3.TRIGONOMETRIC FUNCTIONS
SYNOPSIS

- If in a circle of radius r, an arc of length I subtends and angle θ radians, then l=rθ
- Radian measure = $\frac{\pi}{180}$ × Degree measure
- Degree measure = $\frac{\pi}{180}$ x Radian measure
- $cos^2x + sin^2x = 1$

- $\cos(2n\pi + x) = \cos x$
- $\sin(2n\pi + x) = sinx$

- Sin(-x) = -sin x
- Cos(-x)= cos x

- Cos(x+y) = cos x cos y – sin x sin y
- Cos(x-y) = cos x cos y + sin x sin y
- Sin(x+y) = sin x cos y + sin y cos x
- Sin(x-y) = sin x cos y – sin y cos x

- Tan(x+y) = $\frac{\tan x+\tan y}{1-\tan x \tan y}$, (if none of the angles x,y and x±y is an odd multiple of $\frac{\pi}{2}$)
- Tan(x-y) = $\frac{\tan x-\tan y}{1+\tan x \tan y}$ (if none of the angles x,y and x±y is an odd multiple of $\frac{\pi}{2}$
- $cot(x+y) = \frac{\cot x \cot y-1}{coty+\cot x}$ (if none of the angles x,y and x±y is a multiple of π)
- $cot(x-y) = \frac{\cot x \cot y+1}{coty-\cot x}$ (if none of the angles x,y and x±y is a multiple of π)

- $cos\left(\frac{\pi}{2}+x\right) = -\sin x$ $cos\left(\frac{\pi}{2}+x\right) = \cos x$
- $cos(\pi - x) = -\cos x$ $sin(\pi - x) = \sin x$
- $cos(\pi + x) = -\cos x$ $sin(\pi + x) = -\sin x$
- $cos(2\pi - x) = \cos x$ $sin(2\pi - x) = -\sin x$

- $cos2x = cos^2x - sin^2x = 2cos^2x - 1 = 1 - 2sin^2x = \frac{1-tan^2x}{1+tan^2x}$
- $sin2x = 2\sin x \cos x = \frac{2\tan x}{1+tan^2x}$
- $tan2x = \frac{2\tan x}{1-tan^2x}$
- $sin3x = 3\sin x - 4\sin x$ $\cos 3x = 4cos^3x - 3\cos x$ $tan3x = \frac{3\tan x - tan^3x}{1-3tan^2x}$
- $\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$

- $\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$
- $\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta)$

- $2\cos x\, cos\, y = \cos(x+y) + \cos(x-y)$
- $-2\sin x \sin y = \cos(x+y) - \cos(x-y)$
- $2\sin x \cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x \sin y = sin(x+y) - \sin(x-y)$

GENERAL SOLUTIONS

- $\sin x = 0\ gives\ x = n\pi, \forall n \in Z$
- $\cos x = 0\ gives\ x = (2n+1)\frac{\pi}{2}, \forall n \in Z$
- $\sin x = \sin y\ implies\ x = n\pi + (-1)^n, \forall n \in Z$
- $\cos x = \cos y\ implies\ x = 2n\pi \pm y, \forall n \in Z$
- $\tan x = \tan y\ implies\ x = n\pi + y, \forall n \in Z$

Sine rule

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k,$ a=ksinA, b=ksinB, c=ksinC

Cosine rule

- $a^2 = b^2 + c^2 - 2bc\cos A$, $CosA = \frac{b^2+c^2-a^2}{2bc}$
- $b^2 = c^2 + a^2 - 2ca\cos B$, $CosB = \frac{c^2+a^2-b^2}{2ca}$
- $c^2 = a^2 + b^2 - 2ab\cos C$, $CosC = \frac{a^2+b^2-c^2}{2ab}$

SECTION A (1 Mark)

1. The value of $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) =$
   1) 1 2) -1 3) 0 4) None
2. $\frac{\sin^3\theta}{1+\cos\theta} + \frac{\cos^3\theta}{1-\sin\theta} =$
   1) $\cos\theta - \sin\theta$ 2) $\sin\theta - \cos\theta$ 3) $\sin\theta + \cos\theta$ 4) None
3. $\cos\theta = x + \frac{1}{x}$ is valid
   1) for all values of x 2) for all positive values of x

3) for all negative values of x 4) for no value of x

4 If (1-cosA) (1-cosB) (1-cos C) = sinA sinB sinC, then (1+cosA)(1+cosB) (1+cosC) =

1) cosA cosB cosC 2) sinA sinB sinC 3) □ cosA cosB cosC4) None

5 If cot θ = 4/3 then $2\sin^2\theta$ □- $\cos^2\theta$ =

1) 1/25 2) 2/25 3) 0 4) 4/25

6 If θ lies in the third quadrant and $\sin\theta = \frac{-7}{25}$ then $\frac{7\cot\theta - 24\tan\theta}{7\cot\theta + 24\tan\theta} =$

1) $\frac{31}{17}$ 2) $\frac{17}{31}$ 3) $\frac{-17}{31}$ 4) None

7 sinA, cosA, tanA form a G.P., then $\cot^6 A - \cot^2 A$ =

1) –1 2) zero 3) 1 4) 2

8 If 5 cosA + 7 sinA = 7, then the value of 7 cosA -5 sinA =

1) 5 2) -5 3) 7 4) -7

9 If cosθ - sinθ = $\sqrt{2}$ sinθ, then cosθ + sinθ =

1) $\sqrt{2}$ cosθ 2) $1/\sqrt{2}$ cos θ 3) $\sqrt{2}$ sinθ 4) None

10. If secθ + tanθ = $\frac{1}{5}$, then θ lies in

1) first quadrant 2) second quadrant 3) third quadrant 4) fourth quadrant

11. If secθ + tanθ = 4 , then cosθ =

1) 8/17 2) 4/17 3) 15/17 4) None

12. The value of $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$ is

1) 1 2) 2 3)□□1 4) 1/2

13. log tan 5^0 + log tan 25^0 + log tan 65^0 + log tan 85^0 =

1) log tan 180^0 2) 1 3) 0 4) None

14. log tan 1^0 log tan 2^0 log tan 3^0 ..... log tan 89^0 =

1) 1 2) 0 3) 1/2 4) None

15. cos 24^0 + cos55^0 + cos125^0 + cos204^0 + cos300^0 =

1) - $\frac{1}{2}$ 2) $\frac{1}{2}$ 3) 1 4) None

16. sin 25^0 cos 65^0 + $\sin^2 115^0$ $\cos^2 245^0$ + $\sin^2 295^0$ $\cos^2 335^0$ =

1) 1 2) 2 3) 1/2 4) None

17. sin 600^0 cos 330^0 + cos120^0 sin 150^0 =

1) -1 2) 0 3) 1 4) None

18. If $\sin(x-y) = \frac{1}{2} = \cos(x+y)$ then

1) x = 2y 2) y = 2x 3) x = 3y 4) y = 3x

19. 'A' lies in the third quadrant, 'B' lies in fourth quadrant tan A = $\frac{4}{3}$, sinB = $\frac{-8}{17}$, then the quadrant in which (A+B) lies is

1) first quadrant 2) second quadrant 3) third quadrant 4) fourth quadrant

20. If $\frac{\sin 135^0 - \cos 240^0}{\sin 135^0 + \cos 240^0} = a + \sqrt{b}$, then b =

1) 2 2) 8 3) 3 4) None

21. $\cos^2 6^0 - \cos^2 24^0 =$

1) $\frac{1-\sqrt{5}}{8}$ 2) $\frac{\sqrt{5}-1}{8}$ 3) $\frac{\sqrt{5}+1}{8}$ 4) None

22. $\sin^2 75^0 - \sin^2 15^0 =$

1) $1/\sqrt{2}$ 2) 1/2 3) $\sqrt{3}/2$ 4) None

23. The value of $\tan 20^0 + \tan 25^0 + \tan 20^0 \tan 25^0 =$

1) 1 2) 1/2 3) 2 4) None

24. The value of $\frac{1+\tan 22^0}{1+\tan 13^0}\cdot\frac{1+\tan 23^0}{1+\tan 32^0}$ is

1) 1 2) 2 3) -1 4) -2

25. If $A+B+C = 180^0$, then $\sum \frac{\cos(B-C)}{\sin B \sin C} =$

1) 1 2) 2 3) 4 4) None

26. Evaluate

(i) $\sin \frac{31\pi}{3}$ (ii) $\cos \frac{17\pi}{2}$ (iii) $\tan \frac{-25\pi}{3}$

27. Find the value of

(i) Sin 405º (ii) Sec (-1470º) (iii) Tan (-300º)

28. Find the value of

(i) cos480 º (ii) sin 1230º (iii) Cot(-135º) (iv) Cosec(-1410º)

29. Prove that Cos A + Sin (270º +A) – Sin (270º – A) + Cos (180º +A) = 0

30. Prove that Sin (40 º +A) Cos (10 º +A) – Cos (40 º +A) Sin (10 º +A) = ½

31. Evaluate Sin $\frac{\pi}{12}$

32. If Sin A = $\frac{3}{5}$ and Cos B = $\frac{-12}{13}$ where A and B both lie in the second quadrant, find the value of

(i) Sin (A – B) (ii) Cos (A+B) (iii) Tan (A – B)

SECTION B (2 Marks)

33. If $\cos x + \sin x = \sqrt{2}\cos x$, show that $\cos x - \sin x = \sqrt{2}\sin x$.

34 If $\sin \alpha$ and $\cos \alpha$ are the roots of the equation $ax^2 - bx + c = 0$, then find the relation satisfied by a, b and c.

35. Is the equation $2\sin^2 x - \cos x + 4 = 0$ possible?

36. Find the range of $f(x) = 1 + 3\cos 2x$

37. Find the minimum and maximum values of $3 \sin^2 x - 2 \cos^2 x + 9$

38. Prove that:
(i) $\sin(-690^\circ)\cos(-300^\circ) + \cos(-750^\circ)\sin(-240^\circ) = 1$

(ii) $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ = \frac{1}{2}$

39. If $\tan(A+B) = p$ and $\tan(A-B) = q$, show that $\tan 2A = \frac{p+q}{1-pq}$

40. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, show that $\alpha + \beta = \frac{\pi}{4}$

41. If $x + y = z$ and $\tan x = k \tan y$, then prove that $\sin z = \frac{k+1}{k-1}\sin(x-y)$.

42. If $\cos(x + y) = m \cos(x - y)$, then prove that $\tan x = \frac{1-m}{1+m}\cot y$.

43. Prove that $\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ = 4$

44. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then prove that $\cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta) = 0$

45. Prove that :
$\text{Sin } 4x = 4 \sin x \cos^3 x - 4 \cos x \sin^3 x$

SECTION C (4marks)

Prove the following identities:

46. If $\text{Cos } A = \frac{13}{14}$ and $\text{Cos } B = \frac{1}{7}$ where A and B are acute angles

show that $A - B = \frac{\pi}{3}$

47. $\text{Tan}70^\circ = \text{Tan}20^\circ + 2\text{Tan}50^\circ$

48. If $\text{Cos}(A+B) = \frac{4}{5}$, $\text{Sin}(A-B) = \frac{5}{13}$, A and B lie between 0 and $\frac{\pi}{4}$,

prove that $\tan 2A = \frac{56}{33}$

49. $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56$

50. Tan15 º + tan30 º + tan15 º tan30 º = 1

51. tan13A – tan 9A – tan 4A = tan 13A tan9A tan4A

52. If Tan x + tan (x +60 º) + tan (x +120 º) =3, then prove that $\frac{3\tan x - \tan^3 x}{1-3\tan^2 x} = 1$

53. tan20 º tan40 º tan80 º = tan60 º

54. Cos10 º Cos30 º Cos50 º Cos70 º = $\frac{3}{16}$

55. Cos 20 º + Cos100 º + Cos 140 º = 0

56. If Sin 2A = x Sin2B prove that $\frac{\tan(A+B)}{\tan(A-B)} = \frac{x+1}{x-1}$

57. 2 Cos 45 º Cos 15 º = $\frac{\sqrt{3}+1}{2}$

58. If Cos x + Cos y = $\frac{1}{3}$ and Sin x + Sin y = $\frac{1}{4}$, prove that $\tan\left(\frac{x+y}{2}\right) = \frac{3}{4}$

59. Cos x + Cos (120 º – x) + Cos (120 º + x) =0

60. $\frac{\sin 2A}{1+\cos 2A}$ = tan A

61. $\frac{1+\sin 2A+\cos 2A}{1+\sin 2A-\cos 2A}$ = Cot A

62. $\frac{\cos 2A}{1+\sin 2A} = \tan\left(\frac{\pi}{4} - A\right)$

63. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8A}}} = 2\cos A$

64. $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$

65. Prove that $\cos^2(45^o - A) - \sin^2(45^o - A) = \sin 2A$

66. Find the general solutions of the following trigonometric equations:

(i) 4 Cos x Sin x + 2 Sin x + 2Cos x + 1 = 0
(ii) $\tan^3 x - 3\tan x = 0$
(iii) sin x tan x – 1 = tan x – sin x
(iv) cos x – sin x = - 1
(v) $\sqrt{3}\cos x + \sin x = 1$
(vi) cot x + tan x = 2cosecx

SECTION D(6marks)

Prove the following identities:

67. $\cos^2 A + \cos^2(A + 120^\circ) + \cos^2(A - 120^\circ) = 3/2$

68. $(1 + \cos \frac{\pi}{8})(1 + \cos \frac{3\pi}{8})(1 + \cos \frac{5\pi}{8})(1 + \cos \frac{7\pi}{8}) = \frac{1}{8}$

69. $\cos^3 A + \cos^3(120^\circ + A) + \cos^3(240^\circ + A) = \frac{3\cos 3A}{4}$

70. $\tan A + \tan(60^\circ + A) - \tan(60^\circ - A) = 3 \tan 3A$

71. $\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8} = 2$

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SCORING KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2	3	4	2	2	2	3	2	1	4
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	1	3	2	2	1	1	3	3	2
21	**22**	**23**	**24**	**25**					
2	3	1	1	3					

`26.	(i) $\sqrt{3}/2$	(ii) 0	(iii) $-\sqrt{3}$	
27.	(i) $1/\sqrt{2}$	(ii)$2/\sqrt{3}$	(iii)$\sqrt{3}$	
28.	(i) –1/2	(ii) ½	(iii) 1	(iv) 2
31.	$\sqrt{3} - 1 / 2\sqrt{2}$			
32.	(i) –16/65	(ii) 33/65	(iii) -16/63	
34.	$a^2 - b^2 + 2ca = 0$			
35.	No			
36.	[-2, 4]			
37.	Min=7, Max = 12			
66.	(i) $x = 2n\pi \pm 2\pi/3$ or $n\pi + (-1)^n 7\pi/6$			
	(ii) $x = n\pi + \pi/3$ or $n\pi + 2\pi/3$			
	(iii) $x = n\pi + (-1)^n \pi/2$ or $n\pi + 3\pi/4$			
	(iv) $x = n\pi$ or $(2n + 1)\pi/2$			
	(v) $x = 2n\pi + \pi/2$ or $2n\pi - \pi/6$			
	(vi) $x = 2n\pi \pm \pi/3$			