8.BINOMIAL THEOREM
SYNOPSIS

- The expansion of a binomial or any positive integral n is given by Binomial Theorem, which is $(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots\ldots\ldots\ldots {}^nC_{n-1} ab^{n-1} + {}^nC_n b^n$
- The coefficients of the expansions are arranged in an array.This array is called *Pascal's Triangle.*
- The general term of an expansion $(a+b)^n$ is $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$
- In the expansion $(a+b)^n$, if n is even, then the middle term is the $\left(\frac{n}{2}+1\right)^{th}$ *term.* If n is odd, then the middle terms are $\left(\frac{n+1}{2}\right)^{th}$ *and* $\left(\frac{n+1}{2}+1\right)^{th}$ *terms.*
- If n is positive odd integer then
  $(a+b)^n + (a-b)^n$ and $(a+b)^n - (a-b)^n$ both have same number of terms equal to $\frac{n+1}{2}$
- If n is positive even integer , then
  (i) $(a+b)^n + (a-b)^n$ has $(\frac{n}{2}+1)$ terms and
  (ii) $(a+b)^n - (a-b)^n$ has $\frac{n}{2}$ terms.

8.BINOMIAL THEOREM
MCQ

1. The total number of terms in the expansion of $(x+a)^{51} - (x-a)^{51}$ after simplification is
   a) 102 b) 22 c) 25 d) 24

2) In the binomial expansion of $(a+b)^n$, the coefficient of the 4^{th} and 13^{th} terms are equal, then n is equal to
   a) 10 b) 12 c) 15 d) 16

3) If the coefficients of $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal, find r.
   a) 14 b) 15 c) 16 d) 17

4) In the expansion of $(x - \frac{1}{3x^2})^9$, the term independent of x is

a) T_3 b) T_4 c) T_5 d) T_6

5) The middle term in the expansion of $(x^2 + \frac{1}{x^2})^{2n}$ is

a) ${}^{2n}C_n x^{2n}$ b) ${}^{2n}C_n$ c) ${}^{2n}C_n \frac{1}{x^{2n}}$ d) $\frac{(2n)!}{n!}$

6) The constant term in the expansion of $(x^{\frac{1}{6}} - x^{\frac{-1}{3}})^9$ is

a) 84 b) -0.84 c) 8.4 d) -84

7) If T_2/T_3 in the expansion of $(a + b)^n$ and T_3/T_4 in the expansion of $(a+b)^{n+3}$ are equal , then n =---

a) 3 b) 4 c) 5 d) 6

8) If in the binomial expansion of $(1 + x)^n$, the coefficients of 5^{th} , 6^{th} and 7^{th} terms in A.P., then n is equal to

a) 7,11 b) 7, 14 c) 9,16 d) none of these

9) If x^{-17} occurs in r^{th} term in the expansion of $(x^4 - \frac{1}{x^3})^{15}$, then r =

a) 10 b) 11 c) 12 d) 13

10) The total number of terms in the expansion $(x+ a)^{100} + (x-a)^{100}$

a) 50 b) 202 c) 51 d) None of these

Fill In The Blanks

1) The number of terms in the expansion of $(x+ a)^{51} - (x-a)^{51}$ after simplification is --------

2) The number of terms in the expansion of $(1+ 2x+ x^2)$ is ------

3) In expansion of $(2x^2 - \frac{1}{3x})^{9,}$ the coefficient of x is ----------

4) the fourth term from the end in the expansion of $(\frac{3}{x^2} - \frac{x^3}{3})^9$ ----------

5) If in the expansion of $(1+ x)^{20}$, the coefficient of r th and (r+ 4) th terms are equal, then value of r is ----------

VSA (1 MARK)

1. Write the general term in the expansion of

(a) $(x^2 - y)^6$

(b) $\left(\frac{1}{2}x^{\frac{1}{3}} + x^{\frac{-1}{5}}\right)^8$

2. Find the 7th term in the expansion of $\left(3x^2 - \frac{1}{x^3}\right)^{10}$

3. Find the 5th term from the end in the expansion of $\left(3x - \frac{1}{x^2}\right)^{10}$

4. Find the 8th term in the expansion of $\left(x^{\frac{3}{2}}y^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{3}{2}}\right)^{}$

5. Find the 4th term from the end in the expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

6. Find the 7th term from the end in the expansion of $\left(2x^2 - \frac{3}{2x}\right)^8$

7. Find the coefficient of :

a).x^{10} in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$

b)x^7 in the expansion of $\left(x - \frac{1}{x^2}\right)^{40}$

c)x^m in the expansion of $\left(x + \frac{1}{x}\right)^n$

8. If the coefficients of (2r +4)th and (r – 2)th terms in the expansion of $(1 + x)^{18}$ are equal, find r.

9. If the coefficients of (2r + 1)th term and (r + 2)th term in the expansion of $(1+x)^{43}$ are equal , find r.

10. 10. Find the middle term of the expansion $\left(\frac{2x^2}{3} + 3\right)^{10}$

11. Find the middle term of the expansion $\left(x - \frac{1}{x}\right)^{10}$

12. If in the expansion of $(1 + x)^n$, the coefficients of pth and qth terms are equal , prove tha $p + q = n + 2$, where $p \neq q$.

SECTION B (4MARKS)

13. Find the term independent of x in the expansion of :

(a) $\left(2x+\frac{1}{3x^2}\right)^9$

(b) $\left(3x-\frac{2}{x^2}\right)^{15}$

(c) $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$

14.Expand using binomial theorem:

(a)$\left(x+\frac{1}{y}\right)^{11}$

(b)$\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6$

15.Evaluate the following:

(a) $(1+2\sqrt{x})^5 + (1 - 2\sqrt{x})^5$

(b) $(\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$

SECTION C (6 MARKS)

16. If the fourth term in the expansion of $(ax + 1/x)^n$ is 5/2, then find the values of a and n.

17. In the binomial expansion of $(1 + x)^n$, the coefficients of the fifth, sixth and seventh terms in A.P. Find all values of n for which this can happen.

18. If the coefficients of 2^{nd} ,3^{rd} and 4^{th} terms in the expansion of $(1 + x)^{2n}$ are in A.P, show that $2n^2 - 9n + 7 = 0$

19. If the 2^{nd} , 3^{rd} and 4^{th} terms in the expansion of $(x+ a)^n$ are 240,720 and 1080 respectively, find x, a, n.

20. Find the coefficient of x^7 in $(ax^2 + 1/bx)^{11}$ and of x^{-7} in $(ax - 1/bx^2)^{11}$ and find the relation between a and b so that these coefficients are equal .

21. If n is a positive integer, prove that $3^{3n} - 26n - 1$ is divisible by 676.

22. If P is the sum of odd terms and Q that of even terms in the expansion $(x + a)^n$, prove that

(i) $P^2 - Q^2 = (x^2 - a^2)^n$

(ii) $4PQ = (x + a)^{2n} - (x - a)^{2n}$

(iii) $2(P^2 + Q^2) = (x + a)^{2n} + (x - a)^{2n}$

SCORING KEY

MCQ

1) c) 26	2) c) 15	3) a) 14	4) b)T_4	5) b) $^{2n}C_n$
6) a) 84	7) c) 5	8) b) 7, 14	9) c) 12	10) c) 51

Fill In The Blanks

1) 26 2) 23 3) 0 4) $\frac{28}{9}x^{12}$ 5) 9

1.	(a) $(-1)^{r6}C_r x^{n-r} y^r$	(b) $^8C_r(½)^{8-r}(x^{1/15})^{40-5x}$	15(a)	$2(1+40x+40x^2)]$
2.	$\frac{17010}{x^{10}}$		15(b)	152
3.	$\frac{17010}{x^8}$		16.	a= ½ , n= 6
4.	$-120x^8y^{12}$		17.	7, 14
5.	$\frac{10500}{x^3}$		19.	n = 5,x = 2,a = 3
6.	4032 x^{10}		20.	$^{11}C_5\frac{a^6}{b^5}, ^{11}C_6\frac{a^5}{b^6}, ab=1$
7.	$^{20}C_{10}2^{10}$, $^{40}C_{11}$, $\frac{n!}{\left(\frac{n-m}{2}\right)!\left(\frac{n+m}{2}\right)!}$			
8.	6			
9.	14			
10.	8064 x^{10}]			
11.	-252			
13.	$(a)\frac{64}{27}{}^9C_3, (b)-3003.3^{10}.2^5 (c)\frac{5}{4}$			
14(a)	(a) 			
14(b)	(b)