COMPLEX NUMBERS AND QUADRATIC EQUATIONS
SYNOPSIS

- A number of the form ***a+ib*** , where a and b are real numbers, is called a complex number, a is called the ***real part*** and b is called the ***imaginary part.***
- Let $z_1 = a+ ib$ $z_2 = c+id$ then $z_1 + z_2 = (a +c) + i (b +d)$ and $z_1 . z_2 = ac-bd + i(ad+bc).$
- $i^2 = -1,\ \ i^3 = -i\ \ \ i^4 = 1.$
- The conjugate of a complex number $z= a+ib$ denoted by $\bar{z}$ is $\bar{z} = a-ib.$
- The polar form of a complex number z= x+iy is $r (\cos\theta + \sin\theta)$ where $r=\sqrt{x^2 + y^2}$ r is called the modulus and θ is called the ***argument***. The value of θ such that $-\pi < \theta \leq \pi$ is called the principle argument of z.

5.COMPLEX AND QUADRATIC EQUATIONS

MCQ

1) Find the multiplicative inverse of 2-3i

a) $\frac{2}{13}+\frac{3i}{13}$ b) $\frac{2}{13}-\frac{3i}{13}$ c) $\frac{-2}{13}+\frac{3i}{13}$ d) $\frac{-2}{13}+-\frac{3i}{13}$

2) The value of $i^9 + i^{19}$

a) i b) 1 c) 0 d) – i

3) Find the modulus of the complex number 3+4i

a) 5 b) 3 c) 4 d) 7

4) Express in a+ ib form : $\left(-1+\sqrt{3}i\right)^{-1}$

a) $\frac{1+i\sqrt{3}}{4}$ b) $\frac{-1+i\sqrt{3}}{4}$ c) $\frac{1-i\sqrt{3}}{4}$ d) $\frac{1+i\sqrt{3}}{-4}$

5) Find the argument of the given complex number – 1 - √3i

a) 2π/3 b) -2π/3 c) π/3 d) 0

6) If a + ib = $\frac{\sqrt{1+i}}{\sqrt{1-i}}$ then

a) $a^2 - b^2 =1$ b) $a^2 + b^2 =1$ c) $-a^2 + b^2 =1$ d) $-a^2 - b^2 =1$

7) Simplify $\frac{(1+i)(3+i)}{(3-i)}$

a) 14i/5 b) -14i/5 c) $\frac{1+7i}{5}$ d) -5/14i

8) Find the conjugate of the following: $(6+5i)^2$

a) 11 +60i b) 11 -60i c) -19 +10i d) 19 -10i

9) Find the square root of -15-8i

a) $\mp$ (1-4i) b) ± (1+4i) c) (1-4i) d) (1-4i)

10) If $\frac{1-ix}{1+ix} = a + ib$, then $a^2 + b^2$ = ------

a) 1 b) -1 c) 0 d) none of these

Fill in the blanks

1) i^{-39} = ----------

2) $1 + i^2 + i^4 + i^6 + \ldots + i^{20}$ = ----

3) If $(3x-7) + 2iy = -5y + (5 + x)i$ then value of x = --- and y = -----

4) The product of $3 - 2i$ and its conjugate = ---

5) If the complex numbers $-3+ i x^2 y$ and $x^2+ y + 4 i$ are conjugate of each other, then value(s) of x = --- , y = --

VSA (1 mark each)

1. Evaluate (i) i^{998} (ii) $i^{37} + \frac{1}{i^{67}}$ (iii) i^{-71} (iv) i^{-1}

2. Find the value of

(i) $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

(ii) $i^{107} + i^{112} + i^{117} + i^{122}$

(iii) $(1+i)^4 \times \left(1+\frac{1}{i}\right)^4$

3. Simplify

(i) $(-2i)\left(\frac{1}{6}i\right)$

(ii) $(-i)(3i)\left(\frac{-1}{6}i\right)^3$

(iii) $\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}$

(iv) $6i^{54} + 5i^{37} - 2i^{11} + 6i^{68}$

(v) $\frac{1}{i}-\frac{1}{i^2}+\frac{1}{i^3}-\frac{1}{i^4}$

4 (i) If $2y + (3x - y)i = 5 - 2i$. find x and y (ii) $2 + z = 3 - i$ where $z = x + iy$

5. Find the modulus of the following complex nos:
(i) $2 + 3i$ (ii) $-5 - 4i$ (iii) $-i$ (iv) $2+\sqrt{-3}$ (v) $(3i-1)^2$

6. Find the conjugate of the following:
(i) $-3 + \sqrt{-1}$ (ii) i^3 (iii) $(6+ 5i)^2$

7. Express in the form a + ib :
(i) $\frac{i}{1+i}$ (ii) $\left(-1+\sqrt{3}i\right)^{-1}$

SECTION B (2 marks each)

8. Solve:
(i) $9x^2 + 10x + 3 = 0$
(ii) $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$
(iii) $3x^2 + 8ix + 3 = 0$

9 Find the modulus of the following $\frac{(2-3i^2)}{4+3i}$, $(\sqrt{7}-3i)^3$

10 If $z = 3 - \sqrt{7}\,i$, then find $|z^{-1}|$

11 Express in complex number form $\frac{i+i^2+i^4}{1+i^2+i^4}$

12 Express the square of $\frac{i}{1+i}$ in the form a+ib

SECTION C (4marks each)

13. Represent the following in polar form:

(i) $-1 - \sqrt{3}i$ (ii) $\frac{-16}{1+i\sqrt{3}}$ (iii) $-2i$ iv) $\frac{2+i6\sqrt{3}}{5+i\sqrt{3}}$ v) $-4+4\sqrt{3}i$

14. For complex values of z solve: $|z| + z = 2 + i$

15. If $a + ib = \frac{\sqrt{1+i}}{\sqrt{1-i}}$ then prove that $a^2 + b^2 = 1$

16. Reduce to standard form: $\frac{1}{1-2i} + \frac{3}{1+i}$

17 If $\frac{|z-5i|}{|z+5i|} = 1$, then show that z is a real number.

18. Show that $\frac{(1+i)(3+i)}{(3-i)} = \frac{14i}{5}$

19. Show that $\left[\frac{\sqrt{7}+i\sqrt{3}}{\sqrt{7}-i\sqrt{3}} + \frac{\sqrt{7}-i\sqrt{3}}{\sqrt{7}+i\sqrt{3}}\right]$ is real.

20. If $z = x + iy$ and $w = \frac{1-iz}{z-i}$ and $|w| = 1$ show that z is purely real.

21. (i) Find x and y if $\frac{x-1}{3+i} + \frac{y-1}{3-i} = i$

(ii) $(1+i)y^2 + (6+i) = (2+i)x$

(iii) $(x + iy)(2 - 3i) = 4 + i$

22 Reduce to standard form:

(i) $\frac{3+2i}{2-3i} + \frac{3-2i}{2+i}$

(ii) $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

(iii) $\frac{3-4i}{(4-2i)(1+i)}$

SECTION D(6 marks each)

23. If $p + iq = \frac{(a-i)^2}{2a-i}$ show that $p^2 + q^2 = \frac{(a^2+1)^2}{4a^2+1}$

24. If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real, find real values of θ.

25. If $(x+iy)^3 = u+iv$, prove that $\frac{v^2}{y^2}-\frac{u^2}{x^2}=8\left(x^4-y^4\right)$

26. Find the square root of

i) -15-18i ii) 5-12i iii)-15+8i iv) -3-4i v) $-2-2\sqrt{3}\,i$

27. Solve the following quadratic equations

i) $ix^2-x+2i=0$ ii) $x^2-(3\sqrt{2}-2i)x-\sqrt{2}i=0$ iii) $x^2-\left(\sqrt{2}+i\right)x+\sqrt{2}i=0$

iv) $2x^2-(3+7i)x+9i-3=0$ v) $x^2-\left(3\sqrt{2}-2i\right)x-6\sqrt{2}i=0$

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ANSWER KEY

MCQ

1)a) $\frac{2}{13}+\frac{3i}{13}$ 2) c) 0 3) a) 5 4) d) $\frac{1+i\sqrt{3}}{-4}$

5)b) $-2\pi/3$ 6) b) $a^2 + b^2 = 1$ 7) c) $\frac{1+7i}{5}$ 8) b) 11 -60i

9) a) $\mp$ (1-4i) 10) a) 1

Fill in the blanks

1) i 2) 1 3) x = -1 , y = 2 4) 13 5) x = $\pm$1, y = -4

1.	(i) -1	(ii) 2i	(iii) i	(iv) -i	
2.	(i) 0	(ii) 0	(iii)16		
3.	(i)1/3	(ii) i/72	(iii) 0	(iv) 7i	(v) 0
4.	(i) $x=\frac{1}{6}, y=\frac{5}{2}$	(ii) x =1,y = -1			
5.	(i) $\sqrt{13}$	(ii) $\sqrt{41}$	(iii) 1	(iv) $\sqrt{7}$	(v) 10
6.	(i) -3 – i	(ii) i	(iii) 11 – 60i		
7.	(i) $\frac{1}{2}+\frac{1}{2}i$	(ii) $\frac{-1}{4}-\frac{\sqrt{3}}{4}i$			
8.	(i) $\frac{-5\pm\sqrt{2}i}{9}$	(ii) $\frac{\sqrt{2}\pm i\sqrt{34}}{2\sqrt{3}}$	(iii) $\frac{1}{3i}, -3i$		
9	13/5 , 64	10) 1/4	11) 0+i	12) 0 + (1/2) i	
13	(i) 2(cos 240 +isin240)				
	(ii) 8(cos120 +isin120)]				
	(iii) 2(cos270 + isin270)]				
	(iv) 2(cos60 + isin60)]				
	(v) 8(cos120 +isin120)]				
14	$z=\frac{3}{4}+i$				
16	$\frac{17}{10}-\frac{11i}{10}$				

21	(i)x = -4 , y = 6	(ii)x = 5, y = 2 or x = 5,y = -2		(iii) $x=\frac{5}{13}, y=\frac{14}{13}$	
22	(i) $\frac{4}{5}-\frac{2i}{5}$	(ii) $\frac{63}{25}-\frac{16i}{25}$		(iii) $\frac{1}{4}-\frac{3i}{4}$	
26	i) $\pm(1+4i)$	ii) $\pm(3-2i)$	iii) $\pm(1-4i)$	iv) $\pm(1-2i)$	v) $\pm(\sqrt{3}-i)$
27	i) -2i , i	ii) $\frac{(3\sqrt{2}-2i)}{2}\pm\frac{(4-\sqrt{2}i)}{2}$	iii) $\sqrt{2}, i$	iv) $\frac{3+i}{2}, 3i$	v) $3\sqrt{2}$, -2i