11.CONICS – SYNOPSIS

Conic sections or conics are curves that are obtained by intersecting a plane with a double napped right circular cone. Circles, parabolas, hyperbolas and ellipses are examples of conics. In this Chapter the following concepts and generalizations are studied.

CIRCLE:

- A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.
- The equation of a circle with center (h, k) and the radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

PARABOLA:

- A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane.
- The equation of the parabola with focus at $(a, 0)$ $a > 0$ and directrix $x = -a$ is: $\mathbf{y^2 = 4ax}$
- Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola.
- Length of the latus rectum of the parabola $y^2 = 4ax$ is $\mathbf{4a}$.

ELLIPSE:

- An *ellipse* is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.
- The equations of an ellipse with foci on the x-axis is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

- Latus rectum of an ellipse is a line segment perpendicular to the major axis through any of the foci and whose end points lie on the ellipse.
- Length of the latus rectum of the ellipse is: $\frac{\mathbf{2b^2}}{\boldsymbol{a}}$
- The eccentricity of an ellipse is the ratio between the distances from the center of the ellipse to one of the foci and to one of the vertices of the ellipse.

HYPERBOLA:

- A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.
- The equation of a hyperbola with foci on the x-axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

- Latus rectum of hyperbola is a line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola.
- Length of the latus rectum of the hyperbola: $\frac{\mathbf{2b^2}}{\boldsymbol{a}}$
- The eccentricity of a hyperbola is the ratio of the distances from the center of the hyperbola to one of the foci and to one of the vertices of the hyperbola.

CONIC SECTIONS

SECTION A (1 MARK)

MCQ

1. The eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}$ = 1 is

   1) 0.4 2) 0.5 3) 0.6 4) 0.75

2. The foci of the ellipse $4x^2 + 3y^2 = 24$ are the points

   1) (±2, 0) 2) $(0 \pm 2\sqrt{2})$ 3) $(0, \pm\sqrt{2})$ 4) $(\pm 2\sqrt{2}, 0)$

3. The distance between the foci of the ellipse $5x^2 + 9y^2 = 45$ is

   1) 2 2) $2\sqrt{2}$ 3) 4 4) $4\sqrt{2}$

4. Latus rectum of an ellipse is equal to half of its minor axis. Then its eccentricity is

   1) $\frac{1}{\sqrt{2}}$ 2) $\sqrt{\frac{3}{2}}$ 3) $\frac{\sqrt{3}}{2}$ 4) $\frac{1}{2}$

5. If the latus rectum of the ellipse is half of its major axis, then its eccentricity is

   1) 1/2 2) $\frac{1}{\sqrt{2}}$ 3) 1/4 4) 1/3

6. Eccentricity of the hyperbola $9x^2 - 16y^2 = 144$ is

   1) 5/4 2) 4/5 3) 3/5 4) 4/3

7. The foci of the hyperbola $9x^2 – 4y^2 = 36$ are the points

   1) (± 3, 0) 2) (0, ±3) 3) $(\pm\sqrt{13}, 0)$ 4) $(0, \pm\sqrt{13})$

8. The length of the conjugate axis of the hyperbola $49x^2 - 4y^2 = 196$ is

   1) 4 2) 7 3) 14 4) 2

FILL IN THE BLANKS

9. The length of the latus rectum of the hyperbola $49x^2 - 16y^2 = 784$ is

   1) 49/16 2) 784/49 3) 49/4 4) 49/2

10. The sum of the squares of the eccentricities of the conics $\frac{x^2}{4}+\frac{y^2}{3}=1$ and $\frac{x^2}{4}-\frac{y^2}{3}=1$ is

    1) 2 2) $\sqrt{\frac{7}{3}}$ 3) $\sqrt{7}$ 4) 4

11. If the distance between foci of a hyperbola is 36 and transverse axis is 9, then its eccentricity is

    1) 2 2) 4 3) 3 4) 6

12. Give the two ends of the latus rectum, the maximum number of parabolas that can be drawn is

    1) 1 2) 2 3) 0 4) infinite

VSA

1. If parabola $y^2 = 4ax$ passes through (3, 4). Find the value of a.
2. Find the focus of the parabola $x^2 = -16y$.
3. Find the eccentricity of the ellipse $3x^2 + 2y^2 = 18$.
4. Find the latus rectum of the hyperbola $\frac{y^2}{25} - \frac{x^2}{16} = 1$.
5. Find the equation of the circle concentric with $x^2 + y^2 - 4x - 4y - 8 = 0$ and radius 6cm.
6. Determine the equation of the circle if (3, 2) and (-1, 6) are the end points of the diameter of the circle.

SECTION B (4 MARKS)

7. Find the eccentricity of the ellipse if its latus rectum is equal to one half of its minor axis.
8. Find the equation of the ellipse whose foci are $(\pm 3, 0)$ and eccentricity is $1/3$.
9. If major axis and eccentricity of the ellipse are 8 and $\frac{1}{2}$ are respectively, find the equation of the ellipse in standard form.
10. Find the equation of the hyperbola with vertices at $(\pm 4, 0)$ and foci at $(\pm 6, 0)$.
11. Find the equation of the hyperbola having distance between foci is 16 with eccentricity $\sqrt{2}$.
12. Find the equation of the circle which passes through the point (-2, -3) and has its center on negative side of x-axis and is of radius 5 units.
13. Find the equation of the circle whose center is a point (1, -2) and which passes through the center of $2x^2 + 2y^2 + 4y = 5$.
14. Find the equation of the circle concentric with the circle $3x^2 + 3y^2 - 12x - 18y + 5 = 0$ and which touches y-axis.
15. Show that the line $x + y - 5 = 0$ touches the circle $x^2 + y^2 - 2x - 4y + 3 = 0$. Find the point of contact.
16. Find the equation which passes through the center of the circle $x^2 + y^2 + 8x + 10y - 7 = 0$ and concentric with the circles $2x^2 + 2y^2 - 8x - 12y - 9 = 0$.

SECTION C (6 MARKS)

17. Find the equation of the circle passing thorough (1, 0) (-1, 0) and (0, 1).

18. Find the equation of the circle passing thorough (3, -2) and (-2, 0) and having its center on $2x - y - 3 = 0$.
19. Find the equation of the circle which passes through two points on the x-axis which is at distance 4 units from the origin and whose radius is 5.
20. Find the equation of the hyperbola whose foci is $(0, \pm\sqrt{10})$ and passing through (2, 3).
21. The foci of a hyperbola coincide with foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Find the equation of the hyperbola if its eccentricity is 2.

Scoring key

MCQ

1. 0.6
2. $(0, \pm\sqrt{2})$
3. 4
4. $\sqrt{3}/2$
5. $1/\sqrt{2}$
6. 5/4
7. $(\pm\sqrt{13}, 0)$
8. 14

FILL IN THE BLANKS

1. 49/2
2. 2
3. 4
4. 2

VSA 1.	$4/3$
2.	(0, -4)
3.	$\frac{1}{\sqrt{3}}$
4.	$32/5$
5.	$x^2 + y^2 - 4x - 4y - 28 = 0$
6.	$x^2 + y^2 - 2x - 8y - +9 = 0$
7.	$\sqrt{3}/2$
8.	$\frac{x^2}{81} + \frac{y^2}{72} = 1$
9.	$\frac{x^2}{16} + \frac{y^2}{12} = 1$
10.	$5x^2 - 4y^2 = 80$
11.	$x^2 - y^2 = 32$
12.	$x^2 + y^2 + 12x + 11 = 0$
13.	$x^2 + y^2 - 2x - 4y + 3 = 0$
14.	$x^2 + y^2 - 4x - 6y + 9 = 0$
15.	(2, 3)
16.	$x^2 + y^2 - 4x - 6y - 87 = 0$
17.	$x^2 + y^2 = 1$
18	$x^2 + y^2 + 3x + 12y + 2 = 0$
19.	$x^2 + y^2 - 6y - 16$, $x^2 + y^2 + 6y - 16$
20	$y^2 - x^2 = 5$
21.	$3x^2 - y^2 = 12$