1. SETS
SYNOPSIS

- A *set* is well defined collection of objects.
- A set which does not contain any element is called *empty set*.
- A set which contain a definite number of elements is called a *finite* set, otherwise the set is called *infinite set*.
- Two sets A and B are said to be *equal* if they have exactly the same elements.
- A set A is said to be *subset* of a set B if every element of A is also an element of B .Intervals are subsets of R.
- A *power set* of a set A is the set of all subsets of A. It is denoted as P(A).
- The *union* of two sets A and B is the set of all those elements which are common to A and B. It is denoted as AUB.   ∩
- The *intersection* of two sets A and B is the set of all elements which are common to A and B. It is denoted as A∩B.
- The *difference* of two sets A and B in this order is the set of all elements which belongs to A but not to B.
- The *complement* of a subset A of universal set U is the set of all elements of U which are not in A. It is denoted as A'.
- For any two sets A and B (AUB)' = A'∩B', and (A∩B)' = A'UB'.
- If A and B are any two sets such that A∩B = ø then n(AUB)= n(A)+ n(B) and if A∩B ≠ ø then n(AUB)= n(A)+ n(B) – n(A∩B).

1   The number of subsets of a set containing n elements is

    a)   n   b) 2^n-1 c) n^2 d) 2^n

2   If $A = \{1,2,3,4,5\}$ $B = \{2,4,6,8\}$ Then A-B is

    a) $\{1,3,5\}$ b) $\{2,4,\}$ c) $\{6,8,\}$ d) $\{1,3,\}$

3   The null set is represented by

    a) $\{\phi\}$ b) ϕ c) $\{x : x = x\}$ d) $\{x : x \neq x\}$

4   Let $A = \{x : x \in R, x \geq 4\}$ $B = \{x : x \in R, x \angle 5\}$ Then $A \cap B$

    a) $[3,5)$ b) $(4,5]$ c) $[4,5)$ d) $(4,5)$

5   If A and B are two sets such that n(A) =70, n(B) =60, n(A $\cup$ B) =110,Then n(A $\cap$ B)
    a) 240   b) 50   c) 40   d) 20

6   If   A $\cap$ B = B then
    a) A $\subset$ B   b) B $\subset$ A   c) A = ϕ d) B = ϕ

7   Let U be the universal set containing 700 elements. If A,B are subsets of U such that   n(A) = 200 ,
    n(B) =300   n(A $\cap$ B) =100 Then n(A' $\cap$ B') is
    a) 400   b) 600   c) 300   d) 500

8   In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus.
    Then, persons travelling by car or bus is
    b)   40%   c)   60%   d) 70%
    a)   80%

9   In a class of 175 students the following data shows the number of students opting one or more subjects.
    Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30: Mathematics and Chemistry
    28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. How many students have offered
    Mathematics alone?
    a) 35   b) 48   c)   60   d) 22

10.   The set $(A \cap B')'$ is equal to
    a)$A' \cup B'$ b)$A' \cup B$ c)$A' \cap B'$ d)$A' \cap B$

SECTION B (2 marks)

    Write the power set of the following :
    11.   A ={4, φ, 3}
    12.   A ={2,3}
    13.   A={-1,0,1}

    14.   If A = {1,2,3} find P(A) and n[P(A)]

    15. A= {2x : x $\in$ N, and 1≤ x < 4} , B ={ (x +2) : x $\in$ N, and 2≤ x < 5 },
        C = {x : x $\in$ N, and 4< x < 8}find
      (i) A∩B   (ii) AUB
      (iii) (AUB)∩C   (iv) AUBUC

16. A= {2,4,6,8,10,12} ,B = {3,4,5,6,7,8,10}, find
    (i) A- B   (ii) B – A   (iii) (A – B)U(B – A)

17. If n(A)= 27 , n(B) =35 and n(AUB) = 50 find n(A∩B)

SECTION C(4marks)

18. If A ={a, b, c, d, e} ,B ={a ,c, e ,g} ,C ={ b, e, f ,g} verify
    (a) A∩(B – C) = (A∩B) – (A∩C)
    (b) A – (B∩C) = (A – B) U (A – C)

19. If universal set U = {1,2,3,4,5,6,7,8,9} ,A ={2,4,6,8} ,B = {2,3,5,7} verify
    (a)(AUB)' = A'∩B'   (b) (A∩B)' = A' U B'

20. A ={ x: x Є N, and x ≤ 7 } ,   B={ x : x Є N, x is prime and x < 8} ,
    C= {x : x Є N, x is odd and x < 10} verify that
        (a) AU(B∩C) = (AUB) ∩(AUC)
        (b) A∩(BUC) = (A∩B)U(A∩C)

21. A= {1/x : x Є N, and x < 8},   B= { 1/2x : x Є N, and x ≤ 4} find
        (i) AUB   (ii) A∩B   (iii)A – B   (iv) B – A

22. In a group of 50 persons 30 like tea, 25 like coffee and 16 like both. How many
    like (i) Either tea or coffee? (ii) Neither tea or coffee?

SECTION D(6 marks)

23. A class has 175 students. The following gives the number of students studying   one or more of
the subjects in this class: Mathematics: 100, Physics:70,
    Chemistry:46, Math and physics:30, Math and chemistry:28, Physics and
    Chemistry:23, Math ,Physics and Chemistry:18. Find the no. of students who
    (a) take math alone , Physics alone , Chemistry alone
    (b) have not taken any of these subjects.

24. A school awarded 42 medals in hockey, 18 in basketball and 23 in cricket. If these medals were
    bagged by a total of 65 students and only 4 students got medals in all the 3 sports, how many
    students received medals in exactly 2 of the 3 sports?

25.In a survey of 100 students, the number of students studying in various languages
    is found as English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and
    Hindi;8 English 26;   Sanskrit 48; and no language 24. Find
    (i) how many students are studying Hindi;
    (ii) how many students are studying English and Hindi both.

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ANSWER KEY

	1)d	2)a	3)b	4)c	5)d
	6)b	7)c	8)c	9)c	10)b
11.	[{ {4}, { φ}, { 3 },{4, φ },{4,3}, {3,φ },{4,3, φ}, φ}]				
12.	[{{2},{3},{2,3}, φ}]				
13.	[{{-1},{0},{1},{-1,0},{0,1},{-1,1},{-1,0,1}, φ}]				
14.	{ {1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}, φ }; 8				
15.	[{4,6},{2,4,5,6}] , [{5,6},{2,4,5,6,7 }]				
16.	[(i){2,12},(ii){3,5,7},(iii){2,3,5,7,12}]				
17.	[12]				
22.	[{1,1/2,1/3,1/4,1/5,1/6,1/7,1/8},{1/2,1/4,1/6},{1,1/3,1/5,1/7} ,{1/8}]				
23.	[39 , 11]				
24.	[(a) 60,35,13 (b) 32]				
25.	[10]				

2. RELATIONS AND FUNCTIONS
SYNOPSIS

The main features of the chapter are :

- **Ordered Pair**: A pair of elements grouped together in a particular order.
- **Cartesian Product**: $A \times B$ of two sets A and B is given by

  $A \times B = \{(a,b) : a \in A, b \in B\}$

  In particular
  $R \times R = \{(x, y) : x, y \in R\}$ and
  $R \times R \times R = \{(x, y, z) x, y, z \in R\}$

- If $(a, b) = (x, y)$ then $a = x$ and $b = y$
- If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = p\,q$
- $A \times \phi = \phi$
- In general $A \times B \neq B \times A$
- **Relation**: A relation R from a set A to a set B is a subset of the cartesian product $A \times B$ obtained by describing a relationship between the first element x and the second element y of the ordered pairs in $A \times B$.
- The **image** of an element x under a relation R is given by y , where (x,y) belongs to R.
- The **range** of the relation R is the set of all second elements of the ordered pairs in a relation R.
- **Function**: A function f from a set A to a set B is a specific type of relation for which every element x of set A has one and only one image y in set B.
- We write $f : A \to B, where f(x) = y$
- A is the **domain** and b is the **codomain** of f.
- The range of the function is the set of all images.
- A real function has the set of real numbers or one of its subsets both as its domain and as its range.
- **Algebra of Functions**:For functions $f : X \to R and g : X \to R$, we have

  $$(f+g)(x) = f(x) + g(x)$$
  $$(f - g)(x) = f(x) - g(x)$$
  $$(f . g)(x) = f(x) . g(x)$$
  $$(k\,f)(x) = k\,f(x) , \text{ where k is a real number.}$$
  $$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, x \in X, g(x) \neq 0$$

SECTION A (1 mark)

1. If n(A) = 3, n(B) = 2 then the number of relations from A to B =

   a) 6   b) 36   c) 63   d) 64

2. If A = {x : x² – 5x + 6 = 0}, B = {2, 4}, C = {4, 5} then A x (B ∩ C) is

   a) {(2,4),(3,4)}   b) {(4,2),(4,3)}   c) {(2,4), (3,4),(4,4)}   d)
   {(2,2),(3,3),(4,4),(5,5)}

3. If A = {1, 2, 3, 4, 5 } and f:A→R is such that $f(x)$ = 3x-2 then $f(A)$ =

   a){0,4,10,18,23}   b){0,10,24}   c) {10,24,17}   d) {1,4,7,10,13}

4. If $f(x)$ = 2x-1, g(x) = x² , then 3f–2g =

   a) $6x - 3 - 2x^2$ b)$6x + 3 - 2x^2$ c) $6x - 3 + 2x^2$ d) $6x + 3 + 2x^2$

5. If A x B = {(a,1),(b,3),(a,3),(b,1),(a,2),(b,2)}, then A=_____ & B = _____

6. The domain of F(x) = $\frac{1}{\sqrt{x-|x|}}$ is

   a)R   b)R⁺   c)R⁻   d)none of these

7. If f(x) =$x^3 - \frac{1}{x^3}$ then $f(x) + f\left(\frac{1}{x}\right)$ = _____
   a) 2x³   b)2$\frac{1}{x^3}$ c)0   d)1

8. Let A & B be any 2 sets such that n(B) = p & n(A) = q then the total number of functions
   $f: A \to B$ is _____

SECTION B (2 marks)

9.If A = {1,2,3} , B ={3,4} , C = {4,5,6}, find
    (i) A×(BUC)   (ii) A×(B∩C)   (iii) (A×B)∩(B×C)

10. Find a and b when
     (i) (a +3 ,b – 2) = (5,1)
     (ii) (a + b, 2b – 3) = (4, -5)

11.If A = { 2,3 } , B = { 4, 5} ,find A×B. How many subsets will A×B have ?

Find the domain and range of the following relations:

12. R = {(x, 1/x): x is an integer, 0 < x < 6}

13. R = {(x, y): x, y Є Z and x² + y² = 25}

14. R = {(x, y): x, y Є Z and xy = 4}

15. Find the domain of f(x) = [x] + x

SECTION C (4marks)

16. Draw the graph of the function given below and give its equation:

(i)Identity function   (ii) Polynomial function   (iii) Modulus function

17. Draw the graph of the function given below and give its equation:

   (i) Signum function
   (ii) Greatest integer function

18. Find the domain and range of the following real functions:

(i) $f(x) = x^2$ (ii) $f(x) = \dfrac{1}{x-3}$ (iii) $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

(iv) If $f(x) = \begin{cases} x+4, x < 3 \\ 2x+9, 3 \le x \le 9 \\ 5, x > 9 \end{cases}$

19. Find the domain and range of the following functions :

(i) $f(x) = \dfrac{3}{2-x^2}$ (ii) $f(x) = |\, x - 3\,|$

20.Find the domain and range of

   (i) $f(x) = 1 - |\, x - 2\,|$ (ii) $f(x) = \sqrt{16 - x^2}$
   (iii) $f(x) = 2\text{-}3\sin x$

21. Find the domain and range of
       $f(x) = 4 \sin x - 3 \cos x$

22. Find the domain and range of $\dfrac{x^2}{1+x^2}$

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ANSWER KEY

	1.(d) 2. (a) 3(d) 4(a) 5. A= {a,b} & B= {1,2,3} 6.(d) 7. (0) 8. p^q

9.	(i){(1,3),(1,4),(1,5),(1,6),(2,3),(2,4), (2,5),(2,6)(3,3),(3,4),(3,5),(3,6)}	(ii) {(1,4),(2,4),(3,4)}	(iii){(3,4)}

10.	(i) a = 2,b =3	(ii) a = 5,b = -1

11.	{(2,4),(2,5),(3,4),(3,5)} , 16
12.	Dom ={1,2,3,4,5}, range ={1,1/2,1/3,1/4,1/5}
13.	dom = range ={-4,-3,0,3,4,
14.	dom ={-4,-2,-1,1,2,4}= range}
15.	R

16.	(i) y = x	(ii) y = x^2	(iii) y = |x|

17.	i) $f(x) =$ $\begin{cases} 1 \text{ if x>0} \\ 0 \text{ if x=0} \\ -1 \text{ if x<0} \end{cases}$	(ii) $f(x) = [x]$, $x \in R$

18.	(i) dom = R ,
	    range = R
		(ii) dom = R – {3} ,
		    range = R – {0}
			iii) dom= R
			    range = [1/3 , 3]

	(iv)D=$(-\infty, 3)$ ∪ [3,9] ∪ $(9, \infty)$ = $(-\infty, \infty)$ R= $(-\infty, 7)$ ∪ [15,27]		

19.	(i) D= R – { √2,-√2}, R= (-∞,0]U [3/2,∞)	(ii) D=R, Range=R$^+$U {0}

20.	(i) D = R , R = (-∞,1]	(ii) D = [-4,4] R = [0,4]
	(iii) D=R , R=[-1,5]	

21.	D =R , range = [-5,5]
22.	D = R , Range = [0, 1)