CHAPTER 4
QUADRATIC EQUATIONS
SYNOPSIS

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Definitions:

If $a \neq 0$, then $ax^2 + bx + c$ is called quadratic polynomial.

If $a \neq 0$, then the equation $ax^2 + bx + c = 0$ is called quadratic equation.

2. Solution of a Quadratic Equation:

Roots of $ax^2 + bx + c = 0$, $a \neq 0$ are

$$\frac{-b+\sqrt{b^2-4ac}}{2a}, \frac{-b-\sqrt{b^2-4ac}}{2a}$$

3. Relation between the Roots and Coefficients:

i. $\text{sum of roots} = \frac{-b}{a} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

ii. $\text{Product of roots} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

iii. If α and β are the roots of the equation

$ax^2 + bx + c = 0$, then

$ax^2 + bx + c = a(x-\alpha)(x-\beta)$

4. Formation of a Quadratic Equation:

The quadratic equation whose roots are α and β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e., x^2 - (Sum of the roots) x + Product of the roots = 0 or x^2 - Sx + P = 0.

5. Nature of Roots:

Let $ax^2 + bx + c = 0$, $a \neq 0$ (a, b, c are reals) and, let D = (b^2 - $4ac$)

Then:

i.If D > 0, there are two real and distinct roots, given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-b + \sqrt{D}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-b - \sqrt{D}}{2a}$$

ii.If D = 0, the roots are real and equal, each being equal to $\left(\frac{-b}{2a}\right)$

iii.If D < 0, there are no real roots.

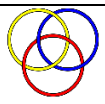
Roots of $ax^2 + bx + c = 0$, are real iff b^2 - $4ac \geq 0$ (or) D $\geq$ 0

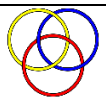
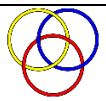
Roots of $ax^2 + bx + c = 0$, are non-real iff $b2$ - $4ac < 0$ (or) D < 0

Roots of $ax^2 + bx + c = 0$, are equal iff b^2 - $4ac = 0$ (or) D = 0.

A.		**Very Short Answer Questions (VSA) 1 Mark**	Level
		Multiple Choice Questions	
1.		Find the value of p for which the quadratic equation $x^2 + 4x + p = 0$ has real and distinct roots a) $p = 4$ b) $p > 4$ c) $p < 4$ d) $p = 0$	U
2.		The discriminant of $(x - 1)(x + 2) = 0$ is a) 9 b) 0 c) 8 d) -9	C
3.		If one root of the equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, find the value of p. a) 8 b) $\frac{3}{2}$ c) $\frac{2}{3}$ d) -8	U

4.		Comment on the nature if the roots of $2(x^2 - x) = 3$. a) Real b) Imaginary c)Real & equal d)Real & distinct	HOT
5.		If $ax^2 + bx + c = 0$ has equal roots, then c is equal to a) $\frac{-b}{2a}$ b) $\frac{b^2}{4a}$ c) $\frac{b}{2a}$ d) $\frac{-b^2}{4a}$	C
6.		Find the value of k so that the equation $x^2 - 4x + k = 0$ has real and distinct roots a) $k > 4$ b) $k = 4$ c) $k < 4$ d) $k \neq 4$	U
7.		If the quadratic equation $9x^2 + 6kx + 4 = 0$ has equal roots, then k is a) 0 b) -2 c) 2 d) ± 2	C
8.		Which of the following is a root of the equation $x^2 - \frac{1}{16} = 0$ a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) 4	C
9.		For what value of k, $x = 2$ is a solution of $kx^2 + 2x - 3 = 0$ a) $\frac{-1}{2}$ b) $\frac{1}{2}$ c) $\frac{-1}{4}$ d) $\frac{1}{4}$	U
10.		If the equation $9x^2 + 6kx + 4 = 0$ has equal roots, then the roots are a) $\pm\frac{2}{3}$ b) $\pm\frac{3}{2}$ c) ± 3 d) 0	U
		FILL IN THE BLANKS	
11.		A quadratic equation has at the most _____ roots.	U
12.		If $x = 2$ is a solution of $kx^2 + 2x - 3 = 0$, then the value of k is _____	C
13.		$x^2 - 8x + 16 = 0$ has _____ roots.	C
14.		$b^2 - 4ac$ is known as _____	C
15.		The discriminant of $3x^2 - 2x + \frac{1}{3} = 0$ is _____	U

16.		The sum of the roots of $-2x^2+5x+4=0$ is _____	C
17.		Nature of the roots $3x^2-5x+2=0$ is _____	C
18.		If x = -a is a root of $x^2+3ax+k=0$, then the value of k is _____	U
19.		_____ should be added and subtracted to solve $4x^2-\sqrt{3}x-5=0$ by the method of completing the square method	HOT
20.		The product of two consecutive odd positive integers is 483. Then the integers are _____	U
		Short Answer Questions I (SA) 2 Marks	level
21.		Find the roots of : $\boldsymbol{p^2x^2+(p^2-q^2)x-q^2=0}$	U
22.		Find the roots of the equation $\mathbf{3x^2-2\sqrt{6}x+2=0}$	C
23.		Find the value(s) of k for which the equation $\mathbf{kx^2+4x+1=0}$ has **real roots**.	U
24.		Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number	HOT
25.		Find the nature of the roots of the equation $\mathbf{x^2+2x+1/3=0}$	C
C		Short Answer Questions II (SA) 3 Marks	Level
26.		Solve x: $a^2b^2x^2-(4b^4-3a^4)x-12a^2b^2=0$	
27.		Solve $\mathbf{2x^2+14x+9=0}$ by completing the square method.	U
28.		Solve by quadratic formula: $\sqrt{5}\,\mathbf{x^2+9x+4\sqrt{5}=0}$	MD

29.		Solve for x: $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$, **x ≠ 1,2**	HOT
30.		A peacock is sitting on the top of a pillar, which is 9m high. From a point 27m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake, the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?	MD
D		Long Answer Questions (LA) 4 Marks	Level
31.		Some students planned a picnic. The budget for food was Rs 500. But, 5 of them failed to go and thus cost of food for each member increased by Rs 5. How many students attended the picnic?	U
32.		A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number	HOT
33.		If the roots of the equation **$(a-b)x^2 + (b-c)x + (c-a) = 0$** are equal, prove that **$2a = b+c$.**	U
34.		Solve **x**: $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$	HOT
35.		In a flight of 2800km an aircraft was slowed down due to bad weather. Its average speed was reduced by 100km/hr, and the time of flight increased by 30 minutes Find the original speed and original duration of the flight.	MD

Answers

QUADRATIC EQUATIONS

1	c
2	a
3	d
4	d
5	b
6	c
7	d
8	a
9	c
10	a

11	2
12	$\frac{-1}{4}$
13	Equal roots
14	Discriminant
15	0
16	$\frac{5}{2}$
17	Real & distinct
18	$2a^2$
19	$\frac{3}{64}$
20	21, 23
21	$x = -1 \ , \ q^2/p^2$
22	$x = \sqrt{2}/3 \ , \sqrt{2}/3$
23	$k < 4$
24	12
25	Real & distinct
26	$4b^2/a^2 \ , \ -3a^2/b^2$
27	$(-7 \pm \sqrt{31})/2$
28	$-\sqrt{5} \ , \ -4/\sqrt{5}$
29	$3 \ , \ 4/3$
30	12 m
31	20
32	92
34	$-a \ , \ -b$
35	3hrs 30 min.
