CHAPTER 5
ARITHMETIC PROGRESSIONS

What is A.P

Some numbers arranged in a definite order, according to a definite rule, are said to form a sequence. A sequence is called an arithmetic progression (A.P.), if the difference of any of its terms and the preceding term is always the same.
i.e., $t_{n+1} - t_n$ = constant. The constant number is called the common difference of the A.P.

If a is the first term and d the common difference of an AP, then the general form of the AP is $a, a + d, a + 2d, ...$

Nth term of an A.P

Let a be the first term and d be the common difference of an AP. Then, its n^{th} term or general term is given by
$t_n = a + (n-1)d$. If l is the last term of the AP, then n^{th} term from the end is the n^{th} term of an AP, whose first term is l and common difference is $-d$.

$\therefore n^{th}$ term from the end = Last term + $(n - 1)(-d)$

$\Rightarrow n^{th}$ term from the end = $l - (n - 1)$ d

REMEMBER

If $a, b, c,$ are in AP, then

1) $(a + k), (b + k), (c + k)$ are in AP

2) $(a - k), (b - k), (c - k)$ are in AP

3) $ak, bk, ck,$ are in AP

4) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP. ($k \neq 0$)

Remember the following while working with consecutive terms in AP.

1) Three consecutive terms in an A.P. $a - d, a, a + d$
   First term = $a - d$, common difference = d.

   Their sum = $a- d + a + a + d = 3a$

2) Four consecutive terms in an AP. $a - 3d, a - d, a + d, a + 3d$
   First term: $a - 3d$, common difference = $2d$
   Their sum = $a- 3d + a - d + a + d+ a + 3d = 4a$
3) Five consecutive terms in an A. P. $a - 2d, a - d, a, a + d, a + 2d$.
   First term = $a - 2d$, common difference = d

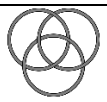
- THE SUM OF n -TERMS
- The sum S_n up to n terms of an AP whose first term is a and common difference d is given by $S_n = \frac{n}{2}[2a + (n-1)d]$
- If the first term and the last term of an AP are t_1 and t_n then $S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}[\text{first term} + \text{last term}]$
- If $t_1 = a$, the first term and $t_n = l$, the last term, then $S_n = \frac{n}{2}(a + l)$
- $S_n - S_{n-1} = t_n$

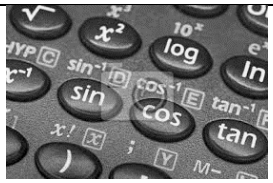
A.		**Multiple choice questions** (1 Mark) SECTION-A	level
1		Find the 10th term from the end of A.P 3, 8, 13, 18...253. a) 204 b) 208 c) 202 d) 206	C
2		Which term of the sequence 4,9,14...is 124? a) n=25 b) n= 24 c) n=26 d) n=23	C
3		For what value of K, the number x,2x+k,3x+6,are three consecutive terms of A.P a) k=2 b) k=6 c) k=3 d).k=4	U

4		How many numbers of two digits are divisible by 8? a)10 b) 11 c)12 d)13	C
5		Find the middle term of A.P: 1,8,15,...,505 a) 37^{th}, middle term = 253 b) 36^{th}, middle term = 246 c) 38^{th}, middle term = 263 d) 35^{th}, middle term = 263	U
6.		Write next term of A.P $\sqrt{8}$, $\sqrt{18}$,$\sqrt{32}$,------ a) $3\sqrt{2},4\sqrt{2},5\sqrt{2}$ b) $4\sqrt{2},5\sqrt{2},6\sqrt{2}$ c) $5\sqrt{2},6\sqrt{2},7\sqrt{2}$ d) $\sqrt{2},3\sqrt{2},5\sqrt{2}$	A
7		What is the common difference of an A.P in which a_{23} - a_{18} = 45 a) d = 9 b) d = 7 c) d=8 d) d=10	U
8.		Find first three terms of an A.P whose n^{th} term is -5 + 2n a) –2,0,2 b) –3,0,3 c) –3,-1,1 d) –5,0,5	A
9.		In the given A.P, find the missing terms 0,_,-8,-12,_ a) -4,-16 b) -2,-24 c) -6,-14 d) -6,-18	C
10.		Find the sum of all odd integers between 1 and 100 which are not multiples of 4 a) 2399 b) 2499 c)2599 d) 2699	C
11.		The common difference of the A.P 3,1,-1,-3 is a) -2 b) 2 c) -1 d) 3	A
12.		An A.P whose first term is 10 and common difference is 3is a)10,13,16, 19…… b) 5,7,9,11……. c)8,12,16,20…….d) All of these	U
13.		The nth term of the A.P2,5,8…….is a)3n-1 b) 2n-1 c)3n-2 d) 2n-3	C

14.		If first term of an A.P is 2 and common difference is -2, then 7^{th} term is a) -8 b)-10 c)-5 d)10	U
15.		Which term of the A.P 4, 9, 14, 19 , ……is109? a) 14^{th} b) 18^{th} c) 22^{nd} d) c. 16^{th}	U
A.		Very Short Answer Questions (VSA) (1 Mark)	level
1		Form the A.P for the following situations  i) Number of students left in the school from the total strength of 1000 students when they leave the school in batches of 25. ii) The cost of digging a well for the first meter is Rs 150 and rises by Rs 20 for each succeeding meter.	C
2		Find the common difference af the A.P and write the next two terms i) 1.8,2.0,2.2,2.4,…… ii) 0,1/4,1/2,3/4,…….	C
3		How many terms of A.P 18, 16, 14...Should be taken so that their sum is zero?	U
4		Find the value of x for which 8x+4,6x-2 and 2x+7 are in A.P	C
5		If the 10^{th} term of an A.P is 47 and first term is 2, find the sum of the first 15 term	U
6		Which term of A.P 121,117,113....is the first negative term?	HOT
B		**Short Answer Questions (SA) (2 marks)**	**level**
1		How many multiples of 4 lie between 10 and 250?	U

2		If a, b, c are in A.P then show that 1/bc,1/ca,1/ab are in A.P	HOT
3		 Solve the equation 2+5+8...+x = 155	U
4		Which term of the A.P 72,68,64,60,…….is 0.	U
C		**Long Answer Questions (LA) (3 Marks)**	
1		Find four numbers in A.P whose sum is 20 and the sum of whose squares is 120	HOT
2		Find the sum of the following A.P $(x-y)^2$,(x^2+y^2),$(x+y)^2$,…….to n terms.	U
3		If the 8^{th} term of an A.P is 0 prove that its 20^{th} term is thrice its 12^{th} term	U
4		Find the 6^{th} term from the end of the A.P 17,14,11,……,-40	HOT
5		If the pth terms of an A.P is q and the qth terms is p,then the find the (p+q)th term of the A.P	HOT
6		Which term of the A.P 5,15,25,……will be 130 more than its 31^{st} term.	U
7		If the sum of n terms of an A.P is n^2+2n, find the A.P and the 20^{th} term.	U
8		For what values of n, n^{th} term of the series 3, 10, 17...and 63,65,67...are equal?	U
9		Which term of the A.P 8,14,20,26,……will be 72 more than its 41^{st} term.	U
D.		**SECTION -D (4 Marks)**	

1		A manufacturer of radio sets produced 600 units in the third year and 700 units in the 7^{th} year .Assuming that the production increases uniformly by a fixed number every year find i) the production in the first year. ii) the total production in 7 years. iii) the production in the 10^{th} year.	C
2		If S_1, S_2, S_3 are the sum of n,2n,3n terms respectively of an A.P then prove that $S_3 = 3(S_2 - S_1)$	HOT
3		If the sum of p terms of an A.P is same as the sum of q terms then show that the sum of (p+q) terms is 0.	HOT
4		Sum of three numbers in A.P is 27 and their product is 405. Find the numbers	HOT
5		Sum of three numbers in A.P is 21 and their product is 231.Find the numbers	U
6		If a^2 ,b^2,c^2 are in A.P then prove that 1/b+c, 1/c+a, 1/a+b are in A.P	U
7		The sum of the 4^{th} and 8^{th} term of an A.P is 24 and the sum of 6^{th} and 10^{th} term is 44.Find the first three terms of the A.P.	U
8		If seven times the seventh term of an A.P is equal to 11 times the eleventh term, show that 18^{th} term of an A.P is zero.	U
9		 If 9^{th} term of an A.P is 0, prove that 29^{th} term is double of the 19^{th} term	U
10		Determine the A.P whose third term is 16 and when 5^{th} term is subtracted from 7^{th} ,we get 12	U

11		Divide 32 into four parts which are in A.P.such that the product of extremes is to the product of means is 7:15.	HOT
12		The ratio of the sums of m and n terms of an A.P is $m^2 : n^2$. Show that the ratio of the mth term and the nth terms is (2m-1) : (2n-1) 	HOT

ARITHMETIC PROGRESSION	
ANSWERS SECTION A	
1) b 2) a 3) c 4) b 5) a 6) c 7) a 8) c 9) a 10) b 11) a 12) a 13) a 14) b 15)c	
1) 1000, 975, 950…..,150,170,190…. 2) d=.2, 2.6,2.8, d= ¼,1,5/4 3) 19 4) 15/2 5) *555* 6) 32nd	
SECTION B	1) 60 3) 29 *4)* 9th
SECTION C	1) 2,4,6,8 2) n{(x-y)2+(n-1)xy} 4) -25 *5)* 0 6) 44^{th} 7) 3,5.7,9,….;41 8) n=13. 9.) 53^{rd}

SECTION D	1) 550,4375,775 4) 3,9,15. *5) 3*,7,11. 7) -13,-8,-3 . . 10) 4,10,16,22. 11).2,6,10,14