CHAPTER II

COMPOSITION OF SIMPLE HARMONIC MOTIONS

Introduction - Composition of two simple harmonic vibrations of same frequency but different phase and amplitude - Composition of two simple harmonic vibrations at right angles to each other having equal frequencies but differing in phases and amplitudes - Lissajous' figures - Composition of two simple harmonic motion at right angles to each other and having time periods in the ratio 1:2 - Experimental determination of Lissajous' figures - Uses of Lissajous' figures - Solved problems - Exercises.

2.1 *Introduction*

Very often problems are encountered in physics where a particle is simultaneously acted upon by more than one simple harmonic vibrations acting either along the same straight line or at right angles to each other. The resultant displacement, velocity, acceleration, etc., of the particle is given by the vector (algebraic) sum of the corresponding quantities due to the individual waves. There are two general methods for solving such problems based on (i) a graphical treatment and (ii) analytical treatment of the dynamics of the particle. The analytical treatment is based on finding the vector sum of the individual motions either with the help of trigonometric functions or writing them as complex quantities. This is usually the easiest to handle and is at the same time most informative. This method of treatment will be followed in the ensuing articles.

2.2 *Composition of two simple harmonic vibrations of same frequency but different phase and amplitude*

Let a particle in a medium be simultaneously acted upon by two simple harmonic vibrations of same frequency but different phase and amplitude given by the following equations.

$$y_1 = a_1 \sin (wt + \alpha_1) \qquad (2.1)$$

$$y_2 = a_2 \sin (wt + \alpha_2) \qquad (2.1)$$

where y_1 and y_2 are the displacements of the particles due to the individual vibrations of amplitudes a_1 and a_2 and angles of epochs α_1 and α_2 respectively. The two vibrations have the same angular frequency w. The resultant displacement y of the particle will be given by the vector sum of the individual displacements so that

$$\begin{aligned} y &= y_1 + y_2 \\ &= a_1 \sin (wt + \alpha_1) + a_2 \sin (wt + \alpha_2) \\ &= a_1 (\sin wt \cos \alpha_1 + \cos wt \sin \alpha_1) \\ &\quad + a_2 (\sin wt \cos \alpha_2 + \cos wt \sin \alpha_2) \\ &= (a_1 \cos \alpha_1 + a_2 \cos \alpha_2) \sin wt \\ &\quad + (a_1 \sin \alpha_1 + a_2 \sin \alpha_2) \cos wt \qquad (2.3) \end{aligned}$$

The amplitudes a_1 and a_2 and the angles of epoch α_1 and α_2 of the two vibrations are constant.

Hence putting

$$a_1 \cos \alpha_1 + a_2 \cos \alpha_2 = A \cos \phi$$

and $a_1 \sin \alpha_1 + a_2 \sin \alpha_2 = A \sin \phi$

the resultant amplitude can be written as

$$\begin{aligned} y &= A \cos \phi \sin wt + A \sin \phi \cos wt \\ &= A \sin (wt + \phi) \qquad (2.4) \end{aligned}$$

Thus the equation of the resultant vibration as given by eqn. (2.4) is simple harmonic and is very much similar to either eqn. (2.1) or (2.2). The amplitude of the resultant vibration is A while the epoch angle is ϕ, the time period of the resultant vibration remaining same as the original vibrations. The values of A and ϕ in eqn. (2.4) can be determined as follows :

$$\begin{aligned} A^2 \sin^2 \phi + A^2 \cos^2 \phi &= a_1^2 \sin^2 \alpha_1 + a_2^2 \sin^2 \alpha_2 + 2a_1 a_2 \sin \alpha_1 \sin \alpha_2 \\ &\quad + a_1^2 \cos^2 \alpha_1 + a_2^2 \cos^2 \alpha_2 + 2a_1 a_2 \cos \alpha_1 \cos \alpha_2 \\ &= a_2^2 (\sin^2 \alpha_1 + \cos^2 \alpha_1) + a_2^2 (\sin^2 \alpha_2 + \cos^2 \alpha_2) + 2a_1 a_2 (\sin \alpha_1 \\ &\quad \sin \alpha_2 + \cos \alpha_1 \cos \alpha_2) \end{aligned}$$

or, $$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos (\alpha_1 - \alpha_2) \qquad (2.5)$$

and

(ii) ÷ (i)

$$\tan\phi = \frac{A\sin\phi}{A\cos\phi}$$

$$= \frac{a_1 \sin\alpha_1 + a_2 \sin\alpha_2}{a_1 \cos\alpha_1 + a_2 \cos\alpha_2}$$

special cases :

(i) *same phase* : if the two simple harmonic vibrations are in the same phase, then $\alpha_1 = \alpha_2 = \alpha$ (say). Thus if the two vibrations acting on the particle are in the same phase or if the phase difference

$(\alpha_1 - \alpha_2) = 0, 2\pi, 4\pi, \ldots\ldots = 2n\pi$ where $n = 0, 1, 2, \ldots\ldots$, then we get from eqn. (2.5)

$\cos(\alpha_1 - \alpha_2) = 1$ and

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2$$

$$= (a_1 + a_2)^2$$

or, $A = a_1 + a_2$

(ii) ÷ (i) when $\alpha_1 = \alpha_2 = \alpha$

$$\text{and } \tan\phi = \frac{(a_1+a_2)\sin\alpha}{(a_1+a_2)\sin\alpha} = \tan\alpha. \qquad \therefore \phi = \alpha$$

cos α

In that case eqn. (2.4) can be rewritten as

$$y = (a_1 + a_2)\sin(wt + \alpha).$$

(ii) *opposite phase* : If the two vibrations acting on the particle are in opposite phase *i.e.*, if the phase difference $(\alpha_1 - \alpha_2) = \pi, 3\pi, 5\pi, \ldots\ldots = (2n+1)\pi$ where $n = 0, 1, 2, \ldots\ldots$, we have $\cos(\alpha_1 - \alpha_2) = -1$ and

$$A^2 = a_1^2 + a_2^2 - 2a_1a_2$$

$$= (a_1 - a_2)^2$$

or, $A = a_1 - a_2$.

If, in addition, the amplitudes of the individual vibrations are equal, *i.e.*,

$a_1 = a_2 = a$ (say), then

for the *same phase* condition

$\therefore y = 2a \sin(\omega t + \alpha)$

$A = 2a$, and $A^2 = 4a^2$

while in case of *opposite phase*

$A = 0$ (*i.e.*, the resultant vibration is zero).

$y = 0$

Example 2.1. *Two simple harmonic motions acting simultaneously on a particle are given by the equations*

$$y_1 = 2\sin(wt + \pi/6)$$

$$y_2 = 3\sin(wt + \pi/3)$$

Calculate (i) amplitude, (ii) phase constant and (iii) time period of the resultant vibration.

What is the equation of the resultant vibration ?

Soln :

$$y_1 = 2\sin(wt + \pi/6) \qquad \text{(i)}$$

$$y_2 = 3\sin(wt + \pi/3) \qquad \text{(ii)}$$

The equations are similar to the equations

$$y_1 = a_1\sin(wt + \alpha_1) \qquad \text{(iii)}$$

$$y_2 = a_2\sin(wt + \alpha_2) \qquad \text{(iv)}$$

The equation of the resultant vibration is given by

$$y = A\sin(wt + \phi) \qquad \text{(v)}$$

$$\text{where } A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(\alpha_1 - \alpha_2) \qquad \text{(vi)}$$

$$\text{and } \phi = \tan^{-1}\left(\frac{a_1\sin\alpha_1 + a_2\sin\alpha_2}{a_1\cos\alpha_1 + a_2\cos\alpha_2}\right) \qquad \text{(vii)}$$

Here $a_1 = 2$, $a_2 = 3$, $\alpha_1 = \pi/6$, $\alpha_2 = \pi/3$

Hence,

(i) the resultant amplitude

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\alpha_2 - \alpha_2)}$$

$$= \sqrt{4+9+(2)(2)(3)\cos\left(-\frac{\pi}{6}\right)}$$

$$= 4.939$$

(ii) $$\phi = \tan^{-1}\left[\frac{(2)(0.5)+(3)(0.866)}{(2)(0.866)+(3)(0.5)}\right]$$

$$= \tan^{-1}[1.114]$$

$$= 48.1^\circ \simeq \frac{4\pi}{15}$$

$\therefore$ phase constant $= (wt + \phi)$

$$= (wt + \frac{4\pi}{15})$$

(iii) the resultant time period is the same as the time period of the individual vibrations. The equation of the resultant vibration is

$$y = 4.939 \sin (wt + \frac{4\pi}{15}).$$

2.3 *Composition of two simple harmonic vibrations at right angles to each other having equal frequencies but differing in phases and amplitudes*

Let us consider two simple harmonic motions of the same frequency (*i.e.*, time period) but of amplitude a and b and having their vibrations mutually perpendicular to one another (*i.e.*, if one vibrates along the X-axis, the other vibrates along the Y-axis). If ϕ is the phase difference between the two vibrations, then their equations can be written as

$$x = a \sin (wt + \phi) \qquad (2.7)$$

and $$y = b \sin wt \qquad (2.8)$$

The vibrations given by eqn. (2.7) leads the vibration given by eqn. (2.8).

From eqn. (2.7) we get

$$\frac{x}{a} = \sin (wt + \phi)$$

$$= \sin wt \cos \phi + \cos wt \sin \phi$$

$$= \sin wt \cos \phi + \sqrt{1-\sin^2 wt}\ \sin \phi \qquad (2.9)$$

Again from eqn. (2.8) we have

$$\sin wt = \frac{y}{b} \qquad (2.10)$$

Substituting this value of sin wt in eqn. (2.9), we get

$$\frac{x}{a} = \frac{y}{b}\cos\varphi + \sqrt{1-\frac{y^2}{b^2}}\,.\sin\phi$$

or, $$\left(\frac{x}{a} - \frac{y}{b}\cos\varphi\right) = \sqrt{1-\frac{y^2}{b^2}}\,.\sin\phi$$

Squaring both sides, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos\varphi - 2.\frac{x}{a}.\frac{y}{b}\cos\varphi = \left(1-\frac{y^2}{b^2}\right)\sin^2\phi$$

or, $$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\varphi + \frac{y^2}{b^2}\sin^2\varphi - 2.\frac{xy}{ab}\cos\varphi = \sin^2\varphi$$

or, $$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2\,xy}{ab}\cos\phi = \sin^2\phi \qquad (2.11)$$

Eqn. (2.11) gives the general equation of the resultant vibration of the two vibrations given by eqns. (2.7) and (2.8). This is the general equation of a conic whose shape will depend upon the value of the phase difference ϕ between the two vibrations.

case I :

$$\phi = 0, 2\pi, 4\pi, \ldots = 2n\pi$$

where $n = 0, 1, 2, \ldots.$

Since there is no phase difference between the two vibrations, $\phi = 0$ and hence $\sin\phi = 0$ and $\cos\phi = 1$. Putting these values in eqn. (2.11), we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2\,xy}{ab} = 0$$

$$\text{or,}\quad \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$\text{or,}\quad \pm\left(\frac{x}{a} - \frac{y}{b}\right) = 0$$

$$\text{or,}\quad y = \frac{b}{a}\,x \qquad (2.12)$$

Eqn. (2.12) is the equation of a straight line passing through the origin and inclined to the direction of first motion i.e., the X-axis, at an angle $\tan^{-1}\frac{b}{a}$ (Fig. 2.1 (i).). The resultant amplitude is $\sqrt{a^2 + b^2}$ If in addition a = b, then the line will be inclined at an angle of 45°.

case II :

$$\phi = \frac{\pi}{4}\ \text{radian}$$

when $\phi = \frac{\pi}{4}$ rad., $\cos\phi = \sin\phi = \frac{1}{\sqrt{2}}$

Putting these values in eqn. (2.11), we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{\sqrt{2}\,xy}{ab} = \frac{1}{2} \qquad (2.13)$$

Eqn. (2.13) represents the equation of an oblique ellipse inscribed in a rectangle whose length parallel to the X-axis is 2a and breadth 2b. The ellipse touches the rectangle at points $\left(\pm a,\ \pm\frac{b}{\sqrt{2}}\right)$ and $\left(\pm\frac{a}{\sqrt{2}},\ \pm b,\right)$ [Fig. 2.1 (ii)]

case III :

$$\phi = \frac{\pi}{2}\ \text{radian.}$$

when $\phi = \frac{\pi}{2}$ rad., $\sin\phi = 1$ and $\cos\phi = 0$.

Hence eqn. (2.11) becomes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (2.14)$$

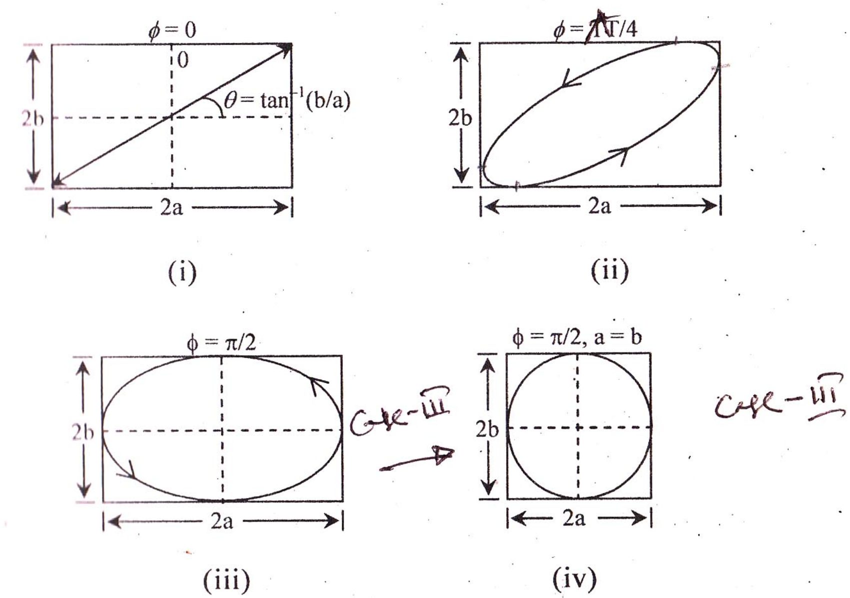
Eqn. (2.14) represents a symmetrical ellipse whose centre coincides with the origin. The semi-major and semi-minor axes of length 2a and 2b respectively coincide with the co-ordinate axes (Fig. 2.11 (iii)]

If in addition a = b, *i.e.*, the amplitudes of the two vibrations are equal; then eqn. (2.14) reduces to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{or,}\quad x^2 + y^2 = a^2 \qquad (2.15)$$

which is the equation of a circle of radius a. The focus of the particle becomes a circle [Fig. 2.1 (iv)].



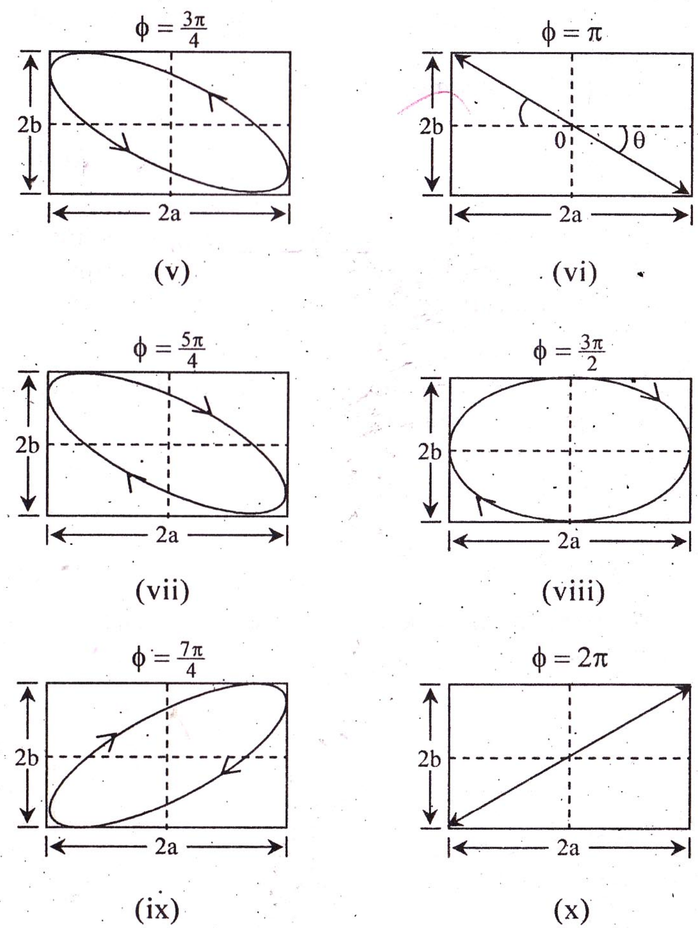


Fig. 2.1

Case IV :

$$\phi = \frac{3\pi}{4} \text{ radian}$$

In that case $\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$ and $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

Hence eqn. (2.11) become

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{\sqrt{2}\,xy}{ab} = \frac{1}{2} \tag{2.16}$$

Eqn. (2.16) represents again the equation of an oblique ellipse with its axes rotated by $\frac{\pi}{2}$ with respect to that in case II. [Fig. 2.1(v)].

case V :

$$\phi = \pi \text{ radian}$$

when $\phi = \pi$ radian, $\sin\phi = 0$ and $\cos\phi = -1$. Hence eqn. (2.11) becomes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

$$\text{or, } \left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0$$

$$\text{or, } \pm\left(\frac{x}{a} + \frac{y}{b}\right) = 0$$

$$\text{or, } y = -\frac{b}{a}x \tag{2.17}$$

Eqn. (2.17) represents again the equation of a straight line with a negative slope as shown in Fig. 2.1 (vi). The line is now inclined with negative X-direction at an angle of $\tan^{-1}(-\frac{b}{a})$. The resultant amplitude is again given by $\sqrt{a^2 + b^2}$. And if a = b, then $\theta = 45°$.

When $\phi = \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$ and 2π respectively the shapes of the curve given by eqn. (2.11) will be as shown in (vii), (viii), (ix) and (x) of Fig. 2.1 respectively.

2.4 *Composition of two simple harmonic motion at right angles to each other and having time periods in the ratio 1 :2.*

Let the equations of the two simple harmonic motion be

$$x = a\sin(2wt + \phi) \tag{2.18}$$

and $y = b \sin wt$ (2.19)

where a is the amplitude for the motion along the X-axis and b is the amplitude for the motion along the Y-axis. The phase difference between the two vibrations is ϕ.

From eqn. (2.18)

$$\frac{x}{a} = \sin(2wt + \phi)$$

$$= \sin 2wt \cos\phi + \cos 2wt \sin\phi$$

$$= 2 \sin wt \cos wt \cos\phi + (1 - 2\sin^2 wt)\sin\phi \qquad (2.20)$$

And from eqn. (2.19) we have

$$\frac{x}{a} = \sin wt$$

$$\therefore \quad \cos wt = \sqrt{1 - \sin^2 wt}$$

$$= \sqrt{1 - \frac{y^2}{b^2}}$$

Substituting these value of *sin wt* and *cos wt* in eqn. (2.20) we get,

$$\frac{x}{a} = 2.\frac{y}{b}.\sqrt{1-\frac{y^2}{b^2}}.\cos\phi + \left(1 - 2.\frac{y^2}{b^2}\right)\sin\phi$$

$$\text{or,} \quad \left[\frac{x}{a} - \left(1 - \frac{2y^2}{b^2}\right)\sin\phi\right] = \frac{2y}{b}\cos\phi\sqrt{1-\frac{y^2}{b^2}}$$

$$\text{or,} \quad \left[\left(\frac{x}{a} - \sin\phi\right) + \frac{2y^2}{b^2}\sin\phi\right] = \frac{2y\cos\phi}{b}\sqrt{1-\frac{y^2}{b^2}}$$

Squaring both sides,

$$\left(\frac{x}{a} - \sin\varphi\right)^2 + \frac{4y^4}{b^4}\sin^2\varphi + 2\left(\frac{x}{a} - \sin\varphi\right)\frac{2y^2}{b^2}\sin\varphi = \frac{4y^2\cos^2\varphi}{b^2}\left(1 - \frac{y^2}{b^2}\right)$$

$$\text{or,} \quad \left(\frac{x}{a} - \sin\varphi\right)^2 + \frac{4y^4}{b^4}\left(\sin^2\varphi + \cos^2\varphi\right) - \frac{4y^2}{b^2}\left(\sin^2\varphi + \cos^2\varphi\right) + \frac{4y^2}{b^2}.\frac{x}{a}\sin\varphi = 0$$

$$\text{or,} \quad \left(\frac{x}{a} - \sin\varphi\right)^2 + \frac{4y^2}{b^2}\left(\frac{y^2}{b^2} + \frac{x}{a}\sin\varphi - 1\right) = 0 \qquad (2.21)$$

Eqn. (2.21) represents the general equation of a curve having two loops, for any difference in phase and amplitude; the actual shape of the curve will of course depend upon the phase difference ϕ between the two vibrations. The resulting curves for different values of ϕ are shown in Fig. 2.2. Some of these cases are discussed below :

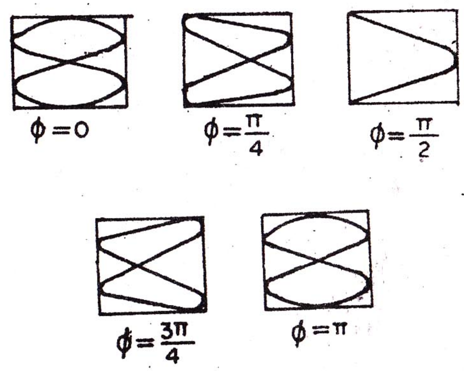


Fig. 2.2

case (I) :

If $\phi = 0, \pi, 2\pi$, etc., $\sin\phi = 0$.

Eqn. (2.21) then becomes

$$\frac{x^2}{a^2} + \frac{4y^2}{b^2}\left(\frac{y^2}{b^2} - 1\right) = 0 \qquad (2.22)$$

The above equation represents the figure of eight and has two loops (Fig. 2.2).

case (II) :

If $\phi = \frac{\pi}{2}$, $\sin\phi = +1$

Then eqn. (2.21) becomes

$$\left(\frac{x}{a}-1\right)^2 + \frac{4y^2}{b^2}\left(\frac{y^2}{b^2}+\frac{x}{a}-1\right) = 0$$

$$\text{or, } \left(\frac{x}{a}-1\right)^2 + \frac{4y^2}{b^2}\left(\frac{x}{a}-1\right) + \frac{4y^4}{b^4} = 0$$

$$\text{or, } \left[\left(\frac{x}{a}-1\right) + \frac{2y^2}{b^2}\right]^2 = 0$$

$$\text{or, } \left(\frac{x}{a}-1\right) + \frac{2y^2}{b^2} = 0$$

$$\text{or, } \frac{2y^2}{b^2} = -\left(\frac{x}{a}-1\right)$$

$$\text{or, } y^2 = -\frac{b^2}{2}\left(\frac{x}{a}-1\right)$$

$$\text{or, } y^2 = -\frac{b^2}{2a}(x-a) \qquad (2.23)$$

Eqn. (2.23) represents the equation of a parabola with vertex at (a, 0).

2.5 *Lissajous' figures*

As can be seen from the discussion in articles 2.3 and 2.4 that the composition of two simple harmonic vibrations in mutually perpendicular directions gives rise to an *elliptical path.* The actual shape of the curve will, however, depend upon the phase difference ϕ between the two vibrations – and also on the ratio of the frequencies of the component vibrations. These figures or curves were first produced optically by Lissajous by reflecting a beam of light from two mirrors, in turn attached to two forks vibrating at right angles to one another. These figures are now known as *Lissajous' figures.*

When two rectangular simple harmonic vibrations whose periods are nearly but not exactly equal act simultaneously on a particle, then the pattern of Lissajous' figures migrates slowly through the sequence shown in Figs. 2.1 and 2.2. The number of such complete sequences gone through per second is equal to the difference of frequencies of the component simple harmonic vibrations.

Example 2.2. *In an experiment to obtain Lissajous' figures, one tuning fork is of frequency 256 Hz and a circular figure occurs after every ten seconds. What deductions may be made about the frequency of the other tuning fork ?*

Soln.

Frequency of A = 256 Hz.

Time for one complete cycle = 10 seconds

$\therefore$ difference in frequencies $= \frac{1}{10} = 0.1$ Hz.

So, the possible frequency of B is

either $256 + 0.1 = 256.1$ Hz

or $256 - 0.1 = 255.9$ Hz.

Example 2.3. *Two tuning forks A and B are used to produce Lissajous' figures. The frequency of A is slightly greater than that of B and is 200 Hz. It is found that the figure completes its cycle in 5 seconds. What is the frequency of B ?*

Soln.

Frequency of A = 200 Hz.

Time for one complete cycle = 5 seconds

$\therefore$ the difference in frequencies $= \frac{1}{5} = 0.2$ Hz.

Since the frequency of A is larger than that of B, the frequency of B

$= 200 - 0.2 = 199.8$ Hz.

Example 2.4. *Two tuning forks A and B are of nearly equal frequencies. Frequency of A is 256 Hz. When the two tuning forks are used to obtain Lissajous' figures, it is found that the complete cycle of changes takes place in 20 seconds. When the tuning fork B is loaded with a little wax, the time taken for one complete cycle of change is 10 seconds. Calculate the original frequency of B.*

Soln.

Frequency of A = 256 Hz.

Time for one complete cycle = 20 secs.

$\therefore$ Difference in frequencies $= \dfrac{1}{20} =$ **0.05 Hz.**

So, possible frequency of B is

either 256 + 0.05 = 256.05 Hz.

or, 256 – 0.05 = 255.95 Hz.

After loading, time for a complete change of cycle is 10 seconds, i.e., the time decreases. Suppose the frequency of B is 256.05 Hz. After loading, the frequency of B will be lowered and its difference with the frequency of A becomes less. Therefore, the time taken for a complete change of cycle will be more than 20 seconds. Hence the frequency of B cannot be 256.05 Hz.

Suppose the frequency of B = 255.95 Hz. After loading its difference with the frequency of A is increased. Therefore, the cycle of change will take place in less time.

Hence the original frequency of B = 255.95 Hz.

2.6 Experimental determination of Lissajous' figures

A and B are two turning forks with frequencies in the ratio of 1:2. The prongs of A vibrate in a horizontal plane while the prong of B vibrate in a vertical plane. A strong beam of light from an electric are S is made to converge on a mirror M_1 attached to one of the prongs of the fork A with the help of a convergent lens. The arrangement is such that after reflection from the mirror M_1, the light is again reflected from the mirror M_2 attached to one of the prongs of B. It is further adjusted so that after reflection from M_1 and M_2, the spot of light is obtained at O, the centre

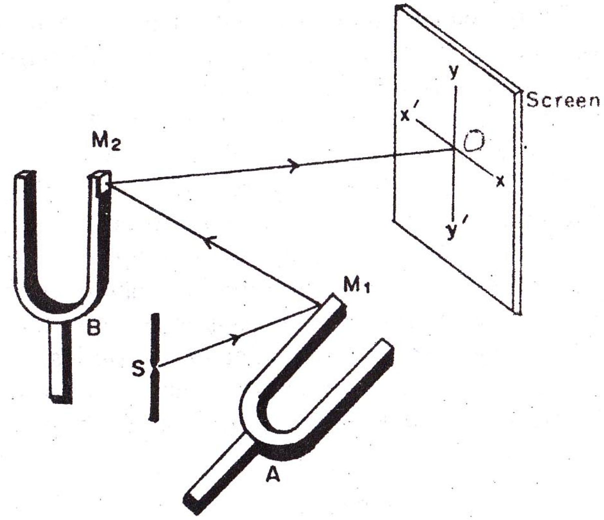


Fig. 2.3

of a white opaque screen (Fig. 2.3). When only the tuning fork A vibrates, the spot of light moves along XX'. The spot of light moves along YY' when only the tuning fork B vibrates. When both the forks vibrate in directions at right angles to one another, the resulting motion of the beam of light will trace out a figure of eight on the screen. However, the shape of the figure will be different for different phase difference of the vibrations of the tuning forks and for different frequency ration of the forks.

2.7 Uses of Lissajous' figures

(1) To determine the ratio of time periods : Lissajous' figures can be used to determine the ratio of the time period of two constituent vibrations. The number of times the curve touches the horizontal and the vertical sides of a rectangle bounding the Lissajous' figures is found. If the curve touches the horizontal side m times and the vertical side n times, the ratio of the time period is m:n. In the figure of eight in Fig. 2.2, the curve touches the horizontal side once and the vertical side twice. Hence the time periods of the two component vibrations are in the ratio of 1:2.

(2) Determination of the frequency of a tuning fork : Lissajous' figures can be demonstrated on the screen of a cathode ray oscilloscope by applying audio – frequency voltage signals on the X- and Y- plates. Stable figures are obtained only when the two frequencies bear a whole number ratio. As this ratio goes on increasing, the figures get more and more crowded but always remain closed curves. Lissajous' figures provide a very sensitive test for the determination of any unknown frequency. The unknown frequency can be determined by combining it with a known frequency in a perpendicular direction. It may be mentioned that composition of simple harmonic motions is an important problem in physics and is very frequently encountered in optics and acoustics; the general approach remaining the same as outlined above. Described below is a *method for determining the frequency of a tuning fork* :

Suppose the given tuning fork has an unknown frequency n_1. Let B be another tuning fork of frequency n which is nearly the same as that of A. If the two tuning forks are set into vibration in perpendicular planes in the manner described above, Lissajous' figures are obtained on the screen. As the tuning forks differ slightly in frequency, the phase difference between the two changes with time. As a result the shapes of the Lissajous' figures changes continuously with the phase changing from 0 to 2π. Suppose the complete cycle of changes take place in t seconds. Then the difference in frequencies of A and B $= \frac{1}{t}$.

Therefore, the frequency of A $= n \pm \frac{1}{t}$.

Now attach a little wax to the tuning fork A. The experiment is repeated and the time taken to complete one cycle of operation is noted. Let it be t_1. If t_1 is greater than t, then the frequency of A

$$= (n - \frac{1}{t}).$$

If t_1 less than t, then the frequency of A

$$= (n + \frac{1}{t}).$$

Thus the frequency of unknown tuning fork can be determined.

EXERCISES

[1] Obtain an expression for the resultant displacement of a particle which is being simultaneously acted upon by two simple harmonic vibrations of same frequency but different phase and amplitude. What happens if the two vibrations are (i) in the same phase, (ii) in opposite phase and (iii) in opposite phase with their amplitudes being equal.

[2] Derive a general expression for the resultant vibration of a particle simultaneously acted upon by two initially perpendicular simple harmonic vibrations, having the same time period but different amplitudes and phase angles. What happens if the phase difference is (i) 0, (ii) $\frac{\pi}{4}$, (iii) $\frac{3\pi}{4}$ and (iv) π radians ?

[3] What are Lissajous' figures? Describe an optical method of obtaining Lissajous' figures. Explain how these figures are useful in the laboratory.

[4] Explain how two simple harmonic vibrations acting simultaneously on a particle in mutually perpendicular directions can be compounded. Deduce an expression for the resultant vibrations.

[5] Two mutually perpendicular simple harmonic vibrations of amplitudes a and b are acting simultaneously on a particle. Their time periods are in the ratio of 1:2 and their phase difference is ϕ. Derive an expression for the resultant vibration of the particle.

[6] Find the resultant of two simple harmonic motions of equal periods when they act at right angles to one another. Analytically discuss the different important cases. How can these be demonstrated ?

[7] Find the resultant of two simple harmonic motions of periods in the ratio 1:2 and having (i) zero phase difference and (ii) phase difference of 45°.

[8] Two simple harmonic motions acting simultaneously on a particle are given by

$$y_1 = \sin(wt + \frac{\pi}{3})$$

$$y_2 = 2 \sin wt$$

Find the equation of the resultant vibration.

[9] The lower of the two tuning forks has a frequency of 380Hertz. If the time required for a complete cycle of changes is 10 seconds, calculate the frequency of the other fork.

[10] In an experiment to obtain Lissajous' figures, one tuning fork is of frequency 250 Hz. and a circular figure appears after every five

seconds. What deductions may be made about the frequency of the other tuning fork ?

[11] Two tuning forks A and B are nearly of equal frequencies. Frequency of A is 256. When the two tuning forks are used to obtain Lissajous' figures, the complete cycle of changes takes place in 10 seconds. When the tuning fork B is loaded with a little wax, the time taken is 20 seconds. Calculate the original frequency of B.

[12] Two tuning forks A and B are nearly of equal frequencies. The frequency of A is 288. When the two tuning forks are used to obtain Lissajous' figures, the complete cycle of changes takes place in 20 seconds. When the tuning fork B is loaded with a little wax, the time taken for one compete change of cycle is 10 seconds. Calculate the original frequency of B.

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