POLYNOMIALS

SYNOPSIS

Polynomials of degree 1 , 2 , 3 are called as linear , quadratic and cubic polynomials respectively

A linear polynomial in x is of the form **ax + b** where a,b are real numbers and a≠0

A quadratic polynomial in x is of the form $\mathbf{ax^2 + bx + c}$ where a,b,c are real numbers and $a \neq 0$

A cubic polynomial in x is of the form $\mathbf{ax^3 + bx^2 + cx + d}$, where a,b,c are real numbers and $a \neq 0$

A linear polynomial can have at most 1 zero.

A quadratic polynomial can have at most 2 zeroes

A cubic polynomial can have at most 3 zeroes

If α and β are zeroes of the quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ then

$$\boldsymbol{\alpha + \beta = -\frac{b}{a} \quad , \quad \alpha\beta = \frac{c}{a}}$$

If α , β and γ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, $a \neq 0$ then

$$\boldsymbol{\alpha + \beta + \gamma = -\frac{b}{a} \quad , \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \quad , \quad \alpha\beta\gamma = -\frac{d}{a}}$$

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials, with $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

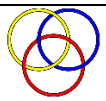
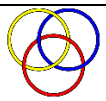
$$f(x) = d(x) \cdot q(x) + r(x).$$

Dividend | Divisor | Quotient | Remainder

where degree of r(x) < degree d(x) or degree r(x) = 0

1.		A quadratic polynomial whose roots are -3 and 4 is (a) x^2- x + 12 b) x^2 + x + 12 c) $x^2/2 - x/2 - 6$ d) $2x^2 + 2x - 24$	C
2.		If α and β are zeroes of $p(x) = x^2 + x - 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ equal to (a) -1 (b) 1 (c) 2 (d) 0	U
3.		If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is (a) 2 (b) 4 (c) -2 (d) -4	U
4.		If α and β are the zeroes of the polynomial $f(x) = x^2 - 3x + 2$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ equal to (a) 3 (b) -1 (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$	C
5.		If the product of zeroes of the polynomial $f(x) = ax^3 - 6x^2 + 11x - 6$ is 4 ,then a = (a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$	MD
6.		If one zero of the polynomial $f(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other, then k = (a) 2 (b) -2 (c) 1 (d) -1	HOT
7.		If α and β are the zeroes of the polynomial $f(x) = x^2 - p(x + 1) - c$,then $(\alpha + 1)(\beta + 1)$ = (a) c-1 (b) 1-c (c) c (d) 1+c	HOT

8.		Zeroes of $p(x) = x^2 + 2x + 1$ are (a) -1 , 1 (b) 1 , 1 (c) -1 , -1 (d) 1 , 2	C
9.		The quadratic polynomial whose zeroes are 5 and -5 can be (a) x^2- 25 (b) x^2 - 25x + 1 (c) x^2 + 25 (d) x^2 + 25x + 1	U
10.		$(x + 2)^3 = x (x^2 – 1)$ is a (a) bi quadratic equation (b) cubic equation (c) quadratic equation (d) linear equation	C
		FILL IN THE BLANKS	
11.		If the degree of a polynomial p(x) is n, then the minimum number of zeroes is _____	U
12.		If the graph of the polynomial p(x) neither touches nor intersects the x-axis, then the number of zeroes of p(x) is _____	C
13.		If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2 ,then k = _____	C
14.		If 1 is a zero of $x^3 - 7x + 6$,then other zeroes are _____	U
15.		The remainder of the polynomial $x^4 + x^3 – 2x^2 + x + 1$ divided by (x-1) is _____	U
16.		If α , β , γ are the zeroes of a cubic polynomial, then $\alpha\beta\gamma$ is _____	C
17.		_____ is a quadratic polynomial with given numbers as the sum and product -3 , -2.	C
18.		If -3 is a zero of the quadratic polynomial $(k – 1)x^2 + kx - 3$, then the value of k is _____	U
19.		If (x-1) and (x +3) are factors of $x^3 + 3x^2 – x – 3$, then the other factor is _____	U

20.		If the product of two zeroes of the polynomial $p(x) = 2x^3 + 6x^2 - 4x + 9$ is 3 then its third zero is _____	HOT
		Short Answer Questions (SA) (2 marks)	level
21.		Form a quadratic polynomialwhose zeroes are $(7 + 2\sqrt{2})$ and $(7 - 2\sqrt{2})$	C
22.		What should be added to the quadratic polynomial $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$	U
23.		If α and β are zeroes of the quadratic polynomial $f(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k	HOT
24.		If α and β are zeroes of the quadratic polynomial $2x^2 + 3x - 6$ then find the value of $\alpha^2 / \beta + \beta^2 / \alpha$	HOT
25.		If the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x + k$ are in AP then find k	MD
		Long Answer Questions (LA) (3 Marks)	
26.		Find the zeroes of the polynomial $p(x) = x^2 - 5$ and verify the relationship between the zeroes and their coefficients.	C
27.		If α and β are zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate (i) $\alpha^4 + \beta^4$ (ii) $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$	HOT
28.		If the zeroes of the polynomial $p(x) = x^3 - 9x^2 + 23x - 15$ are in AP then find the zeroes	MD
29.		If α , β and γ are the zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$ then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$	U
30.		If $p(x) = (6 + 19x + x^2 - 6x^3)$ indicates the amount of money collected from grade 10 students for their excursion and $g(x) = (2 + 5x - 3x^2)$ indicates the	U

		number of students who have paid , then find out the amount paid by each student for the excursion. Also verify by Division Algorithm	
		Long Answer Questions (VLA) (4 Marks)	
31.		Find all the zeroes of the polynomial $2x^4 - 3x^3 - 3x^2 + 6x - 2$, given that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$	U
32.		Find all the zeroes of the polynomial $x^4 - 5x^3 - 4x^2 + 16x - 8$ if two of its zeroes are $3+\sqrt{5}$ and $3-\sqrt{5}$	U
33.		In a class room , students are sitting in rows . If $f(x)= (x^4 + x^3 + 8x^2 + ax + b)$ represents the total number of students in the class and $g(x)= (x^2 + 1)$ represents the number of students sitting in each row then find number of rows in the class. Also find the value of a , b	HOT
34.		If the remainder on division of $p(x) = x^3 + 2x^2 + kx + 3$ by $(x-3)$ is 21, find the quotient and the value of k . Hence find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$	HOT

Answers

POLYNOMIALS

1	c
2	a
3	b
4	b
5	a
6	a
7	b
8	b
9	a
10	c
11	n
12	0
13	10
14	2,3
15	2
16	$\frac{-d}{a}$
17	$X^2 +5x+6$
18	2
19	X+1

20	-3/2
21	K (x^2 - 14x + 41) where k is any non zero real number
22	x -2
23	K=2
24	45/8

25	k = -28
26	$\sqrt{5}$, - $\sqrt{5}$
27	(($b^2 - 2ac$)^2 – 2a^2 c^2) / a^4 , (($b^2 - 2ac$)^2 – 2a^2 c^2) / a^2 c^2
28	1 , 3 , 5
29	5
30	2x + 3
31	$\sqrt{2}$, - $\sqrt{2}$, 1 , ½
32	3 + $\sqrt{5}$, 3 - $\sqrt{5}$, 1 , -2
33	a = 1 , b = 7
34	k = -9 , zeroes are 3 , -2 , -3