[10] A thin converging lens and a thin diverging lens are placed co-axially in air at a distance of 5 cm. If the focal length of each lens is 10 cm. find for the combination (i) the focal length, (ii) the power and (iii) position of the principal points.

[f = 20 cm. P = + 5 dioptres, α = – 10 cm. β = – 10 cm.]

[11] An object is placed at a distance of 60 cm from a thin convex lens of focal length 20 cm. There is a second thin convex lens of 30 cm focal length 10 cm away from the first. Calculate the distance of the final image from the second lens. **[12 cm to the right of the second lens].**

[12] Two identical thin convex lenses of focal lengths 8 cm each are placed coaxially in air separated by a distance of 4 cm. Find the equivalent focal length and the positions of the principal points. Also find the position of the object for which the image is formed at infinity. **[f = 5.3 cm. α = + 2.66 cm. β = – 2.6 cm, u = – 2.67 cm from the first lens].**

[13] Two thin converging lenses of focal lengths 30 cm and 40 cm are placed coaxially 20 cm apart in air. An object is located at a distance of 48 cm from the first lens. Find (i) equivalent focal length, (ii) the position of the focal points and the principal points and (iii) the position of the image. Indicate these positions in a diagram.

[f = 245 cm, α = + 12 cm, β = – 16 cm, F_1 = 12 cm to the left of the first lens. F_2 = 8 cm to the right of the second lens. v = + 24 cm from the second lens].

[14] Two thin converging lenses each of 20 cm focal length are set coaxially 5 cm apart. An upright pole 10 metres high is placed at a distance of 200 metres from the first lens. Find the position of the principal planes and the image. Also find the size of the image.

[f = 11.43 cm, α + 2.85 cm, β = – 2.85 cm. F_1 = 8.58 cm to the left of the first lens, F_2 = 8.58 cm to the right of the second lens, v = + 8.55 cm from the second lens. size of the image = 0.57 cm.].

[15] A concavo-convex lens has an index of refraction 1.5 and the radii of curvature of its surfaces are 10 cm and 20 cm. The concave surface is upwards and is filled with an oil of refractive index 1.60. Calculate the focal length of the oil-glass combination. **[18.18 cm.].**

[16] Two thin converging lenses of focal lengths 3 cm and 4 cm are placed coaxially 2 cm apart in air. An object is placed 4 cm in front of the first lens. Find the position and nature of the image and its lateral magnification. **[v = 2.86 from the second lens, image is real, magnification = 0.86].**

CHAPTER XXV

DEFECTS OF IMAGES : SPHERICAL AND CHROMATIC ABERRATIONS

Aberrations – Spherical aberration of a lens – Reduction of spherical aberration – Coma – Abbe's sine condition – Aplanatic lens – Astigmatism – Curvature – Distortion – Dispersion by prism – Refraction through a prism – Dispersive power of an optical medium – Chromatic aberration in a lens – Axial chromatic aberration in a lens – Achromatism – Condition of achromatism of two lenses in contact – Achromatic doublet – Condition of achromatism of two lenses separated by a distance – Solved problems – Exercises.

25.1 Aberrations

The simple equations connecting object and image distances, focal lengths etc. derived and discussed earlier, were based on the fundamental assumption that the rays were paraxial – the rays of light diverging from the object point were confined to a narrow cone of a small angular opening so as to allow us to replace the sine of the slope angle θ of the ray by the angle itself in the following expansion :

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots \qquad (25.1)$$

The paraxial condition led to the formation of a point image of a point object and this theory of image formation was called the *first order theory* because of the inclusion of only the first order term of the slope angle in the above expansion.

In practice, objects have finite dimensions and lenses are used to form images of points situated off the axis. Furthermore, considerations of brightness of the image demand the use of lenses with large apertures. All these lead to the conclusion that the rays, in reality, are not paraxial; they form a cone of such a wide angular opening that light from an object do not meet at a single point after refraction through the lens. Thus it becomes essential to take into account, in our discussions of the image formation, at least the first two terms of the expansion of $\sin\theta$. Due to the inclusion of the third order term in θ, this theory has come to be known as the *third order theory*. The *deviations from the actual size, shape and position of an image* as calculated from the earlier simple equations, are called *aberrations* produced by a lens. These aberrations may be classified as *(i) spherical aberration, (ii) coma (iii) astigmatism, (iv) curvature and (v) distortion*. The aberrations are also known as *Seidal aberrations* as the third order theory was developed by *Ludwig von Seidal* in 1855. These aberrations are present in the images formed by ordinary lenses, even when light of only one wavelength is employed. Consequently, they are also known as *monochromatic aberrations (monos* and *chroma* are greek words for *single* and *colour* respectively). Each of the five monochromatic aberrations is dependent upon the wavelength of light but, leaving aside spherical aberration, the variation is negligible in the others.

The focal length of a lens depends upon the refractive index of the lens material, which varies with the wavelength of light. Therefore, even in the first order theory, if the light diverging from the object is not monochromatic, a mumber of coloured images of different sizes will be formed by the lens at different positions. Thus another type of aberration, known as *chromatic aberration* because of its dependene on the wavelength, *i.e.*, colour of light, is encountered.

A rigorous mathematical study of these aberrations is beyond our scope, but wherever possible simple mathematical deductions will be given while describing these aberrations. *It must, however, be pointed out that lens aberrations are just the consequence of the refraction laws at the spherical surfaces and not due to defective construction of a lens such as the surfaces being not spherical, etc.*

25.2 Spherical aberration of a lens

The presence of spherical aberration in the image formed by a lens, for which the ratio of the aperture to the focal length is relatively large, is illustrated in Fig. 25.1. O is a point object on the axis of the lens and rays coming from this object are incident on the lens at different heights from the axis. The lens may be imagined to be divided into a number of annular zones of increasing radius with the optical centre as the common centre. As the curvature of the lens surface increases with increasing distances from the axis of the lens,

each annular zone will have a different focal length; the greater the radius of the zone the smaller the focal length along the axis. This means that the rays suffering refraction at different zones will suffer

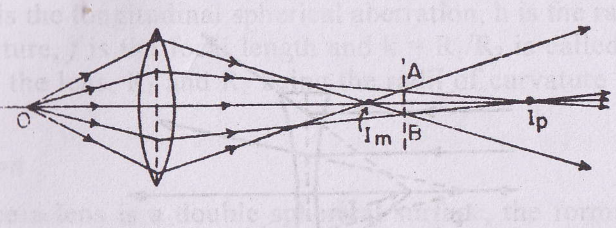

Fig. 25.1

deviation through different extent. The paraxial rays which suffer refraction at the zone close to the principal axis are least deviated and consequently come to focus at a point I_p farthest from the lens. The marginal rays on the other hand suffer refraction at the periphery of the lens and are the most deviated and come to focus at a point I_m closest to the lens. The rays which are refracted at any other zone in between these two extreme zones, in general come to focus at one image point between I_m and I_p. However, each annular zone will give rise to a different axial image point. The lens, therefore, does not produce one image point of the point object; the image is rather extended over the length $I_m \sim I_p$. The image is, therefore, not sharp at any point on the axis. This failure of the lens to provide a sharp image is known as *spherical aberration*. The cross-section of the refracted beam is everywhere circular and if a screen is placed perpendicular to the axis at any point between I_m and I_p, a circular patch of light will be obtained on the screen as the image. However, it will be seen from the figure that there is one plane, *AB*, at which the cross-section has least diameter. At positions on both sides of *AB*, the image patch has a larger diameter. This smallest cross-section is known as the *circle of least confusion* and the best image of the object O is obtained if a screen is placed perpendicular to the lens axis at this point. The spherical aberration produced by a concave lens for parallel rays is illustrated in Fig. 25.2.

The separation between the marginal focus I_m and the paraxial focus I_p is taken as a measure of the *longitudinal spherical aberration* of the lens, while the radius of the circle of least confusion is taken as a measure of the *lateral or transverse spherical aberration.* The longitudinal spherical aberration is considered *positive* if the marginal focus I_m lies on the left of I_p and

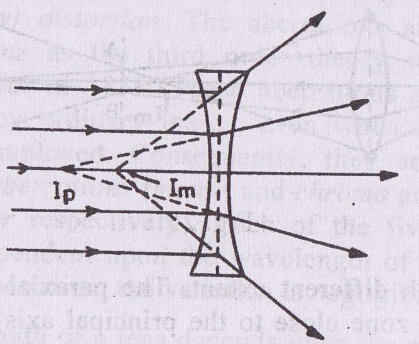

Fig. 25.2

is considered *negative* if I_m lies on the right of I_p. Accordingly, longitudinal spherical aberration produced by a convex lens is positive while that produced by a concave lens is negative. The spherical aberration produced by a lens varies approximately as the square of the height of the incident ray above the axis and it also depends upon the distance of the object point.

25.3 Reduction of spherical aberration

Spherical aberration produced by lenses can be reduced by the following methods :

(i) ***Use of a stop :*** Since the spherical aberration produced by a given lens depends on the radius of the lens aperture, it can be reduced by stopping down the lens aperture with the help of coaxial aperture stop. The stop used can be such as to permit either the paraxial rays or marginal rays of light to fall on the lens. However the image will appear less bright as the amount of light passing through the lens is reduced by the use of the stop.

(ii) Use of crossed lens : The longitudinal spherical aberration of a thin lens for parallel incident rays is given by

$$x = -\frac{h^2}{f}\left[\frac{2-2\mu^2+\mu^3+k(\mu+2\mu^2-2\mu^3)+k^2\mu^3}{2\mu(\mu-1)^2(1-k^2)}\right]$$

where x is the longitudinal spherical aberration, h is the radius of the lens aperture, f is the focal length and $k = R_1/R_2$ is called the *shape factor* of the lens, R_1 and R_2 being the radii of curvature of the lens surfaces.

Derivation

Since a lens is a double spherical surface, the formula will be derived for one surface and then for the other. Let PO be the curved surface under question having C as its centre of curvature (Fig. 25.3). A marginal ray of light from the source S strikes the surface at the point P and undergoes refraction in a direction which when extended backwards intersects the axis OS at point I. Let SO = u, IO = v', OP = h and CO = r_1. If μ be the refractive index of the material of the lens, then we have

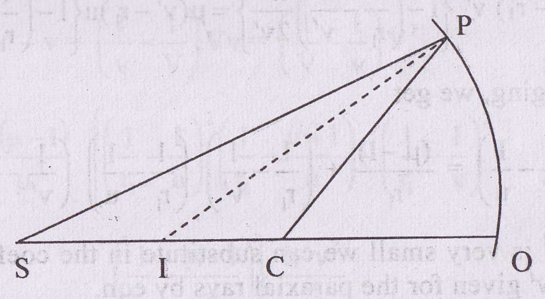

Fig. 25.3

from Δ IPC, $\dfrac{IC}{IP} = \dfrac{\sin\angle IPC}{\sin\angle ICP}$

and from Δ SPC, $\dfrac{SC}{SP} = \dfrac{\sin\angle SPC}{\sin\angle SCP}$

Also, $\mu = \dfrac{\sin\ i}{\sin\ r} = \dfrac{\sin\angle SPC}{\sin\angle IPC}$

whence, SC.IP = μ SP.IC (i)

Also from Δ PCS,

$SP^2 = SC^2 + CP^2 + 2SC.CP \cos OCP$

$$= (u - r_1)^2 + r_1^2 + 2r_1 (u - r_1) \cos \frac{h}{r_1}$$

$$= u^2 - (u - r_1) \frac{h^2}{r_1}\text{, neglecting higher powers of h}$$

$$\therefore \quad SP = u \left[1 - \left(\frac{1}{r_1} - \frac{1}{u}\right)\frac{h^2}{2u}\right]$$

Similarly, $IP = v' \left[1 - \left(\frac{1}{r_1} - \frac{1}{v'}\right)\frac{h^2}{2v'}\right]$

Substituting these values in eqn. (i), we get

$$(u - r_1)\, v' \left\{1 - \left(\frac{1}{r_1} - \frac{1}{v'}\right)\frac{h^2}{2v'}\right\} = \mu(v' - r_1)\, u \left\{1 - \left(\frac{1}{r_1} - \frac{1}{u}\right)\frac{h^2}{2u}\right\}$$

Rearranging, we get

$$\left(\frac{\mu}{v'} - \frac{1}{u}\right) = \frac{(\mu - 1)}{r_1} + \left(\frac{1}{r_1} - \frac{1}{v'}\right)\left(\frac{1}{r_1} - \frac{1}{u}\right)\left(\frac{1}{v'} - \frac{\mu}{u}\right)\frac{h^2}{2}$$

Since h^2 is very small we can substitute in the coefficient of h^2 the value of v' given for the paraxial rays by eqn.

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{r_1}$$

We then get

$$-\frac{\mu}{v'} + \frac{1}{u} = \frac{\mu - 1}{-r_1} + \frac{\mu - 1}{\mu^2}\left(\frac{1}{r_1} - \frac{1}{u}\right)^2 \left(\frac{1}{r_1} - \frac{\mu + 1}{u}\right)\frac{h^2}{2} \qquad \text{(ii)}$$

Proceeding for the second surface in similar manner, we can get

$$-\frac{\mu}{v'} + \frac{1}{v} = \frac{\mu - 1}{-r_2} + \frac{\mu - 1}{\mu^2}\left(\frac{1}{r_2} - \frac{1}{v}\right)^2 \left(\frac{1}{r_2} - \frac{\mu + 1}{v}\right)\frac{h^2}{2} \qquad \text{(iii)}$$

Subtracting eqn. (ii) from eqn. (iii), we get

$$\left(\frac{1}{v} - \frac{1}{u}\right) = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \frac{\mu - 1}{\mu^2}\left\{\left(\frac{1}{r_1} - \frac{1}{u}\right)^2\left(\frac{1}{r_1} - \frac{\mu + 1}{u}\right) - \left(\frac{1}{r_2} - \frac{1}{v}\right)^2\left(\frac{1}{r_2} - \frac{\mu + 1}{v}\right)\right\}\frac{h^2}{2} \qquad \text{(iv)}$$

Eqn. (iv) gives the distance v of the intersection of marginal ray with the axis (*i.e.,* image distance for marginal rays); that for the paraxial rays say V is given by

$$\frac{1}{V} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

So the longitudinal spherical aberration x is given by

$$x = V - v = \left(\frac{1}{v} - \frac{1}{V}\right)Vv = \left(\frac{1}{V} - \frac{1}{v}\right)V^2$$

$$= \frac{(\mu - 1)}{\mu^2}\left\{\left(\frac{1}{r_1} - \frac{1}{u}\right)^2\left(\frac{1}{r_1} - \frac{\mu + 1}{u}\right) - \left(\frac{1}{r_1} - \frac{1}{v}\right)^2\left(\frac{1}{r_2} - \frac{\mu + 1}{v}\right)\right\}\frac{h^2 v^2}{2} \qquad \text{(v)}$$

In case of parallel rays *i.e.,* when the object is at infinity, $u = \infty$ and $v = f$. Eqn. (v), which is a general expression for x, then reduces to

$$x = \frac{(\mu - 1)}{\mu^2}\left[\frac{1}{r_1^3} - \left(\frac{1}{r_2} - \frac{1}{f}\right)^2\left(\frac{1}{r_2} - \frac{\mu + 1}{f}\right)\right]\frac{h^2 f}{2} \qquad \text{(vi)}$$

Substituting $\frac{r_1}{r_2} = k$ and $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

We get $\frac{1}{f} = \frac{(\mu - 1)(1 - k)}{r_1} = \frac{(\mu - 1)(1 - k)}{kr_2}$ (vii)

Eliminating r_1 and r_2 between (vi) and (vii), we get

$$x = -\frac{h^2}{f}\left[\frac{2-2\mu^2+\mu^3+k\left(\mu+2\mu^2-2\mu^3\right)+k^2\mu^3}{2\mu(\mu-1)^2(1-k)^2}\right] \qquad (25.2)$$

The ratio $\frac{h}{f}$ is called the *relative aperture* of the lens.

It appears from eqn. 25.2 that when k is constant the ratio of longitudinal spherical aberration to the focal length is directly proportional to the square of the relative aperture of the lens. The lens is said to be *undercorrected* for spherical aberration when the focus of the marginal rays is nearer to the lens than the focus of the paraxial rays; otherwise it is called *overcorrected*. In the former case the longitudinal spherical aberration is called negative and in the latter positive.

For given value of h, f and μ we can always find a value of k for which spherical aberration is minimum. Thus differentiating eqn. 25.2 with respect to k and equating the result to zero the value of k corresponding to minimum spherical aberration is given by

$$k = \frac{R_1}{R_2} = \frac{\mu(2\mu-1)-4}{\mu(2\mu+1)} \qquad (25.3)$$

It is evident from eqn. (ix) that the spherical aberration for a single lens can never be made to vanish.

For a lens, the refractive index of whose material is $\mu = 1.5$, $k = -\frac{1}{6}$. Thus the lens giving minimum spherical aberration must be *bi-convex* or *bi-concave*, as indicated by the minus sign, and the radius of curvature of its first surface (surface facing the rays) should be $1/6^{th}$ the radius of curvature of its second surface. For a given focal length of the lens, one can therefore, control the spherical aberration by changing the value of *k*. This procedure is called the *bending of the lens* and a lens having the desired value of *k* for which spherical aberration is minimum is called *crossed* lens. It should, however, be emphasized that the spherical aberration cannot be completely removed by any bending of lens with spherical surfaces. However, spherical aberration can be further reduced in a lens of the same

focal length and radius of aperture by simply constructing it from a material of higher refractive index. As for example, for a lens with *f* = 100 cm and radius of aperture h = 10 cm, x = 1.07 cm when μ and *k* are 1.5 and – 1/6 respectively. If for the same values of *f* and *h*, μ is now taken to be equal to 2, *k* becomes equal to + 1/5 and the longitudinal spherical aberration reduces to 0.44 cm. A crossed lens (μ = 1.5 and k = – 1/6) is shown in Fig. 25.4. The physical reason for the minimum longitudinal spherical aberration to occur at k = – 1/6 is as follows. It has already been mentioned that for a convergent lens, longitudinal spherical aberration arises because the marginal rays are much more deviated than the paraxial rays. Therefore, spherical aberration will be minimum when the deviation of marginal rays are minimum. In the case of a prism the deviation is minimum when the incident and emergent rays make equal angles with its faces *i.e., the total deviation suffered is equally divided between the two surfaces*. As a lens may be regarded as a number of truncated prisms placed one above the other, the deviation of marginal rays will be minimum when they enter the first surface and leave the second surface at more or less equal angles. Thus, as a general rule, when a lens is *so designed or used that the total deviation of a given ray is divided equally between the two refractions suffered at the two surface, spherical aberration will be minimum.*

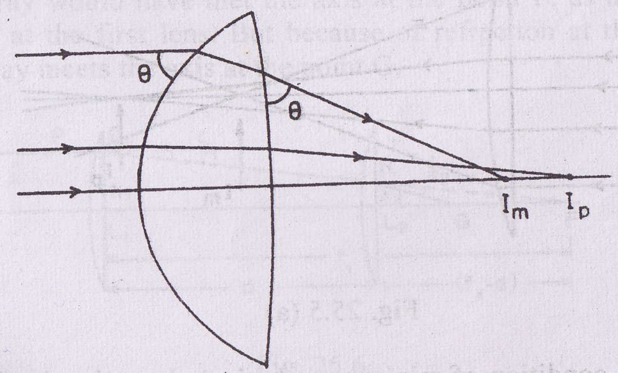

Fig. 25.4

Looking at Fig. 25.3, it is clear that for $k = -\frac{1}{6}$ for the given lens, the deviations suffered at each of the surface are equal and the paraxial and marginal rays of light come to focus with minimum of spherical aberration.

(iii) Use of plano-convex lens : Crossed lenses are expensive. For this reason, plano-convex lenses which are much cheaper to make, are employed in many optical instruments to reduce spherical aberration. When the curved surface of a plano-convex lens faces the light, the longitudinal spherical aberration is very nearly the same as that produced by a crossed lens of the same foal length and radius of the lens aperture. In reality the spherical aberration of the crossed lens is only 8% less than that of the plano-convex lens. However, if the plane side is turned towards the object the spherical aberration is very large. The reason is not far to seek. As already mentioned, spherical aberration will be minimum when a lens is so designed or used that the total deviation of a given ray is divided equally between the two refractions. When a plano-convex lens is so used that its plane surface faces the object as illustrated in Fig. 25.5 (a) the deviation of a given ray is simply produced at the curved surface

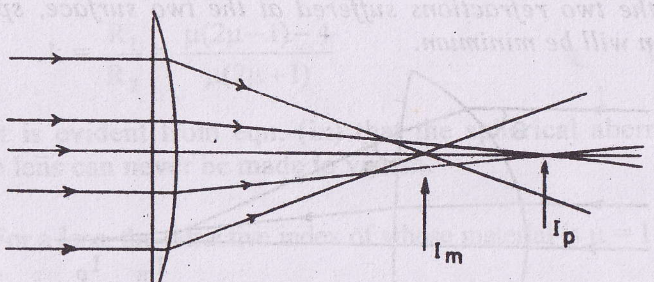

Fig. 25.5 (a)

and thus the condition of minimum spherical aberration is violated, which explains the presence of a large spherical aberration. On the other hand when the curved surface faces the light, the deviation is almost equally divided between the two surfaces, as shown in Fig. 25.5 (b) thus accounting for the presence of minimum spherical aberration.

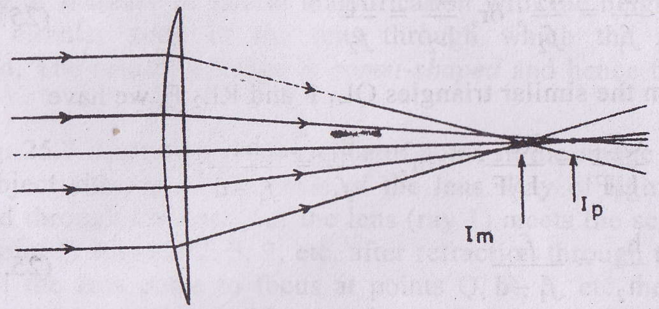

Fig. 25.5 (b)

(iv) Use of two convergent lenses separated by a fixed distance : Spherical aberration can be minimised by extending the above principle of equal distribution of the deviation between the two surfaces of the lens to two convergent lenses separated by a distance. In this arrangement, two convergent lenses of focal lengths f_1 and f_2 are placed coaxially at a distance d apart as shown in Fig. 25.6.

Let a ray PQ parallel to the principal axis be incident on the first lens at a height h_1 above the axis. In the absence of the second lens, the ray would have met the axis at the point F, as a result of refraction at the first lens. But because of refraction at the second lens, the ray meets the axis at the point G.

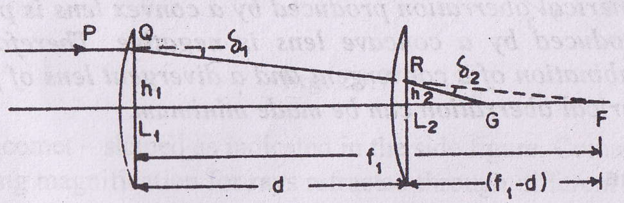

Fig. 25.6

Let δ_1 and δ_2 be the deviations produced by the first and second lens respectively. For minimum spherical aberration $\delta_1 = \delta_2$

Now $\delta_1 = \dfrac{h_1}{f_1}$ and $\delta_2 = \dfrac{h_2}{f_2}$

$$\text{or,} \quad \frac{h_1}{f_1} = \frac{h_2}{f_2}; \text{ or, } \frac{h_1}{h_2} = \frac{f_1}{f_2} \tag{25.4}$$

From the similar triangles QL_1 F and RL_2 F, we have

$$\frac{QL_1}{L_1F} = \frac{RL_2}{L_2F} \quad \text{or,} \quad \frac{h_1}{f_1} = \frac{h_2}{f_1 - d};$$

$$\text{or,} \quad \frac{h_1}{h_2} = \frac{f_1}{f_1 - d} \tag{25.5}$$

Equating the values of h_1/h_2 from eqns. (25.4) and (25.5) we have

$$\frac{f_1}{f_2} = \frac{f_1}{f_1 - d}; \quad \text{or,} \quad \frac{1}{f_2} = \frac{1}{f_1 - d}$$

$$\text{or,} \quad f_1 - f_2 = d \tag{25.6}$$

Thus the spherical aberration for a combination of two convergent lenses is minimum when the distance between the two lenses is equal to the difference of their focal lengths. It is essential that the incident ray should suffer refraction first through the lens of longer local length and then through the one of smaller focal length. *Spherical aberration can be further minimized by using plano-convex lenses instead of bi-convex lenses.*

(v) Spherical aberration produced by a convex lens is positive and that produced by a concave lens is negative. Therefore, by suitable combination of a convergent and a divergent lens of proper shapes, spherical aberration can be made minimum.

25.4 Coma

Rays coming from an object point not situated on the axis of the lens suffers another type of aberration called *coma* (Greek *kome*, meaning *hair*). Comatic aberration is similar to spherical aberration in that both are due to the failure of the lens to bring all rays from a point object to focus at the same point. Spherical aberration refers to object points situated on the axis whereas comatic aberration refers to object points situated off the axis. Comatic aberration arises from the fact that for these non-axial point objects there is either a

decrease or increase of lateral magnification with the height of the narrow circular zone of the lens through which the rays are refracted. The resulting image is *comet-shaped* and hence the name *coma.*

Fig. 25.7 illustrates the presence of coma in the image due to a point object situated off the axis of the lens. Ray of light getting refracted through the centre of the lens (ray 1) meets the screen XY at the point P. Rays 2, 2, 3, 3, etc. after refraction through the outer zones of the lens come to focus at points Q, R, S, etc. nearer the lens. Thus the lateral magnification for outer zones is less than that for the central zone and on the screen overlapping circular patches of gradually increasing diameter are formed. The resultant image of the

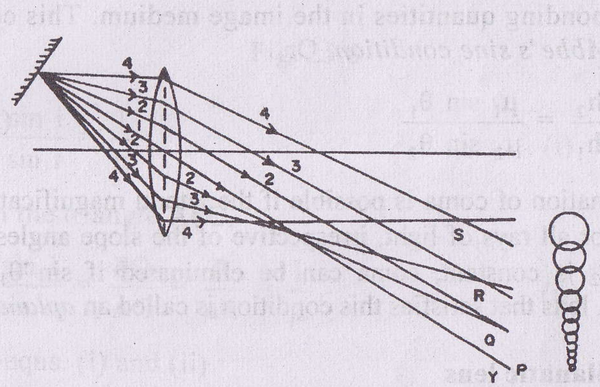

Fig. 25.7

point is comet – shaped as indicated in the side figure. Coma is the result of varying magnification for rays refracted through different zones of the lens and can be positive or negative. As can be seen in Fig. 25.7 for example, rays of light getting refracted through the outer zones come to focus at points nearer the lens. hence the magnification of the image due to the outer zones is smaller than the inner zones and in this case coma is said to be *negative*. Coma is said to be *positive* when the magnification produced in an image due to the outer zones is greater.

Like spherical aberration comatic aberration produced by a single lens can also be corrected by properly choosing the radii of curvature of the lens surfaces. Coma can be altogether eliminated for a given pair of object and image points whereas spherical aberration cannot be completely corrected. Further, a lens corrected for coma will not be free from spherical aberration and *vice versa*. Use of a stop or a diaphragm at the proper position eliminates coma.

25.5 Abbe's sine condition

Abbe, a German optician, showed that coma can be eliminated if a lens satisfies the condition

$$\mu_1 h_1 \sin \theta_1 = \mu_2 h_2 \sin \theta_2 \qquad (25.7)$$

where μ_1, h_1 and θ_1 refer to the refractive index of the object medium, height of the object above the axis and the slope angle of the incident ray of light respectively. Similarly μ_2, h_2 and θ_2 refer to the corresponding quantities in the image medium. This condition is known as *Abbe's sine condition*. Or

$$\frac{h_2}{h_1} = \frac{\mu_1 \sin \theta_1}{\mu_2 \sin \theta_2} \qquad (25.8)$$

Elimination of coma is possible if the lateral magnification h_1/h_2 is the same for all rays of light, irrespective of the slope angles θ_1 and θ_2. Since μ_1/μ_2 is constant, coma can be eliminated if $\sin \theta_1 / \sin \theta_2$ is constant. A lens that satisfies this condition is called an *aplanatic lens*.

25.6 Aplanatic lens

A lens which is free from the defects of spherical aberration and coma is called an *aplanatic lens* and the pair of conjugate points free from these defects are called *aplanatic points*. Fig. 25.8 illustrates the *aplanatic lens* and aplanatic points. O and R are the centre and radius of curvature of the lens respectively, the refractive index of whose material is μ. P is a point on the axis of the lens such that $PO = \frac{R}{\mu}$. It can be shown that all rays passing through the point P appear to diverge from the point Q irrespective of the slope angle made by the incident rays. In the figure PA is the incident ray; the corresponding refracted ray appears to diverge from the point Q which, therefore, is the image of P. Let i and r be the angles of

incidence and refraction and α and β the slope angles made by the incident and refracted rays.

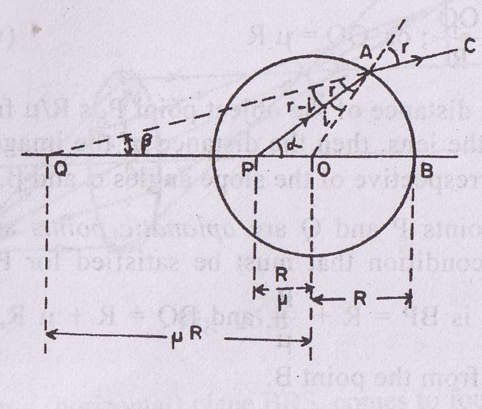

Fig. 25.8

$$\text{Now} \quad \frac{\sin i}{\sin r} = \frac{1}{\mu} \qquad (i)$$

But in the triangle APO,

$$\frac{\sin i}{\sin \alpha} = \frac{PO}{AO} = \frac{R}{\mu R} = \frac{1}{\mu} \qquad (ii)$$

From eqns. (i) and (ii)

$$\frac{\sin i}{\sin r} = \frac{\sin i}{\sin \alpha}; \quad \text{hence } \angle r = \angle \alpha \qquad (iii)$$

Again in the triangle APQ,

$$\alpha = \beta + (r - i) \qquad (iv)$$

Substituting $\angle\alpha = \angle r$ in equation (iv)

$$r = \beta + r - i; \quad \text{or, } i = \beta \qquad (v)$$

In the triangle AQO

$$\frac{\sin r}{\sin \beta} = \frac{\sin r}{\sin i} = \mu$$

$$\text{But } \frac{\sin r}{\sin i} = \frac{OQ}{OA} = \frac{OQ}{R}$$

$$\text{Hence } \mu = \frac{OQ}{R}; \text{ or, } OQ = \mu R \qquad \text{(vi)}$$

Thus, if the distance of the object point P is R/μ from the centre of curvature of the lens, then the distance of the image point Q will always be μR irrespective of the slope angles α and β.

Thus the points P and Q are *aplanatic points* and the lens is *aplanatic*. The condition that must be satisfied for P and Q to be aplanatic points is $BP = R + \frac{R}{\mu}$ and $BQ = R + \mu R$, the distances being measured from the point B.

The high power microscope objective, called the oil immersion objective uses an aplanatic lens as the front lens. As it is not possible to place an object inside a solid spherical lens, the lens is ground a little and the object embedded in between a drop of oil and the lens surface. The oil should be chosen in such a manner that it has the same refractive index as that of the lens.

25.7 Astigmatism

When a lens is corrected for spherical aberration and coma, it will form sharp images of object points lying on or very near the axis. But the image of an object point lying at appreciable distance from the lens axis is not a point but a pair of mutually perpendicular lines, some distance apart. This aberration is known as *astigmatism*. Both coma and astigmatism are aberrations in the images of object points off the axis. The difference between astigmatism and coma, however, is that in coma the spreading of the image takes place in a plane perpendicular to the lens axis and in astigmatism the spreading takes place along the lens axis. *Astigmatism discussed here is different from that in defective vision.*

Fig. 25.9 illustrates the defect of astigmatism in the image of a point B situated off the axis. For the point B, far away from the axis, the lens is not perfectly symmetrical. The cone of the rays of light refracted through the *tangential or meridional* (vertical) plane BMN comes to focus at a point P_1 nearer the lens. The cone of rays refracted

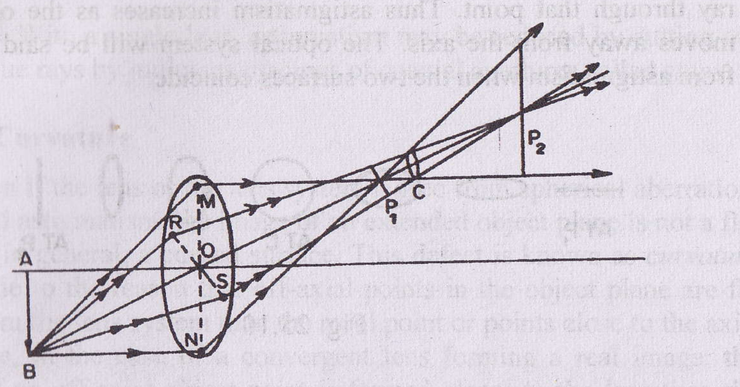

Fig. 25.8

through the *sagittal* (horizontal) plane BRS, comes to focus at a point P_2 away from the lens. Since at the point P_1, the rays in the sagittal plane have not still focussed, one in fact has a focal line which is normal to the meridional plane. This focus line at P_1 is called the *tangential focal line*. Similarly, since at P_2, the rays at the meridional plane have defocussed, one obtains a focal line which lies in the tangential plane and is called the *sagittal focal line*. All rays pass through a horizontal line passing through P_1 called the *primary image* and also through a vertical line passing through P_2 called the *secondary image*. The cross-section of the refracted beam is elliptical which ends to a horizontal line at P_1 and a vertical line at P_2. At some point between P_1 and P_2 the cross-section of the refracted beam is circular and this is called the *circle of least confusion*. If a screen is held perpendicular to the refracted beam between the points P_1 and P_2 the shape of the image at different positions is as shown in Fig. 25.10. The distance between P_1 and P_2 is a measure of astigmatism.

Each point on the extended object gives rise, in the manner described above, to its corresponding primary image, secondary image and the circle of least confusion. Their respective loci are surfaces of revolution, about the lens axis, paraboidal in form and called the *primary image surface, the secondary image surface and the surface of best focus* respectively. These surfaces are tangential to one another at a point on the axis of the lens, the point being the paraxial image of the conjugate axial point A in the object space. The amount of astigmatism present corresponding to any object point is measured by the difference between the primary and secondary image surfaces measured along the principal ray through that point. Thus astigmatism increases as the object point moves away from the axis. The optical system will be said to be free from astigmatism when the two surfaces coincide.

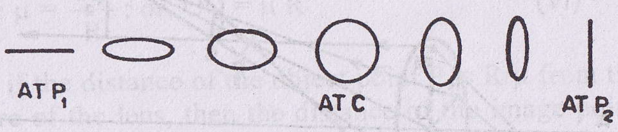

Fig. 25.10

Reduction or elimination of astigmatism

(a) In case of a divergent lens the sagittal focus P_2 is nearer the lens than the meridional focus P_1 and astigmatism is said to be *negative*. For a convergent lens, the meridional focus P_1 is nearer the lens than the sagittal focus P_2 and the astigmatism is said to be *positive*. Fig. 25.11 illustrates positions of the primary and the secondary images for a convergent as well as a divergent lens.

Since astigmatism is positive in case of a convergent lens and negative in case of a divergent lens, by proper spacing of convergent and divergent lenses and suitable choice of their focal lengths, a combination may be designed in which the astigmatic differences compensate for one another to some extent and the images are formed on a single paraboloid surface, shown by the thick line. Such a combination is called an *astigmat*.

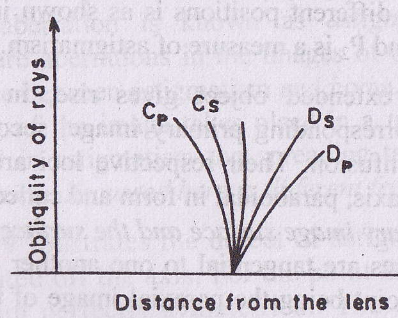

Fig. 25.11

(b) With a single lens, astigmatism may be reduced by cutting out the oblique rays by judicious placings of coaxial apertures called *stops*.

25.8 Curvature

Even if the lens or the lens system is free from spherical aberration, coma and astigmatism, the image of an extended object plane is not a flat one but, in general, a curved surface. This defect is known as *curvature* and is due to the reason that off-axial points in the object plane are far away from the lens system than the axial point or points close to the axis. Therefore, in the case of a convergent lens forming a real image, the image of an off-axial object point is formed closer to the lens than the image of an axial object point. Thus the marginal focal length is shorter than the paraxial focal length. This accounts for the curvature of image shown in Fig. 25.12. In a convergent system the radius of curvature of the image surface is negative and hence the curvature is regarded as *negative*. Since the converse holds true for a divergent system, the resultant curvature is regarded as *positive*. As a consequence of this, by a suitable combination of convergent and divergent systems the image can be flattened.

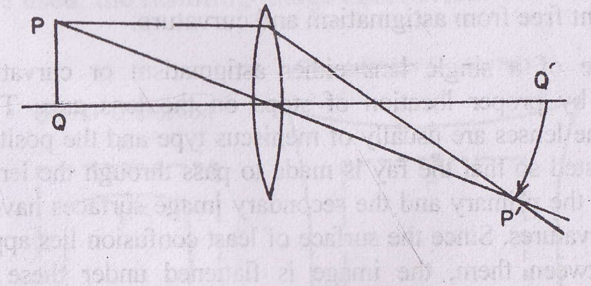

Fig. 25.12

From theoretical considerations it can be shown that for a system of this lenses, the curvature in the final image is given by

$$\frac{1}{R} = \Sigma \frac{1}{\mu_n f_n}$$

where R is radius of curvature of the final image, μ_n and f_n are respectively the refractive index of the material and focal length of the n^{th} lens. It follows from the relation that for no curvature *i.e.*, for the image to be flat, R must be infinity.

$$\therefore \quad \frac{1}{R} = \Sigma \frac{1}{\mu_n f_n} = \frac{1}{\infty} = 0$$

In the case of two lenses the above condition reduces to

$$\frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} = 0, \quad \text{or,} \quad \mu_1 f_1 + \mu_2 f_2 = 0$$

Since μ_1 and μ_2 are positive, f_1 and f_2 should be of opposite signs. This means that the above condition will be satisfied if one of the lenses is convex and the other concave. The condition holds good if the lenses are separated by a distance or placed in contact. The condition is known as *Petzwal's condition* for no curvature. If f_1 refers to the convergent component, then for the combination to be also convergent, f_2 must be greater than f_1. Therefore, for Petzwal's condition to be satisfied, it is essential that μ_2 must be less than μ_1. Petzwal's condition can also be satisfied by forming a combination of convergent and divergent lenses made of the same glass, of equal focal lengths and separated by a distance less than the focal length of either of the lenses. Their combined focal length f, under such condition, is positive, that is, the combination is convergent free from astigmatism and curvature.

In case of a single lens either astigmatism or curvature can be minimized, by proper location of stops on the lens axis. To eliminate curvature, the lenses are usually of meniscus type and the position of front stop is adjusted so that the ray is made to pass through the lens in such a manner that the primary and the secondary image surfaces have equal and opposite curvatures. Since the surface of least confusion lies approximately midway between them, the image is flattened under these conditions. Astigmatism, however, is more pronounced in the outer parts of the field.

25.9 Distortion

Even if it is possible to eliminate or minimize spherical aberration, coma, astigmatism and curvature, there may remain another type of aberration termed *distortion.* This aberration arises because of variation in the magnification with lateral distance produced by a lens for object points at different distances from the lens axis. This aberration is not due

to lack of sharpness in the image and is of two types *viz.*, (a) *pin-cushion distortion* and (b) *barrel-shaped distortion*. In the pin-cushion distortion, the magnification increases with the increase in the lateral distance. Owing to the disproportionately high magnification of the corners as compared with other points, the image of a *square like object* situated transverse to the axis will be of the form resembling a *pin-cushion*.

On the other hand, if the magnification decreases with increasing axial distance, the image of the diagonal is shortened relatively more than the images of the side of the square, that is, the opposite effect is produced. The resulting image of the side of the square is of the form resembling a *barrel* giving rise to a distortion known as barrel-shaped distortion. In Fig. 25.13 the *ideal image is shown by the dotted line* and the *distorted image by the solid line.*

Reduction or elimination of distortion

A little amount of distortion may be present in the case of optical instruments intended mainly for visual observation. But photographic camera lens, where the magnification of the various regions of the object should be the same, must be completely free from distortion. A single thin lens, without any stop to limit the rays, is free from distortion practically for all object distances. But if stops are used, the resulting image is distorted. The distortion will

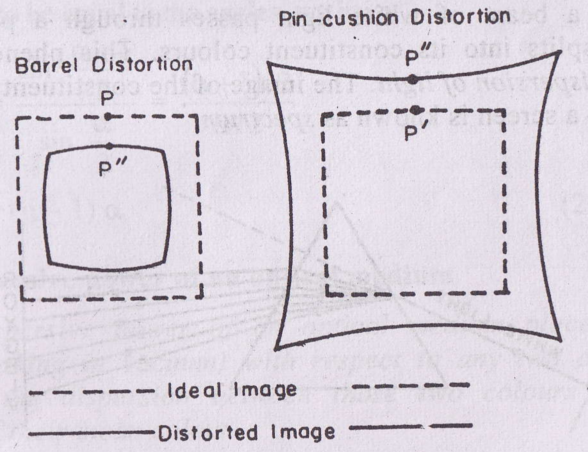

Fig. 25.12

be of barrel-shaped or of pin-cushion type depending on whether the stop is placed in between the object and the lens or the lens and the image.

To eliminate distortion, a stop is placed in between two symmetrical lenses, so that the pin-cushion distortion produced by the first lens is compensated by the barrel-shaped distortion produced by the second lens (Fig. 25.14). Camera and projection lenses are constructed in this manner.

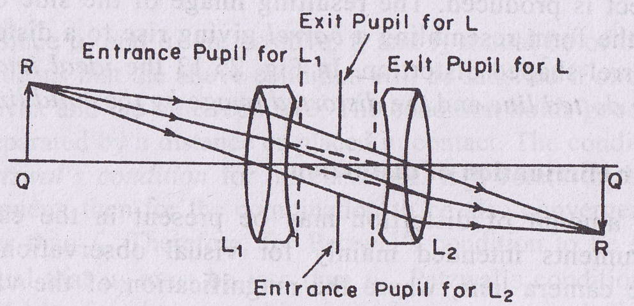

Fig. 25.14

25.10 Dispersion by prism

When a beam of white light passes through a prism (Fig. 25.15), it splits into its constituent colours. This phenomenon is known as *dispersion of light*. The image of the constituent colours as received on a screen is known as *spectrum*.

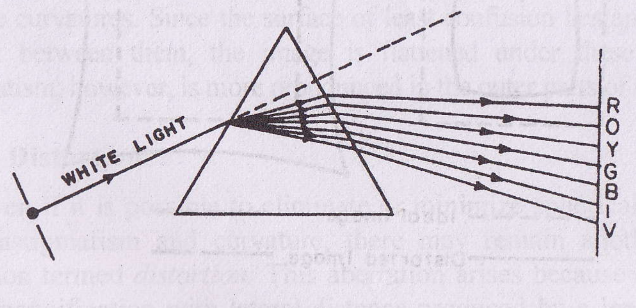

Fig. 25.15

The spectrum consists of both visible and invisible regions. The colours in the visible region are represented by VIBGYOR (violet, indigo, blue, green, yellow, orange and red). The region of the spectrum of wavelengths longer than red is called *infrared* and the region of wavelengths shorter than violet is called *ultraviolet*.

The refractive index for the material of a prism (or of a lens) depends on the wavelength (or colour) of light. The refractive index for violet ray of light is greater than that of red ray. Hence violet is deviated more than the red. A ray which has the average wavelength of two rays near the two ends of the spectrum is taken as the *mean ray* of these two colours. In spectroscopy, yellow ray is taken as the mean ray of the red and blue rays.

25.11 Refraction through a prism

The refractive index of the material of a prism for a particular colour is given by

$$\mu = \frac{\sin\left[(A+D)/2\right]}{\sin\,(A/2)}$$

where A is the angle of the prism and D is the angle of minimum deviation for the colour, say yellow. For a small angled prism, A is small and so is D. Writing α and δ for A and D respectively, and taking sines of the angles to be equal to the angles, we have

$$\mu = \frac{\sin\dfrac{\alpha+\delta}{2}}{\sin\dfrac{\alpha}{2}} = \frac{(\alpha+\delta)/2}{\alpha/2}$$

or, $$\delta = (\mu - 1)\,\alpha \tag{25.9}$$

25.12 Dispersive power of an optical medium

The dispersive power of an optical medium placed in air (strictly speaking in vacuum) with respect to any two colours is defined as the dispersion between those two colours per unit deviation of their mean colour.

Let δ_r, δ_b and δ be the angles of deviation produced in the red, blue and their mean colour yellow rays respectively by a prism (Fig. 25.16).

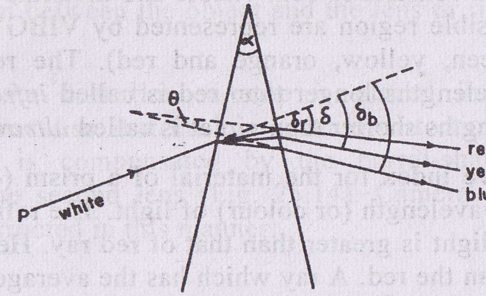

Fig. 25.16

From eqn. (25.15)

$$\delta_b = (\mu_b - 1)\,\alpha\ ;\ \delta_r = (\mu_r - 1)\,\alpha$$

and $\delta = (\mu - 1)\,\alpha$

where α is the angle of the prism and μ_b, μ_r and μ are the respective refractive indices of the blue, red and yellow rays for the material of the prism. The difference in deviation between any two colours is called the *angular dispersion* for those two colours. The angular dispersion between the blue and the red ray, is therefore,

$$\delta_b - \delta_r = (\mu_b - 1)\,\alpha - (\mu_r - 1)\,\alpha$$
$$= (\mu_b - \mu_r)\,\alpha \qquad (25.10)$$

Dividing equation (25.10) by δ, the deviation for the mean ray, we have

$$\frac{\delta_b - \delta_r}{\delta} = \frac{(\mu_b - \mu)\alpha}{(\mu - 1)\alpha} = \frac{\mu_b - \mu_r}{\mu - 1} = \frac{d\mu}{\mu - 1} \qquad (25.11)$$

where $d\mu$ is the change in refractive index of the material of the prism as the colour changes from red to blue. But $(\delta_b - \delta_r)/\delta$ by definition is the dispersive power of the optical medium (material of the prism) with respect to the blue and red colours. Hence, the dispersive power of an optical medium, usually denoted by ω, is given by

$$\omega = \frac{\mu_b - \mu_r}{\mu - 1} \qquad (25.12)$$

The dispersive power of an optical medium is constant for two colours considered. The reciprocal of the dispersive power is called the *constringence*.

Note : *If the dispersion through a prism does not follow the order given by vibgyor, it is said to be anomalous dispersion.*

25.13 Chromatic aberration in a lens

Every refracting medium has a different refractive index for different wavelengths of light. The focal length of a thin lens is related to the refractive index of the optical material by the relation,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The focal length of a lens, therefore, depends upon the wavelength of light and since $\mu_v > \mu_r$, the focal length f_v for the violet ray is less than the focal length f_r of the red ray. Thus the focal length decreases as we pass from red to the violet end of the spectrum. A single lens, therefore, forms not merely one image of an object point but a series of coloured images at varying distances from the lens, one for each of the colours constituting the incident beam. Thus, even if the lens or the lens system is somehow corrected simultaneously for the monochromatic aberrations (spherical aberration, coma, curvature, astigmatism and distortion), which is practically impossible, there will be present another aberration called *chromatic aberration* in the images formed by lenses, when instead of being monochromatic, the light diverging from the object is polychromatic. A beam of white light parallel to the principal axis of a convex lens will converge to different foci at various distances from the lens. The point F_v (Fig. 25.17) where the violet rays converge is known as the *violet focus*, the point F_r where the red rays meet as the *red focus* and so on. The distances f_v, f_r, etc. from the optical centre of the lens to the different foci are known as the violet focal length, red focal length, etc. If a screen is placed at F_v perpendicular to the axis, the image would be a violet point surrounded by annular coloured patches. If the screen is placed at F_r, the image will be a red point surrounded by

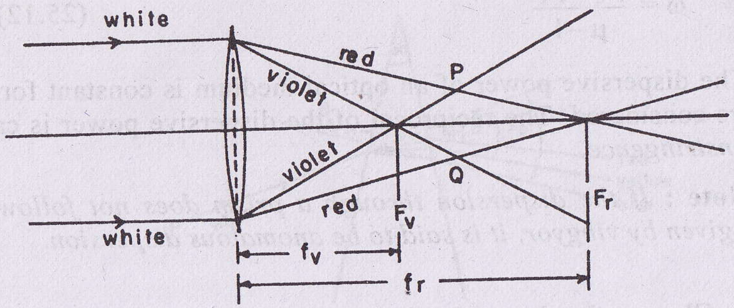

Fig. 25.17

annular coloured patches. But at PQ the image will be an approximately white circular patch. This is called the *circle of least confusion* and is the nearest approach to the image.

Figs. 25.18 (a) & (b) show the image of an extended white object formed by a convex and a concave lens respectively. Since $f_v < f_r$ the violet image is formed nearest to the lens and is also the smallest, while the red image is farthest away and also the largest. The image of other colours (not shown in the figure) are formed at the intermediate positions and are of intermediate sizes. Evidently there is no one plane where all the images are simultaneously in good focus. Therefore, on a screen, we shall get a blurred image due to superposition of numerous coloured images out of focus, the best possible image being obtained at near about the yellow image.

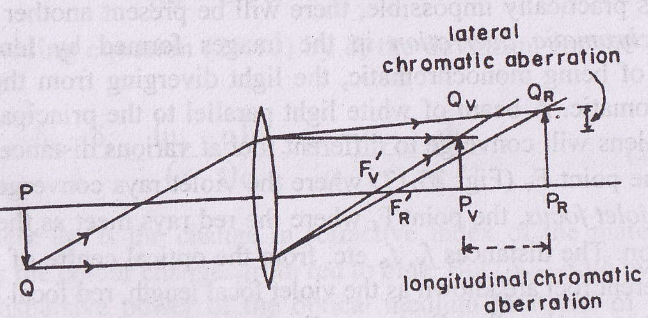

Fig. 25.18 (a)

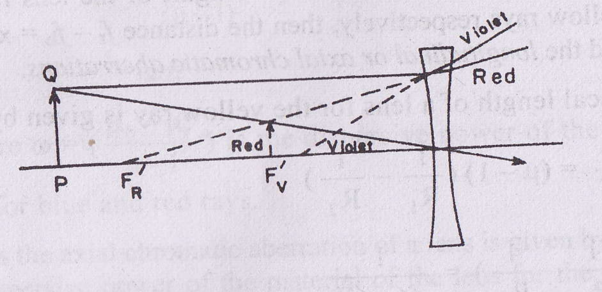

Fig. 25.18 (b)

The variation of the image distance along the axis of the lens due to change in wavelength, $\frac{dv}{d\lambda}$, is called the *axial or longitudinal chromatic aberration*. It is measured by the linear distance along the axis between the extreme images and is said to be *positive* when the violet image is situated to the left of the red image. The variation in the image size with wavelength, $\frac{dy}{d\lambda}$, is called the *lateral chromatic aberration* and is measured by the difference in the lateral sizes of the extreme images. It is said to be *positive* when the red image is more magnified than the violet.

Convergent lens produces positive longitudinal and lateral chromatic aberrations while divergent lens produces negative longitudinal chromatic aberration but positive lateral chromatic aberration.

25.14 Axial chromatic aberration in a lens

When a parallel beam of white light is passed through a lens, the beam gets refracted as well as dispersed and rays of light of different colours come to focus at different points along the axis. The violet light is closest to the lens while the red light is the farthest away from the lens (Fig. 25.17). The refractive indices of glass are usually given for Fraunhofer lines *viz.* C-Red (λ_C = 6563 A.U.), D-yellow (λ_D = 5893 A.U.) and F-Blue (λ_F = 4862 A.U.). The wavelength of the yellow light may, therefore, be regarded as the approximate average of the blue and red rays. Hence in the consideration of the chromatic aberration in a lens, blue and red rays are taken as the two extreme rays instead of the violet and red rays. If f_r, f_b and f are the focal lengths of the lens for the red, blue and yellow rays respectively, then the distance $f_r - f_b = x$ along the axis is called the *longitudinal or axial chromatic aberrations*.

The focal length of a lens for the yellow ray is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\text{or,}\quad \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f(\mu-1)}$$

The focal length of the lens for the blue ray is given by

$$\frac{1}{f_b} = (\mu_b - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(\mu_b - 1)}{f(\mu-1)} \tag{25.13}$$

Similarly, the focal length for the red ray is

$$\frac{1}{f_r} = \frac{(\mu_r - 1)}{f(\mu-1)} \tag{25.14}$$

From equations (25.13) and (25.14)

$$\frac{1}{f_b} - \frac{1}{f_r} = \frac{(\mu_b - 1)}{f(\mu-1)} - \frac{(\mu_r - 1)}{f(\mu-1)}$$

$$\text{or,}\quad \frac{f_r - f_b}{f_r f_b} = \frac{1}{f(\mu-1)}(\mu_b - 1 - \mu_r + 1)$$

$$= \frac{(\mu_b - \mu_r)}{f(\mu-1)} \tag{25.15}$$

Since, f is the focal length of the mean ray, it is possible to write $f_r f_b \simeq f^2$. Eqn. 25.15 may, therefore, be written as

$$\frac{f_r - f_b}{f_r f_b} = \frac{(\mu_b - \mu_r)}{f(\mu-1)}$$

$$\text{or,}\quad f_r - f_b = \frac{(\mu_b - \mu_r) f^2}{f(\mu-1)}$$

$$= \frac{(\mu_b - \mu_r)}{(\mu-1)} \cdot f$$

$$= \omega . f \tag{25.16}$$

where $\omega = \left(\frac{\mu_b - \mu_r}{\mu - 1}\right)$ is the dispersive power of the material of the lens for blue and red rays.

Thus the axial chromatic aberration of a lens is given by the product of the dispersive power of the material of the lens for the two extreme colours and their mean focal length. From this it is clear that *a single lens cannot form an image free from chromatic aberration.*

25.15 Achromatism

An optical instrument free from chromatic aberrations is called an *achromatic* instrument. A perfectly achromatic system is that in which all the coloured images should be formed at the same place and the heights of all these images should also be same. This complete achromatism demands that not only the focal points must be made the same but also the principal planes must be made to coincide for different wavelengths *i.e.*, both axial chromatism ($dv/d\lambda$) and lateral chromatism ($dy/d\lambda$) should be zero. These conditions should be fulfilled not for one position of the object but for its every position. However, this ideal achromatism is extremely difficult to accomplish in the case of any actual system. In practice, only partial achromatism is achieved by removing either axial or lateral chromatic aberration but not both simultaneously. In making the lens system partially achromatic, it is essential to keep in mind the aperture and the field of view of the optical system in deciding the type of chromatic aberration to be eliminated because the type of achromatism which is disadvantageous in one type of instrument may be entirely advantageous in the other. For example, if the optical system has a large field of view but small aperture, lateral chromatic aberration should be eliminated so as to provide uniform magnification for all wavelengths although the images would be formed at different planes. On the other hand, if the optical system has a small field of view but large aperture then it should be corrected for axial chromatic aberration. In that case rays of different wavelengths are brought to focus at the same plane but since the principal points of the system are different for different wavelengths, the focal lengths would not be equal with the result that magnification would be different for different wavelengths. Attainment of even this partial achromatism is possible for two colours and their mean colour but not for all colours.

25.16 Condition of achromatism for two lenses in contact

Chromatic aberration is ordinarily corrected by suitably combining two lenses such that the combination is free from axial chromatic aberration, for the two given colours (say, blue and red), the combination being itself considered as a thin lens. If two lenses of focal length f_1 and f_2 are placed in contact then the equivalent focal length f is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad (25.17)$$

where f_1, f_2 and f refer to the respective focal lengths for the mean ray (yellow) of the colours considered.

Now

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= k_1 (\mu_1 - 1) \qquad (25.18)$$

and $$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R'_1} - \frac{1}{R'_2}\right)$$

$$= k_2 (\mu_2 - 1) \qquad (25.19)$$

where μ_1 and μ_2 are the refractive indices of the mean ray for the materials of the first and second lenses respectively.

Eqn. 25.17 then reduces to

$$\frac{1}{f} = k_1 (\mu_1 - 1) + k_2 (\mu_2 - 1) \qquad (25.20)$$

Eqn. (25.20) shows that the equivalent focal length f is a function of wavelength (λ), since both μ_1 and μ_2 are functions of wavelength. Differentiating eqn. (25.20) with respect to λ, we get

$$\frac{d}{d\lambda}\left(\frac{1}{f}\right) = k_1 \frac{d\mu_1}{d\lambda} + k_2 \frac{d\mu_2}{d\lambda} \qquad (25.21)$$

Achromatism of the combination is attained when the focal length f or, $\frac{1}{f}$ does not change with colour, *i.e.*, wavelength. Hence for achromatism, $\frac{d}{d\lambda}\left(\frac{1}{f}\right)$ should be zero. Thus the condition of achromatism is given by

$$\frac{d}{d\lambda}\left(\frac{1}{f}\right) = k_1 \frac{d\mu_1}{d\lambda} + k_2 \frac{d\mu_2}{d\lambda} = 0 \qquad (25.22)$$

From eqns. 25.18 and 25.19, we have

$$k_1 = \frac{1}{f_1 (\mu_1 - 1)} \quad \text{and} \quad k_2 = \frac{1}{f_2 (\mu_2 - 1)}$$

Eqn. (25.22), therefore, becomes

$$\frac{1}{f_1 (\mu_1 - 1)} \cdot \frac{d\mu_1}{d\lambda} + \frac{1}{f_2 (\mu_2 - 1)} \cdot \frac{d\mu_2}{d\lambda} = 0 \qquad (25.23)$$

If we deal with a finite change in refractive index (= dμ) for a finite change in wavelength (= dλ) of light between the two given colours (say, blue and red), then we may write,

$$\frac{1}{f_1} \cdot \frac{d\mu_1}{(\mu_1 - 1)} + \frac{1}{f_2} \cdot \frac{d\mu_2}{(\mu_2 - 1)} = 0 \times d\lambda = 0 \qquad (25.24)$$

But $$\frac{d\mu_1}{\mu_1 - 1} = \omega_1 \quad \text{and} \quad \frac{d\mu_2}{\mu_2 - 1} = \omega_2$$

where w_1 and w_2 are the dispersive powers of the materials of the two lenses for the blue and red rays of light. Eqn. (25.24), therefore, becomes

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \qquad (25.25)$$

Eqn. 25.25 is the required condition of achromatism for two lenses in contact.

From eqn. 25.25 it follows that

(i) *One of the lens is convergent and the other divergent.*

Since $\mu_b > \mu_r$, $d\mu$ is positive and since $\mu > 1$, the dispersive powers ω_1 and ω_2 are always positive. Hence, in order that the eqn. (25.25) be satisfied f_1 and f_2 should be of opposite signs.

(ii) *The lenses should be made of different materials* : If we take two lenses of the same material, then $\omega_1 = \omega_2 = \omega$ (say). Then the condition (25.25) reduces to

$$\omega\left(\frac{1}{f_1} + \frac{1}{f_2}\right) = 0 \quad \text{or, } \omega \cdot \frac{1}{f} = 0.$$

But since w cannot be zero, $\frac{1}{f} = 0$ or, $f = \infty$. The combination will behave like a parallel plate of glass and will not act like a lens. Thus achromatism cannot be achieved by taking two lenses of the same material in contact.

(iii) *The choice of the focal length and dispersive power of the lens is governed by eqn. (25.25).*

In practice a convergent lens of crown glass and a divergent lens of flint glass are used. The convergent lens is more strongly convergent for the blue rays than for the red rays while the divergent lens is more strongly divergent for the blue rays than for the red rays. Hence if a combination is made of these lenses satisfying eqn. (25.25), the coloured images formed by the combination for different wavelengths will fold on one another and the effects of chromatic aberration will be absent. Since the relation (25.25) is independent of the position of the object, achromatism will be obtained for all object distances.

25.17 Achromatic doublet

Since the removal of chromatic aberration only restricts the choice of focal lengths and not the radii of curvature of the lenses (eqn. 25.25), the latter may be chosen to make spherical aberration minimum as well. In practice a double convex lens of crown glass is combined with a plano-concave lens of flint glass. The double convex lens is a crossed lens, its radii of curvature R_1 and R_2 being in the ratio of 1:6. The radius of curvature of the concave surface of the plano-concave lens is made equal to the surface of the convex lens with the larger radius. The two equal surfaces are then put in

contact to form what is known as *achromatic doublet* (Fig. 25.19). To avoid any loss of light the surface in contact are cemented together by *Canada Balsam*. To reduce spherical aberration the curved

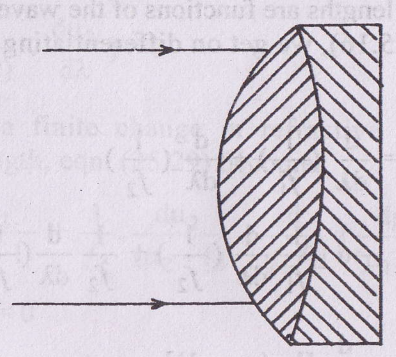

Fig. 25.19

surface of the convex lens with the smaller radius is made to face the beam of light. The focal lengths, f_1 and f_2 are chosen to satisfy the relation $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$, ω_1 and ω_2 being the dispersive powers for the materials of the two lenses. The achromatic doublet is commonly used as an *objective* in telescopes.

In the above cases achromatism has been achieved only in a restricted sense in that the combination has been corrected only for two extreme colours. But if the two colours are suitably chosen the combination will be practically achromatic for the other colours also.

25.18 Condition of achromatism of two lenses separated by a distance (separated doublet)

Let two lenses of focal lengths f_1 and f_2 (for the mean rays) and of dispersive powers ω_1 and ω_2 respectively, are separated by a distance d. Then the equivalent focal length f of the combination is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad (25.26)$$

As in Art. 25.16 the focal lengths of the individual lenses are given by

$$\frac{1}{f_1} = k_1(\mu_1 - 1) \text{ and } \frac{1}{f_2} = k_2(\mu_2 - 1)$$

Since the focal lengths are functions of the wavelengths λ (already discussed in Art. (25.16), we get on differentiating eqn. (25.26) with respect to λ

$$\frac{d}{d\lambda}\left(\frac{1}{f}\right) = \frac{d}{d\lambda}\left(\frac{1}{f_1}\right) + \frac{d}{d\lambda}\left(\frac{1}{f_2}\right)$$

$$- d\left[\frac{1}{f_1}\frac{d}{d\lambda}\left(\frac{1}{f_2}\right) + \frac{1}{f_2}\frac{d}{d\lambda}\left(\frac{1}{f_1}\right)\right] \quad (25.27)$$

But $$\frac{d}{d\lambda}\left(\frac{1}{f_1}\right) = \frac{d}{d\lambda}\left[k_1(\mu_1 - 1)\right]$$

$$= k_1\frac{d\mu_1}{d\lambda} = \frac{1}{f_1(\mu_1 - 1)}\frac{d\mu_1}{d\lambda}$$

Similarly

$$\frac{d}{d\lambda}\left(\frac{1}{f_2}\right) = \frac{1}{f_2(\mu_2 - 1)}\frac{d\mu_2}{d\lambda}$$

Eqn. (25.27), therefore, may be written as

$$\frac{d}{d\lambda}\left(\frac{1}{f}\right) = \frac{1}{f_1(\mu_1 - 1)}\frac{d\mu_1}{d\lambda} + \frac{1}{f_2(\mu_2 - 1)}\frac{d\mu_2}{d\lambda}$$

$$- d\left[\frac{1}{f_1} \cdot \frac{1}{f_2(\mu_2 - 1)}\frac{d\mu_2}{d\lambda} + \frac{1}{f_2}\frac{1}{f_1(\mu_1 - 1)}\frac{d\mu_1}{d\lambda}\right] \quad (25.28)$$

For the combination to be achromatic,

$$\frac{d}{d\lambda}\left(\frac{1}{f}\right) = 0$$

Hence

$$\frac{1}{f_1(\mu_1 - 1)} \cdot \frac{d\mu_1}{d\lambda} + \frac{1}{f_2(\mu_2 - 1)} \cdot \frac{d\mu_2}{d\lambda} - \frac{d}{f_1 f_2}\left[\frac{1}{(\mu_1 - 1)} \cdot \frac{d\mu_1}{d\lambda}\right.$$

$$\left. + \frac{1}{(\mu_2 - 1)} \cdot \frac{d\mu_2}{d\lambda}\right] = 0 \quad (25.29)$$

Considering a finite change in refractive index for a finite change in wavelength, eqn. (25.29) becomes

$$\frac{1}{f_1} \cdot \frac{d\mu_1}{(\mu_1 - 1)} + \frac{1}{f_2} \cdot \frac{d\mu_2}{(\mu_2 - 1)} - \frac{d}{f_1 f_2}\left[\frac{d\mu_1}{(\mu_1 - 1)} + \frac{d\mu_2}{(\mu_2 - 1)}\right]$$

$$= 0 \times d\lambda = 0$$

or, $$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{d}{f_1 f_2}(\omega_1 + \omega_2) = 0$$

or, $$\frac{d}{f_1 f_2}(\omega_1 + \omega_2) = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{\omega_1 f_2 + \omega_2 f_1}{f_1 f_2}$$

or, $$d = + \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2} \quad (25.30)$$

Eqn. (25.30) gives the condition of achromatism of a *separated doublet* when the lenses are made of different material. If the lenses are of the same material $\omega_1 = \omega_2 = \omega$ (say), eqn. (25.30) then becomes

$$d = + \frac{1}{2}(f_1 + f_2) \quad (25.31)$$

Thus if the lenses are of the same material, chromatic aberration can be eliminated by keeping them separated by half the sum of their focal lengths. As d is positive $(f_1 + f_2)$ should be positive. Hence *both the lenses or the lens with the longer focal length must be convergent.* The combination will then be achromatic for all the colours near those for which the mean focal lengths f_1 and f_2 have been calculated. This principle is utilised in the construction of *eye-pieces* for optical instruments.

Example 25.1. The focal lengths of a convex lens are 100 cm and 98 cm for red and blue rays respectively. Calculate the dispersive power of the material of the lens.

Soln.

In terms of the principal focal lengths the dispersive power may be written as

$$\omega = \frac{f_r - f_b}{f} \qquad \text{(eqn. 25.16)}$$

where f is the focal length of the mean ray. Since f is intermediate in value between f_b and f_r

$$f = \sqrt{f_r \times f_b}$$

Hence

$$\omega = \frac{f_r - f_b}{\sqrt{f_r \times f_b}} = \frac{100 - 98}{\sqrt{100 \times 98}} = \frac{2}{99} = \mathbf{.02}$$

Example 25.2. An achromatic objective of focal length 50 cm is to be made of different kinds of glass shown below. Find the focal length of each lens, stating whether it is convergent or divergent.

	Glass A	*Glass B*
μ_{red}	*1.51*	*1.64*
μ_{blue}	*1.52*	*1.66*

Soln.

The refractive index of the mean ray in glass A,

$$\mu_A = \frac{1.51 + 1.52}{2} = 1.515$$

Hence the dispersive power of glass A,

$$\omega_A = \frac{\mu_b - \mu_r}{\mu_A - 1} = \frac{1.52 - 1.51}{1.515 - 1} = 0.0194$$

The refractive index of the mean ray in glass B,

$$\mu_B = \frac{1.64 + 1.66}{2} = 1.65$$

Hence the dispersive power of the glass B,

$$\omega_B = \frac{1.66 - 1.64}{1.65 - 1} = 0.0307$$

Now, the condition for lenses in contact to be achromatic for red and blue light is

$$\frac{\omega_A}{f_A} + \frac{\omega_B}{f_B} = 0$$

$$\text{or,} \quad \frac{f_B}{f_A} = -\frac{\omega_B}{\omega_A} = -\frac{0.0307}{0.014} = -1.582$$

$$\text{or,} \quad f_B = -1.582 f_A$$

Again from the condition of equivalent focal length of two lenses in contact,

$$\frac{1}{f} = \frac{1}{f_A} + \frac{1}{f_B}; \quad \text{or,} \quad \frac{1}{50} = \frac{1}{f_A} - \frac{1}{1.582 f_A}$$

$$\text{or,} \quad 1.582 f_A = 29.1; \quad \text{or,} \quad f_A = \mathbf{+18.4\ cm}$$

focal length is +ve, hence the lens is convergent.

$$f_B = -1.582 f_A = -1.582 \times 18.4 = \mathbf{-19.1\ cm}$$

focal length is – ve, hence the lens is divergent.

Example 25.3. A converging lens of focal length 35.0 cm and achromatic for the C and F lines is to be made with two lenses, one of hard crown and the other of dense flint glass, placed in contact. Find the focal length of these lenses.

Hard crown : $\mu_D = 1.5175,$

$\mu_F - \mu_C = 0.00856$

Dense flint : $\mu_D = 1.6264,$

$\mu_F - \mu_C = 0.01722$

μ_D *is the refractive index for mean ray.*

Soln.

Let the focal length and dispersive power of the hard crown glass lens be f_1 and w_1 respectively, the corresponding quantities for the dense flint glass lens being f_2 and w_2.

$$\omega_1 = \frac{d\mu}{\mu - 1} = \frac{0.00856}{1.5175 - 1} = 0.0165$$

$$\omega_2 = \frac{d\mu}{\mu - 1} = \frac{0.01722}{1.6264 - 1} = 0.0275$$

From the condition of achromatism for two lenses in contact

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0;$$

or, $$\frac{f_2}{f_1} = -\frac{\omega_2}{\omega_1} = -\frac{0.0275}{0.0165}$$

or, $$f_2 = -1.666 f_1$$

The equivalent focal length of the combination

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

or, $$\frac{1}{35} = \frac{1}{f_1} - \frac{1}{1.666 f_1} = \frac{0.666}{1.666 f_1}$$

or, $1.666 f_1 = 23.3$

or, $f_1 = $ **+ 13.98 cm**

Hard crown glass lens is convex; focal length + 13.98 cm

$$f_2 = -1.666 f_1$$

$= -1.666 \times 13.98$ cm $=$ **– 23.3 cm.**

Dense flint glass lens is concave, focal length – 23.3 cm

Example 25.4. *The dispersive powers for crown and flint glasses are in the ratio of 1:2. Calculate the focal lengths of the lenses made of crown and flint glass which form an achromatic combination of focal length 20 cm when placed in contact.*

Soln.

Let f_1 and f_2 be the focal lengths of the crown and flint glass lens respectively, the corresponding dispersive powers being ω_1 and ω_2.

From condition of achromatism

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0;$$

or, $$\frac{f_2}{f_1} = -\frac{\omega_2}{\omega_1} = -\frac{2}{1}$$

or, $$f_2 = -2 f_1$$

Equivalent focal length,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}; \quad \text{or,} \quad \frac{1}{20} = \frac{1}{f_1} - \frac{1}{2f_1} = \frac{1}{2f_1}$$

$\therefore \quad f_1 = +10$ cm

and $f_2 = -2f_1 = $ **– 20 cm**

Example 25.5. *The dispersive power of crown and flint glasses are 0.01 and 0.02 and their refractive indices are 3/2 and 5/2 respectively. Calculate the focal lengths and radii of curvature of the lenses required to design an achromatic doublet of focal length 100 cm with these glasses.*

Soln.

Let f_1 and f_2 be the focal lengths of the crown and flint glass respectively; the corresponding dispersive powers being ϖ_1 and ϖ_2.

From condition of achromatism,

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0; \qquad \frac{f_2}{f_1} = -\frac{\omega_2}{\omega_1} = -\frac{0.02}{0.01}$$

or, $f_2 = -2 f_1$

Equivalent focal length,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}; \quad \text{or,} \quad \frac{1}{100} = \frac{1}{f_1} - \frac{1}{2f_1} = \frac{1}{2f_1}$$

or, $2f_1 = 100$; $f_1 = $ **+ 50 cm**

$\therefore$ $f_2 = -2f_1 = $ **-100 cm.**

Achromatic doublet means that the crown glass lens is double convex and the flint glass lens is plano-concave. Further the second surface of the convex lens and the concave surface of the plano-concave lens are in contact with each other and the radii of curvature of these surfaces are same.

For the flint glass lens we have $R_2' = \infty$,

$$\mu_2 = \frac{5}{3}, \; f_2 = -100 \text{ cm.}$$

If R_1' is the radius of curvature of the first surface of the lens, then

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right)$$

or, $$-\frac{1}{100} = \left(\frac{5}{3} - 1\right)\frac{1}{R_1'} = \frac{2}{3R_1'}$$

or, $R_1' = -66.66$

Now for the crown glass lens,

$$R_2 = R_1' = -\frac{200}{3} \text{ cm}, \; \mu_1 = \frac{3}{2}$$

$$f_1 = +50 \text{ cm}$$

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $$\frac{1}{50} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} + \frac{3}{200}\right)$$

$$= \frac{1}{2R_1} + \frac{3}{400}$$

whence $R_1 = +40$ cm.

Thus the required condition for the design of an achromatic doublet is

crown glass lens : $f_1 = +50$ cm,

$R_1 = +40$ cm, $R_2 = -66.66$ cm.

flint glass lens : $f_2 = -100$ cm,

$R_1' = -66.66$ cm, $R_2' = \infty$.

Example 25.6. *A converging achromat of 60 cm focal length is to be constructed out of a thin crown glass lens, the surfaces in contact having a common radius of curvature of 40 cm. If the refractive indices and dispersive powers are 1.5 and 0.018 for crown glass, and 1.6 and 0.036 for flint glass, what will be the radius of curvature of the other surface of each lens?*

Soln.

For crown glass lens : $\omega_1 = 0.018$, $\mu_1 = 1.5$, $f_1 = ?$ $R_2 = -40$ cm.

For flint glass lens : $\omega_2 = 0.036$, $\mu_2 = 1.6$, $f_2 = ?$ $R_1' = -40$ cm.

Now

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0;$$

or, $$\frac{f_2}{f_1} = -\frac{\omega_2}{\omega_1} = -\frac{0.036}{0.018} = -2$$

or, $f_2 = -2f_1$

Again

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}; \quad \text{or,} \quad \frac{1}{6} = \frac{1}{f_1} - \frac{1}{2f_1} = \frac{1}{2f_1}$$

or, $2f_1 = 60$ cm; or, $f_1 = $ **$+30$ cm**

$$f_2 = -2f_1 = \mathbf{-60\ cm}$$

From

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{30} = (1.5 - 1)\left(\frac{1}{R_1} + \frac{1}{40}\right) = \frac{1}{2R_1} + \frac{1}{80}$$

or, $R_1 = +$ **24 cm**

and $\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right)$

$$\text{or, } -\frac{1}{60} = (1.6 - 1)\left(-\frac{1}{40} - \frac{1}{R_2'}\right)$$

$$= -\left(\frac{6}{400} + \frac{6}{10R_2'}\right)$$

or, $R_2' = +$ **360 cm**

$R_1 = +$ **24 cm**, $R_2' = +$ **360 cm.**

Example 25.7. *Two convex lenses of focal length 12 cm and 4 cm and of the same material are placed at a certain distance apart so as to satisfy the condition for minimum spherical aberration. Is the combination achromatic ?*

Soln.

Let the distance of separation be d. For minimum spherical aberration, $d = f_1 - f_2 = 12 - 4 = 8$ cm

Since the lenses are of the same material, for the combination to be achromatic,

$$d = \frac{1}{2}(f_1 + f_2) = \frac{1}{2}(12 + 4) = 8 \text{ cm}$$

Hence the combination is achromatic.

Example 25.8. *Two thin lenses of focal lengths f_1 and f_2 separated by a distance d have an equivalent focal length of 50 cm. The combination satisfies the conditions for minimum spherical aberration and is also achromatic. Find the values of f_1, f_2 and d. Assume that both the lenses are of the same material.*

Soln.

The equivalent focal length of the combination

$$f = \frac{f_1 f_2}{f_1 + f_2 - d}; \quad \text{or, } 50 = \frac{f_1 f_2}{f_1 + f_2 - d} \qquad \text{(i)}$$

$$\text{For achromatism, } d = \frac{f_1 + f_2}{2} \qquad \text{(ii)}$$

For minimum spherical aberration,

$$d = f_1 - f_2 \qquad \text{(iii)}$$

From (ii) and (iii)

$$f_1 = \frac{3d}{2} \quad \text{and} \quad f_2 = \frac{d}{2}$$

Substituting these values of f_1 and f_2 in equation (i).

$$50 = \frac{\frac{3d}{2} \times \frac{d}{2}}{\frac{3d}{2} + \frac{d}{2} - d} = \frac{3d}{4}$$

or, $d = \frac{200}{3} =$ **66.67 cm**

$\therefore \; f_1 = \frac{3d}{2} =$ **100 cm** and $f_2 = \frac{d}{2} =$ **33.33 cm.**

Example 25.9. *Find the ration of two radii of curvature of a crossed lens to exhibit minimum spherical aberration. Given $\mu = 1.6$.*

Soln.

The ratio R_1/R_2 for spherical aberration to be minimum is given by the relation

$$k = \frac{R_1}{R_2} = \frac{2\mu^2 - \mu - 4}{\mu(2\mu + 1)}$$

$$= \frac{2\times(1.6)^2 - 1.6 - 4}{1.6\ (2\times 1.6 + 1)}$$

$$= \frac{1.6\ (3.2 - 1) - 4}{1.6\times 4.2}$$

$$= \frac{1.6\times 2.2 - 4}{6.72}$$

$$= \frac{3.52 - 4}{6.72}$$

$$= -\frac{0.48}{6.72} = -\frac{1}{16}$$

Hence **k = – 1/16.**

Example 25.10. *If two convex lenses made of same glass and of focal lengths 32 cm and 20 cm are to be used to exhibit minimum spherical aberration ; find the distance between the two. If $\mu = 1.5$, find the radii of curvature of the lens surfaces. One surface of each lens is plane.*

Soln.

For minimum spherical aberration, we have

$$d = f_1 - f_2$$

$$= 32 - 20 = \mathbf{12\ cm.}$$

From the relation

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

we get, for a plano-convex lens

$$\frac{1}{32} = (1.5 - 1)\ \frac{1}{R_1}$$

($R_2 = \infty$ for a plano-convex lens)

$$= \frac{1}{2R_1}$$

or, $\mathbf{R_1 = 16\ cm.}$

Similarly for the second lens,

$$\frac{1}{20} = (1.5 - 1)\ \frac{1}{R_1'}$$

$$= \frac{1}{2R_1'}$$

or, $\mathbf{R_1' = 10\ cm.}$

Example 25.11. *Find the focal length of an equi-curved lens to reduce curvature of field to zero. The refractive indices of the two lenses are 1.5 and 1.65 respectively and the focal length of the first lens is 33 cm.*

Soln.

Petzval condition for zero curvature of field is given by

$$\frac{f_1}{f_2} = -\frac{\mu_2}{\mu_1}$$

Hence $\mu_1 = 1.5, \mu_2 = 1.65, f_1 = 33$ cm

$$\therefore\ f_2 = -\frac{\mu_1}{\mu_2}\cdot f_1 = -\frac{1.5\times 33}{1.65}$$

or, $f_2 = -1.5 \times 20 = -30$ cm.

Thus a combination of convex lens of focal length 33 cm has to be made with a concave lens of 30 cm focal length to reduce curvature of field to zero. Evidently such a combination shall be a divergent combination.

EXERCISES

[1] Explain with suitable diagrams what you mean by spherical aberration. Define the terms longitudinal and lateral spherical aberrations and circle of least confusion.

[2] Explain the causes of spherical aberration fully and the methods to minimize it.

[3] Outline the simple methods used for correction of spherical aberration in the case of an ordinary lens.

[4] What is spherical aberration? How is it minimized when two thin lenses are placed at a distance from each other?

[5] Describe astigmatism coma, curvature and distortion. How they may be reduced to a minimum?

[6] What is an aplanatic lens? Outline the property of an aplanatic lens and the condition that must be satisfied for the lens to be aplanatic.

[7] Explain what is dispersive power of a substance.

Deduce the relation, $\omega = \dfrac{d\mu}{\mu - 1}$

[8] Explain what is meant by chromatic aberration in lenses. Describe, with suitable diagrams, the lateral and longitudinal chromatic aberration. Derive an expression for the axial chromatic aberration for a thin lens.

[9] What do you understand by the term achromatism?

Derive and discuss the condition of achromatism for two thin lenses of focal lengths f_1 and f_2 (a) when they are made of different materials but placed in contact and (b) when they are made of same material but separated by a distance d.

[10] What do you understand by the term achromatism? Derive and discuss the condition of achromatism for two thin lenses in contact.

[11] Describe and achromatic doublet and explain its principle.

[12] Describe and explain chromatic aberration. Deduce a condition for achromatism of two lenses separated by a distance.

Explain what is meant by an achromatic system.

[13] Write a brief essay on aberrations of optical image.

[14] What are the two main defects of optical images as formed by ordinary lenses? How will you achromatise a system of two thin lenses (a) of different materials and (b) of the same materials. Is such a combination of lenses truly achromatic?

[15] Derive a condition for minimum spherical aberration and for achromatism of two thin co-axial lenses placed in air a certain distance apart.

[16] The focal lengths of a thin convex lens are 100 cm and 96.8 cm for red and blue rays respectively. Calculate the dispersive power of the material of the lens. **(0.0325)**

[17] Calculate the dispersive powers for crown and flint glass from the following data :

	Red	**Yellow**	**Blue**
crown	1.5145	1.5170	1.55230
flint	1.6444	1.6520	1.6637

(Crown : 0.01644; flint : 0.02961)

[18] The refractive indices of two kinds of glass for violet and red rays are as follows :

For glass A	For glass B
$\mu_v = 1.523$	$\mu_v' = 1.664$
$\mu_r = 1.517$	$\mu_r' = 1.650$

What should be the focal length of two lenses made of these glasses which will form an achromatic doublet of focal length 50 cm?

(glass A = + 22.93 cm, glass B = – 42.93 cm)

[19] An achromatic converging combination of focal length 60 cm is formed with a convex lens of crown glass and a concave lens of flint glass placed in contact. Calculate their focal lengths if the dispersive power of crown glass is 0.03 and that of flint glass is 0.05. **(crown glass : + 24 cm; flint glass = – 40 cm).**

[20] It is desired to make a converging achromatic lens of mean focal length 20 cm by using two lenses of materials A and B. If the dispersive powers of A and B are in the ratio of 2:3, find the focal length of each lens. **(A = + 6.67 cm, B = – 10 cm).**

[21] The object glass of a telescope is made of a convex lens of crown glass (dispersive power 0.012) and a concave lens of flint glass (dispersive power 0.020). If the focal length of the object glass is 30 cm, calculate the focal lengths of the component lenses. **(crown glass = + 12 cm, flint glass = – 20 cm).**

[22] A thin crown glass lens is in contact with a thin flint glass lens, the radius of curvature of the common surface being 25 cm. The combination forms an achromatic combination of 40 cm focal length, find the radii of curvature of the other face of the two lenses.

μ for crown glass = 1.50; μ for flint glass = 1.60

dispersive power of crown glass = 0.021,

dispersive power of flint glass = 0.045,

(crown glass : f_1 = + 21.33, R_1 = +18.5 cm, R_2 = − 25 cm. flint glass : f_2 = − 45.7 cm, R_1' = − 25 cm. R_2' = 286 cm.)

[23] A convex lens of crown glass and a concave lens of flint glass are combined to form an achromatic doublet of focal length 80 cm. The dispersive powers of crown and flint glass are 0.015 and 0.030 respectively, their respective refractive indices being 1.52 and 1.65. Calculate the focal lengths and the radii of curvature of both the surfaces of the lens. **(crown glass : f_1 = + 40 cm. R_1 = + 34.67 cm. R_2 = − 52 cm, flint glass : f_2 = − 80 cm, R_1' = − 52 cm, R_2' = ∞).**

[24] A converging achromat of 40 cm focal length is to be constructed out of a thin crown glass lens and a thin flint glass lens, the surfaces in contact having a common radius of curvature of 25 cm. Calculate the radius of curvature of the other surface of each lens, given that the values of the dispersive powers and refractive indices are 0.017 and 1.5 for crown glass, and 0.034 and 1.7 for flint glass. **(crown glass : f_1 = + 20 cm, R_1 = + 16.67 cm, R_2 = − 25 cm, flint glass : f_2 = − 40 cm, R_1' = − 25 cm, R_2' = − 233.33 cm).**

[25] At what distance apart should two lenses of focal lengths 24 cm and 8 cm be placed so as to form an achromatic combination? Assume the lenses to be of the same material. Will the combination also satisfy the condition for minimum spherical aberration? **(16 cm; yes, the combination satisfies the condition of minimum spherical aberration).**

[26] Find the value of shape factor to reduce spherical aberration to minimum for μ = 1.686, μ = 1.66 and μ = 1.5. **[0, − 1/6.5, − 1/6]**

[27] The focal length of a lens of refractive index μ_1 = 1.70 is 30 cm. Find the focal length of another lens of refractive index μ_2 = 1.5 to form a doublet to reduce curvature of field. **[f_2 = − 34 cm].**

[28] A convergent doublet of separated lenses, corrected for spherical aberration, has an equivalent focal length of 10 cm. The lenses of the doublet are separated by 2 cm. What are the focal lengths of the component lenses ? **[18 cm, 20 cm].**

CHAPTER XXVI

OPTICAL INSTRUMENTS

Photographic camera – Visual angle – Simple microscope – Compound microscope – Entrance and exit pupil of a compound microscope – Microscope objective – Eye-pieces or oculars – Ramsden eye-piece – Huygens' eye-piece – Comparison of Ramsden eye-piece and Huygens, eye-piece – Binocular microscope – Telescope – Astronomical telescope – Terrestrial telescope – Reflecting telescope – The prism binocular – The spectrometer – Solved problems – Exercises.

26.1 Photographic camera

The term camera eomes from the latin phrase *camera obscura* or dark chamber, for all picture taking instruments have a dark chamber to protect the sensitive film from light.

A photographic camera essentially consists of the following parts (Fig. 26.1)

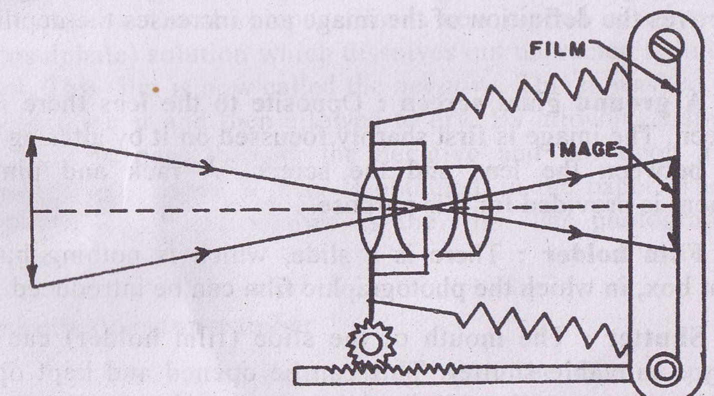

Fig. 26.1

(i) **A light-tight box :** The inside of this box is painted black to absorb the light which falls on it. The sides of the box are made