# Corpuscular Theory of Light (1704)

Isaac Newton proposed that light consists of a stream of small particles, because it travels in straight lines at great speeds and also reflected from mirrors in a predictable way. This theory only explained

reflection and refraction. Characteristics of a particle is that it has mass, m, position, x, and momentum, p.



## The Wave Theory

- •Discovered by *Christian Huygens*, a Dutch scientist, also in the seventeenth century States that light is emitted in a series of waves that spread out from a light source in all directions. These waves are not affected by gravity.
- •100 years later (1802), *Thomas Young* completely disproved the corpuscular theory by showing that light waves can interfere with each other.

## Maxwell's Electromagnetic Wave theory of light

Maxwell (1865-1873) proposed that light is an electromagnetic disturbance created by extremely high frequency oscillators. It was assumed from this theory that these oscillators (resonators) were able to emit light of frequency equal to their own.

Electromagnetic waves are waves that are capable of traveling through a vacuum. They consist of oscillating electric and magnetic fields. The wave speed equation is:  $c = v \lambda$ , where c is the speed of light.

This explained all the properties of light such as reflection, refraction, diffraction, interference and polarization; it do not explain the photoelectric effect or radiation produced by an incandescent light.

#### Planck's Quantum Postulate (1900)-Quantum theory

Stated that light waves travel as separate packets of energy called **quanta** or **photons**. This merged the subjects of the Corpuscular, Wave, and Electromagnetic Theories together.

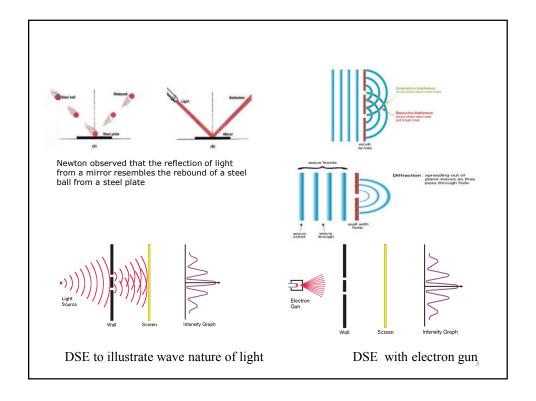
#### Duel dual nature

In 1905 Albert Einstein had proposed a solution to the problem of observations made on the behaviour of light having characteristics of both wave and particle theory. From work of Planck on emission of light from hot bodies, Einstein suggested that light is composed of tiny particles called *photons*, and each photon has energy.

when light is transmitted through space or matter, it behaves like a wave; when light is emitted or absorbed, it behaves like a particle called a *photon*.

#### **Everyday Evidence for Photons**

- •Red light is used in photographic darkrooms because it is not energetic enough to break the halogen-silver bond in black and white films.
- •Ultraviolet light causes sunburn but visible light does not because UV photons are more energetic.
- •Our eyes detect color because photons of different energies trigger different chemical reactions in retina cells



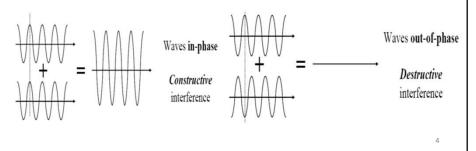
# **Interference**

When waves from two sources overlap, the resultant wave is the sum of the waves from each source. A difference in the path lengths from each of the two sources to a given point causes a phase difference between the two waves at that point. The resultant wave has amplitude that is different in different directions from the sources. This effect is called interference.

 $\blacktriangleright$  Modification of intensity (I  $\alpha$  A<sup>2</sup>) obtained by the superposition of two or more beams of light is known as interference.

If the resultant amplitude is zero or in general less than that of separate intensities, we have destructive interference

If the intensity is greater we have constructive interference.



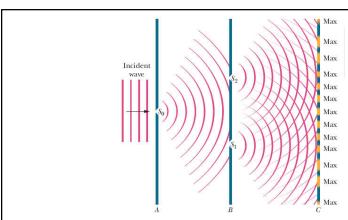
# **Principle of superposition**

When two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at that point by the individual waves if each were present alone. This is known as the principle of superposition, and was first clearly stated by Young in 1802.

## Young's experiment

In the year 1802, Young demonstrated the experiment on the interference of light. He allowed sunlight to fall on a pinhole,  $S_0$ , and then at same distance away on two pinholes  $S_1$  and  $S_2$ . The  $S_1$  and  $S_2$  are equidistant from  $S_0$  and are close to each other. Spherical waves spread out from  $S_0$  (according to Huygen's principle each point on a wave front may be regarded as a new source of waves). Spherical wave also spread out from  $S_1$  and  $S_2$ . These waves are of the same amplitude and wavelength. On the screen interference bands are produced which are alternatively dark and bright.

When the crest due to one wave coincides with the crest of due to the other, they reinforce each other. When the crest of one falls on the trough of the other and they neutralize the effect of each other. If the slits are close together, for example, 0.2 mm apart they give widely spaced fringes, whereas slits farther apart, e.g. 1.0 mm, give very narrow fringe. A piece of red glass placed adjacent to the source and another green glass in front of the lamp will show that red light will produced wider fringes than the green.



#### **Coherent sources**

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. Two sources must emit radiation of the same colour.

#### **Analytical treatment of interference**

Consider a monochromatic source of light,  $S_0$ , emitting waves of wavelength,  $\lambda$ , and two narrow pinholes  $S_1$  and  $S_2$ . The  $S_1$  and  $S_2$  are equidistant from  $S_0$  and act as two virtual sources. Let a be the amplitude of the waves. The phase difference between the two waves reaching the point P at any

Incident

instant is  $\delta$ . If  $y_1$  and  $y_2$  are the displacements,

$$y_1 = a \sin \omega t$$
  
 $y_2 = a \sin (\omega t + \delta)$ 

$$y = y_1 + y_2 = a \sin \omega t + a \sin (\omega t + \delta)$$

$$= a \sin \omega t + a \sin \omega t \cos \delta + a \cos \omega t \sin \delta$$

$$= a \sin \omega t (1 + \cos \delta) + a \cos \omega t \sin \delta$$

$$=R\sin\omega t\cos\theta + R\cos\omega t\sin\theta$$

$$=R\sin(\omega t+\theta)$$

Where, 
$$a(1+\cos\delta) = R\cos\theta$$
 (1)  
 $a\sin\delta = R\sin\theta$  (2)

which represents the equation of simple harmonic vibration of amplitude R. Squaring (1) and (2) and adding

$$R^{2} \sin^{2} \theta + R^{2} \cos^{2} \theta = a^{2} \sin^{2} \delta + a^{2} (1 + \cos \delta)^{2}$$

$$or, R^{2} = a^{2} (\sin^{2} \delta + \cos^{2} \delta) + a^{2} + 2a^{2} \cos \delta$$

$$= 2 a^{2} \cdot 2 \cos^{2} \frac{\delta}{2}$$

$$= 4 a^{2} \cos^{2} \frac{\delta}{2}$$

The intensity at any point is given by the square of the amplitude,

$$I = 4a^2 \cos^2(\delta/2) \tag{3}$$

Special cases:

(i)  $\delta = 0, 2\pi, 2(2\pi), \dots n(2\pi);$ 

or, path difference, 
$$x=0$$
,  $\lambda$ ,  $2\lambda$   $n\lambda$ 

then, I=4  $a^2$ , Intensity is maximum when the phase difference is a whole number multiple of  $2\pi$ , or the path difference is a whole number multiple of wavelength.

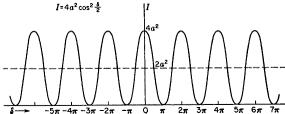
(ii) If 
$$\delta = \pi$$
,  $3\pi$ , .....(2n +1)  $2\pi$ 

or, path difference, 
$$x = \lambda/2$$
,  $3\lambda/2$   $(2n+1)\lambda/2$ 

Intensity is minimum, when the path difference is an odd number multiple of half wavelength.

# (iii) Energy distribution:

From eq 3 it is found that the intensity at bright point is  $4a^2$  and at dark point it is zero. According to the law of conservation of energy, the energy cannot be destroyed but only transferred from the points of minimum intensity to the points of maxin



Intensity distribution for the interference fringes from two waves of the same frequency.

Now, each beam acting separately would contribute a<sup>2</sup>, and so without interference we would have a uniform intensity of  $2 a^2$ .

We know that average of 
$$\cos^2(\delta/2) = 1/2$$

Using (4), we see that the average intensity  $=2a^2$ .

(4)

# Theory of interference fringes

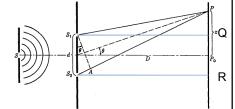
Let d= distance between two sources

D= distance of the screen from the sources.

The point C on the screen is equidistant from S<sub>1</sub> and S<sub>2</sub>. Therefore, the path difference is zero. Thus the point C has a maximum intensity. Consider a point P at a distance x from C.

PQ=x-d/2; PR=x+d/2

$$(S_2P)^2 - (S_1P)^2 = [D^2 + (x+d/2)^2] - [D^2 + (x-d/2)^2]$$
  
=2xd  
Or,  $S_2P - S_1P = \frac{2xd}{S_2P + S_1P} \approx \frac{2xd}{2D} = \frac{xd}{D}$ 



Therefore, the path difference is xd/D.

*The path difference*  $\lambda$  *corresponds to phase difference*  $2\pi$ . *For path difference* x, the phase difference,

the phase difference,  $\delta = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times (path \ difference)$ Therefore, Phase difference =  $\frac{2\pi}{\lambda} \left(\frac{xd}{D}\right)$ (i)Bright fringes,  $\frac{xd}{D} = n\lambda \qquad n = 0, 1, 2, \qquad \dots = \frac{n\lambda D}{d} \qquad n = 0, 1, 2,$ 

$$x_2 - x_1 = \frac{\lambda D}{d}$$

(ii) Dark fringes,

$$x = \frac{xd}{D} = (2n+1)\frac{\lambda}{2} \qquad n = 0,1,2, \qquad \dots \dots$$

$$x = \frac{(2n+1)\lambda D}{2d} \qquad n = 0,1,2, \qquad \dots \dots$$

$$x = \frac{(2n+1)\lambda D}{2d}$$
  $n = 0,1,2,$  ......

For, n=1,  $x_1=3\lambda D/2d$ , n=2,  $x_2=5\lambda D/2d$ , ......  $x_n=(2n+1)\lambda D/2d$ 

$$x_2 - x_1 = \frac{\lambda D}{d}$$

The distance between two consecutive bright or dark fringes is known as fringe width. Therefore, alternate bright or dark fringes are formed on both side of p<sub>0</sub>. The fringe width is

$$\beta = \frac{\lambda D}{d}$$

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# Determination of the thickness of a thin sheet of transparent material

We would like to investigate the effect of interference pattern due to introducing a thin transparent plate in the path of one of the two interfering beams of monochromatic light.

T is the thickness of the plate and  $\mu$  is the refractive index for the monochromatic light that is used for interference

Time required (T) for light to travel AP distance is

$$T = \frac{AP - t}{c_0} + \frac{t}{c} = \frac{1}{c_0} (AP - t + \frac{c_0}{c}t)$$
$$= \frac{1}{c_0} [AP + (\mu - 1)t]$$

$$c_0T=AP+(\mu-1)t$$

Is the effective path in air from A to P.

Similarly, the effective path in air from A to C, the point equidistant from A and B become  $AC+(\mu-1)t$ 

Since  $AC + (\mu - 1)t > BC$ , the central bright fringe of zero order is not formed at C.

Suppose in the presence of the plate central fringe of zero optical path difference is formed at O,

$$BO = AO + (\mu - 1)t$$
 or, 
$$BO - AO = (\mu - 1)t \dots (1)$$

If 
$$CO = x_0$$
 then

$$BO - AO = \frac{x_0 d}{D} \dots (2)$$

Using Eq. 2 in Eq. 1,

$$(\mu - 1)t = \frac{x_0 d}{D}$$

$$or x_0 = \frac{(\mu - 1)tD}{d}$$

$$or, t = \frac{x_0 d}{D(\mu - 1)}$$

Therefore knowing  $x_0$  (distance through which the central fringe is shifted), D, d and  $\mu$  the thickness of the transparent plate can be calculated.

We also know that

$$\frac{x_0 d}{D} = n\lambda$$

*Therefore*  $(\mu - 1)t = n\lambda$ 

for bright fringes.

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### Interference due to reflected light

Consider a transparent film of thickness t and refractive index  $\mu$ . A ray SA incident on the upper surface of the film is partly reflected along AT and partly refracted along AB. At B part of it is reflected along BC and finally emerges out along CQ.

The difference in path between the two rays AT and CQ can be calculated. Draw CN normal to AT and AM normal to BC. The angle of incidence is i and the angle of refraction is r, Also produce CB to meet AE produced at P.

Here <APC=r. The optical, path difference is

 $x = \mu.PM = 2 \mu t \cos r$ 

$$x = \mu(AB + BC) - AN$$
Here, 
$$\mu = \frac{\sin i}{\sin r} = \frac{AN}{CM}$$

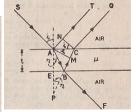
$$\therefore AN = \mu.CM$$

$$\therefore x = \mu (AB + BC) - \mu.CM$$

$$= \mu (AB + BC - CM) = \mu(PC - CM)$$

$$= \mu.PM$$
In the  $\triangle APM$ ,

 $\triangle APM$ ,  $\cos r = \frac{PM}{AP}$   $PM = AP. \cos r = (AE + EP) \cos r$   $= 2t \cos r$   $(\therefore AE = EP = t)$ 



It has been established on the basis of electromagnetic theory that when light is reflected from the surface of an optically denser medium (air-medium interface) a phase change  $\pi$ , equivalent to a path difference  $\lambda/2$  occurs.

Therefore, the correct path difference in this case,

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2}$$

Condition for bright fringe

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2} = n\lambda \qquad n = 0,1,2.....$$

$$or \quad 2\mu t \cos r = (n + \frac{1}{2})\lambda = (2n + 1)\frac{\lambda}{2}$$

Condition for dark fringe

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$n = 0,1,2.....$$

$$2\mu t \cos r = n\lambda$$
  $n+1$  is another integer  $n$ 

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# **Newton's rings**

When a plano-convex lens of long focal length is placed on a plane glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is very small at the point of contact and gradually increases from the centre outwards. The fringes produced with monochromatic light are circular. The fringes are concentric circles. When viewed with white light fringes are colored. These were first studied by Sir Isaac Newton

(1642-1627) and are called Newton's rings

S – The monochromatic source

L<sub>1</sub>- Convex lens for parallel rays:

L- Experimental lens

G-The plane glass Plate.

G WAIR FILM

L and G are in contact with each other light from the source passing through a

convex lens. Then parallel rays fall on the glass plate, B, placed at 45°. The glass plate reflects part at the incident light toward the air film enclosed by L and G. The reflected beam from the air film is viewed with a microscope(M). Interference take place and bright and circular fringes are produced.

This is due to the interference between the light reflect from the lower surface of the lens and the upper surface of the glass plates.

**Theory:** suppose, the radius of curvature of the lens is R and the air film of thickness t is at a distance of OQ=r=PE, from the point of contact O.

From the adjacent figure

From  $\triangle$ CEP, we can write CP<sup>2</sup>=CE<sup>2</sup>+PE<sup>2</sup>

$$t = R - CP$$
 where,  $CP = \sqrt{R^2 - r^2}$ ,  $R = radius$  of curvature

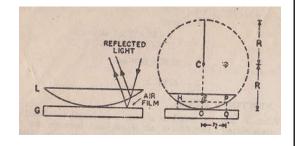
and r = radius of circular Newton's ring

$$t = R - (R^{2} - r^{2})^{\frac{1}{2}}$$

$$= R - R \left[ 1 - \left( \frac{r}{R} \right)^{2} \right]^{\frac{1}{2}}$$

$$= R - R \left( 1 - \frac{1}{2} \frac{r^{2}}{R^{2}} \dots \right)$$

$$= R - R + \frac{1}{2} \frac{r^{2}}{R^{2}} R$$



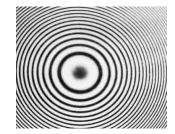
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If we consider the interference due to reflected light then condition for bright fringe

$$2t\cos\theta = (2n+1)\frac{\lambda}{2}$$
, where,  $n = 0,1,2$ 

For normal incidence,  $\cos\theta=1$ 

$$2t = (2n+1)\frac{\lambda}{2}$$
or 
$$\frac{2r^2}{2R} = (2n+1)\frac{\lambda}{2}$$
or 
$$\frac{r^2}{R} = (2n+1)\frac{\lambda}{2}$$
or 
$$\left(\frac{D}{2}\right)^2 = (2n+1)\frac{\lambda}{2}R$$



$$or, \ D^2 = (2n+1)2\lambda R$$

or, 
$$D = \sqrt{(2n+1)2\lambda R}$$

Expression for the diameter of the bright ring

$$or, \ r = \sqrt{\frac{(2n+1)\lambda R}{2}}$$

Expression for the radius of the bright ring

# For dark fringe,

$$2t = n\lambda$$

$$or \frac{r^{2}}{R} = n\lambda$$

$$or, r^{2} = n\lambda R$$

$$or, r = \sqrt{n\lambda R}$$

Expression for the radius of dark ring

$$or, \left(\frac{D}{2}\right)^2 = n\lambda R$$

or, 
$$D^2 = 4n\lambda R$$

or, 
$$D = \sqrt{4n\lambda R}$$

Expression for the diameter of dark ring

$$n = 0, 1, 2, 3 \dots \dots$$

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# Newton's Rings by transmitted light

For bright Fringe,

$$2t = n\lambda$$
or  $D^2 = 4\pi\lambda R$ 

$$n = 0,1,2,3...$$
or  $r = \sqrt{\lambda nR}$ 

For dark fringes,

$$2t = (2n+1)\frac{\lambda}{2}$$
or  $D^2 = (2n+1)2\lambda R$ 

$$n = 0,1,2....$$
or  $r = \sqrt{\frac{(2n+1)\lambda R}{2}}$ 

# Refractive index of a liquid using Newton's Rings

For dark fringes,

$$2\mu t = n\lambda$$

$$or, \frac{2\mu r^2}{2R} = n\lambda$$

$$or, r^2 = \frac{n\lambda R}{\mu}$$

$$or, D^2 = \frac{4n\lambda R}{\mu}$$

If  $D'_n$  is the diameter of the n<sup>th</sup> ring and  $D'_{n+m}$  is the diameter of the (n+m)th

ring then

$$D_n'^2 = \frac{4n\lambda R}{\mu}$$

$$D_{n+m}'^2 = \frac{4(n+m)\lambda R}{\mu}$$

$$(D'_{n+m})^2 - (D'_n)^2 = \frac{4n\lambda R}{\mu}$$

or, 
$$\mu = \frac{4m\lambda R}{(D'_{n+m})^2 - (D'_n)^2}$$
 If  $\lambda$  and R is known then  $\mu$  can be calculated

or, 
$$R = \frac{\mu (D'_{n+m})^2 - (D'_n)^2}{4m\lambda}$$
 If  $\lambda$  and  $\mu$  is known then R can be calculated

or, 
$$\lambda = \frac{\mu (D'_{n+m})^2 - (D'_n)^2}{4mR}$$
 If R and  $\mu$  is known then  $\lambda$  can be calculated

Example 1: Two waves of the same frequency have amplitudes 1.00 and 2.00. They interfere at a point where their phase difference is 60.0°.What is the resultant amplitude? Solution:

The intensity is proportional to the square of the resultant field amplitude. Let the electric field components of the two waves be written as

$$E_1 = E_{10} \sin \omega t$$
  

$$E_2 = E_{20} \sin(\omega t + \phi),$$

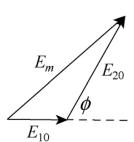
where  $E_{10} = 1.00$ ,  $E_{20} = 2.00$ , and  $\phi = 60^{\circ}$ . The resultant field is  $E = E_1 + E_2$ .

The phasor diagram is shown on the right. The resultant amplitude  $E_m$  is given by the trigonometric law of cosines:

$$E_m^2 = E_{10}^2 + E_{20}^2 - 2E_{10}E_{20}\cos(180^\circ - \phi)$$
.

Thus,

$$E_{\rm m} = \sqrt{\left(1.00\right)^2 + \left(2.00\right)^2 - 2\left(1.00\right)\left(2.00\right)\cos 120^{\circ}} = 2.65$$
.



#### Alternative solution:

$$\sum E_h = E_{10} \cos 0 + E_{20} \cos 60^{\circ} = 1.00 + (2.00) \cos 60^{\circ} = 2.00$$
.

Similarly, the sum over the vertical components is

$$\sum E_v = E_{10} \sin 0 + E_{20} \sin 60^\circ = 1.00 \sin 0^\circ + (2.00) \sin 60^\circ = 1.732$$
.

The resultant amplitude is

$$E_{\rm m} = \sqrt{\left(2.00\right)^2 + \left(1.732\right)^2} = 2.65$$
,

which agrees with what we found above. The phase angle relative to the phasor representing  $E_1$  is

$$\beta = \tan^{-1}\left(\frac{1.732}{2.00}\right) = 40.9^{\circ}$$
.

Thus, the resultant field can be written as  $E = (2.65) \sin(\omega t + 40.9^{\circ})$ .

#### Example 2;

Find the sum y of the following quantities:

$$y_1 = 10 \sin \omega t$$
 and  $y_2 = 8.0 \sin(\omega t + 30^\circ)$ .

Solution: With t=0

$$y_h = 10\cos 0^\circ + 8.0\cos 30^\circ = 16.9$$
  
 $y_v = 10\sin 0^\circ + 8.0\sin 30^\circ = 4.0$ 

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 17.4$$
  
 $\beta = \tan^{-1} \left(\frac{y_v}{y_h}\right) = 13.3^{\circ}$ .

Thus,

$$y = y_1 + y_2 = y_R \sin(\omega t + \beta) = 17.4 \sin(\omega t + 13.3^\circ).$$

Quoting the answer to two significant figures, we have  $y \approx 17 \sin(\omega t + 13^{\circ})$ .

**Example 3:** A thin flake of mica (n=1.58) is used to cover one slit of a double-slit interference arrangement. The central point on the viewing screen is now occupied by what had been the seventh bright side fringe (m=7). If  $\lambda=550$  nm, what is the thickness of the mica?  $(\mu-1)t=n\lambda$ 

Solution: We know that

for 
$$n=7$$
,

$$t = \frac{7\lambda}{\mu - 1} = \frac{7 \times 550 \times 10^{-9} \, m}{1.58 - 1} = 6.64 \times 10^{-6} \, m$$

**Example 4:** A light of wavelength 550 nm from a very small light source incident on a double slit. If the overall separation of 15 fringes on a screen 2.5 m away is 5 cm. Find the slit separation.

$$\beta = \frac{\lambda D}{d}$$

Here 
$$\beta = \frac{5}{15} \times 2$$
;  $\lambda = 550 \times 10^{-9} \text{ m}$ ;  $D = 2.5 \text{ m}$ 

substituting these values 
$$d = \frac{\lambda D}{\beta} = \frac{550 \times 10^{-9} \text{ m} \times 2.5 \text{ m}}{\frac{5}{15} \times 2} = 2.06 \times 10^{-6} \text{ m}$$

**Example 5** In a Newton's ring experiment the diameter of the 10<sup>th</sup> ring changes from 1.4 cm to 1.27 cm. When a liquid in introduced between the lens and the plate. Calculate the refractive index of the liquid.

Solution, For liquid medium,  $D^2_L = \frac{4n\lambda R}{\mu}$ 

For Air medium,  $D^2_A = 4n\lambda R$ 

$$\therefore \quad \mu = \frac{D^2_A}{D^2_L} \qquad D_A = 1.4, \ D_L = 1.27$$

$$= \frac{(1.4)^2}{(1.27)^2}$$

$$= 1.215$$

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Example 6: In a Newton's rings experiment, the radius of curvature R of the lens is 5.0 m and the lens diameter is 20 mm. (a) How many bright rings are produced? Assume that  $\lambda$ =589nm. (b) How many bright rings would be produced if the arrangement were immersed in water (n=1.33)?

Solution (a)

$$r = \sqrt{\frac{(2n+1)\lambda R}{2}}$$

$$or, r^2 = \frac{(2n+1)\lambda R}{2}$$

$$or, (2n+1)\lambda R = 2r^2$$

or, 
$$n = \frac{r^2}{\lambda R} - \frac{1}{2}$$

where, 
$$r = \frac{20 \times 10^{-3} \text{ m}}{2}$$
;  $R = 5 \text{ m}$ ; and  $\lambda = 589 \text{ nm}$ , substituting, we get  $n \approx 33$ 

Since first bright fringe occurs for m=0, therefore m=33 corresponds to 34th bright fringe

or, 
$$n = \frac{\mu r^2}{\lambda R} - \frac{1}{2}$$
  
where,  $r = \frac{20 \times 10^{-3} \text{ m}}{2}$ ;  $R = 5 \text{ m}$ ;  $\lambda = 589 \text{ nm}$ , and  $\mu = 1.33$   
substituting, we get  $n \approx 45$ 

m=45 corresponds to 46th bright fringe

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**Example 7:** A Newton's rings apparatus is to be used to determine the radius of curvature of a lens. The radii of the nth and (n+20)th bright rings are measured and found to be 0.162 and 0.368 cm, respectively, in light of wavelength 546nm. Calculate the radius of curvature of the lower surface of the lens.

$$n = \frac{r^2}{\lambda R} - \frac{1}{2}$$
and 
$$n + 20 = \frac{r'^2}{\lambda R} - \frac{1}{2}$$

Taking the difference between the two equations above, we eliminate n and find

$$R = \frac{r'^2 - r^2}{20\lambda} = \frac{(0.368 \,\mathrm{cm})^2 - (0.162 \,\mathrm{cm})^2}{20(546 \times 10^{-7} \,\mathrm{cm})} = 100 \,\mathrm{cm}.$$