9. SEQUENCES AND SERIES
SYNOPSIS

- A succession of numbers arranged in a definite order according to a given rule is called a sequence.
- Arithmetic progression: It is a sequence in which each term except the first one differs from its preceding term by a constant(common difference)
- **n-th** term of an A.P**:** a_n **= a+(n-1)d.**
- Sum of n terms of an A.P:i) $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ ii) $S_n = \frac{n}{2}\left[a+l\right]$
- **Arithmetic Means** between two given numbers: If $x_1, x_2, x_3, \ldots\ldots x_n$ are n arithmetic means between a and b , then a, $x_1, x_2, x_3, \ldots\ldots x_n$,b form anA.P whose common difference is obtained by **d =** $\frac{(b-a)}{(n+1)}$
- **Geometric progression:** A sequence $a_1, a_2, a_3, a_4 \ldots\ldots\ldots\ldots a_n$ is called

  geometric progression iff the ratio of any term to the preceding term is constant. This constant ratio is called the common ratio (r).
- General term or nth term of a G.P: $a_n = ar^{n-1}$
- Sum to n terms of a G.P: S S_n $\frac{a(r^n-1)}{(r-1)}$ $r \neq 0$, The formula can also be written as

  S_n $\frac{a(1-r^n)}{(1-r)}$ $r \neq 0$

Special series:

- **i)** Sum of first n natural numbers $S_n = \frac{n(n+1)}{2}$
- **ii)** Sum of Squares of n natural numbers $S_n = \frac{n(n+1)(2n+1)}{6}$
- iii) Sum of cubes of n natural numbers $S_n = \frac{[n(n+1)]^2}{4}$

SECTION A (1 mark)

MCQ

1. If log 2, log $(2^x - 1)$, log $(2^x + 3)$ are in A.P. then x =
   a) $\frac{5}{2}$ b) $\log_2^5$ c) $\log_3^2$ d) $\frac{3}{2}$
2. The sum of the first n elements of an A.P. is $5n^2 + 2n$, then its second element is
   a) 7 b) 17 c) 24 d) 42
3. If the p^{th} element of an A.P. is q and the sum of q elements is p, then the sum of $p + q$ elements will be
   a) 0 b) $p + q$ c) $p - q$ d) $-(p + q)$
4. The number of elements of the sequence 20, $19\frac{1}{3}, 18\frac{2}{3}$ …. Of which the sum is 300 is
   a) 25 or 36 b) 36 and 35 c) Only 25 d) Only 36
5. The n^{th} element of a series $\frac{3+n}{4}$, then the sum of 105 elements is
   a) 1470 b) 1360 c) 1530 d) 1435
6. The sum to n elements of the sequence, log a, log ar, log ar^2 ….. is
   a) $\frac{n}{2}\log a^2 r^{n-1}$ b) $n\log a^2 r^{n-1}$ c) $\frac{3n}{2}\log a^2 r^{n-1}$ d) None of these
7. Sum of two digit numbers which when divided by 4 yields unity as remainder is
   a) 1200 b) 1210 c) 1250 d) 1350
8. The sum of first 24 elements of A.P., a_1, a_2, a_3….., if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, is
   a) 865 b) 930 c) 950 d) 900
9. The interior angles of a polygon are in A.P. If the smallest angle is 120^0 and the common difference is 5, then the number of sides of the polygon is
   a) 16 b) 8 c) 9 d) 10
10. The sum of n elements of an A.P is $3n^2 + 5n$ then 164, is (c)
    a) 16 b) 8 c) 9 d) 10
11. If S_1 is the sum of an arithmetic series of 'n' odd number of elements and S_2, the sum of the elements of the series in odd places, then $S_1:S_2$ is
    a) n : n + 1 b) n + 1: n c) n + 1 : 2n d) 2n: n + 1
12. The sum of an A.P is 525. If its first element is 3 and last element is 39, then the common difference is

a) $\frac{3}{2}$ b) 1 c) $\frac{1}{2}$ d) $\frac{5}{4}$

13. a, b, c are in A.P., p is the A.M between a and b, q is the A.M between b and c, then
    a) a is the A.M between p and q b) b is the A.M between p and q
    c) c is the A.M between p and q d) No such relations
14. If the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers, then k =
    a) $\frac{1}{n}$ b) $\frac{n-1}{n}$ c) $\frac{n+1}{2n}$ d) $\frac{n+1}{n}$
15. If x, y, z are in A.P then, (x + 2y – z) (2y + z – x) (z + x + y) =
    a) 4xyz b) 2xyz c) xyz d) 3xyz

16. Which element of the G.P., 5, 10, 20, 40, ….. is 5120?
    a) 9 b) 10 c) 11 d) 12
17. If n^{th} element of a G.P is 128 and the sum of its n term is 225. If its common ratio is 2, then its first term is
    a) 1 b) 2 c) 3 d) 4
18. If the product of three elements of a G.P, is 216, and the sum of their product, taken in pair is 156, then the greatest element is
    a) 54 b) 18 c) 6 d) 2
19. The product of 10 geometric means between 1 and 9 is
    a) 7^9 b) 7^8 c) 7^{10} d) 7^{11}
20. The m^{th} term of an A.P is n and its n^{th} terms is m. Its p^{th} term is
    a) m – n + p b) m + n – p c) m + n + p d) m – n – p

VSA

1,Find the arithmetic mean between (i) 9 and 19 (ii)12 and –8 (iii) –13 and –7

2,Which term of the GP 5, 10, 20, 40, …….is 5120?

3.Which terms of the GP √3 , 3 , 3√3, ……is 729

4,For what values of x are the numbers –2/7 , x and –7/2 in GP

SECTION B (2 marks)

5.If (x+9) ,(x – 6) and 4 are in GP ,what is x?

6.Find the 6^{th} term from the end in the GP: 8 , 4 , 2 , ……..1/1024?

7. How many terms of the series 1 + 4 +16 + 64 + …….will make the sum 5461?

8.Find sum to 8 terms of the GP 3, 6 , 12, 24,……

9.Find the GM between the numbers (i) 5 and 125 (ii) 1 and 9/16

10.Form a quadratic equation in x so that AM of its roots is A and GM is G .

11.Find the GM between the numbers (i) 0.15 and 0.0015 (ii) 27 and 243

12. Find two numbers whose AM = 34 and GM = 16.

SECTION C (4 marks each)

13. If a , b , c are in GP and $a^{1/x} = b^{1/y} = c^{1/z}$ then show that x , y, z are in AP.

14. Three numbers whose sum is 21 are in A.P. if 2,2,14 are added to them respectively, the resulting numbers are in G.P. Find the numbers.

15. The AM of 2 nos. exceeds their GM by 2 and the ratio of the numbers is 4.Find the nos.

16. Find S_n of $x(x + y) + x^2(x^2 + y^2) + x^3(x^3 + y^3) + \ldots\ldots$

17. The AM of 2 positive numbers, a and b is twice their GM .Prove that $a : b = 2+\sqrt{3} : 2 - \sqrt{3}$

18. How many terms of the sequence $3, \frac{3}{2}, \frac{3}{4}, \ldots\ldots\ldots$ are needed to give the sum $\frac{3069}{512}$?

19. (i) Determine the 12^{th} term of the GP whose 8^{th} term is 192 and common ratio is 2.

(ii) Find the number of terms of a GP whose first term is ¾ , common ratio is 2 and last term is 384.

(iii)Find the GP whose 4^{th} term is 54 and 7^{th} term is 1458.

(iv) The 4^{th} term of a GP is the square of its second term and first term is -3. Find the 7^{th} term

20. If $a_n = n^2 + 2^n$ find S_n.

21. Find the sum to n terms of the following series:

(i) $3 . 8 + 6 . 11 + 9 . 14 + \ldots\ldots\ldots\ldots\ldots\ldots$to n terms .

(ii) $1^2 + 3^2 + 5^2 + \ldots\ldots\ldots\ldots\ldots\ldots$

22. Find 3 nos. in GP whose sum is 52 and sum of whose products in pairs is 624.

23. Find 3 nos in GP whose sum is 13 and sum of whose squares is 91 .

24. If p, q, r are in AP show that the pth , qth , rth terms of any GP are in GP.

25. If a, b, c, d are four distinct positive quantities in A.P. , then show that bc > ad

26. If one geometric mean G and two arithmetic means p and q are inserted between two numbers,

Show that $G^2 = (2p - q)(2q - p)$

27. The ratio of the sums of n terms of two AP's is (7n+1) : (4n +27).Find the ratio of their 11^{th} terms.

28. Sum the following to n terms:

i)9 + 99 + 999 + ……..to n terms

ii)0.3 + 0.33 + 0.333 + ……to n terms

29. Find three numbers in G.P whose sum is 52 and the sum of their products in pairs is 624.

30. Find the sum to n terms of the series: $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \cdots$

31. A geometric sequence has all positive terms. The sum of the first two terms is 15 and the sum to infinity is 27. Find the value of (a) the common ratio; (b) the first term.

32. The first four terms of an arithmetic sequence are 2, a – b, 2a +b + 7, and a – 3b, where a and b are constants. Find a and b.

33. The second term of an arithmetic sequence is 7. The sum of the first four terms of the arithmetic sequence is 12. Find the first term, a, and the common difference, d, of the sequence.

34. Cara works as a newspaper delivery agent and initially earns R15 000 in her first year. Thereafter her salary increases by R1 500 per year. Her expenses are R13 000 during the first year, and then they increase by R900 in each subsequent year. Use a formula to determine how long it would take her to save R21 000.

35. Find the sum to n terms of the following series:

(i) $1 \cdot 2^2 + 2 \cdot 3^2 + 3 \cdot 4^2 +$ ……

(ii) $2^3 + 4^3 + 6^3 + 8^3 +$ …….

SECTION D (6marks each)

36. Between 2 nos whose sum is 13/6 an even number of AM's are inserted. If the sum of the means exceeds their number by unity find the number of means.

37. There are n AM's inserted between 1 and 51 such that the ratio of the 4^{th} mean to the 7^{th} mean is 3 : 5 .Find n.

38. The sum of 4 nos. in GP is 60 and AM of first and last is 18.Find the numbers.

39. In an increasing G.P., the sum of the first and the last term is 66, the product of the second and the last but one term is 128. If the sum of the series is 126 find the number of terms in the series.

40. Find the sum to n terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.....$

41. Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7^{th} and $(m\text{-}1)^{th}$ numbers is 5 : 9. Find the value of m .

42. The ratio of the sums of m and n terms of an A.P. is m^2: n^2. Show that the ratio of m^{th} and n^{th} term (2m – 1): (2n – 1).

43. The ratio of the A.M. and G.M. of two positive numbers a and b is n. Show that $a: b = \left(m+\sqrt{m^2-n^2}\right): \left(m-\sqrt{m^2-n^2}\right)$.

44. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

45. If $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P, then show that either ab + bc + ca = 0 or a, b, c are in A.P

46. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio: $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$

47. Show that the ratio of the sum of first n- terms of a G.P to the sum of terms from $(n+1)^{th}$ to (2n)th term is $\frac{1}{r^n}$

48. If S_1 ,S_2 & S_3 denote respectively the sum of first n- natural numbers, their squares & their cubes then show that $9S_2^2 = S_3(1+ 8 S_1)$

49. If a ,b , c are in G .P & the equations $a x^2 + 2 b x + c = 0$ & $dx^2 + 2ex + f = 0$ have a common root then show that d/a , e/b , f/c are in A.

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SCORING KEY

MCQ

1. b	2. b	3. a	4. b	5. a	6. a	7. b	8. d	9. c	10. c
11. d	12. a	13. a	14. d	15. c	16. c	17. a	18. b	19. c	20. b

1.	14,2,-10	2) 11
3.	12	4) -1 or 1
5.	0 or 16	6) 1/32
7.	7	8) 765
9.	i)25 ii)3/	
10.	$x^2 - 2Ax + G^2 = 0$	
11.	i)0.015 ii)81	
12.	4,64	
14.	25, 7, -11	15) 16,4
16.	$\frac{x^2(x^{2n}-1)}{x^2-1} + \frac{xy\{(xy)^n-1\}}{(xy-1)}$	
18.	10	
19.	i.	3072
	ii.	10
	iii.	2,6,18,54,…..
	iv.	-2187
20.	$\frac{n(n+1)(n+2)}{6} + 2(2^n - 1)$	
21.	i) 3n(n+1)(n+3)	ii) $\frac{n(4n^2-1)}{3}$
22.	36,12,4 **or** 4,12,36	
23.	1,3,9 **or** 9,3,1	
25.	Hint: A.M. ≥ G.M. for two positive and real numbers	
27.	148/111	
28.	i)$10/9(10^n - 1) - n$ ii) $1/3 [n - 1/9(1 - (0.1)^n]$	
29.	The numbers are 36,12,4	
30.	$S_n = \frac{n(n+1)(3n^2+5n+1)}{6}$	
31.	a. 2/3 b. 9	
32.	a = 2 and b = - 3	
33.	a = 15 and d = -8	
34.	6 yrs	
35,	i) $\frac{n(n+1)(n+2)(3n+5)}{12}$	ii) $2n^2(n+1)^2$
36.	12	
37.	24	
38.	4,8,16,32 **or** 32,16,8,4	
39.	6	

40.	$\frac{n}{24}\left(2n^2+9n+13\right)$
41.	m = 14
44.	7