Modern Physics

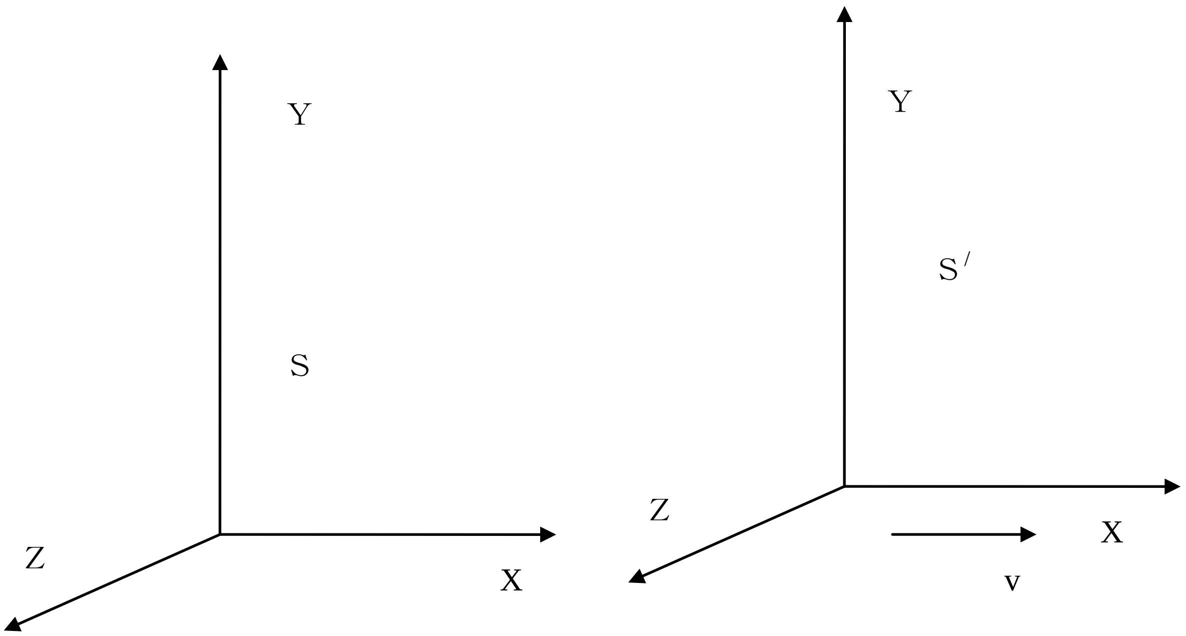
Relativity

Postulates of the special relativity

Postulates 1: The laws of physics have the same form in all inertial reference frames.

Postulates 2: The speed of light in free space (vacuum) is always constant, c, and is independent of the speed of the source or observer or the relative motion of the inertial system.

The Galilean Transformation



- Frame S' moves in the + x direction with the speed v relative to frame S
- Coordinates of some events that occurs at time t in *S* frame are x, y and z.
- An observer located in a different frame of reference S' will find the same event occurs at the time t' and has the coordinates x', y' and z'.
- Problem is to find out the relation between x, y,z, t and x', y', z', t'.
- Suppose at t=0, two frames coincided each other.
- Measurement in the x direction made in S will exceed those made in S' by the amount vt.

$$x=x' + vt, \text{ i.e. } \quad x'=x-vt \qquad (1)$$

there is no relative motion in the y and z direction, so

$$y'=y \qquad (2)$$

$$z'=z \qquad (3)$$

In the absence of any indication to the contrary in our everyday experience, we further assume that

$$t^{/}=t \qquad (4)$$

The set of equations 1-4 is known as the Galilean transformation.

To convert velocity components measured in the *S* frame to their equivalents in the $S^{/}$ frame according the Galilean transformation,

$$v^{/}_{x}=v_{x}-v \qquad \text{(from } dx^{/}/dt\text{)} \qquad (5)$$

$$v^{/}_{y}=v_{y} \qquad (6)$$

$$v^{/}_{z}=v_{z} \qquad (7)$$

- Now the Galilean transformation and the velocity transformation violate both of the postulates of special relativity.
- The fundamental equations of electricity and magnetism assume very different froms when eqs 1-4 are used to convert quantities measured in one frame into their equivalents in the other.
- If we measure the speed of light in the x directionin the *S* system to be *c*, however, in the $S^{/}$ system

$$c^{/}=c-v \qquad \text{according to Eq. 5.}$$

A different transformation is required if the postulates of special relativity are to be satisfied.

The Michelson-Morley experiment

How to prove the existence of ether?

Perform measurements of the speed of light in moving coordinate systems. Problem is how to do this. Speed of light is $c=3\times10^{8}$ m/s. The fastest one could move in Michelson's time (who performed the pioneering experiment of measuring the speed of light in moving coordinate systems) was around 100 km/h i.e 30m/s. (train). This is seven orders of magnitude smaller than the speed of light.

i.e. one needed to detect a deviation of about $1/10^{7}$. In fact, the situation is even worse. Most relativistic effects are proportional to v^{2}/c^{2}, thus they would be of (10^{-14}). This was way beyond the experimental capabilities of anyone at that time. So why don't we use the earth as a moving lab? But even the earth's speed around the sun, 3×10^{4} m/s is paltry compared to the speed of light. To detect a deviation of $1/10^{4}$ Michelson needed to use the power of interferometry.

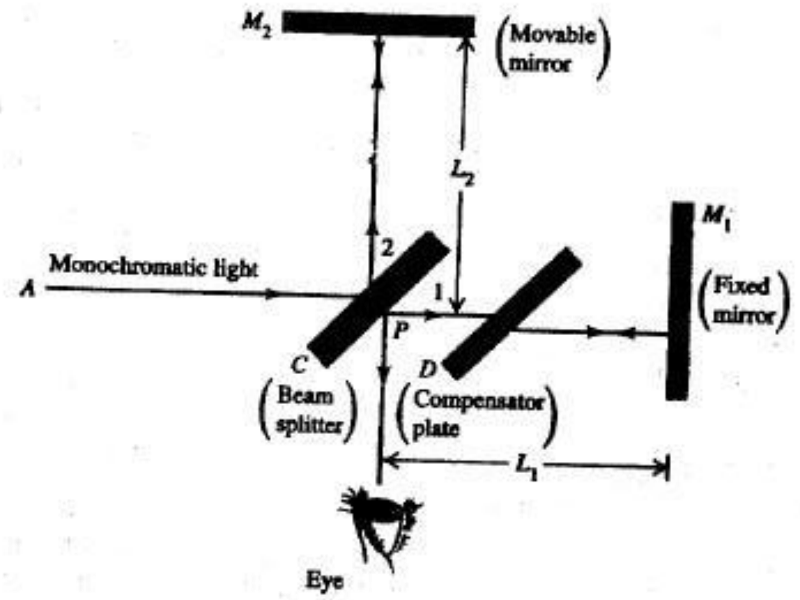


Fig. A schematic Michelson interferometer. The observer sees an interference pattern that results from the difference in path lengths for rays 1 and 2.

Michelson interferometer has two perpendicular arms. Light is split into two beams by a semitransparent mirror. The two light rays, one traveling along the direction of motion of the earth, one perpendicularly are coherent. So after they are reflected back to the mirror they interfere constructively or destructively, depending on their relative phase. The interference pattern of the combined rays is detected though an optical device.

Let us calculate the time it takes for these light rays to come back to the semitransparent mirror where they were originally separated, **assuming** all the time that ether exists and **light travels with a velocity of c only in ether**. The time a light ray takes traveling parallel to the direction of motion of the earth and going back and forth along an arm of length L_1 is

$$t_1 = L_1 / (v+c) + L_1 / (c-v) = 2 L_1 c / (c^2-v^2) = 2L_1/c[1-v^2/c^2]^{-1} \qquad (6)$$

This is so because the speed of light is c in the system tied to the ether, so it changes to c+v and c-v when the light ray is observed in a coordinate system moving with a velocity, v (which is the velocity of earth on its orbit).

Now in the perpendicular arm, of length L_2, of the interferometer the light ray again moves with velocity c in the ether's system, but if the movement is perpendicular in the earth's system then it's velocity vector is at an angle, slightly deviating from the perpendicular, in the ether's system. So the speed in the coordinate system moving with earth (traveling in both directions) is

$$c' = (c^2-v^2)^{1/2} = c[1-v^2/c^2]^{1/2}. \qquad (7)$$

As a result, the time it takes to travel back and forth between the two mirrors is

$$t_2 = 2 L_2 / c[1-v^2/c^2]^{1/2} \qquad (8)$$

The time difference between the return of the two coherent light rays is

$$\Delta t = t_1 - t_2 = 2/c[\{L_1 / (1-v^2/c^2)\} - \{L_2 / (1-v^2/c^2)^{1/2}\}] \qquad (9)$$

Unlike it is indicated in the book it is impossible to make two arms to have equal length to a precision of up to a fraction of the wavelength of the light, so we must assume that L_1 and L_2 are not equal. Fortunately, the result of the experiment is not affected by this.

Michelson and Morley slowly rotated the arms of the interferometer by 90^o. Then the role of the two arms got exchanged. The time difference between the return of the two light rays became

$$\Delta t' = t'_1 - t'_2 = 2/c[\{ L_2 / (1-v^2/c^2)\} - \{ L_1 / (1-v^2/c^2)^{1/2}\}] \qquad (10)$$

Michelson and Morley watched the change of fringes in their observing microscope during the rotation. By this they checked whether $t'_1 - t'_2$ ($\Delta t'$) was different from $t_1 - t_2$ (Δt) or not. That difference is

$$\Delta t'' = (\Delta t) - (\Delta t')$$

$$= 2/c[\{L_1 / (1-v^2/c^2)\} - \{L_2 / (1-v^2/c^2)^{1/2}\}] - 2/c[\{ L_2 / (1-v^2/c^2)\} - \{ L_1 / (1-v^2/c^2)^{1/2}\}]$$

$$= 2(L_1+L_2)/c[\{1-v^2/c^2\}^{-1} - \{1-v^2/c^2\}^{-1/2}]$$

$$= 2(L_1+L_2)/c[1+ v^2/c^2 - 1 - v^2/2c^2]$$

$$= 2(L_1+L_2)/c[v^2/2c^2]$$

$$= (L_1+L_2)v^2/c^3] \qquad (11)$$

Using $v << c$ we obtain an approximate form of the square bracket if we expand in v/c.

The time difference leads to a path difference,

$$d = c\ (\Delta t'')$$

$$= (L_1 + L_2)v^2/c^2 \qquad (12)$$

If d corresponds to the shifting of n fringes, then d=n λ, where λ is the wavelength of the light used.

$$n = d/\lambda$$

$$= (L_1 + L_2)v^2/c^2\lambda \qquad (13)$$

In the actual experiment Michelson and Morley were able to make D about 10 m in effective length through the use of multiple reflections and wavelength of the light they used was 500 nm. The expected fringe shift in each path when the apparatus is rotated by 90° is therefore, n=0.2 fringe.

Since the both paths experience this fringe shift, the total shift should amount to 2n or 0.4 fringe. A shift of this magnitude is readily observable since the apparatus used in the experiment was capable of observing a fringe shift as small as 0.01 fringe. Therefore, Michelson and Morley looked forward to establishing directly the existing of the ether.

However, in their experiment no fringe shift was observed. The experiment was repeated at different places at different times of the day and in different seasons and no significant fringe shift was observed.

The negative result of the Michelson-Morley experiment had two consequences:

(i) It rendered untenable the hypothesis of the ether by demonstrating that the ether has no measurable properties-an ignominious end for what had once been a respected idea.

(ii) It suggested a new physical principle: the speed of light in free space is the same everywhere, regardless of any motion of source or observer.

The Lorentz transformation

We shall now develop a set of transformation equations directly from the postulates of the special relativity. A reasonable guess is to the kind of relationship between x and $x^{/}$ is

$$x^{/}=k(x-vt) \tag{14}$$

where k is a factor of proportionality that does not depend upon either x or tbut may be a function of v. The choice of eq. 14 follows from following considerations:

(i) It is linear in x and $x^{/}$, so that a single event in frame S corresponds to a single event in frame $S^{/}$.

(ii) It is simple, and a simple solution to a problem should explored first.

(iii) It has the possibility of reducing to eq. 1($x^{/}$=x-vt), which we know to be correct in ordinary mechanics.

From eq. 14, equation for x in terms of $x^{/}$ and $t^{/}$

$$x=k(x^{/}+vt^{/}) \tag{15}$$

The factor k must be same in both frames of reference since there is no difference between S and $S^{/}$ frame other than in the sign of v.

As in the case of Galilean transformation, there is nothing to indicate that there might be differences between the corresponding coordinates y, $y^{/}$, and z, $z^{/}$ which are normal to the direction of v.Hence we again take

$$y^{/}=y \tag{16}$$

$$z=z^{/} \tag{17}$$

The time coordinates t and $t^{/}$, however, are not equal. We can see this substituting the value of $x^{/}$ (from 14) in eq. 15. We obtain

$x=(x-vt)+kvt$

or, $t^{/}=kt+[(1-k^2)/kv]x$ (18)

Equations 14, 16, 17, 18 constitute a coordinate transformation that satisfies first postulates of special relativity.

The second postulate of relativity unable us to evaluate k. At the instant t=0, the origins of the two frames of reference S and $S^{/}$ are in the same place, and $t^{/}=0$ then also. Suppose that a flare is set off at the common origin of S and $S^{/}$ at $t=t^{/}=0$, and the observer in each system proceed to measure the speed with which the light from it spreads out. Both observers must find the same speed c, which means that in the *S* frame

$$x=ct \tag{19}$$

while in the $S^{/}$ frame

$$x^{/}=ct^{/} \tag{20}$$

Substituting $x^{/}$ and $t^{/}$ from eqs. 14 and 18

or, $k(x-vt)=ckt+[(1-k^2)/kv]cx$

solving for x,

$x=[ckt+vkt]/[k-\{(1-k^2)/kv\}c]$

$$=ct\frac{k+\frac{v}{c}k}{k-\frac{1-k^2}{kv}c}=ct\left[\frac{1+\frac{v}{c}}{1-(\frac{1}{k^2}-1)\frac{c}{v}}\right]$$

This expression for x will be the same as x=ct, provided the quantity in the brackets equals 1.

Therefore , $\dfrac{1+\frac{v}{c}}{1-(\frac{1}{k^2}-1)\frac{c}{v}}=1$

and $$k=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{21}$$

Inserting the value of *k* in eqs. 14, 16, 17and 18

$$x^{/} = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (22)$$

$$y^{/} = y \qquad (23)$$

$$z^{/} = z \qquad (24)$$

$$t^{/} = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (25)$$

Equations 22-25comprise the *Lorentz Transformation.*

In order to transform measurements from $S^{/}$ to S is known as Inverse Lorentz transformation

$$x = \frac{x^{/} + vt^{/}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (22)$$

$$y = y^{/} \qquad (23)$$

$$z = z^{/} \qquad (24)$$

$$t = \frac{t^{/} + \frac{vx^{/}}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (25)$$

The Lorentz-FitzGerald Contraction

- Suppose a rod is lying along the x' axis of a moving frame of reference S'. An observer in this frame determines the coordinates of its ends to be x'_1 and x'_2.
- To him the length L_0 of the rod is

$$L_0 = x_2' - x_1'$$

L_0 is the rod's length as measured in a frame of reference in which it is at rest.

- Suppose the same quantity is determined from a frame of reference *S* relative to which the rod is moving with the velocity *v*.
- Will the length *L* measured in *S* be the same as the length L_0 that was measured in *S′*?

In more familiar terms, if a yardstick is in a speeding car (the S′ *frame), does it seem to have the same length to someone standing beside the road (the S frame) as it does to someone inside the car?*

$$x_2' = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

In order to find *L*, we use the Lorentz transformation to go from the coordinates x'_1 and x'_2. in the moving frame S′ to the corresponding coordinates x_1 and x_2 in the stationary frame S. Since the measurements of x_1 and x_2. are made at the same time t,

$$x_1' = \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

and so $$L_0 = x'_2 - x'_1$$

$$= \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

By definition the length *L* measured in the stationary frame *S* is given by

$$L = x_2 - x_1$$

i.e. $$L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

or $$L = L_0\sqrt{1-\frac{v^2}{c^2}} \qquad (26)$$

The length of an object in motion with respect to an observer appears to the observer to be shorter than when it is at rest with respect to him, a phenomenon known as *Lorentz-*

FitzGerald Contraction.

Because the relative velocity of the two frames S and $S^{/}$ appears only as v^2 in Eq. 26, it does not matter which frame we call S and which $S^{/}$. If we find that the length of a space ship is *Lo* when it is on its launching pad, we will find from the ground that its length L when moving with the speed v is

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

while to a man in the spaceship, objects on the earth behind him appear shorter than they did when he was on the ground by the same factor $\sqrt{(1-v^2/c^2)}$. The length of an object is a maximum when measured in a reference frame in which it is stationary, and its length is less when measured in a reference frame in which it is moving.

Problem:

The relativistic length contraction is negligible for ordinary speeds, but it is an important effect at speeds close to the speed of light. A speed of 1,000 mi/sec seems enormous to us, and yet it results in a shortening in the direction of motion to only

$$\frac{L}{L_0} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{(1000mi/s)^2}{(186000mi/s)^2}} = 0.999985 = 99.9985\%$$

of the length at rest. On the other hand, a body traveling at 0.9 the speed of light is shortened to

$$\frac{L}{L_0} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{(0.9c)^2}{(c)^2}} = 0.436 = 43.6\%$$

of the length at rest, a significant change.

Time Dilation

Time intervals, too, are affected by relative motion. Clocks moving with respect to an observer appear to tick less rapidly than they do when at rest with respect to him. If we, in the S frame, observe the length of time t some event requires in a frame of reference $S^{/}$ in motion relative to us, our clock will indicate a longer time interval than the t_o determined by a clock in the moving frame. This effect is called *time dilation.*

To see how time dilation comes about, let us imagine a clock at the point $x^{/}$ in the moving frame $S^{/}$. When an observer in $S^{/}$ finds that the time is $t^{/}_1$, an observer in S will find it to be t_1,

$$t_1 = \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

After a time interval of t_o (to him), the observer in the moving system finds that the time is now t'_2 according to his clock. That is,

$$t_o = t'_2 - t'_1$$

The observer in *S*, however, measures the end of the same time interval to be

$$t_2 = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

So to him the duration of the interval **t** is

$$t = t_2 - t_1$$

$$= \frac{t_2' - t_1'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or,
$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

A stationary clock measures a longer time interval between events occurring in a moving frame of reference than does a clock in the moving frame.

Meson Decay

A striking illustration of both the time dilation and the length contraction occurs in the decay of unstable particles called μ *mesons.* A μ *meson* decays into an electron an average of 2×10^{-6} *sec* after it comes into being. Now μ *mesons* are created high in the atmosphere by fast cosmic-ray particles arriving at the earth from space, and reach sea level in profusion. Such mesons have a typical speed of 2.994×10^{8} *m/sec.* which is *0.998* of the velocity of light *c.* But in $t_0 = 2\times10^{-6}$ *sec*, the meson`s mean lifetime, they can travel a distance of only

$$y = vt_o$$

$$= 2.994\times10^{8}\ m/sec \times 2\times10^{-6}\ sec$$

$= 600\ m$

while they are actually created at altitudes more than 10 times greater than this.

We can resolve the meson paradox by using the results of the special theory of relativity. Let us examine the problem from the frame of reference of the meson in which its life time is 2×10^{-6} sec. While the meson life time is unaffected by the motion, its distance to the ground appears shortened by the factor

$$\frac{y}{y_0} = \sqrt{1 - \frac{v^2}{c^2}}$$

If we let 600 *m*, the maximum distance the meson can go in its own frame of reference at the speed 0.998*c* before decaying, we find that corresponding distance y_0 in our reference frame is

$$y_0 = \frac{y}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{600}{\sqrt{1 - \frac{(0.998c)^2}{c^2}}} = \frac{600}{\sqrt{1 - 0.996}} = \frac{600}{0.063} = 9500m$$

Hence, despite their brief life spans, it is possible for the meson to reach the ground from the considerable altitudes at which they are actually formed.

Now let us examine the problem from the frame of reference of an observer on the ground. From the ground the altitude at which the meson is produced is y_0, but its life time in our reference frame has been extended, owing to the relative motion, to the value

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2 \times 10^{-6}}{\sqrt{1 - \frac{(0.998c)_2}{c^2}}} = \frac{2 \times 10^{-6}}{0.063} = 31.7 \times 10^{-6}\, s$$

almost 16 times greater then when it is at rest with respect to us.

In 31.7×10^{-6}s a meson whose speed is 0.998*c* can travel a distance 9500 m, the same distance obtained before.

The Twin Paradox

Consider two identical twin sisters. Let one of the twins go to a long space journey at a high speed in a rocket and the other stay behind on the earth.

The clock in the moving rocket will appear to go slower than the clock on the surface of the earth according to $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Therefore, when the traveler returns back to the earth, she will find herself younger than the twin who stayed behind the earth.

Suppose Twin *A* takes off when she is *20 y* old and travels at $v=0.8c$ to a star $L_0=20$ *light years* away. She ten turns around and returns home. To her sister *B* on the ground *A* seems to be living more slowly during the voyage at rate $\sqrt{(1-v^2/c^2)}=0.60=60\%$.

Finally after *50 y* have elapsed by B`s reckoning her sister *A* is back from a trip that has taken $50\times0.6=30$ *y*.

So, *B*=70 Years old lady

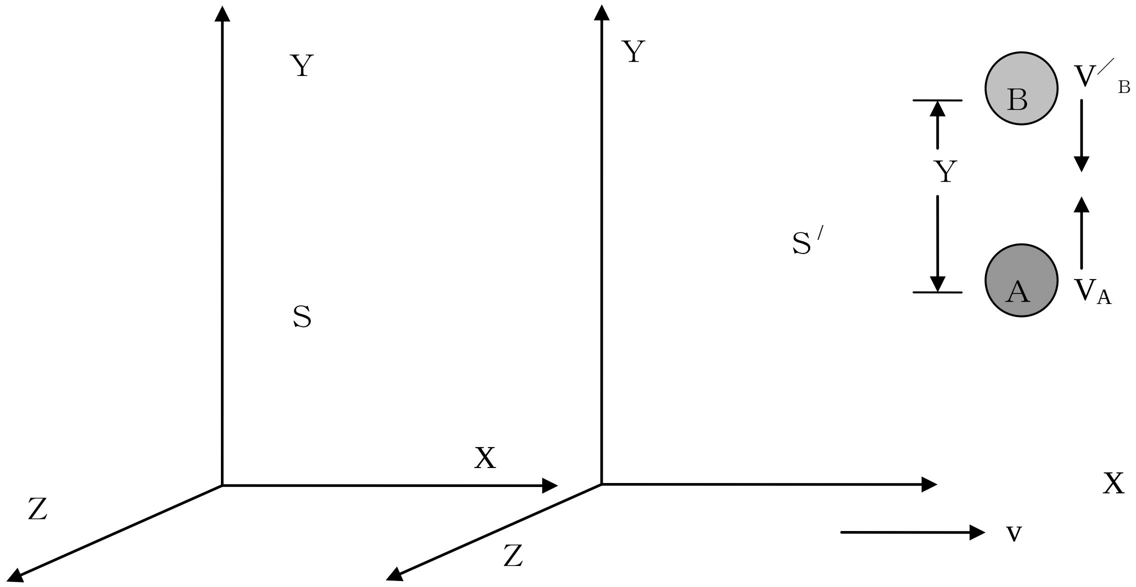
A = 50 years of age.

Problems

1. A rocket ship is 100 m long on the ground. When it is in flight, its length is 99 m to an observer on the ground. What is its speed?
2. How fast would a rocket have to go relative to an observer for its length to be contracted to 99% of its length at rest? 4.24×10^7 *m/s*.

The Relativity of Mass

Consider an elastic collision (that is, a collision in which kinetic energy is conserved) between two particles *A* and *B*, as witnessed by observers in the reference frames S and S' which are in uniform relative motion. The properties of *A* and *B* are identical when determined in reference frames in which they are at rest. The frames S and S' are oriented as in Fig. below, with S' moving in the +x direction with respect to S at the velocity *v*.



Before the collision, particle A has been at rest in frame S and particle B in frame S'.Then at the same instant, A is thrown in the $+y$ direction at the speed V_A while B is thrown in the $-y$ direction at the speed V'_B, where

$$V_A = V'_B \tag{26}$$

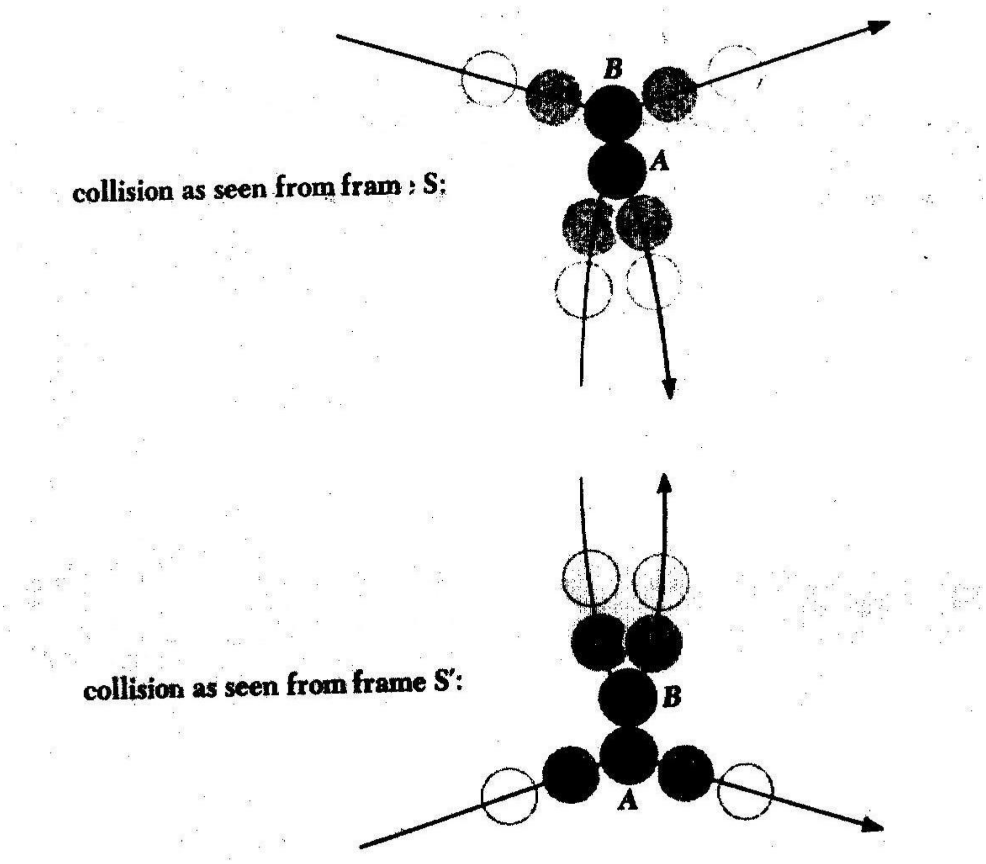


Fig. An elastic collision as observed in two different frames of reference.

Hence the behavior of A as seen from S is exactly the same as the behavior of B as seen from S'. When the two particles collide, A rebounds in the $-y$ direction at the speed $\boldsymbol{V_A}$**,** while B rebounds in the $+y$ direction at the speed V'_B. If the particles are thrown from positions $\boldsymbol{Y}$ apart, an observer in S finds that the collision occurs at y = ½Y and one in S' finds that it occurs at y'=½ Y. The round-trip time $\boldsymbol{T_0}$ for $\boldsymbol{A}$ as measured in frame S ~ therefore

$$T_0=Y/V_A \qquad (27)$$

and it is the same for B in S'

$$To=Y/V'_B$$

If momentum is conserved in the S frame, it must be true that

$$m_AV_A = m_BV_B \qquad (28)$$

where m_A and m_B are the masses of A and B**,** and $\boldsymbol{V_A}$ and V_B their velocities as measured in the S frame. In S the speed $\boldsymbol{V_B}$ is found from

$$\boldsymbol{V_B} = Y/T \qquad (29)$$

where T is the time required for B to make its round trip as measured in S. In S', however, B*'s* trip requires the time T_0, where

$$T = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (30)$$

according to our previous results. Although observers in both frames see the same event, they disagree as to the length of time the particle thrown from the other frame requires to make the collision and return.

Replacing T in eq. 28with its equivalent in terms of $\boldsymbol{T_0}$, we have

$$V_B = \frac{Y\sqrt{1-\frac{v^2}{c^2}}}{T_0}$$

From Eq. 26 , $V_A=Y/T_0$

Inserting these expressions for $\boldsymbol{V_A}$ and $\boldsymbol{V_B}$ in Eq. 28, we see that momentum is conserved if

$$M_A = M_B\sqrt{1-\frac{v^2}{c^2}} \qquad (31)$$

Our original hypothesis was that A and B are identical when at rest with respect to an observer; the difference between m_A and m_B therefore means that measurements of mass, like those of space and time, depend upon the relative speed between an observer and whatever he is observing.

In the above example both A and B are moving in S. In order to obtain a formula giving the mass m of a body measured while in motion in terms of its mass m_0 when measured at rest, we need only consider a similar example in which V_A and V_B are very small. In this case an observer in S will see B approach A with the velocity v, make a glancing collision (since $V'_B << v$), and then continue on. In S

$$m_A = m_0 \quad \text{and} \quad m_B = m$$

and so

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c_2}}} \qquad (32)$$

The mass of a body moving at the speed v relative to an observer is larger than its mass when at rest relative to the observer by the factor $1/\sqrt{(1 - v^2/c^2)}$. This mass increase is reciprocal; to an observer in S'

$$m_A = m \quad \text{and} \quad m_B = m_o$$

Measured from the earth, a spaceship in flight is shorter than its twin still on the ground and its mass is greater. To somebody on the spaceship in flight the ship on the ground also appears shorter and to have a greater mass. (The effect is, of course, unobservably small for present-day rocket speeds.)

Relativistic Momentum

Momentum is defined as $mv = \dfrac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$ is valid in special relativity just as in classical physics. However, Newton's second law of motion is correct only in the form

$$F = \frac{d}{dt}(mv) = \frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right) \qquad (33)$$

This is not equivalent to saying that *F = ma=m(dv/dt)* even with m given by equation 32 because *d/dt(mv)=m(dv/dt)+v(dm/dt)*

and *dm/dt* does not vanish if the speed of the body varies with time. The resultant force on a body is always equal to the time rate of change of its momentum.

Relativistic mass increases are significant only at speeds approaching that of light. At a speed one-tenth that of light the mass increase amounts to only 0.5 percent, but this increase is over 100 percent at a speed nine-tenths that of light. Only atomic particles such as **electrons, protons**, **mesons,** and so on have sufficiently high speeds for relativistic effects to be measurable, and in dealing with these particles the "ordinary" laws of physics cannot be used.

Historically, the first confirmation of Eq. 32 was the discovery by Bücherer in 1908 that the ratio *e/m* of the electron's charge to its mass is smaller for fast electrons than for slow ones

Mass and Energy

The most famous relationship Einstein obtained from the postulates of special relativity concerns mass and energy. This relationship can be derived directly from the definition of the kinetic energy *T* of a moving body as the work done in bringing it from rest to its state of motion. That is,

$$T = \int_0^s F ds$$

where *F is* the component of the applied force in the direction of the displacement *ds* and *s* is the distance over which the force acts. Using the relativistic form of the second law of motion *F=d/dt(mv)*

we have

$$T = \int_0^s \frac{d(mv)}{dt} ds = \int_0^{mv} v d(mv) = \int_0^v v d\left(\frac{m_0 v}{\sqrt{1 - \frac{v^2}{c_2}}} \right)$$

$$T = \frac{m_0 v^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 \int_0^v \frac{v dv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0 v^2}{\sqrt{1-\frac{v^2}{c^2}}} + m_0 c^2 \sqrt{1-\frac{v^2}{c^2}} \Big|_0^v = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 c^2$$

i.e $T=mc^2-m_0c^2$ (34)

Equation 34 states that the kinetic energy of a body is equal to the increase in its mass consequent upon its relative motion multiplied by the square of the speed of light.

Equation 34 may be rewritten

$$mc^2 = T + m_0c^2 \quad (35)$$

If we interpret mc^2 as the *total energy E* of the body, it follows that, when the body is at rest and $T = 0$, it nevertheless possesses the energy m_0c^2. Accordingly m_oc^2 is called the *rest energy* E_0 of a body whose mass at rest is m_0. Equation 35 therefore becomes

$$E=E_0+T \quad (36)$$

where

$$E= mc^2 \quad (37)$$

and

$$E_0= m_0c^2 \quad (38)$$

In addition to its kinetic, potential, electromagnetic, thermal, and other familiar guises (appearance), then, energy can manifest itself as mass. The conversion factor between the unit of mass (kg) and the unit of energy (joule) is c^2, so 1 kg of matter has an energy content of 9×10^{16} joules. Even a minute bit of matter represents a vast amount of energy, and, in fact, the conversion of matter into energy is the source of the power liberated in all of the exothermic reactions of physics and chemistry.

Since mass and energy are not independent entities, the separate conservation principles of energy and mass are properly a single one, the principle of conservation of mass energy. Mass *can* be created or destroyed, but only if an equivalent amount of energy simultaneously vanishes or comes into being, and vice versa.

When the relative speed *v* is small compared with *c*, the formula for kinetic energy must reduce to the familiar $½mov^2$, which has been verified by experiment at low speeds. Let us see whether this is true.

The relativistic formula for kinetic energy is

$$T = \mathrm{mc}^2 - m_o c^2$$

$$T = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2 = m_0 c^2 (1 + \frac{1}{2}\frac{v^2}{c^2}) - m_0 c^2 = \frac{1}{2} m_0 v^2$$

Hence at low speeds the relativistic expression for the kinetic energy of a moving particle reduces to the classical one. The total energy of such a particle is

$$E = m_0 c^2 + ½ m_o v^2$$

In the foregoing calculation relativity has once again met an important test; it has yielded exactly the same results as those of ordinary mechanics at low speeds, where we know by experience that the latter are perfectly valid. It is nevertheless important to keep in mind that, so far as is known, the correct formulation of mechanics has its basis in relativity, with classical mechanics no more than an approximation correct only under certain circum-stances.

Velocity transformation

To see how velocities transform one needs to see first how infinitesimal distances transform when the time change is also infinitesimal. In other words one needs a differential form of the Lorentz transformation formula

$dx' = \gamma(dx - v\, dt)$, where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$dy' = dy,$
$dz' = dz,$
$dt' = \gamma(dt - v\, dx / c^2).$

Dividing the first three equations by the fourth we obtain

$V'_x = V_x - v) / (1 - V_x v / c^2).$
$V_y' = V_y / \gamma(1 - V_x v / c^2).$
$V_z' = V_z / \gamma(1 - V_x v / c^2).$

Where both the denominator and the numerator on the right hand side were divided by *dt.*

We also itroduced the notation V_I and V'_I for the components of the velocities in the unprimed and primed coordinate systems, respectively. Obviously, when the velocities are much smaller than the velocity of light this formula reduces to the Galilean transformation, because the denominators are equal to 1. At the other limit, when $V_x=c$, then we obtain that $V'_x=c$,, as well.

Let us find the inverse transformation for the velocities. Multiplying by the denominator we obtain

$$V_x' (1-V_x v / c^2) = V_x - v.$$

Expressing u_x from the above equation we obtain

$$V_x = (V'_x + v)/(1+V'_x v / c^2).$$

This is exactly the same equation as the expression of V'_x by V_x, except that the velocities are exchanged and the sign of v is changed. This is consistent with the requirements that the two coordinate systems are completely equivalent.

It is also easy to show that once a velocity is smaller than c, as it should be, it will also smaller than c in every coordinate system. See what is the condition for $V'_x < c$:

$$(V_x - v)/(1-V_x v / c^2) < c$$

Expressing V_x we obtain the following constraint for it:

$$V_x < 2c/(1 + v/c).$$

Since the right hands side is larger than c, V_x certainly satisfies this constraint, consequently V'_x is certainly smaller than c.

In other words, if one adds two velocities, the velocity of the object and the velocity of the coordinate system, where both may be close to the velocity of light, still the resulting velocity will be smaller than the velocity of light. This would not happen if we used the Galilean transformation formula for velocities, because then velocities would just be added.

Take an example. Suppose $V_x=0.8\ c$ and $v = 0.8\ c$.. Then we have

$$V'_x = (0.8\ c+ 0.8\ c)/(1 + 0.8x0.8)= 1.6\ c/1.64 = 0.976c < c.$$

Problems:

1. At what speed will the mass of a body be 20% greater than its rest mass? [Ans. 0.553c]
2. A man has a mass of 100 Kg on the ground. When he is in a rocket ship in flight, his mass is 101 Kg as determined by an observer in the ground. What is the speed of the rocket ship? [Ans. 4.2×10^7 m/s]

Compton Effect

The most spectacular proof and direct proof of the existence of photons came about through the investigations of A. H. Compton. The Compton effect is a scattering of photons by electrons.

The quantum theory of light postulates that photons behave like particles except for the absence of any rest mass. If this is true, then it should be possible for us to treat collisions between photons and, say, electron in the same manner as billiard ball collisions are treated in elementary mechanics.

Consider an X-ray photon striking an electron (which is at rest) and being scattered away from its original direction of motion while the electron receives an impulse and begins to move. In the collision the photon may be regarded as having lost an amount of energy that is the same as the kinetic energy T gained by the electron, though actually separate photons are involved. If the initial photon has the frequency ν associated with it, the scattered photon has the lower frequency ν', where

Loss in photon energy= Gain in electron energy

$$h\nu - h\nu' = T$$

Photon has no rest mass, its total energy is E=pc [$E=\sqrt{(m_0^2c^4+p^2c^2)}$ for photon $m_0=0$]

Since $E=h\nu$, for a photon its momentum is

$p=h\nu/c$

Momentum is a vector quantity, incorporating direction as well as magnitude, and in the collision momentum must be conserved in each of two mutually perpendicular directions. The direction we choose here are that of the original photon and one perpendicular to it in the plane containing the electron and the scattered photon.

The initial photon momentum is $h\nu/c$ and the scattered photon momentum is $h\nu'/c$, and the initial and final electron momenta are 0 and p respectively.

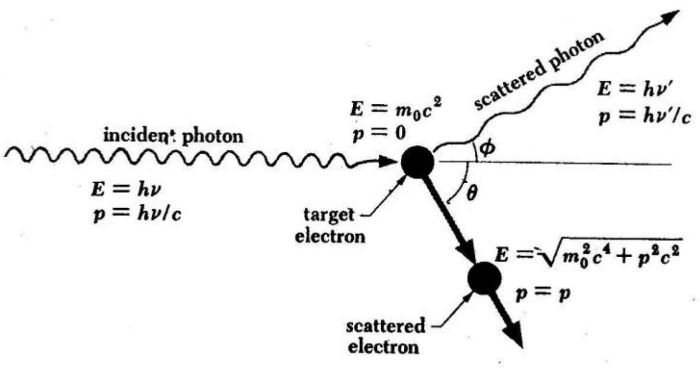


Fig. Compton Effect. Collision of a massless photon photon with an electron.

In the original photon direction

Initial momentum = final momentum

$$\frac{h\upsilon}{c} + 0 = \frac{h\upsilon^{/}}{c}\cos\phi + p\cos\theta \qquad \text{(i)}$$

and the perpendicular to this direction

Initial momentum = final momentum

$$0 = \frac{h\upsilon^{/}}{c}\sin\phi - p\sin\theta \qquad \text{(ii)}$$

The angle ϕ is that between the direction of the initial and scattered photons, and $\theta$is that between the direction of the initial photon and the recoil electron.

From equations (i) and (ii) we can write

$$pc\cos\theta = h\upsilon - h\upsilon^{/}\cos\phi$$
$$pc\sin\theta = h\upsilon^{/}\sin\phi$$

By squiring each of these equations and adding the new ones together, the angle θ is eliminated, leaving

$$p^2c^2 = (h\upsilon)^2 + (h\upsilon^{/})^2 - 2(h\upsilon)(h\upsilon^{/})\cos\phi \qquad \text{(iii)}$$

If we equate the two expressions for the total energy of a particle

$$E = T + m_0c^2$$
$$E = \sqrt{m_0^2c^4 + p^2c^2}$$

we have

$$(T + m_0c^2)^2 = m_0^2c^4 + p^2c^2$$
$$p^2c^2 = T^2 + 2m_0c^2T$$

Since $T=h\nu-h\nu^{/}$

We have

$$p^2c^2 = (h\upsilon)^2 - 2(h\upsilon)(h\upsilon^/) + +(h\upsilon^/)^2 + 2m_0c^2(h\upsilon - h\upsilon^/) \qquad \text{(iv)}$$

Substituting the value of p^2c^2 in equation (iii), we finally obtain

$$2m_0c^2(h\upsilon - h\upsilon^/) = 2(h\upsilon)(h\upsilon^/)(1-\cos\phi)$$

$$\Rightarrow m_0c(\frac{\upsilon}{c} - \frac{\upsilon^/}{c}) = \frac{\upsilon\upsilon^/}{cc}(1-\cos\phi)$$

$$\Rightarrow m_0c(\frac{1}{\lambda} - \frac{1}{\lambda^/}) = \frac{(1-\cos\phi)}{\lambda\lambda^/} \qquad \text{(v)}$$

$$\Rightarrow \lambda - \lambda^/ = \frac{h}{m_0c}(1-\cos\phi)$$

This was first derived by Arthur H. Compton in the early 1920s, and the phenomenon it describes is known as Compton effect.

It gives the change of the wavelength of the electromagnetic radiation (photon) as a function of the scattering angle of the radiation.

It is observed that the change of wavelength is independent of the incident wavelength of the incident photon. The quantity h/m_0c is called the Compton wavelength of the scattering particle, which for an electron is 0.0024 nm. We can see that the greatest wavelength change that can occur will take place for ϕ=180°

Problems

1. What is the frequency of an X-ray photon whose momentum is 1.2×10^{-23} Kg-m/s?
2. An X-ray photon of initial frequency 3×10^{19} Hz collides with an electron and is scattered through 90°. Find its new frequency.
3. A beam of X-rays is scattered by free electron. At 45° from the beam direction the scattered X-rays have a wavelength 0.0022 nm. What is the wavelength of the X-ray in the direct beam?
4. An X-ray photon whose initial frequency was 1.5×10^{19} Hz emerges from a collision with an electron with a frequency of 1.2×10^{19} Hz. How much kinetic energy was imparted to the electron? *[Hints $T=h\nu-h\nu^/$]*
5. A monochromatic X-ray beam whose wavelength is 0.0558 nm is scattered through 46°. Find the wavelength of the scattered beam?
6. Threshold frequency for copper is 1.2×10^{15}Hz. Find maximum energy of the photoelectron when light of 1.5×10^{15} Hz is directed on a copper photocathode.
7. Calculate the work function of sodium in eV when threshold wavelength is 680 nm. [Ans. 1.827 eV]
8. Photoelectric threshold wavelength for a metal is 300 nm. Find the kinetic energy of an electron ejected from it by radiation of wavelength 120 nm. [Ans. 6.2 eV]

Nuclear Physics

Nuclear Composition, Forces, and Size

Nuclei have been known to be composed of **nucleons**, neutrons and protons, since 1930. Neutrons and protons have almost equal masses m_p = 938.3 MeV/c^2, m_p =939.6 MeV/c^2. Neglecting the less then 1% binding energies, the mass of nuclei is obtained from the total mass of neutrons and protons. The number of protons in a nucleus is **Z, the atomic number** characterizing elements. The number of protons, Z, equals to the number of electrons in a neutral atom, thus it determines chemistry.

The mass number, A, is the total number of nucleons in the nucleus, A=Z+N, where **N is the number of neutrons**. The number of neutrons does not affect chemistry, but only nuclear mass, and thus physical density. Nuclei of the same element (Z values are equal) with different values of A, and consequently N, are called **isotopes**. Nuclei of different isotopes can have vastly different properties.

Numerous experiments (scattering and other) have shown that neutrons and protons interact with each other the same way as protons and protons (same the Coulomb repulsion) and neutrons and neutrons. Nuclear forces are oblivious (insensible) of the charge of the nucleon. **Nuclear forces are of very short distance**. Their range is about 1 fermi= 10^{-15}m, five orders of magnitude smaller then the Bohr radius. At the same time nuclear forces are very strong. The Coulomb potential completely overwhelms the nuclear potential, at short distances the nuclear potential is much stronger and attractive, thus able to create a bound state of positively charged protons.

The very short range of nuclear forces implies that if use the classical picture of nucleons that in the nucleus a nucleon only interacts with its neighbors. Inside the nucleus every nucleon sees the same types of potential, created by its neighbors. In other words, the potential energy for a nucleon inside the nucleus can be well represented by a flat potential well. The collection of nucleons inside the nucleus is called **nuclear matter**. Due to the short range of nuclear forces nuclear matter is fairly homogeneous inside nuclei. It follows then the volume of nuclei is proportional to the total number of nucleons, A. Then the volume is $V = (4\pi/3) R^3$ and we obtain that the nuclear radius is proportional to $A^{1/3}$, $R = R_0 A^{1/3}$. Empirically, R_0 turns out to be 1.2 fermi. This shows that the radii of all nuclei are between 1 and 6 f.

Let us determine the density of nuclear matter. The best guess is that, since the radius of nuclei is five orders of magnitude smaller then those of atoms, and atoms packed together create measured densities, the density of nuclear matter is $(10^5)^3 = 10^{15}$ times as large as that of ordinary matter. Since we know the nuclear mass and size the calculation can be made more precise.

$\mathbf{d_n= m_pA/(4\,\pi\, R_0^3\, A/3) =3\, m_p/\, 4\pi R_0^3 =2.3 \times 10^{17}\ kg\ m^{-3}}$. This is indeed 2.3×10^{14} times as large as the density of water.

The binding energy

The binding energy is defined as the total mass energy of constituent nucleons minus the mass energy of the nucleus. This is the total energy one needs to invest to decompose the nucleus into nucleons.

$B(Z,N) = c^2 [Z m_p + N m_n - M(Z,N)]$

where M(Z,N) is the mass of the nucleus. The interesting quantity is the binding energy per nucleon. B(Z,N)/A. In the roughest nuclear model it is a constant (8 MeV). Experimentally for light nuclei it is much less than 8 MeV, it reaches its maximum in the range of $50<A<80$ and starts to decrease for heavier nuclei again.

Radioactivity

It is a spontaneous and self disruptive activity exhibited by heavy elements of atomic weight greater then 206 occurring in nature.

A nucleus undergoing radioactive decay spontaneously emits a ${}_2He^4$ nucleus (alpha particle), an electron (beta particle) or a energetic photon through nucleus excitation, then approaches stability.

Radioactive substance

The substances which possess the property of emitting radiations are known as radioactive substances.

- Natural radioactivity
- Artificial radioactivity (produced in reactions)

Statistics of Radioactive decay

The activity of a sample of any radioactive material is the rate at which the nuclei of its constituent atoms decay. If *N* is the number of nuclei present at a certain time in the sample, its activity *R* is given by

$R=-(dN/dt)$

The minus sign is inserted to make R positive quantity, since dN/dt is intrinsically negative.

The SI unit of activity is named after Henri Becquerel.

1 Becquerel= 1 Bq=1 event/s.

The traditional unit of activity is Curie and its submultiples, the millicurie (mc) and micro curie (μc).

1 curie = 3.7×10^{10} disintegration/s=37 GBq.

Activities and Half lives

The radioactive decay is a statistical process; there is no way to predict when any individual nucleus will decay.

Let $N(t)$ be the (very large) number of radioactive nuclei in a sample at time t, and let $dN(t)$ be the (negative) change in that number during a short time interval dt. The number of decays during the interval dt is $–dN(t)$.The rate of change of $N(t)$ is the negative quantity $dN(t)/dt$; thus $–dN(t)/dt$ is called the decay rate or the activity of the specimen. The larger the number of nuclei in the specimen, the more nuclei in the specimen, the more nuclei decay during any time interval. That is the activity is directly proportional to $N(t)$; it equals a constant λmultiplied by $N(t)$;

$$-\frac{dN(t)}{dt} = \lambda N(t)$$

The constant λ *i*s called the decay constant, and it has different values for different nuclides. A large value of λ corresponds to rapid decay; a small value corresponds to slower decay.

This equation is separable, giving

$dN(t) / N(t) = - dt\, \lambda$

or after integration

$\log N(t) = \log N_0 - \lambda t$

where $\log N_0$ is an integration constant. Exponentiating gives the exponential decay law

$\boldsymbol{N(t) = N_0 e^{-\lambda t}}$**,**

where the significance of the integration constant N_0 is clear: it is the number of nuclei in the given state at $t=0$. The significance of λ can be understood easily if we calculate the time needed for half of a certain sample of radioactive nuclei to decay.

The half life $T_{1/2}$ is the time required for the number of radioactive nuclei to decrease to one half of the original number N_0. Then half of the remaining radioactive nuclei decay during a second interval $T_{1/2}$ and so on.

At $t= T_{1/2}$ we have

$N_0/2 = N_0 \exp\{- \lambda T_{1/2} \}$

or taking the log of both sides

$\boldsymbol{T_{1/2} = \log 2 / \lambda}$.

in other words, $\lambda= \log 2 / T_{1/2}$. After k half lives, i.e. $t=k\, T_{1/2}$ the size of the sample decreases to

$\boldsymbol{N = N_0 \exp\{-\lambda k\, T_{1/2}\} = N_0 \exp\{-(\log 2/T_{1/2})kT_{1/2}\} = N_0/2^k}$.

When one measures radioactive decays then one cannot measure $N(t)$ directly. Using counters one measures the number of decaying nuclei per unit time, $dN(t)/dt = R$, called **activity.** Using the exponential decay law one obtains

$|dN(t)/dt| = R = N_0\, \lambda e^{-\lambda t} = R_0\, e^{-\lambda t}$,

or, $\boldsymbol{R = R_0\, e^{-\lambda t}}$, *or*, $\boldsymbol{R=\lambda N(t)}$

where R_0 is the activity at $t=0$.

Radioactive carbon dating is based on the following facts:

1. Carbon has a radioactive isotope, $_6C^{14}$, with a half-life of 5730 years.
2. The proportion of this isotope, created by cosmic radiation in CO_2 in the air is constant, a fraction of a percent (1.3×10^{-12}). Plants metabolize carbon, animals eat plants and create carbon compounds in their body.
3. Fossils buried under ground are not affected by the types of cosmic radiation that creates $_6C^{14}$, so the proportion of this isotope in fossils is slowly decreasing.
4. By measuring the activity of a fossil fragment, and knowing the total amount of carbon in the fragment, one can calculate the percentage of the isotope $_6C^{14}$ in the sample and deduce its age.

   Example: *Chemical analysis shows that a bone fragment contains 10 g of C. Its activity is 50 counts/min= 0.833 / s. How old is the bone fragment?*

Solution: The total number of C atoms in the fragment is $N = (6 \times 10^{23}\ \text{mole}^{-1} / 12\ \text{g mole}^{-1})\ 10\ \text{g} = 5 \times 10^{23}$ Nuclei. The initial number of ^{14}C nuclei is $N_0 = 5\times10^{23} \times 1.3\times10^{-12} = 6.5\times10^{11}$

Initial activity is $R_0 = N_0\, \lambda = 6.5 \times 10^{11} \log 2 /(5730 \times 365 \times 24 \times 3600) = 2.53\ s^{-1}$

Since $R = R_0\, e^{-\lambda t}$ we obtain $\lambda\, t = \log (2.53/0.833) = 1.11$

Then we obtain $t = 1.11/\lambda = 1.11 \times 5730 / \log(2) = 9180$ years.

Mean life time

The average life time of a radioisotope is defined as the ratio of total life time of the entire radioactive nucleus to the total number of such nuclei in it.

Let N_0 be the initial number of nuclei and any time t later be N(t). The number of nuclei that will disintegrate between time t and t+dt be $N(t)\lambda dt$. Where λ is the decay constant of the radioisotope.

$$\tau = \frac{\int_0^\infty N(t)\lambda dt}{N_0} = \frac{\int_0^\infty N_0 \lambda e^{-\lambda t} dt}{N_0} = \lambda \int_0^\infty e^{-\lambda t} dt = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44 T_{1/2}$$

Types of Radioactivity

The most frequent radioactive decays are α, β and γ decays. These are briefly described as follows:

1. α **decay**. These are decays of heavier nuclei in which a *He* nucleus is emitted from the parent nucleus. The decay has the form

$$_{Z}X^{A} \text{-->} \, _{2}He^{4} + {}_{Z-2}Y^{A-4}$$

Here *X* is called the parent nucleus, while *Y* is the daughter nucleus. Note that the number of neutrons and the number of protons is conserved in the reaction. This is not a real requirement in nuclear reaction, only the total number of nucleons and the total charge must be conserved. Since no other charged particles are present in the reaction this implies the separate conservation of neutron and proton numbers.

The decay happens because by this decay the system goes into a lower energy state. The energy of the state is lower because, for nuclei with $A > 80$ the binding energy/nucleon increases if the nucleus becomes lighter.

The alpha decay produces energy. The system goes into a lower energy state. The total energy produced can be calculated as $Q = (m_X - m_{\square} - m_Y)c^2$. As an example

$_{92}U^{236}$ --> $_{2}He^{4}$ + $_{90}Th^{232}$

The total energy produced is

Q=(236.045561-4.002603-232.03805)931.4943MeV=4.57 MeV

β **decay**. These decays are characteristic of isotopes that have an excess number of neutrons. The way the nucleus tries to address this problem and go to a lower energy state is that one of the neutrons of the nucleus decays into a proton. The general form of the decay is $_{Z}X^{A}$ --> e^{-} +ν+ $_{Z+1}Y^{A}$.

Again, note that the number of nucleons is conserved and charge is conserved, because one of the neutrons is transformed into a proton and an electron. These reactions happen after α particle decays because for the lighter a nucleus is the smaller the value of N/Z becomes for the most stable isotopes. At the same time if N/Z>1, as it is true for heavy nuclei then the ratio N/Z **increases** α decay. It should decrease, so the daughter nucleus becomes unstable and undergoes β decay to decrease N/Z ratio. The presence of the neutrino can be clearly seen experimentally. Their presence is also needed due to angular momentum conservation. The "integer-ness" of the spin of nuclei *X* and *Y* is the same. i.e. they are either both fermions or they are both bosons, having the same number of spin 1/2 nucleons. Electrons are however fermions with s=1/2. Thus total angular momentum can only be conserved if an additional s=1/2 particle, the neutino is produced.

Properties of neutrinos: massless (or rather have masses that are less then a few eV/c^2) Very weakly interact with matter (earth is essentially transparent to them). Detection is very difficult, need a very large number of them to see a few. Neutrinos are different from antineutrinos that have opposite polarizations (right handed vs. left handed). Experimentally shown to be different. Sometimes nuclei decay through electron capture. They absorb some of their inner electrons that have wave functions with large overlaps and undergo a reaction ${}_{Z}X^{A} + e^{-} \rightarrow \nu + {}_{Z-1}Y^{A}$. Example ${}_{4}Be^{7} + e^{-} = \nu + {}_{3}Li^{7}$

Types of β decay

In negative β decay a neutron is transformed in to a proton and an electron is emitted

$n \rightarrow p + e^{-}$

In positive β decay, a protons becomes a neutron and positron is emitted

$p \rightarrow n + e^{+}$

Thus negative β decay decreases the proportion of neutrons and positive β decay increases.

2. **γ decay.** In these decays neither Z nor N changes. This is the preferred decay mode of excited states of nuclei. **γ** particles are photons, only much more energetic than those emitted in atomic or molecular decays. Many times a **γ** decay follows a *β* decay, which lands the nucleus in an excited state.

Radioactive Series

There are many isotopes that are naturally radioactive, especially of heavy elements. All isotopes of all elements Z> 83 (heavier then Pb and Bi) are radioactive. All of them follow chains of decays that end up in stable isotopes of either Pb or Bi. Typical example is the most stable isotope of U_{92}^{238}. Its lifetime is 4.5 billion years, so much of it created at the formation of the solar system is still with us. After decaying into Th, as shown above after a series of decays 8α decays and 4 β decays) it reaches the final state of ${}_{80}Pb^{206}$. This is called a radioactive series. There are several known series, all combinations of α decays and β decays.

Nuclear Energy

Nuclear Reactions

A nuclear reaction is induced by the bombardment of nuclei with a particle beam. The schematics of such a reaction are

$$a + X \rightarrow b + Y,$$

where X and Y are nuclei, a is the incoming particle and b is an outgoing particle. These reactions are also written sometimes as X(a,b)Y. An example is ${}^{13}C(p,n)\,{}^{13}N$.

Nuclear reactions are governed by conservation laws: Energy, momentum, nucleon number (A), and charge (Q).

The incoming particle carries kinetic energy K_a. The outgoing nucleus and particle also carry kinetic energies K_Y and K_b. The target nucleus, X, is supposed to be at rest. So the energy balance of the reaction is

$$K_a + c^2(M_X + M_a) = K_b + K_Y + c^2(M_Y + M_b)$$

The energy produced in the reaction is $Q = K_b + K_Y - K_a$.

If Q>0 then the reaction converts mass into energy and is **exothermic.** If Q<0, then energy is converted into mass and the reaction is **endothermic**. Endothermic reactions have a threshold energy (minimum K_a at which the reaction occurs), which is larger than the mass energy difference

$c^2(M_Y+M_b-M_X-M_a)$, because of momentum conservation.

Fission

Nuclei in the range 50<A<80 have the highest binding energies per nucleon, approximately 8.5 MeV/nucleon. Lighter nuclei have smaller binding energies because of surface effects. Their surface to volume ratio is much larger, consequently a larger fraction of the nucleons are near the surface and have smaller binding energies. Binding energy/ nucleon goes as low as 1 MeV for the deuteron. Nuclei heavier than those in the above range also have smaller binding energies because the positive Coulomb repulsion energy. Thus, when forming larger nuclei from smaller ones (such as p or d) or when splitting heavy nuclei (such as U, or Pt) mass energy is transformed into kinetic energy.

There is another difference between heavy nuclei and medium range nuclei, besides the differing binding energy per nucleon. The ratio of neutrons/protons is lower in the medium range therefore at splitting some of the neutrons become superfluous and emitted as free neutrons. The easiest way to split a heavy nucleus is by neutrons. Neutrons are not repulsed by the positively charged nucleus and as such they penetrate the nucleus with ease. Typical neutron induce fission reaction of ${}^{235}_{92}U$ are

$${}_0n^1 + {}_{92}U^{235} \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\ {}_0n^1,$$

$${}_0n^1 + {}_{92}U^{235} \rightarrow {}_{50}Sn^{132} + {}_{42}Mo^{101} + 3\ {}_0n^1,$$

$${}_0n^1 + {}_{92}U^{235} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\ {}_0n^1.$$

It is easy to find out the number of neutrons produced if we require that both the total A and total Z balance on the side of the equation. It is natural to produce several neutrons in fission reactions because the fission products have much lower Z (thought the total Z is obviously conserved) and at lower Z the ratio of N/Z is smaller for the most stable isotopes.

The energy production can be roughly estimated by taking 7.5 as the average binding energy for the U nucleus and 8.5 for the fission products. Then in the last reaction we have

$$8.5\times(141+92)-7.5\times235 = 208 \text{ MeV}$$

In other words every fission produces 208 MeV heat. How much is produced by a kg of $_{92}U^{235}$? The number of $_{92}U^{235}$ nuclei in one kg is N = N_A 1000g /(235 g) = $2.56 x 10^{24}$ nuclei, each producing 208 MeV provides

E = $208 MeV \times 2.56 \times 10^{24} = 2.37 \times 10^7$ kWh. This would supply 10MW for 2370 hrs. This is the same energy as that of 20,000 tons of TNT.

Notice that in the above fission reactions more neutrons are produced than used. The factor between the two is denoted by K. Averaging over all fission reactions K=2.5 for $_{92}U^{235}$. The fact that K>1 makes a chain reaction possible. Neutrons produced by one nucleus can be used to split more nuclei. One problem is that the neutrons produced carry energies up to 100 MeV or more. Such energetic neutrons are not likely to cause fission. Neutrons need to be slowed down before they can split other nuclei. In a sufficiently large chunk of $_{92}U^{235}$ they can be slowed down by multiple collisions so that they produce new fissions. The minimum size of such a chunk is called the critical mass.

In nature the abundance of $_{92}U^{235}$ is only 0.7%. Most of the nuclei are of the isotope $_{92}U^{238}$. If a nucleus is split and neutrons are produced then chances are they are absorbed by $_{92}U^{238}$ nuclei, which do not likely undergo fission and do not produce new neutrons. They rather produce the elements Pt or Np. In a nuclear reactor one wishes to keep K slightly above 1. This can be easily done if the concentration of $_{92}U^{235}$ is slightly increased from natural levels.

Fusion

Due to surface effects light nuclei, with the notable exception of He^4, usually have much smaller binding energies per nucleon then heavier ones. This implies that if they fuse together they go into a lower energy state and kinetic energy is produced. That is the basis of fusion reactions. Fusion reactions produce the energy radiation of stars, among others of the sun. Starting mostly from *H* heavier atoms are fused together to create more stable nuclei, mostly *He*. The creation of *He* nuclei happens in a series of nuclear reaction which, obviously, should contain beta-decays to transform some of the protons into neutrons that are required in *He* and other heavier atoms. Examples for such chains are the following:

$p + p \rightarrow {}_1d^2 + e^+ + \nu$

$p + {}_1d^2 \rightarrow {}_2He^3 + \gamma$

$p + {}_2He^3 \rightarrow {}_2He^4 + e^+ + \nu$ or

${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + p + p$

This is called the proton cycle that produces ${}_2He^4$ = α-particles from protons. Such a cycle is equivalent to the following single reaction

$p + p + p + p \rightarrow {}_2He^4 + e^+ + \nu + e^+ + \nu + 24MeV$

In other words, the production of every α-particle releases 24MeV kinetic energy. This is the energy that produces the heat of the sun, the energy of *H*-bombs and of future fusion reactors.

In future fusion reactors a more promising series of nuclear reactions would start from deuterium, heavy hydrogen that can be found in sea water in abundance. The reaction one would use are

$d + d \to {}_2He^3 + n + 3.3$ MeV

$d + d \to {}_1H^3 + p + 4$ MeV (${}_1H^3$ is tritium a heavy, radioactive isotope of hydrogen)

$d + {}_2He^3 \to {}_2He^4 + n + 17.6$ MeV

The major obstacle of producing fusion power is the coulomb repulsion of all of the particles on the left hand side of all of the listed reactions. Before nuclear forces start to act between the two fusing nuclei they are repelled by each other. This repulsion must be overcome by a sufficient amount of kinetic energy.

Suppose that the two deuterons must approach each other to a distance of 10f so that the attractive nuclear forces would start to act between them. What should be the temperature of the plasma, formed from electrons and deuterons (no deuterium can exist at that temperature, so atoms get ionized), that the fusion reaction would start? The repulsion energy at a distance of 10f is $E_C = ke^2/R = 1.44$ MeV f / 10 f = 0.144 MeV. To have a substantial fraction of deuterons half of that energy the temperature of the plasma must be $3k_BT/2\sim 0.07$ MeV, T=560 Million K, that is a pretty high temperature, much higher than the one inside the sun.

Photoelectric Effect

In 1888, Hallwachs made the important observation that

(i) when ultra-violet light falls on a neutral zinc plate, the plate becomes positively charged.

(ii) when ultra-violet light falls on a negatively charged zinc plate, it loses its negative charge. and

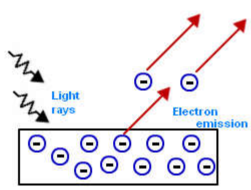
(iii) when ultra-violet light falls on a positively charged zinc plate, it becomes more positively charged.

Therefore, Hallwach came to the conclusion that only negatively charged particles can be emitted by the zinc plate when it is irradiated with ultra-violet light. [Afterwards, it was discovered that alkali metals like lithium, sodium, potassium, rubidium and cesium eject electrons even when ordinary light i.e., visible light falls on them].

1

Photoelectric effect

The phenomenon of ejection of electrons from a metal surface when light of suitable frequency falls upon it is known as photoelectric effect.
The electrons so emitted are called photoelectrons. J J Thomson showed that these photoelectrons are not different from ordinary electrons.



Work Function

The minimum amount of energy required to eject an electron out of a metal surface is called work function of the metal. The work function of a metal depends on the nature of the metal.

Threshold Frequency

The emission of electrons from a particular material does not occur for light of all frequencies. Below a certain frequency photoelectric effect is not observed, no matter how intense the light may be. This frequency is known as threshold frequency. If ν_0 is the threshold frequency of radiation, then work function of the metal is related with the threshold frequency by the relation, $W = h\nu_0$.

2

Stopping Potential

The retarding potential at which the collector is maintained at a negative potential with respect to the emitter, no electron will reach the collector and the current will become zero. This potential is called stopping potential. Work done by the stopping potential is equal to the maximum kinetic energy of the electrons.

If V_0 is the stopping potential applied between the anode and the cathode, then it is related with the maximum kinetic energy of the electron by the relation,

$$eV_0 = \frac{1}{2} mV_{max}^2$$

where e= electronic charge,
m= mass of electron and
V_{max}=maximum velocity of photoelectrons.

3

LAWS OF PHOTOELECTRIC EFFECT

Following are the characteristics of photoelectric emission:

(i) The emission of photoelectrons takes place when the frequency of the incident radiation is above a certain critical value known as threshold frequency, which is an intrinsic characteristic of that metal.

(ii) The emission of photoelectrons is instantaneous. It has been found that the time lag between the incidence of photon and the emission of electron is less than 10^{-9} seconds.

(iii) The number of photoelectrons emitted from a metal surface depends only on the intensity of the incident light and is independent of its frequency.

(iv) The maximum kinetic energy with which photoelectrons are emitted from a metal surface depends only upon the frequency of the incident light and is independent of its intensity.

4

Experimental study of photoelectric effect

The photoelectric effect was investigated by the German physicists Wilhelm Hallwachs and Philipp Lenard during the year 1886-1900. Hallwachs and Lenard constructed a photoelectric vacuum tube consisting of a cathode irradiated by ultraviolet light and an anode collecting electrons emitted by the cathode. A voltage, V, was applied between the anode and cathode to repel the electrons emitted by the cathode. The number of electrons arriving at the anode was measured by the current in an outside circuit connecting the anode and the cathode. Hallwachs and Lenard found that when monochromatic light fell on the cathode, no photoelectrons at all were emitted unless the frequency of the light was greater than some minimum value called the **threshold frequency.** This minimum frequency depends on the material of the cathode. For most metals the threshold frequency is in the ultraviolet (wavelength between 200 and 300 nm) but for potassium and cesium oxides it is in the visible spectrum (wavelength between 400 and 700 nm).

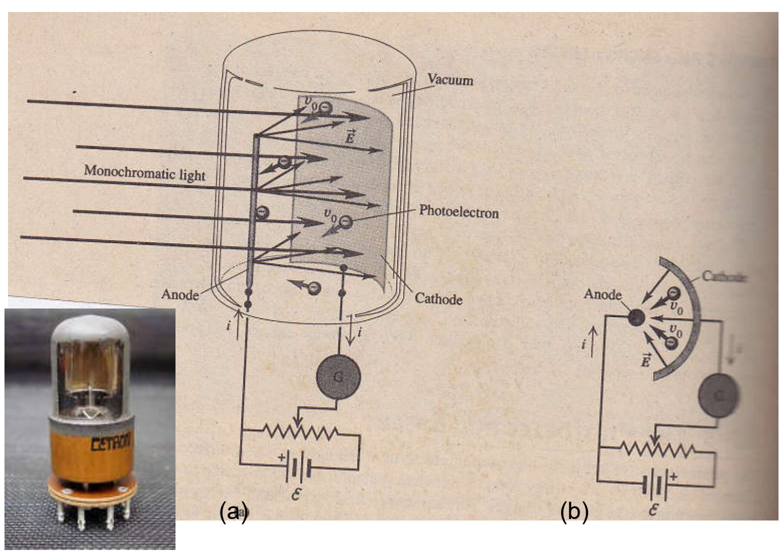


Fig 1. Schematic diagram of a phototube circuit (a) Electrons emitted from the cathode are pushed towards the anode by the electric field force. (b) Even when the direction of the field is reversed, some electrons still reach the anode.

When the frequency ν is greater than the threshold frequency, some electrons are emitted from the cathode with substantial initial speeds. This can be shown by reversing the polarity of the battery so that the electric field force on the electrons is back towards the cathode. If the magnitude of the field is not too large, the highest energy emitted electrons still reach the anode and there is still a current. We can determine the maximum kinetic energy of the emitted electrons by making the potential of the anode relative to the cathode, $V_{AC,}$ just negative enough so that the current stops. This occurs for V_{AC} =$-V_0$, where V_0 is called the **stopping potential**.

As an electron moves from the cathode to the anode, the potential decreases by V_0 and negative work $-eV_0$ is done on the electron, the most energetic electron leaves the cathode with kinetic energy $K_{max}=½mv_{max}^2$ and has zero kinetic energy at the anode. U sing the work energy theorem, we have

$$W_{tot}= - eV_0=\Delta K$$
$$=0-K_{max}$$

Therefore, $$K_{max}=½m\, v_{max}^2 = eV_0$$

Hence by measuring the stopping potential V_0, we can determine the maximum kinetic energy with which electrons leave the cathode.

7

Some experimental results:

Figure 2 shows graphs of photocurrent as a function of potential difference V_{AC} for a light of constant frequency and two different intensities. When V_{AC} is sufficiently large and +ve, the curves levels off, showing that all the emitted electrons are being collected by the anode. The reverse potential $-V_0$ needed to reduce the current to zero is shown.

If the intensity of the light is increased while its frequency is kept the same, the current levels off at a higher value, showing that more electrons are being emitted per time. But the stopping potential V_0 is found to be the same.

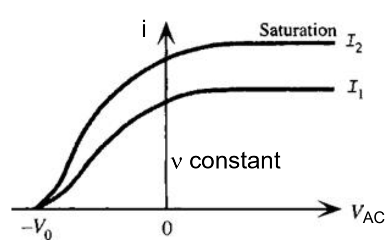


Fig 2. Photocurrent *i* as a function of the potential V_{AC} of the anode with respect to the cathode for a constant light frequency. The stopping potential V_0 is independent of the light intensity *I*, but the photocurrent for large positive V_{AC} is directly proportional to the intensity.

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Figure 3 shows current as a function of potential difference for two different frequencies, with the same intensity in each case. We see that when the frequency of the incident monochromatic light is increased, the potential V_0 is increases. In fact V_0 turns out to be a linear function of the frequency ν.

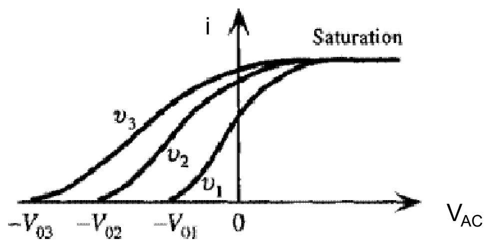

Fig 3. Photocurrent i as a function of the potential V_{AC} of an anode with respect to a cathode for two different light frequencies, ν_1 and ν_2 with the same intensity. The stopping potential V_0 increases linearly with frequency.

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Failure of Classical Physics in case of photoelectric effect

These results are hard to understand on the basis of the classical physics.

- When the intensity (average energy per unit area per unit time) increases, electrons should be able to gain more energy, increasing the stopping potential V_0. But V_0 was found independent of intensity.
- Also, classical physics offer no explanation for the threshold frequency. We know that the intensity of the electromagnetic wave such as light does not depend on frequency, so an electron should be able to acquire its needed escape energy from light of any frequency. Thus there should not be any threshold frequency, ν_0.
- Finally, we would expect it to take a while for an electron to collect enough energy from extremely faint light. But experiment shows that electrons are emitted as soon as any light with $\nu \geq \nu_0$ hits the surface.

10

Einstein's photoelectric equation

The correct analysis of the photoelectric effect was developed by Albert Einstein in 1905. Based on assumptions made by Max Planck in 1900, Einstein postulated that a beam of light consist of small packages of energy called photons or quanta. The energy E of a photon is equal to $h\nu$, where h is Planck's constant.

A photon arriving at the surface is absorbed by an electron. This energy transfer is an all-or-nothing process, in contrast to the continuous transfer of energy in the classical theory; the electron gets all the photon's energy or none at all. If this energy is greater than the work function φ (minimum energy needed to remove an electron from the surface), the electron may escape from the surface.

Greater intensity as a particular frequency means a proportionally greater number of photons per second absorbed, thus a proportionally greater number of electrons emitted per second and the proportionally a greater current is seen.

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Thus Einstein applied conservation of energy to find that the maximum kinetic energy $K_{max}=½mv_{max}^2$ for an emitted electron is the energy hν gained from a photon minus the work function φ.

$$½mv_{max}^2=h\nu-\varphi$$

If ν_0 is the threshold frequency then $\varphi=h\nu_0$

$$\text{Then } ½mv_{max}^2=h\nu-h\nu_0\text{ ; or}$$

$$eV_0=h\nu-h\nu_0$$

We can measure the stopping potential V_0 for each of several values of frequency ν for a given cathode material. A graph of V_0 as a function of ν turns out to be a straight line and from such graph we can determine both the work function φ for the material and the value of the quantity h/e. After the electron charge $-e$ was measured by Robert Millikan in 1909, Planck's constant h could also be determined from these measurements.

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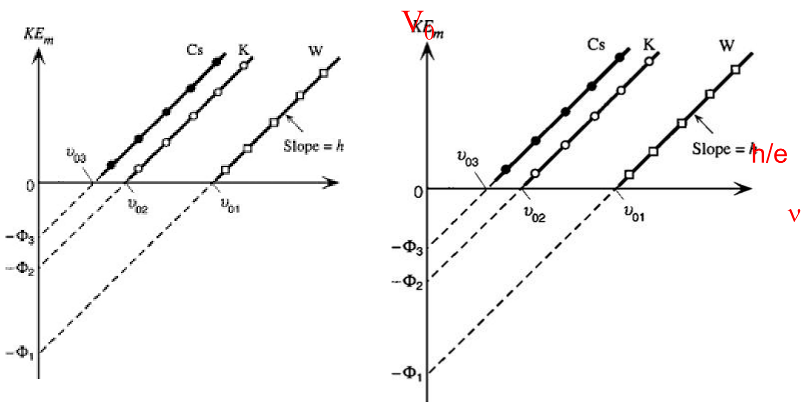

Figure 4. Determination of value of Planck’s constant, work function and threshold frequency

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Photoelectric Effect Applications

Below are the applications of Photoelectric effect:

1. The photoelectric effect is used in the photoelectric cell which converts a light energy into electrical energy.
2. In cinematography photoelectric effect has a application of reproducing the sound.
3. Photoelectric effect also has an application in street lights for automatic switch on and off.
4. Traffic signals are using this effect for automatic controls and for count the machines.
5. Working of burglar alarm uses this photoelectric effect.
6. Television transmission is one of the applications of this photoelectric effect.

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The Photocell

The photoelectric effect has many practical applications which include the photocell, photoconductive devices and solar cells.

A photocell is usually a **vacuum tube** with two electrodes. One is a photosensitive **cathode** which emits electrons when exposed to light and the other is an **anode** which is maintained at a positive voltage with respect to the cathode. Thus when light shines on the cathode, electrons are attracted to the anode and an electron current flows in the tube from cathode to anode. The current can be used to operate a relay, which might turn a motor on to open a door or ring a bell in an alarm system. The system can be made to be responsive to light, as described above, or sensitive to the removal of light as when a beam of light incident on the cathode is interrupted, causing the current to stop. Photocells are also useful as exposure meters for cameras in which case the current in the tube would be measured directly on a sensitive meter.

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Closely related to the photoelectric effect is the photoconductive effect which is the increase in **electrical conductivity** of certain non metallic materials such as cadmium sulfide when exposed to light. This effect can be quite large so that a very small current in a device suddenly becomes quite large when exposed to light. Thus photoconductive devices have many of the same uses as photocells.

Solar cells, usually made from specially prepared silicon, act like a **battery** when exposed to light. Individual solar cells produce voltages of about 0.6 volts but higher voltages and large currents can be obtained by appropriately connecting many solar cells together. **Electricity** from solar cells is still quite expensive but they are very useful for providing small amounts of electricity in remote locations where other sources are not available. It is likely however that as the cost of producing solar cells is reduced they will begin to be used to produce large amounts of electricity for commercial use.

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Problems

P-1:Sodium has a work function of 2.36 eV. If a scientist illuminates a piece of sodium with a 450 nm wavelength light, what is the:

a.stopping potential;

b.maximum kinetic energy of the emitted electrons; and

c.emitted electrons maximum speed?

P-2: Gold has a work function of 5.10 eV. What is the minimum frequency of light to emit photoelectrons? What is the longest corresponding wavelength?

P-3: The work function of copper is 4.7 eV. What is the maximum velocity of photoelectrons produced by ultraviolet light of wavelength 100nm?

P-4: What should be the wavelength of monochromatic radiation to produce photoelectrons of maximum velocity 10^6m/s from iron? The work function for iron is 4.5 eV.

P-5: While conduction photoelectric effect experiment with light of a certain frequency, it was observed that reverse potential difference of 1.25 V is required to reduce the photocurrent to zero. Find (a) the maximum kinetic energy; (b) the maximum speed of the emitted photoelectrons.

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6. Threshold frequency for copper is 1.2×10^{15}Hz. Find maximum energy of the photoelectron when light of 1.5×10^{15} Hz is directed on a copper photocathode.
7. Calculate the work function of sodium in eV when threshold wavelength is 680 nm. [Ans. 1.827 eV]
8. Photoelectric threshold wavelength for a metal is 300 nm. Find the kinetic energy of an electron ejected from it by radiation of wavelength 120 nm. [Ans. 6.2 eV]

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9. A metal of work function of 4 electron volts is exposed to radiation of the wavelength 140 nm. What is the corresponding stopping potential?

$$\frac{hc}{\lambda} = \phi + K.E$$

$$K.E = \frac{hc}{\lambda} - \phi = \frac{6.625\times10^{-34}\times3\times10^{8}}{140\times10^{-9}} - 4\times1.6\times10^{-19}$$

$$= 7.796\times10^{-19}\ joule$$

$$= 4.87 eV$$

We know that

$$K.E = eV_0$$

Therefore,

$$V_0 = 4.87\ V$$

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When a radiation of certain wavelength is incident on a metallic surface, the stopping potentially is found to be 4.8 V. If the same surface is illuminated by the radiation of double the wavelength, the stopping potential is found to be 1.6 V. What is the threshold wavelength of the surface?

$$\frac{hc}{\lambda} = \phi + 4.8 \times e$$

Or, $\frac{hc}{\lambda} - \frac{hc}{\lambda 0} = 4.8 \times e$ (i)

$$\frac{hc}{2\lambda} = \phi + 1.6 \times e$$

0r, $\frac{hc}{2\lambda} - \frac{hc}{\lambda 0} = 1.6 \times e$ (ii)

Therefor, dividing (i) by (ii) , we obtain, $\frac{\frac{hc}{\lambda}-\frac{hc}{\lambda 0}}{\frac{hc}{2\lambda}-\frac{hc}{\lambda 0}}=3$

Or, $\frac{hc}{\lambda} - \frac{hc}{\lambda 0}=3[\frac{hc}{2\lambda} - \frac{hc}{\lambda 0}]$

Or, $-\frac{hc}{2\lambda} + \frac{2hc}{\lambda 0}=0$

Or, $\frac{hc}{2\lambda} = \frac{2hc}{\lambda 0}$; Or , $\lambda_0=4\ \lambda$

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11. If the wavelength of the incident radiation changes from one value to other value, the corresponding kinetic energy emitted by the photo electrons also changes from one value to other value. What is the work function of the metal surface?

$$\frac{hc}{\lambda_1} = \phi + E_1 \quad \Rightarrow hc = \varphi\lambda_1 + E_1\lambda_1$$

$$Again\, for\, another\, wavelength,$$

$$\frac{hc}{\lambda_2} = \phi + E_2 \quad \Rightarrow hc = \varphi\lambda_2 + E_2\lambda_2$$

$$Therefore, \;\; \varphi\lambda_1 + E_1\lambda_1 = \varphi\lambda_2 + E_2\lambda_2$$

$$or, \varphi(\lambda_1 - \lambda_2) = E_2\lambda_2 - E_1\lambda_1$$

$$Therefore, \; \varphi = \frac{E_2\lambda_2 - E_1\lambda_1}{\lambda_1 - \lambda_2}$$

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12. For a certain metal threshold frequency is given. When the incident frequency is doubled the threshold frequency, electrons comes out with a velocity 4×10^6 m/s. If the incident frequency is five times threshold frequency, what is the velocity with which the electrons come out?

$$\frac{1}{2}mv^2 = h(\upsilon - \upsilon_0)\, becomes, \; \frac{1}{2}mv_1^{\,2} = h(2\upsilon_0 - \upsilon_0) = h\upsilon_0$$

$$This\, gives, v_1 = \sqrt{\frac{2h\upsilon_0}{m}} = 4\times10^6\, m/s$$

$$and \;\; \frac{1}{2}mv_2^{\,2} = h(5\upsilon_0 - \upsilon_0) = 4h\upsilon_0$$

$$\frac{v_2^{\,2}}{v_1^{\,2}} = 4, \; or, v_2 = 2v_1 = 8\times10^6\, m/s$$

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13. Photoelectric effect from a metallic surface is observed from two different frequencies where first frequency is greater than that of the second frequency. If the ratio of the maximum kinetic energy emitted by the photo electrons in both the cases is given, find the expression of threshold frequency of the metal surface?

$$h\upsilon_1 = h\upsilon_0 + 1E, \quad and\ for\ second\ case\ \ h\upsilon_2 = h\upsilon_0 + pE$$

$$This\ gives,\ \ E = \frac{h\upsilon_2 - h\upsilon_0}{p}$$

$$Substituting\ this\ value, we\ get, h\upsilon_1 = h\upsilon_0 + 1(\frac{h\upsilon_2 - h\upsilon_0}{p})$$

$$or,\ h\upsilon_1 = h\upsilon_0 + \frac{h\upsilon_2}{p} - \frac{h\upsilon_0}{p},\ or,\ h\upsilon_1 - \frac{h\upsilon_2}{p} = h\upsilon_0(1 - \frac{1}{p})$$

$$or, \frac{ph\upsilon_1 - h\upsilon_2}{p} = h\upsilon_0(\frac{p-1}{p}),\ \ or, ph\upsilon_1 - h\upsilon_2 = h\upsilon_0(p-1)$$

$$or,\ \upsilon_0 = \frac{p\upsilon_1 - \upsilon_2}{p-1}$$

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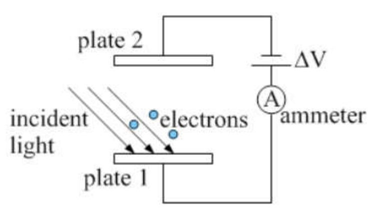
14. when ultraviolet light with a wavelength of 240 nm shines on a particular metal plate, electrons are emitted from plate 1, crossing the gap to plate 2 and causing a current to flow through the wire connecting the two plates. The battery voltage is gradually increased until the current in the ammeter drops to zero, at which point the battery voltage is 1.40 V.

(a) What is the energy of the photons in the beam of light, in eV?
(b) What is the maximum kinetic energy of the emitted electrons, in eV?
(c) What is the work function of the metal, in eV?
(d) What is the longest wavelength that would cause electrons to be emitted, for this particular metal?
(e) Is this wavelength in the visible spectrum? If not, in what part of the spectrum is this light found?

Solution:



(a)

$$E = \frac{hc}{\lambda} = \frac{6.625\times10^{-34}\times3\times10^{8}}{240\times10^{-9}} = 8.28\times10^{-19}\,Joue = 5.17eV$$

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(b) Maximum kinetic energy

$$(K.E)_{\max} = eV_0 = e \times 1.40V = 1.40eV$$

(c) $$\phi = \frac{hc}{\lambda} - (K.E)_{\max} = 5.17eV - 1.40eV = 3.77eV$$

(d) $$\phi = \frac{hc}{\lambda_0},\ or, \lambda_0 = \frac{hc}{\phi} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{3.77 \times 1.6 \times 10^{-19}} = 3.29 \times 10^{-7} m$$

(e) This wavelength lies in UV region of the electromagnetic spectrum.

15. With a particular metal plate, shining a beam of red light on the metal causes electrons to be emitted. (a) If we replace the red light by blue light, do we know that electrons will be emitted? (b) If the two beams have the same intensity and are incident on equal areas of the plate, do we get the same number of electrons emitted per second in the two cases?

The work function for tungsten (W) is $\phi_W = 4.52$ eV.

(a) What is the longest light wavelength (sometimes referred to as the cutoff wavelength, λ_c) that can result in production of a photocurrent?

To eject an electron, a photon must have at least as much energy as ϕ_W. Using $E_\gamma = hc/\lambda_c = \phi_W$ gives $\lambda_c = hc/\phi_W = 1240/4.52$ eV-nm/eV $= 274.3$ nm.

(b) What is the maximum kinetic energy, K_{max}, of emitted electrons when light of wavelength $\lambda = 200$ nm is used to irradiate a piece of W?

If the enegy of the photon is greater than the work function, $hc/\lambda > \phi$, then the extra energy goes into kinetic energy of the photoelectrons. In this case, we have $K_{max} = hc/\lambda - \phi_W = 1240/200 - 4.52 = 1.68$ eV.

(c) What is the stopping potential (voltage) for this case ($\lambda = 200$ nm)?

The potential energy of an electron in an electric potential is $q \cdot V = e \cdot V$, where $q = e$ is the charge of an electron, and V is the applied voltage. The stopping potential is given by the voltage needed to stop the electrons with kinetic energy K_{max} when that voltage is applied across two electrodes, one of which is the photocathode - the metal plate from which the electrons are emitted in the photoelectric process. As the electrons travel from the photocathode to the other electrode, they increase their electric potential energy, and thus their kinetic energy decreases - they slow down. If the voltage is adjusted so that the potential energy increases by exactly K_{max} just before the electrons reach the second electrode, they will stop and thus no current will be recorded. Thus we have $K_{max} = 1.68$ eV $= e \cdot V$, so the stopping potential is exactly 1.68 V.