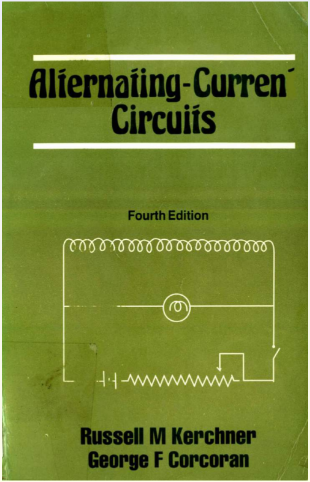
ALTERNATING-CURRENT CIRCUITS, RUSSELL M KERCHNER, GEORGE F CORCORAN, FOURTH EDITION



Syllabus

AC Currents: General concepts and definitions, instantaneous current, voltage and power; R, L, C, RL, RC, and RLC branches. Effective and average value, form factor, crest factor, real and reactive power.

AC Circuits (Steady State Analysis): Impedance in polar and Cartesian forms. Sinusoidal single phase circuit analysis. Impedance in series, parallel branches, series parallel circuits. Network analysis: Thevenin's theorem, Norton's Theorem. Balanced poly phase circuits: Three phase, four wire system of generated EMFs, three phase three wire systems, balanced Y loads and balanced delta loads. Power in balanced systems and power factor. Balanced three phase circuit analysis and power measurement

CHAPTER II - Oscillating Current

An oscillating current is a current which alternately increases and decreases in magnitude with respect to time accordingly to some definitive law.

Periodic Current

A periodic current is an oscillating current the values of which recur at equal interval of time. Thus,

$$i = I_0 + I_1 sin(\omega t + \alpha_1) + I_2 sin(2\omega t + \alpha_2) + ... \quad (1)$$

where i = instantaneous value of a periodic current at a time t
I_0, I_1, α_1, α_2, = Constants (Positive negative or zero)
$\omega = \frac{2\pi}{T}$
An alternating current is a periodic current, the average value of which over a period is zero. The equation for an alternating current is the same as that for a periodic current except that, $I_0 = 0$

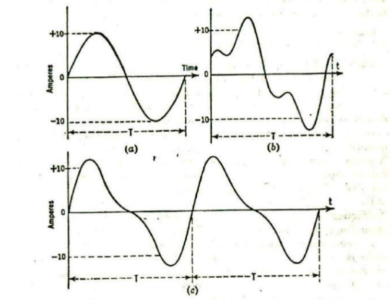


FIG. 3. Wave forms of three a-c variations. T is the period (or duration) of one cycle.

In Fig (a), $i = 10sin\omega tA$
In Fig (b), $i = 10sin\omega t + 4sin(\omega t + 90^{\circ})A$
In Fig (c), $i = 10sin\omega t + 4sin2\omega tA$

Periods and Cycles

1. Smallest value of time that separate the recurring values
2. One complete set of negative and positive values.
3. ω represents the angular velocity
4. One complete cycle is to extend over 360° or 2π radian

Frequency

1. Number of cycles per second
2. since T is the time of one cycle, $f = \frac{1}{T}$

Wave form

The shape of curve resulting from a part of instantaneous values of voltage or current as ordinate against time as abscissa is its waveform or wave shape.

Angular Velocity or Angular Frequency

$\omega = \frac{2\pi}{T} = 2\pi \times \frac{1}{T} = 2\pi f$

Sine Wave

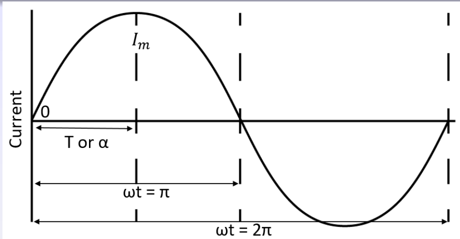


FIGURE: Sine wave may be expressed as $I_m \sin\alpha$ or as $I_m \sin\omega t$

The equation of sine wave is:
$i = I_m sin\omega t$
Here, $\omega t =$ Time angle, expressed in radians.
$i =$ Instantaneous value of current. $I_m =$ Maximum value of sinusoidal variation.
Since ωt represents an angle ,
$i = I_m sin\alpha$,
Where α is in degrees or radians
Alternating Potential Difference

It may take the form of a generated e.m.f or the form of a potential drop, sometimes abbreviated as p.d. Instantaneous values of generated or induced emf's will be designated by e, and instantaneous values of potential drops by the symbol v. Similarly E_m and V_m will be used to distinguish a maximum value of induced voltage from a maximum value of potential drop.

Phase

Phase is the fractional part of a period through which time or the associated time angle t has advanced from an arbitrary reference. In the case of a simple sinusoidal variation, the origin is usually taken as the last previous passage through zero from the negative to the positive direction.

In accordance with the above definition, the phase angle of a single wave is the angle from the zero point on the wave to the value at the point from which time is reckoned. Thus,

$$i = I_m sin(\omega t + \theta)$$

represents a sine wave of current with a phase angle θ. The phase of the wave from which time is reckoned (i.e., when t $= 0$) is

$i = I_m sin\theta$. The angle θ is the phase angle of the current with respect to the point where i = 0 as a reference.

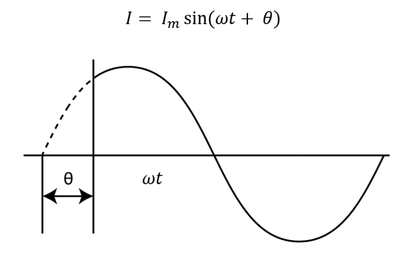


FIGURE: Phase angle θ of sine wave

Phase Difference

The phase angle is a very important device for properly locating different alternating quantities with respect to one another. For example, if the applied voltage is $V = V_m sin\omega t$, and it is known from the nature and magnitude of the circuit parameters that the current comes to a corresponding point on its wave before the voltage wave by θ degree, the current can be expressed as,

$i = I_m sin(\omega t + \theta)$

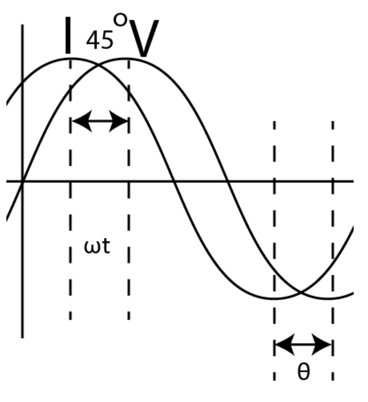


FIGURE: Illustrating a case where i wave leads the v wave by $\theta = 45^o$

The figure illustrating a case where the i wave leads the v wave by $\theta = 45^\circ$

Thus, if $V = 100sin(\omega t + 45^\circ)$ and $i = 10sin(\omega t - 15^\circ)$, the angle of phase difference is $45^\circ - (-15^\circ) = 60^\circ$.

Inspection of the oscillogram will show that the current lags the voltage in this particular case by approximately 60°

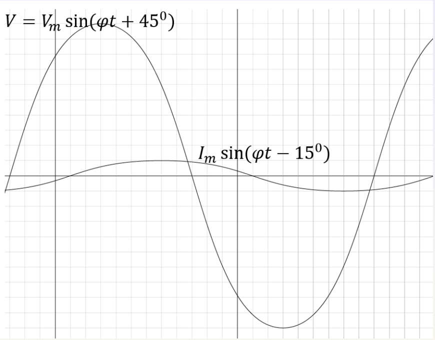
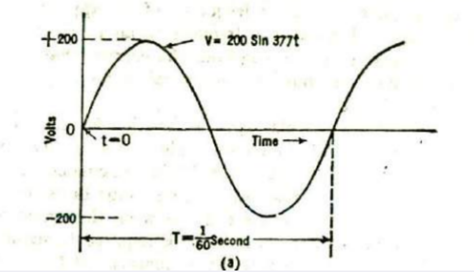


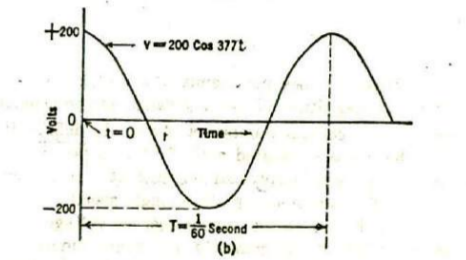
FIGURE: General Concept

Example-1. A voltage is described as having sinusoidal wave form, a maximum value of 200 volts, and an angular frequency of 377 radians per second.(60 cycles per second)



the mathematical expression for the alternating voltage as a function of time, t, is:
$v = 200 sin 377t$ volts

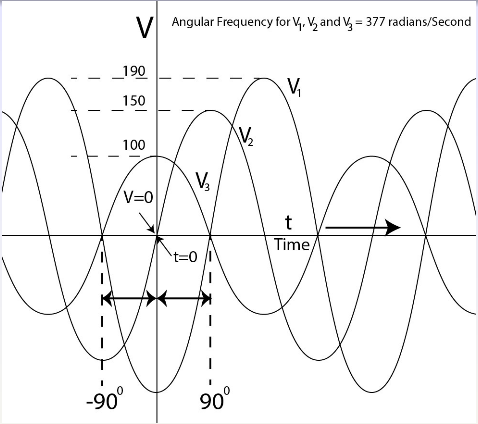
Example-2. A voltage is described as having sinusoidal wave form, a maximum value of 200 volts, and an angular frequency of $377 + 90^{\circ}$ radians per second.(60 cycles per second)



the mathematical expression for the alternating voltage as a function of time, t, is:

$v = 200sin(377t + 90^{\circ}) = 200cos377t$ volts

Problem 3



Write down the expression for V_1, V_2 and V_3

$V_1 = 190sin(377t - 90^{\circ})$

$V_2 = 150sin(377t)$

$V_3 = 100sin(377t + 90^{\circ})$

- Write down the numerical expression of $V_1, V_2 and V_3$ from the given figure

Impedance Function

Mathematically a particular type of function is required to relate voltage and current in an a-c circuit. The one generally employed is called the impedance function or simply the impedance of the circuit. called the impedance function or simply the impedance of the circuit. The impedance function must tell two important facts:
(1) The ratio of V_m to I_m
(2) The phase angle between the waves of voltage and current. A special type of notation is required to signify the two properties of the impedance function in abbreviated form. One such type of notation is:
$Z\angle angle$

1. The above expression does not signify the multiplication of Z and $angle$angle. Z is the magnitude of the impedance and in a particular case is represented by a certain number of ohms.
2. It defines the ratio of V_m to I_m. The angle associated with Z, if it is positive, defines the lead of the voltage with respect to the current.

3. In accordance with the convention thus adopted a positive angle specifies the number of degrees or radians by which the current lags the voltage.
4. The determination of the complete impedance function for various combinations of R, L, and C is the first step in a-c circuit analysis.

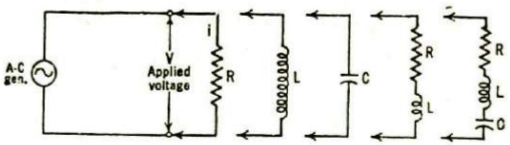


FIG. 8. Elementary circuit arrangements of R, L, and C.

The 'R' Branch

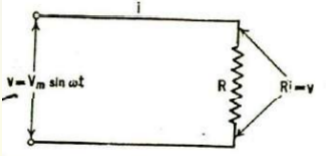


FIG. 9. The R branch.

Here,

$v = V_m sin\omega t = Ri$ ——————(1)

$i = I_m sin\omega t = \frac{V_m sin\omega t}{R}$ ——————(2)

So $\frac{V_m}{I_m}$ = R

the current wave is in time phase with the voltage wave.

Impedance:

$Z_R = R\angle 0^\circ$

Power

The instantaneous power is given by:$p = vi$

$p = vi = V_m sin\omega t \times I_m sin\omega t$

$p = V_m I_m sin^2\omega t$

From trigonometric formula we know, $sin^2\omega t = \frac{1}{2}$ - $\frac{1}{2}cos2\omega t$

so, $p = V_m I_m \frac{1}{2}$ - $V_m I_m \frac{1}{2} cos2\omega t$

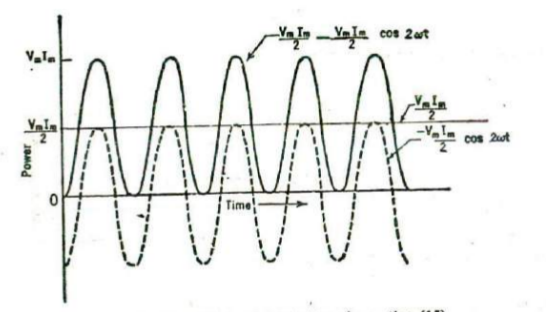


Fig. 10. Graphical representation of equation (15).

The 'L' Branch

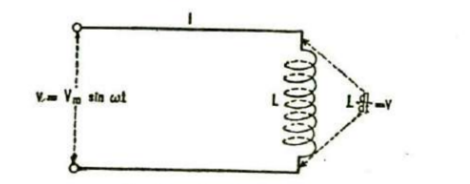


Fig. 11. The L branch.

Here,

$$v = L\frac{di}{dt} = V_m sin\omega t \text{ ——————— } (1)$$

$di = \frac{V_m}{L} sin\omega t dt$ ——————(2)

$\int di = \int \frac{V_m}{L} sin\omega t dt$

$i = -\frac{V_m}{\omega L} cos\omega t + C_1$

So $i = \frac{V_m}{\omega L} sin(\omega t - 90^\circ) = I_m sin(\omega t - 90^\circ)$... Considering $C_1 = 0$ the current wave lags voltage by 90°.

Impedance:

$Z_L = \omega L \angle 90^\circ$

1. The magnitude of the above impedance, ωL, is called inductive reactance.
2. When ω, is expressed in radians per second and L is expressed in henrys, the inductive reactance, X_L, is in ohms. Mathematically, $X_L = \omega L = 2\pi f L$

Power and Energy

The instantaneous power is given by: $p = vi$

$p = vi = V_m sin\omega t \times I_m sin(\omega t - 90^\circ)$

$p = V_m I_m (-sin\omega t \, cos\omega t)$

$p = -\frac{V_m I_m}{2} sin 2\omega t$

The exact amount of energy delivered to the circuit during a quarter of a cycle may be obtained by integrating any positive loop of the power wave, for example, integrating p between the limits of $t = \frac{T}{4}$ and $t = \frac{T}{2}$

$$W_L = \int_{T/4}^{T/2} -\frac{V_m I_m}{2} sin2\omega t dt$$

$$W_L = \frac{V_m I_m}{2(\frac{4\pi}{T})}[cos\frac{4\pi}{T}t]_{T/4}^{T/2}$$

$$W_L = \frac{V_m I_m}{2\omega}$$

Since, $V_m = \omega L I_m$, [Because, $\frac{V_m}{\omega L} sin(\omega t - 90^{\circ}) = I_m sin(\omega t - 90^{\circ})$]

$$W_L = \frac{(\omega L I_m) I_m}{2\omega} = \frac{L I_m^2}{2}$$

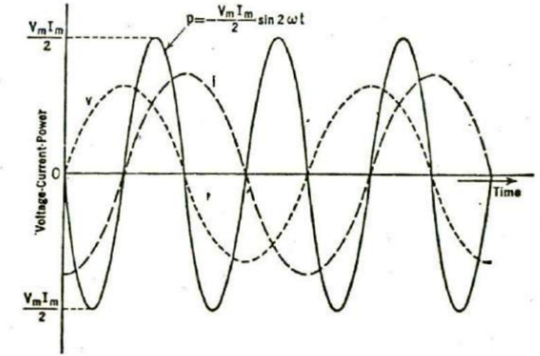


FIG. 12. Voltage, current, and power variations in a purely inductive branch.

The 'C' Branch

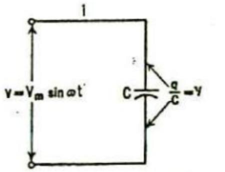


FIG. 13. The C branch.

Here,

$v = \frac{q}{C} = V_m sin\omega t$ ——————(1)

$\frac{dq}{dt} = V_m \omega C cos\omega t$ ——————(2)

$i = I_m sin(\omega t + 90^{\circ}) = \frac{V_m}{\frac{1}{\omega C}} sin(\omega t + 90^{\circ})$

the current wave leads voltage by 90°.

Impedance:

$Z_C = \frac{1}{\omega C} \angle -90^{\circ}$

1. The magnitude of the above impedance, $\dfrac{1}{\omega C}$, is called capacitive reactance.
2. When ω, is expressed in radians per second and C is expressed in farads, the capacitive reactance, X_C, is in ohms. Mathematically, $X_C = \frac{10^6}{\omega C_{\mu f}}$

Power and Energy

The instantaneous power is given by:$p = vi$

$p = vi = V_m sin\omega t \times I_m sin(\omega t + 90^{\circ})$

$p = V_m I_m (sin\omega t \, cos\omega t)$

$p = \frac{V_m I_m}{2} sin2\omega t$

The exact amount of energy delivered to the circuit during a quarter of a cycle may be obtained by integrating any positive loop of the power wave, for example, integrating p between the limits of $t = 0$ and $t = \frac{T}{4}$

$$W_C = \int_0^{T/4} \frac{V_m I_m}{2} sin(2\omega t) dt$$

$$W_C = \int_0^{T/4} \frac{V_m I_m}{2} sin(2 \times 2\pi f t) dt$$

$$W_C = \int_0^{T/4} \frac{V_m I_m}{2} sin(2 \times 2\pi \times \frac{1}{T} t) dt$$

$$W_C = \frac{V_m I_m}{2(\frac{4\pi}{T})} [-cos \frac{4\pi}{T} t]_0^{T/4}$$

$$W_C = \frac{V_m I_m}{2\omega}$$

Since, $I_m = \omega C V_m$,

$$W_C = \frac{(\omega C V_m) V_m}{2\omega} = \frac{C V_m^2}{2}$$

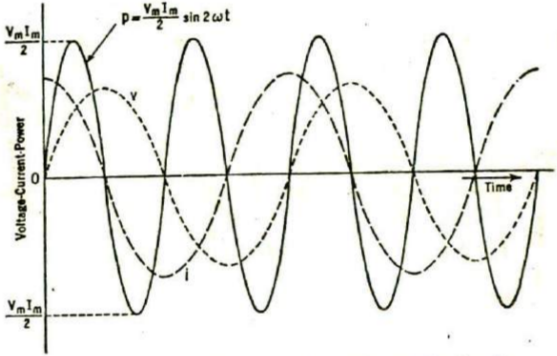


FIG. 14. Voltage, current, and power in a purely capacitive branch.

• In case of a purely resistive,inductive, capacitive circuit write down the general expression of i, v, p, Z and W_L, W_C where the symbols mean their usual meaning

The 'RL' Branch

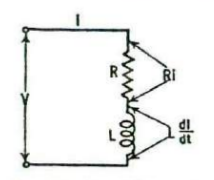


FIG. 15. The *RL* branch.

Here,

$$v = Ri + L\frac{di}{dt} = V_m sin\omega t \text{ ——————(1)}$$

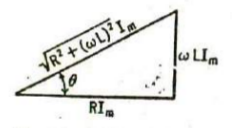
$$v = RI_m sin\omega t + \omega LI_m cos\omega T \text{ ——————(2)}$$

dividing both sides by $\sqrt{R^2 + (\omega L)^2}$

$$\frac{v}{\sqrt{R^2+(\omega L)^2}} = \frac{RI_m sin\omega t + \omega LI_m cos\omega T}{\sqrt{R^2+(\omega L)^2}}$$

$$\frac{v}{\sqrt{R^2+(\omega L)^2}} = I_m[sin\omega t(\frac{R}{\sqrt{R^2+(\omega L)^2}} + cos\omega t(\frac{\omega L}{\sqrt{R^2+(\omega L)^2}}]$$

Representing the above equation by the following figure:



$$cos\theta = \frac{R}{\sqrt{R^2+(\omega L)^2}}$$

$sin\theta = \frac{\omega L}{\sqrt{R^2+(\omega L)^2}}$

$\frac{v}{\sqrt{R^2+(\omega L)^2}} = I_m[sin\omega t cos\theta + cos\omega t sin\theta]$

From which, $v = I_m\sqrt{R^2 + (\omega L)^2}[sin\omega t cos\theta + cos\omega t sin\theta]$

$v = I_m Z(sin\omega t + \theta) = V_m(sin\omega t + \theta)$

Impedance:

$Z_{RL} = \sqrt{R^2 + (\omega L)^2}\angle tan^{-1}\frac{\omega L}{R}$

Like wise In case of R-C Branch $Z_{RC} = \sqrt{R^2 + (-\frac{1}{\omega C})^2}\angle tan^{-1}\frac{(-\frac{1}{\omega C})^2}{R}$

1. The above expression for Z_{RL} implies that the numerical ratio of V_m to I_m in the RL branch is $\sqrt{R^2 + (\omega L)^2}$.
2. Current lags the applied voltage by an angle whose tangent is $\frac{\omega L}{R}$

Power and Energy

The instantaneous power delivered to the RL branch is obtained by:

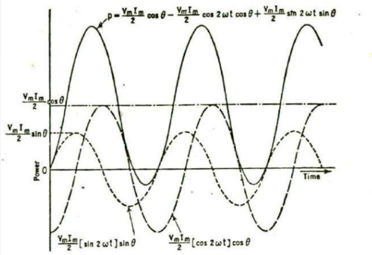
$p = vi$

$p = vi = V_m(sin\omega t + \theta) \times I_m sin(\omega t)$

$p = V_m I_m(sin\omega t \cos\theta + cos\omega t \sin\theta)$

$$p = V_m I_m \sin^2 \omega t \cos\theta + V_m I_m (\cos\omega t \sin\omega t) \sin\theta$$

$$p = \frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} [\cos 2\omega t] \cos\theta + \frac{V_m I_m}{2} [\sin 2\omega t] \sin\theta$$



the average value of the power is given by:

$$P_{av} = \frac{1}{T} \int_0^T V_m \sin(\omega t + \theta) I_m \sin\omega t . dt$$

$$\Rightarrow P_{av} = \int_0^T \left[\frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} (\cos 2\omega t) \cos\theta + \frac{V_m I_m}{2} (\sin 2\omega t) \sin\theta\right]$$

$$\Rightarrow P_{av} = \int_0^T \left[\frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} (\cos 2.2\pi f t) \cos\theta + \frac{V_m I_m}{2} (\sin 2.2\pi f t) \sin\theta\right]$$

$$\Rightarrow P_{av} = \int_0^T \left[\frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} (\cos 4\pi \frac{1}{T} t) \cos\theta + \frac{V_m I_m}{2} (\sin 4\pi \frac{1}{T} t) \sin\theta\right]$$

$$\Rightarrow P_{av} = \frac{V_m I_m}{2} cos\theta$$

Real Power and Reactive power

We already know:

$$p = \frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} [\cos 2\omega t] \cos\theta + \frac{V_m I_m}{2} [\sin 2\omega t] \sin\theta$$

In the equation above $\frac{V_m I_m}{2}\cos\theta$ - $\frac{V_m I_m}{2}[\cos 2\omega t]\cos\theta$ is known as REAL POWER and $\frac{V_m I_m}{2}[\sin 2\omega t]\sin\theta$ is known as REACTIVE POWER.

considering $\frac{V_m I_m}{2}[\cos 2\omega t]\cos\theta$ and $\sin 2\omega t = 0$ and 1 respectively we get:

Real Power $= \frac{V_m I_m}{2}\cos\theta$

Reactive power $= \frac{V_m I_m}{2}\sin\theta$

The 'RLC' Branch

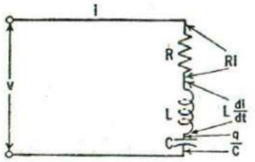


FIG. 19. The *RLC* branch.

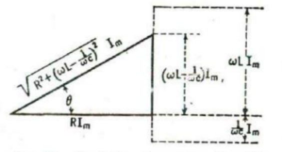
Here,

$v_R = Ri = RI_m \sin\omega t$ ———————(1)

$v_L = L\frac{di}{dt} = \omega L I_m \cos\omega t$ —————(2)

$v_C = \frac{1}{C} = \frac{\int I_m \sin\omega t dt}{C} = \frac{-I_m}{\omega C}\cos\omega t$———(3)

$$\text{So,} v = v_R + v_L + v_C$$
$$\text{So,} v = RI_m \sin \omega t + \omega L I_m \cos \omega t + \frac{-I_m}{\omega C} \cos \omega t$$
$$v = RI_m \sin \omega t + (\omega L - \frac{1}{\omega C}) I_m \cos \omega t$$



Impedance:

$$Z_{RLC} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \angle tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$$

Power and Energy

The instantaneous power delivered to the RLC branch is obtained by:

$$p = vi$$
$$p = vi = V_m(sin\omega t + \theta) \times I_m sin(\omega t)$$
$$p = V_m I_m(sin\omega t \cos\theta + cos\omega t \sin\theta)$$
$$p = V_m I_m \sin^2 \omega t \cos\theta + V_m I_m (\cos\omega t \sin\omega t) \sin\theta$$
$$p = \frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2}[\cos 2\omega t] \cos\theta + \frac{V_m I_m}{2}[\sin 2\omega t] \sin\theta$$

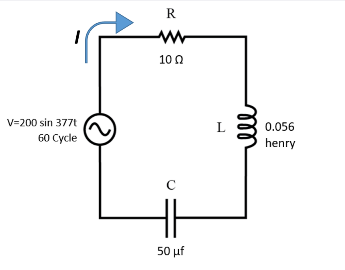
(The equation is similar to that obtained in case of R-L branch)

- In case of an RLC network as shown in the circuit proof the followings:

(a) $Z_{RLC} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \angle tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$

(b) $p = \frac{V_m I_m}{2} \cos\theta$ - $\frac{V_m I_m}{2} [\cos 2\omega t] \cos\theta + \frac{V_m I_m}{2} [\sin 2\omega t] \sin\theta$

Mathematical Problems 1



Find out the followings from the given RLC series network.

(1) Total impedance of the network.

(2) Expression for i

(3) Expression for instantaneous power delivered to the network.

(4) Average power delivered to the network.

Impedance:

$$Z_{RLC} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \angle tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow Z_{RLC} = \sqrt{10^2 + (377 \times .056 - \frac{10^6}{377 \times 50})^2} \angle tan^{-1} \frac{377 \times .056 - \frac{10^6}{377 \times 50}}{10}$$

$$\Rightarrow Z_{RLC} = 33.40 \angle tan^{-1} \frac{21.10 - 53.00}{10}$$

$$\Rightarrow Z_{RLC} = 33.40 \angle tan^{-1}(-3.19)$$

$$\Rightarrow Z_{RLC} = 33.40 \angle -72.60$$

The expression of i:

$$i = \frac{200 \sin 377t}{33.4 \angle -72.6}$$

$$\Rightarrow i = \frac{200 \sin 377t}{33.4 \angle -72.6}$$

$$\Rightarrow i = 5.988 \sin(377t + 72.60^{\circ})$$

Expression of instantaneous power delivered to the network:

Power, $p = \frac{V_m I_m}{2} \cos\theta$ - $\frac{V_m I_m}{2} [\cos 2\omega t] \cos\theta + \frac{V_m I_m}{2} [\sin 2\omega t] \sin\theta$

$\Rightarrow p = \frac{200 \times 5.988}{2} \cos 72.6$ - $\frac{200 \times 5.988}{2} [\cos(2 \times 377)t] \cos 72.6 + \frac{200 \times 5.988}{2} [\sin(2 \times 377)t] \sin 72.6$

$\Rightarrow p = 598.8(.299 - .299\cos 754t + .954\sin 754t)$

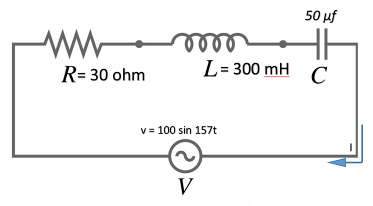
Average power power delivered to the network:

$p_{av} = \frac{V_m I_m}{2}\cos\theta$

$\Rightarrow p_{av} = \frac{200\times 5.988}{2}\cos 72.6$

$\Rightarrow p_{av} = 179.065W$

Mathematical Problems 2



(1) Total impedance of the network.

(2) Expression for i

(3) Expression for instantaneous power delivered to the network.

(4) Average power delivered to the network.

$Z_{RLC} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}\angle tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R}$

$\Rightarrow Z_{RLC} =$

$$\sqrt{30^2 + (157 \times 300 \times 10^{-3} - \frac{10^6}{157\times 50})^2} \angle tan^{-1} \frac{157\times 300\times 10^{-3} - \frac{10^6}{157\times 50}}{30}$$

$$\Rightarrow Z_{RLC} = \sqrt{900 + (-80)^2} \angle tan^{-1} \frac{-80}{30}$$

$$\Rightarrow Z_{RLC} = 85.71\angle -69.44$$

$$i = \frac{100 \sin 157t}{85.71\angle -69.44}$$

$$\Rightarrow i = 1.166 \sin(157t + 69.44^{\circ})$$

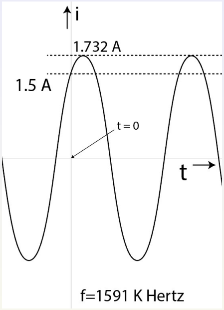
Average power power delivered to the network:

$$p_{av} = \frac{100\times 1.1667}{2} \cos 69.44$$

$$\Rightarrow p_{av} = 20.48$$

Mathematical Problem 3

From the following figure find out the followings: (a) The expression of the sine wave current, i. (b) The value of phase angle, θ.



The $t = 0$ reference is at a point where $\frac{di}{dt}$ is positive and $i = +1.5$ amperes.

i $= i_m \sin(\omega t + \theta)$

where, at t = 0, i = 1.5A, $i_{max} = 1.732$ and f = 1591 KHz

so, $1.5 = 1.732 \sin(2\pi \times 1591 \times 10^3 \times 0 + \theta)$

$= 1.732 \sin\theta$

so, $\sin\theta = \dfrac{1.5}{1.732} = 0.866$

$\theta = 60.0029 = 60^o$

Mathematical Problem 4

Express an alternating current of 10 amperes maximum value which has an angular velocity of 377 radians per second as a cosine function of time. Determine the frequency of this wave.

2.6 If we express the sine wave described in the question.

$$i = i_{max} \sin(2\pi ft + \theta)$$
$$= 10 \sin(2\pi \times ft + \theta) = 10 \sin(377t + \theta)$$

where, $\omega = 377$ rad/s and $i_{max} = 10$ amp

frequency $f = \dfrac{\omega}{2\pi} = \dfrac{377}{2\pi} = 60.0014 = 60$ Hz

Mathematical Problem 5

If $v = 100\sin(\omega t - 30^\circ)$ and $i = 10 sin(\omega t - 60^\circ)$, what is the angle of phase difference between the current and voltage waves? Show graphically which wave leads?

2.12

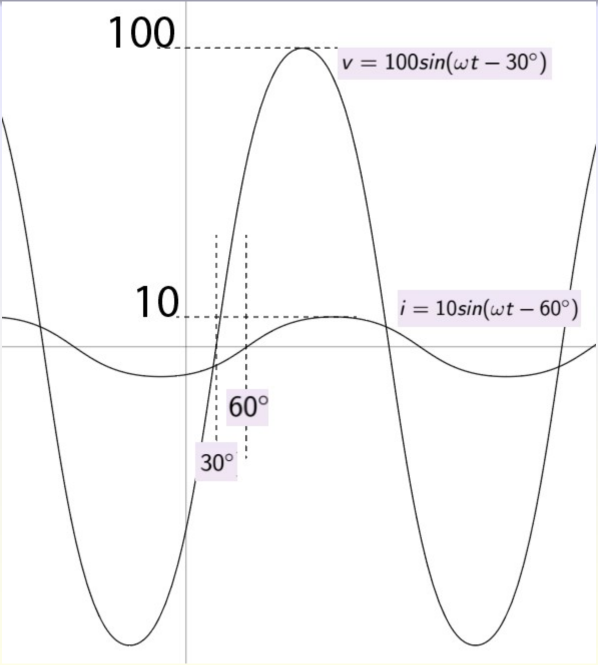
$$v = 100 \sin(\omega t - 30^\circ) \quad \text{phase} = \omega t - 30^\circ$$

and $i = 10 \sin(\omega t - 60^\circ)$ phase $= \omega t - 60^\circ$

$$\therefore \text{ phase difference} = \omega t - 30^\circ - \omega t + 60^\circ$$

$$= 30^\circ$$

So the voltage leads the current by 30°.


100
$v = 100sin(\omega t - 30°)$
10
$i = 10sin(\omega t - 60°)$
60°
30°

Mathematical Problem 6

A current $i = 5sin(110t + 30^{\circ})$ amperes flows in a purely resistive branch of 20 Ohms:

(a) Write the expression for v as a function of time employing numerical coefficients. (b) What is the frequency of the voltage variation? (c) Write the expression for p as a function of times employing numerical coefficients. (d) What is the frequency of the power variation? $i = 5\sin(110t + 30^{\circ})$ and $R = 20\Omega$

(a) $\mathrm{v} = 5\sin(110t + 30^{\circ} \times 20 = 100\sin(110t + 30^{\circ}$ v

(b) $f_v = \frac{110}{2\pi} = 17.5$ Hz

(c) $\mathrm{P} = \mathrm{vi} = 100\sin(110t + 30^{\circ}) \times 5\sin(110t + 30^{\circ})$

$= 500\sin^2(110t + 30^{\circ})$

$= 250 \times 2\sin^2(110t + 30^{\circ})$

$= 250 \times 1 - \cos 2(110t + 30^{\circ})$

$= 250 \times 1 - \cos(220t + 60^{\circ})$

$= 250 - 250\cos(220t + 60^{\circ})$

(d) $f_p = \frac{220}{2\pi} = 35$ Hz

Mathematical Problem 7

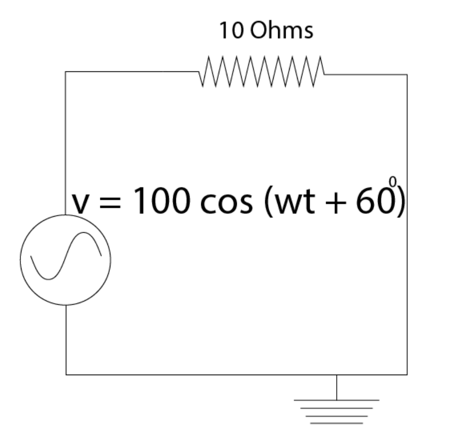
A voltage $v = 100cos(\omega t + 60^{\circ})$ volts is impressed upon a pure resistive circuit of 10 Ω as shown in the figure. (a) Write the equation with respect to time of the current wave and employ numerical coefficients.

(b) Find the equation with respect to time of the power wave.

(c) What is the maximum instantaneous power?

(d) What is the minimum instantaneous power?

(e) What, is the average value of the power wave



2.20 Given,

$v = 100 \cos(\omega t + 60^\circ)$, $R = 10\Omega$

(a) $i = \frac{100}{10} \cos(\omega t + 60^\circ) = 10 \cos(\omega t + 60^\circ)$

(b)
$$
\begin{aligned}
p = vi &= 100 \cos(\omega t + 60) \times 10 \cos(\omega t + 60^\circ) \\
&= 1000 \cos^2(\omega t + 60) \\
&= 500 \times 2 \cos^2(\omega t + 60) \\
&= 500 \times [1 + \cos 2(\omega t + 60)] \\
&= 500 + 500 \cos(2\omega t + 120) \text{ watts}
\end{aligned}
$$

(c) Maximum power, when $\cos(2\omega t + 120) = 1$

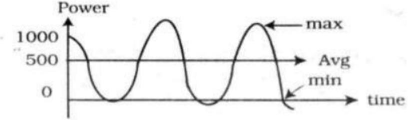
$\therefore$ $P_{max} = 500 + 500 = 1000$ watts

(d) Minimum power, when $\cos(2\omega t + 120°) = -1$

$P_{min} = 500 - 500 = 0$ watts.

(e) $P_{av} = \frac{v_m I_m}{2} \cos\theta = \frac{100 \times 10}{2} \cos\theta$ [where $\theta = 0°$ means phase shift $= 0$]

$= 500$ watts

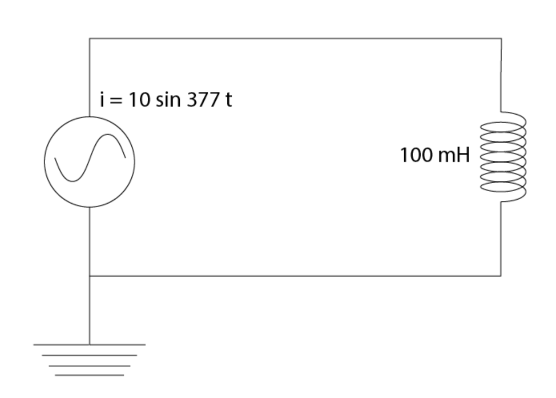


Mathematical Problem 8

For a 60Hz alternating current of sine form with maximum value of 10 amperes as shown in the figure find the followings: (a) The maximum time rate of change of current

(b) The maximum value of the voltage across the terminals of the inductance.

(c) The voltage equation.



(a) Given i $= I_{max} \sin \omega t = 10 \sin 2 \times \pi \times 60 \times t$
$= 10 \sin 377t$
Rate of change of current, $\frac{di}{dt} = \frac{d(10 \sin 377t)}{dt}$
$=10 \times 377 \cos 377t$
Maximum time rate of change current takes place when
$\cos 377t = 1$
So, $(\frac{di}{dt})_{max} = 10 \times 377 \times 1 = 3770$ A

(b)The inductive reactance, $X_L = \omega$L

$= 377 \times 100 \times 10^{-3} = 37.7\ \Omega$

So, $V_{max} = I_{max} \times X_L = 10 \times 37.7 = 377$ V

(c) The voltage equation is, v = $V_{max} \sin(\omega t + 90^{\circ})$

V $= 377 \sin(377t + 90^{\circ})$

Mathematical Problem 9

A voltage drop of v = 100 $sin(377t + 30^{\circ})$ volts is across a pure inductance of 0.02654 H.

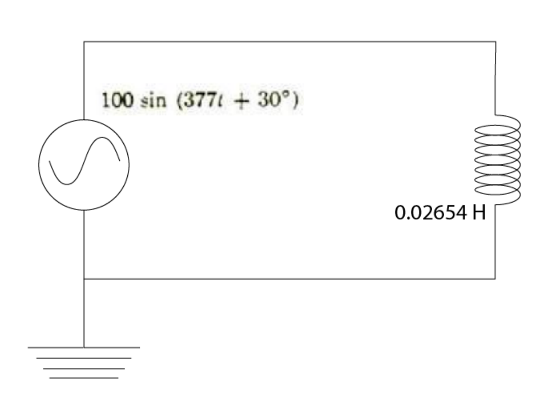
(a) Use numerical coefficient and express the current through the coil as a function of time.

(b) Find the equation with respect to time of the power wave. Express the result as a single sine function.

(c) What is the average power?

(d) What is the first value of time at which maximum energy is stored in the inductance?

(e) What is the maximum amount of energy stored in the inductance during a cycle?



$v = 100\sin(377t + 30^{\circ})$
$L = .025654H$
So, $X_L = \omega_L = 377 \times 0.02654 = 10\Omega$
(a) We know $v = iX_L$
So, $i = \frac{v}{x_L} = \frac{100}{10}\sin(377t + 30^{\circ} - 90^{\circ})$ .... (Current lags from voltage)
$= 10\sin(377t - 60^{\circ})$

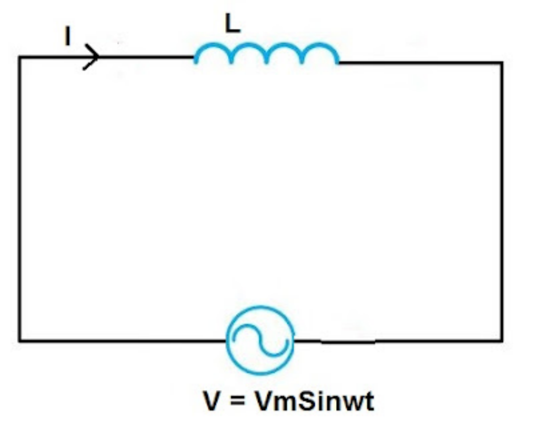
(b) $P = vi = 100\sin(377t + 30^{\circ}) \times 10\sin(377t - 60^{\circ})$
$= -1000\sin(377t + 30^{\circ})\cos(377t - 60^{\circ} + 90^{\circ})$
$= -1000\sin(377t + 30^{\circ})\cos(377t + 30^{\circ})$
$= -\frac{1000\sin(377t+30^{\circ})}{2}$ ... As $2\sin A\cos A = \sin 2A$
$= -500\sin(754t + 60^{\circ})$
(c) $P_{av} = \frac{V_m I_m}{2}\cos\theta = \frac{100\times10}{2}\cos 90^{\circ} = 0$ W
P = -500$\times$ 754 $\cos(754t + 60^{\circ})$
(d) Finding differentiation,
$\frac{dp}{dt} = -500 \times 754\cos(754t + 60^{\circ})$
Finding maximum value,
$\frac{dp}{dt} = 0$, we find ,
$\cos(754t + 60^{\circ}) = 0$
(e) Energy Stored $= \frac{V_m I_m}{2\omega} = \frac{100\times}{2\times377} = 1.326J$

Problem 10

A current of $5\sin 300t$ amperes flows through a pure inductive branch of 0.2 H.
(a) Find the impedance function and express numerically.
(b) How many joules are stored in the magnetic field about the

inductance when t = = 0.05 second ?

c) Write the expression for v as a function of time employing numerical coefficients.



Given $i = 5\sin 300t$

Impedance function for an Inductive circuit

$Z_L = \omega L\angle 90^{\circ}$

$= 300 \times 0.2\angle 90^{\circ}$

(b) $P = vi$

Where,

$$V = L\frac{di}{dt} = L\frac{d(5\sin 300t)}{dt} = .2 \times 5 \times 300 \times cos300t$$

$$= 300\cos 300t$$

so,

$$P = 300\cos 300t \times 5\sin 300t$$

$$= 1500\sin 300t\cos 300t$$

$$= 750\sin 600t$$

Stored Energy $= \int_0^{.05} 750\sin 600tdt$

$$= \frac{-750}{600}\cos\Big|_0^{0.05}$$

$$= \frac{-750}{600}(\frac{\sqrt{3}}{2} - 1)$$

$= 0.16750$ Joules.

(c) $v = V_m\cos 300t$

$$= 300\sin(300 - 90^{\circ})$$

Problem 11

A voltage $v = 100sin377t$ volts is impressed on a pure capacitance of 530.50 μf.

(a) Write the expression for i as a function of time employing numerical coefficients.
(b) Find the expression for the power wave as a function of time, employing numerical coefficients.
(c) How many joules are stored in the condenser when the current is zero and when the current is a maximum?
Here, $v = V_m \sin \omega t = 100 \sin 377t$
$c = 530.50\mu f$ and $\omega = 377$

$$X_c = \frac{10^6}{377 \times 530.50\mu} = 50\Omega$$

(a) $\mathrm{i} = \dfrac{V_m}{\frac{1}{\omega c}\sin(\omega t + 90^\circ)}$

$$\mathrm{i} = \frac{100}{\frac{1}{377 \times 530.50 \times 10^{-06}}} \sin(\omega t + 90^\circ) = \frac{100}{5} \sin(\omega t + 90^\circ)$$

$= 20 \sin(\omega t + 90^\circ)$
(b) $\mathrm{P} = vi = 100 \sin 377t \times 20 \sin(377t + 90^\circ)$
$= 2000 \sin 377t \times \sin(377t + 90^\circ)$
$= 2000(-\cos(377t + 90^\circ)) \times \sin(377t + 90^\circ)$

$$= \frac{2000}{2} 2(-\cos(377t + 90^{\circ})) \times \sin(377t + 90^{\circ})$$
$$= -1000 \sin(754t + 180^{\circ})$$
$$= -1000(\sin 754t . \cos 180 + \cos 754t . \sin 180^{\circ})$$
$$= 1000 \sin 754t$$

(c) When current is maximum, that is $I_m = 20$ A,

Energy stored in the condenser,

$$W_c = \frac{V_m I_m}{2\omega} = \frac{100 \times 20}{2 \times 377} = 2.65 \text{ Joules}$$

Verification of the result

Energy stored in the condenser,

$$W_c = \frac{C \times V_m^2}{2}$$
$$W_c = \frac{530 \times 10^{-6} \times 100^2}{2}$$
$$= \frac{530 \times 100^2}{2 \times 10^6}$$
$$= 2.65 \text{ Joules (Verified)}$$

When current is minimum, that is $I_m = 0$ A,

Energy stored in the condenser,

$$W_c = \frac{V_m I_m}{2\omega}$$
$$= \frac{100 \times 0}{2 \times 377}$$
$$= 0 \text{ Joules}$$

Problem 12

$R = 10$ ohms and $L = 0.05$ henry are connected in series and energized by a 25-cycle sinusoidal voltage, the maximum value of which is 150 volts.

(a) Find the complete impedance expression for the RL branch.

(b) Write the expression for the supply voltage as a function of time, making v = 0, $(\frac{dv}{dt} positive)$ at t=0.

(c) Write the expression for current as a function of time, assuming that the voltage in (b) is applied to the branch. Employ numerical coefficients.

(d) Write the expression for the instantaneous power delivered to the branch as a function of time. Express the result in three terms - a constant term, a single cosine term, and a single sine term. What is the average power?

(e) What are the real and reactive power?

Given, R $= 10\Omega$

$X_L = 2\pi fL$

$=2 \times \pi \times 25 \times .05$

$= 7.854\Omega$

$V_m = 150v$

(a) $X_{RL} = \sqrt{R^2 + X_L^2}\angle tan^{-1}\frac{7.854}{10}$

$= 12.71\angle 38.15^{\circ}$

(b) v $= 150\sin\omega t = 150\sin 157t$

(c) i $= \dfrac{V\sin\omega t}{X_{RL}}$

$= \dfrac{150\sin 157t}{12.71\angle 38.15^{\circ}}$

$= \dfrac{150\sin(157t - 38.15^{\circ})}{12.71}$

$= 11.8\ \sin(157t - 38^{\circ})$

(d) $p = \frac{V_m I_m}{2}\cos\theta$ - $\frac{V_m I_m}{2}[\cos 2\omega t]\cos\theta + \frac{V_m I_m}{2}[\sin 2\omega t]\sin\theta$

$p = \frac{150 \times 11.8}{2} \cos 38.15^\circ$ - $\frac{150 \times 11.8}{2} [\cos(2 \times 157t)] \cos 38.15^\circ$ + $\frac{150 \times 11.8}{2} [\sin(2 \times 157t)] \sin 38.15^\circ$

$p = 885 \cos 38.15^\circ$ - $885[\cos 314t] \cos 38.15^\circ$ + $885[\sin 314t] \sin 38.15^\circ$

$p = 695.96$ - $695.96[\cos 314t] + 546.68[\sin 314t]$ watts

$$P_{av} = \frac{V_m I_m}{2} \cos\theta$$
$$= \frac{150 \times 11.8}{2} \cos 38.15^\circ$$
$$= 695.96 \text{ Watts}$$

(e)Real Power $= \frac{V_m I_m}{2} \cos\theta$
$P_{Real} = \frac{150 \times 11.80}{2} \cos 38.15^\circ$
$= 695.96$ W
Reactive power $= \frac{V_m I_m}{2} \sin\theta$
$P_{reactive} = \frac{150 \times 11.80}{2} \sin 38.15^\circ$

$= 546.68$ W

Problem - 13

A resistive element of 30 Ω is connected in series with an inductance coil, the self-inductance of which is 50 mH and the ohmic resistance of which is 4.5 Ω. A voltage $v = 100 \cos 377t$ volts is connected to the series branch. Draw the circuit diagram find the followings

(a) Evaluate the expression for I.

(b) Evaluate the expression for P.

(c) Write the expression for the real power and reactive power as a function of time, employing numerical coefficients.

(d)What is the average value of the instantaneous power?

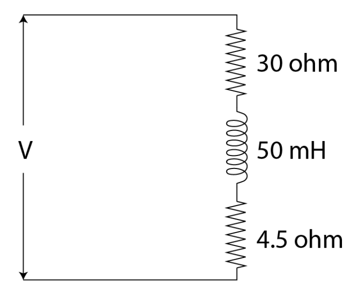


FIGURE: Circuit Diagram

Given, Total Resistance $R = 30 + 4.5 = 34.50\Omega$, Inductive Reactance $X_L = 377 \times 50 \times 10^{-3} = 18.85\Omega$

Impedance,

$Z_{RL} = \sqrt{(R)^2 + (X_{RL})^2}$

$Z_{RL} = \sqrt{(34.50R)^2 + (18.85)^2} \angle \tan^{-1} \frac{18.85}{34.50}$

$= 39.31\angle 28.65$

(a) $\mathrm{i} = \dfrac{V}{Z_{RL}}$

$$= \frac{100 \cos 377t}{39.31 \angle 28.65}$$
$$= 2.54 \cos(377t - 28.65)$$

(b) $p = \frac{V_m I_m}{2} \cos\theta$ - $\frac{V_m I_m}{2} [\cos 2\omega t] \cos\theta + \frac{V_m I_m}{2} [\sin 2\omega t] \sin\theta$

$p = \frac{100 \times 2.54}{2} \cos 28.65$ - $\frac{100 \times 2.54}{2} [\cos 2 \times 377t] \cos 28.65 + \frac{100 \times 2.54}{2} [\sin 2 \times 377t] \sin 28.65$

$p = 127 \cos 28.65$ - $127 [\cos 754t] \cos 28.65 + 127 [\sin 754t] \sin 28.65$

$p = 111.450$ - $111.450 [\cos 754t] + 60.89 \sin 754t$

Real Power $= 111.450$ - $111.450 [\cos 754t]$
Reactive Power $= 60.89 \sin 754t$

(d) Average Power, $P_{Average} = \dfrac{V_m I_m}{2} \cos\theta$

$$= \frac{100 \times 2.54}{2} \cos 28.65$$
$$= 111.450$$

Problem 14

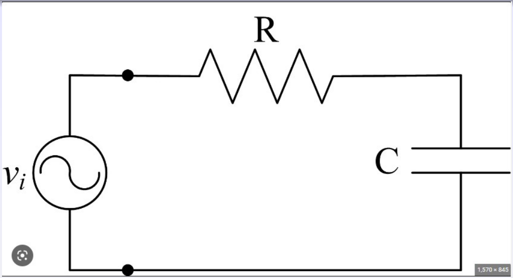
A resistive element of 151 Ω is connected in series with a capacitor of $4\mu f$ capacitance. A 500 cycle sinusoidal voltage, the maximum value of which is 15 volts, energizes the RC branch.

(a) Write the expression for the supply voltage, choosing the $t = 0$ reference at the point of maximum positive voltage.

(b) Evaluate Z_{RC} completely.

(c) Evaluate the expression for i.

(d) Evaluate the expression for p which corresponds to the product of voltage and current employed here, and express all trigonometric terms with exponents no higher than unity.



Capacitive reactance, $X_c = \dfrac{1}{\omega C}$

$= \dfrac{1}{2\pi fc} = \dfrac{1}{2\pi \times 500 \times 4 \times 10^{-6}}$

$= 79.577$

(a) $v = 15 \cos (2\pi \times 500t) = 15 \cos 3141.6\, t$

(b) $z_{RC} = \sqrt{151^2 + (-x_c)^2} \angle \tan^{-1} \frac{-X_c}{R} = 170.69 \angle -27.79°$

(c) $i = \frac{15}{170.69} \cos (319.6t + 27.79)$

$= 0.0879 \cos (314.6t + 27.79)$

(d) $P = vi = 15 \cos 3141.6t \times 0.0879 \cos (3141.6t + 27.79)$

$= 1.3185 \cos 3141.6t\, (0.8847 \cos 3141.6t - 0.466 \sin$
$v\, 3141.6t)$

$= 0.583 + 0.583 \cos 6282\, t - 0.307 \sin 6282t$

(a) $v = V_m \cos \omega t = 15 \cos 2\pi \times 500t = 15 \cos 3141.60t$

(b) $Z_{RC} = \sqrt{R^2 + (-\frac{1}{\omega C})^2} \angle tan^{-1} \frac{-\frac{1}{\omega C}}{R}$

$Z_{RC} = \sqrt{151^2 + (-79.577)^2} \angle \tan^{-1} \frac{-79.577}{151}$

$= 170.685 \angle - 27.788$

(c) $i = \frac{15 \cos 3141.60t}{170.685 \angle - 27.788}$

$= 0.08788 \cos(3141.60t + 27.788)$

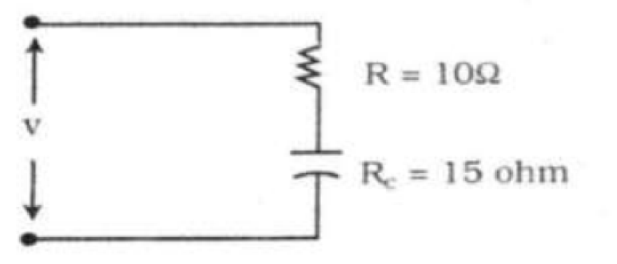
(d) $p = vi = (15 \cos 3141.60t)(0.08788 \cos(3141.60t + 27.788))$

$$= 1.3182(\cos 3141.60t)(\cos(3141.60t + 27.788))$$
$$= 1.3182 \cos 3141.60t[.8846 \cos 3141.60t - .4662 \sin 3141.60t]$$
$$=1.166 \cos^2 3141.60 - .6145 \cos 3141.60t \sin 3141.60t$$
$$= .583 + .583 \cos 6283.20 - .307 \sin 6283.2t$$

$[2 \sin \theta \cos \theta = \sin 2\theta$
and $2 \cos^2 \theta = 1 + \cos 2\theta]$

Problem 15

A resistance of 10 Ω is in series with a 303 μf capacitor. If the voltage drop across the capacitor is $150 \sin(220t - 60^\circ)$ volts, find the equation with respect to time of the voltage drop across the entire series circuit. Find also the expression for the current at any time t.



Capacitive Reactance, $X_c = \dfrac{1}{\omega c} = \dfrac{1}{2\pi fc}$

$\dfrac{1}{220 \times 303 \times 10^{-6}} = 15\Omega$

Impedance, $Z_{RC} = \sqrt{10^2 + (-15)^2} \angle \tan^{-1} \frac{-15}{10}$

$= 18.023 \times \angle - 56.309$

$V_c = 150 \sin(220t - 60^{\circ})$

$i = \frac{150 \sin(220t - 60^{\circ} + 90^{\circ})}{15}$

$i = 10 \sin(220t + 30^{\circ})$

Voltage across the entire circuit, $\mathrm{V} = Z_{RC} \times i$

$\mathrm{V} = 18.023 \times \angle - 56.309 \times 10 \sin(220t + 30^{\circ})$

$= 180.23\sin(220t - 26.309)$

Problem 16

Consider a series RLC branch wherein R $= 10\ \Omega$, L $= 0.10$ henry, and C is 200 μf. Assume that the current $i = 10\sin(157t)$ amperes flows through the RLC branch.

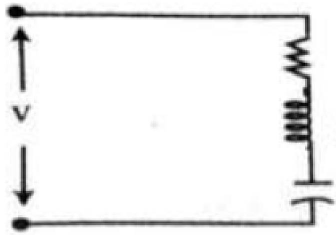
(a) Write the expression for the voltage drop across R, namely, Ri, employing numerical coefficients.

(b) Write the expression for the voltage drop across L, namely, $L = \dfrac{di}{dt}$ employing numerical coefficients.

(c) Write the expression for the voltage drop across C, namely, $\dfrac{q}{C}$ employing numerical coefficients.

(d) Add (a), (b), and (c) to find the voltage drop across the RLC branch. Express the result as a single sine function of time.

(e) What is the numerical value of the impedance of the series RLC branch?



(a) i $= I_m \sin \omega t$

$= 10 \sin 157t$

$V_R = iR$

$= 10 \sin 157t \times 10$

$= 10 \sin 157t$

(b) $V_L = L\dfrac{di}{dt}$

$= 0.01 \times \dfrac{d(10 \sin 157t)}{dt}$

$= 157 \cos 157t$

(c) $V_C = \dfrac{q}{C} = \int \dfrac{idt}{C}$

$$= \int \frac{(10 \sin 157t)dt}{200 \times 10^{-6}}$$

$$= 10 \frac{-\cos 157t}{157} \times \frac{10^6}{200}$$

$$= -318.5 \cos 157t$$

Total voltage drop,

(d) $V_{Drop} = V_R + V_L + V_C =$
$(100 \sin 157t) + (157 \cos 157t) + (-318.5 \cos 157t)$
$= 100 \sin 157t - 161.5 \cos 157t$

(e) $X_{RLC} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

$= \sqrt{10^2 + (157 \times .01 - \frac{10^6}{157 \times 200})^2}$

$= \sqrt{100 + (15.7 \times -31.85^2}$

$= 18.99 = 19\Omega$

Problem 17

Assume that the current $I = I_m \cos \omega t$ flows through a given RLC branch. Show that the voltage across the branch is:

$$v = I_m \left[R \sin \omega t + (\omega L - \frac{1}{\omega C}) \cos \omega t \right]$$

Here,

$v_R = Ri = RI_m \sin\omega t$ ———————(1)

$v_L = L\frac{di}{dt} = \omega L I_m \cos\omega t$ —————(2)

$v_C = \frac{1}{C} = \frac{\int I_m \sin\omega t dt}{C} = \frac{-I_m}{\omega C}\cos\omega t$———(3)

So, $v = v_R + v_L + v_C$

So, $v = RI_m \sin\omega t + \omega L I_m \cos\omega t + \frac{-I_m}{\omega C}\cos\omega t$

$v = RI_m \sin\omega t + (\omega L - \frac{1}{\omega C}) I_m \cos\omega t$

$$v = I_m\left[R\sin\omega t + (\omega L - \frac{1}{\omega C})\cos\omega t\right]$$

Problem 18

A resistive element of 20 ohms, an inductance coil of L = 300 millihenrys and $R_L = l0\ \Omega$, and a condenser of 50 μf capacitance are connected in series to form an RLC branch. A voltage $v = 100\sin 157t$ volts is applied to the RLC branch.

(a) What is the numerical value of Z_{RLC}?

(b) Write the expression for i, employing numerical coefficients.

(c) Write the expression for p, employing numerical coefficients

(d) What is the average value of the power delivered to the branch?

(e) What is the maximum value of the reactive volt. amperes?
(f) Write the expression for the voltage drop across the 20 Ω resistive element as a function of time, employing numerical coefficients.
(g) Write the expression for the instantaneous power delivered to the 20 Ω resistor as a function of time, employing numerical coefficients.

(a) $Z_{RLC} = \sqrt{(R)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tan^{-1} \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$

$= \sqrt{(20+10)^2 + \left(157 \times 300 \times 10^{-3} - \frac{1}{157 \times 50 \times 10^{-6}}\right)^2} \tan^{-1} \frac{-80.288}{30}$

$= \sqrt{900 + 6446.2488} \angle - 69.51$

$= 85.71 \angle - 69.51$

(b) $\mathrm{i} = \frac{V}{Z_{RLC}} = \frac{100 \angle - 90}{85.71 \angle - 69.51}$

$= 1.166 \angle - 20.49$

(c) $\mathrm{P} = vi$

$= 100\angle -90 \times 1.166\angle -20.49$

$= 116.67\angle -110.49$

(d) Average Power,

$$p_{av} = \frac{V_m I_m}{2} \cos\theta$$

$$p_{av} = \frac{100 \times 1.166}{2} \cos(-20.49)$$

$$p_{av} = 58.335 \times (-.9367)$$

$$p_{av} = -54.610$$

(e) Maximum Value of the reactive volt amperes,

$$p_{max-react} = \frac{V_m I_m}{2} \sin\omega t \sin\theta$$

$$= \frac{100 \times 1.166}{2} \sin 157t \sin(-20.49)$$

$$= 58.335 \sin 157t \times (-0.35)$$

$$= \text{-}20.41 \sin 157t$$

(f) $V_R = i \times R = 1.166\angle -20.49 \times 20$

$= 23.32\angle -20.49$

(g) $P_R = V_R \times i$

$= 23.32\angle -20.49 \times 1.166\angle -20.49 = 27.19\angle -40.98W$

Problem 19

A voltage $v = 282.80 \sin 500t$ volts is applied to a series circuit, and the resulting current is found to be $i = 5.656 \sin(500t - 36.87^{\circ})$ amperes. One element of this series combination is known to be a capacitor which has a capacitance of 200 μf. Determine the magnitudes of the other series elements present.

Here Given,

$v = 282.80 \sin 500t$

$i = 5.656 \sin(500t - 36.87^{\circ})$

From the impedance formula,

$$Z_{RLC} = \sqrt{R^2 + (X_L - X_C)^2} \tan^{-1} \frac{(X_L - X_C)}{R} = \frac{v}{i}$$

$$\sqrt{R^2 + (X_L - X_C)^2} \tan^{-1} \frac{(X_L - X_C)}{R} = \frac{282.80 \sin 500t}{5.656 \sin(500t - 36.87^{\circ})}$$

$$\sqrt{R^2 + (X_L - X_C)^2} \tan^{-1} \frac{(X_L - X_C)}{R} = 50\angle 36.87$$

So, $\sqrt{R^2 + (X_L - X_C)^2} = 50$ —(1)

and

$$\tan^{-1}\frac{(X_L - X_C)}{R} = 36.87 \text{———————} (2)$$

Solving (2)

$$\tan^{-1}\frac{(X_L - X_C)}{R} = 36.87$$

$$\frac{(X_L - X_C)}{R} = \tan 36.87$$

$$\frac{(X_L - X_C)}{R} = .75$$

$$\frac{(X_L - X_C)^2}{0.5625} = R^2 \text{———} (3)$$

Putting the value of R^2 in Eqn (1),

$$\frac{(X_L - X_C)^2}{0.5625} + (X_L - X_C)^2 = 2500$$

$$(X_L - X_C)^2 = \frac{2500}{2.7777}$$

$$(X_L - X_C)^2 = 900$$

$$(X_L - X_C) = 30 \text{————————-} (4)$$

WE also know,

$$X_C = \frac{1}{\omega C}$$

$$X_C = \frac{10^{-6}}{500 \times 200}$$
$= 10\Omega$ ——————————————(5)

Putting the value of $X_C =$ in equation (4)

$(X_L - 10) = 30$

$X_L = 40\ \Omega$

Putting the value of X_L and X_C in equation (1)

$R^2 + (X_L - X_C)^2 = 50^2$

$\Rightarrow R = \sqrt{50^2 - 30^2}$

$\Rightarrow R = \sqrt{50^2 - 30^2}$

$\Rightarrow R = \sqrt{50^2 - 30^2}$

$\Rightarrow R = 10\Omega$

So, $X_C = 10\Omega$, $X_L = 40\Omega$ and $R = 10\Omega$

IMPORTANT QUESTION

Explain Resonance in an RLC Circuit.

Resonance of an RLC circuit refers to the condition when the voltage across the inductor is the same as the voltage across the capacitor, or $V_L = V_C$. As a result, the EMF of the battery is entirely consumed by the resistor and the current achieves its maximum value.

Proof the in the state of Resonance in an RLC circuit, the resonance frequency,

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

As $V_L = V_C$

$$iX_L = iX_C$$

$$L\omega_R = \frac{1}{C\omega_R}$$

$$\omega_R = \frac{1}{\sqrt{LC}}$$

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

Ampere Value of Alternating Current.

An alternating current which produces heat in a given resistance at the same average rate as I amperes of direct current is said to have a value of I amperes. The average rate of heating produced by an alternating current during one cycle is,

$$\frac{1}{T}\int_0^T Ri^2dt$$

The average rate of producing heat by I amperes of direct current in the same resistance is RI^2. Hence by definition,

$$RI^2 = \frac{1}{T}\int_0^T Ri^2dt = \sqrt{Averagei^2} \text{——} (1)$$

The current given in equation (1) which defines the alternating current in terms of its average rate of producing heat in a resistance is called the root mean square (abbreviated RMS) value. It is also called the effective or virtual value. The graphical evaluation of the rms value of an alternating current is illustrated in the following fig.

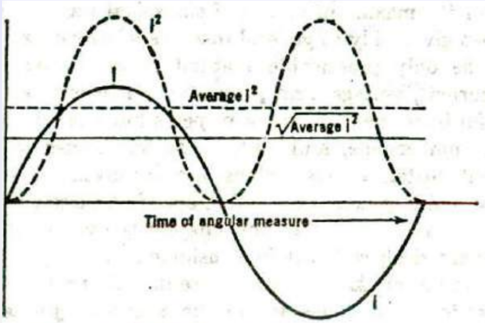


FIGURE: Graphical Representation of RMS value

Alternating Volt.

An alternating volt is the value of a wave of alternating potential which maintains an alternating current of 1 RMS ampere through a non-inductive resistance of 1 ohm. It therefore follows that the volt value of a wave is measured by the square root of the average square of the instantaneous values of the voltage wave.

Average Values. The average value of any ac wave which is symmetrical about the zero axis is zero. However, when average value is applied to alternating quantities, it usually means the average of either the positive or negative loop of the wave. This value represents the dc equivalent for electrolytic action of the alternating wave abcde, in the following Fig, if the wave were commutated (or rectified) and made the same as the wave abcfe. Since the average ordinate multiplied by the base is equal to the area under the curve, it follows directly that,

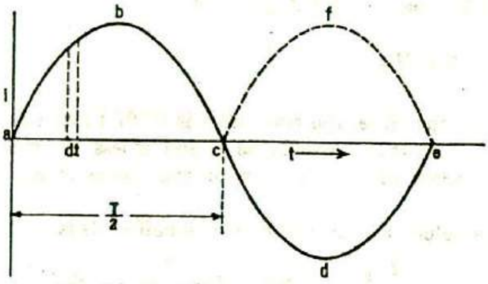


FIGURE: Rectified ac wave shown by dotted line

$$\text{Average Value} = \frac{2}{T}\int_0^{\frac{T}{2}} idt \text{ ——}(2)$$

Equation (2) is applicable only when the wave passes through zero at the time t = 0. For any other condition the time t_1 at which the instantaneous value of the wave is zero must be determined and the average value found from,

$$\text{Average Value} = \frac{2}{T}\int_{t_1}^{t_1+\frac{T}{2}} idt \text{ ——}(3)$$

RMS value of an alternating current

$$I_{(rms)} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \sqrt{average\ i^2}$$

Average value of an alternating current

$$I_{average} = \frac{2}{T}\int_0^{T/2} i dt$$

Effective and Average Values of it Sinusoid

We know, $i = I_m \sin \omega t$

and

$$I_{(rms)} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

$$\Rightarrow I_{(rms)}^2 = \frac{1}{T}\int_0^T (I_m \sin \omega t)^2 dt$$

$$\Rightarrow I_{(rms)}^2 = \frac{I_m^2}{2T}\int_0^T (2\sin^2 \omega t)^2 dt$$

$$\Rightarrow I_{(rms)}^2 = \frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 - \cos 2\omega t) dt$$

$$\Rightarrow I_{(rms)}^2 = \frac{I_m^2}{T}\int_0^T (\frac{1}{2} - \frac{1}{2}\cos 2\omega t) dt$$

$$\Rightarrow I_{(rms)}^2 = \frac{I_m^2}{T}[(\frac{t}{2} - \frac{1}{4\omega}\sin 2(\frac{2\pi}{T})t]_0^T = \frac{I_m^2}{2}$$

$$\Rightarrow I_{(rms)} = \frac{I_m}{\sqrt{2}} = .707 I_m$$

For a sine wave, therefore, the rms value is 0.707 times the maximum. In general, $I_{(rms)}$ is written simply as I, and unless otherwise specified the symbol I refers to the effective or rms value of an alternating current.

The average value of a sinusoid over one-half cycle is:

$$I_{average} = \frac{2}{T}\int_0^{T/2} idt$$

$$\Rightarrow I_{average} = \frac{2}{T}\int_0^{T/2} I_m \sin\omega t dt$$

$$\Rightarrow I_{average} = -\frac{2}{T}\frac{I_m}{\omega}[\cos\omega t]_0^{T/2}$$

$$\Rightarrow I_{average} = -\frac{2}{T}\frac{I_m}{2\pi f}[\cos(2\pi\frac{1}{T}\times\frac{T}{2}) - 1]$$

$$\Rightarrow I_{average} = -\frac{2}{T}\frac{I_m}{2\pi\frac{1}{T}}[\cos(2\pi\frac{1}{T}\times\frac{T}{2}) - 1]$$

$$\Rightarrow I_{average} = -\frac{I_m}{\pi}[\cos\pi - 1]$$

$$\Rightarrow I_{average} = -\frac{I_m}{\pi}[-1 - 1]$$

$$\Rightarrow I_{average} = -\frac{I_m}{\pi}[-2] = \frac{2}{\pi}I_m = .636 I_m$$

Form Factor (Sinusoidal wave)

$$\text{Form Factor} = \frac{\sqrt{\frac{1}{T}\int_0^T e^2 dt}}{\frac{2}{T}\int_0^2 e^2 dt} = \frac{I_{(rms)}}{I_{(average)}} = \frac{.707 I_m}{.636 I_m} = 1.11$$

1. Form factor does give some indication of the relative hysteresis loss that will exist when a voltage is impressed on a coil wound on an iron core.

2. Also some use is made of form factor in determining effective voltages induced in such coils when a known non-sinusoidal flux wave is present in the iron core.

Crest Factor (Sinusoidal wave)

Crest Factor $= \frac{E_m}{.707E_m} = \sqrt{2} = 1.414$

The crest, peak, or amplitude factor is the ratio of the maximum value of a voltage wave to the effective value.

TRIANGULAR WAVE

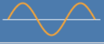
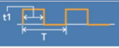
Form Factor (Triangular Wave) $= \frac{\sqrt{\frac{1}{T}\int_0^T e^2 dt}}{\frac{2}{T}\int_0^2 e^2 dt} = \frac{I_{(rms)}}{I_{(average)}} = \frac{.577I_m}{.5I_m} =$ 1.154

Crest Factor $= \frac{E_m}{.577E_m} = \sqrt{3} = 1.732$

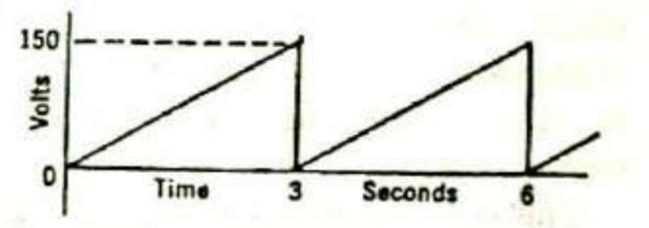
Summary (Form Factor and Crest Factor of Sinusoidal and Triangular Wave)

1. For a pure sinusoidal waveform the Form Factor will always be equal to 1.11
2. For a pure sinusoidal waveform the Crest Factor will always be equal to 1.414
3. For a pure triangular waveform the Form Factor will always be equal to 1.154
4. For a pure triangular waveform the Crest Factor will always be equal to 1.732

Form Factor and Crest Factors of Different kinds of waveform

Waveform		Effective value (RMS value)	Mean value	Waveform factor	Crest factor
DC		1	1	1	1
Sine wave		$\frac{1}{\sqrt{2}} \fallingdotseq 0.707$	$\frac{2}{\pi} \fallingdotseq 0.637$	$\frac{\pi}{2\sqrt{2}} \fallingdotseq 1.11$	$\sqrt{2} \fallingdotseq 1.414$
Full wave rectified sine wave		$\frac{1}{\sqrt{2}} \fallingdotseq 0.707$	$\frac{2}{\pi} \fallingdotseq 0.637$	$\frac{\pi}{2\sqrt{2}} \fallingdotseq 1.11$	$\sqrt{2} \fallingdotseq 1.414$
Half-wave rectified sine wave		$\frac{1}{2} = 0.5$	$\frac{1}{\pi} \fallingdotseq 0.318$	$\frac{\pi}{2} \fallingdotseq 1.571$	2
Triangle wave		$\frac{1}{\sqrt{3}} \fallingdotseq 0.577$	$\frac{1}{2} \fallingdotseq 0.5$	$\frac{2}{\sqrt{3}} \fallingdotseq 1.154$	$\sqrt{3} \fallingdotseq 1.732$
Sawtooth wave		$\frac{1}{\sqrt{3}} \fallingdotseq 0.577$	$\frac{1}{2} \fallingdotseq 0.5$	$\frac{2}{\sqrt{3}} \fallingdotseq 1.154$	$\sqrt{3} \fallingdotseq 1.732$
Square wave		1	1	1	1
PWM signal		$\sqrt{\frac{t1}{T}}$	$\frac{t1}{T}$	$\sqrt{\frac{T}{t1}}$	$\sqrt{\frac{T}{t1}}$

For the figure given find out the $V_{average}$, form factor and crest factor:



Average value $(V_{average}) = \frac{1}{2} \times V_{max} = 75$ V

Form Factor $= \frac{V_{(rms)}}{V_{(average)}} = \frac{\frac{V_{max}}{\sqrt{3}}}{\frac{V_{max}}{2}} = \frac{\frac{150}{\sqrt{3}}}{\frac{150}{2}} = 1.1547$

and

Crest Factor $= \frac{V_{(max)}}{V_{(rms)}} = \frac{V_{(max)}}{\frac{V_{max}}{\sqrt{3}}} = \sqrt{3} = 1.732$

The sine wave of current $I = I_m \sin \omega t$ is shown in Fig. All the ordinates of this wave at the various times t may be represented by the projection of the revolving vector OA on the vertical axis of the Fig. This projection is $I_m \sin \omega t$ if OA has a magnitude of I_m. This is the equation of the wave shown in the figure.

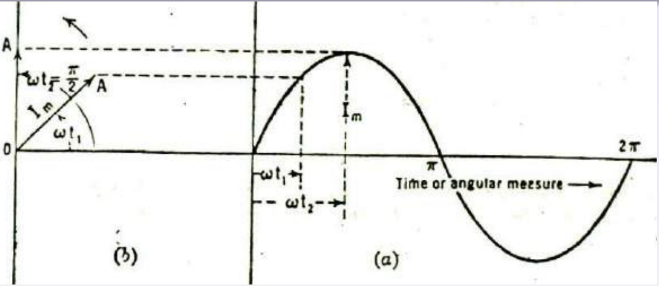


FIGURE: Vector representation of sine wave

PROBLEM 1

Find out the resultant current by adding the following currents as waves and as vectors:

$i_1 = 5\sin\omega t$

$i_2 = 10\sin(\omega t + 60^\circ)$

As waves:

Sum $= i_0 = i_1 + i_2 = 5\sin\omega t + 10\sin(\omega t + 60^\circ)$

$= 5\sin\omega t + 10\sin\omega t cos 60^\circ + 10\cos\omega t sin 60^\circ$

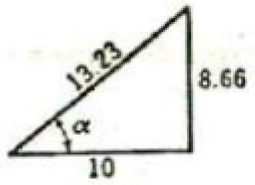
$= 5\sin\omega t + \frac{10}{2} \times \sin\omega t + \frac{10}{\sqrt{3}} \times \cos\omega t$

$= 10\sin\omega t + 8.66\cos\omega t$

If the previous equation is multiplied and divided b y 13.23:

$i_0 = 13.23[\frac{10}{13.23}\sin\omega t + \frac{8.66}{13.23}\cos\omega t]$ Consider the following figure

The above equation becomes:



(a)

$i_0 = 13.23[\frac{10}{13.23}\sin\omega t + \frac{8.66}{13.23}\cos\omega t]$

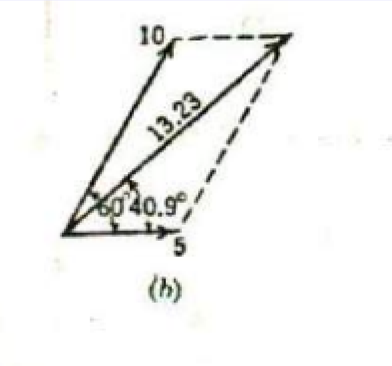
$i_0 = 13.23[\cos\alpha\sin\omega t + \sin\alpha\cos\omega t]$

$i_0 = 13.23[\sin(\omega t + alpha)$

$i_0 = 13.23[\sin(\omega t + 40.89^{\circ})$

where the value of $\alpha = cos^{-10/13.23} = sin^{-8.66/13.23} = 40.89^{\circ}$

As vectors



$\sum x = 5 \sin \omega t = 10$
$\sum y = 10 \sin(\omega t + 60^{\circ}) = 8.66$
Sum $= \sqrt{x^2 + y^2} = \sqrt{10^2 + 8.66^2} = 13.23$
$\alpha = tan^{-1} \frac{\sum y}{\sum x} = \tan^{-1} \frac{8.66}{10} = 40.89^{\circ}$
$i_0 = 13.23[\sin(\omega t + 40.89^{\circ})$

PROBLEM 2

Draw the vector representation and find the resultant vector corresponding to points 'A' and 'B' of the sine waves given in the following figure.

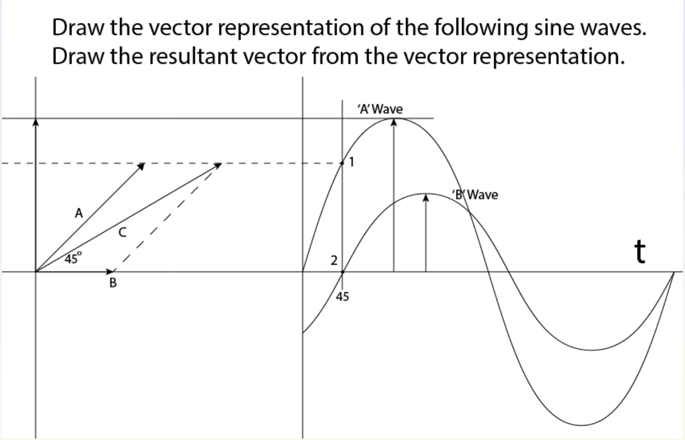
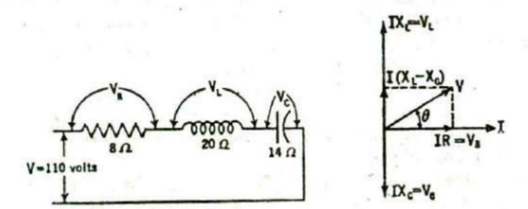


FIGURE: Vector representation of SIne Wave

PROBLEM 3



Page 100 (ALTERNATING-CURRENT CIRCUITS, Russell M Kerchner, George F Corcoran)

calculate current, power, power factor, vars, reactive factor, volt-amperes, voltage drop across each circuit element. Solution:

$X_L = 2\pi fL = 2\pi 60 \times 0.053 = 20\Omega$

$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi 60 \times 180.70 \times 10^{-06}} = 14\Omega$

$X = X_L - X_C = (20 - 14)\Omega = 6\Omega$

$R = 8\Omega$

$Z = \sqrt{R^2 + X^2} = \sqrt{8^2 + 6^2} = 10\Omega$

$I = \frac{110}{10} = 11 Amperes$

$P.f = \cos\theta = \frac{IR}{V} = \frac{IR}{IZ} = \frac{R}{Z} = \frac{8}{10} = 0.80$

$P = I^2 R = 11^2 \times 8 = 968\Omega$

$Reactiveva = V\sin\theta = 110 \times 11 \times \frac{XI}{ZI} = 110 \times 11 \times \frac{6}{10} = 726vars$

$va = VI = 110 \times 11 = 1210vars =$

$\sqrt{REAL\ POWER^2 + REACTIVE\ POWER^2} = \sqrt{968^2 + 726^2}$

$V_R = IR = 11 \times 8 = 88$ V

$V_L = IX_L = 11 \times 20 = 220$ V

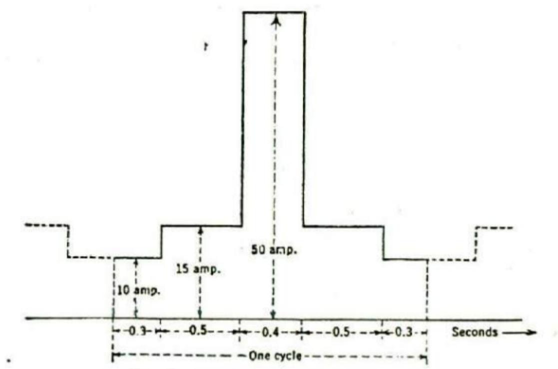
$V_C = IX_C = 11 \times 14 = 154$ V

PROBLEM 4

Page 103 (ALTERNATING-CURRENT CIRCUITS, Russell M Kerchner, George F Corcoran)

(a) What is the average value of the pulsating current shown in the following figure?

(b) What is the effective value?



Time period $= 0.3 + 0.5 + 0.4 + 0.5 + 0.3 = 2$ sec

Average Current.

$$I_{avg} = \frac{1}{2}\ [10 \times 0.03 + 15 \times 0.05 + 50 \times 0.04 + 15 \times 0.05 + 10 \times 0.03]$$

$$\Rightarrow I_{avg} = 20 Amps]$$

$$I_{avg} = \sqrt{\frac{1}{2}[10^2 \times 0.03 + 15^2 \times 0.05 + 50^2 \times 0.04 + 15^2 \times 0.05 + 10^2 \times 0.03]}$$

$$\Rightarrow I_{avg} = 25.35 Amps]$$

Problem 5

A current in a circuit starts at zero and increases Linearly until a value of 12 amperes is attained. It then drops to zero in negligible time and repeats the cycle. What will an effective reading a-c ammeter in this circuit read?

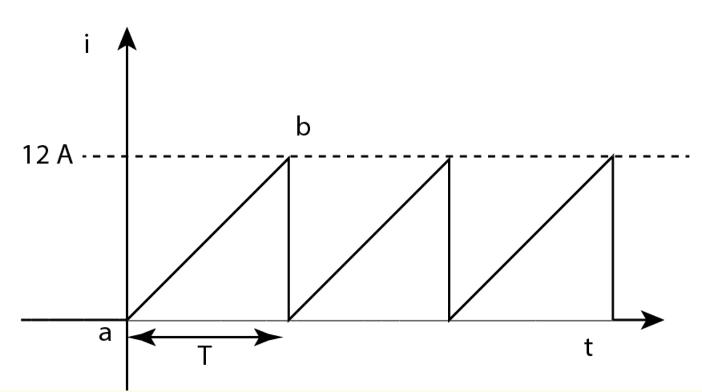


FIGURE: CHAPTER 3 PROBLEM 14

$$\mathrm{I} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

$$\Rightarrow I = \sqrt{\frac{1}{T}\int_0^T \left(\frac{12}{T}t\right)^2 dt}$$

$$\Rightarrow I = \sqrt{\frac{1}{T}\int_0^T \left(\frac{144}{T^2}t\right) dt}$$

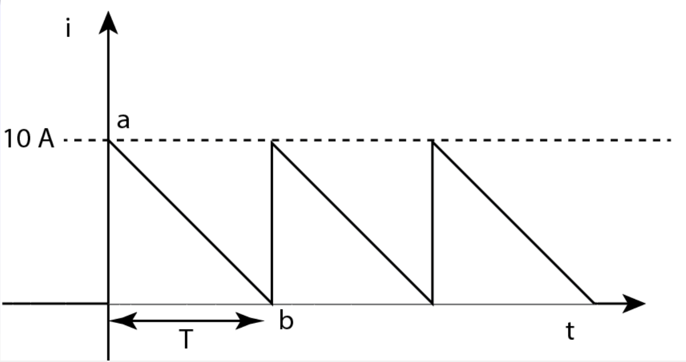
$$\Rightarrow I = \sqrt{\frac{1}{T} \times \frac{144}{T^2} \times \frac{t^3}{3}\Big|_0^T}$$

$$\Rightarrow I = \sqrt{\frac{1}{T} \times \frac{144}{T^2} \times \frac{T^3}{3}}$$

$$\Rightarrow I = 6.93A$$

Problem 6

A current starts abruptly at 10 amperes and decreases linearly to zero and then repeats this cycle. Find the rms value without changing the orientation of the wave from that given.



$$I_{rms} = \sqrt{\frac{1}{T} i^2 dt}$$

$$\Rightarrow I_{rms} = \sqrt{\frac{1}{T}\left(10 - \frac{10t}{T}\right)^2 dt}$$

$$\Rightarrow I_{rms} = \sqrt{\frac{1}{T}\int_0^T (100 - 2 \times 10 \times \frac{10t}{T}) + \frac{100t^2}{T^2} dt}$$

$$\Rightarrow = \sqrt{\frac{1}{T}(100t - \frac{200t^2}{2T} + \frac{100t^3}{3T^2})}\Big|_0^T$$

$$\Rightarrow = \sqrt{\frac{1}{T}(100T - 100T + \frac{100T}{3})}$$

$$\Rightarrow = \sqrt{\frac{100}{3}} = 5.77A$$

Problem 7

Calculate the form factor of the current, wave in the figure below.

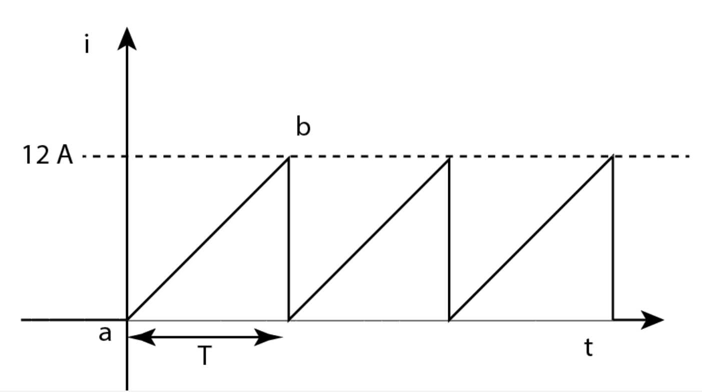


FIGURE: CHAPTER 3 PROBLEM 14

We know,

Form Factor $= \dfrac{I_{rms}}{I_{average}}$

Here, $I_{rms} = \sqrt{\dfrac{1}{T}\int_0^T i^2 dt}$

$$\Rightarrow I = \sqrt{\frac{1}{T}\int_0^T \left(\frac{12}{T}t\right)^2 dt}$$

$$\Rightarrow I_{rms} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{144}{T^2}t\right) dt}$$

$$\Rightarrow I_{rms} = \sqrt{\frac{1}{T} \times \frac{144}{T^2} \times \frac{t^3}{3}\Big|_0^T}$$

$$\Rightarrow I_{rms} = \sqrt{\frac{1}{T} \times \frac{144}{T^2} \times \frac{T^3}{3}}$$

$$\Rightarrow I = 6.93A$$

$$I_{avg} = \frac{1}{T}\int_0^T idt$$

$$\Rightarrow I_{avg} = \frac{1}{T}\int_0^T \left(\frac{12}{T}t\right) dt$$

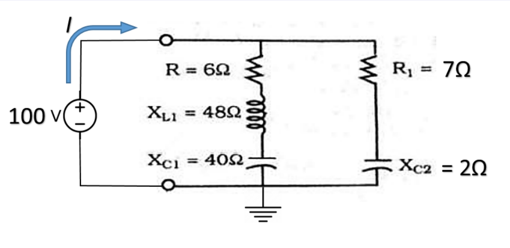
$$\Rightarrow I_{avg} = \frac{1}{T}\left(\frac{12t^2}{2T}\right)\Big|_0^T$$

$$\Rightarrow I_{avg} = \frac{1}{T^2}\left(6t^2\right)\Big|_0^T$$

$$= 6$$

So form factor $= \dfrac{6.93}{6} = 1.55$ (Ans)

PROBLEM



Page 105 (ALTERNATING-CURRENT CIRCUITS, Russell M Kerchner, George F Corcoran)

1. Find the current delivered to the combination.
2. Calculate the total power.
3. Calculate power consumed by each branch.

$Z_1 = 6 + j(48 - 40) = 10\angle 53.13$

$Z_2 = 7 - j2 = 7.28\angle{-15.95}$

$$Z_{circuit} = \frac{Z_1 \times Z_2}{Z_1 + Z_2} = \frac{\angle 37.18}{14.32\angle 24.78}$$

$$Current = \frac{100}{5.08} = 19.88$$

$$Total\ Power = VI\cos\theta = 100 \times 10 \times \cos 12.40 = 1,922.09W$$

$$I_1 = \frac{V}{Z_1} = \frac{100}{10} = 10A$$

$$P_1 = VI_1\cos\theta = 100 \times 10\cos 53.13 = 600W$$

$$I_2 = \frac{V}{Z_2} = \frac{100}{7.28} = 13.74A$$

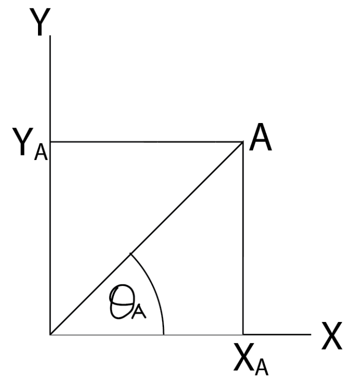
$$P_2 = VI_1\cos\theta = 100 \times 13.74\cos(-15.95) = 1321.10W$$

Chapter - IV

Phasor Algebra (as Applied to A-C Circuit Analysis)

The Phasor A

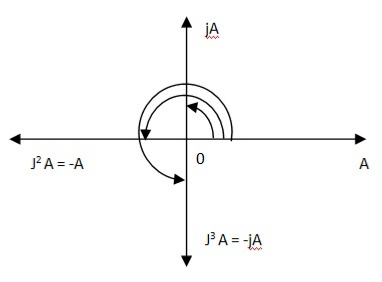
The Phasor A is expressed as $A = \sqrt{x_A^2 + x_B^2}$



here, $\theta_A = \tan^{-1} \frac{y_A}{x_A}$

The Operator j

The value of $j = \sqrt{-1}$



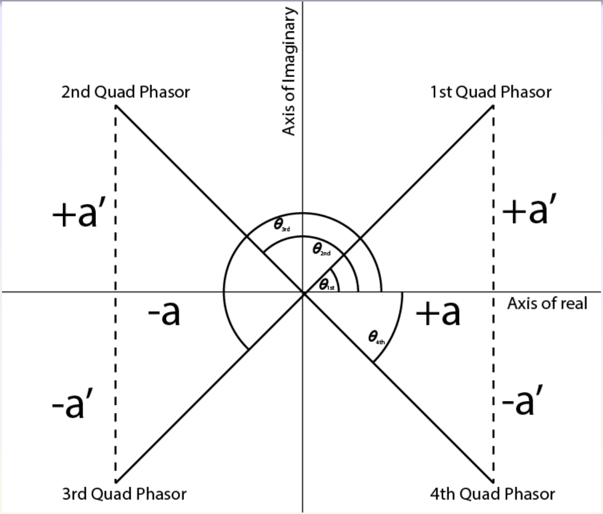
The Cartesian Form of Notation

A phasor in any quadrant can be completely specified in a Cartesian or rectangular form of notation, as shown below:

$A = \pm a + \pm a^{'}$

where a is the x-axis projection and a' is the y-axis projection of the phasor. In any case the magnitude of the phasor A is:

$A = \sqrt{a^2 + a'^2}$



$$\theta_{1st} = \tan^{-1}\frac{(+a^{'})}{(+a)}$$

$$\theta_{2nd} = \tan^{-1}\frac{(a^{'})}{(-a)}$$

$$\theta_{3rd} = \tan^{-1}\frac{(-a^{'})}{(-a)}$$

$\theta_{4th} = \tan^{-1}\frac{(-a^{'})}{(+a)}$

The Operator $(\cos\theta \pm j\sin\theta)$

$A = A(\cos\theta \pm j\sin\theta) = \pm a + \pm a^{'}$

The plus sign is used if θ is measured counter-clockwise from the reference axis, the minus sign if θ is measured clockwise.

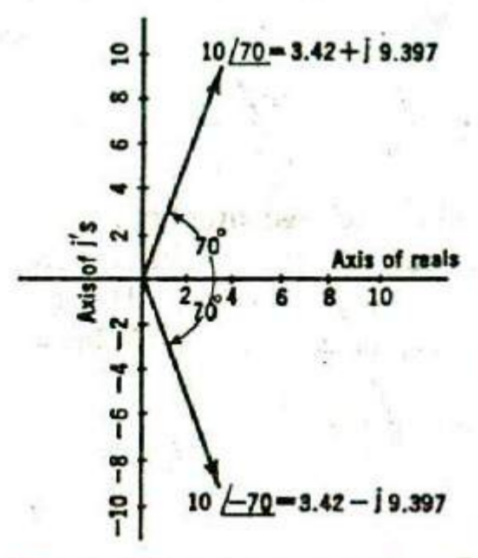
Exponential Form of the Operator $(cos\theta \pm jsin\theta) = e^{\pm j\theta}$

<u>All Forms</u>

$Ae^{j\theta} = A\angle\theta = (cos\theta + jsin\theta)$

$Ae^{-j\theta} = A\angle -\theta = (cos\theta - jsin\theta)$

Exponential form, Polar form, Cartesian form/Rectangular form



$$10\angle 70 = 10e^{j70} = 10(cos70 + jsin70) = 3.42 + j9.39$$

Problem 1. Write the equivalent polar form of the phasor $3 + j4$ where the numbers refer to unit lengths. Illustrate the phasor by means of a diagram.

Ans.: $5\epsilon^{j53.1°} = 5\angle 53.1°$.

Problem 2. A phasor is given in the form of $10\epsilon^{-j120°}$. Write the symbolic polar and cartesian forms of the phasor, and illustrate, by means of a phasor diagram, the magnitude and phase position of the phasor.

Ans.: $10\angle -120° = -5 - j8.66$.

Phasor Addition

Example. Let it be required to add

$$\mathbf{A} = 10\angle 36.9° = 8 + j6 \quad \text{and} \quad \mathbf{B} = 6\angle 120° = -3 + j5.20$$

$$\mathbf{A} + \mathbf{B} = \mathbf{C} = (8 - 3) + j\,(6 + 5.2)$$

$$\mathbf{C} = 5 + j11.2$$

The magnitude of the **C** phasor is

$$C = \sqrt{5^2 + 11.2^2} = 12.27 \text{ units}$$

The position of the phasor **C** with respect to the $+x$-axis is

$$\theta_C = \tan^{-1}\frac{11.2}{5} = \tan^{-1} 2.24 = 65.95°$$

Phasor Subtraction

$$\mathbf{A} = 30\angle 60^\circ = 30\ (\cos 60^\circ + j\sin 60^\circ) = 15 + j26$$

$$\mathbf{B} = 21\ (\cos 160^\circ - j\sin 160^\circ) = -19.75 - j7.18$$

$$\mathbf{A} - \mathbf{B} = (15 + j26) - (-19.75 - j7.18)$$

$$= 34.75 + j33.18 = 48\angle 43.6^\circ$$

Phasor Multiplication

$$\mathbf{AB} = 2\epsilon^{j40^\circ} \cdot 3\epsilon^{j100^\circ} = 6\epsilon^{j(40^\circ+100^\circ)} = 6\epsilon^{j140^\circ}$$

$$\mathbf{AB} = 2 \times 3\angle 40^\circ + 100^\circ = 6\angle 140^\circ$$

Example. Given the phasors:

$$\mathbf{A} = 2\,(\cos 40^\circ + j\sin 40^\circ) = 1.532 + j1.286$$

$$\mathbf{B} = 3\,(\cos 100^\circ + j\sin 100^\circ) = -0.521 + j2.954$$

let it be required to find the product of **A** and **B** by the algebraic multiplication of the cartesian forms.

$$\begin{aligned}
\mathbf{F} = \mathbf{AB} &= (1.532 + j1.286)\,(-0.521 + j2.954) \\
&= -0.799 + j4.525 - j0.670 + j^2 3.798 \\
&= (-0.799 - 3.798) + j(-0.670 + 4.525) \\
&= -4.597 + j3.855
\end{aligned}$$

$$\begin{aligned}
\mathbf{F} &= \sqrt{-4.597^2 + 3.855^2}\;\Big/\tan^{-1}\frac{3.855}{-4.597} \\
&= 6.0\,\underline{/180^\circ - 40^\circ} = 6\,\underline{/140^\circ}
\end{aligned}$$

Phasor Division

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{A\,\underline{/\alpha_A}}{B\,\underline{/\alpha_B}} = \frac{A}{B}\,\underline{/\alpha_A - \alpha_B} \tag{40a}$$

Examples. The processes of division in two particular cases are shown below:

$$\mathbf{F} = \frac{\mathbf{A}}{\mathbf{B}} = \frac{20\,\underline{/60^\circ}}{5\,\underline{/30^\circ}} = 4\,\underline{/30^\circ}$$

$$\mathbf{G} = \frac{\mathbf{C}}{\mathbf{D}} = \frac{12\epsilon^{j90^\circ}}{4\epsilon^{-j30^\circ}} = 3\epsilon^{j120^\circ}$$

Example. If $\mathbf{A} = 10 + j17.3$ and $\mathbf{B} = 4.33 + j2.5$, let it be required to find $\mathbf{A}/\mathbf{B}$ by the method given in equations (41) and (42).

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{10 + j17.3}{4.33 + j2.5} = \frac{(10 + j17.3)\,(4.33 - j2.5)}{(4.33 + j2.5)\,(4.33 - j2.5)}$$

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{(43.3 + 43.3) + j(75 - 25)}{4.33^2 + 2.5^2}$$

Reduced to polar form

$$\frac{\mathbf{A}}{\mathbf{B}} = \sqrt{3.465^2 + 2.0^2}\,\underline{/\tan^{-1}\frac{2.0}{3.465}} = 4.0\,\underline{/30^\circ}$$

Reduced to polar form

$$\frac{\mathbf{A}}{\mathbf{B}} = \sqrt{3.465^2 + 2.0^2} \angle \tan^{-1}\frac{2.0}{3.465} = 4.0 \angle 30^\circ$$

Raising a Phasor to a Given Power

$$\mathbf{a} = 1\angle 120^\circ$$

$$\mathbf{a}^2 = 1\angle 240^\circ$$

$$\mathbf{a}^3 = 1\angle 360^\circ = 1\angle 0^\circ$$

$$\mathbf{a}^4 = 1\angle 480^\circ = 1\angle 120^\circ$$

$$A = \sqrt{3.08^2 + 8.455^2} \angle \tan^{-1}\frac{8.455}{3.08}$$

The first root is $\sqrt{9.0} \angle \frac{70^\circ}{2} = 3\angle 35^\circ$.

The second root is $\sqrt{9.0} \angle \frac{70^\circ + 360^\circ}{2} = 3\angle 215^\circ$.

Extracting the Roots of a Phasor

$$A = \sqrt{3.08^2 + 8.455^2} \angle \tan^{-1}\frac{8.455}{3.08}$$

The first root is $\sqrt{9.0} \angle \frac{70^\circ}{2} = 3\angle 35^\circ$.

The second root is $\sqrt{9.0} \angle \frac{70^\circ + 360^\circ}{2} = 3\angle 215^\circ$.

Impedance Expressed in Polar Form

$$\mathbf{Z} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \Big/ \tan^{-1} \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

The abbreviated form is

$$\mathbf{Z} = Z\underline{/\theta}$$

Where θ represents a lead of the voltage with respect to the current or a lag of the current with respect to the voltage. If a phasor voltage V $= V\angle\alpha$ is applied to the above branch the resulting current is

$i = \frac{V}{I} = \frac{V\angle\alpha}{Z\angle\theta} = [\frac{v}{Z}]\angle\alpha - \theta$

Problem

An RLC series branch consists of R $=$ 12.9 ohms, L $=$ 0.056 henry, and C $=$ 78 μf

(a) What is the complex impedance of the RLC branch at 60 cycles

(b) If a 60-cycle current, I$=$ 10$\angle$30 deg amperes, flows through the branch, find the voltage phasor V across the terminals of the series branch.

(a) $Z = R + j(X_L - X_C)$

$\Rightarrow R + j\left(2\pi fL - \dfrac{1}{2\pi f \times C}\right)$

$\Rightarrow 12.90 + j\left(2 \times \pi \times 60 \times 0.056 - \dfrac{10^6}{2 \times \pi \times 60 \times 78}\right)$

$\Rightarrow 12.9 + j(21.11 - 34.00) =$

$\sqrt{12.9^2 + (21.11 - 34.00)^2} \tan^{-1} \dfrac{21.11 - 34.00}{12.9}$

$=18.24\angle - 45^{\circ}$

(b) $V = Zi = 18.24\angle - 45^{\circ} \times 10\angle 30^{d} egree$

$= 182.40\angle - 15^{\circ}$

Problem

Express the following as a single complex number in Cartesian, polar forms and exponential form:

$$= \frac{24 + j36}{5\angle(-39^{\circ})} \times \frac{e^{j45^{\circ}}}{\cos 39 + j\sin 39}$$

$$= \frac{\sqrt{24^2 + 36^2} \tan^{-1} \dfrac{36}{24}}{5e^{-j39}} \times \frac{e^{j45^{\circ}}}{e^{j39}}$$

$$
\begin{aligned}
&= \frac{\sqrt{576 + 1296}\angle 56.309}{5e^{-j39}} \times \frac{e^{j45^\circ}}{e^{j39}} \\
&= \frac{43.266\angle 56.309}{5e^{-j39}} \times \frac{e^{j45^\circ}}{e^{j39}} \\
&= \frac{43.266e^{56.309}}{5e^{-j39}} \times \frac{e^{j45^\circ}}{e^{j39}} \\
&= 8.6533e^{j(56.309+39+45-39)} \\
&= 8.6533e^{j101.309} \text{ (Exponential Format)} \\
&= 8.6533\angle 101.309 \text{ (Polar Format)} \\
&= 8.6533 \times (\cos 101.309 + j \sin 101.309) \\
&= 8.6533(-0.197 + j0.98) \\
&= -1.697 + j8.4852 \text{ (Cartesian or Rectangular format)}
\end{aligned}
$$

Impedance in Series

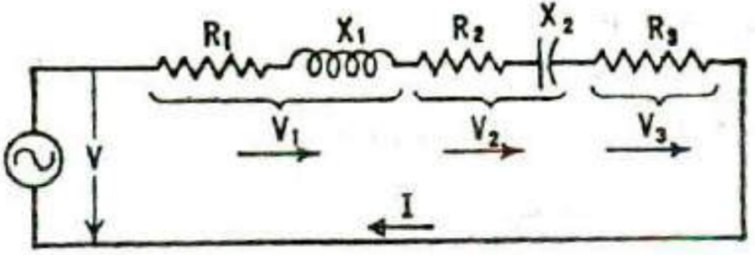


FIG. 1. Impedances in series.

$$V = V_1 + V_2 + V_3$$

$$V = IZ_1 + IZ_2 + IZ_3$$

$$V = I(Z_1 + Z_2 + Z_3) = IZ$$

$$Z = Z_1 + Z_2 + Z_3 = (R_1 + jX_1) + (R_2 + jX_2) + (R_3 + j0)$$
$$Z = (R_1 + R_2 + R_3) + j(X_1 + X_2) = R + jX$$

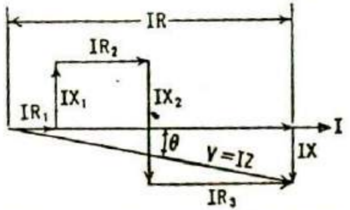


FIG. 2. Funicular or string vector diagram of circuit in Fig. 1.

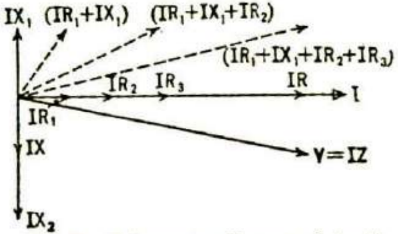


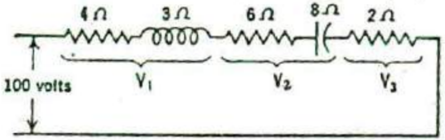
FIG. 3. Polar vector diagram of circuit in Fig. 1.

$$Z = \sqrt{(R_1 + R_2 + R_3 + \cdots + R_n)^2 + (X_1 + X_2 + X_3 + \cdots + X_n)^2} \Big/ \tan^{-1}\frac{X_1 + X_2 + X_3 + \cdots + X_n}{R_1 + R_2 + R_3 + \cdots + R_n} \qquad (7)$$

$$\text{Power factor} = \cos\theta = \frac{IR}{IZ} = \frac{R}{Z}$$

$$= \frac{R_1 + R_2 + R_3 + \cdots + R_n}{\sqrt{(R_1 + R_2 + R_3 + \cdots + R_n)^2 + (X_1 + X_2 + X_3 + \cdots + X_n)^2}}$$

Example 1. Calculate the current, voltage drops V_1, V_2, and V_3, power consumed by each impedance, and the total power taken by the circuit with the constants shown in Fig. 4. The impressed voltage will be taken along the reference axis.



$$I = \frac{V}{Z} = \frac{100 + j0}{4 + j3 + 6 - j8 + 2} = \frac{100\,(12 + j5)}{(12 - j5)\,(12 + j5)} = 7.1 + j2.96 \text{ amperes}$$

$$V_1 = IZ_1 = (7.1 + j2.96)\,(4 + j3) = 19.53 + j33.14 \text{ volts}$$
$$V_2 = IZ_2 = (7.1 + j2.96)\,(6 - j8) = 66.27 - j39.06 \text{ volts}$$
$$V_3 = IZ_3 = (7.1 + j2.96)\,(2 + j0) = 14.2 + j5.92 \text{ volts}$$

Check: $V = 100 + j0$ volts

Note that the drops are added *vectorially* to check the impressed voltage.

$$P_1 = RI^2 = 4(\sqrt{7.1^2 + 2.96^2})^2 = 4 \times 7.69^2 = 237 \text{ watts}$$
$$P_2 = 6 \times 7.69^2 = 355 \text{ watts}$$
$$P_3 = 2 \times 7.69^2 = 118 \text{ watts}$$
$$\text{Total power} = 710 \text{ watts}$$

Difference between String vector diagram and polar vector diagram

The distinguishing characteristic of a string vector diagram is that certain component vectors are combined head-to-tail to form a resultant vector as, for example, the component voltages IR_1, IX_1, IR_2, IX_2 *and* IR_3 are combined head-to-tail to form the resultant voltage vector V.

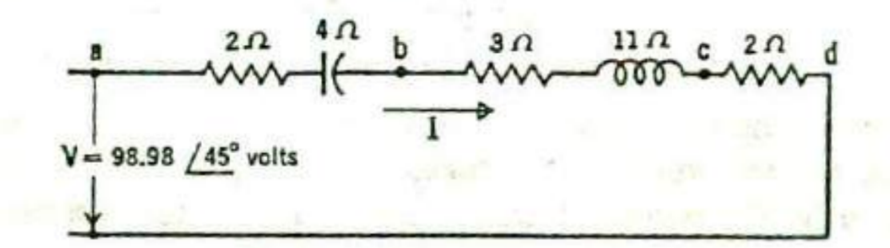
In a polar vector diagram, all vectors are started from a common origin.Either type of diagram may be used since they represent the same thing.

The one which appears to be the simpler in any particular case should be used. In certain cases the funicular diagram shows the quantities to better advantage, whereas for others the polar diagram is more suggestive of the relationships and more convenient to use.

Problem 1.

(a) Find the current through the circuit in Fig Below and the voltage drops V_{ab}, V_{bc}, Vcd.

(b) Draw a string vector and polar diagram of V_{ab}, V_{bc}, Vcd , including both V and I on the diagram.



(c) Calculate the total power dissipated applying the following formulas

(a) Here,

$V = ZI$

$V = [(2 - j4) + (3 + j11) + (2 + j0)]I$

$\Rightarrow 98.98\angle 45 = [(7 + j7)]I$

$\Rightarrow \dfrac{98.98\angle 45}{9.89\angle 45} = I$

$\Rightarrow \mathrm{I} = 10\angle 0$

$V_{ab} = IZ_1 = (2 - j4) \times 10\angle 0 = 20 - j40 = I\sqrt{R_1^2 - X_1^2}\tan^{-1}\dfrac{X_1}{R_1}$

$= 44.721\angle - 63.434$

$V_{bc} = IZ_2 = (3 + j11) \times 10\angle 0 = 30 + j110 =$
$I\sqrt{R_2^2 + X_2^2} \tan^{-1} \dfrac{X_2}{R_2} = 114.017\angle 74.74$

$V_{cd} = IZ_3 = (2 + j0) \times 10\angle 0 = 20 + j0 = I\sqrt{R_3^2 - X_3^2} \tan^{-1} \dfrac{X_3}{R_3}$
$= 20\angle 0$

(b) Here,

$IR_1 = 20$

$IR_2 = 30$

$IR_3 = 20$

$IX_1 = -j40$

$IX_2 = j110$

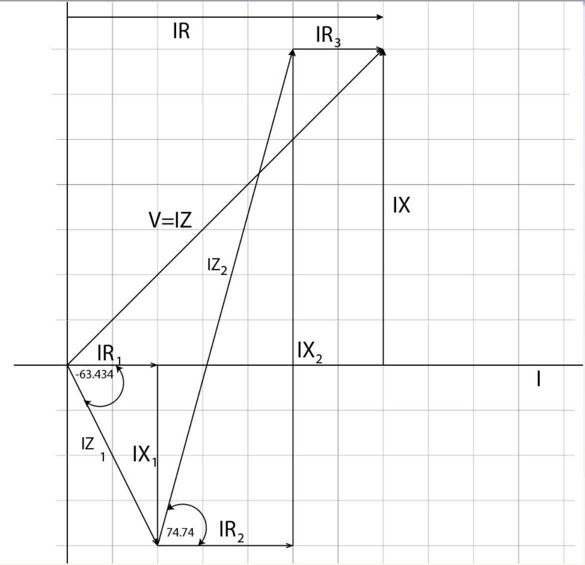


FIGURE: String Vector Diagram

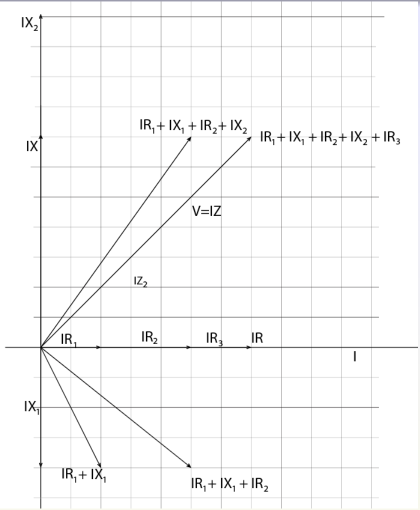


FIGURE: Polar Vector Diagram

(b) Total Power dissipated,
$P = VI = I^2R = VI\cos\theta = vi + v^{'}i'$

Here V $= 98.98\angle 45 = 69.989 + j69.989$

$\Rightarrow 10^2(7) = 700$ Watt

$$\Rightarrow 98.98\angle 45 \times 10\angle 0 \cos\angle 45 = \frac{989.80}{\sqrt{2}} = 699.89$$

$$\Rightarrow vi + v^{'}i^{'} = 69.989 \times 10 + 69.989 \times 0 = 699.89$$

SERIES RESONANCE

A series circuit containing R, L, and C is in resonance when the resultant reluctance is zero

In case of a series resonance $X_L = X_C$

So $2\pi fL = \dfrac{1}{2\pi fC}$

$\Rightarrow = f_m = \frac{1}{2\pi\sqrt{LC}}$

Here $f_m =$ Series resonance frequency

The vector diagram corresponding to a series circuit in resonance is as follows:

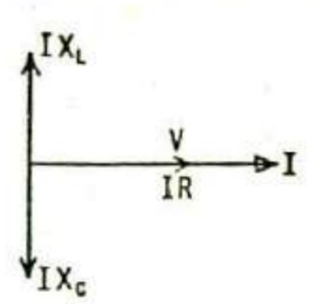


FIGURE: Vector diagram of Series Resonance

Series resonance can be produced in a series circuit by varying either L, C, or f. The current is always given by:

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{V}{\sqrt{R^2+(2\pi f_L-\frac{1}{2\pi f_C})^2}}$$

For any value of current the drop across the resistance, inductance and capacitance is respectively:

$$V_R = \text{IR} = \frac{VR}{\sqrt{R^2+(2\pi f_L-\frac{1}{2\pi f_C})^2}}$$

$V_L = \mathrm{IL} = \frac{VL}{\sqrt{R^2+(2\pi f_L - \frac{1}{2\pi f_C})^2}}$

$V_C = \mathrm{IC} = \frac{VC}{\sqrt{R^2+(2\pi f_L - \frac{1}{2\pi f_C})^2}}$

For instance in all cases the power factor at resonance is 1. The power is simply the impressed voltage times the current.

The current is $\frac{V}{R}$, the maximum possible value for the resistance which is in the circuit.

Effect of resistance on series resonance

In the following diagram resonance occurs at the point C. The current at the resonant point C will be large if the resistance is small.

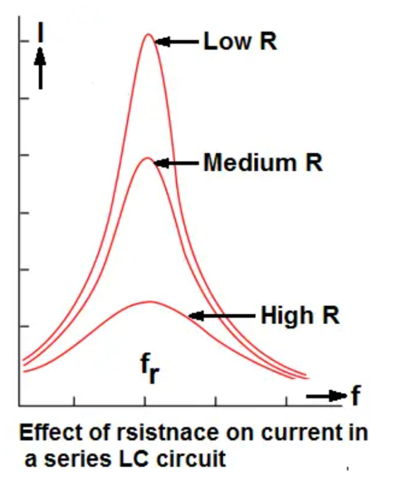


FIGURE: Effect of resistance on a series resonant circuit

- When the resultant reactance is large as it is at point A there will be only a small current flowing. Hence there is a rapid rise in current from point A to point C.

- Conversely,when the resistance is large, the amount of the change in current from point A to C will be small.
- In the former case the current peak will be sharper than in the latter, as illustrated in Fig.
- Hence the small resistance is said to give sharp tuning and the large resistance broad tuning.

Illustrate the effect of varying 'L' in a series RLC circuit and proof that maximum V_L occurs after resonance when .

$$X_L = \frac{R^2 + X_C^2}{X_C}$$

Answer

When L is varied to produce resonance, a series of curves shown in the following Fig is obtained.. It will be noted that V_C becomes maximum at resonance whereas the maximum value of V_L occurs after resonance. (WHY?)

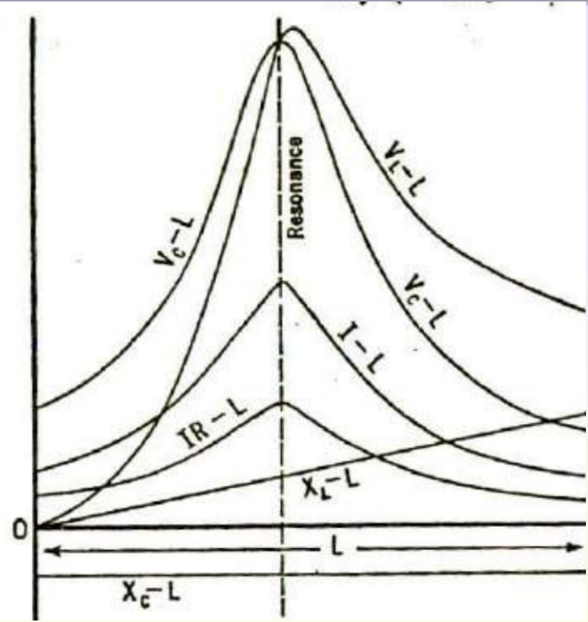


FIGURE: Series resonance by varying L

- Since $V_C = IX_C$ and X_C is constant, the maximum drop across the condenser will occur when the current is a maximum.
- In the case of $V_L = IX_L$, both I and X_L are increasing before resonance and the product must be increasing.
- At resonance, I is not changing but XL is increasing, and hence the drop is increasing.
- The drop continues to increase until the reduction in the current offsets the increase in X_L. The point at which V_L becomes maximum is calculated in the following manner.

We already know in case of resonance,
$V_L = \mathrm{IL} = \frac{VL}{\sqrt{R^2+(2\pi f_L - \frac{1}{2\pi f_C})^2}}$
Differential the above equation w.r.t X_L and equation it equal to $=$ 0, we get.

$$\frac{dV_L}{dX_L} = \frac{d\left(\frac{VL}{\sqrt{R^2+(X_L - \frac{1}{2\pi f_C})^2}}\right)}{dX_L} = 0$$

$$\Rightarrow \frac{d\left(\frac{VL}{\sqrt{R^2+(X_L-X_C)^2}}\right)}{dX_L} = 0$$

$$\Rightarrow$$

$$\frac{\sqrt{R^2+(X_L-X_C)^2}V - VX_L\frac{1}{2}(R^2+(X_L-X_C)^2)^{-\frac{1}{2}}2(X_L-X_C)}{R^2+(X_L-X_C)^2}$$

$$= 0$$

Solving the above equation yields :

$$X_L = \frac{R^2+X_C^2}{X_C} \text{ (Proved)}$$

and in case of resonance we already know,

$$X_L = X_C = 2\pi fL = \frac{1}{2\pi fC}$$

So,

$$L = \frac{1}{2\pi f}\left(\frac{R^2+X_C^2}{X_C}\right) = C(R^2+X_C^2)$$

Example 2. As L is varied to produce resonance in a series circuit containing $R = 100$ ohms, $X_C = 200$ ohms, and $f = 60$ cycles, find the voltage drop across L at resonance and also when the drop across L is a maximum if 1000 volts are impressed.

Conductance, Suceptance and admittance

When impedances are connected in parallel, as in Fig. the same voltage V is impressed across each impedance.

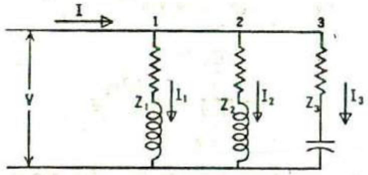


Fig. 21. Impedances in parallel.

The current in each impedance is therefore

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_1}, \quad \mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z}_2}, \quad \text{and} \quad \mathbf{I}_3 = \frac{\mathbf{V}}{\mathbf{Z}_3}$$

From Kirchhoff's current law,

$$\begin{aligned} \mathbf{I} &= \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 \\ &= \frac{\mathbf{V}}{\mathbf{Z}_1} + \frac{\mathbf{V}}{\mathbf{Z}_2} + \frac{\mathbf{V}}{\mathbf{Z}_3} = \mathbf{V}\left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right) \\ &= \mathbf{V}(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) = \mathbf{V}\mathbf{Y}_0 \end{aligned}$$

where the symbol Y represents the reciprocal of impedance and is called admittance

$$Y = \frac{1}{R_s + jX_s} = \frac{1}{R_p} + \frac{1}{jX_p} = \frac{1}{R_P} - j\frac{1}{X_P} = g - jb$$

(Here g = conductance and b = suseptance)

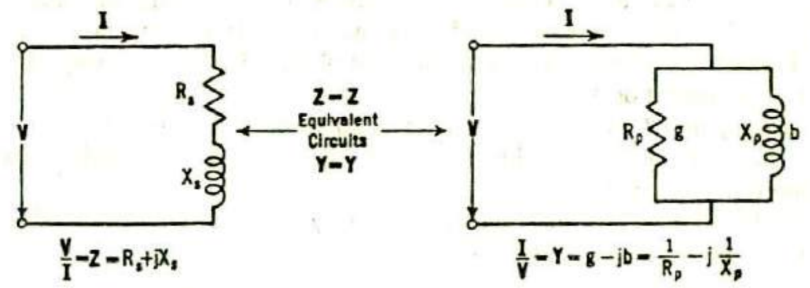


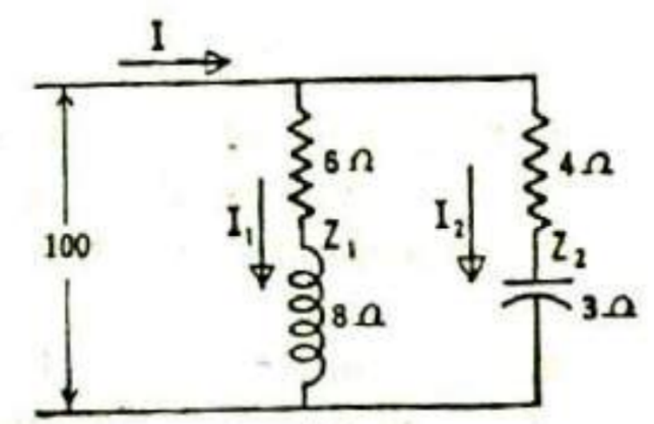
FIG. 22. The parallel equivalent of a series impedance, $R_s + jX_s$.

Problem

For the circuit of Fig. with the parameters shown, the followings are desired:

(a) conductance and susceptance of each branch;

(b) the resultant conductance and susceptance;



$$I_1 = \frac{100 + j0}{6 + j8} = 6 - j8 = 10\underline{/-53.2^\circ} \text{ amperes}$$

$$I_2 = \frac{100 + j0}{4 - j3} = 16 + j12 = 20\underline{/36.9^\circ} \text{ amperes}$$

$$I = I_1 + I_2 = 22 + j4 = 22.35\underline{/10.3^\circ} \text{ amperes}$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{(6 + j8)}\frac{(6 - j8)}{(6 - j8)} = 0.06 - j0.08 \text{ mho}$$

From which $g_1 = 0.06mho$ $b_1 = 0.08mho$

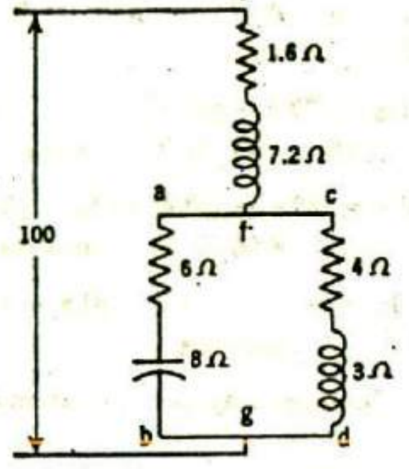
$$Y_2 = \frac{1}{Z_2} = \frac{1}{(4 - j3)}\frac{4 + j3}{(4 + j3)} = 0.16 + j0.12 \text{ mho}$$

$$g_2 = 0.16 \text{ mho}, \quad b_2 = -0.12 \text{ mho}$$

$$g = g_1 + g_2 = 0.06 + 0.16 = 0.22 \text{ mho}$$
$$b = b_1 + b_2 = 0.08 - 0.12 = -0.04 \text{ mho}$$

Problem

Calculate current, power, and power factor for each impedance shown in Fig., and the total current and power and the power factor of the whole combination.



$$Y_{ab} = \frac{1}{6 - j8} = 0.06 + j0.08 \text{ mho}$$

$$Y_{cd} = \frac{1}{4 + j3} = 0.16 - j0.12 \text{ mho}$$

$$Y_{fg} = Y_{ab} + Y_{cd} = 0.22 - j0.04 \text{ mho}$$

$$Z_{fg} = \frac{1}{Y_{fg}} = \frac{1}{(0.22 - j0.04)} \frac{(0.22 + j0.04)}{(0.22 + j0.04)} = 4.4 + j0.8 \text{ ohms}$$

$$Z_{eg} = Z_{ef} + Z_{fg} = 1.6 + j7.2 + 4.4 + j0.8 = 6 + j3 \text{ ohms}$$

$$I = \frac{100\angle 0^\circ}{6 + j8} = 6 - j8 = 10\angle -53.2^\circ \text{ amperes}$$

$$P = vi + v'i' = 6 \times 100 + 0 \times 8 = 600 \text{ watts}$$

$$\text{P.f.} = \frac{600}{100 \times 10} = 0.6 \quad \text{or} \quad \frac{R}{Z} = \frac{6}{10} = 0.6 \text{ lag}$$

Power factor $= \cos\theta = \cos -53.2 = .599 \quad .600(Lagging)$

$$V_{ef} = I_{ef}Z_{ef} = (6 - j8)(1.6 + j7.2) = 67.2 + j30.4$$
$$= 73.8\angle 24.4^\circ \text{ volts}$$

$$V_{fo} = V - I_{ef}Z_{ef} = 100 - 67.2 - j30.4 = 32.8 - j30.4$$
$$= 44.7\angle -42.8^\circ \text{ volts}$$

$$\mathbf{I}_{ab} = \mathbf{V}_{fg}\mathbf{Y}_{ab} = (32.8 - j30.4)\,(0.06 + j0.08)$$
$$= 4.4 + j0.8 = 4.48\,\underline{/10.3^\circ} \text{ amperes}$$

$$\mathbf{I}_{cd} = \mathbf{V}_{fg}\mathbf{Y}_{cd} = (32.8 - j30.4)\,(0.16 - j0.12)$$
$$= 1.6 - j8.8 = 8.95\,\underline{/-79.7^\circ} \text{ amperes}$$

$$\mathbf{I}_{cd} = \mathbf{I} - \mathbf{I}_{ab} = 6 - j8 - 4.4 - j0.8 = 1.6 - j8.8$$
$$= 8.95\,\underline{/-79.7^\circ} \text{ amperes}$$

The powers in the various branches may now be determined in terms of principles previously considered.

$$P_{ab} = vi + v'i' = (32.8)\,(4.4) + (-30.4)\,(0.8)$$
$$= 144.32 - 24.32 = 120 \text{ watts}$$

$$P_{cd} = (32.8)\,(1.6) + (-30.4)\,(-8.8)$$
$$= 52.48 + 267.52 = 320 \text{ watts}$$

$$P_{ef} = (67.2)\,(6) + (30.4)\,(-8) = 403.2 - 243.2 = 160 \text{ watts}$$

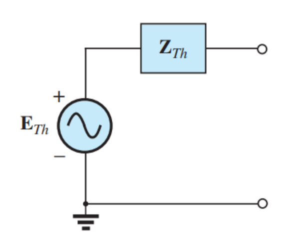
or $$P_{ef} = I^2r = (6^2 + 8^2)\,(1.6) = 160 \text{ watts}$$

$$P_{eg} = 100 \times 6 = 600 \text{ watts}$$

Check: $P = P_{ab} + P_{cd} + P_{ef} = 120 + 320 + 160 = 600$ watts

Thévenin's Theorem

Thévenin's theorem, as stated for sinusoidal ac circuits, is changed only to include the term impedance instead of resistance; that is, any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series, as shown in Fig.



Procedure

- Remove that portion of the network across which the

Thévenin equivalent circuit is to be found.

- Mark the terminals of the remaining two-terminal network.
- Calculate Z_{Th} by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.
- Calculate E_{Th} by first replacing the voltage and current sources and then finding the open-circuit voltage between the marked terminals.
- Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thévenin equivalent circuit.

PROBLEM

Find the Thévenin equivalent circuit for the network external to branch $a - a^{'}$ in the following Fig.

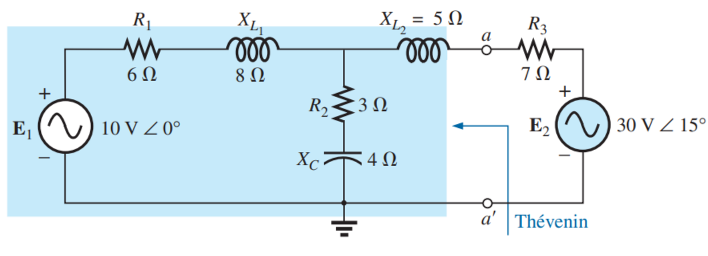
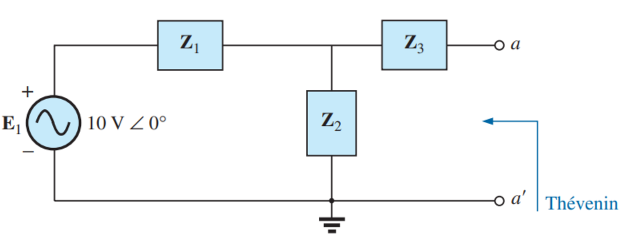


FIGURE: THEVENIN's THEOREM

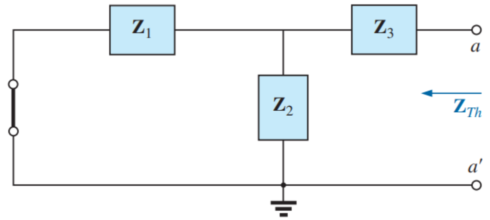
simplifying the above fig we get,//



Where,

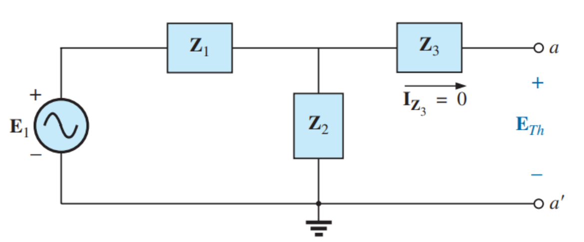
$$\mathbf{Z}_1 = R_1 + jX_{L_1} = 6\,\Omega + j\,8\,\Omega$$
$$\mathbf{Z}_2 = R_2 - jX_C = 3\,\Omega - j\,4\,\Omega$$
$$\mathbf{Z}_3 = +jX_{L_2} = j\,5\,\Omega$$

Assigning the subscripted impedances for the network and finding Z_{Th} :



$$\mathbf{Z}_{Th} = \mathbf{Z}_3 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = j\,5\,\Omega + \frac{(10\,\Omega\,\angle 53.13^\circ)(5\,\Omega\,\angle -53.13^\circ)}{(6\,\Omega + j\,8\,\Omega) + (3\,\Omega - j\,4\,\Omega)}$$
$$= j\,5 + \frac{50\,\angle 0^\circ}{9 + j\,4} = j\,5 + \frac{50\,\angle 0^\circ}{9.85\,\angle 23.96^\circ}$$
$$= j\,5 + 5.08\,\angle -23.96^\circ = j\,5 + 4.64 - j\,2.06$$
$$\mathbf{Z}_{Th} = \mathbf{4.64\,\Omega + j\,2.94\,\Omega = 5.49\,\Omega\,\angle 32.36^\circ}$$

Since $a - a'$ is an open circuit, Then E_Th is the voltage drop across Z_2:

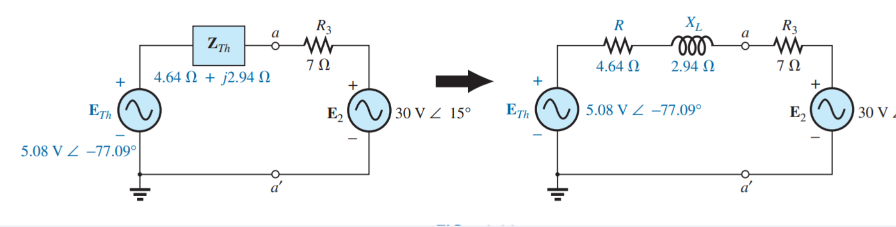


$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$

$$= \frac{(5\ \Omega \angle -53.13^\circ)(10\ \text{V} \angle 0^\circ)}{9.85\ \Omega \angle 23.96^\circ}$$

$$\mathbf{E}_{Th} = \frac{50\ \text{V} \angle -53.13^\circ}{9.85 \angle 23.96^\circ} = \mathbf{5.08\ V \angle -77.09^\circ}$$

So the thevenin's equivalent circuit can be formed as below,:



Norton's Theorem

Norton's theorem allows us to replace any two-terminal linear bilateral ac network with an equivalent circuit consisting of a current source and an impedance, as shown in the following Fig.

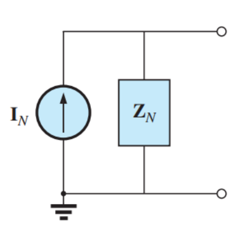


FIGURE: Norton's Theorem

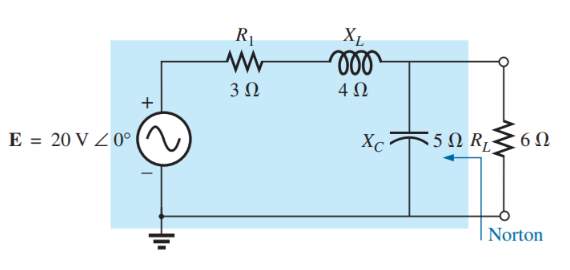
Procedure

- Remove that portion of the network across which the Norton equivalent circuit is to be found.
- Mark the terminals of the remaining two-terminal network.
- Calculate Z_N by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.

- Calculate I_N by first replacing the voltage and current sources and then finding the short-circuit current between the marked terminals.
- Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Norton equivalent circuit.

Problem

Determine the Norton equivalent circuit for the network external to the 6 Ω resistor in the following Fig:



Assigning the subscripted impedances to the network:

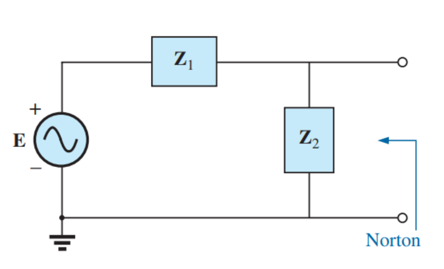


FIGURE: Assigning the subscripted impedances to the network

$$\mathbf{Z}_1 = R_1 + jX_L = 3\,\Omega + j4\,\Omega = 5\,\Omega\,\angle 53.13^\circ$$
$$\mathbf{Z}_2 = -jX_C = -j5\,\Omega$$

For Z_N,

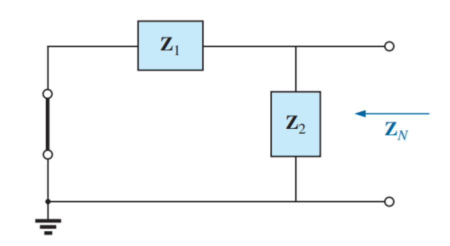


FIGURE: Finding out Z_N

$$\mathbf{Z}_N = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(5\ \Omega \angle 53.13^\circ)(5\ \Omega \angle -90^\circ)}{3\ \Omega + j\,4\ \Omega - j\,5\ \Omega} = \frac{25\ \Omega \angle -36.87^\circ}{3 - j\,1}$$

$$= \frac{25\ \Omega \angle -36.87^\circ}{3.16 \angle -18.43^\circ} = 7.91\ \Omega \angle -18.44^\circ = \mathbf{7.50\ \Omega - j\,2.50\ \Omega}$$

For I_N,

$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{20\ \text{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = \mathbf{4\ A\ \angle -53.13^\circ}$$

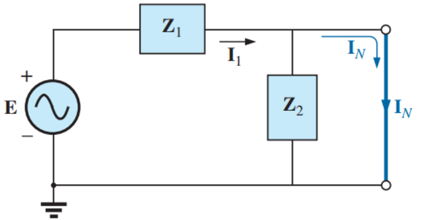
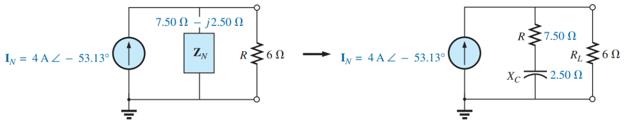


FIGURE: Finding out I_N

Hence the Norton equivalent circuit becomes:



Chapter VIII
BALANCED POLYPHASE CIRCUITS

A polyphase system is simply several single-phase systems which are displaced in time phase from one another.

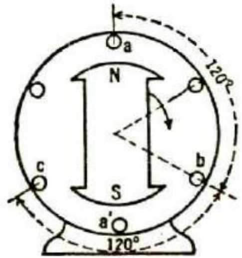


FIG. 1. Elementary three-phase generator.

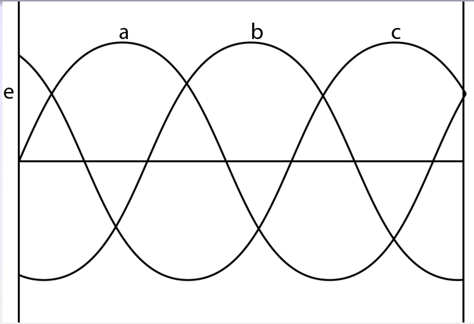


FIGURE: Waves of emf generated by a Three Phase generator

This system is called three-phase because there are three waves of different time phases. In general, the electrical displacement between phases for a balanced n-phase system is 360/n electrical degrees. Three-phase systems are the most common, although for certain special applications a greater number of phases is used.

three-Phase, Four-Wire Systems of Generated E.m.f's

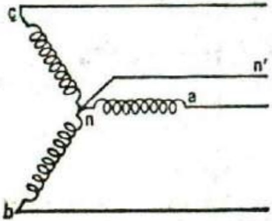


FIG. 19. Three-phase four-wire system.

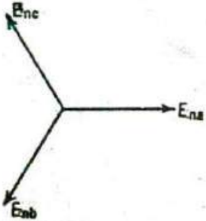


FIG. 20. Line-to-neutral voltages of Fig. 19.

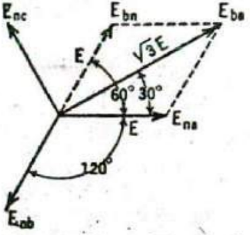


FIG. 21. Line voltage equals phase voltage times $\sqrt{3}$ in the wye connection.

The above connection is also called is called a wye connection. The voltages between terminals a, b, and c are called the line or terminal voltages. Under balanced conditions they are definitely related to the phase voltages, as the following shows:

$E_{ba} = E_{bn} + E_{na}$ or $-E_{ab} = -E_{nb} - E_{an} = E_{bn} - E_{an}$ or

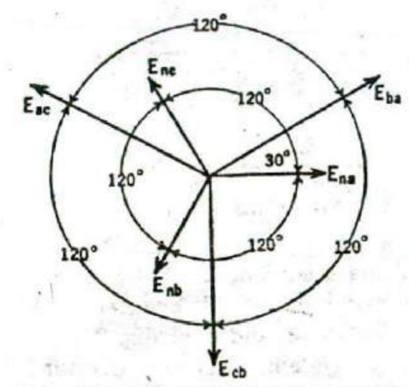
$E_{ab} = E_{an} - E_{bn}$

$E_{ac} = E_{an} + E_{nc}$ or $E_{ca} = E_{cn} - E_{an}$

$E_{cb} = E_{cn} + E_{nb}$ or $E_{bc} = E_{bn} - E_{cn}$

Here magnitude of the phase voltage is considered as E. Hence line voltage in the balanced three phase star or wye connection is the

$\sqrt{3}$ times the phase voltage and makes an angle with the component phase voltages of either 30° or 150° depending upon which are considered. The complete vector diagram showing all line voltages is given in Fig.



When the system is balanced, the currents in the three phases are all equal in magnitude and differ by 120° in time phase, as shown in Fig:

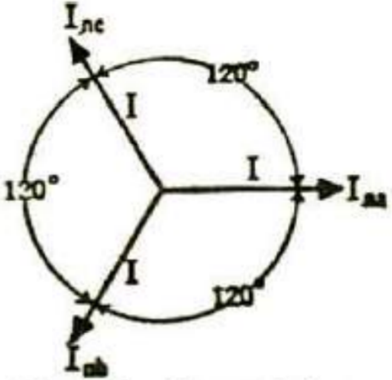


FIG. 24. Currents in a balanced-wye system.

Under balanced condition, $I_{nn'} = I_{na} + I_{nb} + I_{nc} = 0$

Three-Phase, Three-Wire Systems

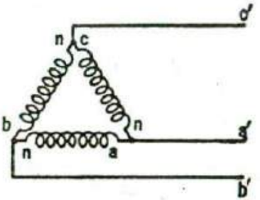


FIG. 25. Delta connection of the coils shown in Fig. 19.

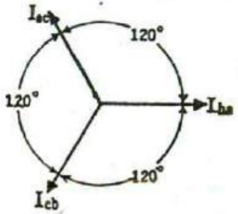


FIG. 26. Phase currents for the balanced delta of Fig. 25.

Here, $E_{na} + E_{nb} + E_{nc} = 0$ In this event loads are not placed between the lines and neutral, and the neutral wire is therefore not brought out.

Power Calculations in Balanced Systems If the voltage per phase is V_P , the phase current I_P,, and the angle between them θ_P, the power per phase is:

$P_P = V_P \times I_P \cos\theta_P$

The power for all phases of the n-phase system is:

$P_t = nP_P = nV_P \times I_P \cos\theta_P$

In case of DELTA Connection:

$$P_t = 3P_P = 3V_P \times I_P \cos\theta_P = 3\frac{V_L}{\sqrt{3}} \times I_L \cos\theta_P = \sqrt{3}V_L I_L \cos\theta_P$$

$$P_t = 3P_P = 3V_P \times I_P \cos\theta_P = 3\frac{V_L}{\sqrt{3}} \times I_L \cos\theta_P = \sqrt{3}V_L I_L \cos\theta_P$$

- The equations for power in terms of line voltages and line currents for balanced three-phase loads whether delta or wye-connected are identical and equal to: $\sqrt{3}V_L I_L \cos\theta_P$
- In this expression, $\sqrt{3}V_L I_L \cos\theta_P$ for balanced three-phase power, it must be remembered that θ_P is the angle between phase voltage and phase current and not between line voltage and line current.

Volt-Ampere

$v_{a_t} = \sqrt{3}V_L I_L$ (For both wye and delta connection)

Reactive Volt-Amperes

$\sqrt{3}V_L I_L \sin\theta_P$ (For both wye and delta connection)

Example - 2

The line voltage, V_L of the following balanced three phase system is 220 volts. R and X of each phase are 6 Ω resistance and 8 Ω inductive reactance. Find the followings:

(a) Value of line and phase and line voltage.

(b) Value of phase and line current.

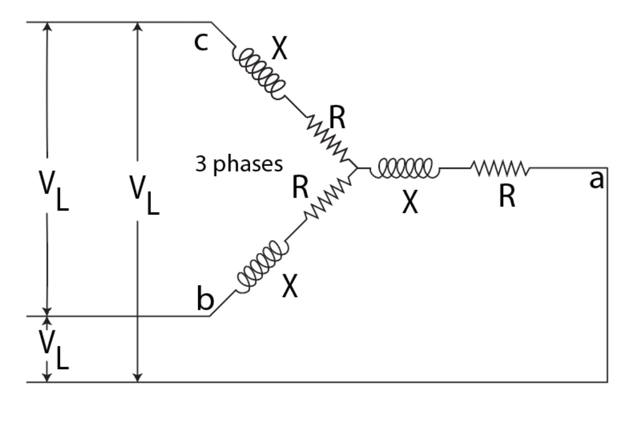
(c) Power per phase.

(d) Total power.

(e) The vector expressions for the line and phase voltages.

(f) vector expression of phase and line current.

(g) Vector diagram showing the line voltages, phase voltages and phase current.



Solution

(a) Value of line and phase and line voltage.

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127 \text{ Volts}$$

(b) Value of phase and line current.

$$I_L = I_P = \frac{127}{\sqrt{6^2 + 8^2}} = \frac{127}{10} = 12.70 \text{ A}$$

(c) Power per phase.

Power per phase $= I_P^2 \times R_P = 12.70^2 \times 6 = 968\ watts$

(d) Total power.

Total Power $= 3 \times 968 = 2904$ Watts

(e) The vector expressions for the line and phase voltages.

$V_{na} = 127 + \text{j}0$ volts

$V_{nb} = 127\angle-120^o = 127(\cos 120^o - j\sin 120^o) = -63.50 - j110\ Volts$

$V_{nc} = 127\angle 120^o =$ -63.50 +j110 volts

$V_{ba} = V_{bn} + V_{na} = 63.5+\text{j}110 + 127 + \text{j}0 = 190.5 + \text{j}110$

$V_{cb} = V_{cn} + V_{nb} = 63.50 - \text{j}110\text{-j}110 = \text{-j}220$

$V_{ac} = V_{an} + V_{nc} = \text{-}127 - \text{j}0 \text{-}63.50 + \text{j}110 = \text{-}190.50 + \text{j}110$

(f) Vector expression of line and phase current.

$$I_{na} = \frac{V_{na}}{Z_{na}} = \frac{127 + j0}{6 + j8} = 7.62 — \text{j}10.16 = 12.70\ \angle-53.13^o$$

$$I_{nb} = \frac{V_{nb}}{Z_{nb}} = \frac{-63.5 - j110}{6 + j8} = \frac{127\angle-120^o}{10\angle 53.13^o} = 12.70\ \angle-173.13^o$$

$$I_{nc} = \frac{V_{nc}}{Z_{nc}} = \frac{127\angle 120^o}{10\angle 53.13^o} = 12.7\angle 66.87^o$$

Power per phase (Vector solution).

$P_{na} = vi + v^{'} i^{'} = 127 \times 7.62 = 968$ watts

The vector representation of the given circuit is given below:

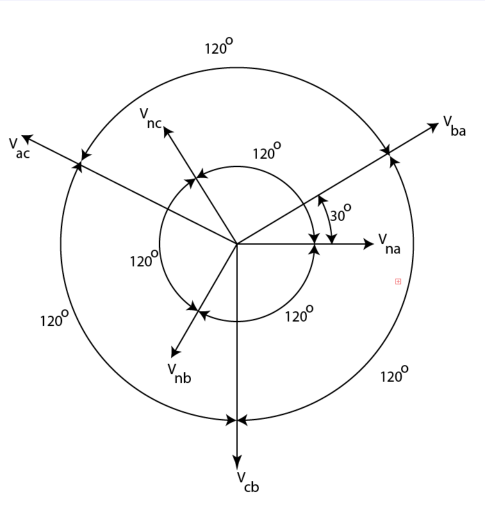


FIGURE: Vector representation of the given circuit

$P_{nb} = 127 \times 12.70 \cos(120^o - 173.13^o) = 968$

Example 3

Calculate phase current, line current, phase power, and total power from the given delta connected network.

Solution

$V_L = V_P = 220 Volts$

$I_P = \frac{220}{\sqrt{R^2 + X^2}}$

$I_P = \frac{220}{\sqrt{6^2 + 8^2}} = 22A$

$I_L = \sqrt{3} \times 22 = 38.10A$

Power per phase $= \sqrt{22^2} \times 6 = 2904$ *watts*

Total power $= 3 \times 2904 = 8712$ *watts*

(g) Vector diagram showing the line voltages, phase voltages and phase current.

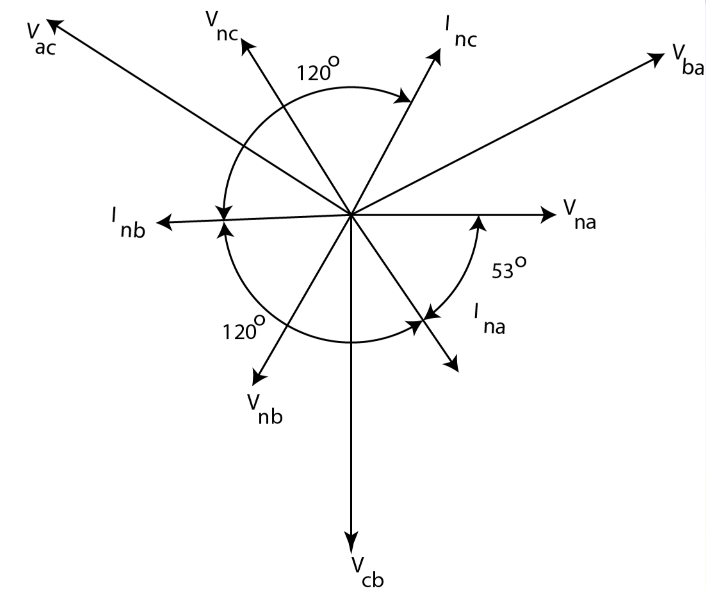


FIGURE: Vector representation

Example - 3

In the given delta connection as shown in the following figure calculate the followings: (a) Phase current, line current, phase power, and total power.

(b) Vector expression of the phase voltages.

(c) Vector representation of the phase current.

(d) Vector representation of the line current.

(e) Draw the vector vector diagram corresponding to the vector solutions.

Here R = 6 *ohm* and X = 8 *ohm*

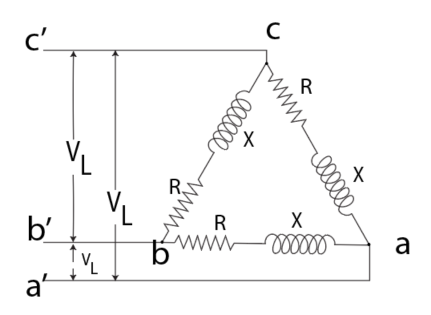


FIGURE: Delta connected network

(a) <u>Phase current, line current, phase power, and total power</u>.

$V_L = V_P = 220 \ Volts$

$I_P = \dfrac{220}{\sqrt{R^2 + x^2}} = \dfrac{220}{\sqrt{6^2 + 8^2}} = 22$ A.

$I_L = \sqrt{3} \times 22 = 38.10 \ Amperes$

Power per phase $= 22^2 \times 6 = 2904 \ Watts$

Total Power, $3 \times \ 2904 = 8712$ Watts.

(b) Vector expression of the phase voltages.

$V_{ba} = 220\angle 0^o$ Volts

$V_{cb} = 220\angle -120^o$ Volts

$V_{ac} = 220\angle 120^o$ Volts

(c) Vector representation of the phase current.

$$Z = \sqrt{R^2 + X^2}\angle \tan^{-1}\frac{X}{R} = \sqrt{6^2 + 8^2}\angle \tan^{-1}\frac{8}{6} = 10\angle 53.13^o$$

$$I_{ba} = \frac{220\angle 0^o}{10\angle 53.13^o} = 22\angle -53.13^o = 13.2 — j17.6 \text{ amperes}$$

$$I_{cb} = \frac{220\angle -120^o}{10\angle 53.13^o} = 22\angle -173.13^o = -21.85 — j2.63 \text{ amperes}$$

$$I_{ac} = \frac{220\angle 120^o}{10\angle 53.13^o} = 22\angle 66.87^o = 8.65 + j20.2 \text{ amperes}$$

$$P_{ba} = \text{VI}\cos\theta = 220 \text{ X } 22 \cos 53.13 = 2904 \text{ watts}$$

Total Power, $P_{Total} = 3 \times 2904 = 8712$ watts.

(d) Vector representation of the line current.

$I_{c'c} = I_{cb} + I_{ca} =$ -21.85 — j2.63 + (- 8.65 - j20.2) = -30.5 - j22.8

$= 38.10\angle - 143.13^{o}$

$I_{b'b} = I_{ba} + I_{bc} =$ 13.2 — j17.6 + 21.85 + j2.63 = 35.05 - j15 =

38.1 $\angle - 23.13^{o}$

$I_{a'a} = I_{ab} + I_{ac} =$ -13.2 + j17.6 + 8.65 + j20.2 = — 4.55 + j37.8

38.1 $\angle 96.87^{o}$

(e) Draw the vector vector diagram corresponding to the vector solutions.

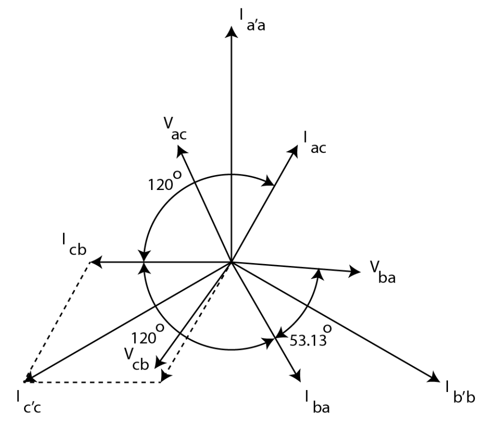


FIGURE: Vector Diagram

Power Calculations in Balanced System

if the voltage per phase is V_P, the phase current I_L, and the angle between them θ_P, the power per phase is $P_P = V_P I_P \cos\theta_P$

The power for all phases of an n-phase system is,

$P_t = nP_P = V_P I_P \cos\theta_P$

For Wye connection,

$$P_t = 3P_P = 3V_P I_P \cos\theta_P = 3\frac{V_L}{\sqrt{3}} I_L \cos\theta_P = \sqrt{3} V_L I_L \cos\theta_P$$

For the delta connection,

$$P_t = 3P_P = 3V_P I_P \cos\theta_P = 3\frac{I_L}{\sqrt{3}} V_L \cos\theta_P = \sqrt{3} V_L I_L \cos\theta_P$$

The equations for power in terms of line voltages and line currents for balanced three-phase loads whether delta- or wye-connected are identical and equal to $\sqrt{3} V_L I_L \cos\theta_P$

Volt-Amperes. The volt-amperes of a balanced three-phase system are defined as the sum of the volt-amperes of the separate phases or three times the number of volt-amperes per phase: Hence

$va_t = 3va_P = 3V_P I_P$

In terms of line voltage and line current, volt-amperes are,

For delta: $3V_P I_P = 3V_L \frac{I_L}{\sqrt{3}} = \sqrt{3} V_L I_L$

For wye: $3V_P I_P = 3\frac{V_L}{\sqrt{3}} I_L = \sqrt{3} V_L I_L$

Reactive Volt-Amperes

The reactive volt-amperes for a balanced three-phase system are defined as the sum of the reactive volt-amperes for each phase, or three times the reactive volt-amperes per phase. In terms of line voltage and line current the reactive volt-amperes or reactive power is,

For Wye connection,

$$P_X = 3P_X = 3V_P I_P \sin\theta_P = 3\frac{V_L}{\sqrt{3}} I_L \sin\theta_P = \sqrt{3} V_L I_L \sin\theta_P$$

For the delta connection,

$$P_X = 3P_X = 3V_P I_P \sin\theta_P = 3\frac{I_L}{\sqrt{3}} V_L \sin\theta_P = \sqrt{3} V_L I_L \sin\theta_P$$

QUESTION -

Write down the interrelationship between the followings in case of both wye and delta connected balanced poly phase network:

(a) Phase voltage and line voltage.

(b) Phase current and line current.

Basing on the above relationships, proof that the Reactive

volt ampere for both the wye and delta connected network is identical. That is:

$P_x = \sqrt{3} V_L I_L \sin\theta_P$

Where the notations symbolize their usual meaning.

Power Factor

The power factor of a balanced three-phase system is defined as the cosine of the angle between phase voltage and phase current independent of whether the connection is delta or wye.

Summary

- For a balanced Wye and Delta connected balanced polyphase network write down the expression for total power, reactive volt ampere, volt ampere and power factor in terms of line voltage (V_L), line current (I_L) and phase angle θ_P.

For balanced 3-phase network (Both Delta and Wye):

Total Power, $P_t = \sqrt{3} V_L I_L \cos\theta_P$

Volt-Ampere, $v_a = \sqrt{3} V_L I_L$

Reactive Volt-Ampere, $P_X = \sqrt{3} V_L I_L \sin\theta_P$

- Write down the numerical expression of $V_1, V_2 and V_3$ from the given figure
- In case of a purely resistive,inductive, capacitive circuit write down the general expression of i, v, p, Z and W_L, W_C where the symbols mean their usual meaning
- In case of an RLC network as shown in the circuit proof the followings:

  (a) $Z_{RLC} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \angle tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$

  (b) $p = \frac{V_m I_m}{2} \cos\theta$ - $\frac{V_m I_m}{2} [\cos 2\omega t] \cos\theta + \frac{V_m I_m}{2} [\sin 2\omega t] \sin\theta$
- For a balanced Wye and Delta connected balanced polyphase network write down the expression for total power, reactive volt ampere, volt ampere and power factor in terms of line voltage (V_L), line current (I_L) and phase angle θ_P.

  For balanced 3-phase network (Both Delta and Wye):

  Total Power, $P_t = \sqrt{3} V_L I_L \cos\theta_P$

  Volt-Ampere, $v_a = \sqrt{3} V_L I_L$

  Reactive Volt-Ampere, $P_X = \sqrt{3} V_L I_L \sin\theta_P$