مـــدرسـة دلهـــي الخـاصـة ذ.م.م.

DELHI PRIVATE SCHOOL L.L.C.

Affiliated to C.B.S.E., DELHI

(Approved & Recognized By Ministry of Education - United Arab Emirates)

D P S SHARJAH

PERIODIC TEST I (2023-24)MATHEMATICS	
Section A(1 mark each)	
1.	Option-c
2.	Option-b
3.	Option-b
4.	Option-a
Section B(2marks)	
5	400
6	Domain: $[-3, 3]$ Range: $[0, 3]$ **Explanation:** The value under a square root cannot be negative, or else the solution is imaginary. So, we need $9 - x^2 \geq 0$, or $9 \geq x^2$, so $x \leq 3$ and $x \geq -3$, or $[-3.3]$. As x takes on these values, we see that the smallest value of the range is 0, or when $x = \pm 3$ (so $\sqrt{9-9} = \sqrt{0} = 0$), and a max when $x = 0$, where $y = \sqrt{9-0} = \sqrt{9} = 3$

7

$$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$$

$$= 2\left(\frac{\frac{\sqrt{3}}{2}}{\sin 20^\circ} - \frac{\frac{1}{2}}{\cos 20^\circ}\right)$$

$$= 2\left(\frac{\sin 60^\circ}{\sin 20^\circ} - \frac{\cos 60^\circ}{\cos 20^\circ}\right)$$

$$= 2\left(\frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}\right)$$

$$= \frac{2\sin(60^\circ - 20^\circ)}{\frac{1}{2}(2\sin 20^\circ \cos 20^\circ)}$$

$$= \frac{4\sin 40^\circ}{\sin 40^\circ}$$

$= 4$

Section C (3 marks)

8

Solution Let F, B and C denote the set of men who received medals in football, basketball and cricket, respectively.

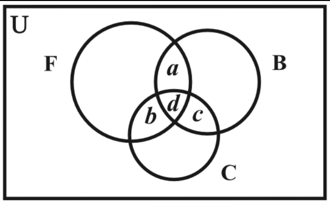

Fig 1.14

Then $n(\text{F}) = 38$, $n(\text{B}) = 15$, $n(\text{C}) = 20$

$n(\text{F} \cup \text{B} \cup \text{C}) = 58$ and $n(\text{F} \cap \text{B} \cap \text{C}) = 3$

Therefore, $n(\text{F} \cup \text{B} \cup \text{C}) = n(\text{F}) + n(\text{B}) + n(\text{C}) - n(\text{F} \cap \text{B}) - n(\text{F} \cap \text{C}) - n(\text{B} \cap \text{C}) + n(\text{F} \cap \text{B} \cap \text{C})$,

gives $n(\text{F} \cap \text{B}) + n(\text{F} \cap \text{C}) + n(\text{B} \cap \text{C}) = 18$

Consider the Venn diagram as given in Fig 1.14

Here, a denotes the number of men who got medals in football and basketball only, b denotes the number of men who got medals in football and cricket only, c denotes the number of men who got medals in basket ball and cricket only and d denotes the number of men who got medal in all the three. Thus, $d = n(\text{F} \cap \text{B} \cap \text{C}) = 3$ and $a + d + b + d + c + d = 18$

Therefore $a + b + c = 9$,

which is the number of people who got medals in exactly two of the three sports.

9

For D_f, $f(x)$ must be a real number $\Rightarrow \frac{x^2}{1+x^2}$ must be a real number

$\Rightarrow \quad D_f = \mathbf{R}$ $\qquad (\because x^2 + 1 \neq 0 \text{ for all } x \in \mathbf{R})$

For R_f, let $y = \frac{x^2}{1+x^2} \Rightarrow x^2y + y = x^2$

$\Rightarrow \quad (y-1)x^2 = -y \Rightarrow x^2 = -\frac{y}{y-1}, y \neq 1.$

But $x^2 \geq 0$ for all $x \in \mathbf{R} \Rightarrow -\frac{y}{y-1} \geq 0, y \neq 1$

(Multiply both sides by $(y-1)^2$

$\Rightarrow \quad -y(y-1) \geq 0 \Rightarrow y(y-1) \leq 0$

$\Rightarrow \quad (y-0)(y-1) \leq 0 \Rightarrow 0 \leq y \leq 1$ but $y \neq 1$

$\Rightarrow \quad 0 \leq y < 1 \Rightarrow R_f = [0, 1).$

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$$\sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \frac{3}{5} \text{ and}$$

$$\cos(\alpha-\beta) = \sqrt{1-\sin^2(\alpha-\beta)} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}.$$

$$\therefore \quad \tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\text{and} \quad \tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

Now $\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta))$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}$$

$$= \frac{36+20}{48-15} = \frac{56}{33}.$$

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$$= \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}$$

$$= \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2(4\theta)}}}$$

$$= \sqrt{2+\sqrt{2+2\cos(4\theta)}}$$

$$= \sqrt{2+\sqrt{2(1+\cos(4\theta))}}$$

$$= \sqrt{2+\sqrt{2\times 2\cos^2(2\theta)}}$$

$$= \sqrt{2+2\cos 2\theta}$$

$$= \sqrt{2(1+\cos 2\theta)}$$

$$= \sqrt{2\times 2\cos^2\theta}$$

$$= 2\cos\theta$$

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Section D (4 Marks)

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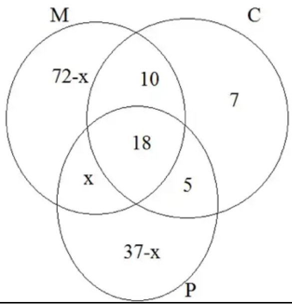
<tr><td>12</td><td>

x=12

i) only maths=72-x=72-12=60
ii) maths and physics but not chem=x=12
iii) only chem=7
iv) all the three= 18



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13	$\Rightarrow \cos A = \frac{-4}{5}$ and $\sin B = \frac{5}{13}$ Now, $\sin(A+B) = \sin A \cos B + \cos A \sin B$ $= \frac{3}{5} \times \frac{-12}{13} + \frac{-4}{5} \times \frac{5}{13}$ $= \frac{-36}{65} - \frac{20}{65}$ $= \frac{-56}{65}$
SectionE(5 marks)	
14	**Solution.** L.H.S. $= \frac{1+\cos 2x}{2} + \frac{1+\cos 2\left(x+\frac{\pi}{3}\right)}{2} + \frac{1+\cos 2\left(x-\frac{\pi}{3}\right)}{2}$ $\left(\because \cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}\right)$ $= \frac{1}{2}\left[3+\cos 2x + \left(\cos\left(2x+\frac{2\pi}{3}\right)+\cos\left(2x-\frac{2\pi}{3}\right)\right)\right]$ $= \frac{1}{2}\left[3+\cos 2x + 2.\cos\frac{2x+\frac{2\pi}{3}+2x-\frac{2\pi}{3}}{2}\cos\frac{2x+\frac{2\pi}{3}-\overline{2x-\frac{2\pi}{3}}}{2}\right]$ $= \frac{1}{2}\left[3+\cos 2x + 2\cos 2x \cos\frac{2\pi}{3}\right] = \frac{1}{2}\left[3+\cos 2x+2\cos 2x\cos\left(\pi-\frac{\pi}{3}\right)\right]$ $= \frac{1}{2}\left[3+\cos 2x - 2\cos 2x\cos\frac{\pi}{3}\right] = \frac{1}{2}\left[3+\cos 2x - 2\cos 2x \cdot \frac{1}{2}\right]$ $= \frac{3}{2}$ = R.H.S.