Diffraction

When a beam of light passes through a narrow slit, it spreads out to a certain extent into the region of the geometrical shadow. This effect is one of the simplest example of diffraction, i.e., of **the failure of light to travel in straight lines**.

This can be satisfactorily explained only by a assuming a wave character for light.

Classification of Diffraction

Diffraction phenomena are conveniently divided into two general classes:

(i) Fraunhofer diffraction
(ii) Fresnel diffraction

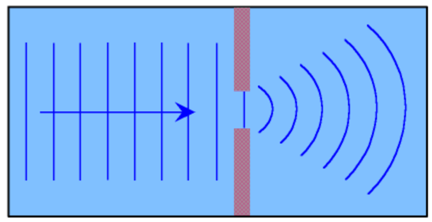

Fig 1. Diffraction of wave passing through a small aperture.

Distinction between Fraunhofer and Fresnel diffraction

1. Fraunhofer diffraction: If the source of light and the screen on which the pattern is observed are effectively at infinite distances from the aperture causing the diffraction is called Fraunhofer diffraction.

Fresnel diffraction: If either the source or screen or both, are at finite distances from the aperture is called Fresnel diffraction.

2. **Fraunhofer diffraction** is much simpler to treat theoretically, Fresnel diffraction is difficult to treat theoretically.

(3) In **Fraunhofer diffraction** lenses are required; on other hand no lenses are necessary for **Fresnel diffraction**.

(4) In **Fraunhofer diffraction** wave fonts are plane and in **Fresnel diffraction** wave fonts are divergent.

Diffraction through single slit

A slit is a rectangular aperture of length large compared to its breadth.

Consider a slit ‘s’ to be setup with its long dimension perpendicular to the plane of the plane, and to be illuminated by parallel monochromatic light from the narrow slit ‘s’ at the principle focus of lens L_1. The light focused by another lens L_2 on a screen or photographic plate P at its principle focus will form a diffraction pattern.

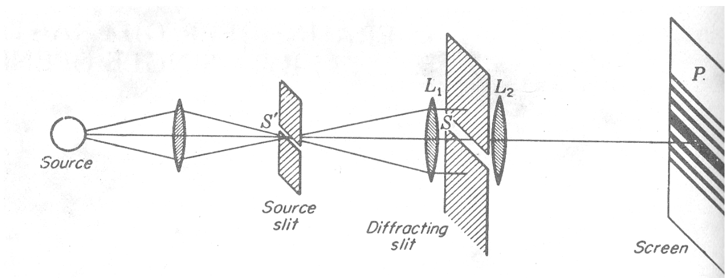

Fig. 2. Experimental arrangement for obtaining the diffraction pattern of a single slit Fraunhofer diffraction.

The explanation of the single slit pattern lies in the interference of the Huygens secondary wavelets which can be thought of as sent out from every point on the wave front. If we assume wavelets to be uniform spherical waves, the emission of which stops abruptly at the edges of the slit.

We consider a slit of width *b*, illuminated by parallel light from the left. Let *ds* be an element of width of the wave front in the plane of the slit, at a distance *s* from the center *O*, which we shall call the origin.

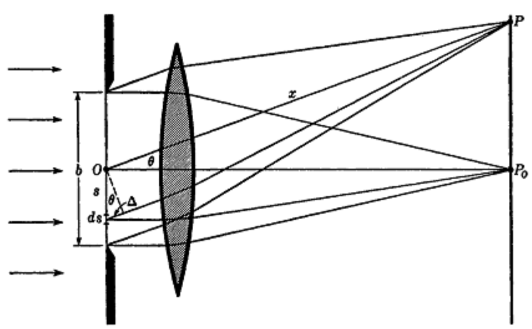

Fig. 3. Geometrical construction for investigating the intensity in the single slit Fraunhofer diffraction pattern.

The parts of each secondary wave which travel normal to the plane at the slit will be focused at P_0, while those which travel at any angle θ will reach at P. Consider first the wavelet emitted by the element ds situated at the origin, its amplitude will be directly proportional to the length *ds* and inversely proportional to *x*.

At P it will produced an infinitesimal displacement which, for spherical wave, may be expressed as

$$dy_0 = a\frac{ds}{x}\sin(\omega t - kx)$$

As the position of *ds* is varied, the displacement it produces will vary in phase because of the different path length to P. When it is at a distance *s* below the origin, the contribution will be

$$dy_s = \frac{a\,ds}{x}\sin\left[\omega t - k(x + \Delta)\right]$$

$$= \frac{a\,ds}{x}\sin(\omega t - kx - ks\sin\theta)$$

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We now wish to sum the effects of all elements from one edge of the slit to the other. This can be done by integrating from s = - *b/2* to *b/2*. The simplest way is to integrate the contributions from pairs of elements symmetrically placed at s and *-s,* each contribution being

$$dy = dy_{-s} + dy_s$$

$$= \frac{a\,ds}{x}\left[\sin(\omega t - kx - ks\sin\theta) + \sin(\omega t - kx + ks\sin\theta)\right]$$

By the identity $\sin\alpha + \sin\beta = 2\cos\frac{1}{2}(\alpha - \beta)\sin\frac{1}{2}(\alpha + \beta)$, we have

$$dy = \frac{a\,ds}{x}\left[2\cos(ks\sin\theta)\sin(\omega t - kx)\right]$$

which must be integrated from $s = 0$ to $b/2$. In doing so, x may be regarded as constant, insofar as it affects the amplitude. Thus

$$y = \frac{2a}{x}\sin(\omega t - kx)\int_0^{b/2}\cos(ks\sin\theta)\,ds$$

$$= \frac{2a}{x}\left[\frac{\sin(ks\sin\theta)}{k\sin\theta}\right]_0^{b/2}\sin(\omega t - kx)$$

$$= \frac{ab}{x}\,\frac{\sin(\frac{1}{2}kb\sin\theta)}{\frac{1}{2}kb\sin\theta}\sin(\omega t - kx)$$

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The resultant vibration will therefore be a simple harmonic one, amplitude of which varies with the position of P, since this is determined by θ.

We may represent the amplitude as

$$A = A_0 \frac{\sin\beta}{\beta}$$

where,

$$\frac{ab}{x} = A_0$$

$$\frac{1}{2} kb \sin\theta = \beta$$

The intensity on the screen is then

$$I \cong A^2 = A_0^{\,2} \frac{\sin^2\beta}{\beta^2}$$

If the light (instead of being incident on the slit perpendicular to its plane) make an angle i, then

$$\beta = \frac{\pi b(\sin i + \sin\theta)}{\lambda}$$

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Further investigation of the single slit diffraction pattern

$$A = A_0 \frac{\sin\beta}{\beta} \qquad I = A_0^{\,2} \frac{\sin^2\beta}{\beta^2}$$

where $A_0 = \frac{ab}{x}$

$$\beta = \frac{1}{2} kb \sin\theta = \frac{1}{2} kb \sin\theta = \frac{1}{2} \frac{2\pi}{\lambda} b \sin\theta = \frac{\pi b}{\lambda} \sin\theta$$

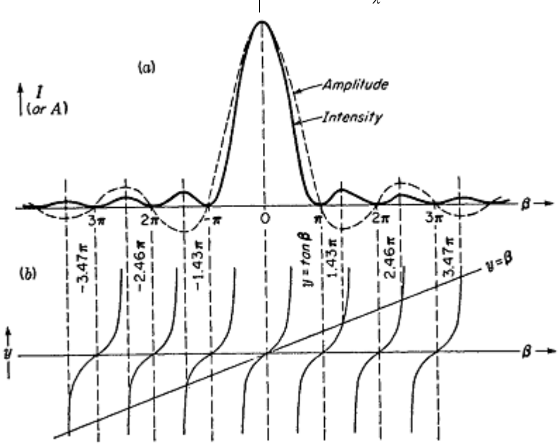

Fig. 4. Amplitude and intensity contour for Fraunhofer diffraction of single slit showing position of maximum and minimum.

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The maximum intensity of the strong central band comes at the point P_0, where all the secondary wavelets will arrive in phase because the path difference $\Delta = 0$.

For this point $\beta = 0; then \quad \sin\beta = \beta$

Therefore $\frac{\sin\beta}{\beta} = 1$

$$I \cong A_0^{\ 2} \frac{\sin^2\beta}{\sin\beta} = A_0^{\ 2}\left(\frac{\sin\beta}{\beta}\right)^2 = A_0^{\ 2}$$ is the maximum intensity (at β=0)

From this principal maximum the intensity fall to zero at then passes through several secondary maximum with equally spaced points at zeros intensity at

$$\beta = \pm\pi, \quad \beta = \pm\pi, \ \pm 2\pi, \ \pm 3\pi, \ ....$$

Or in general $\beta = m\pi$ $\quad m = \pm 1, \ \pm 2 \ .... \pm m$

$$\frac{\pi b \sin\theta}{\lambda} = m\pi$$

$$b\sin\theta = m\lambda$$ for minima.

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The secondary maximum do not fall half way between these points, but are displaced toward the center of the pattern by an amount which decreasing with increasing m. The exact values of β for these maxima can be found differentiating $A = A_0 \frac{\sin\beta}{\beta}$ with respect to β and equating zero.

$$\frac{\sin\beta}{\beta} = \frac{A}{A_0}$$

$$\frac{d}{d\beta}\left(\frac{\sin\beta}{\beta}\right) = 0$$

$$or, \frac{\beta\cos\beta - \sin\beta}{\beta^2} = 0$$

$$or, \ \beta\cos\beta = \sin\beta$$

$$or, \frac{\sin\beta}{\cos\beta} = \beta$$

$$or, \tan\beta = \beta$$

The values of β satisfying this relation are easily found graphically as the intersection of the curve $y = \tan\beta$ and straight line $y = \beta$. These points of intersection lie directly below the corresponding secondary maxima. The intensity of the secondary maxima can be calculated to a very close approximation by finding the values of $\frac{\sin^2\beta}{\beta^2}$ at the half way positions, where

$$\beta = 3\pi/2, \ 5\pi/2, \ 7\pi/2, \ . \ . \ . \ .$$

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This gives $4/9\pi^2$, $4/25\pi^2$, $4/49\pi^2$, . . . , or 1/22.2, 1/61.7, 1/121, . . . , of the intensity of the principal maximum.

The first secondary maximum is only 4.72 percent the intensity of the central maximum, while the second and third secondary maxima are only 1.65 and 0.83 percent respectively.

Figure 5. shows plots of the intensity of a single-slit diffraction pattern, calculated with intensity equation for three slit widths: $d = \lambda$, $d=5\lambda$, and $d= 10\lambda$. Note that as the slit width increases (relative to the wavelength), the width of the *central diffraction maximum* (the central hill-like region of the graphs) decreases; that is, the light undergoes less flaring by the slit. The secondary maxima also decrease in width (and become weaker). In the limit of slit width *a* being much greater than wavelength λ, the secondary maxima due to the slit disappear; we then no longer have single-slit diffraction (but we still have diffraction due to the edges of the wide slit, like that produced by the edges of the razor blade.

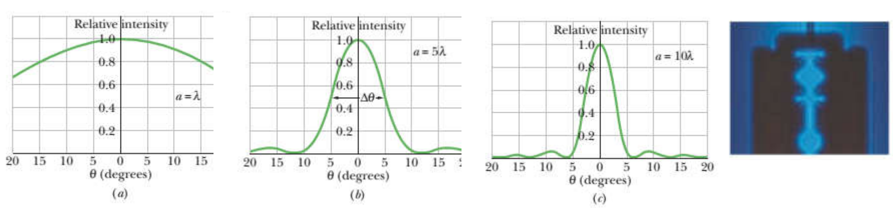

Fig. 5 .The relative intensity in single-slit diffraction for three values of the ratio a/λ. The wider the slit is, the narrower is the central diffraction maximum.

Diffraction by double slit

Now, we have two equal slits of width b, separated by an opaque space of width c, origin may be chosen at the centre of c, and the integration extended from s=d/2-b/2 to s= d/2+b/2.

Fraunhofer Diffraction by double slit

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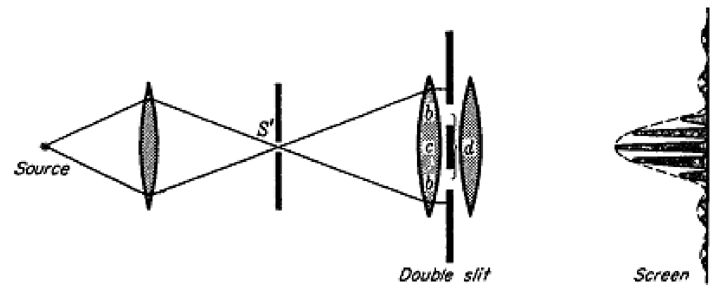

Fig. 6 **Apparatus for observing Fraunhofer diffraction from a double slit. Drawn for $2b = c$, that is, $d = 3b$.**

This gives,

$$y = \frac{2a}{xk \sin\theta}\{\sin[\tfrac{1}{2}k(d+b)\sin\theta] - \sin[\tfrac{1}{2}k(d-b)\sin\theta]\}[\sin(\omega t - kx)]$$

The quantity in braces is of the form $\sin(A + B) - \sin(A - B)$, and when it is expanded, we obtain

$$y = \frac{2ba}{x}\frac{\sin\beta}{\beta}\cos\gamma \sin(\omega t - kx)$$

where, as before,

$$\beta = \tfrac{1}{2}kb\sin\theta = \frac{\pi}{\lambda}b\sin\theta$$

and where

$$\gamma = \tfrac{1}{2}k(b + c)\sin\theta = \frac{\pi}{\lambda}d\sin\theta$$

Therefore,

$$I = 4A_0^{\,2}\frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

The factor $\frac{\sin^2\beta}{\beta^2}$ in this equation is just that derived for the single slit of width b. the second factor $\cos^2\gamma$ is characteristics of the interference pattern produced by two beams of equal intensity and phase difference δ.

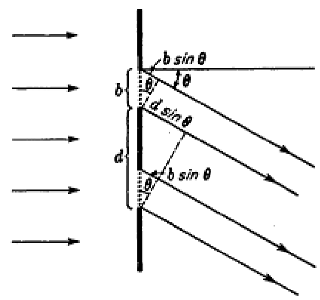

Fig. 7. Path difference of parallel rays leaving a double slit.

The resultant intensity will be zero when either of the two factors is zero. For the first factor this will occur when

$\beta=\pi, 2\pi, 3\pi$………..

For the second factor this will occur when,

$\gamma=\pi/2, 3\pi/2, 5\pi/2$………

The two variables γ and β are not independent. The difference in path from the two edges of a given slit is bsinθ. The corresponding phase difference is $\delta = \frac{2\pi}{\lambda} b \sin\theta$, which equals to 2β.

The path difference from any two corresponding points in the two slits, as illustrated for the two points at the lower edges of the slits, is $d \sin\theta$, and the phase difference is $\delta = (2\pi/\lambda)d\sin\theta = 2\gamma$. Therefore, in terms of the dimensions of the slits,

$$\frac{\delta}{2\beta} = \frac{\gamma}{\beta} = \frac{d}{b}$$

POSITIONS OF THE MAXIMA AND MINIMA, MISSING ORDERS

As shown in Sec. 16.2, the intensity will be zero wherever $\gamma = \pi/2, 3\pi/2, 5\pi/2, \ldots$ and also when $\beta = \pi, 2\pi, 3\pi, \ldots$. The first of these two sets are the minima for the interference pattern, and since by definition $\gamma = (\pi/\lambda)d \sin\theta$, they occur at angles θ such that

- $$d \sin\theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots = (m + \tfrac{1}{2})\lambda \qquad \textit{Minima} \qquad (16e)$$

m being any whole number starting with zero. The second series of minima are those for the diffraction pattern, and these, since $\beta = (\pi/\lambda)a \sin\theta$, occur where

- $$b \sin\theta = \lambda, 2\lambda, 3\lambda, \ldots = p\lambda \qquad \textit{Minima} \qquad (16f)$$

the smallest value of p being 1. The exact positions of the *maxima* are not given by any simple relation, but their approximate positions can be found by neglecting the variation of the factor $(\sin^2\beta)/\beta^2$, a justified assumption only when the slits are very narrow and when the maxima near the center of the pattern are considered [Fig. 16A(*b*)]. The positions of the maxima will then be determined solely by the $\cos^2\gamma$ factor, which has maxima for $\gamma = 0, \pi, 2\pi, \ldots$, that is, for

$$d \sin\theta = 0, \lambda, 2\lambda, 3\lambda, \ldots = m\lambda \qquad \textit{Maxima} \qquad (16g)$$

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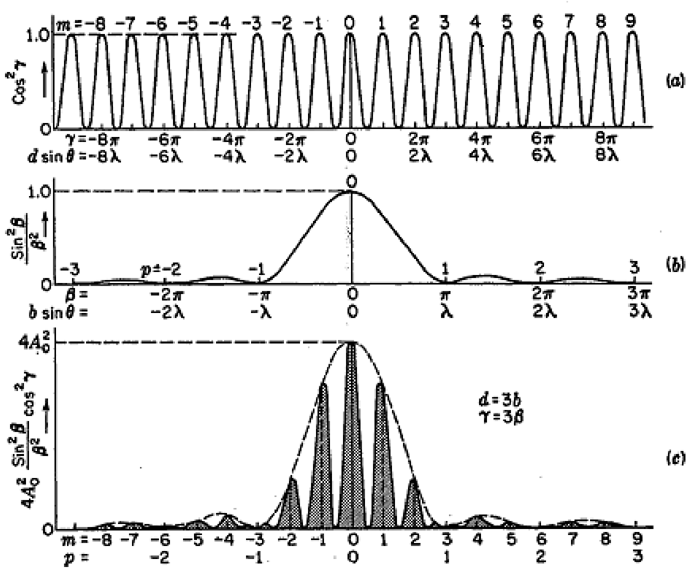

FIGURE 16D
Intensity curves for a double slit where $d = 3b$.

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The whole number m represents physically the number of wavelengths in the path difference from corresponding points in the two slits (see Fig. 16C) and represents the *order* of interference.

Figure 16D(a) is a plot of the factor $\cos^2 \gamma$, and here the values of the order, of half the phase difference $\gamma = \delta/2$, and of the path difference are indicated for the various maxima. These are all of equal intensity and equidistant on a scale of $d \sin \theta$, or practically on a scale of θ, since when θ is small, $\sin \theta \approx \theta$ and the maxima occur at angles $\theta = 0, \lambda/d, 2\lambda/d, \ldots$. With a finite slit width b the variation of the factor $(\sin^2 \beta)/\beta^2$ must be taken into account. This factor alone gives just the single-slit pattern discussed in the last chapter, and is plotted in Fig. 16D(b). The complete double-slit pattern as given by Eq. (16c) is the product of these two factors and therefore is obtained by multiplying the ordinates of curve (a) by those of curve (b) and the constant $4A_0^2$. This pattern is shown in Fig. 16D(c). The result will depend on the relative scale of the abscissas β and γ, which in the figure are chosen so that for a given abscissa $\gamma = 3\beta$. But the relation between β and γ for a given angle θ is determined, according to Eq. (16d), by the ratio of the slit width to the slit separation. Hence if $d = 3b$, the two curves (a) and (b) are plotted to the same scale of θ. For the particular case of two slits of width b separated by an opaque space of width $c = 2b$, the curve (c), which is the product of (a) and (b), then gives the resultant pattern. The positions of the maxima in this curve are slightly different from those in

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curve (a) for all except the central maximum ($m = 0$), because when the ordinates near one of the maxima of curve (a) are multiplied by a factor which is decreasing or increasing, the ordinates on one side of the maximum are changed by a different amount from those of the other, and this displaces the resultant maximum slightly in the direction in which the factor is increasing. Hence the positions of the maxima in curve (c) are not exactly those given by Eq. (16g) but in most cases will be very close to them.

Let us now return to the explanation of the differences in the two patterns (b) and (c) of Fig. 16A, taken with the same slit separation d but different slit widths b. Pattern (c) was taken for the case $d = 3b$, and is seen to agree with the description given above. For pattern (b), the slit separation d was the same, giving the same spacing for the interference fringes, but the slit width b was smaller, such that $d = 6b$. In Fig. 13D, $d = 14b$. This greatly increases the scale for the single-slit pattern relative to the interference pattern, so that many interference maxima now fall within the central maximum of the diffraction pattern. Hence the effect of decreasing b, keeping d unchanged, is merely to broaden out the single-slit pattern, which acts as an envelope of the interference pattern as indicated by the dotted curve of Fig. 16D(c).

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If the slit-width b is kept constant and the separation of the slits d is varied, the scale of the interference pattern varies, but that of the diffraction pattern remains the same. A series of photographs taken to illustrate this is shown in Fig. 16E. For each pattern three different exposures are shown, to bring out the details of the faint and the strong parts of the pattern. The maxima of the curves are labeled by the order m, and underneath the upper one is a given scale of angular positions θ. A study of these figures shows that certain orders are missing, or at least reduced to two maxima of very low intensity. These *missing orders* occur where the condition for a maximum of the interference, Eq. (16g), and for a minimum of the diffraction, Eq. (16f), are both fulfilled for the same value of θ, that is for

$$d \sin \theta = m\lambda \qquad \text{and} \qquad b \sin \theta = p\lambda$$

so that

$$\frac{d}{b} = \frac{m}{p} \qquad \text{(16h)}$$

Since m and p are both integers, d/b must be in the ratio of two integers if we are to have missing orders. This ratio determines the orders which are missing, in such a way that when $d/b = 2$, orders 2, 4, 6, ... are missing; when $d/b = 3$, orders 3, 6, 9, ... are missing; etc. When $d/b = 1$, the two slits exactly join, and all orders should be missing. However, the two faint maxima into which each order is split can then be shown to correspond exactly to the subsidiary maxima of a single-slit pattern of width $2b$.

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Our physical picture of the cause of missing orders is as follows. Considering, for example, the missing order $m = +3$ in Fig. 16D(c), this point on the screen is just three wavelengths farther from the center of one slit than from the center of the other. Hence we might expect the waves from the two slits to arrive in phase and to produce a maximum. However, this point is at the same time one wavelength farther from the edge of one slit than from the other edge of that slit. Addition of the secondary wavelets from one slit gives zero intensity under these conditions. The same holds true for either slit, so that although we may add the contributions from the two slits, both contributions are zero and must therefore give zero resultant.

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Example 1: What must be the ratio of the slit width to the wavelength for a single slit to have the first diffraction minimum at θ=45.0°?

We know that $d \sin\theta = m\lambda$

Therefore, $d/\lambda = m/\sin\theta = 1/\sin 45 = 1.41$ *(Ans.)*

Example 2: The distance between the first and fifth minima of a single-slit diffraction pattern is 0.35 mm with the screen 40 cm away from the slit, when light of wavelength 550 nm is used. (a) Find the slit width. (b) Calculate the angle θ of the first diffraction minimum.

(a)

$$\Delta y = D\Delta \sin\theta$$

We know that $d \sin\theta = m\lambda$; Or, $\sin\theta = (\lambda/d)m$

Or, $\Delta\sin\theta = (\lambda/d)\Delta m = (\lambda/d)(m_2 - m_1)$

Therefore, $\Delta y = D\Delta\sin\theta = D(\lambda/d)(m_2 - m_1)$

Or, $d = (D\lambda)(m_2 - m_1)/\Delta y$ $= \dfrac{(400\,\text{mm})(550\times10^{-6}\,\text{mm})(5-1)}{0.35\,\text{mm}} = 2.5\,\text{mm}$.

(b) $\sin\theta = (\lambda/d)m$

$$= \frac{(1)(550\times10^{-6}\,\text{mm})}{2.5\,\text{mm}} = 2.2\times10^{-4} .$$

The angle is $\theta = \sin^{-1}(2.2\times10^{-4}) = 2.2\times10^{-4}$ rad.

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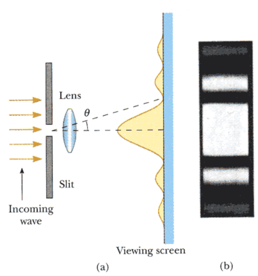
Example 3: Monochromatic light of wavelength 441 nm is incident on a narrow slit. On a screen 2.00 m away, the distance between the second diffraction minimum and the central maximum is 1.50 cm. (a) Calculate the angle of diffraction θ of the second minimum. (b) Find the width of the slit.

(a)

$$\theta = \sin^{-1}(1.50\text{ cm}/2.00\text{ m}) = 0.430°.$$

(b) For the mth diffraction minimum, $a \sin\theta = m\lambda$. We solve for the slit width:

$$a = \frac{m\lambda}{\sin\theta} = \frac{2(441\,\text{nm})}{\sin 0.430°} = 0.118\,\text{mm} .$$



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Example 4: Light of wavelength 633 nm is incident on a narrow slit. The angle between the first diffraction minimum on one side of the central maximum and the first minimum on the other side is 1.20°. What is the width of the slit?

The condition for a minimum of a single-slit diffraction pattern is

$$a\sin\theta = m\lambda$$

where a is the slit width, λ is the wavelength, and m is an integer. The angle θ is measured from the forward direction, so for the situation described in the problem, it is 0.60° for $m = 1$. Thus,

$$a = \frac{m\lambda}{\sin\theta} = \frac{633\times10^{-9}\text{ m}}{\sin 0.60°} = 6.04\times10^{-5}\text{ m}.$$

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Example 5: Monochromatic light with wavelength 538 nm is incident on a slit with width 0.025 mm. The distance from the slit to a screen is 3.5 m. Consider a point on the screen 1.1 cm from the central maximum. Calculate (a) θ for that point, (b) α, and (c) the ratio of the intensity at that point to the intensity at the central maximum.

(a) $$\theta = \sin^{-1}(0.011\text{ m}/3.5\text{ m}) = 0.18°.$$

(b) $$\alpha = \left(\frac{\pi a}{\lambda}\right)\sin\theta = \frac{\pi(0.025\text{ mm})\sin 0.18°}{538\times10^{-6}\text{ mm}} = 0.46\text{ rad}.$$

(c) $$\frac{I(\theta)}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0.93.$$

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