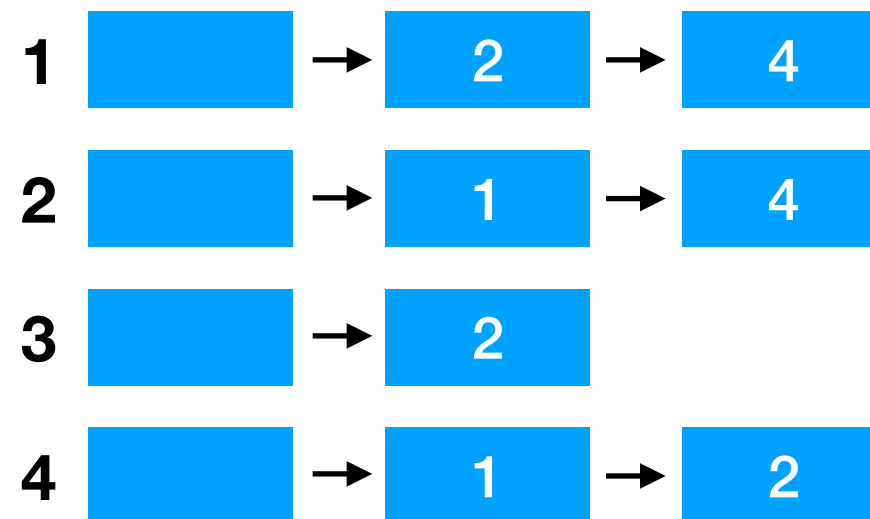
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