

12. INTRODUCTION TO 3-DIMENSIONAL GEOMETRY

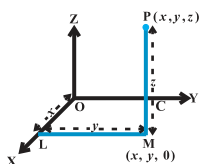
SYNOPSIS

COORDINATE AXES AND COORDINATE PLANES

- In three dimensions, the coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called the x , y and z -axes.
- The three planes determined by the pair of axes are the coordinate planes, called XY , YZ and ZX -planes.
- The three coordinate planes divide the space into eight parts known as *octants*

COORDINATES OF A POINT

- The coordinates of a point P in three-dimensional geometry is always written in the form of triplet like (x, y, z) . Here x , y and z are the distances from the YZ , ZX and XY -planes. The coordinates of the origin O are $(0,0,0)$.



Octants Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

- The sign of the coordinates of a point determine the octant in which the point lies. The following table shows the signs of the coordinates in eight octants.

Coordinates of points lying on different axes

- Any point on x -axis is of the form $(x, 0, 0)$
- Any point on y -axis is of the form $(0, y, 0)$
- Any point on z -axis is of the form $(0, 0, z)$.

Coordinates of points lying in different planes

- The coordinates of any point in the YZ -plane will be as $(0, y, z)$.
- The coordinates of any point in the XY -plane will be as $(x, y, 0)$.
- The coordinates of any point in the ZX -plane will be as $(x, 0, z)$.

- Distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}. \text{ This is known as } \mathbf{distance \text{ formula.}}$$

- The coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally and externally in the ratio $m : n$ are given by

$$\left(\frac{mx_2 \pm nx_1}{m \pm n}, \frac{my_2 \pm ny_1}{m \pm n}, \frac{mz_2 \pm nz_1}{m \pm n} \right)$$

This is known as **section formula**.

- The coordinates of the mid point of the line joining the points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

This is known as **midpoint formula**

- The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$

INTRODUCTION TO 3-D GEOMETRY

SECTION A (1 MARK)

MCQ

- The length of the foot of the perpendicular drawn from the point P(3, 4, 5) on the Y axis is
a) 10 b) $\sqrt{34}$ c) $\sqrt{113}$ d) $5\sqrt{2}$
- What is the perpendicular distance of the point P(6, 7, 8) from XY plane?
a) 8 b) 7 c) 6 d) None of these
- L is the foot of the perpendicular drawn from a point P (6, 7, 8) On the XY plane. What are coordinates of L?
a) (6, 7, 0) b) (0, 7, 0) c) (0, 0, 8) d) None of these
- What is the locus of a point for which $y = 0, z = 0$?
a) X axis b) Y axis c) Z axis d) None of these
- L is the foot of the perpendicular drawn from a point P(3, 4, 5) on the XZ plane. What are the coordinates of L?
a) (3, 0, 0) b) (0, 4, 5) c) (3, 0, 5) d) (3, 4, 0)
- The ratio in which the line joining (2, 4, 5) and (3, 5, -9) is divided by the YZ plane is
a) 2:3 b) 3:2 c) -2:3 d) 4:-3
- If P(0, 1, 2), Q(4, -2, 1) and O(0, 0, 0), then $\angle POQ$ is equal to

a) $\frac{\pi}{6}$

b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{2}$

8. If the extremities of the diagonal of a square are (1, -2, 3) and (2, -3, 5), then the length of the side is

a) $\sqrt{6}$

b) $\sqrt{3}$

c) $\sqrt{5}$

d) $\sqrt{7}$

9. XOZ plane divides the join of (2, 3, 1) and (6, 7, 1) in the ratio

a) 3:7

b) 2:7

c) -3:7

d) -2:7

10. The points (5, -4, 2), (4, -3, 1), (7, 6, 4) and (8, -7, 5) are the vertices of

a) A rectangle

b) a square

c) a parallelogram

d) None of these

Fill in the blanks:

11. A line is parallel to XY plane if all the points on the line have equal _____.

12. The equation $x = b$ represents a plane parallel to _____ plane.

13. The perpendicular distance of the point P(3, 5, 6) from the Y axis is _____.

14. The coordinates of the foot of the perpendicular from the point P(3, 4, 5) on the ZX plane are _____.

15. The length of the foot of the perpendicular from the point P(a, b, c) on the Z axis is _____.

16. If the centroid of the triangle is the origin and two of its vertices are (1, 2, -3) and (3, 0, 1) then the third vertex is _____.

17. If the midpoints of the sides of a triangle are (1, 2, -3), (3, 0, 1) and (-1, 1, -4) then the centroid is _____.

VSA

18. Find the distance between P (-2, 4, 1) and Q (1, 2, -5).

19. Centroid of a triangle whose vertices are (a, 1, 3), (-2, b, -5) and (4, 7, c) is origin. Find the value of a, b, c.

20. Three vertices of a parallelogram ABCD are A (3, 4, 6), B (-1, 0, 6) and C (5, 2, 0). Find coordinates of D.

21. Find the ratio in which line segment joining the points A (1, 4, 6) and B (-4, -3, 5) is divided by yz-plane.

22. Find p, if distance between (4, 5, p) and (7, 1, -13) is 13.

SECTION B (2 MARKS)

23. Show that, if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1 - x^2 - y^2})$ is at a unit distance from the origin.

24. Three vertices of a parallelogram ABCD are A (1, 2, 3), B (-1, -2, -1) and C (2, 3, 2) find the fourth vertex D.

25. Find the length of the foot of the perpendicular from the point P (3, 4, 5) on y-axis?

26. If the distance between the point (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, find a.

27. Find x so that the point (6, 5, -3) is at a distance of 13 units from the point (x, -7, 0)

SECTION C (4 MARKS)

28. If L is the foot of the perpendicular drawn from a point (3, 4, 5) on X-axis, find the coordinates of L.
29. Find the coordinates of the points which trisect the line-segment joining the points P (4, 2, -6) and Q (10, -16, 6).
30. Find the ratio in which the line-segment joining the point A (1, 2, 3) and B(-4, 5, -2) is divided by the plane $x+2y-z=4$.
31. A (5, 4, 6), B (1, -1, 3) and C (4, 3, 2) are three points forming $\triangle ABC$. AD bisector of angle BAC meeting BC at D. Find the coordinates of D.
32. Given that P (3, 2, -4), Q (5, 4, -6) and R (9, 8, -10) are collinear. Find the ratio in which Q divides the joining of P and R.

SECTION D (6 MARKS)

33. Find the coordinates of a point equidistant from four points (a, 0, 0), (0, b, 0),
a. (0, 0, c) and (0, 0, 0).
34. Find the ratio in which the line-segment joining the points (2, 4, -3) and (-3, 5, 4) is divided by
i. The xy-plane
ii. The plane $x + y + z = 8$
35. Find the distance of the point (1, 2, 0) from the point where the line joining A (2, -3, 1) and B (3, -4, -5) cuts the plane $2x + y + z = 7$.
36. Show that the points (3, 2, 2), (-1, 1, 3), (0, 5, 6) and (2, 1, 2) lie on a sphere whose centre is (1, 3, 4). Find the radius of sphere.
37. Prove that the lines joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent.

VALUE BASED QUESTION

38. Three students are standing in a park with sign boards "SAVE ENVIRONMENT", "DON'T LITTER" and "KEEP YOUR PLACE CLEAN". Their positions are marked by the points A(0, 7, 10), B(-1, 6, 6) and C(-4, 9, 6). The three are holding green colored ribbon together. Does the ribbon form the sides of a right-angled triangle? Do you feel the need to promote what is written on sign boards?

HOTS

39. Two vertices of a parallelogram are (2, 5, -3), (3, 7, -5) and its diagonal meet in (4, 3, 3). Find the remaining vertices of a parallelogram.

SCORING KEY

1	b
2	a
3	a
4	a
5	c
6	c
7	d
8	b
9	c
10	a
11	z coordinates
12	YZ plane
13	$\sqrt{45}$ units
14	(3, 0, 5)
15	$\sqrt{a^2 + b^2}$
16	(-2, -2, -1)
17	(1, 1, -2)
20	(9, 6, 0)
21	1:4
22	-25 or 1
24	(4, 7, 6)
25	$\sqrt{34}$
26	± 5
27	x = 2 or 10
28	(3, 0, 0)

29	(6, -4, -2) (8, -10, 2)
30	1:3
31	
32	1:2
33	$\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$
34	i)3:4 ii) -5:2
35	$\sqrt{65}$
36	3
38	Yes there is need for immediate attention.
39	(6, 1, 9) & (5, -1, 11)