



Introduction to Scientific Computation

Lecture 6

Fall 2023

Data related problems, Supervised learning
Perceptron

Basic definitions

We have a set of objects: X

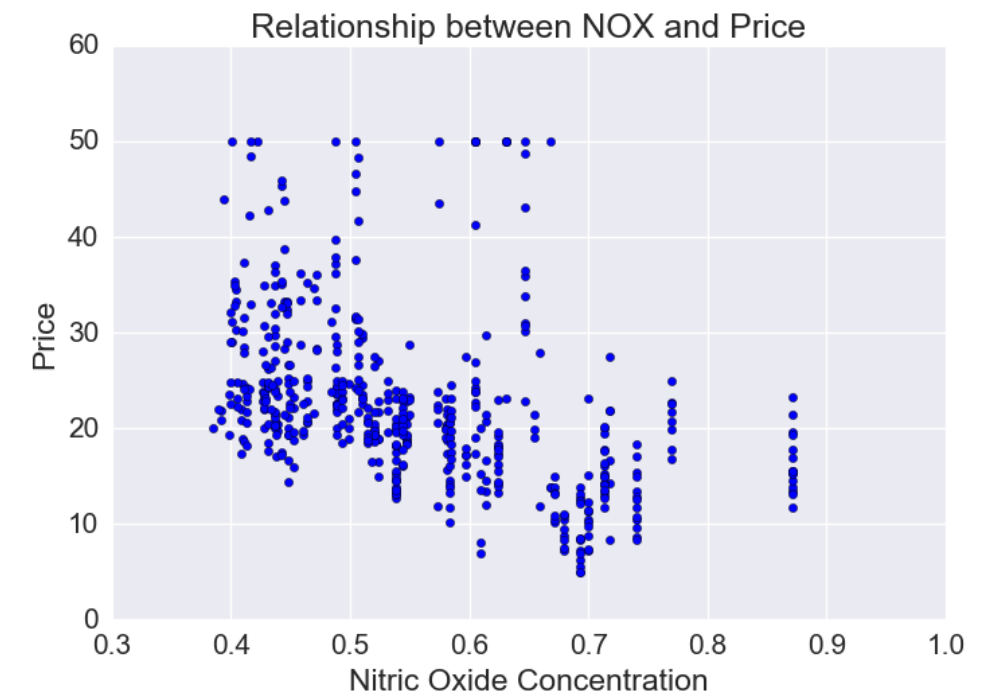
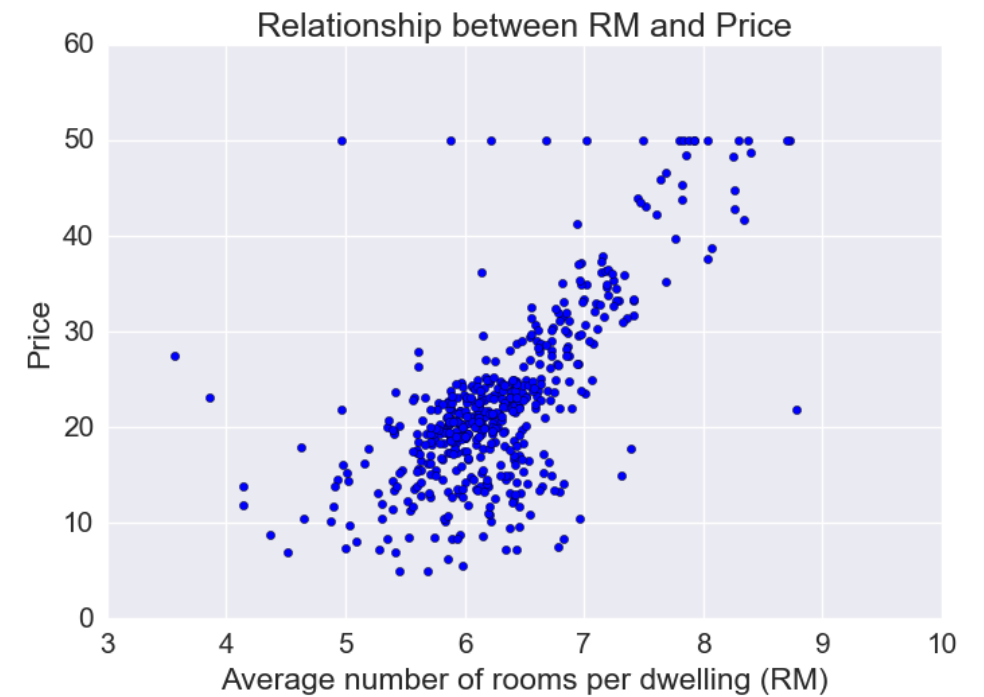
And the set of possible answers: Y

Basic definitions

We have a set of objects: X

And the set of possible answers: Y

We define the target function: $y^* : X \rightarrow Y$

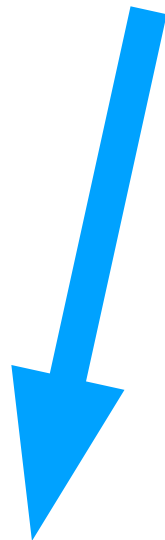


Basic definitions

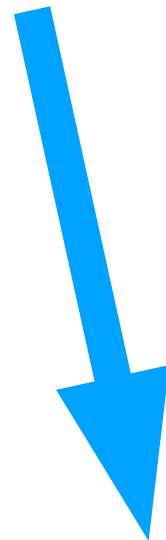
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Supervised



Unsupervised $Y \in \emptyset$

Supervised

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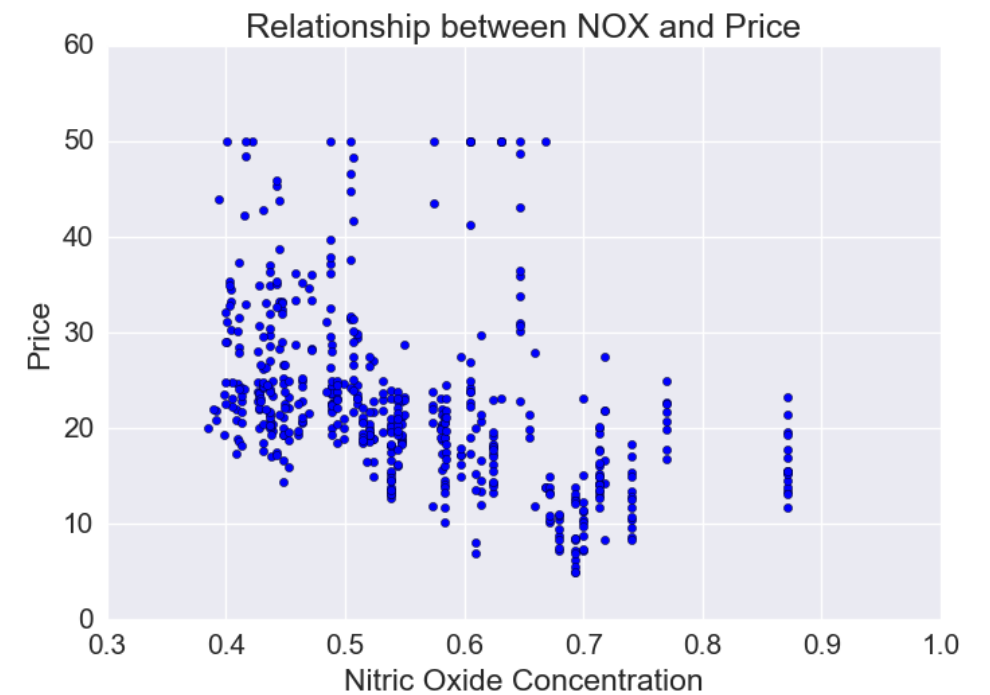
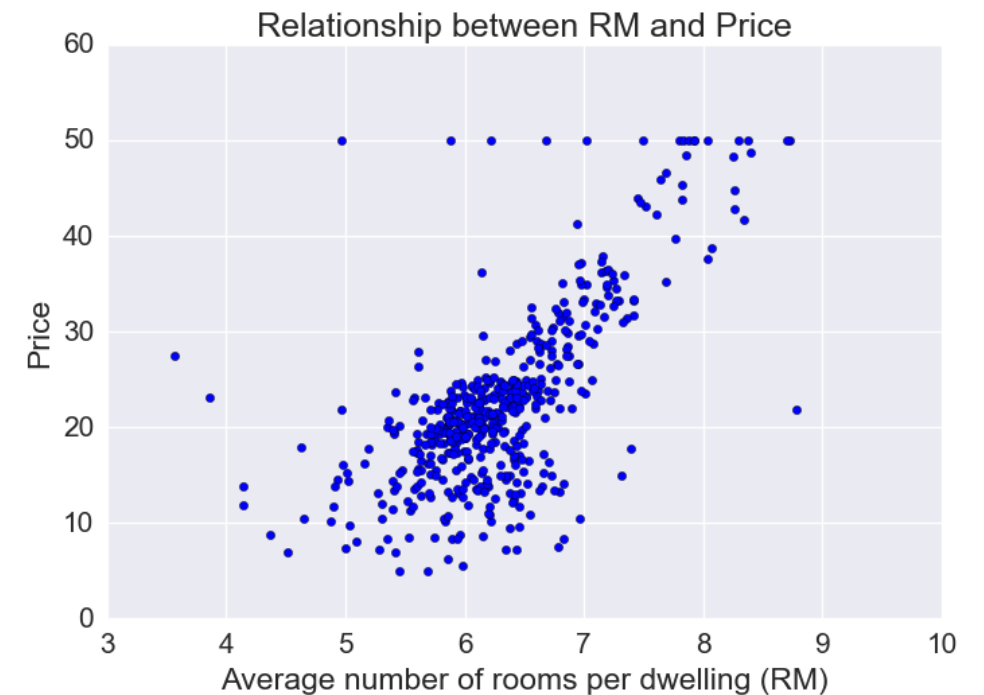
We define the target function: $y^* : X \rightarrow Y$

And we know values of y^* only on a finite subset $\{x_1, \dots, x_l\} \subset X$

Training sample:

$$X^l = (x_i, y_i)_{i=1}^l$$

Precedent



Let's look into this guy



↓

$$X^l = (x_i, y_i)_{i=1}^l$$

Let's look into this guy



↓

$$X^l = (x_i, y_i)_{i=1}^l$$

	N_Rooms	Floor	Smokers?
Hous1	1	1	Yes
...
Hous N	3	-	No

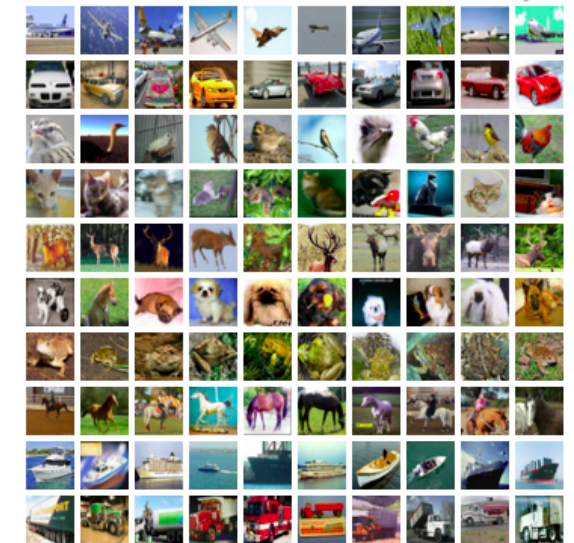
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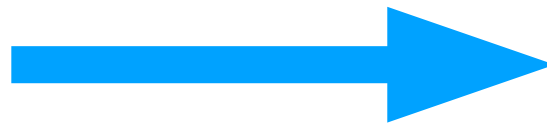
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If Y is discrete $\{1, \dots, N\}$

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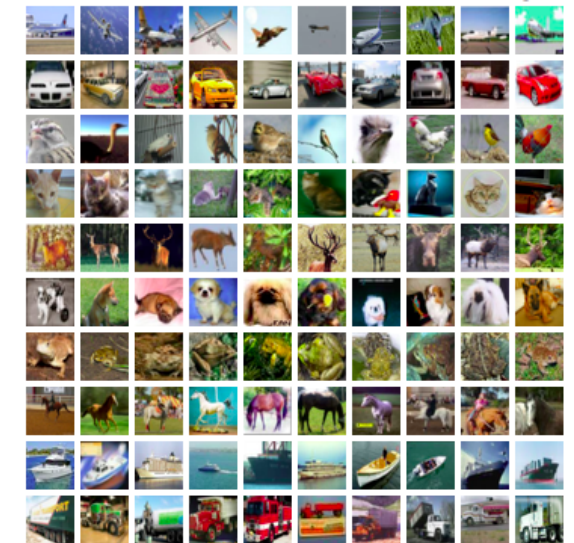


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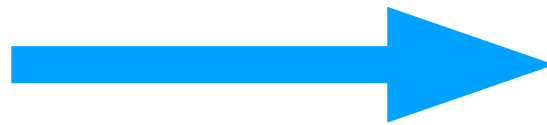


Classification

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And the set of possible answers: Y
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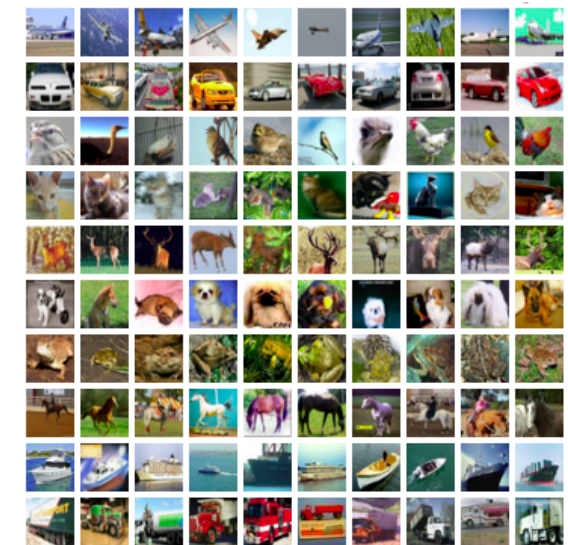


If Y is discrete $\{1, \dots, N\}$



Classification

If Y is continuous $[-25, 100]$



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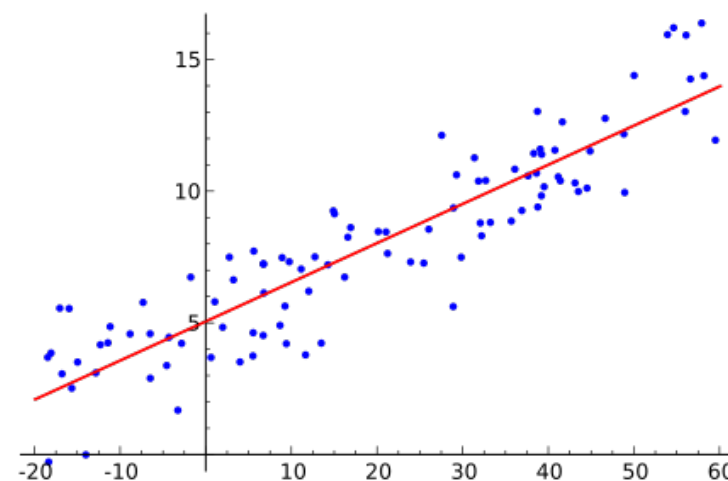


Classification

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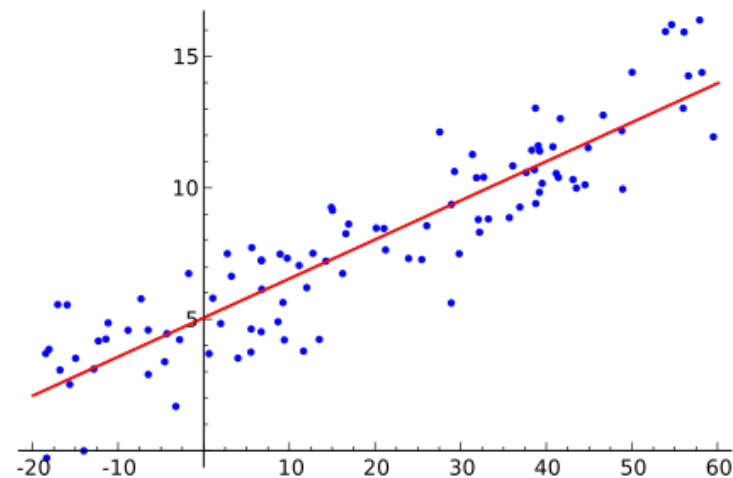


Regression



How to say if one target function is better?

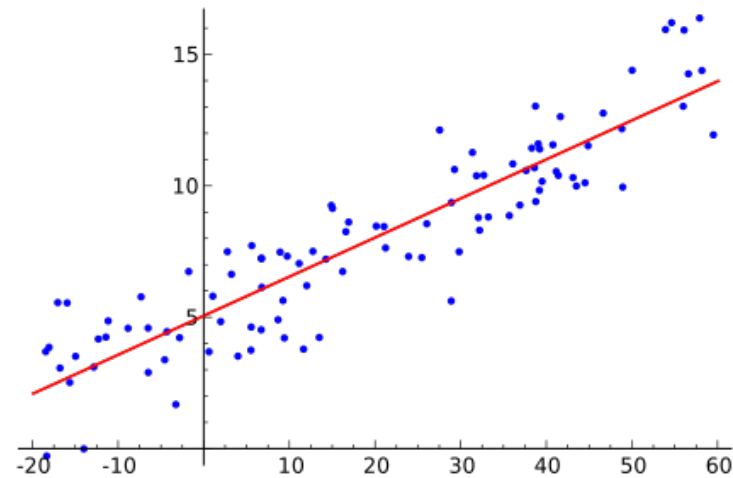
$$y_1^* = 5x + 6 \quad \text{or} \quad y_2^* = 4x^3 - 2x + 1$$



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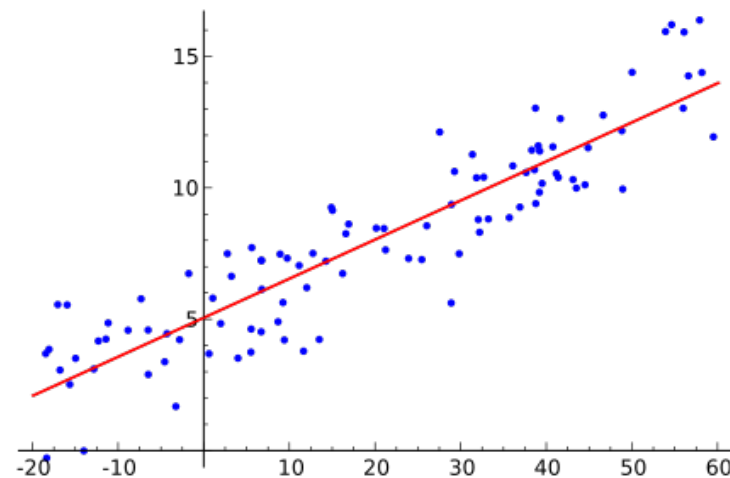
We need to compare them.



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$$y_1^* = 5x + 6 \quad \text{or} \quad y_2^* = 4x^3 - 2x + 1$$

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$$L(\theta, x_i, y_i)$$

Loss Function

Popular loss functions:

Classification



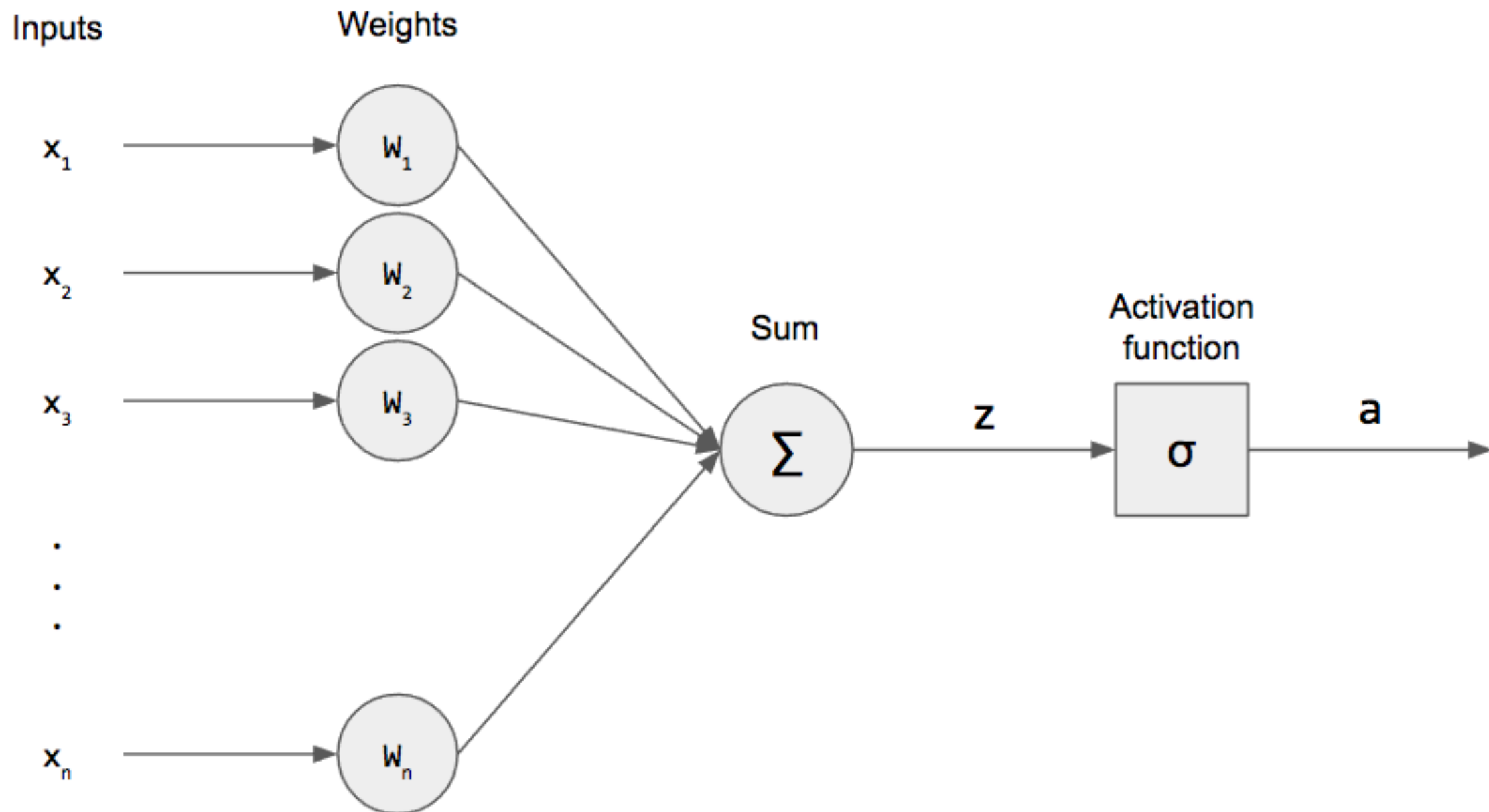
$$-(y_i \log(p_i) + (1 - y_i) \log(1 - p_i))$$

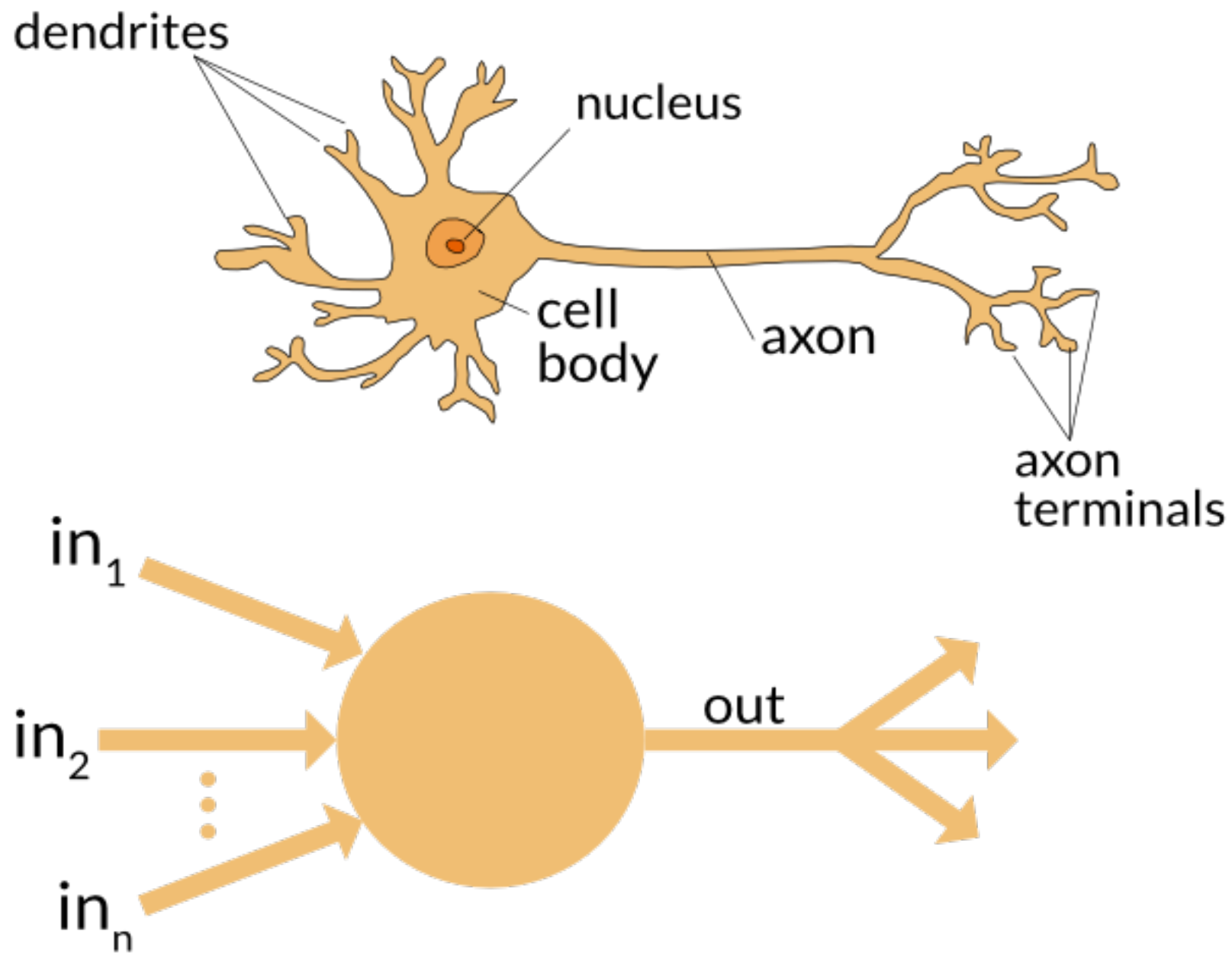
Regression



$$(y_i - p_i)^2$$

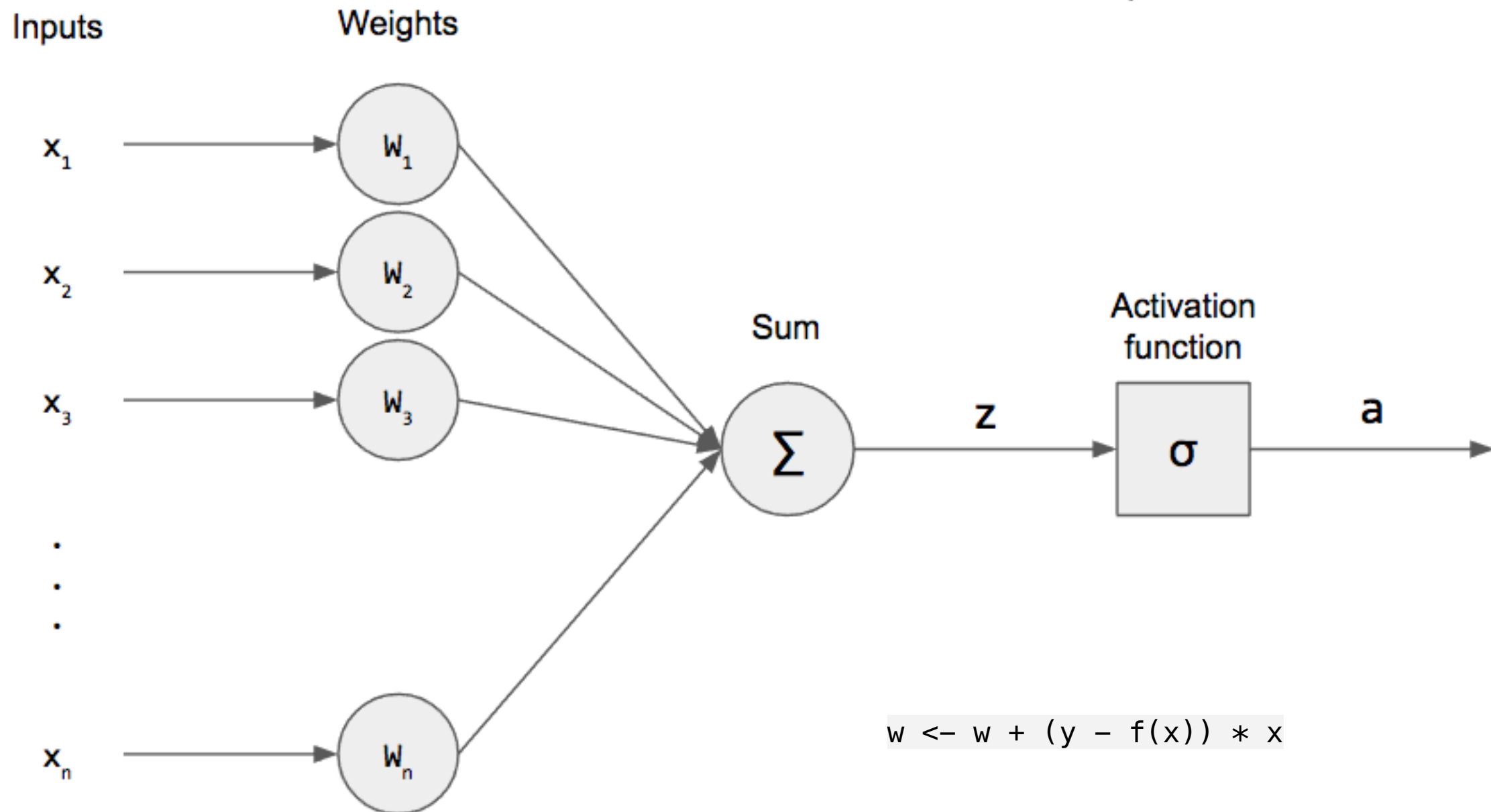
Perceptron:





Perceptron:

$$f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$



$$w \leftarrow w + (y - f(x)) * x$$

Perceptron:

