CHAPTER 12

AREAS RELATED TO CIRCLES

SYNOPSIS

- Perimeter (circumference) of a circle with diameter d ($d = 2r$, where r is the radius) is given by $\mathrm{C} = \pi d = 2\pi r$

- Perimeter of semicircle with radius $r = 2r + \pi r = r(\pi + 2)$

- Area of a circle with radius r is given by $\mathrm{A} = \pi r^2$.

- Area of a semicircle of radius $r = \dfrac{\pi r^2}{2}$

- Area of a ring whose outer and inner radii are R and r respectively
  $= \pi(\mathrm{R}^2 - r^2) = \pi(\mathrm{R} + r)(\mathrm{R} - r)$

- If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

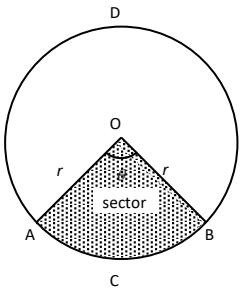
- If two circles touch externally, then the distance between their centres is equal the sum of their radii.

- The distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

- The number of revolutions completed by a rotating wheel in one minute:
  $= \dfrac{\text{Distance moved in one minute}}{\text{Circumference of the wheel}}$

- Length of an arc which subtends an angle of $\theta°$ at the center $= \dfrac{2\pi r\theta^\circ}{360^\circ} = \dfrac{\pi r\theta^\circ}{180^\circ}$

- Sector of a circle is a region enclosed by an arc of a circle and its two bounding radii.



  (i) Area of sector $\text{OACBO} = \frac{\pi r^2 \theta^\circ}{360^\circ}$.

  (ii) Perimeter of sector $\text{OACBO} = 2r + \frac{2\pi r \theta^\circ}{360^\circ}$.

- Minor sector: A sector of a circle is called a minor sector if the minor arc of the circle is a part of its boundary. In the figure above minor sector is OACB.

- Major sector: A sector of a circle is called a major sector, if the major arc of the circle is a part of its boundary. In the above figure, OADB is the major sector.

- The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.

- The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.

- The area of a sector is given by $\text{A} = \frac{1}{2} lr$, where $l = \left(\frac{\theta r}{180^\circ} \times \pi \right)$

- Angle described by minute hand in 60 minutes =360°.

  $\therefore$ angle described by minute hand in one minute $= \left(\frac{360}{60} \right)^\circ = 6^\circ$.

  Thus, the minute hand rotates through an angle of 6° in one minute.

- Angle described by hour hand in 12 hours = 360°.

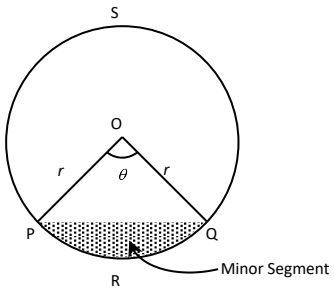
  $\therefore$ angle described by hour hand in 1 hour $= \frac{360^\circ}{12} = 30^\circ$

  Angle described by hour hand in one minute $= \frac{30^\circ}{60} = \frac{1}{2}^\circ$

  Thus, hour hand rotates through $\frac{1}{2}^\circ$ in 1 minute.

- A segment of a circle is the region bounded by an arc and a chord, including the arc and the chord.

- Minor segment: If the boundary of a segment is a minor arc of a circle, then the corresponding segment is called a minor segment. In the figure, segment PQR (the area which is shaded) is a minor segment.



- Major segment: A segment corresponding a major arc of a circle is known as the major segment. In the figure, segment PQSP is a major segment.

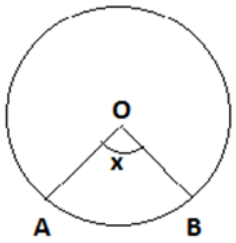
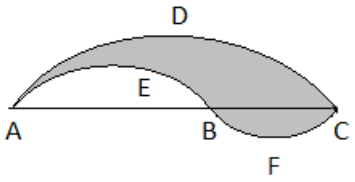
- Area of minor segment PRQS $= \frac{\pi r^2 \theta^\circ}{360} - \frac{1}{2} r^2 \sin\theta$

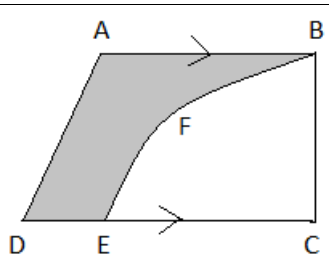
- Area of major segment PQSP $= \pi r^2 -$ area of minor segment PRQS.

A.		MCQ (1 Mark)	Level
1.		The ratio of the outer and inner perimeter of a circular path is 23:22.If the path is 5m wide, the diameter of the inner circle is a) 55m b)110m c) 220m d) 230	MD
2.		If the circumference of a circle increases from 4 π to 8π, then area is a) halved b) doubled c) tripled d) quadrupled	U
3.		If the radius of the circle is diminished by 10 % then the area is diminished by a) 10 % b) 19 % c) 20 % d) 36%	MD
4.		If the perimeter of a semicircular protractor is 108 cm, find the diameter of the protractor(π = 22 / 7) a) 42 cm b) 24 cm c) 44 cm d) 40 cm	U
5.		A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, then its area will be a) 3520 cm^2 b) 6400 cm^2 c) 7744 cm^2 d) 8800 cm^2	U

6.		The cost of fencing the circular field at the rate of Rs 24 per meter is Rs 5280. Find the circumference of the circular field a) 202 b) 251 c)220 d) 280	HOT
7		If the circumference of two circles are in the ratio 2:3, what is the ratio of their Areas a) 2 :3 b) 4: 9 c) 16: 9 d) 3;2	U
8.		What is the length (in terms of π) of the arc that subtends an angle of 36 º at the center of a circle of radius 5 cm a) π b) 2π c) $3\ \pi$ d) 4π	C
9.		If the circumference of a circle exceeds the diameter by 16.8 cm Find the radius of the circle a) 4.32 b) 3.92 c) 9. 32 d) 2.93	U
10.		The difference between the circumference and the radius of a circle is 37cm.The area of the circle is a) 149 cm ^{2} b) 154 cm^2 c) 121 cm ^{2} d) 169 cm^2	HOT
11.		The area of the largest circle that can be drawn inside a square of side 14 cm in length is a) 121 cm ^{2} b) 154 cm^2 c) 169 cm ^{2} d) 196 cm^2	C
12.		A circular wire of radius 42 cm is cut and bent into the form of a rectangle whose sides are in the ratio of 6: 5. The smaller side of the rectangle is a) 30 cm b) 60 cm c) 70 cm d) 80 cm	HOT
13.		Hour hand rotates through in 1 minute a) $\frac{1}{2}$ º b) 2 º c) 22 º d) $\frac{22}{7}$ º	U
14.		The racetrack is in the form of a ring whose inner and outer circumference are 437 m and 503 m respectively. The width of the track is	U

		a) 10.5 m b) 20.5 m c) 21 m d) 30 m	
		FILL IN THE BLANKS (1 MARK)	
1.		The portion of the circular region enclosed by two radii and the corresponding arc is called a ---------	C
2.		Perimeter of a quadrant of a circle of radius r is equal to …….	C
3.		A region in the circle, bounded by an arc and a chord, including the arc and the chord is called ……..	C
4.		Area of a quadrant of a circle whose circumference is 44cm is ……..	U
5.		A wheel makes 1000 revolutions in covering a distance of 0.88 Km.The radius of the wheel is ……..	HOT
		Very Short Answer Questions (VSA) (1 Mark)	
1		The area of a circle is 394.24cm2. Then find the radius of the circle.	U
2		If the perimeter of a circle is equal to that of a square, then find the ratio of their area .	HOT
3		The circumference of a circle exceeds its diameter by 180cm. Then find its radius	U
4		Find the area of incircle of an equilateral triangle of side 42cm.	U
5		Find angle of the sector of radius 5cm with corresponding chord as $5\sqrt{3}$cm	MD
6		The radius of the wheel is 0. 25m. Find the number of revolutions it will make to travel a distance of 11 km.	U
B.		Short Answer Questions (SA) (2 marks)	
7.		What is the perimeter of the sector of radius 10.5 cm and angle is 60 º	C
8		The radius of a radius of a circle is 20cm Three more concentric circles are drawn inside it in such a manner that it is divided into four parts of equal area. Find the radius of the largest of the three concentric circle	HOT

9.		If O is the centre of a circle. The area of sector OAB is 5/ 18 of area of circle. Find x 	U
10.		The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 am and 9.35 am	U
C.		Long Answer Questions (LA) (3 Marks)	
11.		The circumference of a circle A is 132 cm. It is equal to sum of circumferences of 2 circles B and C. The radius of circle B is 14 cm. Find the radius of the circle C .	C
12.		PQ = 24 cm , PR = 7 cm and O is the centre of the circle. Find the area of the shaded region (take $\pi = 3.14$) if $\angle$ QPR = 90 º .	U
13.		The three cows tethered to the corners of a triangular plot whose sides are 40 m , 50 m and 60 m with ropes of lengths 7 m each. Find the area of grass grazed by 3 cows together.	HOT
14		A square park has each side of 100 m .At each corner of the park there is a flower bed in the form of a quadrant of radius 14 m . Find the area of the remaining part of the park (π = 22/	MD
D.		V Long Answer Questions (VLA) (4 Marks)	
15		In the figure drawn, find the perimeter of shaded region where ADC and AEB and BFC are semi-circles on diameter AC and AB and BC respectively. If AB = 2.8 cm BC = 1.4 cm 	U
16.		From a thin metallic piece in the shape of trapezium ABCD in which AB is parallel to CD and $\angle$ BCD = 90 º , a quarter circle BFEC removed .	U

			Given AB = BC = 3.5 cm and DE = 2 cm. Calculate the area of the remaining piece of metal sheet.	

AREAS RELATED TO CIRCLES

ANSWERS

SECTION A

ANSWERS of MCQ

1) 220 m 2) Quadruplet 3) 19 % 4) 42 cm 5) 7744
6) 220 7) 4: 9 8) π 9) 3.92 10) 154 cm^2 11) 154 cm^2

12) 60 cm 13)1/2 14) 10.5

Fill in the blanks

1) sector 2) $2r + \pi r/2$ 3) segment 4) 38.5 cm^2 5) 14 cm

VSA

1. 11.2 cm.
2. 14:11
3. 42 cm
4. 462cm^2
5. 120^o
6. 7000

SECTION B

7. 32
8. $10\sqrt{3}$cm
9. 100 ^{o}
10. 183.3cm^2

SECTION C

11. 7cm
12. 161.3 cm^2
13. 77
14. 9384 cm^2

SECTION D

15. 13.2 cm
16. 6.125 cm^2