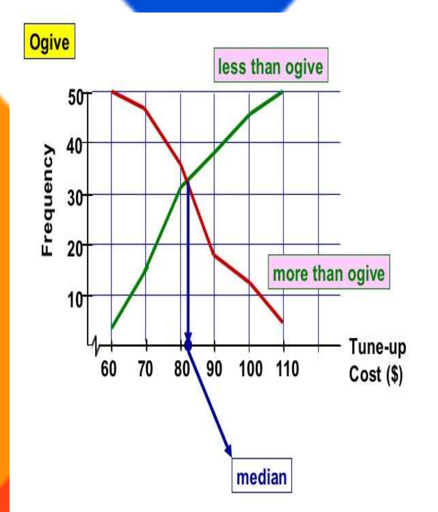
DELHI PRIVATE SCHOOL

MATHEMATICS



ACADEMIC WINDOW

2023–24

GRADE XII

Foreword

Frustration, anxiety, anguish, ... Words rarely used by competent adults in ordinary life , stream forth when people are asked to describe feelings evoked by Mathematics. It is strange that Mathematics, which is thought to be a rational subject , should elicit such emotion-laden language.

This book has been compiled keeping in mind the student’s need to understand the concept and to reinforce it in a gradual manner. Most of us do realise that practice makes a man perfect and to avoid the above said anxiety and anguish it befits us to put in enough hours of work as drill and practice to achieve excellence.

The very idea of reinforcement of concepts and practice of questions from previous board examinations has been kept foremost before making the assignments.A synopsis at the beginning of each chapter helps you recall the chapter at a glance.

A successful completion of the assignments will be very beneficial as it will help you to tackle any question with confidence and clarity.

On behalf of the Mathematics Department I wish the students success in all their endeavours.

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RELATIONS AND FUNCTIONS

SYNOPSIS

In this chapter we studied different types of relations and equivalence relations, composition of functions and invertible functions. The main features of the chapter are as follows:

- ***Empty relation*** is the relation in X given by R = $\phi \subset X \times X$.
- ***Universal relation*** is the relation R in X given by R = $X \times X$.
- ***Reflexive relation*** R in X is a relation with (a, a) $\in R \forall a \in X$.
- ***Symmetric relation*** R in X is a relation satisfying (a, b) $\in R \Rightarrow (b,a) \in R$.
- ***Transitive relation*** R in X is a relation satisfying $(a,b) \in R and (b,c) \in R \Rightarrow (a,c) \in R$.
- ***Equivalence relation*** R in X is a relation which is reflexive symmetric and transitive.
- ***Equivalence class*** [a] containing $a \in X$, for an equivalence relation R in x is the subset of X containing all elements b related to a.
- A function $f : X \to Y$ is ***one-one or injective*** if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2, \forall x_1, x_2 \in X$.
- A function $f : X \to Y$ is ***onto or surjective*** if given any $y \in Y, \exists x \in X$ such that f (x) = y.
- A function $f : X \to Y$ is ***one-one and onto or bijective***, if f is both injective and surjective.
- The ***composition of functions*** $f : A \to B$ and $g : B \to C$ is the function $gof : A \to C$ given by gof(x) = g [f (x)] $\forall x \in A$.
- A function $f : X \to Y$ is ***invertible*** if $\exists g : Y \to X$ such that gof = I_x and fog = I_y.
- A function $f : X \to Y$ is invertible if and only if f is one-one and onto.
- Given a finite set X, a function $f : X \to Y$ is one-one (respectively onto) if and only if f is onto (respectively one-one). This is the characteristic property of a finite set. This is not true for an infinite set.

SECTION A (1 mark)

MCQ

1. Let R be a relation on the set N of natural numbers defined by *n* R *m* if *n* divides *m*. Then R is
   (a) Reflexive and symmetric (b) Transitive and symmetric
   (c) Equivalence (d) Reflexive, transitive but not symmetric

2. Let L denote the set of all straight lines in a plane. Let a relation R be defined by *l* R *m* if and only if *l* is perpendicular to *m* ∀ *l*, *m* ∈ L. Then R is
   (a) Reflexive (b) Symmetric (c) Transitive (d) None of these

3. Let **N** be the set of natural numbers and the function *f* : **N** → **N** be defined by *f(n)* = 2*n* + 3 ∀ *n* ∈ **N**. Then *f* is

(a) Surjective (b) Injective (c) Bijective (d) None of these

4. Set A has 3 elements and the set B has 4 elements. Then the number of injective mappings that can be defined from A to B is
   (a) 144 (b) 12 (c) 24 (d) 64

5. Let f : **R** → **R** be defined by $f(x) = \sin x$ and g : **R** → **R** be defined by $g(x) = x^2$, then $f \circ g$ is
   (a) $x^2 sin\ x$ (b) $(\sin x)^2$ (c) $\sin x^2$ (d) $\frac{\sin x}{x^2}$

6. Let f : **R** → **R** be defined by $f(x) = 3x - 4$. Then $f^{-1}(x)$ is given by
   (a) $\frac{x+4}{3}$ (b) $\frac{x}{3} - 4$ (c) $3x + 4$ (d) None of these

7. Let f : **R** → **R** be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 and – 3, respectively, are
   (a) ϕ, {4, – 4} (b) {3, – 3}, ϕ (c) {4, –4}, ϕ (d) {4, – 4}, {2, – 2}

8. For real numbers x and y, define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is
   (a) Reflexive (b) Symmetric (c) Transitive (d) None of these

9. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b, $a, b \in$ T. Then R is
   (a) Reflexive but not transitive (b) Transitive but not symmetric
   (c) Equivalence (d) None of these

10. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is
    (a) Symmetric but not transitive (b) Transitive but not symmetric
    (c) Neither symmetric nor transitive (d) Both symmetric and transitive

11. The maximum number of equivalence relations on the set A = {1, 2, 3} are
    (a) 1 (b) 2 (c) 3 (d) 5

12. If a relation R on the set {1, 2, 3} be defined by R = {(1, 2)}, then R is
    (a) Reflexive (b) Transitive (c) Symmetric (d) None of these

13. Let us define a relation R in **R** as aRb if $a \geq b$. Then R is
    (a) An equivalence relation (b) Reflexive, transitive but not symmetric
    (c) Symmetric, transitive but not reflexive (d) neither transitive nor reflexive but symmetric.

14. Let A = {1, 2, 3} and consider the relation R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}. Then R is
    (a) Reflexive but not symmetric (b) Reflexive but not transitive

(c) Symmetric and transitive (d) neither symmetric, nor transitive

15. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is
(a) 720 (b) 120 (c) 0 (d) none of these

16. Let A = {1, 2, 3, ...n} and B = {a, b}. Then the number of surjections from A into B is
(a) ${}^nP_2$ (b) $2^n - 2$ (c) $2^n - 1$ (d) None of these

17. Let f : **R** → **R** be defined by $f(x) = \frac{1}{x}$, $x \in$ R. Then f is
(a) one-one (b) onto (c) bijective (d) f is not defined

18. Let f : **R** → **R** be defined by $f(x) = 3x^2 - 5$ and g : **R** → **R** by $g(x) = \frac{x}{x^2+1}$. Then $g \, o \, f$ is
(a) $\frac{3x^2-5}{9x^4-30x^2+26}$ (b) $\frac{3x^2-5}{9x^4-6x^2+26}$ (c) $\frac{3x^2}{x^4+2x^2-4}$ (d) $\frac{3x^2}{9x^4+30x^2-2}$

19. Which of the following functions from **Z** into **Z** are bijections?
(a) $f(x) = x^3$ (b) $f(x) = x + 2$ (c) $f(x) = 2x + 1$ (d) $f(x) = x^2+1$

20. Let f : **R** → **R** be the functions defined by $f(x) = x^3 + 5$. Then $f^{-1}(x)$ is
(a) $(x+5)^{\frac{1}{3}}$ (b) $(x-5)^{\frac{1}{3}}$ (c) $(5-x)^{\frac{1}{3}}$ (d) $5 - x$

21. Let f : **R** – $\{\frac{3}{5}\}$ → **R** be defined by $f(x) = \frac{3x+2}{5x-3}$. Then
(a) $f^{-1}(x) = f(x)$ (b) $f^{-1}(x) = -f(x)$ (c) $(fof)x = -x$ (d) $f^{-1}(x) = \frac{1}{19} f(x)$

22. . Let f : [0, 1] → [0, 1] be defined by $f(x) = \begin{cases} x, if\ x\ is\ rational \\ 1-x, if\ x\ is\ irrational \end{cases}$.
Then $(fof)(x)$ is
(a) Constant (b) 1 + x (c) x (d) none of these

23. Let f : [2, ∞) → **R** be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is
(a) **R** (b) [1, ∞) (c) [4, ∞) (d) [5, ∞)
(a) 1 (b) 1 (c) $\frac{7}{2}$ (d) none of these

Fill in the blanks:

24. Consider the set A = {1, 2, 3} and R be the smallest equivalence relation on A, then R = ________.

25. The domain of the function f : **R** → **R** defined by $f(x) = \sqrt{x^2 - 3x + 2}$ is ________.

26. Consider the set A containing n elements. Then, the total number of injective functions from A onto itself is ________.

27. Let **Z** be the set of integers and R be the relation defined in **Z** such that a R b if $a - b$ is divisible by 3. Then R partitions the set Z into ________ pairwise disjoint subsets.

28. Let the relation R be defined in **N** by a R b if $2a + 3b = 30$. Then R = ______.

29. Let the relation R be defined on the set A = {1, 2, 3, 4, 5} by R = {(a, b): $|a^2 - b^2| < 8$}. Then R is given by _______.

VSA

30. If $f(x) = |x|$ and $g(x) = [x]$. Find$fog\left(-5/3\right) - gof\left(-5/3\right)$.

31. Let f : A→B where A= {1, 2, 3}; b={a, c} defined as $f(1) = a$; $f(2) = c$; $f(3) = a$, find f^{-1} if exists. Give reason for the answer.

32. A= {1, 3, 5}; B {9, 11} R= {(a, b) $\in A \times B$: a-b is odd} Write the relation R.

33. Let $A = \{1, 2, 3\}$. Consider the equivalence relation: $R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ on A. Find the equivalence class of elements of A.

34. Show that the function $f: R \to R$ defined by $f(x) = 2x + 9$ is one-one.

SECTION B (2 Marks)

35. fLet $f : R \to R$ be defined as $f(x) = x^2 + 1$ Find $f^{-1}(26)$.

36. If $f : R \to R$ be defined as $f(x) = x^3 + 3$, then find $f^{-1}(x)$.

<u>SECTION C (4 Marks)</u>

37. Show that $f : N \to N$ defined by $f(x) = x^2 + x + 1;\ x \in N$ is not invertible.

38. Consider $f : R^+ \to [-5, \propto]$ given by $f(x) = 9x^2 + 6x - 5$ Show that f is invertible. Also find f^{-1}

39. Show that the relation R defined by $R = \{(a, b): a - b \text{ } is \text{ } divisible \text{ } by \text{ } 3\}, a, b \in N$ is an equivalence relation.

40. Let N denote the set of all natural numbers and R be the relation on N x N defined by$(a, b)R(c,\ d) \Leftrightarrow ad(b + c) = bc(a + d)$. Show that R is an equivalence relation on N x N.

41. Prove that f: R → R is a bijection given by $f(x) = x^3 + 3$. Find $f^{-1}(x)$.

42. Let A be the set of all students of class XII in a school and R be the relation, having the same sex on A, and then prove that R is an equivalence relation.

43. If **h** denotes the number of honest people and p denotes the number of punctual people and a relation between an honest people and punctual people is given as $h = p + 5$. If P denotes the number of people who progress in life and a relation between number of people who progress and honest people is given as $P = \left(\frac{h}{8}\right) + 5$, find the relation between number of people who progress in life and punctual people.

SCORE KEY

1.	d
2.	b
3.	b
4.	c
5.	c
6.	a
7.	c
8.	a
9.	c
10.	b
11.	d

12. b

13. b

14. a

15. c

16. c

17. b

18. d

19. a

20. b

21 b

22. a

23. a

24. c

25. b

26. d

27. R = {(1, 1), (2, 2), (3, 3)}.

28. $(-\infty, 1] \cup [2, \infty)$

29. n!

30. Three

31. 0

32. R = (3,8), (6,6), (9,4), (12,2)

33. R = {(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3), (3,4), (4,3), (4,4), (5,5)}

34. *gof* = {(1,3), (3,1), (4,3)} and *fog* = {(2,5), (5,2), (1,5)}

35. 1

36. f^{-1} doesn't exist, f is not one-one

37. Empty Relation

38. $[1] = \{1,2\}$, $[2] = \{1,2\}$, $[3] = \{3\}$

40. 2 , 1

41. $gof = \{(1, 3)\,(3, 1)\,(4,3)\}$

42. {-5, 5}

43. $\sqrt[3]{x-3}$

44. -1

45. $(i)4x^2 - 6x + 1, \qquad (ii)2x^2 + 6x - 1,$
$(iii)x^4 + 6x^3 + 14x^2 + 15x + 5$

47. $\dfrac{\sqrt{x-6}}{3} - 1$

50. $f^{-1}(x) = (x-3)^{\frac{1}{3}}$

52. $P = \dfrac{p+45}{8}$

2.MATRICES AND DETERMINANTS

SYNOPSIS

(a) Area (Δ) of a triangle ABC having vertices A (x_1, y_1), B(x_2, y_2) and C(x_3,y_3) is

Δ = absolute value of $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$: If points are collinear, then $\Delta = 0$

(b) $(AB)^T = B^T A^T$

(c) $(AB)^{-1} = B^{-1} A^{-1}$

(d) $(A^T)^{-1} = (A^{-1})^T$

(e) A square matrix A is called a symmetric matrix if $A^T = A$

A square matrix A is called a skew symmetric Matrix if $A^T = -A$

(f) Ax = B (Solve by matrix method)

i. $|A| \neq 0$, then the system of equation is consistent & having unique solution.

ii. $|A| = 0$, then find (adj A) B

If (adj A). B $\neq$ 0, then the system is in consistent i.e. no solution.

If (adj A). B = 0, then the system is consistent with infinitely many solutions

(g) For AX = 0

(i) If $|A| \neq 0$, then x = y = 0 is the only solution.

(ii) If $|A| = 0$, then the system has infinitely y many solutions.

(h) If A=kB where A & B are square matrices of order n, then $|A| = k^n |B|$

(i) If A be any given square matrix of order n, then $A(adjA) = (adjA)A = |A|I$,
Where I is the identity matrix of order n.

(j) If A is a non-singular matrix of order n, then $|adjA| = |A|^{n-1}$

Answer the following MCQ questions-

1. If $\begin{bmatrix} x+3 & 2y+x \\ x-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ -4 & 2a \end{bmatrix}$, then x - y + a =

1) 0 2) 1 3) 2 4) 3

2. A = $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then

1) AB, BA exist and equal 2) AB, BA exist but not equal
3) AB exists but not BA 4) BA exist but not AB

3. Order of the matrix $\begin{bmatrix} 2 & 2 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} \ell \\ m \\ n \end{bmatrix}$ is

1) 3 × 3 2) 3 × 2
3) 2 × 3 4) 3 × 1

4. If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then AA^T =

1) O 2) I
3) A 4) -A

5. If A = $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$, then A^2 =

1) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ 2) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

3) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 4) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

6. If $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$, $C = \begin{pmatrix} -3 & 2 \\ 7 & 1 \end{pmatrix}$ and AB = C, then B =

1) $\begin{pmatrix} 13 & 3 \\ 36 & -7 \end{pmatrix}$ 2) $\begin{pmatrix} -13 & 3 \\ 36 & -7 \end{pmatrix}$

3) $\begin{pmatrix} 13 & 3 \\ 36 & 7 \end{pmatrix}$ 4) $\begin{pmatrix} -13 & 3 \\ -36 & 7 \end{pmatrix}$

7. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then A^p =

1) $\begin{pmatrix} 3p & -4p \\ p & -p \end{pmatrix}$ 2) $\begin{pmatrix} p+2 & p-5 \\ 2p-1 & 2p-3 \end{pmatrix}$

3) $\begin{pmatrix} 1+2p & -4p \\ p & 1-2p \end{pmatrix}$ 4) $\begin{pmatrix} 1-2p & -4p \\ p & 1-2p \end{pmatrix}$

8. If $A = \begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & -3 \\ 1 & -4 & -1 \end{bmatrix}$, then A is

1) symmetric 2) singular 3) non-singular 4) unit matrix

9. The inverse of $\begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}$ is

1) $\begin{pmatrix} 1 & -2 \\ -3 & 5 \end{pmatrix}$ 2) $\begin{pmatrix} -1 & 2 \\ -3 & 5 \end{pmatrix}$ 3) $\begin{pmatrix} -1 & -2 \\ -3 & -5 \end{pmatrix}$ 4) $\begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix}$

10. The inverse of the matrix $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is

1) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ 2) $\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 3) $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ 4) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

11. If $A = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$, $AB = \begin{pmatrix} -1 & 12 & 11 \\ a & 7 & 8 \\ -2 & b & 5 \end{pmatrix}$, then (a, b) =

1) (1, 1) 2) (-1, 1) 3) (1, -1) 4) (-1, -1)

12. If $A = \begin{pmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{pmatrix}$, then A^2 =

1) scalar zero 2) matrix $O_{2\times2}$ 3) Matrix $O_{3\times3}$ 4) I_3

13. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, then (A + I)(A - 5I) =

1) Null matrix 2) Unit matrix 3) Scalar matrix 4) Diagonal matrix

14. If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}$, then A.adjA =

1) Null matrix 2) Unit matrix 3) Scalar matrix 4) Diagonal matrix

15. If $A = \begin{pmatrix} 9 & 1 \\ 4 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 5 \\ 6 & 11 \end{pmatrix}$ and 3A + 5B + 2X = 0, then X =

1) $\begin{pmatrix} -16 & -14 \\ -21 & -32 \end{pmatrix}$ 2) $\begin{pmatrix} -16 & -14 \\ 21 & -32 \end{pmatrix}$ 3) $\begin{pmatrix} -16 & 14 \\ -21 & -32 \end{pmatrix}$ 4) $\begin{pmatrix} -16 & -14 \\ 21 & 32 \end{pmatrix}$

16. A is m × n type matrix such that AB and BA both exist then order of B is

1) m × n 2) n × m 3) n × n 4) m × m

ASSIGNMENT

Fill in the blanks-

1.If a non-singular matrix A satisfies $A^2 - A + 2I = 0$, then A^{-1} =------------

2. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$, then adj (adjA) =-----------------

3. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then (a, b) =-------

4. If $3A + 4B^T = \begin{bmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{bmatrix}$, $2B - 3A^T = \begin{bmatrix} -1 & 18 \\ 4 & -6 \\ -5 & -7 \end{bmatrix}$, then B =--------------

5. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ then A^3 =--------------

1-MARK

1. For what value of k, the matrix $\begin{bmatrix} -1 & -3 \\ 4 & k \end{bmatrix}$ has no inverse.

2. If A= $\begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ find the value of (adj A) A.

3. Find the value of k if the area of the triangle is 7 sq. units and the vertices are (1,3) (0,5) and (k,0)

4. Find the value of k for which the matrix $\begin{bmatrix} 4 & k \\ 8 & 6 \end{bmatrix}$ may be singular.

5. Express $\begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}$ as the sum of symmetric and skew. Symmetric matrix.

6. If A and B are symmetric matrix of same order, then what type of matrix is (AB-BA).

7. If A= $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and A^2 = xA+yI find x and y.

8. Construct a matrix A= $[a\ i\ j]_{3\times 2} = \frac{i-2j}{2}$.

9. A matrix *x* has *a+b* rows and *a+1* column while the matrix *y* has *b+2* rows and *a+3* columns. Both matrix *xy* and *yx* exist? Find *a&b*.

10. Find *x* if $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$.

11. What type of a matrix is A if A is both symmetric and skew Symmetric?

12. If A is a square matrix of order 3, then find the value of k if |3A|=k |A|

13. If A is an invertible matrix of order 3, then find |adj A|.

14. Find the cofactor of all the elements in the second row $\begin{vmatrix} 1 & -2 & 3 \\ 4 & 0 & 1 \\ 2 & -1 & 1 \end{vmatrix}$.

2 Mark

15. Find the inverse of the matrix by using elementary row transformation $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$.

16. Find the matrix ***'P'*** satisfying the equation $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} P \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

17. If the matrix A=$\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ Satisfies the equation A^2 -3A-7I=0 and hence find A^{-1}.

18. Find the matrix A for which $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$ A=$\begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$.

19. Solve $\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix}$=0.

4 Mark

20. For two matrices A and B, A = $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$.Verify that $(AB)^T = B^T A^T$

21. If A= $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ Prove that $A^n = \begin{bmatrix} a^n & b(a^n-1)/a-1 \\ 0 & 1 \end{bmatrix}$ For every positive integer n.

22. Find matrix x and y , if $2x$-y=$\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $x+2y=\begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$.

23. Three shopkeepers A, B, C are using polythene, handmade bags (prepared by prisoners), and newspaper's envelope as carry bags. It is found that the shopkeepers A,B,C are using (20,30,40), (30,40,20), (40,20,30) polythene, handmade bags and newspapers envelopes respectively. The shopkeepers A, B, C spend Rs.250, Rs.220 and Rs 200 on these carry bags respectively. Find the cost of each carry bags using matrices. Keeping in mind the social & environmental conditions, which shopkeeper is better and why?
24. A school wants to award its students for the values of Honesty , Regularity and Hard work with a total cash award of ₹6,000. Three times the award money for Hard work added to that given for Honesty amounts to ₹11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

6 Marks

25. Using elementary transformations, find the inverse of matrix A $\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and use it to solve the following system of linear equations: 8x +4y +3z =19,
2x + y +z =5 x +2y + 2z= 7

26. Solve the system of equation by matrix method.

a. $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$; $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$; $\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$

b. If $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$ find A^{-1} and hence solve the equation

$x + 2y + 5z = 10$; $x - y - z = -2$; $2x + 3y - z = -11$

c. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ find A^{-1} and hence solve the equation
$x + y + 2z = 0$; $x + 2y - z = 9$; x - 3y + 3z = -14

d. $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ find AB. Use this result to solve the equation
2x – y + z = -1; -x +2y - z = 4 ; x – y +2z = -3

Scoring key

MCQ -ANSWERS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
3	2	4	2	2	2	3	2	4	1
11	**12**	**13**	**14**	**15**	**16**				
4	3	1	2	1	2				

II

1. $\frac{1}{2}(I - A)$ **2.** A **3.** (1, 4) **4.** $\begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}$ **5.** $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 64 \end{bmatrix}$

1.	K=12
2.	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
4.	K= $\frac{-9}{2}$
5.	X=3
6.	Symmetric $\begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$
7.	Skew symmetric
8.	X = 4; y = -1
9.	$\begin{bmatrix} \frac{-1}{2} & \frac{-3}{2} \\ 0 & -1 \\ \frac{1}{2} & \frac{-1}{2} \end{bmatrix}$
10	a = 4; b= 3
11.	$x = 5$; $x = -3$
12.	A is zero matrix
13.	K=27
14.	$\|A\|^2$
15.	$A_{21} = -1$ $A_{22} = -5$ $A_{23} = -3$
16.	$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

17.	$x = \begin{bmatrix} 9 & -14 \\ -16 & 25 \end{bmatrix}$
18.	$\frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$
19.	A= $\begin{bmatrix} 6 & 2 \\ 11/2 & 2 \end{bmatrix}$
20.	$X = 4$
26. 27.	$x = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$; $y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$ [Polythene=Re.1] [Handmade bag = Rs.5] [Newspaper's envelop=Rs.2] Shopkeeper A is better for environmental conditions as he is using least no of polythene. Shopkeeper B is better for social conditions as he is using handmade bags (Prepared by prisoners.)
28.	x = 500, y = 2000, z =3500
29	x=1, y=2,z=1
30.	a. x = ½ , y = 1/3 , z = 1/5
	b. $x = -1$; $y = -2$; $z = 3$
	c. $x = 1$; $y = 3$; $z = -2$
	d. $x = 1$; $y = 2$; $z = -1$

INVERSE TRIGONOMETRY

SYNOPSIS:

1. If $\sin\theta = x$, we write $\theta = \sin^{-1} x$.

2. $\sin(\sin^{-1} x) = x$, $\sin^{-1}(\sin\theta) = \theta$ if 'θ' $\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos(\cos^{-1} x) = x$, $\cos^{-1}(\cos\theta) = \theta$ if $\theta \in [0, \pi]$

$\tan(\tan^{-1} x) = x$, $\tan^{-1}(\tan\theta) = \theta$ if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

3. That value of $\sin^{-1} x$ lying between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ is called the principal value of $\sin^{-1} x$.

That value of $\cos^{-1} x$ lying between 0 and π is called the principal value of $\cos^{-1} x$.

That value of $\tan^{-1} x$ lying between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ is called the principal value of $\tan^{-1} x$.

4. If $-1 \le x \le 1$, then i) $\sin^{-1}(-x) = -\sin^{-1} x$ ii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$

5. If $x \varepsilon R$, then i) $\tan^{-1}(-x) = -\tan^{-1} x$ ii) $\cot^{-}(-x) = \pi - \cot^{-1} x$

6. If $x \le -1$ or $x \ge 1$, then

i) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$ ii) $\sec^{-}(-x) = \pi - \sec^{-1} x$

7. $\operatorname{cosec}^{-1} x = \sin^{-1} \frac{1}{x}$ (if $x \ne 0$).

$\sec^{-1} x = \cos^{-1} \frac{1}{x}$ (if $x \ne 0$).

$\cot^{-1} x = \tan^{-1} \frac{1}{x}$ (if $x > 0$).

$= \pi + \tan^{-1} \frac{1}{x}$ (if $x < 0$).

8. $\sin^{-1} x + \cos^{-1} x = \pi/2$, $\tan^{-1} x + \cot^{-1} x = \pi/2$, $\sec^{-1} x + \operatorname{cosec}^{-1} x = \pi/2$.

9. If $\sin^{-1} x + \sin^{-1} y = \pi/2$, then $x^2 + y^2 = 1$.

10. $\sin(\cos^{-1} x) = \sqrt{1-x^2}$, $\cos(\sin^{-1} x) = \sqrt{1-x^2}$

11. $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$ for $0 \le x \le 1$

$= -\cos^{-1}\sqrt{1-x^2}$ for $-1 \le x < 0$

12. $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2}$ for $o \le x \le 1$

$= \pi - \sin^{-1}\sqrt{1-x^2}$ for $-1 \le x < 0$

$= \tan^{-1}\frac{\sqrt{1-x^2}}{x}$ for $0 < x \le 1$, $\pi + \tan^{-1}\frac{\sqrt{1-x^2}}{x}$ for $-1 \le x < 0$

13. $\tan^{-1} x = \sin^{-1} \dfrac{x}{\sqrt{1+x^2}}$ for $x \geq 0$

$= \cos^{-1} \dfrac{1}{\sqrt{1+x^2}}$ for $x \geq 0$

14. $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ for $\dfrac{-1}{\sqrt{2}} \leq x \leq \dfrac{1}{\sqrt{2}}$

$= \pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ for $\dfrac{1}{\sqrt{2}} < x \leq 1$

$= \sin^{-1}\left(2x\sqrt{1-x^2}\right) - \pi$ for $-1 \leq x < \dfrac{-1}{\sqrt{2}}$

15. $2\cos^{-1} x = \cos^{-1}\left(2x^2 - 1\right)$ for $0 \leq x \leq 1$

$= \pi - \cos^{-1}\left(2x^2 - 1\right)$ for $-1 \leq x < 0$

16. $2\tan^{-1} x = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ for $-1 < x < 1$

$= \pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ for $x > 1$

$= -\pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ for $x < -1$

17. $2\tan^{-1} x = \sin^{-1} \dfrac{2x}{1+x^2}$ for $-1 \leq x \leq 1$

$= \cos^{-1} \dfrac{1-x^2}{1+x^2}$ for $0 \leq x < \infty$

$= \tan^{-1} \dfrac{2x}{1-x^2}$ for $-1 < x < 1$

18. $3\sin^{-1} x = \sin^{-1}\left(3x - 4x^3\right)$ for $\dfrac{-1}{2} \leq x \leq \dfrac{1}{2}$

$3\cos^{-1} x = \cos^{-1}\left(4x^3 - 3x\right)$ for $\dfrac{1}{2} \leq x \leq 1$

$3\tan^{-1} x = \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right)$ for $\dfrac{-1}{\sqrt{3}} \leq x \leq \dfrac{1}{\sqrt{3}}$

19. If $o \leq x \leq 1,\ 0 \leq y \leq 1,$ then $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

20. If $0 \leq x \leq 1,\ 0 \leq y \leq 1$ and $x^2 + y^2 \leq 1,$

then $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

21. If $0 \le x \le 1$, $0 \le y \le 1$ and $x^2 + y^2 > 1$, then

$$\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}(\sqrt{1-x^2}\sqrt{1-y^2} - xy)$$

22. If $0 \le x \le 1$, $0 \le y \le 1$ and $x < y$, then

$$\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

23. If $0 \le x \le 1$, $0 \le y \le 1$ and $x > y$, then

$$\cos^{-1}x - \cos^{-1}y = -\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

24. (a) If $-1 < x < 1$, $-1 < y < 1$ and $xy < 1$ then

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

(b) If $x > 0$, $y > 0$ and $xy > 1$ then

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy} + \pi$$

(c)If $x < 0$, $y < 0$ and $xy > 1$, then

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy} - \pi$$

25. (a) If $xy > -1$ then

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

(a)If $x > 0$, $y < 0$ and $xy < -1$, then

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy} + \pi$$

(b) If $x < 0$, $y>0$ and $xy < -1$ then,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy} - \pi$$

26. (a) If $-1 < x < 1$, then

$$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$$

(b) If $x > 1$, then

$$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2} + \pi$$

(c)If $x < -1$, then

$$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2} - \pi.$$

27. $\sin(2\tan^{-1}x) = \dfrac{2x}{1+x^2}$

$\cos(2\tan^{-1}x) = \dfrac{1-x^2}{1+x^2}$

$\tan(2\tan^{-1}x) = \dfrac{2x}{1-x^2}$

28. $\sin(3\sin^{-1}x) = 3x - 4x^3$

$\cos(3\cos^{-1}x) = 4x^3 - 3x$

$\tan(3\tan^{-1}x) = \dfrac{3x - x^3}{1-3x^2}$

I. **Choose the correct option in the following question**:

1. The value of $\sin(\tan^{-1}x)$ is

i)$\frac{x}{\sqrt{1+x^2}}$ ii)$\frac{1}{\sqrt{1+x^2}}$ iii)$\sqrt{1+x^2}$ iv)$\frac{x^2}{\sqrt{1+x^2}}$

2. The value of $\cot(\cos^{-1}\frac{7}{25})$ is

i)24/25 ii)25/24 iii)25/7 IV)7/24

3. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, then value of $\cos^{-1}x + \cos^{-1}y$ is

i)$\frac{\pi}{2}$ ii)π iii)0 iv) $\frac{2\pi}{3}$

4. If $3\tan^{-1}x + \cot^{-1}x = \pi$, then value of *x is :*

i) 1 ii) 0 iii)-1 iv) ½

5. The value of $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3)$ is

i)5 ii) 11 iiii)13 iv)15

6. If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$,then the value of x + y +xy

i) 0 ii)1 iii)-1 iv) 2

7. The value of $\tan^{-1}\left[2\sin^{-1}\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ is

i)$\frac{\pi}{3}$ ii)$\frac{\pi}{4}$ iii)$\frac{\pi}{2}$ iv)π

8. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

i) $\frac{\pi}{3}$ ii)$\frac{\pi}{4}$ iii) $\frac{\pi}{2}$ iv) π

9. The value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

i) 0 ii) π iii) $\frac{\pi}{6}$ iv) $\frac{\pi}{4}$

10. If $\cos\left(\sin^{-1}\frac{3}{5}+\cos^{-1}x\right) = 0$, then x equal to

i) 1/5 ii) 3/5 iii) 0 iv) 1

II. The following question consists of two statement-Assertion(A) and Reason (R). Answer the questions selecting the appropriate option given below:

(a) Both A and R are true, and R is the correct explanation for A
(b) Both A and R are true, and R is not the correct explanation for A
(c) A is true but R is false
(d) A is false but R is true.

1. Assertion (A): Domain of $\cos^{-1}(2x-1)$ is $[0,1]$
Reason (R): Domain of a function f: A→ B is the set of values where the function is defined. Hence A is the domain of the function f.

2. Assertion(A): Principle value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is $\frac{2\pi}{3}$

Reason(R): $\cos^{-1}[-1,1] \rightarrow [0,\pi]$ is a bijection map.

3. Assertion (A): Domain of f(x)=$\sin^{-1}x$ +$\cos^{-1}x$ is $[-1,1]$
Reason(R): Domain of a function is the set of all possible values for which the function is defined.
4. Assertion (A): Function f: R→R given by f(x)=sinx is not a bijection
Reason (R): A function is said to be bijection if it is one-one and onto.

III. Fill in the blanks:

1. The principle value of $\sin\left[2\cos^{-1}\left(\frac{-5}{13}\right)\right]$ is ……………...

2. If $tan^{-1}(1-x), tan^{-1}x$ & $tan^{-1}(1+x)$ are in A.P, then x^3+x^2 is……………………

3. The value of $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3$ is ………………………

4.If cos ($\tan^{-1}x$)=sin($\cot^{-1}\frac{3}{4}$), then x is equal to ………………………….

5.The principal value of $\tan^{-1}\sqrt{3}$ -$\cot^{-1}(-\sqrt{3})$……………………………….

__**1 Mark**__

IV

1. Evaluate $\cos\left(\sin^{-1}\left(3/5\right)\right)$.
2. Evaluate $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$
3. Evaluate for the principle values $\tan^{-1}(-1)+\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.
4. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$ find x.
5. Simplify $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\quad 0<x<\pi$.
6. Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$.
7. Show that $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)=\cos ec^{-1}x$.
8. Prove That $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$.

2 Marks

9. Prove that $2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}=\tan^{-1}\frac{4}{7}$.
10. Prove that $sec^2(tan^{-1}2)+cos\,e\,c^2(cot^{-1}3)=15$.
11. Solve for x $\quad\cot^{-1}x-\cot^{-1}(x+2)=\pi/12$.
12. Prove that $\cos\left[\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right]=\frac{33}{65}$.
13. Solve $\tan^{-1}\frac{1}{4}+2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\pi/4$.
14. Simplify $\cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)$.
15. Simplify $\tan^{-1}\left(\frac{a+bx}{b-ax}\right)$..

4 Marks

16. $\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=\pi$. Prove it.
17. If $cos^{-1}x+cos^{-1}y+cos^{-1}z=\pi$, prove that $x^2+y^2+z^2+2xyz=1$.

Answers of MCQ - assertion and reason- fill in the blank

I. 1(i) 2(ii) 3(i) 4(i) 5(iv) 6.(ii) 7.(i) 8.(iv) 9.(i) 10(ii)

II. 1. (a) 2(b) 3.(a) 4.(a)

III. 1.$\frac{-120}{169}$ 2.1 3. π 4.3/4 5. -$\frac{\pi}{2}$

IV-Answers for question bank-

1.	$\frac{4}{5}$
2.	$\frac{-7}{17}$
3.	$\frac{\pi}{2}$
4.	$x = \frac{1}{5}$
5.	$\frac{x}{2}$
6.	$\frac{1}{\sqrt{3}}$
14.	$x - \tan^{-1}\frac{4}{5}$
15.	$\tan^{-1}\frac{a}{b} + \tan^{-1}x$

CONTINUITY AND DIFFERENTIABILITY

SYNOPSIS

1. Let f(x) be any function of x defined in a particular interval.

Then $f^1(x) = \underset{h \to 0}{Lt} \frac{f(x+h) - f(x)}{h}$.

2. **(a)** $f^1(a) = \underset{h \to 0}{Lt} \frac{f(a+h) - f(a)}{h}$ or $f^1(a) = \underset{x \to a}{Lt} \frac{f(x) - f(a)}{x - a}$.

(b) $L f^1(a) = \underset{h \to 0}{Lt} \frac{f(a-h) - f(a)}{-h}$ (Left hand differentiation)

$R f^1(a) = \underset{h \to 0}{Lt} \frac{f(a+h) - f(a)}{h}$ (Right hand differentiation)

$Lf^1(a) = Rf^1(a)$ then f(x) is differentiable at x = a

3. If u, v are function of x, then

$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

4. $\frac{d}{dx}(uv) = u.\frac{dv}{dx} + v.\frac{du}{dx}$

5. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$

6. If k is a constant and u is any function of x, $\frac{d}{dx}(k\,u) = k.\frac{du}{dx}$.

7. Derivative of a constant is zero.

8. $\frac{d}{dx}(x^n) = n\,x^{n-1}, \frac{d}{dx}\{(ax+b)^n\} = n(ax+b)^{n-1}.\,a$

9. $\frac{d}{dx}(e^x) = e^x, \frac{d}{dx}(a^x) = a^x \log a$

$\frac{d}{dx}(x^x) = x^x(1 + \log x) = x^x \log ex$

10. $\frac{d}{dx}(\log_e x) = \frac{1}{x}, \frac{d}{dx}\{\log_a x\} = \frac{1}{x \log a}$

11. $\frac{d}{dx}\left(\log|x|\right)=\frac{1}{x},\frac{d}{dx}\left\{\log_a|x|\right\}=\frac{1}{x\log a}$

12. $\frac{d}{dx}(x)=1,\frac{d}{dx}\left(\sqrt{x}\right)=\frac{1}{2\sqrt{x}},\frac{d}{dx}\left(\frac{1}{x}\right)=-\frac{1}{x^2}.$

13. $\frac{d}{dx}\left(\frac{ax+b}{cx+d}\right)=\frac{ad-bc}{(cx+d)^2}$

14. $\frac{d}{dx}(\sin x)=\cos x,\frac{d}{dx}\left(\sin ax\right)=\cos ax.\,a$

15. $\frac{d}{dx}(\cos x)=-\sin x,\frac{d}{dx}(\tan x)=\sec^2 x.$

$\frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x,\ \frac{d}{dx}(\operatorname{cosec} x)=-\operatorname{cosec} x.\cot x$

$\frac{d}{dx}(\sec x)=\sec x.\tan x,$

16. $\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}},\frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2},\frac{d}{dx}(\cot^{-1}x)=-\frac{1}{1+x^2}$

$\frac{d}{dx}(\sec^{-1}x)=\frac{1}{|x|\sqrt{x^2-1}},\frac{d}{dx}(\operatorname{cosec}^{-1}x)=-\frac{1}{|x|\sqrt{x^2-1}}$

I. **Choose the correct option in the following question**:

1. Find the value of 'k', f(x) = $\begin{cases}\dfrac{\cos 3x+3\cos x}{\left(\frac{\pi}{2}-x\right)^3}, & x\neq\frac{\pi}{2}\\ k, & x=\frac{\pi}{2}\end{cases}$ is continuous at x = $\frac{\pi}{2}$

   i) 4 ii)8 iii)2 iv) 16

2. If $f(x)=\sqrt{x^2+1}; g(x)=\frac{x+1}{x^2+1}$ & $h(x)=2x-3$ then $f'[h'\{g'(x)\}]$ is:

   i)$\frac{2}{\sqrt{5}}$ ii)2 iii)$\frac{1}{2}$ iv)$\sqrt{5}$

3. Derivative of x^2 w.r.t x^3

$i)\ \frac{5}{3x}$ ii) $\frac{7x}{3}$ iii)$\frac{2}{3x}$ iv)0

4. If x= a $sec^3\theta$ and y=a $tan^3\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$ is

   i)$\frac{2}{\sqrt{3}}$ ii) $\frac{1}{\sqrt{3}}$ iii)$\sqrt{3}$ iv)$\frac{\sqrt{3}}{2}$

5. If y = log tan($\frac{\pi}{4} + \frac{x}{2}$) . then $\frac{dy}{dx}$ is

   i) secx ii) secx/2 iii) sinx/2 iv) sinx/2cosx/2

6. If y = $\sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots to\infty}}}$,then $\frac{dy}{dx}$ is

   i)$\frac{2y}{cosx}$ ii)$\frac{cosx}{2y-1}$ iiii)$\frac{2y-1}{cosx}$ iv)$\frac{2y-1}{sinx}$

7. The function f(x)=$\frac{4-x^2}{4x-x^3}$ is

   i)discontinuous at only one point. ii)discontinuous at exactly two points.

   iii) discontinuous at exactly three points iv) None of these.

8. Differential coefficient of sec $(\tan^{-1} x)\ wrt\ x\ is$

   i)$\frac{x}{\sqrt{1+x^2}}$ ii)$\frac{1}{\sqrt{1+x^2}}$ iii)$\sqrt{1+x^2}$ iv)$\frac{x^2}{\sqrt{1+x^2}}$

9.The function f(x)=$|x| + |x-1|$ is

   i)continuous at x=0 and x=1 ii) continuous at x=1 but not at x=0

   iii) discontinuous at x=0 and x=1 iv) continuous at x=0 but not at x=1.

10.The value of c in mean value theorem for the function f(x)=x(x-2), x$\in [1,2] is$

   i)3/2 ii)2/3 iii)1/2 iv)7/4

II. The following question consists of two statement-Assertion(A) and Reason (R). Answer the questions selecting the appropriate option given below:

(e) Both A and R are true, and R is the correct explanation for A
(f) Both A and R are true, and R is not the correct explanation for A
(g) A is true but R is false
(h) A is false but R is true.

1. Assertion(A): $|sinx|$ is a continuous function.
   Reason(R) : If f(x) and g(x) both are continuous function, the gof(x) is also a continuous function.

2. Assertion: If f(x)=$\sin^{-1} x + \cos^{-1} x$ +2 then f'(1) =0

Reason: $\frac{d(sinx)}{dx}$ = cosx.

3. Assertion: If y=$\sin^{-1}\frac{2x}{1+x^2}$, then$\frac{dy}{dx}$ =$\frac{2}{1+x^2}$

Reason: $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1} x$ and $\frac{d\ (\tan^{-1} x)}{dx}$ =$\frac{1}{1+x^2}$

4. Assertion: If y=$(sinx + cosx)^2$, then $\frac{dy}{dx}\ at\ x = \frac{\pi}{4}$ =0

Reason: $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

III. Fill in the blanks:

1.The derivative of $\log_5 x$ w.r.t x is ……………………

2.The value of c in Rolle's Theorem for the f(x)=$e^x sinx, x\epsilon[0, \pi]$is………

3.If f(x)= $\frac{\sin\ (e^{x-2}-1)}{\log(x-1),}$ x$\neq$ 2 and f(x)=k for x=2,,then value of k for which f is continuous is ………

4.If y= log(tanx) +log(cotx), then $\frac{dy}{dx} is$ … … … … … … ..

5.$\frac{d}{dx}(x^{x})$=⋯………………………

ASSIGNMENT

1. Write the point where the following function is not continuous $f(x) = \dfrac{1}{x-5}$

2. Find the value of 'b' for which $f(x) = \begin{cases} 5x - 4, 0 \langle x \le 1 \\ 4x^2 + 3bx, 1 \langle x \langle 2 \end{cases}$ is continuous at x = 1

3. Differentiate $y = e^{\sin x}$

4.If y = sin[√cos√x] , find dy/dx

5.If $e^y(1 + x) = 1$, show that $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$

6 Differentiate log (x + $\sqrt{(x^2 + a^2)}$) w.r.t x

7 If $y = x^x$, find $\dfrac{dy}{dx}$

8 Differentiate logx3 w.r.t x

9 If $y = \sqrt{(x^2 + a^2)}$ then prove that $y\dfrac{dy}{dx} - x = 0$

10 Differentiate w.r.t x: $\sin^{-1}\sqrt{(x - 1)}$

11 Locate the points of discontinuity for $f(x) = \dfrac{5x+2}{x^2 - 3x - 4}$

TWO-MARKS

1. Differentiate w.r.t x : $\log_7 x + 7^x$

2.Discuss the applicability of Rolle's theorem for $f(x) = 3 + (2 - x)^{2/3}$ in [0,3]

3. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$ show that $\dfrac{dy}{dx}$ = tan t

4. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$ where $-1 < x < 1$, find $\frac{dy}{dx}$

5. Verify Lagrange's mean value theorem for the following function:

$f(x) = x^2 + 2x + 3$, for $[4, 6]$.

6. The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ \frac{k}{2}, x = 0 \end{cases}$ is continuous at x = 0. find k

4 Marks

1. Examine the continuity of the following functions:

(a) $f(x) = \begin{cases} \frac{x^4 - 16}{x^3 - 8}, x \neq 2 \\ \frac{8}{5}, x = 2 \end{cases}$ at $x = 2$

(b) If the function $f(x) = \begin{cases} 3ax + b, x > 1 \\ 11, x = 1 \\ 5ax - 2b, x < 1 \end{cases}$

is continuous at x = 1, find the values of a & b.

2. Discuss the continuity of each of the following at the indicated points:

(i) $f(x) = \begin{cases} x^2 - x - 6, x \neq 3 \\ 5, x = 3 \end{cases}$ at $x = 3$

(ii) $f(x) = \begin{cases} \frac{\tan 7x}{\sin 4x}, x \neq 0 \\ \frac{7}{4}, x = 0 \end{cases}$ at $x = 0$

(iii) $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, x \neq 0 \\ 8, x = 0 \end{cases}$ at $x = 0$

(iv) $f(x) = 2x - |x|$ at $x = 0$

(v) f(x) = $\begin{cases} x-[x], x<2 \\ 0, x=2 \\ 3x-5, x>2 \end{cases}$ at x = 2

(vi) Let f(x) = $\begin{cases} \dfrac{\sin(a+1)x+\sin x}{x}, x<0 \\ c, x=0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}}, x>0 \end{cases}$ which may be continuous at x = 0. Find the values of a,b,c

3. Let f(x) = $\begin{cases} \dfrac{1-\cos 4x}{x^2}, x<0 \\ a, x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, x>0 \end{cases}$

   Determine the value of a, If possible, so that the function is continuous at x = 0

4. Discuss the continuity if f(x)= $\begin{cases} x, x<1 \\ 2-x, 1\le x\le 2 \\ -2+3x-x^2, x>2 \end{cases}$

5. Show that the function f(x) = $|x-3|+|x-4|$ is continuous at x=3 but not differentiable at x=3 and x=4.

6. Let f(x) = x - $|x-x^2|, x\in[-1,1]$. Find the point of discontinuity (if any), of this function in $[-1,1]$

7. Find $\dfrac{dy}{dx}$:

   (a) y = $5^{\log(\sin x)} + (\sin x)^x$

   (b) y = $\tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$

   (c) y = $x^{\tan x} + (\sin x)^{\cos x}$

(d) $y = \log\sqrt{\dfrac{1+\cos^2 x}{1-e^{2x}}}$

(e) $y = \tan^{-1}\left\{\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$

(f) $y = \sin^{-1}\left\{\dfrac{5x+12\sqrt{1-x^2}}{13}\right\}$

(g) $y = (\log x)^x + x^{\log x}$

(h) $y = \tan^{-1}\left\{\dfrac{x^{1/3}+a^{1/3}}{1-x^{1/3}a^{1/3}}\right\}$

7. If $x^y = e^{x-y}$, prove that $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$

8. $y = e^{\theta}(2\sin\theta + \sin 2\theta)$, $x = e^{\theta}(2\cos\theta + \cos 2\theta)$, find $\dfrac{dy}{dx}$

9 . $x = \log t + \sin t$, $y = e^t + \cos t$, find $\dfrac{dy}{dx}$

10 If $x = a\left(\cos\theta + \log\tan \theta/2\right)$ and $y = a\sin\theta$ then show that $\left[\dfrac{dy}{dx}\right]_{\theta=\pi/4} = 1$

11 (a) If $\dfrac{x}{a} = \sin 2t(1+\cos 2t)$ and $\dfrac{y}{b} = \cos 2t(1-\cos 2t)$ then show that $\left[\dfrac{dx}{dy}\right]_{\theta=\pi/4} = \dfrac{b}{a}$

(b) If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + ...\infty}}}$ show that $(2y-1)\dfrac{dy}{dx} = \dfrac{1}{x}$

12. If $y = 3e^{2x} + 2e^{3x}$ then show that $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$

13. Differentiate w.r.t x , if $y = \log\left[e^x\left(\dfrac{x-2}{x+2}\right)^{3/4}\right]$

14. Differentiate w.r.t x , if $y = x^x . \text{Sin}^{-1}\sqrt{x}$

15. Differentiate w.r.t x , if $(\tan^{-1} x)^y + y^{\cot x} = 1$

16. A car driver is driving a car on a dangerous path given by

f(x) = (1- x^m) / (1-x) when $x \neq 1$, and f(x) = (m-1) when x= 1 where m $\in$ N,Find the dangerous point (point of discontinuity) on the path. Whether the driver should pass that point or not? Justify your answers.

6 Marks

1. If $(a+bx)e^{y/x} = x$ then prove that $x^3 \dfrac{d^2y}{dx^2} = \left(x\dfrac{dy}{dx} - y \right)^2$

2. If $y = \sqrt{\dfrac{1-\sin 2x}{1+\sin 2x}}$, show that $\dfrac{dy}{dx} + \sec^2\left(\pi/4 - x\right) = 0$

3. If $y = \tan^{-1} x$ then show that $\left(1+x^2\right)\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = 0$

4. $y = \left(x + \dfrac{1}{x} \right)^x + (x)^{\left(1+\frac{1}{x}\right)}$ find $\dfrac{dy}{dx}$

5. If $\tan\left(\dfrac{x^2-y^2}{x^2+y^2} \right) = a$ then prove that $\dfrac{dy}{dx} = \dfrac{y}{x}$

6. If $y = \left(x + \sqrt{x^2+1}\right)^m$ then show that $\left(x^2+1\right)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - m^2 y = 0$

7. If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$, show that $\left(x^2+1\right)\dfrac{dy}{dx} + xy + 1 = 0$

8. If $y = e^x \tan^{-1} x$ then show that $\left(1+x^2\right)\dfrac{d^2y}{dx^2} - 2\left(1-x+x^2\right)\dfrac{dy}{dx} + (1-x)^2 y = 0$

9. If $y = \left(x + \sqrt{x^2+a^2}\right)^n$ then prove that $\dfrac{dy}{dx} = \dfrac{ny}{\sqrt{x^2+a^2}}$

10. a) If $x \varepsilon \left(\dfrac{1}{\sqrt{2}}, 1 \right)$ then differentiate $\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x} \right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$

b) Differentiate $\tan^{-1}\left(\dfrac{1+ax}{1-ax} \right) w.r.t \sqrt{1+a^2x^2}$

ANSWERS:

IV. 1(i) 2(i) 3. (iii) 4(iv) 5(i) 6. (ii) 7.(iii) 8.(i) 9.(i) 10(ii)

V. 1.(a) 2(b) 3.(a) 4.(b)

VI. 1. $\frac{dy}{dx}=\frac{1}{x\log 7}$ 2.$\frac{3\pi}{4}$ 3. $k=1$ 4.0 5. $x^x(1+logx)$

IV

	1 MARK
1.	x = 5
2.	b = -1
3.	$x^{\sin x} \cdot \text{Cos}x$
4.	$\frac{-\sin\sqrt{x}\cos\sqrt{\cos\sqrt{x}}}{4\sqrt{x}\cos\sqrt{x}}$
6.	$\frac{1}{\sqrt{x^2+a^2}}$
7.	$x^x(1+\log x)$
8.	$\frac{-\log 3}{x(\log)^2}$
10.	$\frac{1}{2\sqrt{2(x-1)}}$
11.	x = 4, x = -1
	2 Marks
1.	$\frac{1}{x\log 7}+7^x\log 7$
3.	$\frac{-2}{1+x^2}$
5.	6
	4 MARKS
1.	: Not continuous
2.	a = 3, b = 2
3.	i) continuous ii) dis continuous iii)continuous iv)continuous v)dis continuous
4.	-3/2 , ½, b ≠ 0
5.	8

6.	a) cot x. $5^{\log(\sin x)}$ $\log_e 5$ + $(\sin x)^x$ {x cotx + log(sinx)} b) $\frac{1}{2(1+x^2)}$
	c) $x^{\tan x}$ (tan x/ x + $\sec^2$x.log x)+$(\sin x)^{\cos x}$ ($\cos^2$x/sin x – sinx.log(sinx)]
	d) $\frac{1}{2}\left(\frac{-\sin 2x}{1+\cos^2 x}+\frac{2e^{2x}}{1-e^{2x}}\right)$
	e) -1/2
	f) $\frac{1}{\sqrt{1-x^2}}$
	g) $(\log)^x\left\{\frac{1}{\log x}+\log(\log x)\right\}$
	h) $\frac{1}{3x^{2/3}\left(1+x^{2/3}\right)}$
8.	$\frac{2\sin\theta+\sin 2\theta+2(\cos\theta+\cos 2\theta)}{2\cos\theta+\cos 2\theta-2(\sin\theta+\sin 2\theta)}$
9.	$\frac{t(e^t-\sin t)}{1+t\cos t}$
13.	$\frac{x^2-1}{x^2-4}$
14.	$x^x\left\{\frac{1}{2\sqrt{x}\sqrt{1-x}}+\sin^{-1}(1+\log x)\right\}$
15.	$\frac{y^{\cot x}\operatorname{cosec}^2 x\log y(1+x^2)-y(\tan^{-1}x)^{y-1}}{(1+x^2)\left[(\tan^{-1}x)^y\log\tan^{-1}x+y^{\cot x-1}\cot x\right]}$
16.	Point of discontinuity at x=1 No, because Life is precious. OrDrive carefully.
	6 MARKS
4.	$\left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right]+x^{1+1/x}\left[\frac{x+1}{x^2}-\frac{\log x}{x^2}\right]$
10.	a) ½ b) $\frac{1}{ax\sqrt{1+a^2x^2}}$

APPLICATIONS OF DIFFERENTIATION

SYNOPSIS

Rate of change

- Rate of change of 'x' w.r.t. time is denoted by $\frac{dx}{dt}$
- If x and y are both function of a third variable t then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and it gives the rate of change of y w.r.t. x
- If s is the displacement, then velocity = $\frac{ds}{dt}$ and acceleration $= \frac{d^2s}{dt^2} = \frac{dv}{dt} = \frac{vdv}{ds}$

Increasing/Decreasing functions

- A function *f(x)* is said to be increasing in *(a, b)*, if for $x_1, x_2 \in (a, b)$
- $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$. The function is strictly increasing if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$
- A function is said to be decreasing in interval (a, b) if $\forall$ $x_1, x_2 \in (a, b)$,
  $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$
- A function is said to be increasing over its domain if $f(x) \ge 0$ $\forall x \in$ its domain.
- If the function is strictly increasing we have $f(x) > 0$ $\forall x \in$ domain of the function.
- A function is said to be decreasing over its domain if $f(x) \le 0$ $\forall x \in$ its domain. If the function is strictly decreasing we have $f(x) < 0$ $\forall x \in$ domain of function.
- A function 'f' is said to be neither increasing nor decreasing over D if it is increasing in D_1 and decreasing in D_2 an vice versa where $D = D_1 \cup D_2$

M.C.Q

I. Multiple Choice Questions:

1. If the rate of change in the circumference of a circle is 0.5cm/sec. Then the rate of change in the area when the circumference is 10πcm is ____

a **2.5 sq.cm/sec** **b** **5 sq.cm/sec** **c** **1.5sq.cm/sec** **d** **2 sq.cm/sec**

2. A spherical balloon is being inflated so that its volume increases at the rate of 40c.c/sec. Then tge rate of increases in the surface area when the radius is 8cm is ______

a **10** **b** **15** **c** **20** **d** **25**

3. If y=$x^3$and x increases at the rate of 3 units per sec; then the rate at which the slope increases when x=3 is _________

a **9** **b** **18** **c** **27** **d** **54**

6. The point at which the tangent of the curve y=x^2+3x passes through the origin is ______

a **(0,3)** **b** **(0,0)** **c** **(2,0)** **d** **(2,3)**

10. The maximum area of a rectangle inscribed in a circle of radius 5 cm is a

a **25 sq.cm** **b** **50 sq.cm** **c** **100 sq.cm** **d** **25/2 sq.cm**

11. The hypotenuse of a right angles triangle is k cm, if the area is maximum then the sides of the triangle are __________

12. The function x+ log($\frac{x}{1+x}$) is increasing when x belongs to the interval _____

13. Water is poured into an inverted conical vessel of depth 20cm and base radius 10cm at the rate of 4 c.c per minute. The rate of increase of water level when the depth is 5 cm is _____________

16. The point on the curve $x^2=2y$ which is nearest to the point (0,5) is ______

17 The sum of two positive numbers is equal to ‘a’ and if the sum of their cubes is least, the numbers are _____

ASSIGNMENT

1Mark

2. An edge of a variable cube is increasing at rate of 5cm/s. How fast is the volume?
   increasing when the side is 15cm.
3. If y = $\log_e x$, then find Δy when x=3 and Δx=0.03.
5. The radius of a spherical soap bubble is increasing @ of 0.2 cm/sec. Find the rate of increase of its surface area when the radius is 7cm.
6. If the Rolle’s theorem is applicable to the function f(x) = $x^4 + 3x^3 + 3x^2 + 2x$ on [k,0] what is the value of k
7. What are the minimum and maximum values of f(x) = 3sinx + 4cosx
8. Find the interval in which f(x) = log (x – 1) is increasing.

(2 MARKS)

1. Rice is being poured on the ground in such a way that the rice on the ground forms a cone of altitude equal to the radius of the base. Rice is being poured @ 20cm^3/sec. Find the rate at which the altitude of the rice cone is increasing when it is 50cm.
2. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y coordinate is changing 8 times as fast as x-coordinate

4 Marks

4. An aeroplane is ascending vertically @ 300 km/hr. If the radius of the earth is 'R', how fast is the area of the earth visible from the plane increasing at 3minutes after it starts ascending ?(visible area $A = \underline{2\pi R^2 h/(R+h)}$, where h is the height of the plane above the earth)

7) A man is moving away from a tower 80m high at a speed of 4m/sec. Find the rate at which his angle of elevation of the top of the tower is changing when he is at 60m from the foot of the tower. Assume that the height of the man is negligible.

8. Water is being pumped into a conical reservoir whose height is $3\frac{1}{3}$ m and radius of its top $1\frac{2}{3}$ m at the constant rate of $\frac{2}{3}$ m^3/ min. How fast is the water level rising when it is 2m deep?

13. Find the intervals in which the following functions (I) increasing (II) decreasing:

(a) $f(x) = x^3 - 12x^2 + 36x + 17$

(b) $f(x) = \sin x - \cos x, 0 \leq x \leq 2\pi$

14. A car parking company has 500 subscribers and collects fixed charges of Rs.300 per subscriber per month. The company proposes to increase the monthly subscription and it is believed that for every increase of Re.1, one subscriber will discontinue the service. What increase will bring maximum income of the company?

15. A farmer wants to construct a circular well and a square garden in his field. He wants to keep sum of their perimeters fixed. Then prove that the sum of their areas is least when the side of square garden is double the radius of the circular well.

16. Prove that the surface area of a solid cuboid, of square base and given volume, is minimum when it is a cube.

<u>(5marks)</u>

1. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

2. A given quantity of metal is to be cast into half cylinder with a rectangular base and semi-circular ends. Show that surface area is minimum when the ratio of length of cylinder to the diameter of its semi-circular ends is $\pi : \pi + 2$

3 A right circular cylinder is inscribed in a right circular cone. Show that the curved surface area of the cylinder is maximum when the diameter of cylinder is equal to the radius of the base of the cone.

4 Find the equation of the line through the point (3,4) which cuts from the first quadrant a triangle of minimum area.

5 An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $c^3/6\sqrt{3}$ cubic units.

6 The sum of the areas of a sphere and a cube is given. Show that when sum of their volumes is least, the diameter of sphere is equal to the edge of the cube.

7 A quadrant of a circle of radius 'a' is bounded by radii OA,OB and the circular arc AB. A triangle is inscribed in this quadrant with one vertex fixed at the midpoint of the arc and two variable vertices are on each of OA and OB equidistant from O. Find the maximum possible area of the triangle.

Answers of MCQ

(1)2.5 sq.cm (2) 10 (3) 54 (4) (2,1) (5) 4 (6) (0,0) (7) 9/2 (8) 0.72π (9)$\frac{\mathbf{0.1}}{\mathbf{2\sqrt{3}}}$

(10) 50 sq.cm (11) $\frac{k}{\sqrt{2}}, \frac{k}{\sqrt{2}}$ (12) $(-\infty.-1) \cup (0,\infty)$ (13) $\frac{16}{2\pi}$ (14) 0.4983

(15). a/2,a/2 (16) $(2\sqrt{2},4)$ (17) a/2,a/2

SCORING KEY

	1MARK
1.	(1,2)
2.	$3375cm^3$
3.	0.01
4.	$\frac{3}{2}, \frac{-17}{2}$
5.	$35.2\ cm^2/s$

6.	k = -2
7.	Min= -5 , max= 5
8.	$(1, \infty)$
	2 MARKS
1.	0.0025cm/sec
2.	(4,11) & $(-4, \frac{-31}{3}))$
3.	i)6572.664 ii)3.0092 iii)6.0833 iv)0.6929
4.	-0.32
	4 Marks
6.	$\frac{600\pi R^3}{(R+15)^2}$)
7.	0.032 rad/sec
8.	$7/33$ m/min
9.	48x – 24y – 23 = 0
10.	$\left(\frac{3}{2\sqrt{10}}, \frac{-1}{3\sqrt{10}}\right) \& \left(\frac{-3}{2\sqrt{10}}, \frac{1}{3\sqrt{10}}\right)$
12.	$4\pi\ cm^2$
13.	a) $(-\infty,2) \cup (6,\infty)$; (2,6) b) $(0, 3\pi/4) \cup (7\pi/4, 2\pi)$; $(3\pi/4, 7\pi/4)$
14.	Increase of Rs.100 monthly subscription for Max. Income of the company.