$\therefore$ Fringe width $\beta = \frac{l}{n} = \frac{1.888}{20}$ cm

$d = 0{\cdot}075$ cm, $D = 120$ cm

$$\lambda = \frac{\beta d}{D} = \frac{1{\cdot}888}{20} \times \frac{0{\cdot}075}{120} = 5900 \times 10^{-8} \text{ cm}$$

$$= 5900 \text{ Å}$$

Example 8·11. *In an experiment with Fresnel's biprism fringes for light of wavelength 5×10^{-5} cm are observed 0·2 mm apart at a distance of 175 cm from the prism. The prism is made of glass of refractive index 1·50 and it is at a distance of 25 cm from the illuminated slit. Calculate the angle at the vertex of the biprism.*

Here, $y_1 = 25$ cm, $y_2 = 175$ cm

$\beta = 0{\cdot}2$ mm $= 0{\cdot}02$ cm

$\lambda = 5 \times 10^{-5}$ cm

$\mu = 1{\cdot}50$

$\alpha = ?$

$$d = 2(\mu - 1)\,\alpha \cdot y_1 \quad \ldots(i)$$

But $$\beta = \frac{\lambda D}{d}$$

or $$d = \frac{\lambda D}{\beta} \quad \ldots(ii)$$

From equations (*i*) and (*ii*)

$$\frac{\lambda D}{\beta} = 2(\mu - 1)\,\alpha \,.\, y_1$$

Also $D = y_1 + y_2$

$$\therefore \quad \frac{\lambda(y_1 + y_2)}{\beta} = 2(\mu - 1)\,\alpha \,.\, y_1$$

or $$\alpha = \frac{\lambda(y_1 + y_2)}{2\beta(\mu - 1)y_1}$$

$$\alpha = \frac{5 \times 10^{-5}(25 + 175)}{2 \times 0{\cdot}02(1{\cdot}5 - 1)25}$$

$$= 0{\cdot}02 \text{ radian}$$

The vertex angle $\theta = (\pi - 2\alpha)$ radian

$\theta = (\pi - 0{\cdot}04)$ radian

$\theta = 177° \; 42'$

Example 8·12. *Calculate the separation between the coherent sources formed by a biprism whose inclined faces make angles of 1 degree with its base. The slit source being 20 cm away from the biprism. μ of the biprism material $= 1{\cdot}5$.*

$$d = 2(\mu - 1)\,\alpha y_1$$

Here $\mu = 1{\cdot}5$, $\alpha = 1°$

$$= \frac{\pi}{180} \text{ radian}$$

$y_1 = 20$ cm

$$d = \frac{2(1{\cdot}5 - 1)\pi \times 20}{180}$$

$$= \frac{2 \times 0{\cdot}5 \times 22 \times 20}{7 \times 180}$$

$$= 0{\cdot}35 \text{ cm}$$

Example 8·13. *Calculate the separation between the coherent sources formed by a biprism whose inclined faces make angles of 2° with its base, the slit source being 10 cm away from the biprism ($\mu = 1{\cdot}50$.)* (*Delhi 1974, 1977*)

$$d = 2(\mu - 1)\,\alpha\, y_1$$

Here $\mu = 1{\cdot}50$

$$\alpha = 2° = \frac{2 \times \pi}{180} = \frac{\pi}{90} \text{ radian}$$

$y_1 = 10$ cm

$$d = \frac{2(1{\cdot}5 - 1) \times 10}{90}$$

$$= \frac{2 \times 0{\cdot}5 \times \pi \times 10}{90}$$

$$= 0{\cdot}35 \text{ cm}$$

8·10 DETERMINATION OF THE THICKNESS OF A THIN SHEET OF TRANSPARENT MATERIAL

The biprism experiment can be used to determine the thickness of a given thin sheet of transparent material *e.g.*, glass or mica.

Suppose A and B are two virtual coherent sources. The point C is equidistant from A and B. When a transparent plate G of thickness t and refractive index μ is introduced in the path of one of the beams (Fig. 8·10), the fringe which was originally a C shifts to P The time taken by the wave from B to P in air is the same as the time taken by the wave from A to P partly through air and partly through the plate. Suppose C_0 is the velocity of light in air and C its velocity in the medium

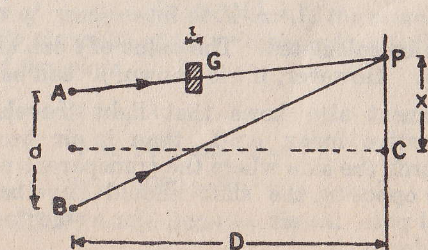

Fig. 8·10.

$$\therefore \qquad \frac{BP}{c_0} = \frac{AP-t}{c_0} + \frac{t}{c}$$

$$BP = AP - t + \frac{c_0}{c}t. \quad \text{But } \frac{c_0}{c} = \mu$$

$$\therefore \qquad BP - AP = \mu t - t = (\mu-1)t$$

If P is the point originally occupied by the nth fringe then the path difference

$$BP - AP = n\lambda$$

$$\therefore \qquad (\mu-1)t = n\lambda \qquad \ldots(i)$$

Also the distance x through which the fringe is shifted

$$= \frac{n\lambda D}{d}$$

where $\frac{\lambda D}{d} = \beta$, the fringe width.

$$\therefore \qquad x = \frac{n\lambda D}{d}$$

Also, $$n\lambda = \frac{xd}{D}$$

or $$(\mu-1)t = \frac{x \cdot d}{D} \qquad \ldots(ii)$$

Therefore, knowing x, the distance through which the central fringe is shifted, D, d and μ, the thickness of the transparent plate can be calculated. If a monochromatic source of light is used, the fringes will be similar and it is difficult to locate the position where the central fringe shifts after the introduction of the transparent plate. Therefore, white light is used. The fringes will be coloured but the central fringe will be white. When the cross wire is at the central white fringe without the transparent plate in the path, the reading is noted. When the plate is introduced, the position to which the central white fringe shifts is observed. The difference between the two positions on the micrometer scale of the eyepiece gives the value of the shift which is equal to x. Now, with the monochromatic source of light, the micrometer eyepiece is moved through the same distance x and the number of fringes that cross the field of view is observed. Suppose n fringes cross the field of view. Then, from the relation

$$(\mu-1)t = n\lambda$$

the value of t can be calculated. The value of t can also be calculated from equation (ii). However, if t is known, μ can be calculated.

This experiment also shows that light travels more slowly in a medium of refractive index $\mu > 1$, than in air because the central fringe shifts towards the side where the transparent plate is introduced. Had it been opposite, the shift should have been to the other side. The optical path in air $= \mu \times t$, for a medium of thickness t and refractive index μ.

Example 8·14. *When a thin piece of glass $3{\cdot}4 \times 10^{-4}$ cm thick is placed in the path of one of the interfering beams in a biprism arrangement, it is found that the central bright fringe shifts through a distance equal to the width of four fringes. Find the refractive index of the piece of glass. Wavelength of light used is $5{\cdot}46 \times 10^{-5}$ cm.* [*Delhi (Hons.) 1975*]

Here
$$t = 3{\cdot}4 \times 10^{-4} \text{ cm}$$
$$n = 4$$
$$\lambda = 5{\cdot}46 \times 10^{-5} \text{ cm}$$
$$\mu = ?$$
$$(\mu-1)\,t = n\lambda$$
$$\mu = \frac{n\lambda}{t} + 1$$
$$\mu = \frac{4 \times 5{\cdot}46 \times 10^{-5}}{3{\cdot}4 \times 10^{-4}} + 1$$
$$\mu = 0{\cdot}6424 + 1$$
or $$\mu = \mathbf{1{\cdot}6424}$$

Example 8·15. *Fringes are produced with monochromatic light of $\lambda = 5450$ Å. A thin plate of glass of $\mu = 1{\cdot}5$ is then placed normally in the path of one of the interfering beams and the central band of the fringe system is found to move into the position previously occupied by the third bright band from the centre. Calculate the thickness of the glass plate.* (*Delhi B.Sc. Hons. 1971*)

Here $$(\mu-1)\,t = n\lambda$$
$$\mu = 1{\cdot}5, \; n = 3,$$
$$\lambda = 5450 \times 10^{-8} \text{ cm}$$
$$t = \frac{n\lambda}{(\mu-1)}$$
$$t = \frac{3 \times 5450 \times 10^{-8}}{(1{\cdot}5-1)}$$
$$t = \mathbf{0{\cdot}000327 \text{ cm}}$$

Example 8·16. *A transparent plate of thickness 10^{-3} cm is placed in the path of one of the interfering beams of a biprism experiment using light of wavelength 5000 Å. If the central fringe shifts by a distance equal to the width of ten fringes, calculate the refractive index of the material of the plate.* (*Mysore 1970*)

$$(\mu-1)t = n\lambda$$

Here, $$t = 10^{-3} \text{ cm}, \; n = 10$$
$$\lambda = 5000 \times 10^{-8} \text{ cm}, \; \mu = ?$$
$$(\mu-1)10^{-3} = 10 \times 5000 \times 10^{-8}$$
$$\mu = \mathbf{1{\cdot}5}$$

with white light, the central fringe is white and the other fringes are coloured because the width of the fringe, $\beta = \frac{\lambda D}{d}$. The width

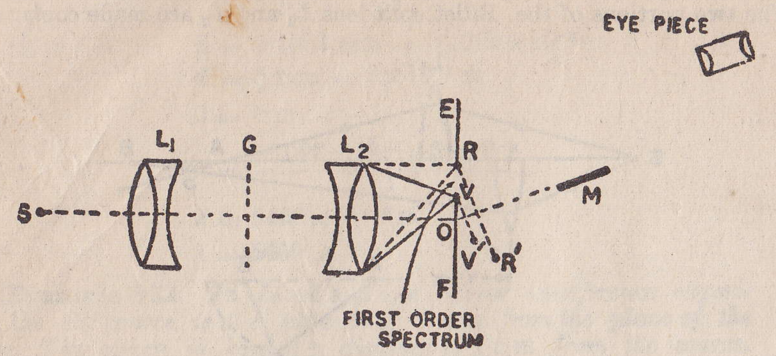

Fig. 8·14.

of the red fringe is more than the blue fringe. Rayleigh designed an experiment to show that it is possible to have white and dark fringes. This can be done if the fringe width is independent of the wavelength of light and is the same for all wavelengths. The fringe width β can be kept constant for all wavelengths, if $\frac{\lambda}{d}$ is the same in all cases.

S is a source of white light (narrow slit) at the focal plane of the converging lens L_1 (Fig. 8·14). The grating G having 800 to 1200 lines per cm is placed normal to light emerging from L_1. Another achromatic lens L_2 is used and the first order spectrum is formed. EF is an opaque screen with a narrow opening in it. The narrow opening is adjusted so that only the first order spectrum (violet to red) is allowed to pass through it. The violet end is nearer to the highly polished Lloyd's mirror M than the red end. The position of M is so adjusted that V' and R' are the images of V and R. Interference occurs between the beams from VR and those from $V'R'$. The violet fringes are produced by V and V' while the red fringes are produced by R ond R'.

Suppose, $VV' = d_1$ and $RR' = d_2$

If $\frac{\lambda_v}{d_1} = \frac{\lambda_r}{d_2}$, the fringe width β will be the same and interference fringes due to different colours will overlap and white achromatic fringes are produced in the field of view. The white and dark fringes are seen through the eyepiece or can be produced on the screen.

Instead of a diffraction grating, a prism of small angle can also be used.

8·15 INTERFERENCE IN THIN FILMS

Newton and Hooke observed and developed the interference phenomenon due to multiple reflections from the surface of thin transparent materials. Everyone is familiar with the beautiful colours produced by a thin film of oil on the surface of water and also by the thin film of a soap bubble. Hooke observed such colours in thin films of mica and similar thin transparent plates. Newton was able to show the interference rings when a convex lens was placed on a plane glass-plate. Young was able to explain the phenomenon on the basis of interference between *light reflected from the top and the bottom surface of a thin film.* It has been observed that interference in the case of thin film takes place due to (1) reflected light and (2) transmitted light.

8·16 INTERFERENCE DUE TO REFLECTED LIGHT (THIN FILMS)

Consider a transparent film of thickness t and refractive index μ. A ray SA incident on the upper surface of the film is partly reflected along AT and partly reflected along AB. At B part of it is reflected along BC and finally emerges out along CQ. The difference in path between the two rays AT and CQ can be calculated. Draw CN normal to AT and AM normal to BC. The angle of incidence is i and the angle of refraction is r. Also produce CB to meet AE produced at P. Here $\angle APC = r$ (Fig. 8·15).

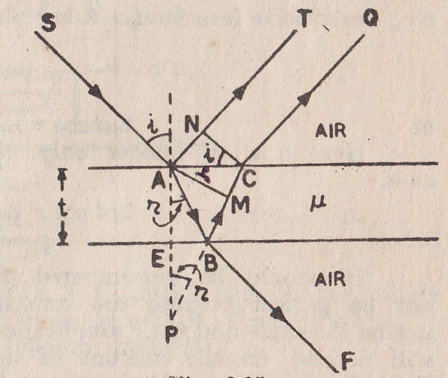

Fig. 8·15.

The *optical* path difference

$$x = \mu(AB+BC)-AN$$

Here, $$\mu = \frac{\sin i}{\sin r} = \frac{AN}{CM}$$

$$\therefore \quad AN = \mu.CM$$

$$\therefore \quad x = \mu\,(AB+BC)-\mu.CM$$
$$= \mu\,(AB+BC-CM) = \mu(PC-CM)$$
$$= \mu.PM$$

In the $\triangle\ APM$,

$$\cos r = \frac{PM}{AP}$$

or $$PM = AP.\cos r = (AE+EP)\cos r$$
$$= 2t\cos r$$
$$(\because\ AE = EP = t)$$

$$\therefore \quad x = \mu.PM = 2\,\mu t\cos r \qquad \ldots(i)$$

This equation (i), in the case of reflected light does not represent the correct path difference but only the apparent. It has been established on the basis of electromagnetic theory that, when light is reflected from the surface of an optically denser medium (air-medium interface) a phase change π, equivalent to a path difference $\frac{\lambda}{2}$, occurs.

Therefore, the correct path difference in this case,

$$x = 2\,\mu t\,\cos r - \frac{\lambda}{2} \quad \ldots(ii)$$

(1) If the path difference $x = n\lambda$, where $n = 0, 1, 2, 3, 4\ldots$ etc., constructive interference takes place and the film appears bright.

$$\therefore \quad 2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

or

$$2\,\mu t\,\cos r = (2n+1)\,\frac{\lambda}{2} \quad \ldots(iii)$$

(2) If the path difference $x = (2n+1)\frac{\lambda}{2}$ where $n = 0, 1, 2\ldots$. etc., destructive interference takes place and the film appears dark.

$$\therefore \quad 2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or

$$2\mu t \cos r = (n+1)\,\lambda \quad \ldots(iv)$$

Here n is an integer only, therefore $(n+1)$ can also be taken as n.

$$\therefore \quad 2\mu t \cos r = n\lambda \quad \ldots(v)$$

where $n = 0, 1, 2, 3, 4\ldots$etc.

It should be remembered that the interference pattern will not be perfect because the intensities of the rays AT and CQ will not be the same and their amplitudes are different. The amplitudes will depend on the amount of light reflected and transmitted through the films. It has been found that for normal incidence, about 4% of the incident light is reflected and 96% is transmitted. There is a small difference in the amplitudes of the rays AT and CQ. Therefore, the intensity never vanishes completely and perfect dark fringes will not be observed for the rays AT and CQ alone. But in the case of multiple reflection, the intensity of the minima will be zero.

Consider reflected rays 1, 2, 3 etc. as shown in Fig. 8·16. The amplitude of the incident ray is a. Let r be the reflection coefficient, t the transmission coefficient from rarer to denser medium and t' the transmission coefficient from denser to rarer medium.

The amplitudes of the reflected rays are, ar, $atrt'$, atr^3t', atr^5t' ...etc. The ray 1 is reflected at the surface of a denser medium. It undergoes a phase change π. The rays 2, 3, 4 etc. are all in phase but out of phase with ray 1 by π.

The resultant amplitude of 2, 3, 4 etc. is given by

$$A = atrt' + atr^3t' + atr^5t' + \ldots$$

$$A = att'r\,[1 + r^2 + r^4 + \ldots]$$

as r is less than 1, the terms inside the bracket form a geometric series.

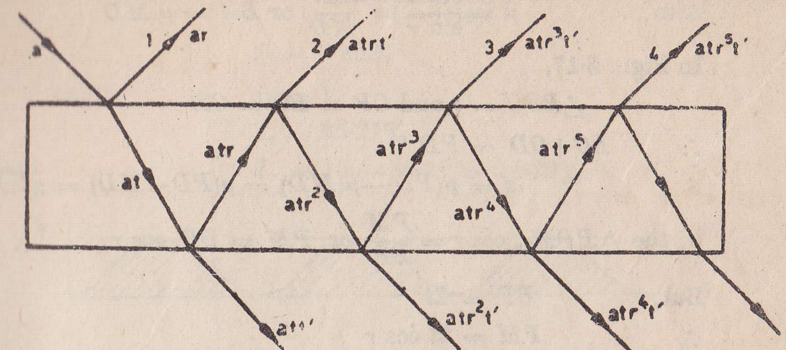

Fig. 8·16.

$$A = att'r\left[\frac{1}{1-r^2}\right]$$

$$A = \left[\frac{att'r}{1-r^2}\right]$$

According to the principle of reversibility,

$$tt' = 1 - r^2$$

$$A = \frac{a(1-r^2)r}{(1-r^2)} = ar$$

Thus, the resultant amplitude of 2, 3, 4,...etc. is equal in magnitude of the amplitude of ray 1 but out of phase with it. Therefore the minima of the reflected system will be of zero intensity.

8·17 INTERFERENCE DUE TO TRANSMITTED LIGHT (THIN FILMS)

Consider a thin transparent film of thickness t and refractive index μ. A ray SA after refraction goes along AB. At B it is partly reflected along BC and partly refracted along BR. The ary BC after reflection at C, finally emerges along DQ. Here at B and C reflection takes place at the rarer medium (medium—air interface). Therefore, no phase change occurs. Draw BM normal to CD and DN normal to BR. The optical path difference between DQ and BR is given by,

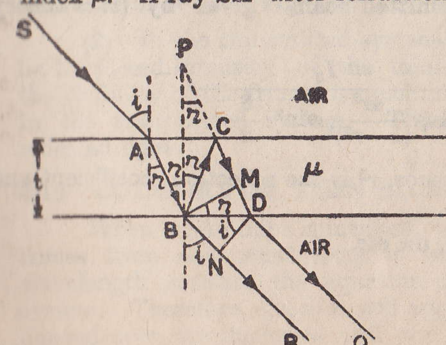

Fig. 8·17.

$$x = \mu(BC+CD)-BN$$

Also $$\mu = \frac{\sin i}{\sin r} = \frac{BN}{MD}, \text{ or } BN = \mu.MD$$

In Fig. 8·17,

$$\angle BPC = r \text{ and } CP = BC = CD$$

$$\therefore \quad BC+CD = PD$$

$$\therefore \quad x = \mu(PD)-\mu(MD) = \mu(PD-MD) = \mu PM$$

In the $\triangle BPM$, $\cos r = \frac{PM}{BP}$ or, $PM = BP.\cos r$

But, $$BP = 2t$$

$$\therefore \quad PM = 2t \cos r$$

$$\therefore \quad x = \mu.PM = 2\mu t \cos r \qquad ...(i)$$

(i) For bright fringes, the path difference $x = n\lambda$

$$\therefore \quad 2\mu t \cos r = n\lambda \qquad ...(ii)$$

where $n = 0, 1, 2, 3...$etc.

(ii) For dark fringes, the path difference $x = (2n+1)\frac{\lambda}{2}$

$$\therefore \quad 2\mu t \cos r = \frac{(2n+1)\lambda}{2}$$

where $n = 0, 1, 2, 3,...$etc.

In the case of transmitted light, the interference fringes obtained are less distinct because the difference in amplitude between BR and DQ is very large. However, when the angle of incidence is nearly 45°, the fringes are more distinct.

8·18 INTENSITIES OF MAXIMA AND MINIMA IN THE INTERFERENCE PATTERN OF REFLECTED AND TRANSMITTED BEAMS IN THIN FILMS

The intensity of the transmitted beam is given by (*vide* theory of Fabry-Perot Interferometer)

$$I_t = \frac{I_0}{1+\frac{4r^2}{(1-r^2)^2}\sin^2\frac{\delta}{2}}$$

Here δ is the phase difference, r^2 is the reflection coefficient and I_0 is the maximum intensity.

For values of $\delta = \pi, 3\pi, 5\pi$ etc.

$$\sin^2\frac{\delta}{2} = 1$$

For $r^2 = 0{\cdot}04$ [*i.e.* Reflectance of 4%]

$$I_t = \frac{I_0}{1+\frac{4\times 0{\cdot}04}{(1-0{\cdot}04)^2}}$$

$$I_t = 0{\cdot}8521\ I_0$$

Taking $I_0 = 1$,

$$I_t = 85{\cdot}21\%$$

and $$I_r = 100-85{\cdot}21 = 14{\cdot}79\%$$

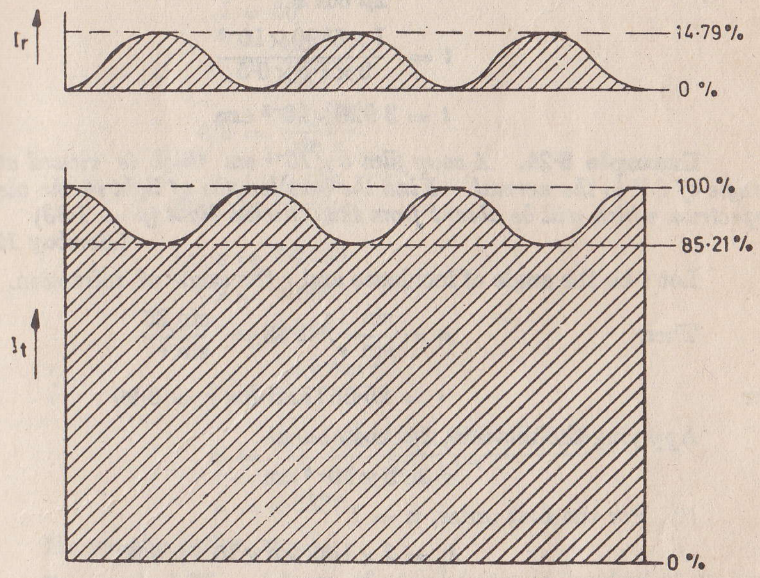

Fig. 8·18.

(1) In the reflected system, the intensity of the interference maxima will 14·79% of the incident intensity and the intensity of the minima will be zero (Fig. 8·18).

(2) In the transmitted system, the intensity of the maxima will be 100% and intensity of the minima will be 85·21%. It means the visibility of the fringes is much higher in the reflected system than in the transmitted system. Thus, the fringes are more sharp in reflected light.

8·19 COLOURS OF THIN FILMS

When white light is incident on a thin film, the light which comes from any point from it will not include the colour whose wavelength satisfies the equation $2\mu t \cos r = n\lambda$, in the reflected system. Therefore, the film will appear coloured and the colour will depend upon the thickness and the angle of inclination. If r and t are constant, the colour will be uniform. In the case of oil on water different colours are seen because r and t vary. This is clear from the following solved example.

Example 8·31. *A square piece of cellophane film with index of refraction 1·5 has a wedge-shaped section so that its thickness at two opposite sides are t_1 and t_2. If with a light of $\lambda = 6000$ Å, the number of fringes appearing in the film is 10, calculate the difference $t_2 - t_1$.*
(*Agra 1965*)

Here
$$x = 10\beta, \qquad \beta = \frac{x}{10}$$
$$\theta = \frac{t_2 - t_1}{x} \qquad \lambda = 6000 \times 10^{-8} \text{ cm}$$
$$\mu = 1 \cdot 50$$

But
$$\beta = \frac{\lambda}{2\theta\mu}$$
$$\lambda = 2\theta\mu\beta$$
$$6000 \times 10^{-8} = \frac{2 \times (t_2 - t_1) \times 1 \cdot 5 \times x}{x \times 10}$$
$$t_2 - t_1 = \frac{6000 \times 10^{-8} \times 10}{2 \times 1 \cdot 5}$$
$$= \mathbf{2 \times 10^{-4} \text{ cm}}$$

Example 8·32. *Fringes of equal thickness are observed in a thin glass wedge of refractive index 1·52. The fringe spacing is 1* mm *and wavelength of light is 5893 Å. Calculate the angle of the wedge in seconds of an arc.*

Here
$$\mu = 1 \cdot 52 \qquad \lambda = 5893 \times 10^{-8} \text{ cm}$$
$$\beta = 1 \text{ mm} = 0 \cdot 1 \text{ cm}$$
$$\beta = \frac{\lambda}{2\mu\theta} \qquad \theta = \frac{\lambda}{2\mu\beta}$$

or
$$\theta = \frac{5893 \times 10^{-8}}{2 \times 1 \cdot 52 \times 0 \cdot 1} \text{ radian}$$
$$\theta = \frac{5893 \times 10^{-8} \times 180 \times 7 \times 60 \times 60}{0 \cdot 304 \times 22} \text{ seconds of an arc}$$
$$\theta = \mathbf{39 \cdot 96 \text{ seconds of an arc.}}$$

Example 8·33. *Interference fringes are produced by monochromatic light falling normally on a wedge shaped film of cellophane of refractive index 1·40. If the angle of the wedge is 20 seconds of an arc and the distance between successive fringes is 0·25* cm. *Calculate the wavelength of light.*
(*Delhi (Sub) 1977*)

Here,
$$\theta = 20 \text{ seconds of an arc}$$

or
$$\theta = \frac{20 \times \pi}{60 \times 60 \times 180} \text{ radian}$$
$$\beta = 0 \cdot 25 \text{ cm}$$

$$\mu = 1 \cdot 40$$
$$\beta = \frac{\lambda}{2\theta\mu}$$
$$\lambda = 2\theta\mu\beta$$
$$\lambda = \frac{2 \times 20 \times 22 \times 1 \cdot 40 \times 0 \cdot 25}{60 \times 60 \times 180 \times 7}$$
$$\lambda = \mathbf{6790 \times 10^{-8} \text{ cm}}$$
$$\lambda = \mathbf{6790 \text{ Å}}$$

8·23. NEWTON'S RINGS

When a plano-convex lens of long focal length is placed on a plane glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is very small at the point of contact and gradually increases from the centre outwards. The fringes produced with monochromatic light are circular. The fringes are concentric circles, uniform in thickness and with the point of contact as the centre. When viewed with white light, the fringes are coloured. With monochromatic light, bright and dark circular fringes are produced in the air film.

S is a source of monochromatic light at the focus of the lens L_1 (Fig. 8·25). A horizontal beam of light falls on the glass plate *B* at

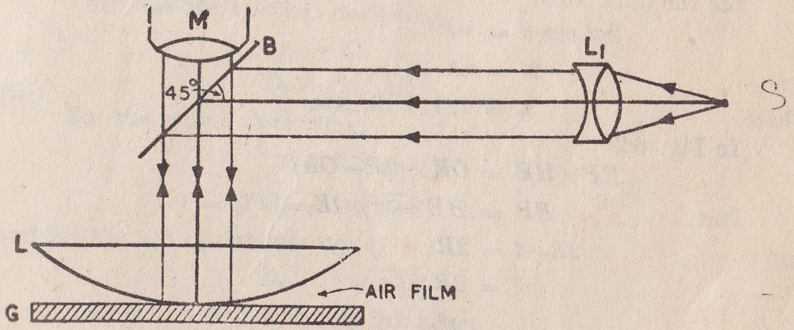

Fig. 8·25.

45°. The glass plate *B* reflects a part of the incident light towards the air film enclosed by the lens *L* and the plane glass plate *G*. The reflected beam from the air film is viewed with a microscope. Interference takes place and dark and bright circular fringes are produced. This is due to the interference between the light reflected from the lower surface of the lens and the upper surface of the glass plate *G*.

Theory. (*i*) **Newton's rings by reflected light.** Suppose the radius of curvature of the lens is *R* and the air film of thickness *t* is at a distance of $OQ = r$, from the point of contact *O*.

Here, interference is due to reflected light. Therefore, for the bright ring

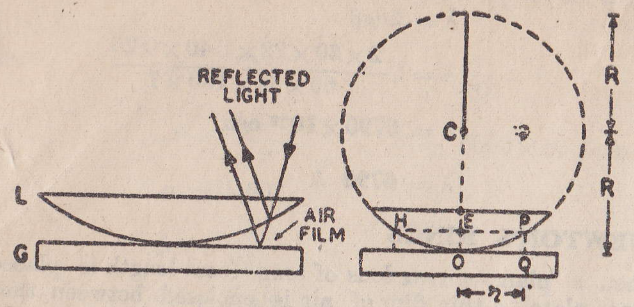

Fig. 8·26.

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2} \quad ...(i)$$

where $n = 1, 2, 3...$etc.

Here, θ is small, therefore $\cos\theta = 1$

For air, $\mu = 1$

$$2t = (2n-1)\frac{\lambda}{2} \quad ...(ii)$$

For the dark ring,

$$2\mu t \cos\theta = n\lambda$$

or $2t = n\lambda$

where $n = 0, 1, 2, 3...$etc. ...(*iii*)

In Fig. 8·26,

$$EP \times HE = OE \times (2R - OE)$$

But, $EP = HE = r,\ OE = PQ = t$

and $2R - t = 2R$ (approximately)

$$r^2 = 2R.t$$

or

$$t = \frac{r^2}{2R}$$

Substituting the value of t in equations (*ii*) and (*iii*),

For bright rings

$$r^2 = \frac{(2n-1)\lambda R}{2} \quad ...(iv)$$

or

$$r = \sqrt{\frac{(2n-1)\lambda R}{2}} \quad ...(v)$$

For dark rings,

$$r^2 = n\lambda R \quad ...(vi)$$

$$r = \sqrt{n\lambda R} \quad ...(vii)$$

when $n = 0$, the radius of the dark ring is zero and the radius of the bright ring is $\sqrt{\frac{\lambda R}{2}}$. Therefore, the centre is dark. Alternately dark and bright rings are produced (Fig. 8·27).

Result. The radius of the dark ring is proportional to (*i*) $\sqrt{n}$ (*ii*) $\sqrt{\lambda}$ and (*iii*) $\sqrt{R}$. Similarly the radius of the bright ring is proportional to

(*i*) $\sqrt{\frac{2n-1}{2}}$ (*ii*) $\sqrt{\lambda}$

and (*iii*) $\sqrt{R}$.

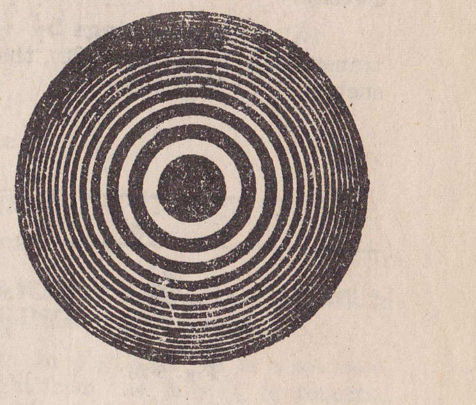

Fig. 8·27.

If D is the diameter of the dark ring,

$$D = 2r = 2\sqrt{n\lambda R} \quad ...(viii)$$

For the central dark ring

$$n = 0$$

$$D = 2\sqrt{n\lambda R} = 0$$

This corresponds to the centre of the Newton's rings. While counting the order of the dark rings 1, 2, 3, etc. the central ring is not counted.

Therefore for the first dark ring,

$$n = 1$$

$$D_1 = 2\sqrt{\lambda R}$$

For the second dark ring,

$$n = 2,$$

$$D_2 = 2\sqrt{2\lambda R}$$

and for the nth dark ring,

$$D_n = 2\sqrt{n\lambda R}$$

Take the case of 16th and 9th rings,

$$D_{16} = 2\sqrt{16\lambda R} = 8\sqrt{\lambda R},$$

$$D_9 = 2\sqrt{9\lambda R} = 6\sqrt{\lambda R}$$

The differerce in diameters between the 16th and the 9th rings,

$$D_{16} - D_9 = 8\sqrt{\lambda R} - 6\sqrt{\lambda R} = 2\sqrt{\lambda R}$$

Similarly the difference in the diameters between the fourth and the first rings,

$$D_4 - D_1 = 2\sqrt{4\lambda R} - 2\sqrt{\lambda R} = 2\sqrt{\lambda R}$$

Therefore the fringe width decreases with the order of the fringe and the fringes get closer with increase in their order.

For bright rings,

$$r^2 = \frac{(2n-1)\lambda R}{2} \qquad \ldots(ix)$$

or

$$D^2 = 2(2n-1)\lambda R \qquad \ldots(x)$$

$$r_n = \sqrt{\frac{(2n-1)\lambda R}{2}} \qquad \ldots(xi)$$

In equation (ix), substituting n = 1, 2, 3 (number of the ring), the radii of the first, second, third etc., bright rings can be obtained directly.

(*ii*) **Newton's rings by transmitted light.** In the case of transmitted light (Fig. 8·28), the interference fringes are produced such that for bright rings,

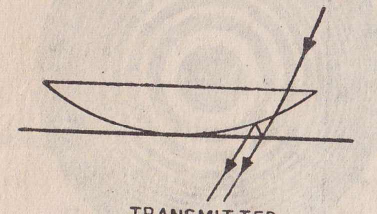

Fig. 8·28.

$$2\mu t \cos\theta = n\lambda \qquad \ldots(xii)$$

and for dark rings

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2} \qquad \ldots(xiii)$$

Here, for air

$$\mu = 1,$$

and $\cos\theta = 1$

For bright rings,

$$2t = n\lambda$$

and for dark rings $2t = (2n-1)\dfrac{\lambda}{2}$

Taking the value of $t = \dfrac{r^2}{2R}$, where r is the radius of the ring and R the radius of curvature of the lower surface of the lens, the radius for the bright and dark rings can be calculated.

For bright rings,

$$r^2 = n\lambda R \qquad \ldots(xiv)$$

For dark rings,

$$r^2 = \frac{(2n-1)\lambda R}{2} \qquad \ldots(xv)$$

where $n = 1, 2, 3\ldots$etc.

When, $n = 0$, for bright rings $r = 0$.

Therefore, in the case of Newton's rings due to transmitted light, the central ring is bright (Fig. 8·29) *i.e.*, just opposite to the rings due to reflected light.

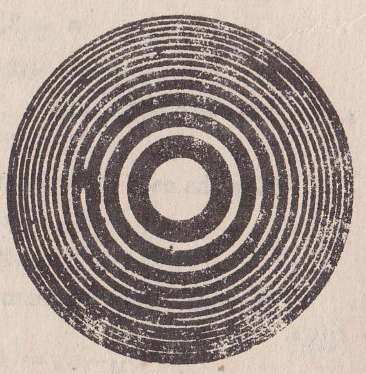
Fig. 8·29.

Example 8·34. *A thin equiconvex lens of focal length 4 metres and refractive index 1·50 rests on and in contact with an optical flat, and using light of wavelength 5460Å, Newton's rings are viewed normally by reflection. What is the diameter of the 5th bright ring?*

The diameter of n th bright ring is given by

$$D_n = \sqrt{2(2n-1)\lambda R}$$

Here $n = 5, \quad \lambda = 5460\times10^{-8}$ cm

$f = 400$ cm, $\mu = 1{\cdot}50$

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here $R_1 = R, \quad R_2 = -R$

$$\therefore \quad \frac{1}{f} = (\mu-1)\left(\frac{2}{R}\right)$$

$$\frac{1}{400} = (1{\cdot}50-1)\left(\frac{2}{R}\right)$$

$$R = 400 \text{ cm}$$

$$\therefore \quad D_n = \sqrt{2\times(2\times5-1)\times5460\times10^{-8}\times400}$$

$$D_n = \mathbf{0{\cdot}627\ cm}$$

8·24 DETERMINATION OF THE WAVELENGTH OF SODIUM LIGHT USING NEWTON'S RINGS

The arrangement used is shown in Fig. 8·25. S is a source of sodium light. A parallel beam of light from the lens L_1 is reflected by the glass plate B inclined at an angle of 45° to the horizontal. L is a plano-convex lens of large focal length. Newton's rings are viewed through B by the travelling microscope M focussed on the air film. Circular bright and dark rings are seen with the centre dark. With the help of a travelling microscope, measure the diameter of the n th dark ring.

Suppose, the diameter of the n th ring $= D_n$

$$r_n^2 = n\lambda R$$

But, $\quad r_n = \dfrac{D_n}{2}$

$$\therefore \quad \frac{(D_n)^2}{4} = n\lambda R$$

or

$$D_n^2 = 4n\lambda R \qquad \ldots(i)$$

Measure the diameter of the $(n+m)$ th dark ring. Let it be D_{n+m}

$$\therefore \quad \frac{(D_{n+m})^2}{4} = (n+m)\lambda R$$

or

$$(D_{n+m})^2 = 4(n+m)\lambda R \qquad \ldots(ii)$$

Subtracting (*i*) from (*ii*)

$$(D_{n+m})^2-(D_n)^2 = 4m\lambda R$$

or
$$\lambda = \frac{(D_{n+m})^2-(D_n)^2}{4mR} \qquad ...(iii)$$

Hence, λ can be calculated. Suppose the diameters of the 5th ring and the 15th ring are determined. Then, $m = 15-5 = 10$.

$$\therefore \qquad \lambda = \frac{(D_{15})^2-(D_5)^2}{4 \times 10R} \qquad ...(iv)$$

The radius of curvature of lower surface of the lens is determined with the help of a spherometer but more accurately it is determined by Boy's method. Hence the wavelength of a given monochromatic source of light can be determined.

Example 8·35. *A plano-convex lens of radius 300 cm is placed on an optically flat glass plate and is illuminated by monochromatic light. The diameter of the 8th dark ring in the transmitted system is 0·72 cm. Calculate the wavelength of light used.*

[Delhi 1976; Delhi B.Sc. (Hons) 1972)

For the transmitted system,

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

Here $n = 8, \quad D = 0{\cdot}72 \text{ cm}, \quad r = 0{\cdot}36 \text{ cm}$

$R = 300 \text{ cm}, \quad \lambda = ?$

$$\lambda = \frac{2r^2}{(2n-1)R} \qquad \lambda = \frac{2 \times (0{\cdot}36)^2}{(2 \times 8-1)\ 300}$$

$$\lambda = 5760 \times 10^{-8} \text{ cm}$$

or $\lambda = 5760$ Å

Example 8·36. *In a Newton's rings experiment the diameter of the 15th ring was found to be 0·590 cm and that of the 5th ring was 0·336 cm. If the radius of the plano-convex lens is 100 cm, calculate the wavelength of light used.* [Delhi 1972]

Here $D_5 = 0{\cdot}336 \text{ cm} \qquad D_{15} = 0{\cdot}590 \text{ cm}.$

$R = 100 \text{ cm} \qquad m = 10,$

$$\lambda = \frac{(D_{n+m})^2-(D_n)^2}{4mR} \qquad \lambda = \frac{D_{15}^2-D_5^2}{4 \times 10 \times R}$$

$$\lambda = \frac{(0{\cdot}590)^2-(0{\cdot}336)^2}{4 \times 10 \times 100} \qquad \lambda = 5880 \times 10^{-8} \text{ cm}$$

or $\lambda = 5880$ Å

Example 8·37. *In a Newton's rings experiment, the diameter of the 5th ring was 0·336 cm and the diameter of the 15th ring = 0·590 cm. Find the radius of curvature of the plano-convex lens if the wavelength of light used is 5890 Å.*

Here $D_5 = 0{\cdot}336 \text{ cm.}, D_{15} = 0{\cdot}590 \text{ cm}, R = ?$

and $m = 10, \quad \lambda = 5890 \times 10^{-8} \text{ cm}$

$$R = \frac{(D_{n+m})^2-(D_n^2}{4m\lambda} = \frac{D_{15}^2-D_5^2}{4 \times 10 \times \lambda}$$

$$R = \frac{(0{\cdot}590)^2-(0{\cdot}336)^2}{4 \times 10 \times 5890 \times 10^{-8}}$$

$$= 99{\cdot}82 \text{ cm}$$

Example 8·38. *In a Newton's rings experiment, find the radius of curvature of the lens surface in contact with the glass plate when with a light of wavelength 5890×10^{-8} cm, the diameter of the third dark ring is 3·2 mm. The light is falling at such an angle that it passes through the air film at an angle of zero degree to the normal.*

[Rajasthan, 1975]

For dark rings

$$r^2 = n\lambda R \ ; \ R = \frac{r^2}{n\lambda}$$

Here $r = \frac{3{\cdot}2}{2} \text{ mm} = 1{\cdot}6 \text{ mm}$

or $r = 0{\cdot}16 \text{ cm}$

$n = 3$

$\lambda = 5890 \times 10^{-8} \text{ cm}$

$$\therefore \qquad R = \frac{(0{\cdot}16)^2}{3 \times 5890 \times 10^{-8}}$$

$$R = 144{\cdot}9 \text{ cm}$$

8·25. REFRACTIVE INDEX OF A LIQUID USING NEWTON'S RINGS

The experiment is performed when there is air film between the plano-convex lens and the optically plane glass plate. These are kept in a metal container C. The diameters of the nth and the $(n+m)$th dark rings are determined with the help of a travelling microscope (Fig. 8·30).

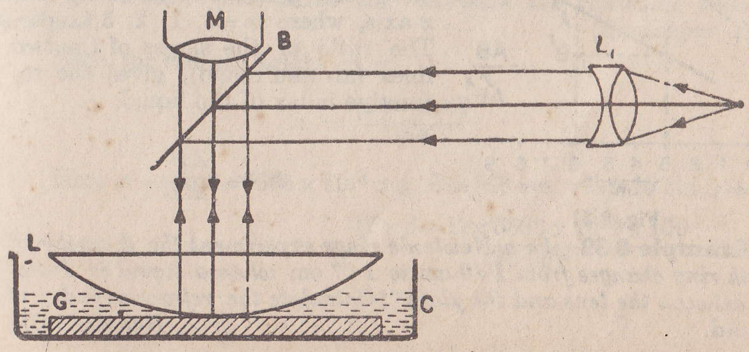

Fig. 8·30.

For air, $(D_{n+m})^2 = 4(n+m)\lambda R \quad ; \quad D_n^2 = 4n\lambda R$

$$D_{n+m}^2 - D_n^2 = 4m\lambda R \qquad \ldots(i)$$

The liquid is poured in the container C without disturbing the arrangement. The air film between the lower surface of the lens and the upper surface of the plate is replaced by the liquid. The diameters of the nth ring and the $(n+m)$ th ring are determined.

For the liquid, $2\mu t \cos\theta = n\lambda$ for dark rings

or $$2\mu t = n\lambda. \quad \text{But,} \quad t = \frac{r^2}{2R}$$

or $$\frac{2\mu r^2}{2R} = n\lambda$$

or $$r^2 = \frac{n\lambda R}{\mu}. \quad \text{But } r = \frac{D}{2} \; ; \; D^2 = \frac{4n\lambda R}{\mu}$$

If D'_n is the diameter of the nth ring and D'_{n+m} is the diameter of the $(n+m)$ th ring

then, $$(D'_{n+m})^2 = \frac{4(n+m)\lambda R}{\mu} \; ; \qquad (D'_n)^2 = \frac{4n\lambda R}{\mu}$$

or $$(D'_{n+m})^2 - (D'_n)^2 = \frac{4m\lambda R}{\mu} \qquad \ldots(ii)$$

or $$\mu = \frac{4m\,\lambda R}{(D'_{n+m})^2 - (D'^2_n)} \qquad \ldots(iii)$$

If m, λ, R, D'_{n+m} and D'_n are known, μ can be calculated. If λ is not known, then divide (iii) by (i)

$$\mu = \frac{(D_{n+m})^2 - (D_n)^2}{(D'_{n+m})^2 - (D'_n)^2} \qquad \ldots(iv)$$

Graphical Method. The diameters of the dark rings are determined for various orders, varying from the n th ring to the $(n+m)$th ring, first with air as the medium and then with the liquid. A graph is plotted between D_{n+m}^2 along the y-axis and m along the x-axis, where $m=0, 1, 2, 3, \ldots$ etc. The ratio of the slopes of the two lines (air and liquid), gives the refractive index of the liquid.

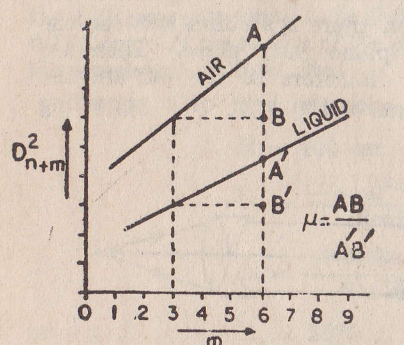

Fig. 8·31

$$\mu = \frac{AB}{A'B'}$$

Example 8·39. *In a Newton's rings experiment the diameter of the 10th ring changes from 1·40 cm to 1·27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.* (*Nagpur 1974*)

For liquid medium $$D_1^2 = \frac{4n\lambda R}{\mu} \qquad \ldots(i)$$

For air medium

$$D_2^2 = 4n\lambda R \qquad \ldots(ii)$$

Dividing (ii) by (i)

$$\mu = \left(\frac{D_2}{D_1}\right)^2$$

Here $D_1 = 1{\cdot}27$ cm, $D_2 = 1{\cdot}40$ cm

$$\therefore \qquad \mu = \left(\frac{1{\cdot}40}{1{\cdot}27}\right)^2 = 1{\cdot}215$$

Example 8·40. *In a Newton's rings arrangement, if a drop of water ($\mu = 4/3$) be placed in between the lens and the plate, the diameter of the 10th ring is found to be 0·6* cm. *Obtain the radius of curvature of the face of the lens in contact with the plate. The wavelength of light used is 6000 Å.* (*Delhi 1973*)

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

or $$R = \frac{\mu D_n^2}{4n\lambda}$$

Here $$\mu = \frac{4}{3}, \; D_n = 0{\cdot}6 \text{ cm}$$

$$n = 10, \quad \lambda = 6000 \text{ A} = 6 \times 10^{-5} \text{ cm}$$

$$R = ?$$

$$R = \frac{4 \times (0{\cdot}6)^2}{3 \times 4 \times 10 \times 6 \times 10^{-5}}$$

or $$R = 200 \text{ cm}$$

Example 8·41. *Newton's rings are formed by reflected light of wavelength 5895 Å with a liquid between the plane and curved surfaces. If the diameter of the 5th bright ring is 3* mm *and the radius of curvature of the curved surface is 100* cm, *calculate the refractive index of the liquid.* (*Gorakhpur 1966*)

Here, for the nth bright ring,

$$\mu = \frac{(2n-1)\lambda R}{2r^2}$$

Here $n = 5$, $\lambda = 5895 \times 10^{-8}$ cm, $R = 100$ cm, $r = 0{\cdot}15$ cm, $\mu = ?$

$$\mu = \frac{(2 \times 5 - 1) \times 5895 \times 10^{-8} \times 100}{2(0{\cdot}15)^2}$$

$$\mu = 1{\cdot}179$$